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Purpose: **Assignment 2 – Set 2**

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Ans.
Given that, mu = 45, sigma = 8 and the probability that service manager cannot meet his commitment is P (X>50) = 1- P (X<=50) Where X is time taken to complete work.

We know that the formula for standard normal distribution is, Z = (X-mu/sigma)

Now, P (X<=50) = P (Z<= (50-45)/8)
= P (Z<= 5/8)
= P (Z<= 0.625) = 0.73237

Let us convert this probability in percentage : P (X<=50) = 0.73237 = 73.237%
Now minus the obtained percentage form 100% :100% - 73.237% = 26.763%
Now again convert this percentage in probability then we get the probability is 0.26763.
Hence, the probability that the service manager cannot meet his commitment is 0.26763.

OR

1-stats.norm.cdf(50, 45, 8)= 0.26598552904870054

Hence the option B is true.

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2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and 44.
B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans.
Given that, mu = 38, sigma = 6

A)

Here mu = 38 and sigma = 6

If we calculate the first standard deviation then, mu + sigma = 38+6 = 44

This implies that 68% of data will fall in one standard deviation.

Hence, this is FALSE.

B)
Here X = 30, mu = 38, sigma = 6

Let us find Z score.

Z = (X-mu)/sigma = 30-38/6 =-8/6 = -1.33 = 0.09176 = 9.176%

If we take 9.176% of 400 then it will approximately 36.

There-fore a training program for employee under the age of 30 at the center would be expected to attract about 36 employee is TRUE.

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3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans.

If we calculate the Moment Generating Function of X1 and X2 then we obtain that,
X1 + X2 ~ N (mu1 + mu2, (sigma1)^2+(sigma2)^2) that is addition of two random variables is normally distributed.

If X1 is normally distributed then 2X1 is also normally distributed.

That is 2X1~N (2mu,4(sigma)^2).

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3) Let X ~ N (100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8

C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Ans.

Given that, X ~ N (100,20^2) and probability of the random variable taking a value between them is 0.99.

We have to find out the values of and b. So the probability of getting value between a & b should be 0.99.

If we convert the given probability in percentage then we get 99%.

This implies that 99% of data will fall within the third standard deviation.

We know that the Imperical rule is ($\mu \pm 3\sigma$) And here μ = 100 & σ = 20 put this value in above formula

Hence, (100±3*20) = (100±60) = (100-60,100+60) = (40,160)

There-fore the option D is correct.

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4) Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans.

Given that, the distribution of annual profit from two divisions are Profit1~N (5,3^2) & Profit2~N (7,4^2).

Let us find out the mean profit from profit1 and profit2.   Mean = 5+7 = 12

We have to given that $1 = 45 rupee then 12*45 = 540

This implies that 540 million is the mean profit.

We have also given that the variance and let add these variances then we get SD.

SD = √3^2 + 4^2 = 5   and 5*45 = 225 that is 225 million is standard deviation.

A) ***Stats. norm. interval (0.95,540,225)***

Then the rupee range in million is Rs **(99.00810347848784, 980.9918965215122).**

B) Here we have to use the Z score formula that is Z = X – mu/sigma

This implies that X = Z * sigma + mu = 540 + (-1.645) * 225 = 540-370.125 = 169.875

Note that -1.645 is the 5^{th} percentile in Z table.
If we round off the value of X then it will 170.

This implies that X = 170.

C) The probability of division 1 making a loss P (X<0) is
Stats. norm .cdf (0,5,3) = 0.04779035227281147

The probability of division 2 making a loss P(X<0) is
Stats. norm. cdf(0,7,4) = 0.040059156863817086

This implies that division 1 has a probability of making a large loss in a given year.