Project Report: Analysis of a Frequency De-mixer System

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1. Objective

To analyze a corrupted audio track, design a digital filter system to remove an unwanted flute solo and low-frequency beats, and verify the system's performance and stability.

2. Methodology & Key Formulas

The project followed a four-step digital signal processing workflow:

- **Analysis:** Identified problem frequencies using Power Spectral Density (PSD).
- **Design:** Created a cascade of multiple Butterworth filters to target the problem areas.
- **Implementation:** Applied the filters sequentially using a zero-phase 'filtfilt' algorithm.
- **Verification:** Analyzed the system with Bode/Pole-Zero plots and evaluated the result.

Key Formulas Used:

- **Butterworth Filter Magnitude Response:** Defines the filter's shape.

$$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}$$

  Where N is the filter order (steepness) and ω_c is the cutoff frequency.

- **Digital Filter Difference Equation:** How the filter is applied.

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$

  Where $y[n]$ is the output, $x[n]$ is the input, and b_k, a_k are filter coefficients.

3. System Design: Filter Cascade

Based on the analysis, the following cascade of six filters was designed:

Table 1: Filter Cascade Design Parameters

Filter Name	Type	Order	Frequency Range (Hz)	Purpose
Filter A	High-Pass	5	Cutoff at 250	Remove low-frequency beats
Filter B	Band-Stop	4	1100 – 1400	Remove flute fundamental
Filter C	Band-Stop	4	1650 – 1850	Remove flute harmonic
Filter D	Band-Stop	4	1420 – 1600	Refine fundamental removal
Wide BS 1	Band-Stop	4	1700 – 17200	Remove flute's "airy" texture
Wide BS 2	Band-Stop	4	4000 – 16000	Suppress high harmonics

4. Analysis and System Verification

A. Power Spectral Density (PSD) of Original Audio

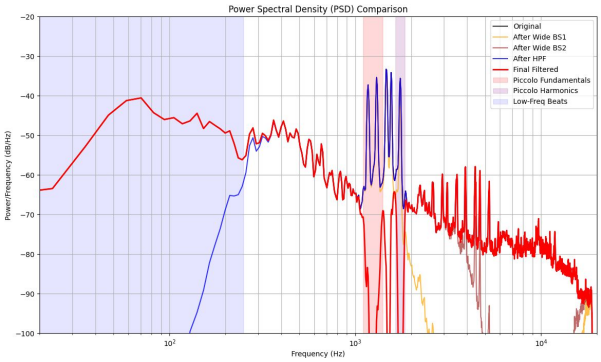

Figure 1: Power Spectral Density Analysis

The PSD plot shows the frequency content of the original audio, justifying the filter design.
Observations:

- High energy below 250 Hz confirms unwanted low-frequency beats.
- Sharp peaks around 1250 Hz and 1750 Hz identify the flute tones.

B. Bode Plot (System Frequency Response)

This plot shows the combined effect of all six filters.

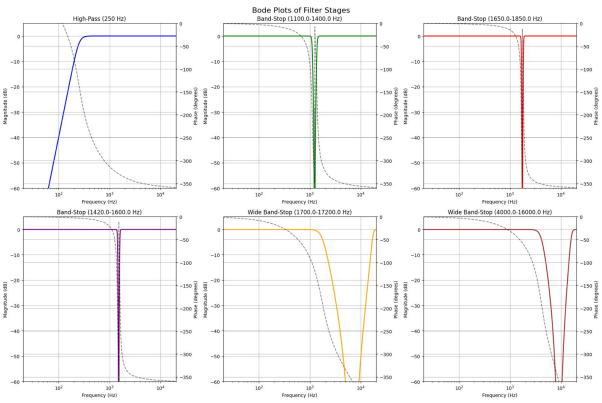

Figure 2: System Frequency Response

Observations:

- The plot shows a steep drop below 250 Hz and deep "notches" at the flute frequencies.
- In all other regions, the magnitude is 0 dB, showing that desired audio is preserved.

C. Pole-Zero Plot (System Stability)

This plot verifies that the filter system is stable. **Observations:**

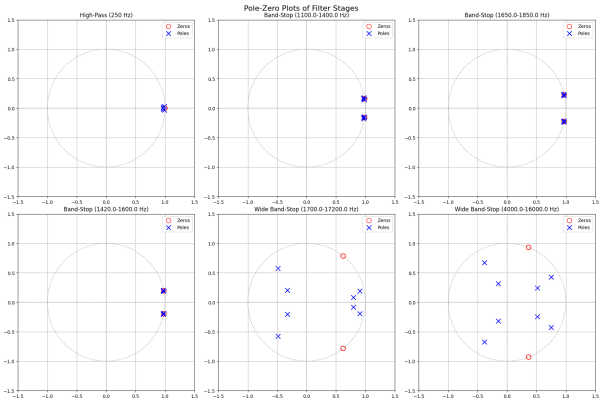

Figure 3: Plot Zero Analysis

- All poles (marked 'x') are located **strictly inside the unit circle**.
- **Conclusion: The designed filter system is stable.**

5. Final Results and Conclusion

The spectrogram comparison provides a clear visual confirmation of the filter's success.

Observations:

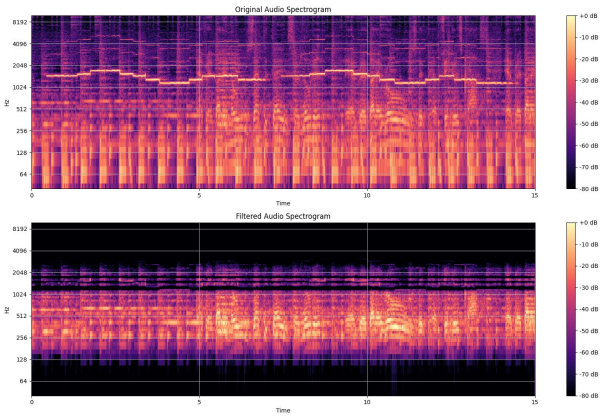

Figure 4: Original vs Restored Audio

- The bright horizontal lines corresponding to the flute in the original spectrogram (top) are effectively erased in the restored spectrogram (bottom).

Conclusion: The multi-stage filter cascade successfully performed the task of frequency demixing. By analyzing the audio signal and designing a series of targeted Butterworth filters, the unwanted flute solo and low-frequency beats were significantly attenuated. The system was

verified to be stable and effective. The final restored audio file, `restored_music.wav`, faithfully preserves the desired musical elements while removing the corrupting sounds.