THEOREMS

1. Sunflower Lemma

Let F be w-uniform family of sets if $|F| > w!(k-1)^w$, then F contains a k-sunflower.

Let $f(k,w)$ denote the minimum no. of w-sized sets required to ensure the presence of k-sunflower.

$$(k-1)^w < f(k,w) \leq w!(k-1)^w + 1$$

2. Erdos-ko-rado

for $n \geq 2k$.

Let F be a k-uniform intersecting family of subsets of $[n]$ that is intersecting.

$$|F| \leq {}^{n-1}C_{k-1}$$

3. Dilworth Theorem

i) let $P = (X, \leq) \Rightarrow$ Poset

if length of longest chain in P is 'r', then the elements of X can be partitioned into 'r' antichains.

ii) if length of longest antichain in P is 'r', then the elements of X can be partitioned into 'r' chains.

4. Hall's Theorem

Let G be a bipartite graph with bipartition {A,B}. Then G has a matching that matches all the vertices of A iff G satisfies the Hall's condition

$$\forall\ S \subseteq A, \quad |N_G(S)| \geq |S|$$

8. Application of Bollobas thm.

Theorem: Let F be a family of size greater than $\left({}^{r+k}C_k\right)$ further every set in F is of size of atmost r. Then $\exists$ some "k+1" sets in F that have a strong system of distinct rep^n.

9. Ramsey Numbers

R(K,K) is the minimum 'n', such that no matter how we colour edges of K_n with 2 colours, we will surely encounter either a Red K_k or blue K_k clique.

$$2^{\lfloor k/2 \rfloor} \le R(K,K) \le 2^{2K-3}$$

10. Tournaments.

if ${}^nC_k\left(1-\frac{1}{2^k}\right)^{n-k} < 1$, then there is a tournament on n vertices that satisfy property P_k

$\forall$ K-sized subsets of T, $\exists$ a vertex which defeats all players in the subset.

11. Dominating set.

Let G be a graph on n vertices, with min degree δ, then the size of the dominating set in G is atmost

$$\frac{n}{\delta+1}\left(1+\log_e(1+\delta)\right)$$

12. Sum-free sets

Every set $B=\{b_1, b_2 \ldots b_n\}$ of n non-zero integers has a sumfree subset A of size atmost

$$|A| > n/_3$$

5. Sperner's theorem

Let F be a family of subsets of $[n]$.
F is an antichain under the containment relation
Then,

$$|F| \le {}^{n}C_{n/2}$$

→ Poset $P = (\text{Powerset } [n], \subseteq)$
largest antichain $\le {}^{n}C_{n/2}$

6. LYM inequality

Let F be a family of subsets of $[n]$.
F is an antichain under the containment relation,
$F = \{A_1, A_2, A_3 \dots, A_m\}$.

$$\sum_{i=1}^{m} \frac{1}{{}^{n}C_{|A_i|}} \le 1$$

7. Bollobas theorem

Let $(A_1, A_2 \dots, A_m)$ and $(B_1, B_2, B_3 \dots B_m)$ be 2 sequences of sets such that $\forall\, i,j \in [m]$, $A_i \cap B_j = \phi$ iff $i = j$, then

$$\sum_{i=1}^{m} \frac{1}{{}^{a_i+b_i}C_{a_i}} \le 1$$

where,

$|A_i| = a_i$; $|B_i| = b_i$

if $|A_i| \le a$ and $|B_i| \le b$ $\forall\, i \in [m]$

$$m \le {}^{a+b}C_a$$