Four-Wheel Suspension Model

Research Project Assignment
Vehicle Dynamics (MECE- 624)

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1. List of Symbols:

Parameters	**Units**	**Symbols**
Sprung Mass	Kg	Ms
Unsprung Mass	Kg	Mus
Rolling Axis moment of inertia	Kgm^2	Iɸ
Pitch axis moment of inertia	Kgm^2	IƟ
Front suspension spring stiffness	N/m	Kf1, Kf2
Rear suspension spring stiffness	N/m	Kr3, Kr4
Non-linear coefficient of damping	N/m	Csh_1,Csh_2,Csh_3,Csh_4
Damping coefficient of front suspension	N/m	Csh1, Csh2
Damping coefficient of rear suspension	N/m	Csh3, Csh4
Tire spring stiffness	N/m	Kt1, Kt2, Kt3, Kt4
Damping coefficient of tire	N/m	Ct1, Ct2, Ct3, Ct4
Length from CG to front wheel	m	a
Length from CG to rear wheel	m	b
Track length for front & Rear	m	Tf, Tr
Vehicle displacement in Z-direction under bounce	m	Z
Vehicle velocity in Z- direction under bounce	m/s	$\dot{Z}$
Vehicle acceleration in Z-direction under bounce	m/s^2	$\ddot{Z}$
Pitch Angle	rad	θ
Pitch Rate	rad/s	$\dot{\theta}$
Roll Angle	rad	ϕ
Roll Rate	rad/s	$\dot{\phi}$

2. Research Project Objectives:

- Develop a higher degree of freedom model for one or more vehicle dynamic scenarios.
- Numerically simulate the vehicle behavior using the model.
- Develop a four wheel vehicle suspension model which accounts for realistic suspension. Geometry, spring and damper coefficients, unsprung mass, etc.
- Research or develop necessary parameters for a vehicle.
- Present results from the numerical simulation of your model for a step impulse.
- Present results of a parametric study on at least one parameter using the numerical simulation of the model.

3. Introduction:

The ride quality of the car is determined by the sensation of the passenger in the environment of passenger. To improve the ride quality of the car is the function of the car suspension. Steering stability with good handling and maximize the friction between the tires and road surface are the important responsibilities of a good suspension system of any vehicle. Ride comfort problems mainly arise from the vibrations of the vehicle body, which are caused by various sources. Sources include surface irregularities, aerodynamic forces, vibration of the engine and driveline and non-uniformities of the tire/wheel assembly. If the roads were perfectly flat then there won't be any need for suspension system. But for even a newly paved road, there are few imperfections which interact with the car.

Figure 1 Effect of bump on the vehicle

A bump on the road causes the wheel to move up and down and it is perpendicular to the surface of the road. If there is no suspension in the vehicle, then due to this bump the wheel movement will be transferred to the frame in the upward direction. As a result of this movement the wheels may lose contact with the ground surface. Due to which entire frame and wheel assembly will fall down at the same time. Suspension if present would have avoided this vibrational transfer from wheels to the frame. It is primary function of the suspension system to control this vibrations of the vehicle so that the passenger's sensation of discomfort does not exceed a certain level.

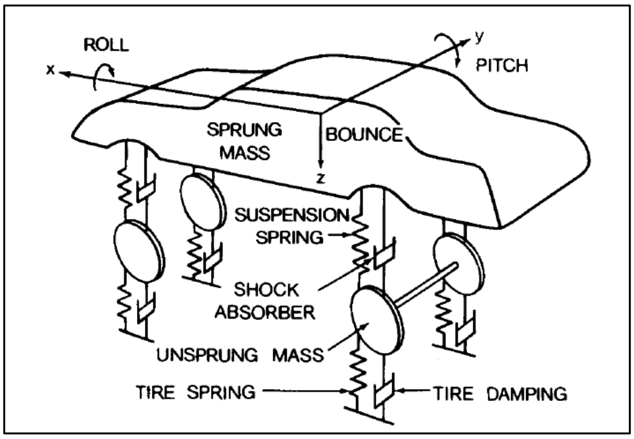


Figure 2 Seven degrees of freedom of a ride model of a passenger car

Suspension systems are classified into types as passive, semi active and active systems.

Passive suspension system can be found in controlling the dynamics of vertical motion of a vehicle. There is no energy supplied by the suspension element to the system. Even though it doesn't apply energy to the system, but it controls the relative motion of the body to the wheel by using different types of damping or energy dissipating elements. Passive suspension has significant limitation in structural applications. The characteristic are determined by the designer according to the design goals and the intended application. The disadvantage of passive suspension system is it has fix characteristic, for example if the designer design the suspension heavily damped it will only give good vehicle handling but at the same time it transfer road input (disturbance) to the vehicle body.

