Project Report: Analysis of a Frequency Mixer System

Shreyas Chougaonkar

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1. 2D Discrete Fourier Transform Equations

The 2D Discrete Fourier Transform (DFT) for an image $I(x, y)$ of size $M \times N$ is defined as:

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} I(x, y) \cdot \exp\left[-i2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)\right]$$

Here:

- $F(u, v)$ denotes the frequency-domain (complex) output
- $u = 0, \ldots, M-1$, $v = 0, \ldots, N-1$: frequency indices

The corresponding inverse DFT to reconstruct the original image is:

$$I(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) \cdot \exp\left[i2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)\right]$$

2. Centering the Magnitude Spectrum

- **Without shifting (default FFT output):**
  - The DC (zero frequency) component $F(0, 0)$ appears in the top-left corner
  - Low frequencies are distributed toward the image corners
- **After shifting (centered spectrum):**
  - Using `np.fft.fftshift` moves the DC component to the center
  - Low frequencies now cluster in the middle for better visualization
- **Visual representation:**
  - Linear scale: directly plots magnitude $|F(u, v)|$
  - Logarithmic scale (in decibels): $\text{dB} = 20 \log_{10} |F(u, v)|$ enhances contrast

3. Effect of Image Rotation on Spectrum

Property:

$$\text{Rotating } I(x, y) \Rightarrow \text{Rotates } |F(u, v)| \text{ by the same angle}$$

Observations after a 90° anti-clockwise rotation:

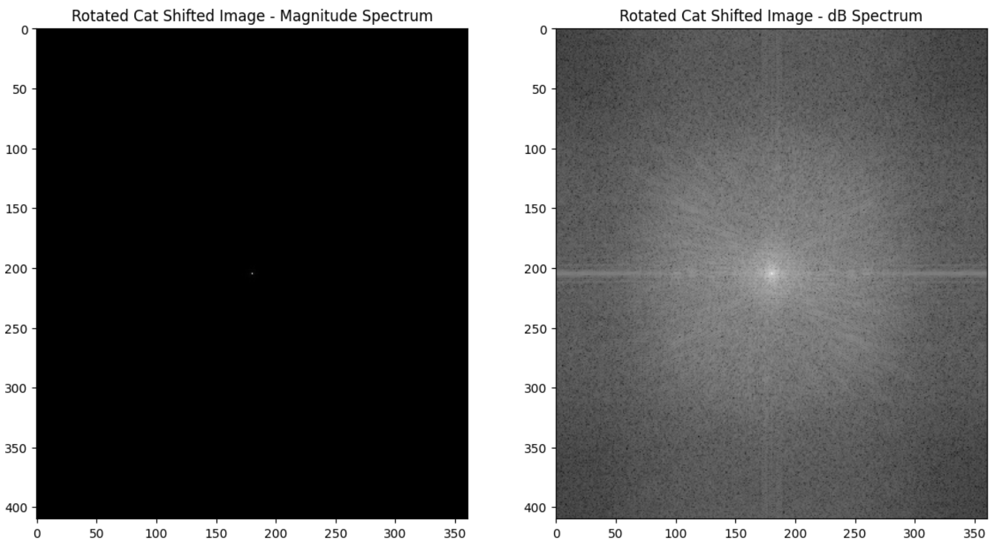

Figure 1: Magnitude spectrum after shifting and rotating the image

- The magnitude spectrum rotates 90° in the same direction
- Energy is conserved but reoriented in frequency space
- Phase content is affected, but magnitude retains structural symmetry
- Low-frequency regions shift accordingly with image axes

4. Frequency Mixer System Design

This system combines:

- Low-frequency (structural) components from Image A
- High-frequency (detail) components from Image B

Frequency-domain filters:

- Low-pass filter (LPF) applied to Image A:

$$H_{\mathrm{LP}}(u,v) = \exp\left(-\frac{D^2(u,v)}{2\sigma^2}\right)$$

- High-pass filter (HPF) applied to Image B (complement of LPF):

$$H_{\mathrm{HP}}(u,v) = 1 - H_{\mathrm{LP}}(u,v)$$

where $D(u,v) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$ is the distance from the frequency origin (DC point)

