Solutions: Chapter 2

Coplanar Forces

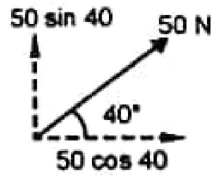
Resolution and Composition of Forces

Exercise 2.1

Resultant of Concurrent Force System

P1. Resolve the given forces into horizontal and vertical components.

a)



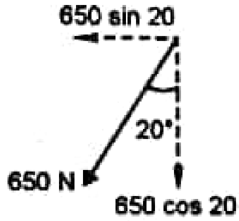
$F_x = 50\cos 40 \text{ N} \rightarrow$

$= 38.3 \text{ N} \rightarrow$

$F_y = 50\sin 40 \text{ N} \uparrow$

$= 32.14 \text{ N} \uparrow$

b)



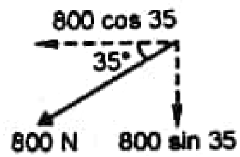
$F_x = 650\sin 20 \text{ N} \leftarrow$

$= 222.3 \text{ N} \leftarrow$

$F_y = 650\cos 20 \text{ N} \downarrow$

$= 610.8 \text{ N} \downarrow$

c)



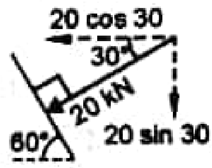
$F_x = 800\cos 35 \text{ N} \leftarrow$

$= 655.3 \text{ N} \leftarrow$

$F_y = 800\sin 35 \text{N} \downarrow$

$= 458.9 \downarrow$

d)



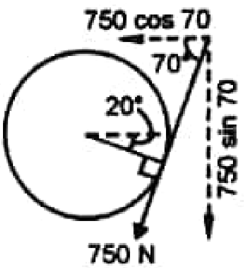
$F_x = 20\cos 30 \text{ kN} \leftarrow$

$= 17.32 \text{ kN} \leftarrow$

$F_y = 20\sin 30 \text{kN} \downarrow$

$= 10 \text{ kN} \downarrow$

e)



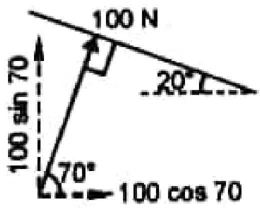
$F_x = 750\cos 70 \text{ N} \leftarrow$

$= 256.5 \text{ N} \leftarrow$

$F_y = 750\sin 70 \text{ N} \downarrow$

$= 704.8 \text{ N} \downarrow$

f)



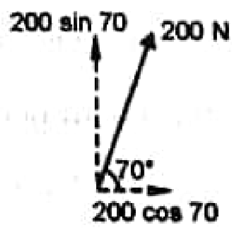
$F_x = 100\cos 70 \text{ N} \rightarrow$

$= 34.2 \text{ N} \rightarrow$

$F_y = 100\sin 70 \text{ N} \uparrow$

$= 93.97 \text{ N} \uparrow$

g)



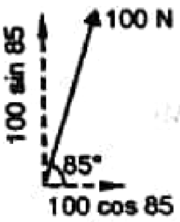
$F_x = 200\cos 70 \text{ N} \rightarrow$

$= 68.4 \text{ N} \rightarrow$

$F_y = 200\sin 70 \text{ N} \uparrow$

$= 187.9 \text{ N} \uparrow$

h)



$F_x = 100\cos 85 \text{N} \rightarrow$

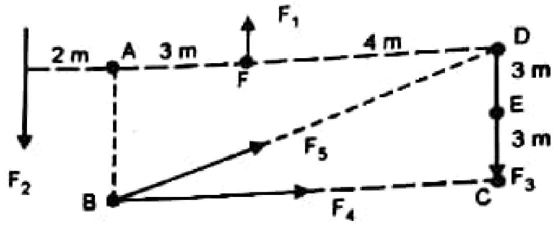
$= 8.71 \text{ N} \rightarrow$

$F_y = 100\sin 85 \text{ N} \uparrow$

$= 99.62 \text{ N} \uparrow$

P2. Given F_1 = 10 N, F_2 = 20 N, F_3 = 30 N, F_4 = 40 N, F_5 = 50 N and taking anticlockwise moments as positive, Find

a) Moment of force F_1 about B and E
b) Moment of force F_2 about A and C
c) Moment of force F_3 about F
d) Moment of force F_4 about B and D
e) Moment of force F_5 about A, F and D.