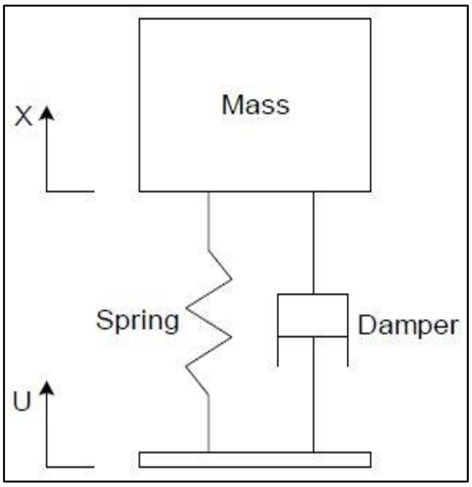


Figure 3 Passive Suspension

Semi active suspension system provides a rapid change in rate of springs damping coefficients. It does not provide any energy into suspension system but the damper is replaced by controllable damper. The controller's determine the level of damping based on control strategy and automatically adjust the damper to the desired levels. This type of suspension system used external power to operate. Sensors and actuator are added to detect the road profile for control input. The most commonly semi-active suspension system is called skyhook damper.

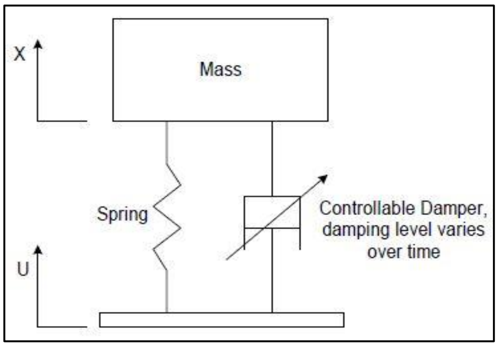


Figure 5 Semi Active suspension

Active suspension system has the ability to response to the vertical changes in the road input. The damper or spring is interceding by the force actuator. This force actuator has own task which is to add or dissipate energy from the system. The force actuator is control by various types of controller determine by the designer. The correct control strategy will give better compromise between comfort and vehicle stability. Therefore active suspension system offer better riding comfort and vehicle handling to the passengers. In this type of suspension the controller can modify the system dynamics by activating the actuators

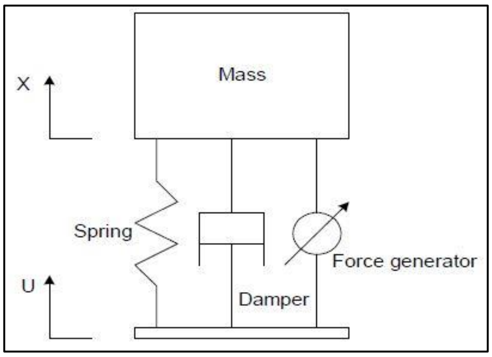


Figure 4 Active Suspension

Considering the scope of the project we have selected to design a four wheel full car model of a passive suspension system and to implement a non linear parameter which would be nonlinear damping.

4. Mathematical Model: -

a. Quarter Car model: -

The purpose of mathematical modeling in this project is to obtain a state space representation of the quarter car model. In this project the suspension system is modeled as a non-linear suspension system. The state variable can be represented as a vertical movement of the car body and a vertical movement of the wheels.

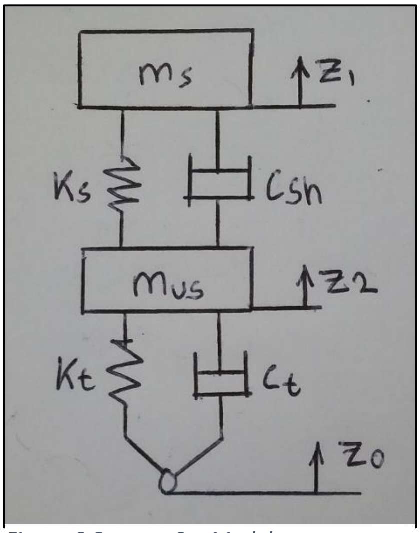


Figure 6 Quarter Car Model

Z- Direction sprung mass equation:

$$m_s\ddot{Z}_s = -C_{sh1}(Z_{S1}-Z_{s2}) - K_s(Z_{S1}-Z_{u1})$$

Z Direction un-sprung mass equation:

Unsprung mass = $m_{us}\ddot{Z}_{u1} + C_{sh1}(\dot{Z}_{s1}-\dot{Z}_{u1}) + K_{f1}(Z_{S1}-Z_{u1}) + C_{t1}(\dot{Z}_0-\dot{Z}_{u1}) + K_{t1}(Z_0-Z_{u1})$ = F (t)

