2.10.42

EE25BTECH11013 - Bhargav

Question:
If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit coplanar vectors, then the scalar triple product
$[2\mathbf{a}-\mathbf{b} \quad 2\mathbf{b}-\mathbf{c} \quad 2\mathbf{c}-\mathbf{a}] =$
Solution:

$$\mathbf{B} = \begin{pmatrix} 2\mathbf{a}-\mathbf{b} & 2\mathbf{b}-\mathbf{c} & 2\mathbf{c}-\mathbf{a} \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix} \tag{0.1}$$

Let

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix} \tag{0.2}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{pmatrix} \tag{0.3}$$

$$\therefore \det(\mathbf{B}) = \det(\mathbf{A}) \cdot \det(\mathbf{M}) \tag{0.4}$$

Since $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar,

$$[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] = 0. \tag{0.5}$$

$$\implies [2\mathbf{a}-\mathbf{b} \quad 2\mathbf{b}-\mathbf{c} \quad 2\mathbf{c}-\mathbf{a}] = \det(\mathbf{M}) \cdot [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] = 0. \tag{0.6}$$

This can be verified by taking an example of 3 coplanar unit vectors.

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{0.7}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{0.8}$$

$$\mathbf{c} = \begin{pmatrix} 0.6 \\ 0.8 \\ 0 \end{pmatrix} \tag{0.9}$$

$$\mathbf{X} = \begin{bmatrix} 2\mathbf{a} - \mathbf{b} & 2\mathbf{b} - c & 2\mathbf{c} - \mathbf{a} \end{bmatrix} \tag{0.10}$$

From the code, it is clear that the value of $\mathbf{X}$ is 0