Solution: Moment of force = ± (Force × ⊥ distance)

Note: Anticlockwise rotation of the force about a point gives + ve moments, while clockwise rotation about a point gives – ve moment.

a) Moment of force F_1 = 10 N about point B

$$M_B^{F_1} = +(10 \times 3) = +30 \text{ Nm}$$

Moment of force F_1 = 10 N about point E

$$M_E^{F_1} = -(10 \times 4) = -40 \text{ Nm}$$

b) Moment of force F_2 = 20 N about point A

$$M_A^{F_2} = +(20 \times 2) = +40 \text{ Nm}$$

Moment of force F_2 = 20 N about point C

$$M_C^{F_2} = +(20 \times 9) = +180 \text{ Nm}$$

c) Moment of force F_3 = 30 N about point F

$$M_F^{F_3} = -(30 \times 4) = -120 \text{ Nm}$$

d) Moment of force F_4 = 40 N about point B

$M_B^{F_4} = 0$ .... since F_4 passes through B.

Moment of force F_4 = 40 N about point D

$$M_D^{F_4} = +(40 \times 6) = +240 \text{ Nm}$$

e) Moment of force F_5 = 50 N about point A

The force F_5 is inclined at 40.6° with the horizontal. We need to resolve F_5 into components and then take moments of the two components.

$$M_A^{F_5} = +(50 \cos 40.6 \times 6) = +227.8 \text{ Nm}$$

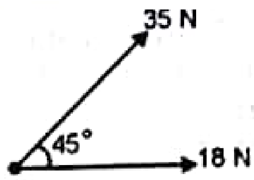
Moment of force F_5 = 50 N at F

$$M_F^{F_5} = +(50 \cos 40.6 \times 6) - (50 \sin 40.6 \times 3) = +130.2 \text{ Nm}$$

Moment of force F_5 = 50 N at D

$M_D^{F_5} = 0$ .... since line of action of force passes through D.

P3. Two forces of 18 N and 35 N acts away from a point. If the angle between the forces is 45°, find the magnitude of the resultant and the angle made by it with the 18 N force.



Solution: This is a system of two concurrent forces.
Method 1: Using Law of Parallelogram of Forces
Let P = 18 N, Q = 35 N, α = 45°

Using $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

$= \sqrt{18^2 + 35^2 + 2 \times 18 \times 35 \cos 45}$

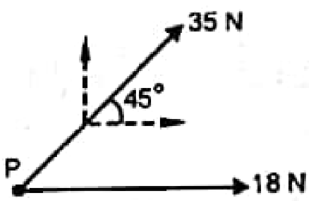
$\therefore$ $R = 49.39$ N ... Magnitude of resultant force

Using $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha} = \dfrac{35\sin 45}{18 + 35\cos 45}$

$\therefore$ $\theta = 30.07°$ ... Direction of resultant force

$\therefore$ $R = 49.39\,N$ at $\theta = 30.07° \measuredangle$ .... **Ans.**

Method 2: Using Method of Resolution



$\Sigma F_x \;\rightarrow +ve$

$= 18 + 35\cos 45$

$= 42.75\,N$

$\therefore$ $\Sigma F_x = 42.75\,N \rightarrow$

$\Sigma F_y \;\uparrow +ve$

$= 35\sin 45$

$= 24.75\,N$

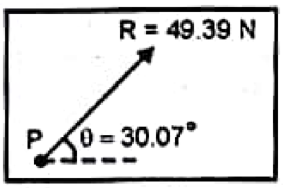
$\therefore$ $\Sigma F_y = 24.75\,N \uparrow$

Using $R = \sqrt{\Sigma {F_x}^2 + \Sigma {F_y}^2} = \sqrt{42.75^2 + 24.75^2}$

= 49.39 N ... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{24.75}{42.75}$

$\therefore$ $\theta = 30.07°$ ... Direction of resultant force



Resultant – Force

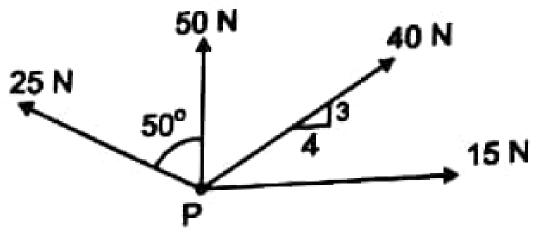
The arrows of ΣF_x and ΣF_y implies that the sense of resultant is first quadrant $\measuredangle$ ... Sense of resultant force

$\therefore$ Resultant is a force R = 49.39 N at $\theta = 30.07° \measuredangle$ acts at a point P as shown.