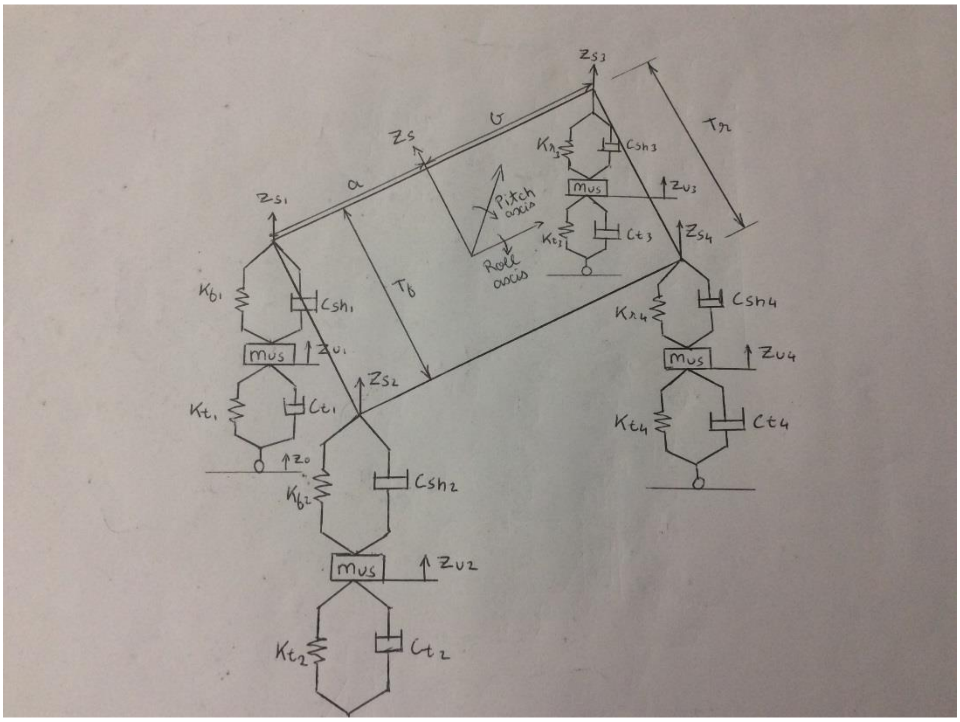


Figure 7 Full Car Model

The purpose of mathematical modeling in this project is to obtain a state space representation of the full car model. In this project the suspension system is modeled as a non-linear suspension system. The state variable can be represented as a vertical movement of the car body and a vertical movement of the wheels.

<u>Bounce Equation of Motion:-</u>

$$m_s\ddot{Z}_s = -C_{sh1}(\dot{Z}_{s1}-\dot{Z}_{u1}) - C_{sh2}(\dot{Z}_{s2}-\dot{Z}_{u2}) - C_{sh3}(\dot{Z}_{s3}-\dot{Z}_{u3}) - C_{sh4}(\dot{Z}_{s4}-\dot{Z}_{u4}) - K_{f1}(Z_{S1}-Z_{u1}) - K_{f2}(Z_{S2}-Z_{u2}) - K_{r3}(Z_{S3}-Z_{u3}) - K_{r4}(Z_{S4}-Z_{u4})$$

<u>Pitch Equation of Motion: -</u>

$$\mathrm{I_p}\ddot{\theta}_s = -C_{sh1}a(\dot{Z}_{s1}-\dot{Z}_{u1}) - C_{sh2}a(\dot{Z}_{s2}-\dot{Z}_{u2}) + C_{sh3}b(\dot{Z}_{s3}-\dot{Z}_{u3}) + C_{sh4}b(\dot{Z}_{s4}-\dot{Z}_{u4}) - K_{f1}a(Z_{S1}-Z_{u1}) - K_{f2}a(Z_{S2}-Z_{u2}) + K_{r3}b(Z_{S3}-Z_{u3}) + K_{r4}b(Z_{S4}-Z_{u4})$$

<u>Roll Equation of Motion: -</u>

$$I_r\ddot{\emptyset}_s = -C_{sh1}T_f(\dot{Z}_{s1}-\dot{Z}_{u1}) + C_{sh2}T_f(\dot{Z}_{s2}-\dot{Z}_{u2}) - C_{sh3}T_r(\dot{Z}_{s3}-\dot{Z}_{u3}) + C_{sh4}T_r(\dot{Z}_{s4}-\dot{Z}_{u4}) - K_{f1}T_f(Z_{S1}-Z_{u1}) + K_{f2}T_f(Z_{S2}-Z_{u2}) - K_{r3}T_r(Z_{S3}-Z_{u3}) + K_{r4}T_r(Z_{S4}-Z_{u4})$$

