

Convergence Condition

There exists an interval $I \equiv [r-c, r+c]$ for some $c > 0$, such that

$$|g'(x)| < 1 \text{ on } I \text{ \& } x_0 \in I$$

$$\begin{array}{l|l} 1) & x - \cos(x) = f(x) \\ & \Rightarrow g(x) = \cos(x) \\ & |g'(x)| = \sin x \end{array} \left| \begin{array}{l} [0.5, 1) \\ \equiv I \\ \Rightarrow \text{converges} \end{array} \right.$$

$$M = \max_{x \in I} |g'(x)| = \sin 1 < 1$$

$$\begin{array}{l} 2) \quad g(x) = e^{-2x(x-1)+x} \\ \quad g'(x) = -2e^{-2x(x-1)+x} + e^{-2x(x-1)+x} \\ \quad g'(1) = e^{-2} + 1 > 1 \\ \quad \Rightarrow \text{Diverges} \end{array}$$