

AI 1110 Assignment 1

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12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution: Let a binomial random variable be:

$$X \sim \text{Bin}(n, p) \quad (1)$$

where, p be the probability of getting odd number on one throw of dice.

n is the number of times dice is rolled.

Let i be the number of times odd number occurs.

$$\therefore \Pr(X = i) = {}^nC_i \cdot p^i \cdot (1 - p)^{n-i} \quad (2)$$

$$\implies p = \frac{1}{2} \quad (3)$$

$$\implies n = 3 \quad (4)$$

Let Cumulative Distribution function be:

$$F_X(i) = \Pr(X \leq i) \quad (5)$$

$$\Pr(X = i) = {}^3C_i \cdot p^i \cdot (1 - p)^{3-i} \quad (6)$$

$$\therefore F_X(i) = \sum_{r=0}^i {}^3C_r p^r (1 - p)^{3-r} \quad (7)$$

$$\implies F_X(0) = {}^3C_0 p^0 (1 - p)^{3-0} \quad (8)$$

$$\implies F_X(0) = {}^3C_0 \cdot p^0 \cdot (1 - p)^{3-0} = \frac{{}^3C_0}{8} = \frac{1}{8} \quad (9)$$

$$F_X(3) = 1 \quad (10)$$

\therefore ,

$$\implies \Pr(X > 0) = \sum_{i=1}^3 \Pr(X = i) \quad (11)$$

$$= F_X(3) - F_X(0) \quad (12)$$

$$= 1 - \frac{1}{8} \quad (13)$$

$$= \frac{7}{8} \quad (14)$$