Bass Diffusion Model

Report by Shreyas Kulkarni under the guidance of Prof. Chintan Amrit

December 2023

For global product managers, one big question is how fast a new product is likely to sell. The Bass diffusion model is used to predict this diffusion of innovation. The Bass model is a special case of the Gamma/shifted Gompertz distribution (G/SG). Here innovation refers to the discovery of our product for the users in the market & diffusion refers to its adoption in the market of concern. Diffusion can also be thought of as penetration in the market just like in the case of a gas from where we pickup the word to use in the marketing context. This simple model is named after Prof. Frank M. Bass of UT, Dallas. The paper describing the model is one of the 10 most influential papers in the 50-year history of management science. In the Bass diffusion model, we encounter 2 processes - **Innovation** & **Imitation**. Innovation refers to the discovery of the product in the market while imitation refers to the act of copying the usage of the product seeing the users (innovators).

- The Bass Model parameter representing the potential market, which is the ultimate number of purchasers of the product, is constant. It is denoted by M.
- A(t-1) denotes the cumulative number of consumers who have already adopted the product before the time t
- a(t) denotes the number of consumers who have adopted the product at a time t
- $F(t) = \frac{A(t)}{M}$ & $f(t) = \frac{a(t)}{M}$
- Parameter of innovation is denoted by p
- Parameter of imitation is denoted by q

To estimate A(t)

$$A(t) = A(t-1) + p(M - A(t-1)) + q\frac{A(t-1)}{M}(M - A(t-1))$$

Now the term 2 in the RHS refers to the number of innovators, it would be proportional to the remaining users in the market which is total users - users at t-1. The term 3 refers to imitators. They will be proportional to the fraction of already adopted consumers as well as remaining users in the market. In both terms, we have the constants also representing the individual rates of processes. A market contains 3 unknowns for a Bass diffusion model - M, p & q, all the other parameters are empirical.
Now we define something called a hazard rate. The hazard rate is the portion that adopts at t given that they have not yet adopted.

$$h(t) = \frac{f(t)}{1-F(t)}$$

Also the probability of adopting by those who have not yet adopted is a linear function of those who had previously adopted.

h(t)=p+qF(t)

Equating both of these relations & substituting a & A

$$a(t) = Mp + (q-p)A(t) - \frac{q}{M}A^2(t)$$

$$\frac{dF}{dT} = p + (q-p)F - qF^2$$

Solving the differential equation & putting F(0)=0 (fraction of adopters at time 0 is 0)

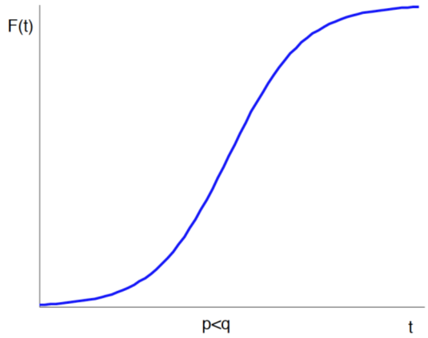
$$F(t) = \frac{1-e^{-(p+q)t}}{1+\frac{qe^{-(q+p)t}}{p}}$$

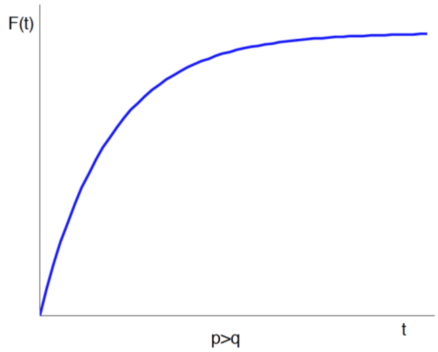
Now we have 2 special cases of the Bass diffusion model

1. q=0: Exponential distribution

2. p=0: Logistic distribution

Also we have p¿q (less risky products & everyday items) & q¿p (risky items so customers are initially hesitant to adopt)





A high value for p indicates that the diffusion has a quick start but also tapers off quickly. A high value of q indicates that the diffusion is slow at first but accelerates after a while and later it saturates to a maximum value. The maximum number of adoptions occur at the following time

$$t_f = ln(\frac{q}{p(p+q)})$$