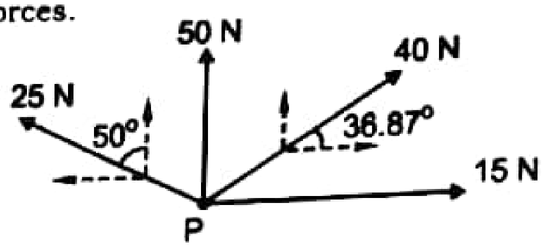
.... **Ans.**

P4. Four concurrent forces act at a point as shown. Find their resultant.

(M.U Dec 14)



Solution: This is a system of four concurrent forces.
Using Method of Resolution



$$\Sigma F_x \quad \rightarrow +ve$$
$$= 15 + 40\cos 36.87 - 25\sin 50$$
$$= 27.85\,N$$
$$\therefore \quad \Sigma F_x = 27.85\,N \rightarrow$$

$$\Sigma F_y \quad \uparrow +ve$$
$$= 40\sin 36.87 + 50 + 25\cos 50$$
$$= 90.07\ N$$
$$\therefore \quad \Sigma F_y = 90.07\ N \uparrow$$

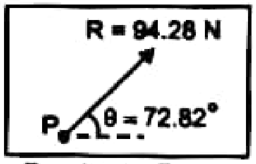
Using $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}} = \sqrt{27.85^2 + 90.07^2}$

$= 94.28$ N ... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{90.07}{27.85}$

$\therefore \quad \theta = 72.82°$ ... Direction of resultant force

The arrows of ΣF_x and ΣF_y, implies that the sense of resultant is second quadrant ... Sense of resultant force

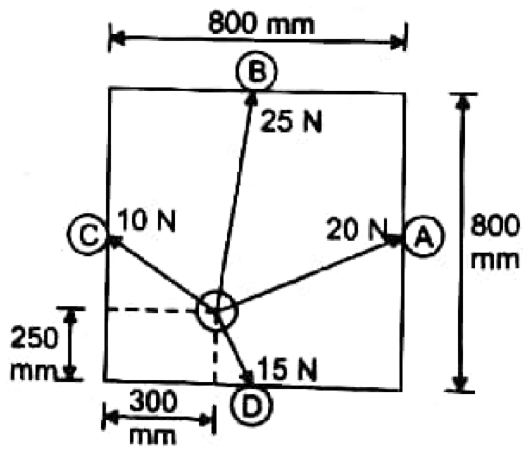


Resultant – Force

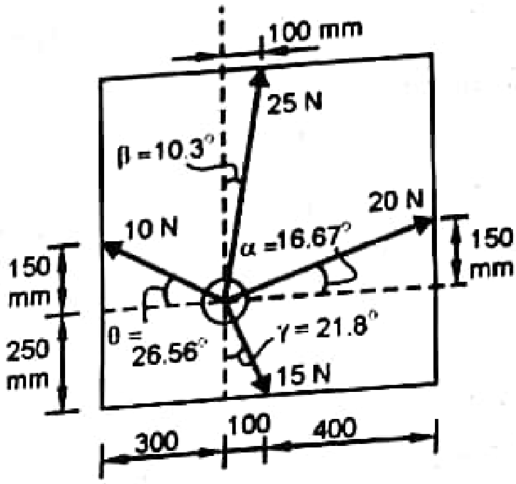
$\therefore$ Resultant is a force $R = 94.28\,N$ at $\theta = 72.82°$ acts at a point P as shown ... **Ans.**

P5. The striker of carom board lying on the board is being pulled by four players as shown in the figure. The players are sitting exactly at the centre of the four sides. Determine the resultant of forces in magnitude and direction.

(M.U. May 08, NMIMS Feb 10, KJS May 15)



Solution: This is a system of four concurrent forces.
Note: Since players are seated in the centre of each side, the distance of each player from the corner of the table is 400 mm.



