AI 1110 Assignment 1

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12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution: Let a binomial random variable be:

$$X \sim Bin(n, p) \tag{1}$$
$$\implies p = \frac{1}{2} \tag{2}$$
$$\implies n = 3 \tag{3}$$

where, p be the probability of getting odd number on one throw of dice.

n is the number of times dice is rolled.

Let i be the number of times odd number occurs.

$$\therefore \Pr(X = i) = {}^{n}C_i p^i (1-p)^{n-i} \tag{4}$$
$$\tag{5}$$

Let Cumulative Distribution function be:

$$F_X(i) = \Pr(X \le i) \tag{6}$$
$$\Pr(X = i) = {}^{3}C_i p^i (1-p)^{3-i} \tag{7}$$
$$\therefore F_X(i) = \sum_{r=0}^{i} {}^{3}C_r p^r (1-p)^{3-r} \tag{8}$$
$$\implies F_X(0) = {}^{3}C_0 p^0 (1-p)^{3-0} \tag{9}$$
$$\implies F_X(0) = {}^{3}C_0 p^0 (1-p)^{3-0} = \frac{{}^{3}C_0}{8} = \frac{1}{8} \tag{10}$$
$$F_X(3) = 1 \tag{11}$$

$\therefore$,

$$\implies \Pr(X > 0) = \sum_{i=1}^{3} \Pr(X = i) \tag{12}$$
$$= F_X(3) - F_X(0) \tag{13}$$
$$= 1 - \frac{1}{8} \tag{14}$$
$$= \frac{7}{8} \tag{15}$$