Hardware Assignment

AI25BTECH11023 - Pratik Raj
AI25BTECH11035 - Sujal Rajani

1 Introduction

This report details the process of determining the voltage across a PT-100 RTD (Resistance Temperature Detector) as a function of temperature. The least squares method was used to estimate the parameters of the Callendar-Van Dusen equation.

2 Collecting Data

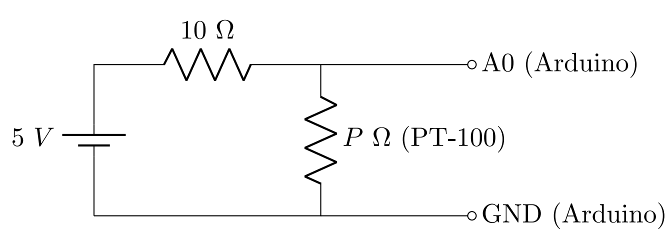
We were provided with the following components

- PT-100 Resistance Temperature Detector
- Arduino Uno and USB Cable
- 10 Ω resistor
- Breadboard
- Wires

We also made use of the following items from the EE Lab to control and monitor the temperature

- Electric kettle
- Digital Thermometer

The 100 Ω resistor and the PT-100 were connected in series between the 5V output pin and the ground pin of the Arduino to create a voltage divider. The other pin of the PT-100 is connected to the A0 pin of the Arduino to measure the voltage across the PT-100.



The PT-100 was immersed in an electric kettle filled with water, and a digital thermometer was used to measure the temperature.

The kettle was turned on for some time, and then turned off. Once the reading of the digital thermometer becaome stable, a reading was taken, and the Temperature was increased again. A total of 30 readings were taken over a range of temperatures from 26.8 °C to 97.8 °C.

17 points were randomly chosen from the data set to form the training set. The remaining 13 points were put in the testing set.