Using Method of Resolution

$\Sigma F_x \quad \rightarrow +ve$

$= 20\cos 16.7 + 25\sin 10.3 - 10\cos 26.56 + 15\sin 21.8$

$= 20.25\,N$

$\therefore \quad \Sigma F_x = 20.25\ N \rightarrow$

$\Sigma F_y \quad \uparrow +ve$

$= 20\sin 16.7 + 25\cos 10.3 + 10\sin 26.56 - 15\cos 21.8$

$= 20.89\,N$

$\therefore \quad \Sigma F_y = 20.89\ N \uparrow$

Using $R = \sqrt{\Sigma F_x{}^2 + \Sigma F_y{}^2} = \sqrt{20.25^2 + 20.89^2}$

$= 29.09\ N$ ... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{20.89}{20.25} = 1.0316$

$\therefore \quad \theta = 45.89°$ ... Direction and sense of resultant force

$\therefore$ Resultant is a force $R = 29.09\,N$ at $\theta = 45.89°$ acts as shown ... **Ans.**

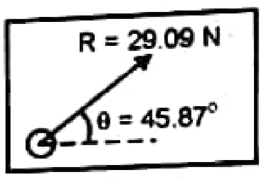
From geometry

$\tan\alpha = \dfrac{150}{500} \quad \therefore \alpha = 16.7°$

$\tan\beta = \dfrac{100}{550} \quad \therefore \beta = 10.3°$

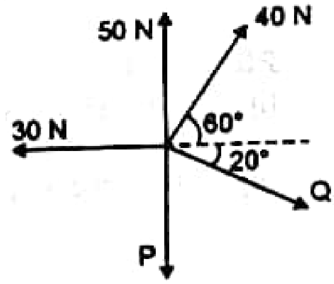
$\tan\theta = \dfrac{150}{300} \quad \therefore \theta = 26.56°$

$\tan\gamma = \dfrac{100}{250} \quad \therefore \gamma = 21.8°$



Resultant - Force

P6. Five concurrent coplanar forces act on a body as shown in figure. Find the force P and Q such that the resultant of the five forces is zero. ***(M.U. Dec 09, May 13)***



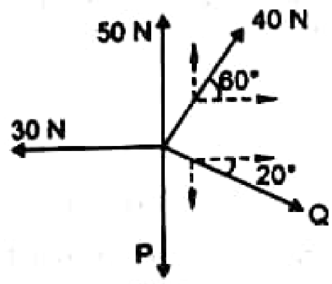
Solution: This is a system of five concurrent forces. Also given that the resultant is zero. This implies $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Using Method of Resolution

$\Sigma F_x \quad \rightarrow +ve$

$= Q\cos 20 + 40\cos 60 - 30 = 0$ .... since $\Sigma F_x = 0$

$\therefore \quad Q = 10.64\,N$ .... **Ans.**



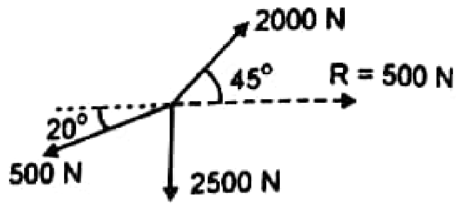
$\Sigma F_y \quad \uparrow +ve$

$= -Q\sin 20 - P + 40\sin 60 + 50 = 0$ .... since $\Sigma F_y = 0$

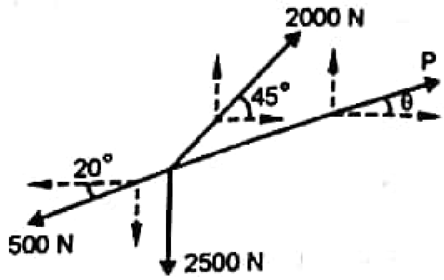
$\therefore \quad -10.64\sin 20 - P + 40\sin 60 + 50 = 0$

$\therefore \quad P = 81\ N$ .... **Ans.**

P7. Figure shows a concurrent system of four forces. Three of the four forces are shown. Find the unknown fourth force 'P' given that the resultant of the system is a horizontal force of 500 N acting to the right.



Solution: This is a system of four concurrent forces. Let the unknown fourth force P lie in the 1st quadrant and directed at an angle θ as shown.



