AI 1110 Assignment 1

Name: Shreyas Premkhede
Roll no.: CS22BTECH11053

12.13.2.12 Question: A die is tossed thrice. Find the probability of getting an odd number at least once.

Answer: $\frac{7}{8}$

Solution:

Let X,Y,Z be three Bernoulli random variables defined as follows -

$$X = \begin{cases} 1 & \text{,if odd number occurs on the } 1^{st} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases} \tag{1}$$

$$Y = \begin{cases} 1 & \text{,if odd number occurs on the } 2^{nd} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases} \tag{2}$$

$$Z = \begin{cases} 1 & \text{,if odd number occurs on the } 3^{rd} \text{ roll of die} \\ 0 & \text{,otherwise} \end{cases} \tag{3}$$

Let A be the event of getting an odd number atleast once and $\Pr(A)$ be its probability.

Then $\Pr(A)$ is equal to the compliment of getting an even number on every die toss.

$\because$

$$\Pr(A) + \Pr\left(\bar{A}\right) = 1 \tag{4}$$

$\implies$

$\Pr(X=1 \cup Y=1 \cup Z=1) = 1 - \Pr(X=0 \cap Y=0 \cap Z=0)$

For this event $(\Pr(X=0 \cap Y=0 \cap Z=0)) = \frac{3}{6} \times \frac{3}{6} \times \frac{3}{6}$

$\therefore$ From Equation (4),

$$\Pr(A) = 1 - \left(\frac{3}{6} \times \frac{3}{6} \times \frac{3}{6}\right) \tag{5}$$

$$= 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = \frac{7}{8} \tag{6}$$