<u>Equation of Motion for Unsprung Mass: -</u>

- $m_{us}\ddot{Z}_{u1} = C_{sh1}(\dot{Z}_{s1}-\dot{Z}_{u1}) + K_{f1}(Z_{S1}-Z_{u1}) + C_{t1}(\dot{Z}_0-\dot{Z}_{u1}) + K_{t1}(Z_0-Z_{u1})$
- $m_{us}\ddot{Z}_{u2} = C_{sh2}(\dot{Z}_{s2}-\dot{Z}_{u2}) + K_{f2}(Z_{S2}-Z_{u2}) + C_{t2}(\dot{Z}_0-\dot{Z}_{u2}) + K_{t2}(Z_0-Z_{u2})$
- $m_{us}\ddot{Z}_{u3} = C_{sh3}(\dot{Z}_{s3}-\dot{Z}_{u3}) + K_{r3}(Z_{S3}-Z_{u3}) + C_{t3}(\dot{Z}_0-\dot{Z}_{u3}) + K_{t3}(Z_0-Z_{u3})$
- $m_{us}\ddot{Z}_{u4} = C_{sh4}(\dot{Z}_{s4}-\dot{Z}_{u4}) + K_{r4}(Z_{S4}-Z_{u4}) + C_{t4}(\dot{Z}_0-\dot{Z}_{u4}) + K_{t4}(Z_0-Z_{u4})$

<u>Displacement of Each Sprung Mass: -</u>

- $Z_{S1} = T_f\emptyset_s + a\theta_s + Z_s$
- $Z_{S2} = -T_f\emptyset_s + a\theta_s + Z_s$
- $Z_{S3} = T_r\emptyset_s - b\theta_s + Z_s$
- $Z_{S4} = -T_f\emptyset_s - b\theta_s + Z_s$

5. Simulink models:

a. Quarter Car model

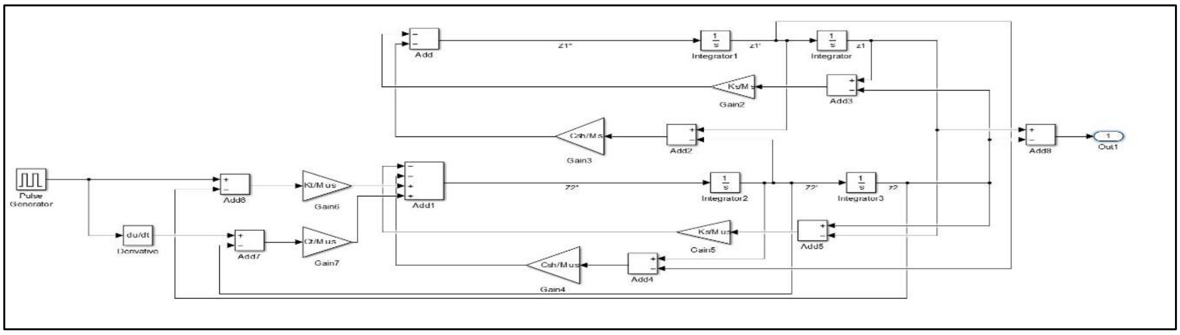
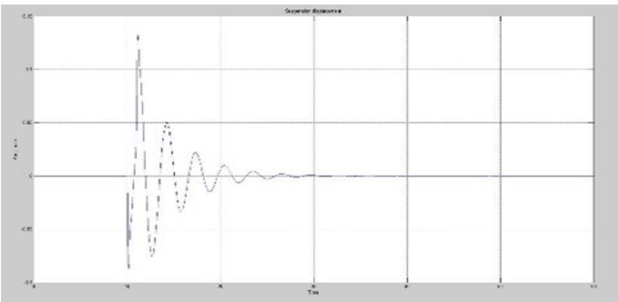


