

Perceptron Learning - Why it works?

What is the intuition behind the Perceptron learning algorithm

1. $W = [w_1, w_2, \dots, w_n]$
2. $X = [x_1, x_2, \dots, x_n]$
3. $\cos \theta = w \cdot x / \|w\| \|x\|$, here the numerator can be replaced with $\sum w_i x_i$
 - a. The denominator is always positive
 - b. Therefore $\cos \theta \propto \sum w_i x_i$
 - c. As θ ranges from 0 to 180° , $\cos \theta$ ranges from 1 to -1
 - d. If $\cos \theta > 0$, it is an acute angle
 - e. If $\cos \theta < 0$, it is an obtuse angle
4. For $x \in P$, if $w \cdot x < 0$, then it means that the angle(α) between this x and the current w is greater than 90° , but we want $\alpha < 90^\circ$
 - a. What happens to the new angle α_{new} when $w_{\text{new}} = w + x$
 - b. $\cos \alpha_{\text{new}} \propto w_{\text{new}}^T x$
 - c. $\propto (w+x)^T x$
 - d. $\propto w^T x + x^T x$ (always +ve)
 - e. $\propto \cos \alpha + x^T x$ (some +ve value)
 - f. This means that the cosine is going to increase, which leads to decrease of α
5. For $x \in N$, if $w \cdot x > 0$, then it means that the angle(α) between this x and the current w is less than 90° , but we want $\alpha > 90^\circ$
 - a. What happens to the new angle α_{new} when $w_{\text{new}} = w - x$
 - b. $\cos \alpha_{\text{new}} \propto w_{\text{new}}^T x$
 - c. $\propto (w-x)^T x$
 - d. $\propto w^T x - x^T x$ (always +ve)
 - e. $\propto \cos \alpha - x^T x$ (some +ve value)
 - f. This means that the cosine is going to decrease, which leads to increase of α