

# Solutions: Chapter 11

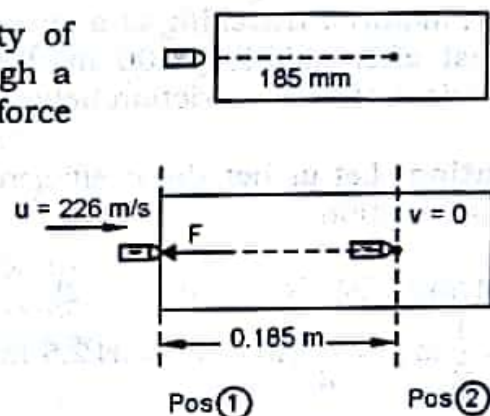
## Kinetics of Particles:

### Work Energy Method

#### Exercise 11.1 Work Energy Principle

**P1.** A bullet of mass 30 gm moving with a velocity of 226 m/s strikes a wooden log and penetrates through a distance of 185 mm. Calculate the average retarding force offered by the log in stopping the bullet.

**Solution:** Let  $F$  be the force of resistance



Applying W E P pos (1) to pos (2).

$$T_1 = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.03 \times 226^2 = 766.14 \text{ J}$$

$T_2 = 0$  .....since bullet comes to rest.

$$U_{1-2} \quad 1) \text{ Work by force} = F \times s \\ = -F \times 0.185 \\ = -0.185 F \text{ J}$$

Between pos (1) and pos (2) only external resistance force  $F$  does work.

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

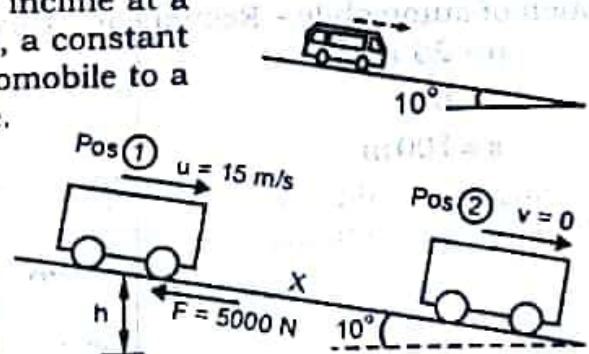
$$766.14 + [-0.185 F] = 0$$

$$\therefore F = 4141.3 \text{ N} \quad \text{..... Ans.}$$

**P2.** A 1200 kg automobile is driven down a  $10^\circ$  incline at a speed of 54 kmph. When the brakes are applied, a constant braking force of 5000 N acts and brings the automobile to a halt within a certain distance. Find this distance.

**Solution:** Let the automobile travel  $x$  metres before it comes to rest.

Applying W E P pos (1) to pos (2).



$$T_1 = \frac{1}{2} mv^2 = \frac{1}{2} \times 1200 \times 15^2 = 135000 \text{ J}$$

$$T_2 = 0 \text{ .....since the automobile comes to rest.}$$

$$U_{1-2} \quad 1) \text{ Work by Weight force} = + mgh \\ = 1200 \times 9.81 \times (x \sin 10) = 2044.2x \text{ J}$$

$$2) \text{ Work by braking force} = F \times s \\ = -5000 \times x = -5000x \text{ J}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$135000 + [2044.2x - 5000x] = 0$$

$$\therefore x = 45.673 \text{ m} \text{ ..... Ans.}$$

Between pos (1) and pos (2), weight and external braking force do work.

Work by weight is +ve since pos (2) is below pos (1)

**P3.** A motorist travelling at a speed of 90 kmph suddenly applies the brakes and comes to rest after skidding 100 m. Determine the time required for the car to stop and coefficient of kinetic friction between the tires and the road. (M.U. May 14)

**Solution:** Let  $\mu_k$  be the coefficient of kinetic friction.

Applying WEP Pos (1) to Pos (2)

$$T_1 = \frac{1}{2} mv^2 = \frac{1}{2} \times m \times 25^2 = 312.5 m \text{ J}$$

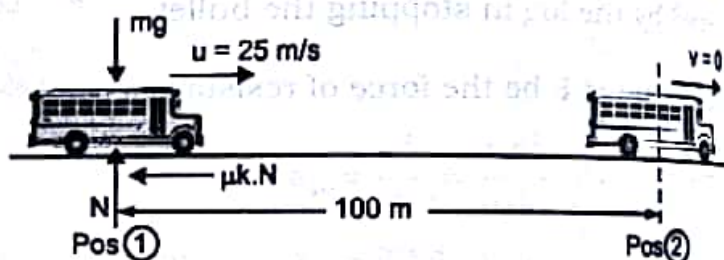
$$T_2 = 0 \text{ ..... Since the automobile comes to rest.}$$

$$U_{1-2} \quad 1) \text{ by friction} = -\mu_k \cdot N \cdot S \\ = -\mu_k \times (m \times 9.81) \times 100 \\ = -981 \mu_k \times m$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$312.5m + [-981\mu_k \times m] = 0$$

$$\text{Or } \mu_k = 0.3185 \text{ ..... Ans.}$$



Between Pos (1) and Pos (2) only friction force does work.

$$N = mg \\ = m \times 9.81$$

### Kinematics

Let  $t$  be the time taken by the automobile to come to rest.

Motion of automobile - Rectilinear - Uniform acceleration

$$u = 25 \text{ m/s}$$

$$v = 0$$

$$s = 100 \text{ m}$$

$$a = a$$

$$t = t$$

$$\text{Using } v^2 = u^2 + 2as$$

$$0 = 25^2 + 2 \times a \times 100$$

$$\text{or } a = -3.125 \text{ m/s}^2$$

$$\text{Using } v = u + at$$

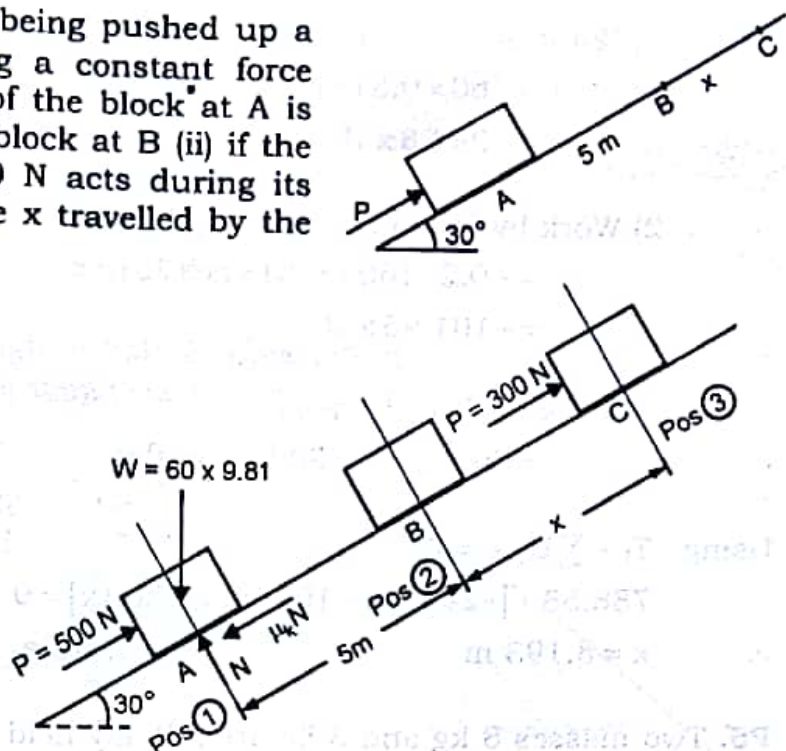
$$0 = 25 - 3.125 \times t$$

$$\text{or } t = 8 \text{ sec ..... Ans.}$$



**P4.** A block of mass 60 kg at A is being pushed up a inclined plane ( $\mu = 0.2$ ) by applying a constant force  $P = 500$  N. knowing that the speed of the block at A is 3 m/s, determine i) the speed of the block at B (ii) if the force  $P$  is now reduced and  $P = 300$  N acts during its motion from B to C, find the distance  $x$  travelled by the block as it comes to a halt at C.

**Solution:** A) Let the speed of the block at position B be  $v$  m/s.



Applying W E P pos (1) to pos (2).

$$T_1 = \frac{1}{2} mv^2 = \frac{1}{2} \times 60 \times 3^2 = 270 \text{ J}$$

$$T_2 = \frac{1}{2} mv^2 = \frac{1}{2} \times 60 \times v^2 = 30 v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = -mgh \\ = -60 \times 9.81 \times (5 \sin 30) \\ = -1471.5 \text{ J}$$

$$2) \text{ Work by friction force} = -\mu_k \cdot N \cdot s \\ = -0.2 \times (60 \times 9.81 \times \cos 30) \times 5 \\ = -509.74 \text{ J}$$

$$3) \text{ Work by force} = F \times s = 500 \times 5 = 2500 \text{ J}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$270 + [-1471.5 - 509.74 + 2500] = 30 v^2$$

$$\therefore v = 5.127 \text{ m/s}$$

..... **Ans.**

B) Let the block travel  $x$  metres from B to C

Applying W E P pos (2) to pos (3).

$$T_2 = \frac{1}{2} mv^2 = \frac{1}{2} \times 60 \times 5.127^2 = 788.58 \text{ J}$$

$$T_3 = 0 \text{ ..... since block comes to rest}$$

Between pos (1) and pos (2), weight, friction and external force  $P$  do work.

Work by weight is -ve since pos (2) is above pos (1)

$$N = mg \cos \theta$$

Between pos (2) and pos (3), weight, friction and external force  $P$  do work.