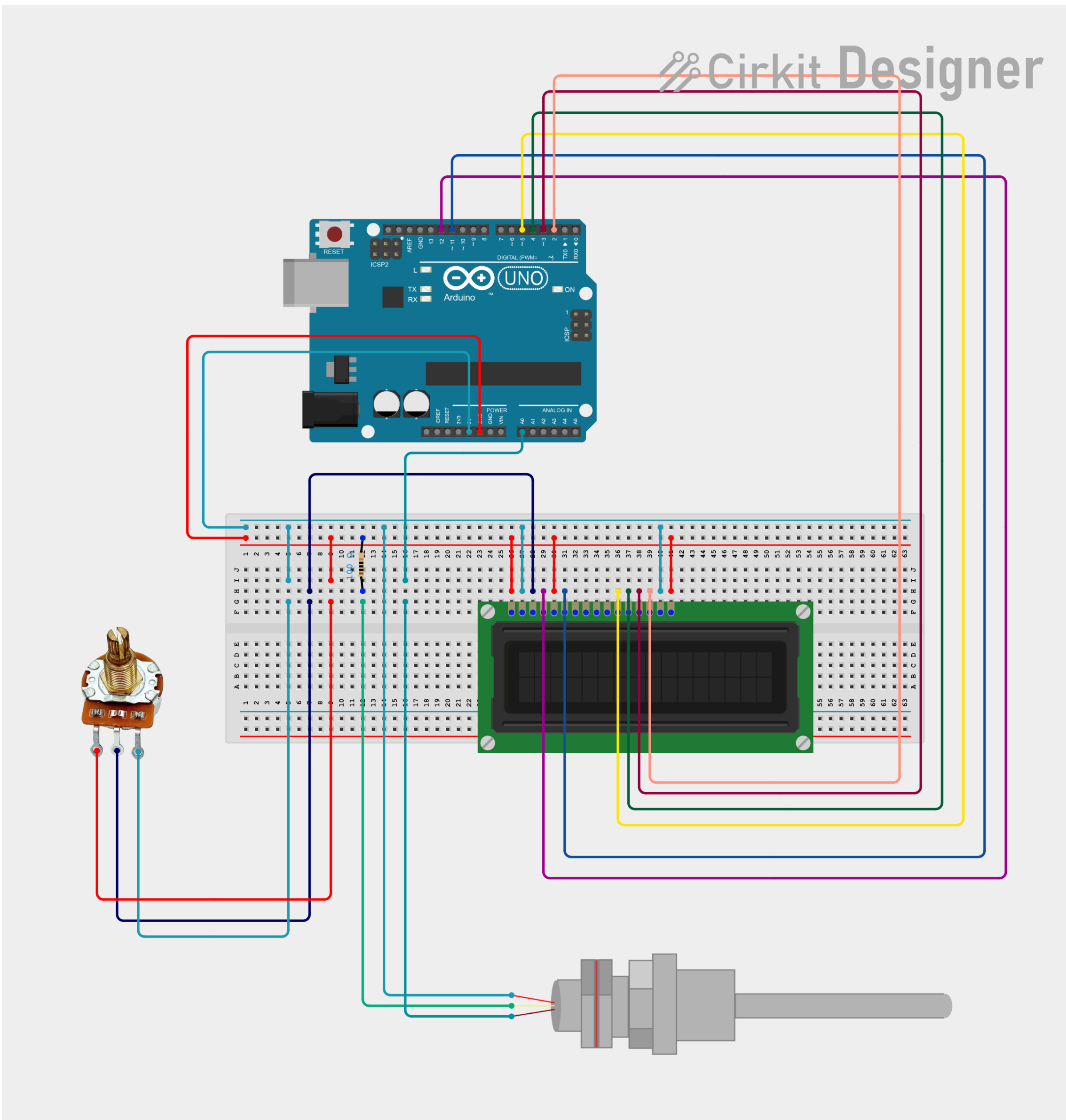


Figure 1: Circuit Diagram

3 Model

We use the Callendar-Van Dusen equation to model the voltage across the PT-100 as a quadratic function of temperature.

$$V(T) = n_0 + n_1 T + n_2 T^2 \tag{1}$$
$$c = \mathbf{n}^\top \mathbf{x} \tag{2}$$
$$\tag{3}$$

where

$$c = V(T), \mathbf{n} = \begin{pmatrix} n_0 \\ n_1 \\ n_2 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 1 \\ T \\ T^2 \end{pmatrix} \tag{4}$$

We can write the equation in matrix form as

$$\mathbf{X}^\top \mathbf{n} = \mathbf{C} \tag{5}$$

where

$$\mathbf{X} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ T_1 & T_2 & \dots & T_n \\ T_1^2 & T_2^2 & \dots & T_n^2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} V(T_1) \\ V(T_2) \\ \vdots \\ V(T_n) \end{pmatrix} \tag{6}$$

Using Least Square Method

$$\mathbf{n} = (\mathbf{X}\mathbf{X}^\top)^{-1}\mathbf{X}\mathbf{C} \tag{7}$$

For the $PT - 100$ data, the approximate model is given by

$$V(T) = 2.23689 + 0.00130T + (-3.07 \times 10^{-5})T^2 \tag{8}$$

$$\mathbf{n} = \begin{pmatrix} 2.23689 \\ 0.00130 \\ -3.07 \times 10^{-5} \end{pmatrix} \tag{9}$$

Table 1: Temperature and Voltage Data

Temperature	Voltage
26.800000	2.270000
38.000000	2.228700
42.500000	2.250000
48.900000	2.240000
54.500000	2.210000
37.100000	2.228700
79.100000	2.150000
85.000000	2.130000
91.600000	2.100000
94.100000	2.080000
97.800000	2.070000
33.300000	2.230000
66.100000	2.200000
76.800000	2.170000
81.100000	2.140000
73.800000	2.150500
69.600000	2.170100

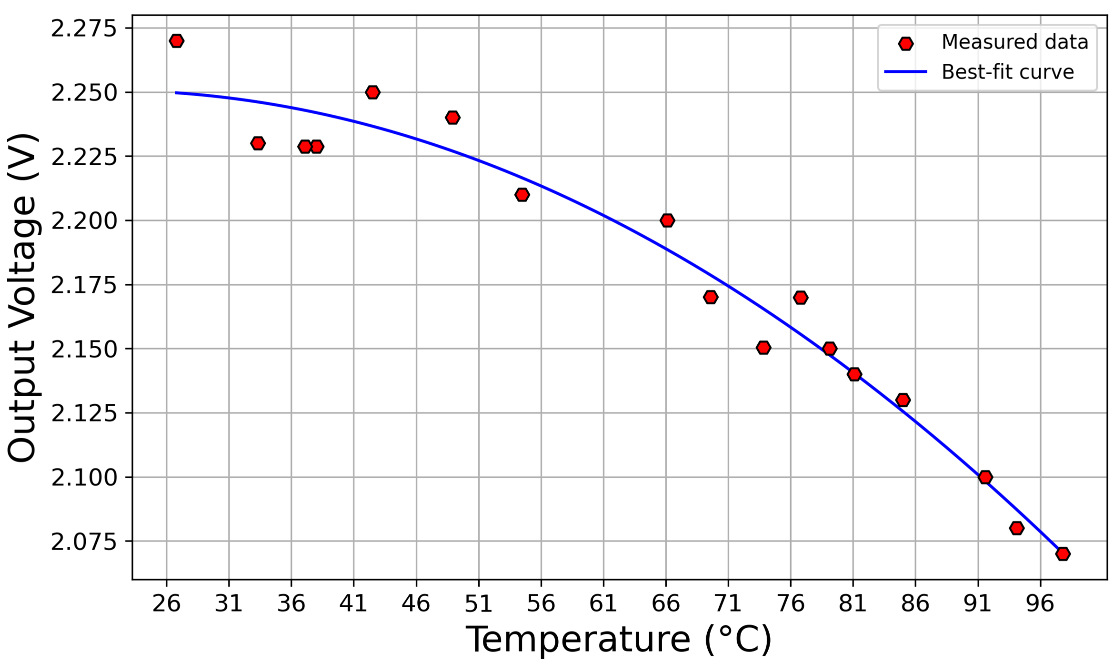


Figure 2: Training regression model

To obtain temperature as a function of voltage, we rearrange or numerically invert the above equation