Image fusion procedure:

1. Compute DFTs: $F_A = \mathcal{F}\{I_A\}$, $F_B = \mathcal{F}\{I_B\}$
2. Apply filters and combine:

$$F_{\mathrm{fused}} = F_A \cdot H_{\mathrm{LP}} + F_B \cdot H_{\mathrm{HP}}$$

3. Perform inverse DFT to get the final hybrid image:

$$I_{\mathrm{fused}} = \mathcal{F}^{-1}\{F_{\mathrm{fused}}\}$$

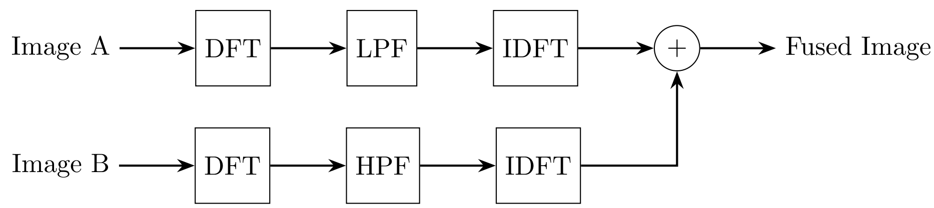

Figure 2: Block diagram of the frequency mixing pipeline

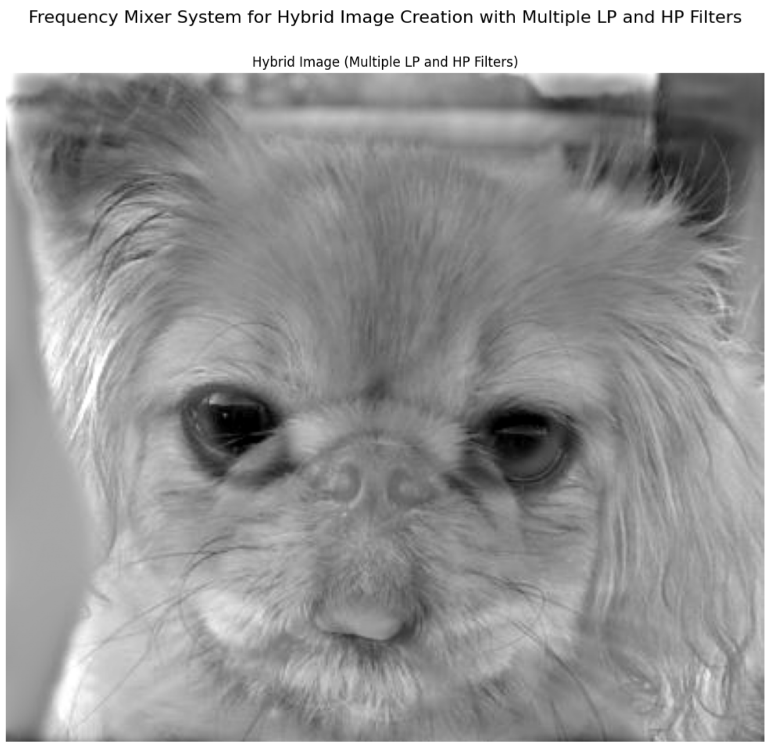

Figure 3: Final image after frequency-based fusion
From a close distance (or when eyes are wide open), the dog is more prominent. From far away (or with eyes squinted), the cat becomes more visible.

5. Summary of Key Results

- **FFT Centering:** Using fftshift relocates the DC component to the center, enhancing interpretability
- **Rotation Effects:** A spatial rotation in the image induces a corresponding rotation in its magnitude spectrum
- **Frequency Fusion:** Gaussian-based LPF and HPF enable seamless fusion of structure and details from two images
- **Visual Perception:** The resulting hybrid image demonstrates how the human brain interprets frequency content differently based on viewing distance