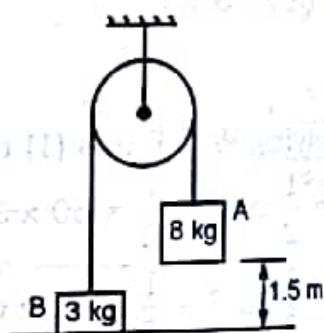
$$\begin{aligned}
 U_{2-3} \quad 1) \text{ Work by weight force} &= -mgh \\
 &= -60 \times 9.81 \times (x \sin 30) \\
 &= -294.3x \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 2) \text{ Work by friction force} &= -\mu_k \cdot N \cdot s \\
 &= -0.2 \times (60 \times 9.81 \times \cos 30) \times x \\
 &= -101.95x \text{ J}
 \end{aligned}$$

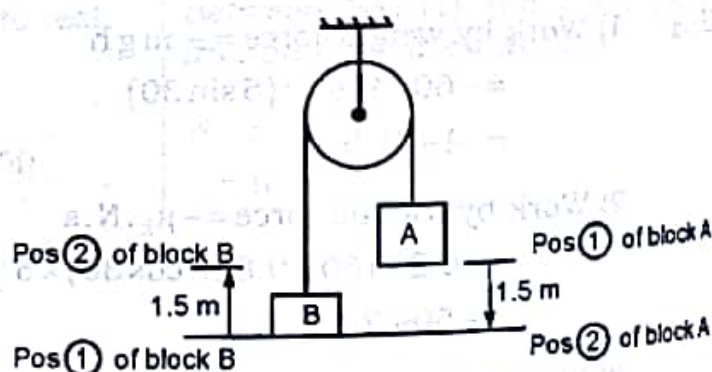
$$\begin{aligned}
 3) \text{ Work by force } P &= F \times s \\
 &= 300 \times x = 300x \text{ Joules}
 \end{aligned}$$

$$\begin{aligned}
 \text{Using } T_1 + \sum U_{1-2} &= T_2 \\
 788.58 + [-294.3x - 101.95x + 300x] &= 0 \\
 \therefore x &= 8.193 \text{ m} \quad \dots \dots \text{Ans.}
 \end{aligned}$$

**P5.** Two masses 8 kg and 3 kg are initially held at rest in the position shown. Determine the speed of the 8 kg block as it hits the ground. Neglect friction at the pulley.



**Solution:** As the 8 kg block A travels 1.5 m to strike the ground, the 3 kg block B travels 1.5 m up. At any instant both will have the same velocity. Let  $v$  be the velocity of block A at it hits the ground.



Applying W E P pos (1) to pos (2) to the system of both blocks.

$T_1 = 0$  ..... since both blocks are at rest

$$\begin{aligned}
 T_2 &= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\
 &= \frac{1}{2} \times 8 \times v^2 + \frac{1}{2} \times 3 \times v^2 = 5.5v^2 \text{ J}
 \end{aligned}$$

$$v_A = v_B = v$$

$$\begin{aligned}
 U_{1-2} \quad 1) \text{ Work by weight of block A} &= +mgh \\
 &= 8 \times 9.81 \times 1.5 = 117.72 \text{ J}
 \end{aligned}$$

Work by weight of block A is +ve since pos (2) is below pos (1)



$$\begin{aligned} 2) \text{ Work by block B} &= -mgh \\ &= -3 \times 9.81 \times 1.5 = -44.145 \text{ J} \end{aligned}$$

Work by weight of block B is - ve since pos (2) is above pos (1)

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

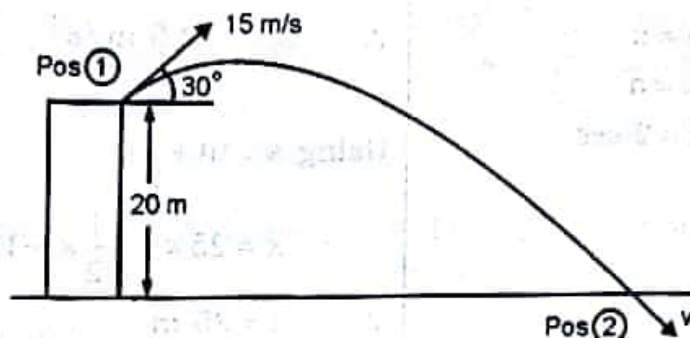
$$0 + [117.72 - 44.145] = 5.5 v^2$$

$$\therefore v = 3.657 \text{ m/s}$$

..... Ans.

**P6.** From the top of a building 20 m high, a ball is projected at 15 m/s at an angle of  $30^\circ$  upwards to the horizontal. Find the magnitude of the velocity of the ball as it hits the ground. Use Work Energy Principle.

**Solution:** Let  $m$  be the mass of the ball.  
Let  $v$  be the velocity of the ball as it strikes the ground.



Applying W E P pos (1) to pos (2).

$$T_1 = \frac{1}{2} mv^2 = \frac{1}{2} \times m \times 15^2 = 112.5 \text{ J}$$

$$T_2 = \frac{1}{2} mv^2$$

$$\begin{aligned} U_{1-2} \quad 1) \text{ Work by weight force} &= + mgh \\ &= m \times 9.81 \times 20 \\ &= 196.2 \text{ m J} \end{aligned}$$

Between pos (1) and pos (2), only weight force does work.

Work by weight is + ve since pos (2) is below pos (1)

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$112.5m + [196.2 \text{ m}] = \frac{1}{2} \times m \times v^2$$

$$\therefore v = 24.846 \text{ m/s}$$

..... Ans.

**Note:** Try solving the problem using projectile motion analysis, to get the same answer.

DJC

**P7.** At a certain instant a body of mass 15 kg is falling vertically down at a speed of 25 m/s. What upward vertical force will stop the body in 2 seconds? (M.U May 08)

**Solution:** Let the body travel  $x$  metres in the 2 sec interval, before coming to a halt. Let  $F$  be the force necessary to bring the body to a halt. Note that the body is not in MUG (Motion Under Gravity) since it is not a free fall motion.

Kinematics - Motion of block - Rectilinear -

Uniform acceleration

$$u = 25 \text{ m/s}$$

$$v = 0$$

$$s = x$$

$$a = a$$

$$t = 2 \text{ sec}$$

$$\text{Using } v = u + at$$

$$0 = 25 + a \times 2$$

$$\therefore a = -12.5 \text{ m/s}^2$$

$$\text{Using } s = ut + \frac{1}{2}at^2$$

$$x = 25 \times 2 + \frac{1}{2} \times (-12.5) \times 2^2$$

$$\therefore x = 25 \text{ m}$$

Applying W E P pos (1) to pos (2).

$$T_1 = \frac{1}{2} mv^2 = \frac{1}{2} \times 15 \times 25^2 = 4687.5 \text{ J}$$

$$T_2 = 0 \text{ ..... since both blocks are at rest}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = + mgh$$

$$= 15 \times 9.81 \times x$$

$$= 15 \times 9.81 \times 25$$

$$= 3678.75 \text{ J}$$

$$2) \text{ Work by force} = F \times s$$

$$= -F \times x$$

$$= -F \times 25$$

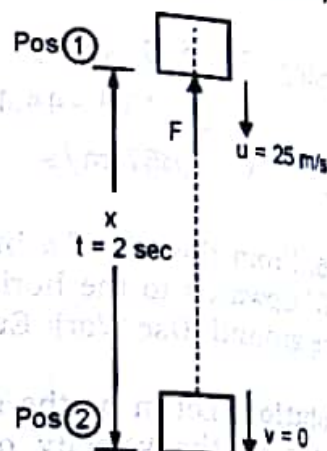
$$= -25F \text{ J}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$4687.5 + [3678.75 - 25F] = 0$$

$$\therefore F = 334.65 \text{ N}$$

..... **Ans.**



Between pos (1) and pos (2), weight and external force  $F$  do work.

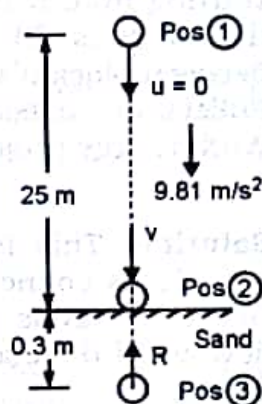
Work by weight is + ve since pos (2) is below pos (1)

Work by force is - ve since it acts opposite to displacement.



**P8.** A stone weighing 15 N dropped from a height of 25 m buries itself 300 mm deep in the sand. Find the average resistance to penetration and the time of penetration.

**Solution:** The stone starts from rest from position (1) and hits the sand at position (2) with a certain velocity  $v$ . The stone now penetrates through the sand for 300 mm and comes to rest at position (3) as shown.



**Kinematics - Motion of stone from pos (1) to pos (2) - M U G ↓ +ve**

$$u = 0$$

$$v = v$$

$$s = 25 \text{ m}$$

$$a = 9.81 \text{ m/s}^2$$

$$t = -$$

$$\text{Using } v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.81 \times 25$$

$$\therefore v = 22.147 \text{ m/s} \quad \dots \text{velocity of stone as it strikes the sand.}$$

**M<sup>o</sup> of stone through the sand, from pos(2) to pos(3) - Rectilinear - Uniform acceleration.**

$$u = 22.147 \text{ m/s}$$

$$v = 0$$

$$s = 0.3 \text{ m}$$

$$a = a$$

$$t = t$$

$$\text{Using } v^2 = u^2 + 2as$$

$$0 = 22.147^2 + 2 \times a \times 0.3$$

$$\therefore a = -817.5 \text{ m/s}^2$$

$$\text{Using } v = u + at$$

$$0 = 22.147 + (-817.5) \times t$$

$$\therefore t = 0.0271 \text{ sec} \quad \dots \text{Ans.}$$

**Kinetics - Applying W E P pos (2) to pos (3).**

$$T_2 = \frac{1}{2} mv^2 = \frac{1}{2} \times \frac{15}{9.81} \times 22.147^2 = 375 \text{ J}$$

$$T_3 = 0 \quad \dots \text{since it comes to rest}$$

$$U_{2-3} \quad 1) \text{ Work by weight force} = + mgh$$

$$= 15 \times 0.3$$

$$= 4.5 \text{ J}$$

$$2) \text{ Work by force of resistance } R = F \times s$$

$$= -R \times 0.3$$

$$= -0.3R \text{ J}$$

Between pos (2) and pos (3), weight and external force  $R$  do work.