Figure 8 Quarter Car model



Graph 1 Suspension displacement plot for quarter car

b. Full Car Model

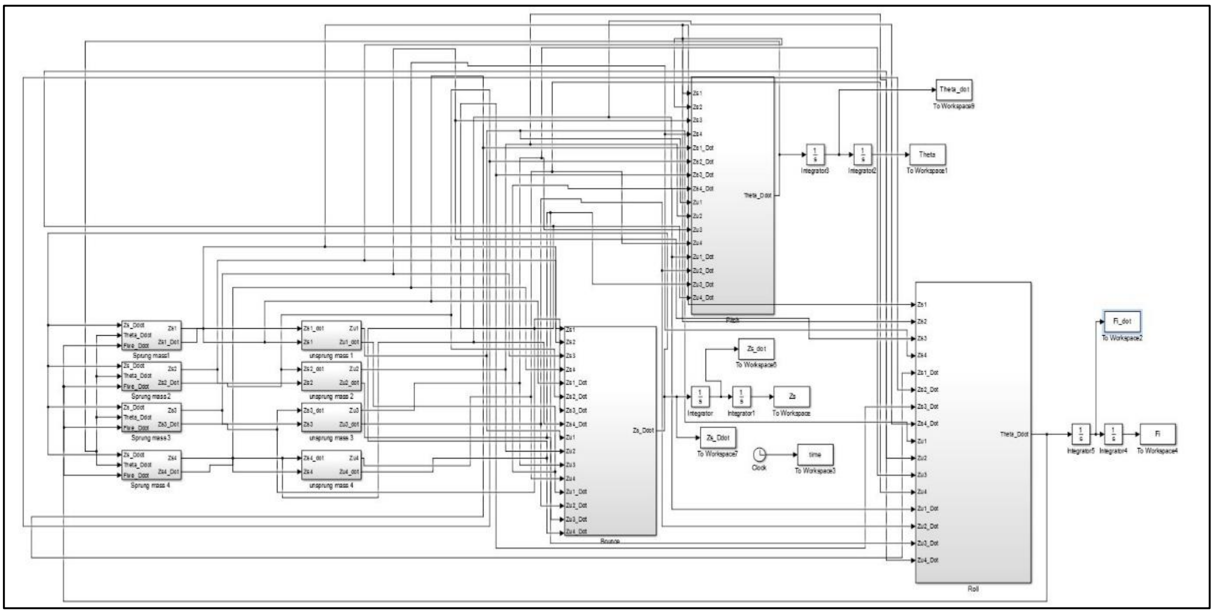


Figure 9 Full Car Simulink Model

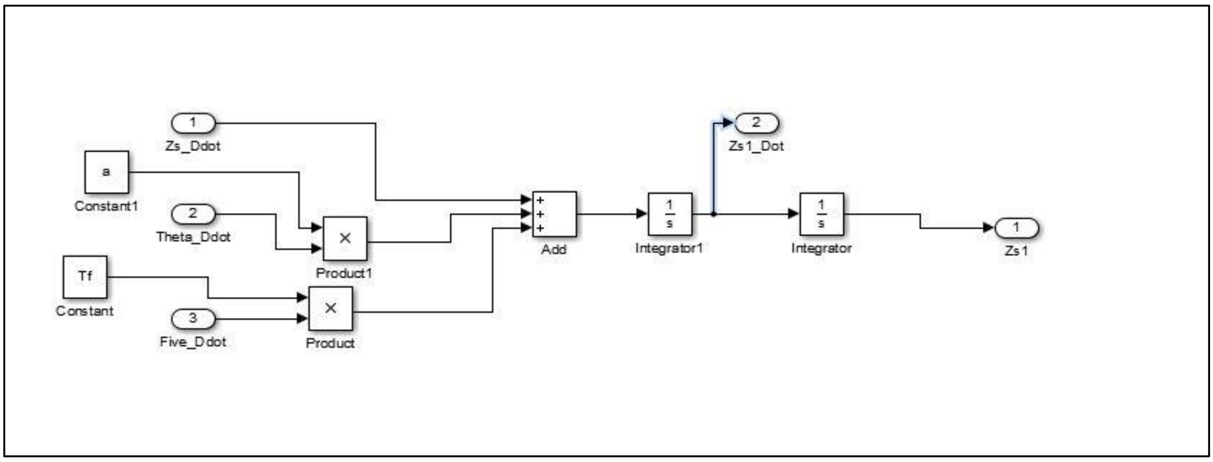


Figure 10 Sprung mass system

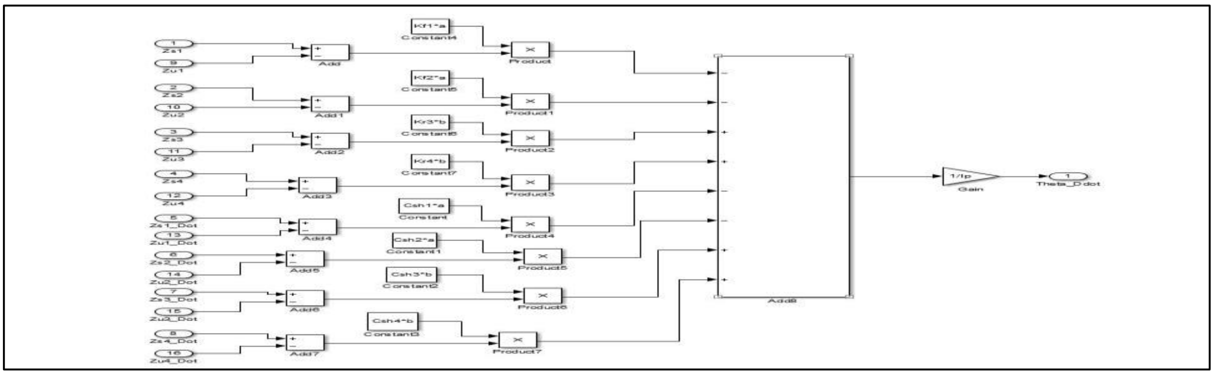


Figure 11 Pitch subsystem

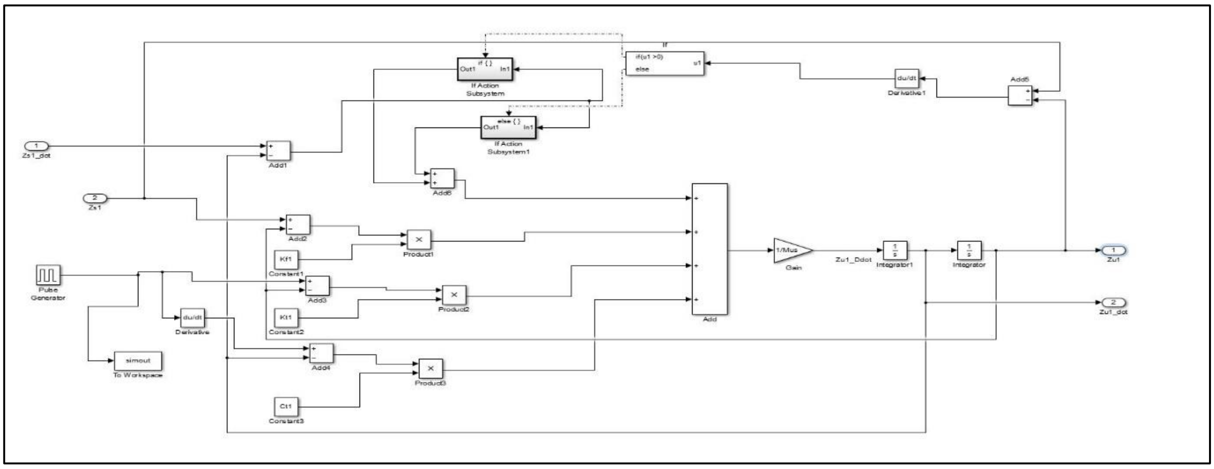


Figure 12 Unsprung mass system

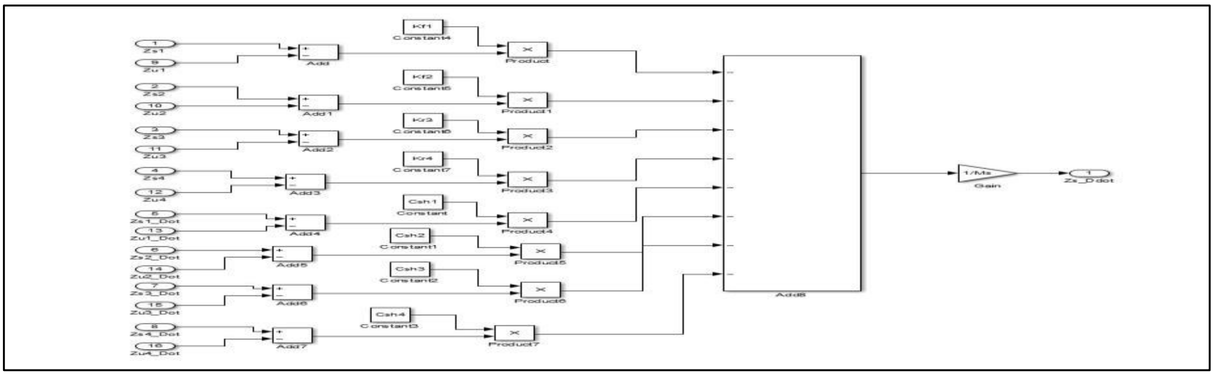


Figure 13 Bounce Subsystem

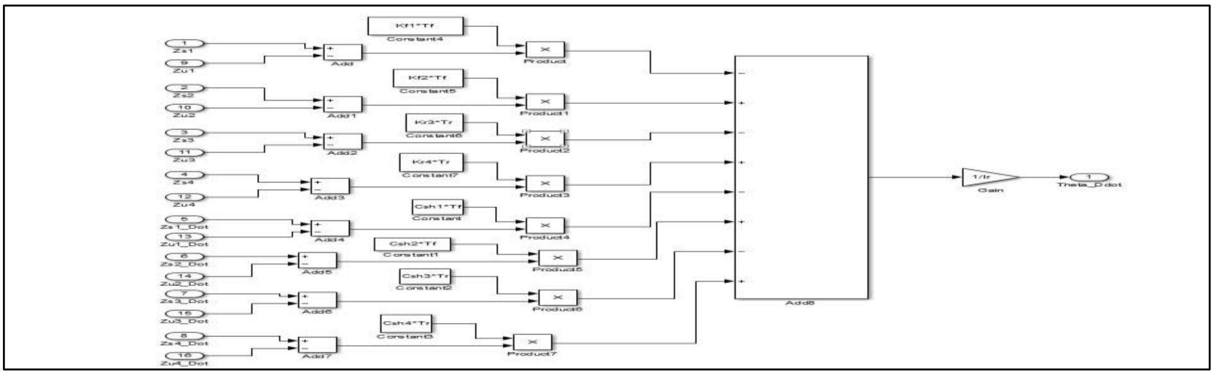


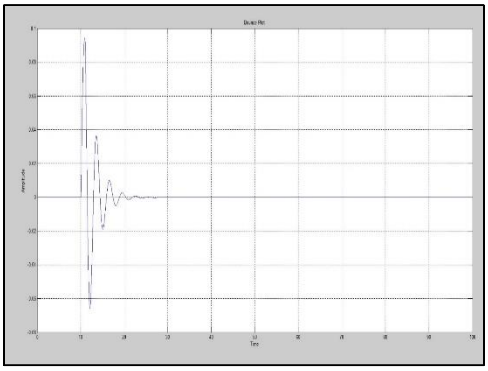
Figure 12 Roll Subsystem

6. Design Inputs:

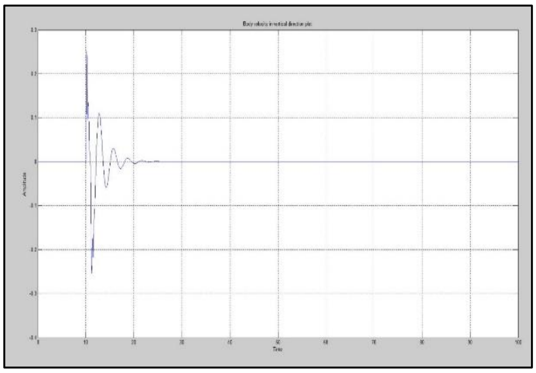
