Department of Electrical Engineering
IIT Hyderabad
ID 1370 – Digital Signal Processing

Lab Exercise - 02 24 January 2019

Discrete-Time Systems: Impulse Response and Frequency Response

1 Objective

In the previous lab, you have learned the basics of digitizing the signals, and studied the artefacts involved in digitizing the continuous-time signals. Now that all the signals are represented by a sequence of (finite precision) numbers, the next step is to develop systems to process the discrete-time signals. The objective of this experiment is to develop discrete-time systems, verify their properties and study their frequency domain characteristics. The major objectives of this experiment are as follows:

- Developing discrete-time systems for specified tasks.
- Verifying linearity and time-invariant properties of an arbitrary discrete-time system.
- Simulating the effect of reverberation using a discrete-time system.
- Studying frequency domain characteristics of discrete-time systems.
- Using moving average filters to enhance speech signals, and studying the trade-offs.

2 Discrete-Time Systems

The concept of *system* is very common in engineering. A physical *system* may be defined as a physical device that performs a specific operation on an input signal. Most of the physical systems are inertial in nature, in the sense that they produce an *output* or *response* only when they are subjected to an external input or *excitation*. In consistent with this definition of the system, a digital system is one that accepts a digital signal as input, and generates another digital signal as its output.

For our purposes, it is convenient to broaden the definition of a system to include not only physical devices, but also software realizations of operations on a signal. In digital processing of signals on a digital computer, the operations performed on a signal consist of a series of mathematical operations as specified by a software program. In this case, the program represents an implementation of the system in software. Thus we have a system that is realized on a digital computer by means of a sequence of mathematical operations; that is we have digital signal processing realized in software. For example, a digital computer can be programmed to reduce noise and interference in the input signal. Alternatively, the digital processing on the signal may be performed by digital hardware (logic circuits) configured to perform the desired operations.

2.1 Exercise - Implementation of discrete-time systems

(a) Simulate a time-reversal system in MATLAB, that takes an arbitrary input $x[n]$, and outputs it time-reversed version. $y[n] = x[-n]$

(b) Simulate a discrete-time system in MATLAB, that takes an arbitrary signal $x[n]$ as input and generates its even component at the output. $y[n] = x_e[n]$. Use the system developed in part (a) to realize this.

(c) Simulate a discrete-time system in MATLAB, that takes an arbitrary signal $x[n]$ as input and generates its odd component at the output. $y[n] = x_o[n]$. Use the system developed in part (a) to realize this.

(d) Test the operation of the systems developed in parts (a), (b) and (c) using an input signal $x[n] = 0.95^n u[n]$.

(e) Simulate an ideal-delay system, that accepts an arbitrary input $x[n]$ and outputs its time-delayed version. $y[n] = x[n - d]$, where d is an integer.

(f) Simulate a moving average system described by the following input-output relation:

$$y[n] = \frac{1}{M+1} \sum_{k=0}^{M} x[n-k]$$

Use the ideal delay system developed in part (e) to realize this.

(g) Verify the operation of the moving average system in part (f) using an input signal $x[n] = 0.95^n u[n]$ for $M = 10$.

(g1) Read your image into MATLAB, and add white Gaussian noise to the image. Process the rows and columns of the noisy image using the moving average filter developed in (f), for different values of M. Explain your obersations.

(h) Simulate a backward difference system described by the following input-output relation

$$y[n] = x[n] - x[n-1]$$

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(i) Simulate a discrete-time system with input-output relation: $y[n] = x[n] - 2x[n-1] + x[n-2]$.

(j) Verify the operation of the systems (h) and (i) using an input signal $x[n] = 0.95^n u[n]$ for $M = 10$.

(j1) Read your image into MATLAB, and process the rows and columns of the image using the differencing systems (h) and (i). Explain your obersations.

(k) Simulate a squarer system with input-output relation $y[n] = x^2[n]$.

(l) Simulate a discrete-time system whose input and output are related by $y[n] = x[n-2] + x[2-n]$

3 Linear Time-Invariant Systems

Linear time-invariant (LTI) systems form an important subset of general systems. Sophisticated mathematical tools are available to analyze the LTI systems. For example, convolution can be used to determine the output of an LTI system to an arbitrary input. In order to make use of the available mathematical tools, we need to determine whether a given system satisfies the properties of linearity and time-invariance or not. The objective of this experiment is to verify the linearity and time-invariance properties of a given LTI system using a digital system.

3.1 Exercise

(a) Generate the following test signals for $-100 \leq n \leq 100$.

$$x_1[n] = 0.9^n u[n]$$

$$x_2[n] = \sin\left(2\pi \frac{200}{8000} n\right)$$

(b) **Linearity**: Apply $x_1[n]$, and $x_2[n$ as inputs to the system and generate the outputs $y_1[n]$ and $y_2[n]$, respectively. Now apply $ax_1[n] + bx_2[n]$ as input to the system to generate the output $y[n]$. To verify linearity check whether $y[n]$ is equal to $ay_1[n] + by_2[n]$ or not. Check for linearity of all the systems simulated in Section 2.1.

(c) **Time-invariance:** Apply $x_1[n]$ as input to the system and generate the output $y_1[n]$. Now apply $x_1[n-d]$ as input and generate the output $y[n]$. To verify time-invariance property check whether $y[n]$ is equal to $y_1[n-d]$ or not. Repeat verification for $x_2[n]$. Check for time-invariance of all the systems simulated in Section 2.1.

(d) For LTI systems, the output of the system for any arbitrary input can be determined using [illegible] function in MATLAB. Repeat 2.1(g) and 2.1(j) using [illegible] function, and compare the results with earlier outputs.

4 Simulation of Reverberation

Reverberation is the persistence of sound in a particular space after the original sound is removed. Reverberation is created when a sound is produced in an enclosed space causing a large number of echoes to build up, and then slowly decay as the sound is absorbed by the walls and air. An echo can be thought of as a single and perceivable reflection of sound. The reverberation can be simulated by superposing several delayed and attenuated components of the source signal. Hence reverberated signal $x_r[n]$ can be expressed in terms of the source signal $x[n]$ as

$$x_r[n] = \sum_{k=0}^{M} a_k x[n - d_k]$$

where M is the total number of reflections, a_k and d_k represents the attenuation coefficient and delay of k^{th} reflected component.

4.1 Exercise

- Generate the reverberated signal if the impulse response of the room is $h[n] = 0.999^n u[n]$. What are your observations.