Work by weight is +ve since pos (3) is below pos (2)

Work by force is -ve since it acts opposite to displacement.

$$\text{Using } T_2 + \sum U_{2-3} = T_3$$

$$375 + [4.5 - 0.3R] = 0$$

$$\therefore R = 1265 \text{ N}$$

$\dots \text{Ans.}$

**P9.** Find the velocity of block A and B when A has traveled 1.2 m up the inclined plane starting from rest. Mass of A is 10 kg and that of B is 50 kg. Coefficient of friction between block A and inclined plane is 0.25. Pulleys are massless and frictionless. Use Work energy principle. (M.U. Dec 07)

**Solution:** This is a dependent system of two blocks connected by a common string. Block B travels under its weight in the downward direction pulling block A up the slope.

Using CSLM, we get  $v_A = 2v_B$  ..... (1)  
[since 1 portion of string holds A, while 2 portions of the same string holds B]

The above equation (1) also implies that, if A travels 1.2 m up the slope, B travels 0.6 m vertically down.

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since both blocks are at rest

$$\begin{aligned} T_2 &= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\ &= \frac{1}{2} \times 10 \times (2v_B)^2 + \frac{1}{2} \times 50 \times v_B^2 \\ &= 45 v_B^2 \text{ J} \end{aligned}$$

$$U_{1-2} \quad 1) \text{ Work by weight of block A} = -mgh \\ = -10 \times 9.81 \times (1.2 \sin 40) = -75.67 \text{ J}$$

$$\begin{aligned} 2) \text{ Work by weight of block B} &= +mgh \\ &= +50 \times 9.81 \times 0.6 = 294.3 \text{ J} \end{aligned}$$

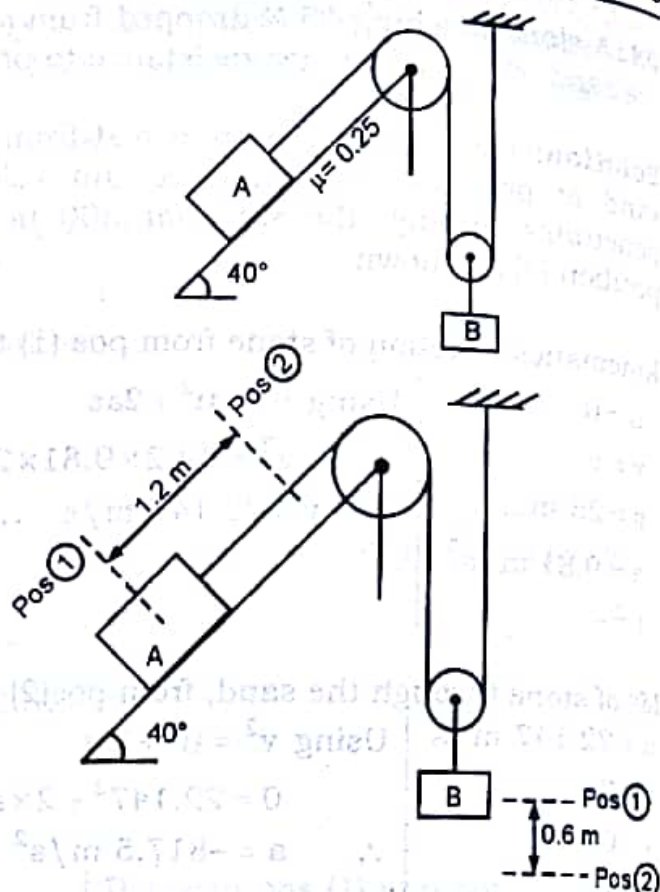
$$\begin{aligned} 3) \text{ Work by friction force} &= -\mu_k \cdot N \cdot s \\ &= -0.25 \times (10 \times 9.81 \times \cos 40) \times 1.2 \\ &= -22.54 \text{ J} \end{aligned}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$0 + [-75.67 + 294.3 - 22.54] = 45 v_B^2$$

$$\therefore v_B = 2.087 \text{ m/s}$$

$$\text{also } v_A = 2v_B = 4.175 \text{ m/s}$$



Between pos (1) and pos (2), weight and friction force do work.

Work by weight is - ve since for block A pos (2) is above pos (1)

Work by weight is + ve since for block B, pos (2) is below pos (1)

$$N = w \cos \theta = mg \cos \theta$$

..... Ans.

..... Ans.



**P10.** The 3 kg smooth collar is attached to a spring of spring constant,  $k = 6 \text{ N/m}$  that has an unstretched length 2.8 m. Determine its speed at A, when it is drawn to point B and released from rest.

**Solution:** For the moving collar, let point B be marked as pos (1) and point A as pos (2).  
Let  $v$  be the speed of collar as it passes point A [pos (2)].

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since collar starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 3 \times v^2 = 1.5 v^2 \text{ J}$$

$$\begin{aligned} U_{1-2} \quad 1) \text{ Work by spring force} &= \frac{1}{2} k (x_1^2 - x_2^2) \\ &= \frac{1}{2} \times 6 \times (2.2^2 - 0.2^2) \\ &= 14.4 \text{ J} \end{aligned}$$

Between pos (1) and pos (2), only spring force does work.

Here

$$x_1 = 5 - 2.8 = 2.2 \text{ m}$$

$$x_2 = 3 - 2.8 = 0.2 \text{ m}$$

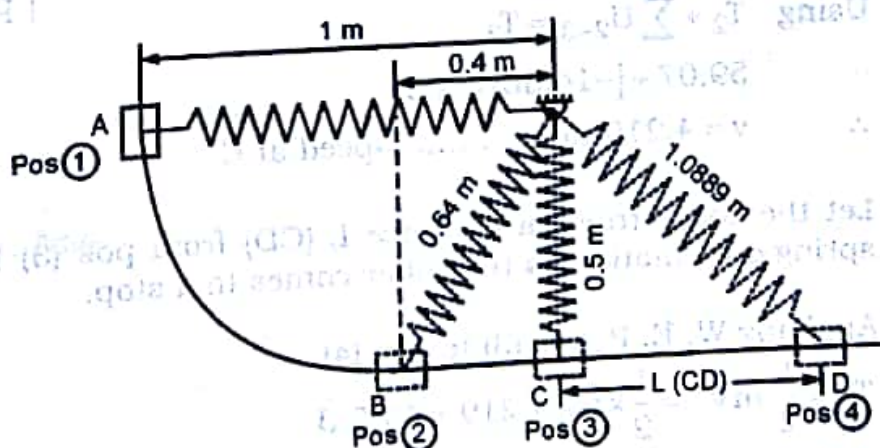
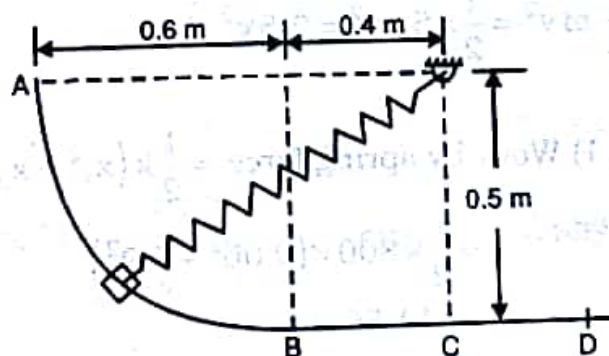
$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$0 + 14.4 = 1.5 v^2$$

$$\therefore v = 3.098 \text{ m/s} \quad \text{..... Ans.}$$

**P11.** A 5 kg steel collar is attached to a spring of  $k = 800 \text{ N/m}$  and a free length of 0.7 m. If the collar is released from rest at A, determine the speed of the collar as it passes through B and C. If the collar finally comes to halt at D, find the distance CD.

**Solution:** Let A, B, C and D be marked as position (1), (2), (3) and (4) respectively.



Applying W.E.P. pos (1) to pos (2)

$T_1 = 0$  ..... since it starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times v^2 = 2.5 v^2 \text{ J}$$

Between pos (1) and pos (2) weight force and spring forces do work.

$U_{1-2}$  1) Work by weight force  $= +mgh$

$$= 5 \times 9.81 \times 0.5$$

$$= 24.525 \text{ J}$$

Work by weight is +ve since pos (2) is below pos (1).

2) Work by spring force  $= \frac{1}{2} k (x_1^2 - x_2^2)$

$$= \frac{1}{2} \times 800 \times (0.3^2 - 0.06^2)$$

$$= 34.56 \text{ J}$$

Here

$$x_1 = 1 - 0.7 = 0.3 \text{ m}$$

$$x_2 = 0.7 - 0.64 = 0.06 \text{ m}$$

Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [24.525 + 34.56] = 2.5 v^2$$

$\therefore v = 4.861 \text{ m/s}$  ..... speed at B

..... Ans.

Applying W E P pos (2) to pos (3).

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times 4.861^2 = 59.07 \text{ J}$$

Between pos (2) and pos (3), only spring force does work.

$$T_3 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times v^2 = 2.5 v^2 \text{ J}$$

$U_{2-3}$  1) Work by spring force  $= \frac{1}{2} k (x_1^2 - x_2^2)$

$$= \frac{1}{2} \times 800 \times (0.06^2 - 0.2^2)$$

$$= -14.56 \text{ J}$$

Here

$$x_1 = 0.7 - 0.64 = 0.06 \text{ m}$$

$$x_2 = 0.7 - 0.5 = 0.2 \text{ m}$$

Note the weight does no work since the collar experiences no level difference 'h' between pos (2) and pos (3).

Using  $T_2 + \sum U_{2-3} = T_3$

$$59.07 + [-14.56] = 2.5 v^2$$

$\therefore v = 4.219 \text{ m/s}$  ..... speed at C

..... Ans.

Let the collar travel a distance L (CD) from pos (3) to pos (4). At pos (4), let x be the spring deformation as the collar comes to a stop.