$$T(V) = a_0 + a_1 V + a_2 V^2 \tag{10}$$

The coefficients can be again found by applying the least square method to given data

$$\begin{pmatrix} 1 & 1 & ... & 1 \\ V_1 & V_2 & ... & V_n \\ (V_1)^2 & (V_2)^2 & ... & (V_n)^2 \end{pmatrix}^\top \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ . \\ . \\ . \\ T_n \end{pmatrix} \tag{11}$$

we get

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -3460.32 \\ 3619.11 \\ -917.98 \end{pmatrix} \tag{12}$$

$$T(V) = -3460.32 + 3619.11V + -917.98V^2 \tag{13}$$

To obtain the temperature, we will use the above equation

4 Validation

The model can be validated by using test dataset

Table 2: Test Dataset

Temperature	Voltage
61.600000	2.200000
66.900000	2.180000
74.800000	2.170000
82.100000	2.150000
79.000000	2.160000
84.600000	2.130000
69.900000	2.175000
43.100000	2.240000
46.800000	2.228700
48.500000	2.214100
50.600000	2.209200
88.700000	2.087000
91.500000	2.077200

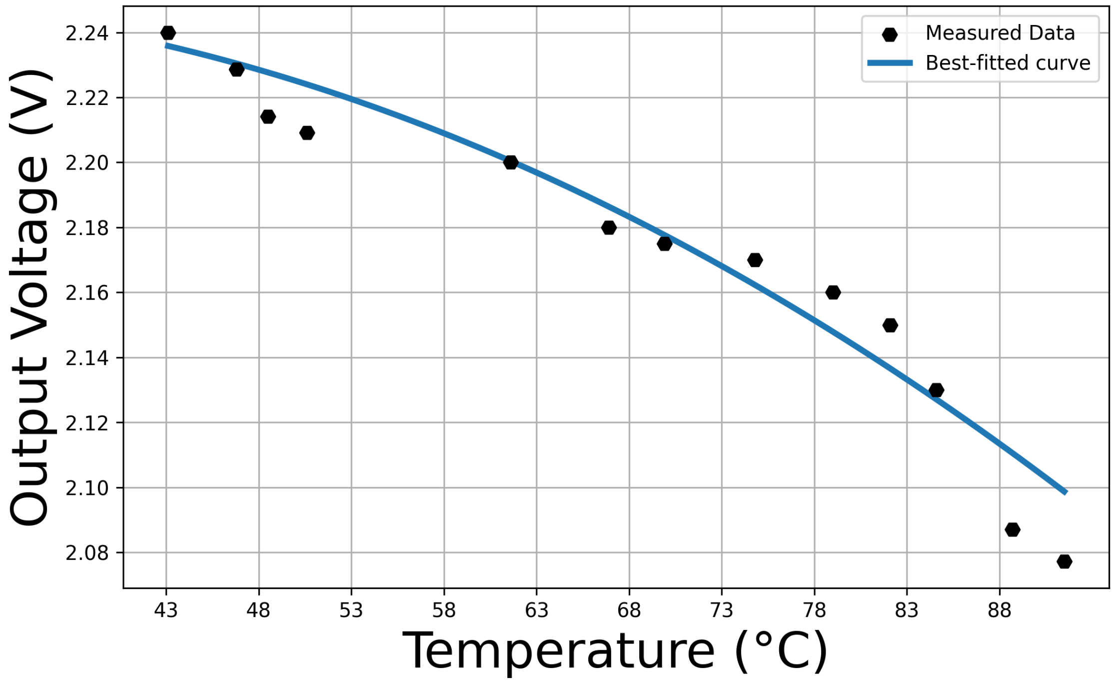


Figure 3: testing the trained model

5 Error & Conclusion

Calculating error by Mean Absolute Error (MAE)

$$MAE = \frac{\Sigma |T_{PT-100} - T_{A_i}|}{13} \tag{14}$$

where

T_{PT-100} is Temperature from PT-100 Model

T_A is Actual Temperature reading

we get

$$MAE = 3.58°C \tag{15}$$

The model produces an error of 3.58°C, demonstrating reliable predictive accuracy for the system.

Source of error may be:

1. Measurement noise in voltage reading
2. Non-linearity in the PT-100 response
3. Approximation errors in the least square method
4. Temperature sensor calibration uncertainties

The PT-100 sensor, when integrated with machine learning calibration, provided consistent and interpretable results suitable for practical applications.