UQ Group Homework1

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1. Develop probabilistic representations for the information detailed above for us as a prior.

For U_C we are given that it approximates to 1.1627, which we take to be the mean. If the error expected is less than 5% with 95% confidence, we take that $2\sigma_{U_C} = 5\%$ of the mean (which we have now decided is 1.1627).

Therefore the prior for U_C:

$$U_C = \mathcal{N}(1.1627, \sigma^2_{U_C}) \tag{1}$$

where $2\sigma_{U_C} = 5\%$ of 1.1627, or $\sigma_{U_C} = 0.0291$ ($\sigma^2_{U_C} = 8.4681\text{e-}04$).

Since we are given that p ranges from 1 to 10, the maximum entropy distribution is that the prior for p is

$$p = \mathcal{U}(1, 10). \tag{2}$$

We are told that we have no reason to think that C is either negative or positive. Because of this, we assume that the mean of C is 0. We take that 95% confidence that the discretization error has magnitude less than 0.5% of U_C gives us that $2\sigma_C = 0.5\%$ of 1.1627 or $\sigma_C = 0.00291$.

$$C = \mathcal{N}(0, \sigma^2_C) \tag{3}$$

where $\sigma^2_C = 8.4681\text{e-}06$.

2. Develop a likelihood function describing the likelihood of obtaining the observed values of $\hat{U}_{ch}$ in the table for specific values of U_C, C, and p.

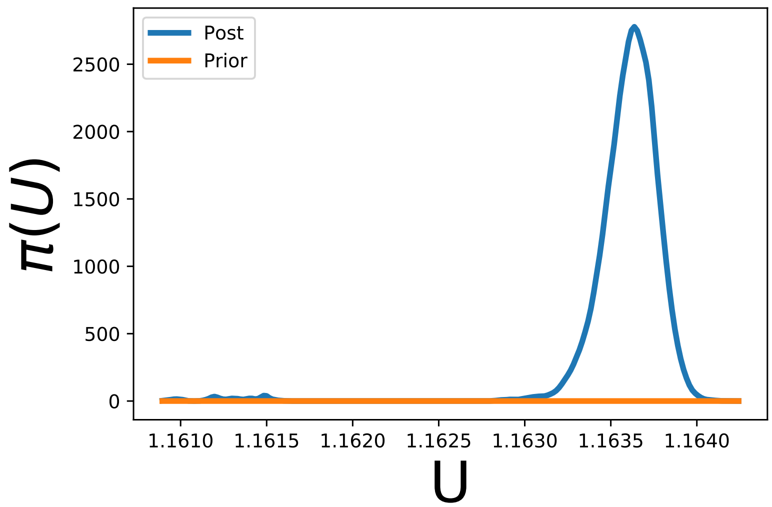
The likelihood of observing $\hat{U}_{ch}$ values of the table is:

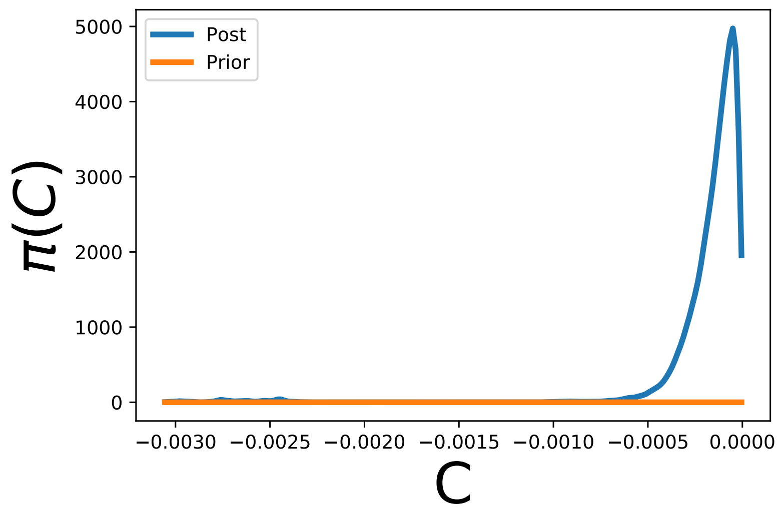
$$L(\hat{U}_{ch}|U, C, p) = \prod_{n=1}^{N_d} \frac{1}{\sqrt{2\pi}\sigma_i} exp(\frac{1}{2}\frac{(\hat{U}_{ch} - (Ch^p))^2}{\sigma_i^2}) \tag{4}$$

where N_d is the number of data points, σ_i is the standard deviation of the i^{th} measurement of $\hat{U}_{ch}$. When taking the log of the product, this becomes a nice manageable sum of log of the constants in front of the exponential and the argument of the exponential.

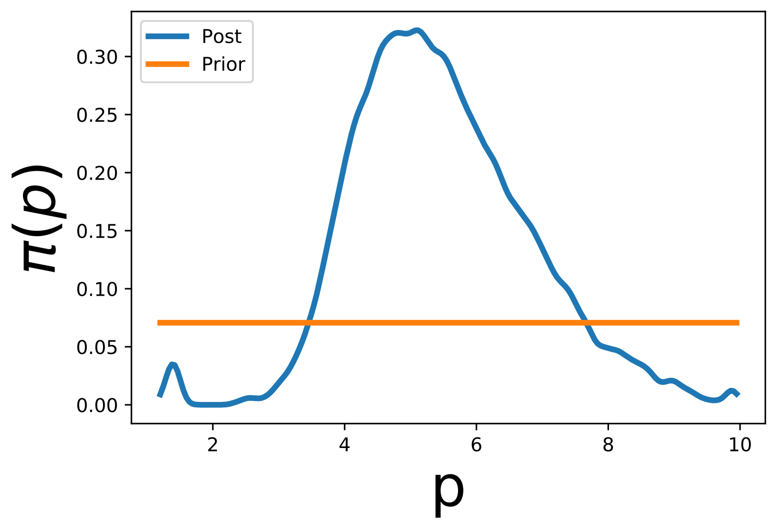
3. Use the results from (1) and (2) to obtain and plot posterior distributions of the U_C, C, and p. 4 What conclusions can be drawn from these results.

We opt to make the figures quite large for clarity and intend to email the homework in.

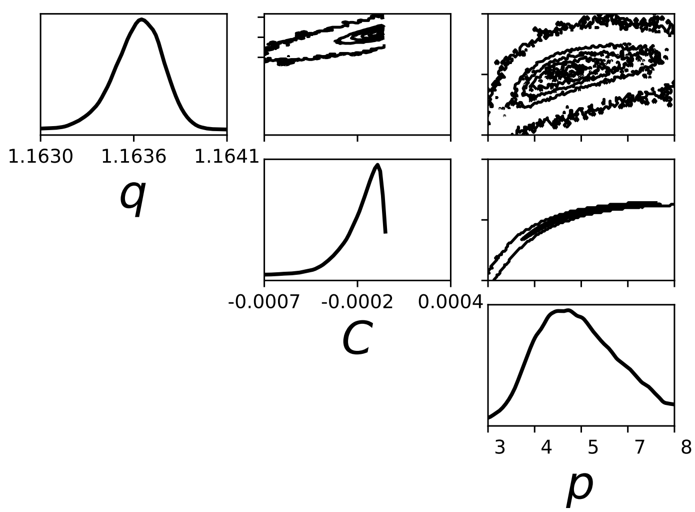




The posterior of C is skewed to the negative.



The posterior of q and p give us a sort-of normal distribution with the maximum values of the pdf at around 1.1636 and 4.5 respectively.



p and C seem to be strongly correlated, almost as if given C you can determine p and vice versa.