

Convergence Analysis

$$\underline{f(r) = 0}$$

$$e_k = |x_k - r|$$

$$e_{k+1} = |x_{k+1} - r|$$

$$= |g(x_k) - r| = \boxed{|g(x_k) - g(r)|}$$

$$e_{k+1} \leq \frac{1}{2} e_k^2$$

$$\boxed{\max_{\xi} |g''(\xi)|}$$

$$e_{k+1} \leq \boxed{M} e_k^2$$

Taylor Expansion of $g(x_k)$
around $x_k = r$

$$g(x_k) = g(r) + \overbrace{(x_k - r)}^0 g'(r) + \frac{1}{2} g''(\xi) (x_k - r)^2$$

$$= g(r) + \frac{1}{2} (x_k - r)^2 \underline{g''(\xi)}$$

$$\Rightarrow x_{k+1} = \boxed{g(r)} + \frac{1}{2} \underline{(x_k - r)^2 g''(\xi)}$$

$$\Rightarrow \underline{e_{k+1}} = \frac{1}{2} e_k^2 \boxed{g''(\xi)}$$