AI 1110 Assignment 2

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12.13.6.04 Question: Suppose that 90 % of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

Answer: 0.0127951893

Solution: Let a binomial random variable be:

$$X \sim Bin(n, p) \tag{1}$$

$$\implies p = \frac{9}{10} \tag{2}$$

$$\implies n = 10 \tag{3}$$

where, p be the probability of a person being right-handed.

n is the number of people.

Let i be the number of times odd number occurs.

$$\therefore \Pr(X = i) = {}^{n}C_i p^i (1-p)^{n-i} \tag{4}$$

Let Cumulative Distribution function be:

$$F_X(i) = \Pr(X \le i) \tag{5}$$

$$\Pr(X = i) = {}^{10}C_i p^i (1-p)^{10-i} \tag{6}$$

$$\therefore F_X(i) = \sum_{r=0}^{i} {}^{10}C_r p^r (1-p)^{10-r} \tag{7}$$

$$\implies F_X(0) = {}^{10}C_0 p^0 (1-p)^{10-0} \tag{8}$$

$$\implies F_X(0) = {}^{10}C_0 p^0 (1-p)^{10-0} = \frac{{}^{10}C_0}{10^{10}} = \frac{1}{10^{10}} \tag{9}$$

$$F_X(1) = \frac{1}{10^{10}} + {}^{10}C_1 p^1 (1-p)^{10-1} \tag{10}$$

$$\implies F_X(1) = \frac{1}{10^{10}} + \frac{9}{10^9} = \frac{91}{10^{10}} \tag{11}$$

$$F_X(2) = \frac{91}{10^{10}} + {}^{10}C_2 p^2 (1-p)^{10-2} \tag{12}$$

$$\implies F_X(2) = \frac{91}{10^{10}} + \frac{3645}{10^{10}} = \frac{3736}{10^{10}} \tag{13}$$

$$F_X(3) = \frac{3736}{10^{10}} + {}^{10}C_3 p^3 (1-p)^{10-3} \tag{14}$$

$$\implies F_X(3) = \frac{3645}{10^{10}} + \frac{8748}{10^9} = \frac{91125}{10^{10}} \tag{15}$$

$$F_X(4) = \frac{91125}{10^{10}} + {}^{10}C_4 p^4 (1-p)^{10-4} \tag{16}$$

$$\implies F_X(4) = \frac{91125}{10^{10}} + \frac{137781}{10^9} = \frac{1468935}{10^{10}} \tag{17}$$

$$F_X(5) = \frac{1468935}{10^{10}} + {}^{10}C_5 p^5 (1-p)^{10-5} \tag{18}$$

$$\implies F_X(5) = \frac{1468935}{10^{10}} + \frac{14880348}{10^{10}} = \frac{16349283}{10^{10}} \tag{19}$$

$$F_X(6) = \frac{16349283}{10^{10}} + {}^{10}C_6 p^6 (1-p)^{10-4} \tag{20}$$

$$\implies F_X(6) = \frac{16349283}{10^{10}} + \frac{11160261}{10^9} = \frac{127951893}{10^{10}} \tag{21}$$

$\therefore$,

$$\implies \Pr(X \le 6) = \sum_{i=0}^{6} \Pr(X = i) \tag{22}$$

$$= F_X(6) \tag{23}$$

$$= \frac{127951893}{10^{10}} \tag{24}$$

$$= 0.0127951893 \tag{25}$$