

AI 1110 Assignment 1

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12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution:

Let X,Y,Z be three Bernoulli random variables defined as follows -

$$X = \begin{cases} 1 & \text{,if odd number occurs on the 1}^{st} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases} \quad (1)$$

$$Y = \begin{cases} 1 & \text{,if odd number occurs on the 2}^{nd} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases} \quad (2)$$

$$Z = \begin{cases} 1 & \text{,if odd number occurs on the 3}^{rd} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases} \quad (3)$$

Let A be the event of getting an odd number atleast once and $\Pr(A)$ be its probability.

Then $\Pr(A)$ is equal to the compliment of getting an even number on every die toss.

\therefore

$$\Pr(A) + \Pr(\bar{A}) = 1 \quad (4)$$

\Rightarrow

$$\Pr(X=1 \cup Y=1 \cup Z=1) = 1 - \Pr(X=0 \cap Y=0 \cap Z=0)$$

Let the number of times even number occurs in the three time roll of the dice is i.

Then probability of getting i times even number

$$= \frac{\binom{3}{i}}{(2)^3}.$$

For this event,

$$(\Pr(i=3)) = \frac{\binom{3}{3}}{(2)^3} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

\therefore From Equation (4),

$$= 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8} \quad (5)$$