Parameters	**Value**
Sprung Mass (Kg)	1000
Unsprung Mass (Kg)	60
Rolling Axis moment of inertia (Kgm^2)	400
Pitch axis moment of inertia (Kgm^2)	2000
Front suspension spring stiffness (N/m)	10000
Rear suspension spring stiffness (N/m)	6000
Non-linear coefficient of damping (N/m)	100
Damping coefficient of front suspension (N/m)	5000
Damping coefficient of rear suspension (N/m)	4000
Tire spring stiffness (N/m)	190000
Damping coefficient of tire (N/m)	10
Length from CG to front wheel (m)	1.15
Length from CG to rear wheel (m)	1.65
Track length for front & Rear (m)	1.5

7. Results: -

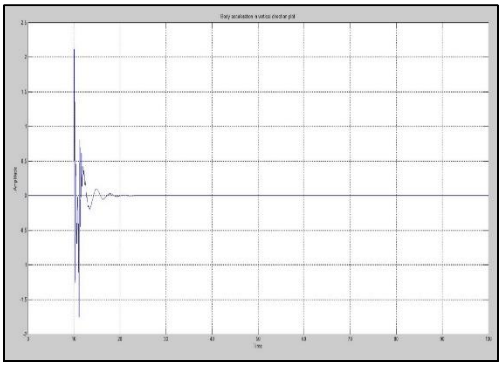
