1.10.10

EE25BTECH11051 - Shreyas Goud Burra

Question The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

(a) $\hat{i} - 2\hat{j} + 2\hat{k}$

(b) $\hat{i} - 2\hat{j}$

(c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

(d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.

Let us assume the given vector to be vector **A**

$$\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \tag{4.1}$$

To check whether the given vector is parallel with the vectors given in the options. Let the vectors in the options be **B1, B2, B3, B4**. For parallel nature, the rank of (**0 A B1**) must be 1.

$$(\mathbf{0\ A\ B1}) = \begin{pmatrix} 0 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & 2 & 2 \end{pmatrix} \tag{4.2}$$

$$(\mathbf{0\ A\ B2}) = \begin{pmatrix} 0 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & 2 & 0 \end{pmatrix} \tag{4.3}$$

$$(\mathbf{0\ A\ B3}) = \begin{pmatrix} 0 & 1 & 3 \\ 0 & -2 & -6 \\ 0 & 2 & 6 \end{pmatrix} \tag{4.4}$$

$$(\mathbf{0\ A\ B4}) = \begin{pmatrix} 0 & 1 & 9 \\ 0 & -2 & -18 \\ 0 & 2 & 18 \end{pmatrix} \tag{4.5}$$

Through rank manipulation we get the matrices **B1, B3 and B4** to have a rank of 1. They are parallel to **A** Here the magnitude(norm) of the vector **A** is given by

$$\mathbf{A}^T\mathbf{A} = ||\mathbf{A}||^2 \implies \begin{pmatrix}1 & -2 & 2\end{pmatrix}\begin{pmatrix}1\\-2\\2\end{pmatrix} = 9 \tag{4.6}$$

$$\implies ||\mathbf{A}|| = 3 \tag{4.7}$$

From 4.7, this gives us

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{||\mathbf{A}||} = \frac{1}{3}\begin{pmatrix}1\\-2\\2\end{pmatrix} \tag{4.8}$$

From 4.8, the vector of magnitude 9 along this direction is given by

$$9\hat{\mathbf{A}} = 9 \times \frac{1}{3}\begin{pmatrix}1\\-2\\2\end{pmatrix} \tag{4.9}$$

$$\implies 3\begin{pmatrix}1\\-2\\2\end{pmatrix} = \mathbf{B3} \tag{4.10}$$

Therefore the required vector is **B3**. This is option (b).

We get the same result by plotting a graph for the following.

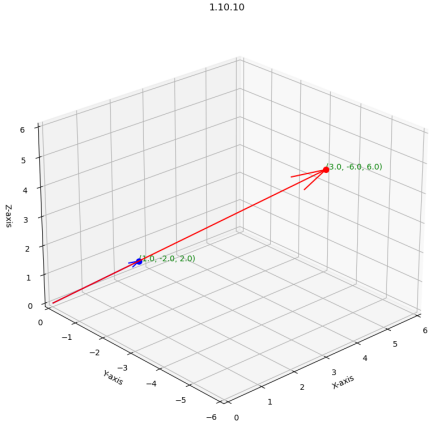

Fig. 4.1: 3D Plot