AI 1110 Assignment 2

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12.13.6.04 Question: Suppose that 90 % of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Answer: 0.0127951893

Solution: Let a binomial random variable be:

$$X \sim Bin(n,p) \tag{1}$$
$$\implies p = \frac{1}{10} = 0.1 \tag{2}$$
$$\implies n = 10 \tag{3}$$

where, p be the probability of a person being left-handed.

n is the number of people.

By Gaussian approximation,

$$P(X \geq 4) = 1 - P(X < 4) \tag{4}$$
$$= 1 - F_X(3) \tag{5}$$

Hence,

$$\mu = np \tag{6}$$
$$\implies \mu = 10 \times 0.1 = 1 \tag{7}$$
$$\sigma^2 = np(1-p) \tag{8}$$
$$\implies \sigma^2 = 10 \times 0.1 \times 0.9 = 0.9 \tag{9}$$
$$\sigma = \sqrt{np(1-p)} \tag{10}$$
$$\implies \sigma = \sqrt{0.9} \tag{11}$$

To calculate error function:

$$x = \frac{3-\mu}{\sqrt{2}\sigma} \tag{12}$$
$$x = \frac{3-1}{\sqrt{1.8}} \tag{13}$$
$$x = 1.490711985 \tag{14}$$

$\therefore$ $erf(x)$ implies,

$$erf(1.490711985) = \int_0^{1.490711985} \frac{1}{\sqrt{2\pi}} e^{-x^2} dx \tag{15}$$
$$erf(1.490711985) = 0.9649836 \tag{16}$$
$$F_X(3) = 0.5 \cdot (1 + erf(1.490711985)) \tag{17}$$
$$F_X(3) = 0.9824918 \tag{18}$$
$$P(X \geq 4) = 1 - F_X(3) \tag{19}$$
$$= 1 - 0.9824918 \tag{20}$$
$$= 0.0175082 \tag{21}$$

So, The answer differs by,
$0.0175082 - 0.0127951893 = 0.004713011$