Applying W. E. P. pos (3) to pos (4)

$$T_3 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times 4.219^2 = 44.5 \text{ J}$$

$T_4 = 0$  ..... since it comes to rest



$$\begin{aligned}
 U_{3-4} \quad 1) \text{ Work by spring force} &= \frac{1}{2}k(x_1^2 - x_2^2) \\
 &= \frac{1}{2} \times 800 \times (0.2^2 - x^2) \\
 &= 16 - 400x^2 \text{ J}
 \end{aligned}$$

here

 $x_1 = 0.2 \text{ m}$  ...deformation at pos (3) $x_2 = x$  ...deformation at pos (4)

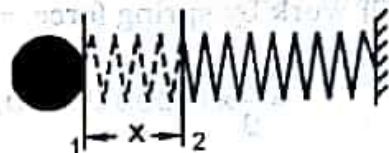
$$\begin{aligned}
 \text{Using } T_3 + \sum U_{3-4} &= T_4 \\
 44.5 + [16 - 400x^2] &= 0
 \end{aligned}$$

$$\therefore x = 0.3889 \text{ m} \quad \dots\dots\dots \text{Ans.}$$

If deformation in spring is  $x = 0.3889 \text{ m}$  it implies that the length of spring in position (4) is (free length + deformation)  $= 0.7 + 0.3889 = 1.0889 \text{ m}$

$$\text{From geometry, } L(CD) = \sqrt{1.0889^2 - 0.5^2} = 0.9673 \text{ m} \quad \dots\dots\dots \text{Ans.}$$

**P12.** A spring of stiffness  $k$  is placed horizontally and a ball of mass  $m$  strikes the spring with a velocity  $v$ . find the maximum compression of the spring. Take  $m = 5 \text{ kg}$ ,  $k = 500 \text{ N/m}$ ,  $v = 3 \text{ m/s}$ . (M.U. Dec 12)



**Solution:** Let  $x$  be the maximum compression of the spring.

Applying W E P pos (1) to pos (2).

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 3^2 = 22.5 \text{ J}$$

$T_2 = 0$  ..... since the ball comes to rest.

$$\begin{aligned}
 U_{1-2} \quad 1) \text{ Work by spring force} &= \frac{1}{2}k(x_1^2 - x_2^2) \\
 &= \frac{1}{2} \times 500 \times (0 - x^2) \\
 &= -250x^2 \text{ J}
 \end{aligned}$$

Here

 $x_1 = 0$   $\therefore$  spring is initially free $x_2 = x$ 

$$\begin{aligned}
 \text{Using } T_1 + \sum U_{1-2} &= T_2 \\
 22.5 + [-250x^2] &= 0
 \end{aligned}$$

$$\therefore x = 0.3 \text{ m} \quad \dots\dots\dots \text{Ans.}$$

**P13.** A 5 kg mass drops 2m upon a spring whose modulus is 10 N/mm. a) What will be the speed of the block when the spring is deformed 100 mm?  
b) What will be the maximum compression of the spring.

(M.U Dec 10)

**Solution:** let  $v$  be the speed of the 5 kg mass as it compresses the spring by 100 mm = 0.1 m

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since block starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times v^2 = 2.5 v^2 \text{ J}$$

$U_{1-2}$  1) Work by weight force =  $+mgh$

$$= 5 \times 9.81 \times 2.1$$

$$= 103 \text{ J}$$

$$2) \text{ Work by spring force} = \frac{1}{2} k (x_1^2 - x_2^2)$$

$$= \frac{1}{2} \times 10000 \times (0 - 0.1^2) = -50 \text{ J}$$

Here

$x_1 = 0$   $\therefore$  spring is free

$x_2 = 0.1 \text{ m}$

$k = 10 \text{ N/mm} = 10000 \text{ N/m}$

Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [103 - 50] = 2.5 v^2$$

$$\therefore v = 4.6 \text{ m/s} \quad \text{..... Ans.}$$

b) Let  $x$  be the maximum compression of the spring

Applying W E P pos (1) to pos (3).

$T_1 = 0$  ..... since the block starts from rest.

$T_3 = 0$  ..... since the block comes to rest.

$U_{1-3}$  1) Work by weight force =  $+mgh$

$$= 5 \times 9.81 \times (2 + x)$$

$$= 98.1 + 49.05 x \text{ J}$$

$$2) \text{ Work by spring force} = \frac{1}{2} k (x_1^2 - x_2^2)$$

$$= \frac{1}{2} \times 10000 \times (0 - x^2) = -5000 x^2 \text{ J}$$

Here

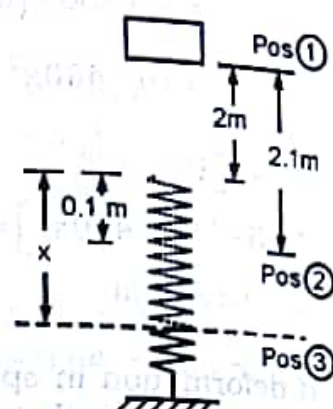
$x_1 = 0$   $\therefore$  spring is free

$x_2 = x$  ... max. spring compression

Using  $T_1 + \sum U_{1-3} = T_3$

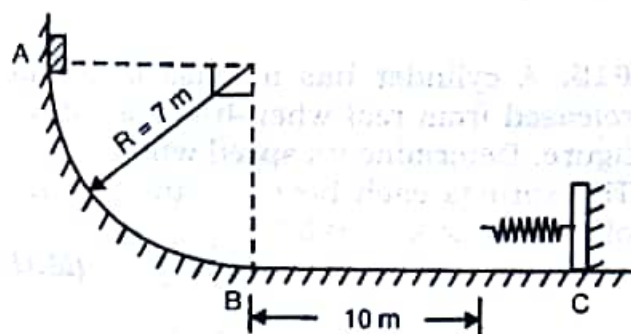
$$0 + [98.1 + 49.05 x - 5000 x^2] = 0$$

$$\therefore x = 0.145 \text{ m} = 145 \text{ mm} \quad \text{..... Ans.}$$





**P14.** A small block of mass 5 kg is released at A from rest on a frictionless circular surface AB. The block then travels on the rough horizontal surface BC whose  $\mu_s = 0.3$  and  $\mu_k = 0.2$ . A spring having stiffness  $k = 900 \text{ N/m}$  is placed at C to bring the block to a halt. Find the maximum velocity attained by the block and also the maximum compression undergone by the spring.



**Solution:** Let A and B be pos (1) and pos (2) of the block. The block starts from rest at A and gains speed due to fall under gravity. The block therefore has max. speed at B.

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since block starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times v^2 = 2.5 v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = +mgh \\ = 5 \times 9.81 \times 7 = 343.35 \text{ J}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$0 + [343.35] = 2.5 v^2 \quad \therefore v = 11.72 \text{ m/s} \dots \text{maximum speed of block} \dots \text{Ans.}$$

Let the block compresses the spring by  $x$  before coming to halt at pos (3).

Applying W E P pos (2) to pos (3).

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times 11.72^2 = 343.35 \text{ J}$$

$T_3 = 0$  ..... since block comes to rest.

$$U_{2-3} \quad 1) \text{ Work by friction force} = -\mu_k \cdot N \cdot s \\ = -0.2 \times (5 \times 9.81) \times (10 + x) \\ = -98.1 - 9.81x \text{ J}$$

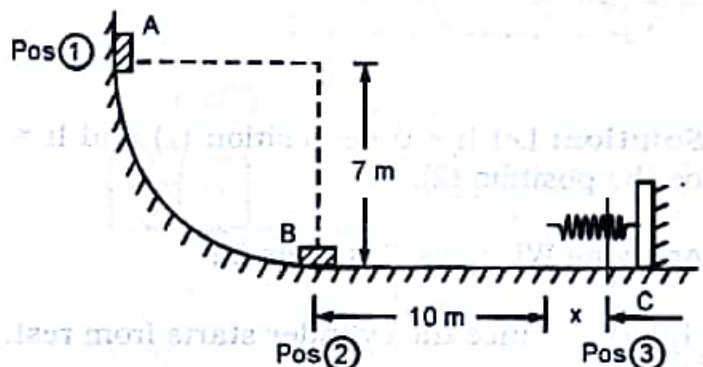
$$2) \text{ Work by spring force} = \frac{1}{2} k (x_1^2 - x_2^2) \\ = \frac{1}{2} \times 900 \times (0 - x^2) = -450x^2 \text{ J}$$

$$\text{sing } T_2 + \sum U_{2-3} = T_3 \\ 343.35 + [-98.1 - 9.81x - 450x^2] = 0$$

$$\therefore -450x^2 - 9.81x + 245.3 = 0$$

Solving the above quadratic equation we get  $x = 0.727 \text{ m}$

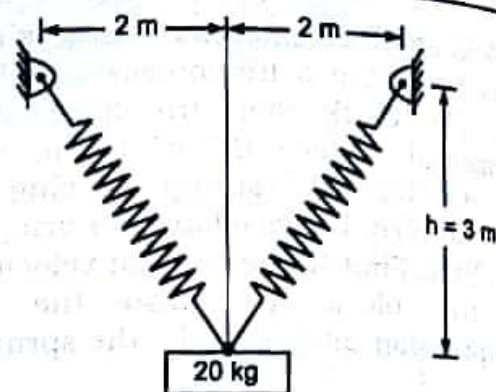
..... **Ans.**



Between pos (2) and pos (3) friction force and spring force do work.

**P15.** A cylinder has a mass of 20 kg and is released from rest when  $h = 0$  as shown in the figure. Determine its speed when  $h = 3$  m. The springs each have an un-stretched length of 2 m. Take  $k = 40$  N/m.

