

# AI 1110 Assignment 3

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**12.13.6.04 Question:** Suppose that 90 % of people are right-handed. What is the probability that atmost 6 of a random sample of 10 people are right-handed.

**Answer:** 0.0127951893

**Solution:** Let a binomial random variable be:

$$X \sim \text{Bin}(n, p) \quad (1)$$

$$\Rightarrow p = \frac{1}{10} = 0.1 \quad (2)$$

$$\Rightarrow n = 10 \quad (3)$$

where,  $p$  be the probability of a person being left-handed.

$n$  is the number of people.

**Proof of Gaussian Approximation on Binomial:** By defination,

$$\Pr(X = k) = {}^nC_k p^k (1-p)^{n-k} \quad (4)$$

By Stirling's formula,

For  $m \rightarrow \infty$ ,

$$m! \sim \sqrt{2\pi m} e^{-m} m^m \quad (5)$$

Hence,

$$\Pr(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \quad (6)$$

$$(7)$$

Hence,

$$\sim \frac{\sqrt{2\pi n} e^{-n} n^n p^k (1-p)^{n-k}}{\sqrt{2\pi k} e^{-k} k^k \sqrt{2\pi(n-k)} e^{-(n-k)} (n-k)^{n-k}} \quad (8)$$

$$= \left(\frac{p}{k}\right) \left(\frac{1-p}{n-k}\right)^{n-k} n^n \sqrt{\frac{n}{2\pi k(n-k)}} \quad (9)$$

$$= \left(\frac{np}{k}\right)^k \left(\frac{n(1-p)}{n-k}\right)^{n-k} \sqrt{\frac{n}{2\pi k(n-k)}} \quad (10)$$

Now by identities,

$$\ln\left(\frac{np}{k}\right) = -\ln\left(1 + \frac{k-np}{np}\right) \quad (11)$$

$$\ln\left(\frac{n(1-p)}{n-k}\right) = -\ln\left(1 - \frac{k-np}{n(1-p)}\right) \quad (12)$$

By approximation,

For  $y \rightarrow 0$

$$\ln(1+y) \sim y - \frac{y^2}{2} + \frac{y^3}{3} \dots \quad (13)$$

Hence,

$$\ln\left(\left(\frac{np}{k}\right)^k \left(\frac{n(1-p)}{n-k}\right)^{n-k}\right) = \quad (14)$$

$$-k \ln\left(1 + \frac{k-np}{np}\right) + (n-k) \ln\left(1 - \frac{k-np}{n(1-p)}\right) \quad (15)$$

$\therefore$ ,

$$\ln\left(\left(\frac{np}{k}\right)^k \left(\frac{n(1-p)}{n-k}\right)^{n-k}\right) \sim -\frac{(k-np)^2}{2np(1-p)} \quad (16)$$

$\therefore$ ,

$$\left(\frac{np}{k}\right)^k \left(\frac{n(1-p)}{n-k}\right)^{n-k} \sim e^{-\frac{(k-np)^2}{2np(1-p)}} \quad (17)$$

By approximating that,  $k-np$  is of order  $\sqrt{n}$

$$k-np \approx \sqrt{n} \quad (18)$$

$$n-k \approx n(1-p) - \sqrt{n} \quad (19)$$

$$k(n-k) \approx n^2 p(1-p) \quad (20)$$

$$\Rightarrow \sqrt{\frac{n}{2\pi k(n-k)}} \sim \frac{1}{\sqrt{2\pi np(n-p)}} \quad (21)$$

$$\Pr(X = x) = \frac{e^{-\frac{(x-np)^2}{2np(1-p)}}}{\sqrt{2\pi np(n-p)}} \quad (22)$$

$$\Pr(X = x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}} \quad (23)$$

By Q-function,

$$\Pr(X \geq 4) = 1 - \Pr(X < 4) \quad (24)$$

$$= 1 - F_X(3) \quad (25)$$

Hence,

$$\mu = np \quad (26)$$

$$\Rightarrow \mu = 10 \times 0.1 = 1 \quad (27)$$

$$\sigma^2 = np(1 - p) \quad (28)$$

$$\Rightarrow \sigma^2 = 10 \times 0.1 \times 0.9 = 0.9 \quad (29)$$

$$\sigma = \sqrt{np(1 - p)} \quad (30)$$

$$\Rightarrow \sigma = \sqrt{0.9} \quad (31)$$

$$\Rightarrow Z = \frac{X - \mu}{\sigma} \quad (32)$$

$$Z = \frac{3 - 1}{\sqrt{0.9}} \quad (33)$$

$$Z = 2.108185107 \quad (34)$$

By Z-score table,  $\Pr(X \leq 3) = F_X(3) = 0.9854$

Hence,

$$\Pr(X \geq 4) = 1 - 0.9854 \quad (35)$$

$$= 0.0146 \quad (36)$$

So, The answer differs by,

$$0.0146 - 0.0127951893 = 0.001804811$$