# 2.5.5

EE25BTECH11051 - Shreyas Goud Burra

**Question** Write the projection of the vector $(\mathbf{b}+\mathbf{c})$ on the vector $\mathbf{a}$, where $\mathbf{a} = 2\hat{i}-2\hat{j}+\hat{k}$, $\mathbf{b} = \hat{i}+2\hat{j}-2\hat{k}$, and $\mathbf{c} = 2\hat{i}-\hat{j}+4\hat{k}$.

**Solution:**

Let us find the solution theoretically first and then verify it computationally.
Let the vectors be represented as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ respectively. Given by

$$\mathbf{A} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \tag{0.1}$$

Mathematically, the projection of the vector $\mathbf{B}+\mathbf{C}$ on the vector $\mathbf{A}$ is given by some vector $\mathbf{D}$,

$$\mathbf{D} = k\mathbf{A}, \text{ such that } ((\mathbf{B}+\mathbf{C}) - \mathbf{D})^{\mathrm{T}}\mathbf{D} = 0 \tag{0.2}$$

yielding,

$$((\mathbf{B}+\mathbf{C}) - k\mathbf{A})^{\mathrm{T}}\mathbf{A} = 0 \tag{0.3}$$

or,

$$k = \frac{(\mathbf{B}+\mathbf{C})^{\mathrm{T}}\mathbf{A}}{\|\mathbf{A}\|^2} \implies \mathbf{D} = \frac{(\mathbf{B}+\mathbf{C})^{\mathrm{T}}\mathbf{A}}{\|\mathbf{A}\|^2}\mathbf{A} \tag{0.4}$$

On substituting the values,

$$\mathbf{D} = \frac{\left(\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right)^{\mathrm{T}} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}}{\left\|\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}\right\|^2} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \tag{0.5}$$

On calculation, this gives us,

$$\mathbf{D} = \begin{pmatrix} 4/3 \\ -4/3 \\ 2/3 \end{pmatrix} \tag{0.6}$$

We get the same result by plotting a graph for the following.

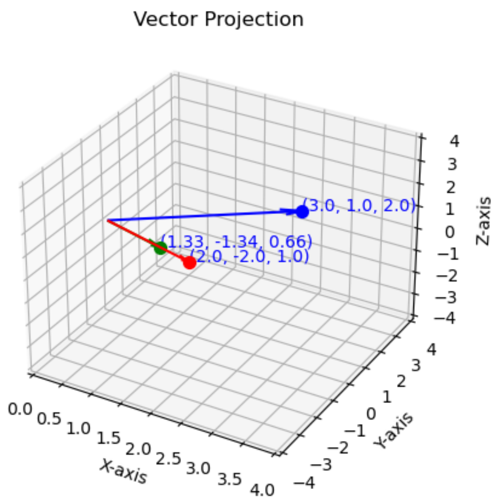


Fig. 0.1: 3D Plot