(M.U Dec 16)



**Solution:** Let  $h = 0$  be position (1) and  $h = 3$  m be the position (2).

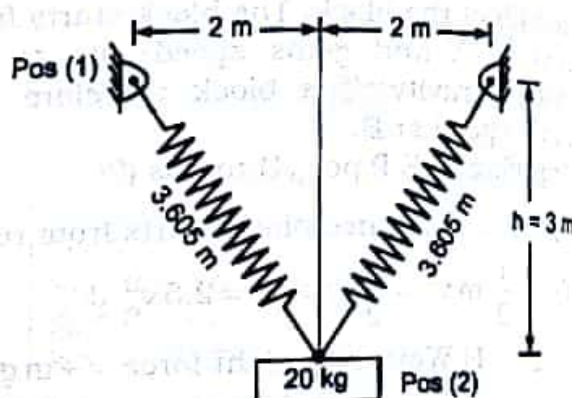
Applying WEP Pos (1) to Pos (2)

$T_1 = 0$  ... since the cylinder starts from rest.

$$T_2 = \frac{1}{2} \times 20 \times v^2 = 10 v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ work by weight force} = + mgh \\ = 20 \times 9.81 \times 3 \\ = 588.6 \text{ J}$$

$$2) \text{ work by spring force} = \frac{1}{2} K (x_1^2 - x_2^2) \times 2 \\ = \frac{1}{2} \times 40 (0 - 1.605^2) \times 2 \\ = -103.04 \text{ J}$$



Since free length = 2 m

$$x_1 = 2 - 2 = 0$$

$$x_2 = 3.605 - 2 = 1.605 \text{ m}$$

\*  $\times 2$  since there are 2 springs.

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

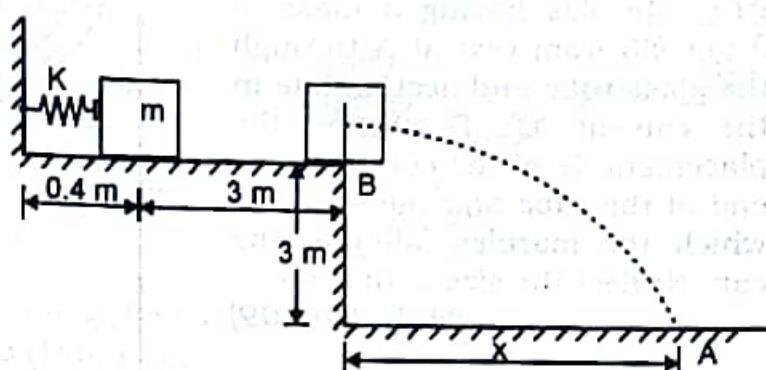
$$0 + [588.6 - 103.04] = 10 v^2$$

$$\therefore v = 6.968 \text{ m/s} \quad \dots \text{Ans.}$$

DJC



**P16.** A block of mass  $m = 80 \text{ kg}$  is compressed against a spring as shown in figure. How far from point B (distance  $x$ ) will the block strike on the plane at point A. Take free length of spring as  $0.9 \text{ m}$ ,  $\mu_k = 0.2$  and spring stiffness as  $K = 40 \times 10^2 \text{ N/m}$ . (M.U May 08)



**Solution:** Let pos (1) and pos (2) be marked as shown.

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since block starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 80 \times v^2 = 40 v^2 \text{ J}$$

$U_{1-2}$  1) Work by spring force

$$= \frac{1}{2} k (x_1^2 - x_2^2)$$

$$= \frac{1}{2} \times 4000 \times (0.5^2 - 0) = 500 \text{ J}$$

Here

$$x_1 = 0.9 - 0.4 = 0.5 \text{ m}$$

$$x_2 = 0 \text{ ... since the spring becomes free.}$$

2) Work by friction force  $= -\mu_k \cdot N \cdot s = -0.2 \times (80 \times 9.81) \times 3 = -470.88 \text{ J}$

Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [500 - 470.88] = 40 v^2$$

$\therefore v = 0.853 \text{ m/s}$  ..... speed of block at B

From B to A, the block is in projectile motion.

Horizontal motion

$$v = 0.853 \text{ m/s}$$

$$s = x$$

$$t = t$$

Using  $v = \frac{s}{t}$

$$0.853 = \frac{x}{t} \text{ ..... (1)}$$

Substituting  $t = 0.7821 \text{ sec}$ , we get  
 $x = 0.6671 \text{ m}$  ..... **Ans.**

Vertical motion  $\downarrow +ve$

$$u = 0$$

$$v = -$$

$$s = 3 \text{ m}$$

$$a = 9.81 \text{ m/s}^2$$

$$t = t$$

Using  $s = ut + \frac{1}{2} at^2 \therefore 3 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$$\therefore t = 0.7821 \text{ sec}$$

**P17.** Marbles having a mass of 5 gm fall from rest at A through the glass tube and accumulate in the can at 'C'. Determine the placement 'R' of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.

(M. U. May 09)

**Solution:** Let A and B be pos (1) and pos (2) of the marble as shown.

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since marble starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.005 \times v^2 = 0.0025 v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = +mgh \\ = 0.005 \times 9.81 \times 1 = 0.04905 \text{ J} \\ (\text{Level difference } h = 3 - 2 = 1 \text{ m})$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2 \\ 0 + [0.04905] = 0.0025 v^2$$

$$\therefore v = 4.429 \text{ m/s} \text{ ..... speed of marble at B}$$

From B to C, the marble performs projectile motion

Horizontal motion

$$v = 4.429 \text{ m/s}$$

$$s = R$$

$$t = t \text{ sec}$$

$$\text{Using } v = \frac{s}{t}$$

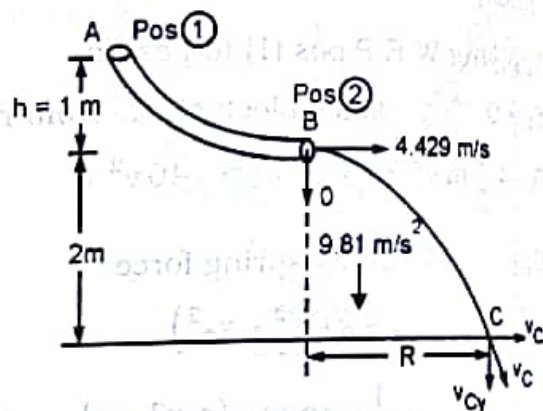
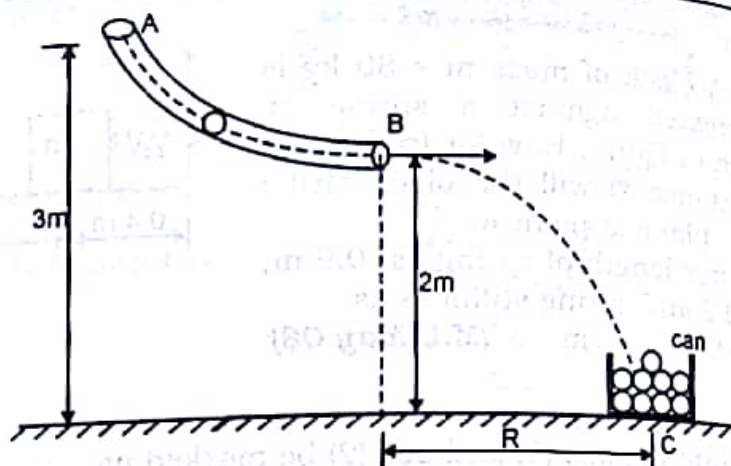
$$4.429 = \frac{R}{t} \text{ ..... (1)}$$

Substituting  $t = 0.6386 \text{ sec}$

$$\therefore R = 2.828 \text{ m} \text{ ..... Ans.}$$

Landing speed of marble at C =

$$v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = \sqrt{4.429^2 + 6.265^2} \\ = 7.672 \text{ m/s} \text{ ..... Ans.}$$



..... Ans.

Vertical motion ↓ +ve

$$u = 0$$

$$v = v_{Cy}$$

$$s = 2 \text{ m}$$

$$a = 9.81 \text{ m/s}^2$$

$$t = t$$

... since the marbles leave horizontally, the vertical component of velocity is zero.

$$\text{Using } s = ut + \frac{1}{2} at^2 \therefore 2 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$\therefore t = 0.6386 \text{ sec}$$

$$\text{Using } v = u + at$$

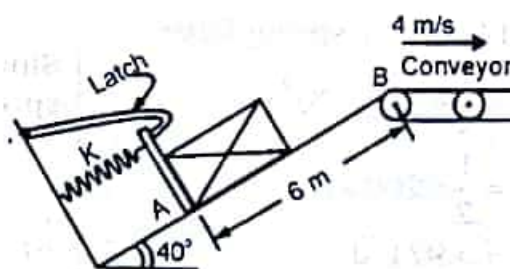
$$v_{Cy} = 0 + 9.81 \times 0.6386$$

$$\therefore v_{Cy} = 6.265 \text{ m/s}$$

(Here  $v_{Cx} = 4.429 \text{ m/s}$ , since velocity remains constant in the horizontal direction.)



**P18.** A spring is compressed by 0.3 m and held by a latch mechanism. When the latch is released it propels a package of 500 N weight from position A to position B on the conveyor. If  $\mu_k = 0.2$  between the package and the incline and the desired speed of the package at B is 4 m/s, determine the stiffness value  $k$  of the spring which should be provided.



**Solution:** Let A and B be pos (1) and pos (2) of the block.  
Applying Work Energy Principle from pos (1) to pos (2).  
 $T_1 = 0$  ..... since the block starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times \left( \frac{500}{9.81} \right) \times 4^2 = 407.75 \text{ J}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = -mgh \\ = -500 \times 6 \sin 40 = -1928.4 \text{ J}$$

$$2) \text{ Work by spring force} \\ = \frac{1}{2} k (x_1^2 - x_2^2) = \frac{1}{2} \times k \times (0.3^2 - 0) \\ = 0.045k \text{ J}$$

$$3) \text{ Work by friction force} \\ = -\mu_k \cdot N \cdot s \\ = -0.2 \times (500 \cos 40) \times 6 = -459.6 \text{ J}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2 \\ 0 + [-1928.4 + 0.045k - 459.6] = 407.75$$

$$\therefore k = 62128 \text{ N/m} \quad \text{..... Ans.}$$

Between pos (1) and pos (2), weight, spring force and friction force do work.

Work by weight is -ve, since pos (2) is above pos (1).

Level difference  $h = 6 \sin 40$ .

