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EE25BTECH11031 - Sai Sreevallabh

Question:

Find the point on the Y-Axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$

Solution:

Given points are

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad (0.1)$$

Let \mathbf{P} be a point on the Y-Axis.

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix} \quad (0.2)$$

\mathbf{P} is equidistant from both \mathbf{A} and \mathbf{B} . Hence the norms of vectors $\mathbf{P} - \mathbf{B}$ and $\mathbf{P} - \mathbf{A}$ are equal.

$$\|\mathbf{P} - \mathbf{B}\| = \|\mathbf{P} - \mathbf{A}\| \quad (0.3)$$

$$\implies \|\mathbf{P} - \mathbf{B}\|^2 = \|\mathbf{P} - \mathbf{A}\|^2 \quad (0.4)$$

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{A} + \mathbf{A}^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{B} + \mathbf{B}^2 \quad (0.5)$$

Simplification of the above results in:

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (0.6)$$

$$\therefore \mathbf{P} = y\mathbf{e}_2 \quad (0.7)$$

where, $\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$y = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_2} \quad (0.8)$$

Substituting the values of \mathbf{A} and \mathbf{B} :

$$y = \frac{\left\| \begin{pmatrix} 5 \\ -2 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} -3 \\ 2 \end{pmatrix} \right\|^2}{2 \begin{pmatrix} 8 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}} \quad (0.9)$$

$$y = -2 \quad (0.10)$$

\therefore The point on the y-axis that is equidistant from the given two points is $\mathbf{P} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