Also given resultant force R = 500 N →
This implies $\Sigma F_x = 500\ N \rightarrow$ and $\Sigma F_y = 0$
Using Method of Resolution

$\Sigma F_x \quad \rightarrow +ve$

$\therefore \quad P\cos\theta + 2000\cos 45 - 500\cos 20 = 500$ .... since $\Sigma F_x = 500 \rightarrow$

$\therefore \quad P\cos\theta = -444.37\ N$ Or $P\cos\theta = 444.37\ N \leftarrow$ .... (1)

$\Sigma F_y \quad \uparrow +ve$

$\therefore \quad P\sin\theta + 2000\sin 45 - 500\sin 20 - 2500 = 0$ .... since $\Sigma F_y = 0$

$\therefore \quad P\sin\theta = 1256.8\ N$ Or $P\sin\theta = 1256.8\ N \uparrow$ .... (2)

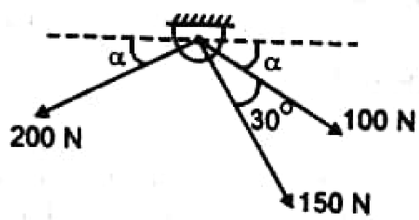
Dividing equation (2) by (1)

$\therefore \quad \dfrac{P\sin\theta}{P\cos\theta} = \dfrac{1256.8}{444.37} \qquad \therefore \quad \theta = 70.53°$

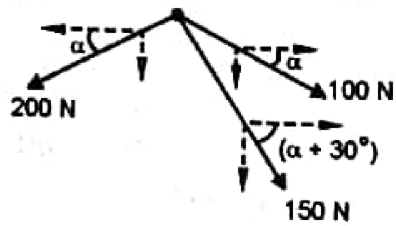
Also $\quad P\sin 70.53 = 1256.8 \qquad \therefore \quad P = 1333\ N$

The arrows of $P\cos\theta$ and $P\sin\theta$ indicate that force P lies in the 2nd quadrant
$\therefore$ Force $P = 1333 N$ acts at $\theta = 70.53°$ .... **Ans.**

P8. For the system shown, determine
(i) The required value of α if resultant of three forces is to be vertical.
(ii) The corresponding magnitude of resultant.
(M.U. Dec 08)



Solution: It is given in the problem that resultant force is vertical. This implies $\Sigma F_x = 0$ and $\Sigma F_y = R$



$\Sigma F_x \quad \rightarrow +ve$

$= 100\cos\alpha + 150\cos(\alpha + 30) - 200\cos\alpha = 0$

.... since $\Sigma F_x = 0$

$\therefore \quad 100\cos\alpha + 150[\cos\alpha \times \cos 30 - \sin\alpha \times \sin 30] - 200\cos\alpha = 0$

$\therefore \quad 100\cos\alpha + 129.9\cos\alpha - 75\sin\alpha - 200\cos\alpha = 0$

$\therefore \quad 29.9\cos\alpha = 75\sin\alpha$

$\therefore \quad \tan\alpha = 0.3987$ Or $\alpha = 21.74°$ .... **Ans.**

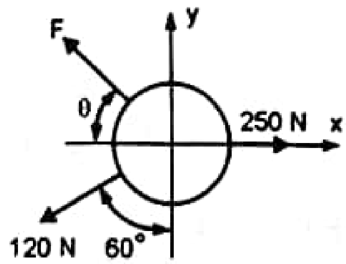
$\Sigma F_y \quad \uparrow + ve$

$= -100\sin\alpha - 150\sin(\alpha+30) - 200\sin\alpha = R$ .... since $\Sigma F_y = R$

$-100\sin 21.74 - 150\sin(21.74+30) - 200\sin 21.74 = R$

$\therefore$ $R = -228.89$ N Or $R = 228.89 \downarrow$ N .... **Ans.**

P9. A ring is pulled by three forces as shown in figure. Find the force F and the angle θ if resultant of these three forces is 100 N acting in vertical direction. ***(M.U. Dec 13)***



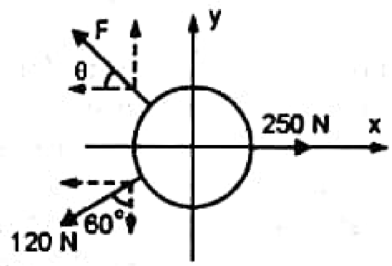
Solution: This is a system of three concurrent forces. Also given that the resultant is vertical.
This implies $\Sigma F_x = 0$ and $\Sigma F_y = R = 100N$

Using Method of Resolution

$\Sigma F_x \quad \rightarrow + ve$

$-F\cos\theta - 120\sin 60 + 250 = 0$

$\therefore$ $F\cos\theta = 146.08$ .... (1)