$x_1 = 0.3 \text{ m}$  given

$x_2 = 0$  ... since the spring becomes free.

$N = W \cos \theta = 500 \cos 40$

$s = \text{distance travelled} = 6 \text{ m}$

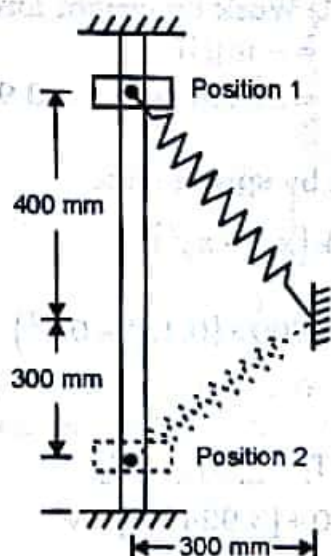
**P19.** A collar of mass 10 kg moves in a vertical guide as shown. Neglecting friction between guide and the collar, find its velocity when it passes through position (2), after starting from rest in position (1). The spring constant is 200 N/m and the free length of the spring is 200 mm.

(M.U. Dec 08)

**Solution:** Applying W E P pos (1) to pos (2).  
 $T_1 = 0$  ..... since the collar starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 10 \times v^2 = 5v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = +mgh \\ = 10 \times 9.81 \times 0.7 = 68.67 \text{ J}$$



2) Work by spring force

$$= \frac{1}{2} k (x_1^2 - x_2^2)$$

$$= \frac{1}{2} \times 200 \times (0.3^2 - 0.224^2)$$

$$= 3.971 \text{ J}$$

Since free length of spring = 0.2 m

$$x_1 = 0.5 - 0.2$$

$$= 0.3 \text{ m}$$

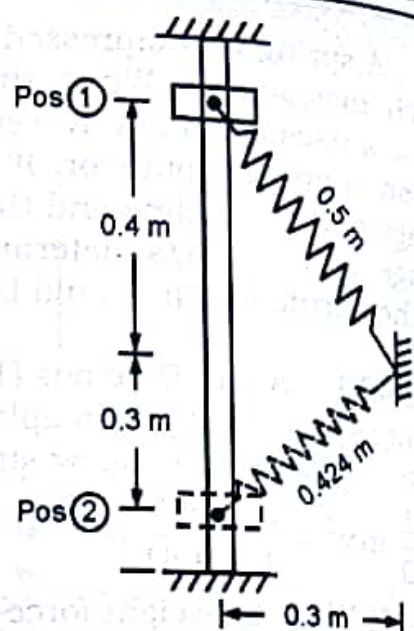
$$x_2 = 0.424 - 0.2$$

$$= 0.224 \text{ m}$$

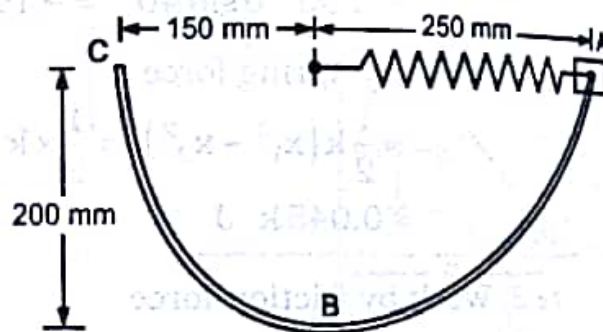
Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [68.67 + 3.971] = 5v^2$$

$$\therefore v = 3.811 \text{ m/s} \quad \text{..... Ans.}$$



**P20.** A 2 kg collar M is attached to a spring and slides without friction in a vertical plane along the curved rod ABC as shown in figure. The spring has an un-deformed length of 100 mm and its stiffness  $k = 800 \text{ N/m}$ . If the collar is released from rest at A, determine its velocity i) as it passes through B ii) as it reaches C.  
(M.U. Dec 15)



**Solution:** Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the collar starts from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 2 \times v^2 = v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ Work by weight force} \\ = + mgh \\ = 2 \times 9.81 \times 0.2 = 3.924 \text{ J}$$

2) Work by spring force

$$= \frac{1}{2} k (x_1^2 - x_2^2)$$

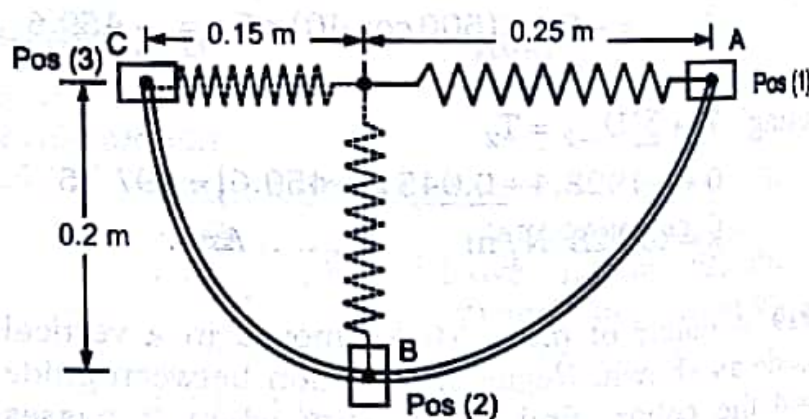
$$= \frac{1}{2} \times 800 \times (0.15^2 - 0.1^2)$$

$$= 5 \text{ J}$$

Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [3.924 + 5] = v^2$$

$$\therefore v = 2.987 \text{ m/s} \quad \text{..... velocity at B ..... Ans.}$$



Since free length of spring

$$= 100 \text{ mm} = 0.1 \text{ m}$$

$$x_1 = 0.25 - 0.1 = 0.15 \text{ m}$$

$$x_2 = 0.2 - 0.1 = 0.1 \text{ m}$$



Applying W E P pos (1) to pos (3).

$T_1 = 0$  ..... since the collar starts from rest.

$$T_3 = \frac{1}{2} m v^2 = \frac{1}{2} \times 2 \times v^2 = v^2 \text{ J}$$

$U_{1-3}$  1) Work by weight force = 0 ..... since pos (1) and pos (3) are at same level.

2) Work by spring force

$$= \frac{1}{2} k (x_1^2 - x_2^2)$$

$$= \frac{1}{2} \times 800 \times (0.15^2 - 0.05^2) = 8 \text{ J}$$

Since free length of spring  
= 100 mm = 0.1 m

$$x_1 = 0.25 - 0.1 = 0.15 \text{ m}$$

$$x_2 = 0.15 - 0.1 = 0.05 \text{ m}$$

Using  $T_1 + \sum U_{1-3} = T_3$

$$0 + [8] = v^2$$

$\therefore v = 2.828 \text{ m/s}$  ..... velocity at C ..... **Ans.**

**P21.** Two blocks  $m_A = 10 \text{ kg}$  and  $m_B = 5 \text{ kg}$  are connected as shown. Determine the velocity of each block when system starts from rest and the block B gets displaced by 2 m. Take  $\mu = 0.2$  between block A and the horizontal surface.

(VJTI Dec 13, M.U. Dec 13)

**Solution:** This is a dependent system of two blocks connected by a common string. Block B travels under its weight in the downward direction pulling block A horizontally to the right.

Using CSLM, we get  $2v_A = v_B$  ..... (1)  
[since 2 portions of string holds A, while 1 portion of the same string holds B]

The above equation (1) also implies that, if B travels 2 m down, A travels 1 m to the right.

Applying W E P pos (1) to pos (2).

$$T_1 = 0$$

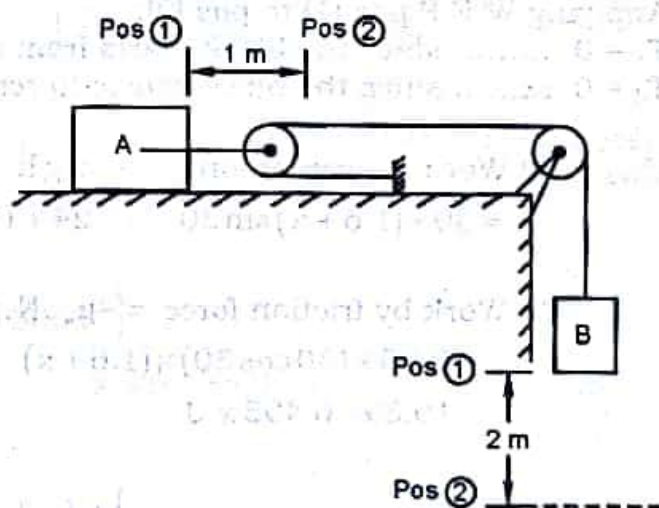
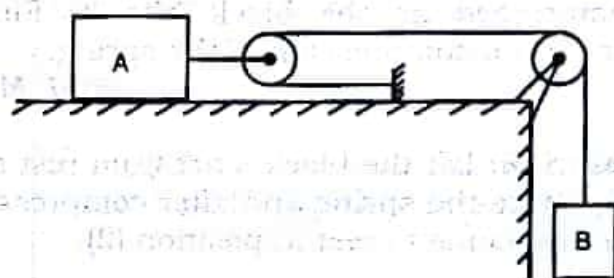
$$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$= \frac{1}{2} \times 10 \times v_A^2 + \frac{1}{2} \times 5 \times v_B^2$$

$$= 5 \times v_A^2 + 2.5 \times (2v_A)^2$$

$$= 15 v_A^2 \text{ J}$$

Between pos (1) and pos (2), weight of block B and friction force do work.



$$U_{1-2} \quad 1) \text{ Work by weight of block B} = + mgh \\ = +5 \times 9.81 \times 2 = 98.1 \text{ J}$$

$$2) \text{ Work by friction force} = -\mu_k \cdot N \cdot s \\ = -0.2 \times (10 \times 9.81) \times 1 = -19.62 \text{ J}$$

Work by weight is + ve since for block B pos (2) is below pos (1)

$$N = mg$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$0 + [98.1 - 19.62] = 15 v_A^2$$

$$\therefore v_A = 2.287 \text{ m/s}$$

$$\text{also } v_B = 2v_A = 4.575 \text{ m/s}$$

..... Ans.

