ANALYTICAL ASSIGNMENT - 1

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① Solve the Recurrence Relation

i) $x(n) = x(n-1)+5$ for $n>1$ $x(1)=0$

$x(1)=0$

$x(2) = x(2-1)+5$

$= \cancel{x+5}\ x(1)+5$

$= 0+5$

$= 5$

$x(3) = x(3-1)+5$

$= x(2)+5$

$= 5+5$

$= 10$

$x(4) = x(4-1)+5$

$= x(3)+5$

$= 10+5$

$= 15$

So, $x(n) = 5(n-1)$

ii) $x(n) = 3x(n-1)$ for $n>1$ $x(1)=4$

$x(1)=4$

$x(2) = 3x(2-1)$

$= 3x(1)$

$= \cancel{3x}\ 3\times4 = 12$

$x(3) = 3x(3-1)$

$= 3x(2)$

$= 3 \times 12$

$= 36$

$x(4) = 3x(4-1)$

$= 3x(3)$

$= 3 \times 36$

$= 108$

So, $x(n) = 4 \times 3^{(n-1)}$

(c) $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$ (solve for $n = 2^k$)

$x(n) = x(n/2) + n$

$n = \frac{n}{2}$

$x\left(\frac{n}{2}\right) = x\left(\frac{\frac{n}{2}}{2}\right) + \frac{n}{2}$

$x\left(\frac{n}{2}\right) = x\left(\frac{n}{2^2}\right) + \frac{n}{2}$

$x(n) = x\left(\frac{n}{2^2}\right) + \frac{n}{2} + n$

$x(n) = x\left(\frac{n}{2^2}\right) + \frac{3n}{2}$

$x\left(\frac{n}{2^2}\right) = x\left(\frac{\frac{n}{2}}{2^2}\right) + \frac{3}{2} \times \frac{n}{2^2}$

$x\left(\frac{n}{2^2}\right) = x\left(\frac{n}{2^3}\right) + \frac{3n}{2^3}$

$x(n) = x\left(\frac{n}{2^3}\right) + \frac{3n}{2^3} + \frac{3n}{2}$

$= x\left(\frac{n}{2^3}\right) + 6\!\!\!/$

$n = 2^k$

$x(2^k) = x\left(\frac{2^k}{2}\right) + n$

$= x(2^k \cdot 2^{-1}) + n$

$= x(2^{k-1}) + n$

$y(k) = x(2^k)$

$y(k) = y(k-1) + 2^k$

$y(1) = x(2) = x(1) + 2 = 1 + 2 = 3$

$y(2) = x(4) = x(2) + 4 = 3 + 4 = 7$

$y(3) = x(8) = x(4) + 8 = 7 + 8 = 15$

$y(k) = 1 + 3 + 7 + 15 \ldots + 2^k$

$y(k) = 2^{k+1} - 1$

$x(2^k) = 2^{k+1} - 1$

(2) Evaluate the following recurrences completely

i) $T(n) = T(\frac{n}{2}) + 1$, where $n = 2k$ for all $k \geq 0$

$n = 2^k$

$T(1) = T(2^0) + 1$

$T(n) = T(\frac{n}{2}) + 1$

$T(2^k) = T(\frac{2^k}{2}) + 1$

$= T(2^{k-1}) + 1$

$T(2^{k-1}) = T(\frac{2^{k-1}}{2}) + 1$

$= T(2^{k-2}) + 1$

$\vdots$

$T(2^k) = T(1) + k \cdot 1$

Since $n = 2^k$

log on both sides

$\log n = \log 2^k$

$\log n = k \log 2$

$k = \log n$

$T(n) = T(1) + \log(n)$

$T(n) = c + \log n$

$T(n) = O(\log n)$

(d) $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$ (solve for $n = 3$

$x(1) = 1$

$n = 3^k$

$$x(3^k) = x\left(\frac{3^k}{3}\right) + 1$$

$$= x\left(3^k \cdot 3^{-1}\right) + 1$$

$$= x\left(3^{k-1}\right) + 1$$

$$y(k) = x(3^k)$$

$$y(k) = y(k-1) + 1$$

$$y(1) = \cancel{y(} x(3) = x(1) + 1 = 1 + 1 = 2$$

$$y(2) = x(9) = x(3) + 1 = 2 + 1 = 3$$

$$y(3) = x(27) = x(9) + 1 = 3 + 1 = 4$$

$$y(k) = k + 1$$

$$x(3^k) = k + 1$$

$n = 3^k$

$$x(n) = n + 1$$

(4) Analyze the order of growth

i) $F(n) = 2n+5$ and $g(n) = 7n$. Use the $\Omega(g(n))$ notation

Ω notation

$$F(n) \geq c \cdot g(n)$$

Applying definition

$F(n) = 2n^2+5$

$g(n) = 7n$

$$2n^2+5 \geq c \cdot 7n \text{ for all } n \geq n_0$$

Analyze

$$2n^2 \geq c \cdot 7n$$

$$2n^{\cancel{2}1} \geq 7c\cancel{n}$$

$$2n \geq 7c \text{ for } n \geq \frac{7c}{2}$$

Choosing Constants

$C = 1$

$$2n \geq 7 \cdot 1$$

$$2n \geq 7$$

$$n \geq \frac{7}{2}$$

$c = 1$ and $n_0 = 4$

$$F(n) = 2n^2+5 = \Omega(n)$$

ii) $T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + cn$

$a = 1$

$b = 3$

$f(n) = cn$

$\log_b^a = \log_3^1 = 0$

$p = 1$

Case 3: $\log_b^a < k$

$\cancel{p \geq 0, \Theta(n^k \log_n^p)}$

$\cancel{\Theta(n^1 \log}^1$

$f(n) = \Omega(n^c)$

$T(n) = \Theta(n^1)$

$= \Theta(n)$

③ Consider the following ~~recurrences~~ ~~compt~~ recursion algorithm

Min1 (A[0] ---- n-1])
if n=1 return A[0]
Else temp = Min1(A[0.... n-2])
if temp <= A[n-1] return temp
Else
Return A[n-1]

a) What does this algorithm compute?
b) Setup a recurrence relation for the algorithms basic operation count and solve it.

ⓐ The given algorithm computes the minimum value in the array.

ⓑ

For $n=1$ $T(1)=0$

For $n>1$ $T(n)=T(n-1)+1$

$n=n-1$

$$T(n-1)=T((n-1)-1)+1$$
$$=T(n-2)+1$$

$n=n-2$

$$T(n-2)=T((n-2)-1)+1$$
$$=T(n-3)+1$$

$T(n-1)+1$

$T(n-2)+1$

$T(n-3)+1$

$\vdots$

$\cancel{T(n)} = T(1)+1$

$T(n) = T(1) + (n-1) \times 1$

$T(1) = 0$

$T(n) = 0 + n-1$

$T(n) = n-1$