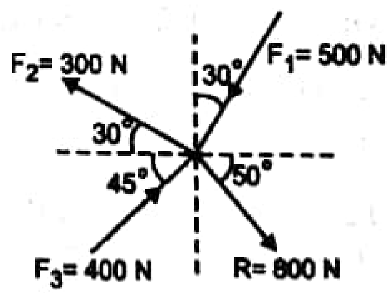
$\Sigma F_y \quad \uparrow + ve$

$F\sin\theta - 120\cos 60 = 100$

$\therefore$ $F\sin\theta = 160$ .... (2)

Solving equations (1) and (2), we get $F = 216.65$ N and $\theta = 47.6°$ .... **Ans.**

P10. Find the force F4 so as to give the resultant of the force systems shown. ***(M.U Dec 16)***

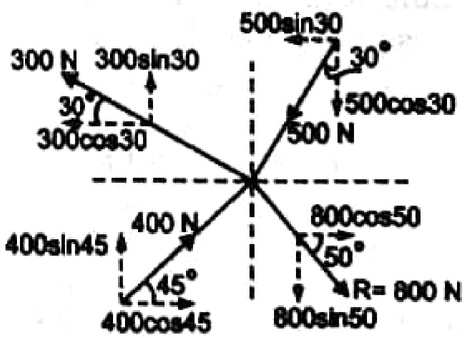


Solution: This is a system of four concurrent forces.
Let $(F_4)_x$ and $(F_4)_y$ be the components of the fourth force
Since it is given R = 800 N at θ = 50° ↘, implies
$\Sigma F_x = 800\cos 50 \rightarrow$ and $\Sigma F_y = 800\sin 50 \downarrow$

$\Sigma F_x \quad \rightarrow \quad + ve$

$-500\sin 30 - 300\cos 30 + 400\cos 45 + (F_4)_x = 800\cos 50$

$\therefore$ $(F_4)_x = 741.19$ N $\rightarrow$



$\Sigma F_y \quad \uparrow \quad + ve$

$-500\cos 30 + 300\sin 30 + 400\sin 45 + (F_4)_y = -800\sin 50$

$\therefore$ $(F_4)_y = -612.66$ N $= 612.66$ N $\downarrow$

Now $F_4 = \sqrt{(F_4)_x^{\,2} + (F_4)_y^{\,2}} = \sqrt{741.19^2 + 612.66^2} = 961.62$ N

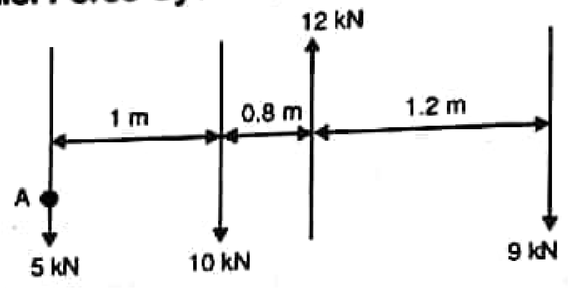
also $\tan\theta = \dfrac{(F_4)_y}{(F_4)_x} = \dfrac{612.66}{741.19}$ $\therefore$ $\theta = 39.58°$ ↘

The fourth force $F_4 = 961.62$ N at $\theta = 39.58°$ ....**Ans.**

Exercise 2.2

Resultant of Parallel Force System

P1. Determine the magnitude and position of the resultant with respect to point A, of the parallel forces shown.



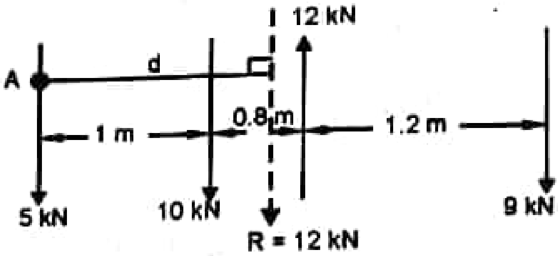
Solution: This is a system of four parallel forces.

Resultant force $R = \sum F \uparrow + ve$

$$= -5 - 10 + 12 - 9$$
$$= -12 \text{ kN} \qquad \text{Or} \qquad R = 12 \text{ kN} \downarrow$$

Location of resultant force

Let the resultant force be located at a perpendicular distance d to the right of A as shown.

