2.2.22

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Find the angle between two vectors **a** and **b** with magnitudes 1 and 2 respectively and when $\mathbf{a} \cdot \mathbf{b} = 1$.

Solution

We use the dot product formula:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta \tag{1}$$

Substitute the given values:

$$1 = (1)(2)\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \tag{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60° \tag{3}$$

Matrix Representation

Let:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{4}$$

Then:

$$\mathbf{a}^T \mathbf{b} = 1 \tag{5}$$

$$\|\mathbf{a}\| = \sqrt{1^2 + 0^2} = 1, \quad \|\mathbf{b}\| = \sqrt{1^2 + \sqrt{3}^2} = 2 \tag{6}$$

So the angle is:

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}^T \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60° \tag{7}$$
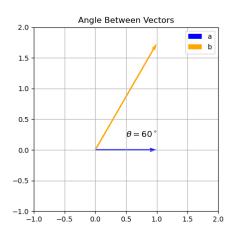
Plot

Figure: