Samyak Gondane-AI25BTECH11029

Question

Find the angle between two vectors $\mathbf{a}$ and $\mathbf{b}$ with magnitudes 1 and 2 respectively and when $\mathbf{a} \cdot \mathbf{b} = 1$.

Solution

Given two vectors $\mathbf{a}$ and $\mathbf{b}$ with magnitudes:

$$||\mathbf{a}|| = 1, \quad ||\mathbf{b}|| = 2 \tag{1}$$

and dot product:

$$\mathbf{a} \cdot \mathbf{b} = 1 \tag{2}$$

We use the matrix formulation of the dot product:

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^T \mathbf{b} = ||\mathbf{a}||||\mathbf{b}|| \cos \theta \tag{3}$$

Substituting the known values:

$$1 = (1)(2) \cos \theta \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1} \left(\frac{1}{2} \right) \Rightarrow \theta = 60^\circ \tag{4}$$

Matrix Representation

Let:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{5}$$

Then:

$$\mathbf{a}^T \mathbf{b} = 1 \tag{6}$$

$$||\mathbf{a}|| = \sqrt{1^2 + 0^2} = 1, \quad ||\mathbf{b}|| = \sqrt{1^2 + \sqrt{3}^2} = 2 \tag{7}$$

So the angle is:

$$\theta = \cos^{-1} \left(\frac{\mathbf{a}^T \mathbf{b}}{||\mathbf{a}||||\mathbf{b}||} \right) = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ \tag{8}$$

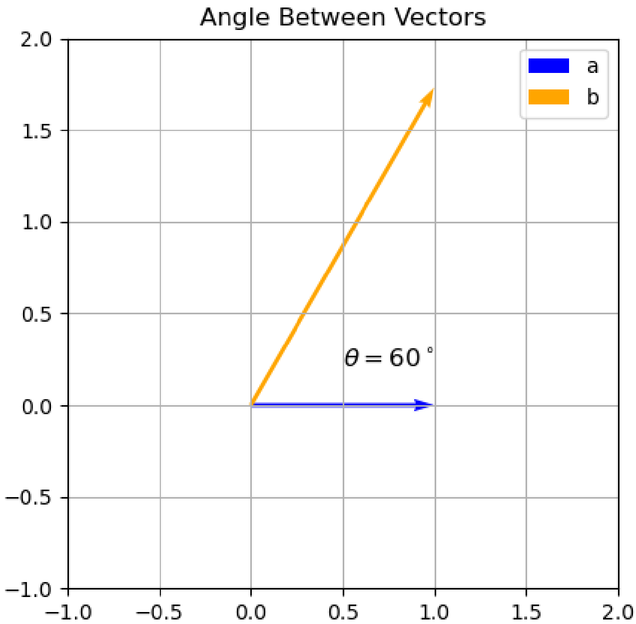
Angle Between Vectors
a
b
$\theta = 60^\circ$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

Figure 1