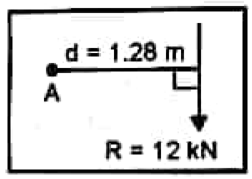


Using Varignon's theorem

$$\sum M_A{}^F = M_A{}^R \quad \circlearrowleft + ve$$
$$-(10\times1)+(12\times1.8)-(9\times3) = -(12\times d)$$

$\therefore \quad d = 1.28\,\text{m}$

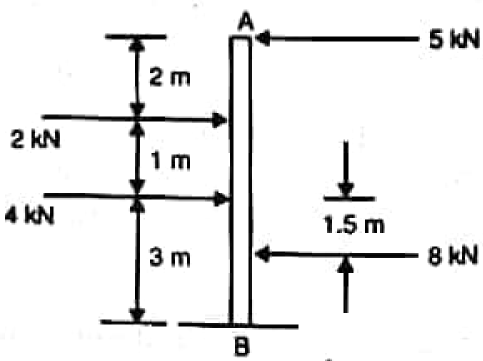
Or $\quad$ d = 1.28 m to the right of A



Resultant - Force

$\therefore$ The resultant is R = 12 kN $\downarrow$ is located at a $\perp^{er}$ distance d = 1.28 m right of A as shown in figure. .... **Ans.**

P2. Find the magnitude, nature and position of the resultant of the four parallel forces from B.



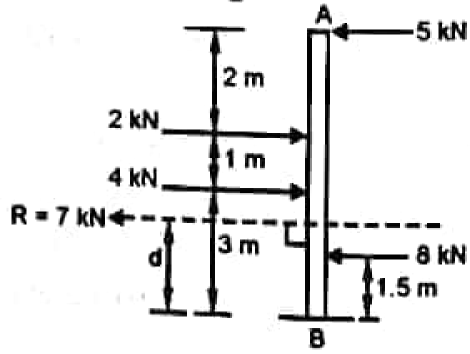
Solution: This is a system of four parallel forces.

Resultant force $R = \sum F \rightarrow + ve$

$$\therefore \quad R = -8 + 4 + 2 - 5$$
$$= -7 \text{ kN}$$
$$\text{Or} \qquad R = 7 \text{ kN} \leftarrow$$

Location of resultant force

Let the resultant force be located at a perpendicular distance d above B as shown.

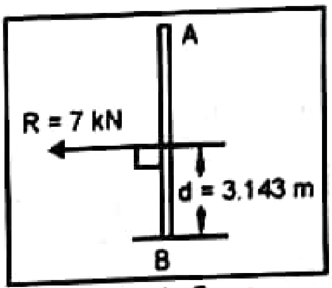


Using Varignon's theorem

$$\sum M_B{}^F = M_B{}^R \quad \circlearrowleft + ve$$
$$+(5\times6)-(2\times4)-(4\times3)+(8\times1.5) = +(7\times d)$$

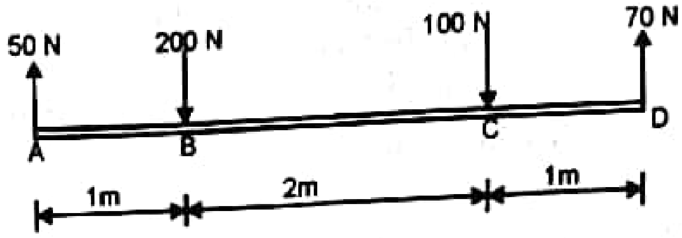
$\therefore \quad d = 3.143\,\text{m} \qquad$ Or $\qquad$ d = 3.143 m above B

∴ The resultant is R = 7 kN ← is located at a ⊥^{er} distance d = 3.143 m above B as shown in figure. .... **Ans.**



Resultant - Force

P3. A system of parallel, non-concurrent forces is acting on a rigid bar. Reduce this system of forces to
i) A single forces R & its position
ii) A single force R & a couple at B
(VJTI Nov 10)



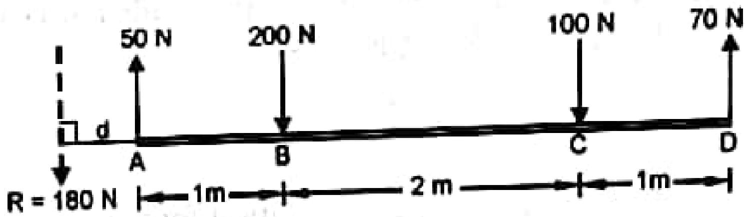
Solution: This is a system of four parallel forces. Resultant force,

$R = \sum F \quad \uparrow + ve$

∴ $R = 50 - 200 - 100 + 70$

$= -180\ N$ Or $R = 180\ N \downarrow$

Location of resultant force
Let us assume the resultant force is located at a perpendicular distance d to the left of A as shown.