This section covers the non-linear passive suspension system's performance as previous sections. Simulation based on the mathematical model for a full car model by using MATLAB/SIMULINK software was performed. Performances of the suspension system in term of ride quality and car handling was observed, where road disturbance is assumed as the input for the system. Parameters that are observed are the vertical displacement of car body, velocity and acceleration body, pitch and roll of the car body.



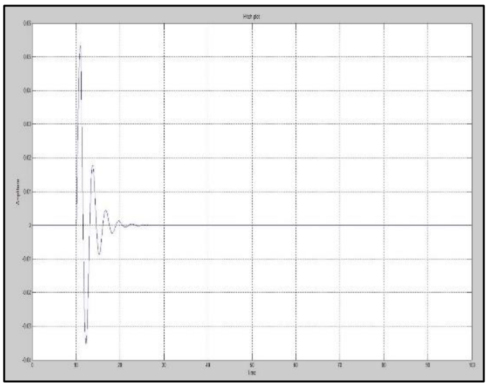
Graph 2. Bounce Plot



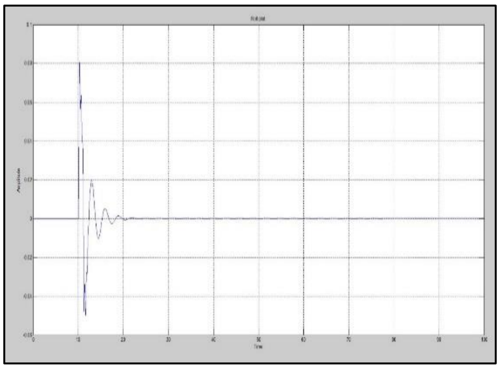
Graph 3 Body velocity plot in vertical direction