..... Ans.

**P22.** A block weighing 30 N is released from rest and slides down a rough inclined plane having  $\mu = 0.25$ . It soon comes in contact with a spring of  $k = 1 \text{ N/mm}$  which gets compressed as the block hits it. Find the maximum compression of the spring.

(M.U May 14)

**Solution:** Let the block start from rest at position (1), strike the spring and after compressing it by  $x$  metres, come to rest at position (2).

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the block starts from rest.

$T_2 = 0$  ..... since the block comes to rest.

$$U_{1-2} \quad 1) \text{ Work by weight force} = + mgh \\ = 30 \times (1.6 + x) \sin 30 = 24 + 15x \text{ J}$$

$$2) \text{ Work by friction force} = -\mu_k \cdot N \cdot s \\ = -0.25 \times (30 \cos 30) \times (1.6 + x) \\ = -10.39 - 6.495x \text{ J}$$

$$N = W \cos \theta = 30 \cos 30$$

$$s = \text{distance travelled} = (1.6 + x) \text{ m}$$

$$3) \text{ Work by spring force} = \frac{1}{2} k (x_1^2 - x_2^2) \\ = \frac{1}{2} \times 1000 \times (0 - x^2) = -500x^2 \text{ J}$$

$$x_1 = 0 \text{ ..... since spring is free.}$$

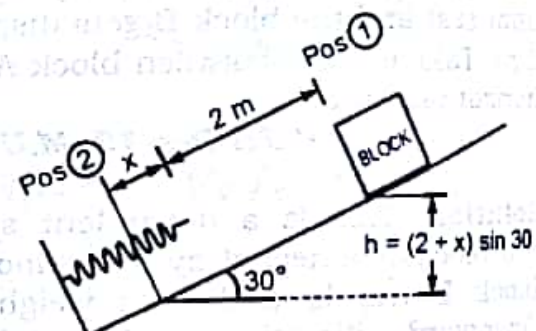
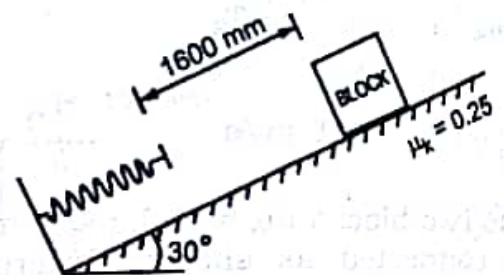
$$x_2 = x$$

$$K = 1 \text{ N/mm} = 1000 \text{ N/m}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

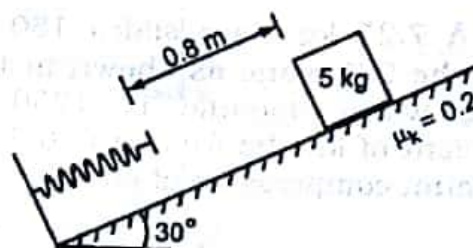
$$0 + [24 + 15x - 10.39 - 6.495x - 500x^2] = 0$$

$$\therefore x = 0.1736 \text{ m ..... Ans.}$$

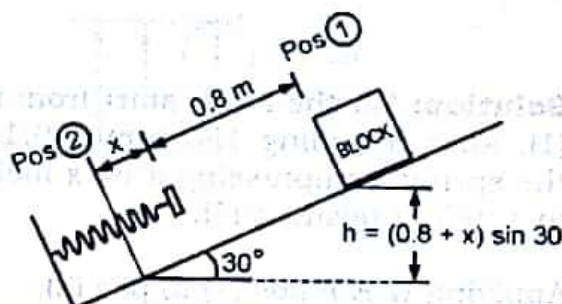




**P23.** A block of mass 5 kg is released from rest on an inclined plane as shown in figure. Find maximum compression of the spring, if the spring constant is 1 N/mm and coefficient of the friction between the block and the inclined plane is 0.2. (VJTI Nov 09)



**Solution:** Let the block start from rest at position (1). After travelling 0.8 m it strikes the spring, compressing it by  $x$  metres and comes to a halt at position (2).



Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the block start from rest.

$T_2 = 0$  ..... since the block comes to rest.

$$U_{1-2} \quad 1) \text{ Work by weight force } = + mgh \\ = 5 \times 9.81 \times (0.8 + x) \sin 30 \\ = 19.62 + 24.525x \text{ J}$$

$$2) \text{ Work by friction force } = -\mu_k \cdot N \cdot s \\ = -0.2 \times (5 \times 9.81 \times \cos 30) \times (0.8 + x) \\ = -6.796 - 8.496x \text{ J}$$

$$3) \text{ Work by spring force } = \frac{1}{2} k (x_1^2 - x_2^2) \\ = \frac{1}{2} \times 1000 \times (0 - x^2) = -500x^2 \text{ J}$$

Work by weight force is + ve, since position (2) is below pos (1).

$$N = mg \cos \theta$$

$$s = \text{distance travelled} = (2 + x) \text{ m}$$

$$k = 1 \text{ N/mm} = 1000 \text{ N/m}$$

$$x_1 = 0 \text{ ..... since spring is free.}$$

$$x_2 = x \text{ ..... max spring compression.}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

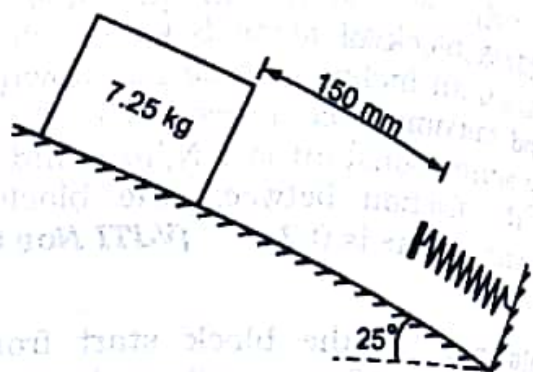
$$0 + [19.62 + 24.525x - 6.796 - 8.496x - 500x^2] = 0$$

$$\therefore -500x^2 + 16.029x + 12.824 = 0 \quad \text{or} \quad x = 0.17698 \text{ m} \quad \text{..... Ans.}$$

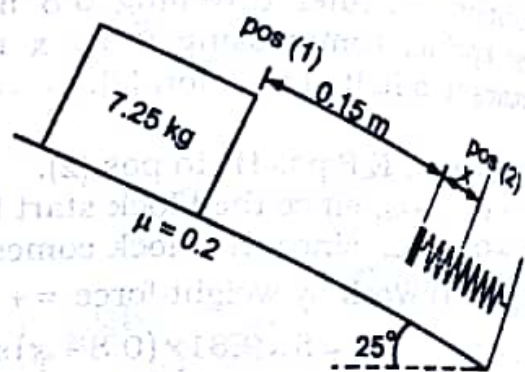
DJC

**P24.** A 7.25 kg mass slides 150 mm from rest down the  $25^\circ$  plane as shown in figure. It hits a spring whose module is 1750 N/m. If the coefficient of kinetic friction is 0.2, determine the maximum compression of spring.

(KJS Nov 15)



**Solution:** Let the block start from rest at position (1), after travelling 150 mm = 0.15 m, it strikes the spring, compressing it by  $x$  metres and comes to a halt at position (2).



Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the block starts from rest.

$T_2 = 0$  ..... since the block comes to rest.

$$\begin{aligned} U_{1-2} \quad 1) \text{ Work by weight force} &= + mgh \\ &= 7.25 \times 9.81 \times (0.15 + x) \sin 25 \\ &= 4.509 + 30.06x \text{ J} \end{aligned}$$

Work by weight force is + ve, since pos (2) is below pos (1).

$$\begin{aligned} 2) \text{ Work by friction force} &= -\mu_k \cdot N \cdot s \\ &= -0.2 \times (7.25 \times 9.81 \cos 25) \times (0.15 + x) \\ &= -1.934 - 12.89x \text{ J} \end{aligned}$$

$$N = mg \cos \theta$$

$$s = \text{distance travelled} = (0.15 + x) \text{ m}$$

$$\begin{aligned} 3) \text{ Work by spring force} &= \frac{1}{2} k (x_1^2 - x_2^2) \\ &= \frac{1}{2} \times 1750 \times (0 - x^2) = -875x^2 \text{ J} \end{aligned}$$

$$k = 1750 \text{ N/m}$$

$$x_1 = 0 \text{ ..... since spring is free.}$$

$$x_2 = x \text{ ..... max. spring compression}$$

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$0 + [4.509 + 30.06x - 1.934 - 12.89x - 875x^2] = 0$$

$$\therefore -875x^2 + 17.17x + 2.575 = 0$$

$$\text{Or } x = 0.0649 \text{ m}$$

..... **Ans.**



**P25.** A ball of weight 25 N is suspended from a 1200 mm long elastic cord. The ball is now pulled vertically down by 200 mm and then released. Determine the speed of the ball as it strikes the ceiling. Neglect initial deformation of the elastic cord due to self wt. of the ball.

**Solution:** Let the ball starting from rest at pos (1) hit the ceiling with a speed  $v$  at position (2)

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the block start from rest.

$$T_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times \frac{25}{9.81} \times v^2 = 1.274 v^2 \text{ J}$$

$$U_{1-2} \quad 1) \text{ by weight force} = -mgh \\ = -25 \times 1.4 = -35 \text{ J}$$

$$2) \text{ by elastic cord} = \frac{1}{2}k(x_1^2 - x_2^2) \\ = \frac{1}{2} \times 2000 \times (0.2^2 - 0) = 40 \text{ J}$$

$$k = 2 \text{ N/mm} = 2000 \text{ N/m}$$

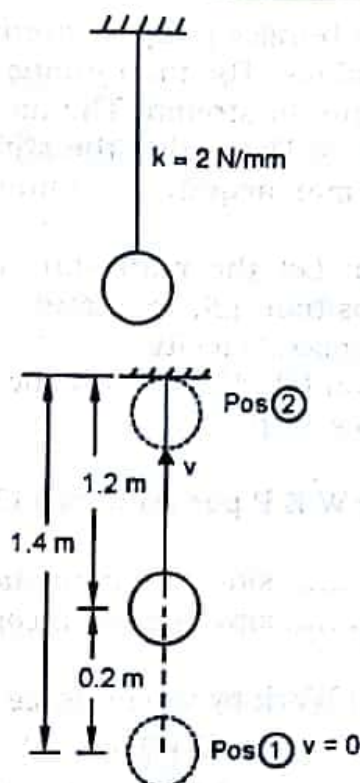
$x_1 = 0.2 \text{ m}$  ..... initial stretch.

