

AI 1110 Assignment 1

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12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution:

Let X,Y,Z be three Bernoulli random variables defined as follows -

X = 1 , if odd number occurs on the first roll of die.

X = 0 , otherwise

Y = 1 , if odd number occurs on the second roll of die.

Y = 0 , otherwise

Z = 1 , if odd number occurs on the third roll of die.

Z = 0 , otherwise

Let 'Pr' be the probability of getting an odd number atleast once.

Then 'Pr' is equal to the compliment of getting an even number on every die toss.

Now since,

$$P(A) + P(\bar{A}) = 1 \quad (1)$$

This implies that,

$$\begin{aligned} &\text{Pr}(\text{getting an odd number at least once on the die}) \\ &= 1 - \text{Pr}(\text{getting an even number on every die toss}) \end{aligned}$$

$$\text{Pr}(X=1 \cup Y=1 \cup Z=1) = 1 - \text{Pr}(X=0 \cap Y=0 \cap Z=0)$$

\therefore For this event, (i.e. of getting an even number on all the three outcomes),

The total favourable outcomes are 3 {2,4,6} out of all the possible outcomes 6 {1,2,3,4,5,6}.

\therefore From Equation 1,

$$\begin{aligned} Pr &= 1 - \left(\frac{3}{6} * \frac{3}{6} * \frac{3}{6} \right) \\ \Rightarrow Pr &= 1 - \left(\frac{1}{2} \right)^3 \\ \Rightarrow Pr &= 1 - \frac{1}{8} \\ \Rightarrow Pr &= \frac{7}{8} \end{aligned}$$

Hence,

Answer: The probability of getting an odd number atleast once on tossing the die three times is equal to $\frac{7}{8}$.