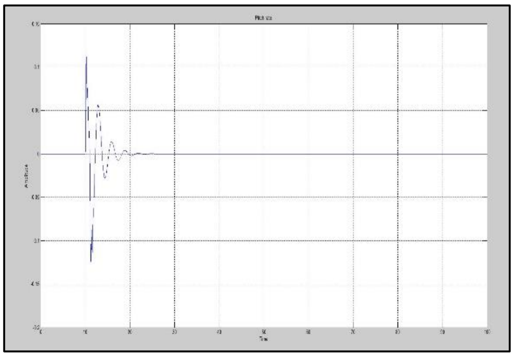
Graph 4 Body acceleration in vertical direction



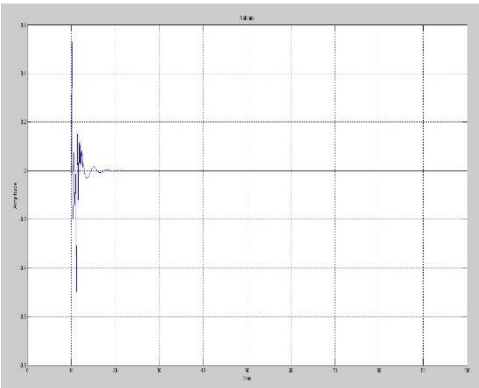
Graph 5 Pitch plot using non-linear damping



Graph 6 Roll plot using non-linear damping



Graph 7 Pitch Rate using non-linear damping

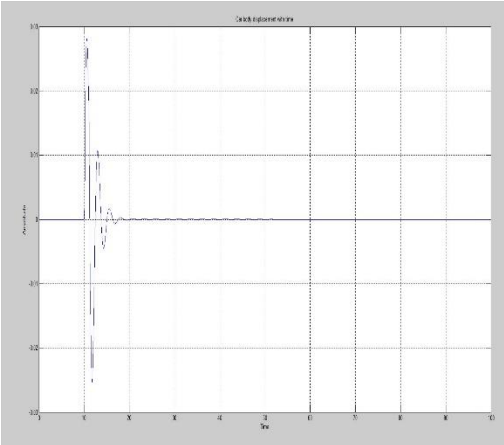


Graph 8 Roll Rate using non-linear damping

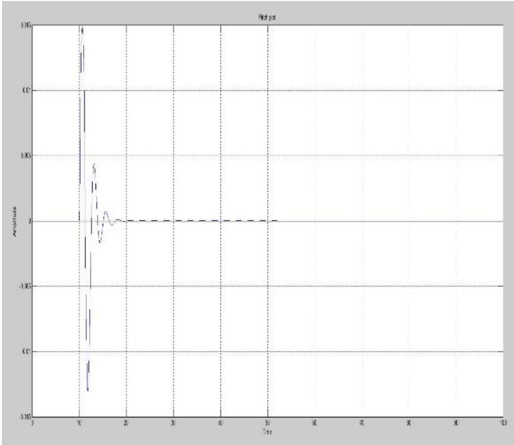
8. Parametric Study

In this we have changed the parameters of the input function to study the behaviors of the suspension system under different condition. The step input is given 5 cm. Some of the observation during the study are as below:-

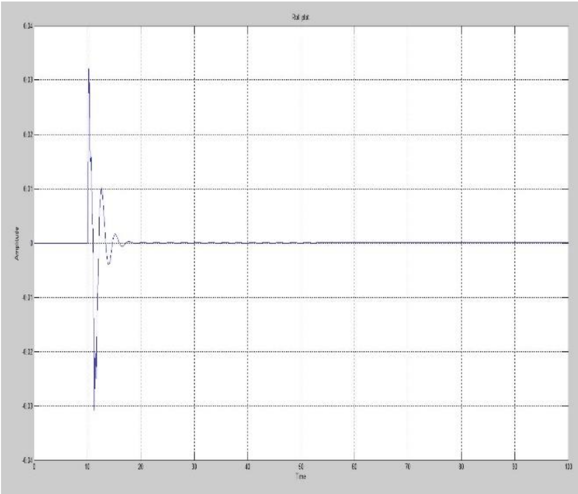
1. Reduced amplitude of oscillation
2. Reduced settling time



Graph 9 Bounce Plot



Graph 10 Pitch Plot



Graph 11 Roll plot

9. Conclusion:-

The objectives of this project have been achieved. Dynamic model for linear full car suspensions systems has been formulated and derived. From computer simulations it shows that the output performance for full car model using non-linear damper is stable. It also has less settling time and amplitude of oscillation compared to linear passive suspension system.

10. Reference:-

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