AI 1110 Assignment 1

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12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution: Let A be the required event.

Let i be the number of times odd number occurs.

Let p be the probability of getting odd number on one throw of dice.

$\therefore \Pr(X = i) = {}^3C_i \cdot p^i \cdot (1-p)^{3-i}$

$p = \frac{1}{2}$

Let Cumulative Distribution function be:

$$F_X(i) = \Pr(X \le i) \tag{1}$$

$$\Pr(X = i) = {}^3C_i \cdot p^i \cdot (1-p)^{3-i} \tag{2}$$

$$\therefore F_X(i) = \sum_{r=0}^{i} {}^3C_r p^r (1-p)^{3-r} \tag{3}$$

$$\implies F_X(0) = {}^3C_0 p^0 (1-p)^{3-0} \tag{4}$$

$$\implies F_X(0) = {}^3C_0 \cdot p^0 \cdot (1-p)^{3-0} = \frac{{}^3C_0}{8} = \frac{1}{8} \tag{5}$$

$$\implies F_X(1) = \frac{1}{8} + {}^3C_1 \times \frac{1}{2} \times \left(1 - \frac{1}{2}\right)^2 = \frac{4}{8} \tag{6}$$

$$\implies F_X(2) = \tfrac{1}{8} + {}^3C_1 \times \tfrac{1}{2} \times \left(1 - \tfrac{1}{2}\right)^2 + {}^3C_2 \times \left(\frac{1}{2}\right)^2 \times \left(1 - \frac{1}{2}\right) = \frac{7}{8} \tag{7}$$

$$F_X(3) = 1 \tag{8}$$

$\therefore$,

$$\Pr(A) = F_X(3) - F_X(0) \tag{9}$$

$$= 1 - \frac{1}{8} \tag{10}$$

$$= \frac{7}{8} \tag{11}$$