$x_2 = 0$  ..... since elastic cord becomes free.

$$\text{Using } T_1 + \sum U_{1-2} = T_2$$

$$0 + [-35 + 40] = 1.274 v^2$$

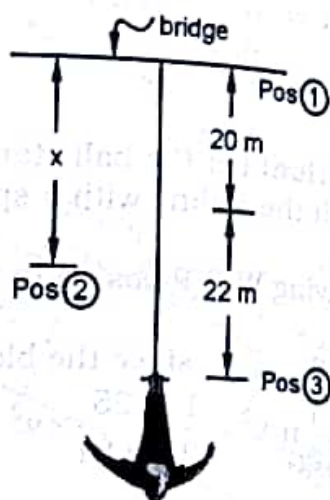
$$\therefore v = 1.981 \text{ m/s} \quad \text{..... Ans.}$$



**P26.** In a bungee jumping event a man of mass 65 kg has a 20 m elastic cord tied to his bound ankles. The man jumps from a bridge and falls freely for 20 m before the elastic cord begins to stretch. The man reaches 42 m below the bridge before he starts rising upwards. a) Determine the stiffness of the elastic cord. b) How much below the bridge does the man acquire maximum velocity and the value of this maximum velocity.

**Solution:** Let the man start from rest at position (1). At some position (2),  $x$  metres below the bridge the man acquires max. velocity.

At position (3), 42 m below the bridge the man momentarily comes to a stop.



Applying W E P pos (1) to pos (3).

$T_1 = 0$  ..... since the man start from rest.

$T_3 = 0$  ..... since the man comes to rest.

$$\begin{aligned} U_{1-3} \quad 1) \text{ Work by weight force} &= + mgh \\ &= 65 \times 9.81 \times 42 \\ &= 26781.3 \text{ J} \end{aligned}$$

$$\begin{aligned} 2) \text{ Work by elastic cord} &= \frac{1}{2} k (x_1^2 - x_2^2) \\ &= \frac{1}{2} \times k \times (0 - 22^2) = -242 k \text{ J} \end{aligned}$$

$$\begin{aligned} x_1 &= 0 \text{ ..... since initially cord is} \\ &\text{unstretched.} \\ x_2 &= 42 - 20 = 22 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Using } T_1 + \sum U_{1-3} &= T_3 \\ 0 + [26781.3 - 242k] &= 0 \end{aligned}$$

$$\therefore k = 110.67 \text{ N/m} \quad \text{..... Ans.}$$

During the jump, the man acquires a maximum velocity at some position (2) distance  $x$  from the top.

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the man start from rest.

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 65 \times v^2 = 32.5 v^2 \text{ J}$$

$$\begin{aligned} U_{1-2} \quad 1) \text{ by weight force} &= + mgh \\ &= 65 \times 9.81 \times x = 637.65 x \text{ J} \end{aligned}$$

$$\begin{aligned} 2) \text{ by elastic cord force} &= \frac{1}{2} k (x_1^2 - x_2^2) \\ &= \frac{1}{2} \times 110.67 \times [0 - (x - 20)^2] \\ &= -55.33x^2 + 2213.4x - 2213.4 \text{ J} \end{aligned}$$

$$\begin{aligned} k &= 110.67 \text{ N/m} \\ x_1 &= 0 \\ x_2 &= x - 20 \end{aligned}$$



Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [637.65x - 55.33x^2 + 2213.4x - 22134] = 32.5v^2$$

$$\text{or } v^2 = -1.702x^2 + 87.72x - 681 \quad \dots\dots\dots (A)$$

The above equation (A) is  $v = f(x)$ , to maximize the value of  $v$ ,  $\frac{dv}{dx} = 0$

Differentiating equation (A) w.r.to  $x$  and equating  $\frac{dv}{dx}$  to zero

$$2v \times \frac{dv}{dx} = -3.404x + 87.72$$

$$\frac{dv}{dx} = \frac{-3.404x + 87.72}{2v} = 0$$

or  $x = 25.77 \text{ m}$  ..... position of man below bridge at max. velocity ... **Ans.**

Substituting value of  $x$  obtained above in equation A to get maximum velocity, we get

$$v_{\max} = 21.19 \text{ m/s} \quad \dots\dots\dots \text{Ans.}$$

**P27.** A 50 kg block kept on a  $15^\circ$  inclined plane is pushed down the plane with an initial velocity of 20 m/s. If  $\mu_k = 0.4$ , determine the distance travelled by the block and the time it will take as it comes to rest. **(M.U. Dec 13)**

**Solution:** The 50 kg block has a initial speed of 20 m/s at position (1) and comes to rest at position (2). Let it travel a distance  $x$  between the two positions.

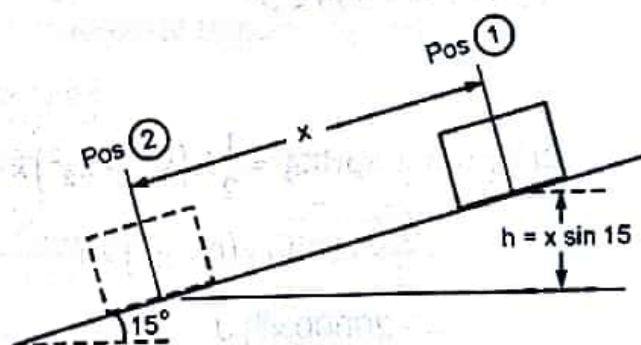
Applying W E P pos (1) to pos (2).

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2} \times 50 \times 20^2 = 10000 \text{ J}$$

$$T_2 = 0$$

$$U_{1-2} \quad 1) \text{ Work by weight force} = + mgh \\ = 50 \times 9.81 \times (x \sin 15) \\ = 126.95x \text{ J}$$

$$2) \text{ Work by friction force} = -\mu_k \cdot N \cdot s \\ = -0.4 \times (50 \times 9.81 \times \cos 15) \times x \\ = -189.51x \text{ J}$$



Between pos (1) and pos (2), weight, friction force do work.

Work by weight is + ve since pos (2) is below pos (1)

$$N = mg \cos \theta$$

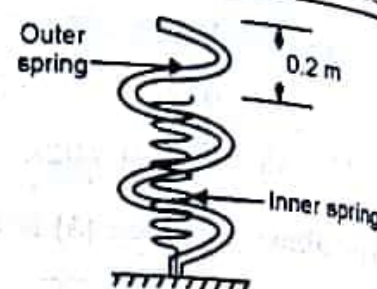
Using  $T_1 + \sum U_{1-2} = T_2$

$$10000 + [126.95x - 189.51x] = 0$$

$$x = 159.83 \text{ m} \quad \dots\dots\dots \text{Ans.}$$

**P28.** A lift of total mass 500 kg at rest snaps off accidentally and falls freely for 6 m till it comes in contact with a set of four nested springs and is soon brought to a halt. Each nested spring consists of an outer spring of stiffness 10 kN/m and an inner spring of stiffness 15 kN/m. The inner spring is lower by 0.2 m than the outer spring as shown.

Find the maximum deformation of the outer spring as the lift is brought safely to a halt.



Details of a single nested spring (there are four such sets)

**Solution:** Let the lift at rest start from position (1), strike the outer spring and then the inner spring, to finally comes to halt at position (2). If the outer spring compress by  $x$  metres, the inner spring compresses by  $(x - 0.2)$  metres.

Applying W E P pos (1) to pos (2).

$T_1 = 0$  ..... since the lift start from rest.

$T_2 = 0$  ..... since the lift comes to rest.

$U_{1-2}$  1) by weight force  $= + mgh$

$$= 500 \times 9.81 \times (6 + x) \dots \text{Level difference } h = (6 + x)$$

$$= 29430 + 4905x \text{ J}$$

$$2) \text{ by outer spring } = \frac{1}{2}k(x_1^2 - x_2^2) \times 4$$

$$= \frac{1}{2} \times 10000 \times (0 - x^2) \times 4$$

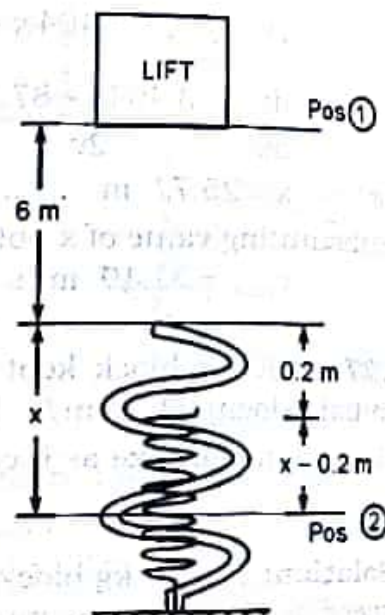
$$= -20000x^2 \text{ J}$$

$$3) \text{ by inner spring } = \frac{1}{2}k(x_1^2 - x_2^2) \times 4$$

$$= \frac{1}{2} \times 15000 \times [0 - (x - 0.2)^2] \times 4$$

$$= -30000x^2 + 12000x - 1200 \text{ J}$$

Note: there are 4 sets of nested spring. This implies there are 4 outer and 4 inner springs. Initial deformation of both the spring is zero since they are initially free.



Using  $T_1 + \sum U_{1-2} = T_2$

$$0 + [29430 + 4905x - 20000x^2 - 30000x^2 + 12000x - 1200] = 0$$

$\therefore x = 0.9392 \text{ m} \dots \dots \dots \text{Ans.}$

