AI 1110 Assignment 1

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12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution:

Let X,Y,Z be three Bernoulli random variables defined as follows -

$$X = \begin{cases} 1 & \text{,if odd number occurs on the } 1^{st} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases}$$

$$(1)$$

$$Y = \begin{cases} 1 & \text{,if odd number occurs on the } 2^{nd} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases}$$

$$(2)$$

$$Z = \begin{cases} 1 & \text{,if odd number occurs on the } 3^{rd} \text{ roll of die} \\ 0 & \text{,othserwise} \end{cases}$$

$$(3)$$

Let A be the event of getting an odd number atleast once and $\Pr(A)$ be its probability.

Then $\Pr(A)$ is equal to the compliment of getting an even number on every die toss.

$\therefore$

$$\Pr(A) + \Pr(\bar{A}) = 1 \qquad (4)$$

$\implies$

Pr(X=1∪Y=1∪Z=1) = 1 - Pr(X=0∩Y=0∩Z=0)

For this event (Pr(X=0∩Y=0∩Z=0)) = $\frac{3}{6} \times \frac{3}{6} \times \frac{3}{6}$

$\therefore$ From Equation (4),

$$\Pr(A) = 1 - \left(\frac{3}{6} \times \frac{3}{6} \times \frac{3}{6}\right) \qquad (5)$$

$$= 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8} \qquad (6)$$