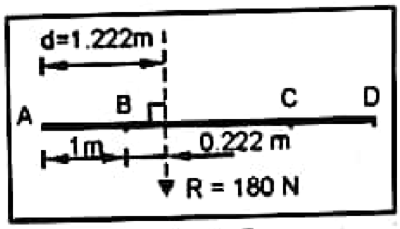
Using Varignon's theorem

$\sum M_A^{\ F} = M_A^{\ R} \quad ↺ + ve$

$-(200\times1)-(100\times3)+(70\times4) = +(180\times d)$

∴ $d = -1.222\,m$

Or $d = 1.222$ .... right of A



Resultant - Force

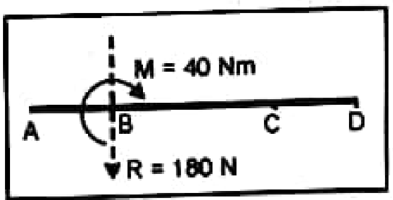
∴ The resultant is R = 180 N ↓ is located at a ⊥ distance d = 1.222 m right of A as shown in figure. .... **Ans.**

b) To replace the system by a couple at B, we need to shift the force R to B by introducing a couple M. The ⊥ distance between point B and force R is 0.222 m

Couple M $= F \times d$

$= -180 \times 0.222 = -40\ Nm$

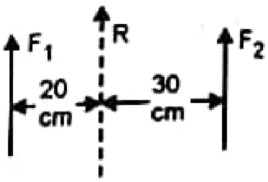
or $M = 40\ Nm$ ↻



Resultant - Force Couple at B

The Resultant force couple at B is as shown in figure.

P4. Find the magnitude of two like parallel forces F_1 and F_2 acting at a distance of 50 cm apart, if their resultant is 300 N and acts at a distance of 20 cm from one of the force.



Solution: This is a system of two parallel forces.

Resultant force $R = \Sigma F \uparrow +ve$

$\therefore \quad 300 = F_1 + F_2 \quad$ .... (1)

Also given resultant force R = 300 N ↑, located at a ⊥ distance of 20 cm from force F_1.

Using Varignon's theorem

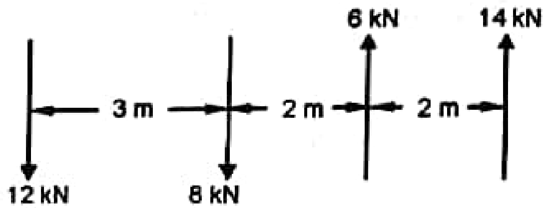
$\Sigma M_A^F = M_A^R \ \circlearrowleft +ve$ .... here let A be a point on force F_1

$F_2 \times 50 = 300 \times 20$

$\therefore \quad F_2 = 120$ N .... **Ans.**

Substituting value of F_2 in equation (1), we get $F_1 = 180$ N .... **Ans.**

P5. Determine the resultant of the parallel forces shown.



Solution: This is a system of four parallel forces.

Resultant force $R = \Sigma F \uparrow +ve$

$\therefore \quad = -12 - 8 + 6 + 14$

$\therefore \quad R = 0$

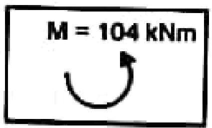
For parallel system if the resultant force is zero, it implies that the resultant is a couple.

Couple $M = \Sigma M_A^F \ \circlearrowleft +ve$ .... here let A be a point on 12 kN force

$= -(8 \times 3) + (6 \times 5) + (14 \times 7) = 104$ kNm

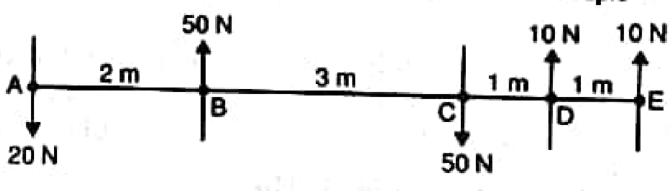
Or couple M = 104 kNm ↺

$\therefore$ The resultant is a couple M = 104 kNm ↺ as shown ... **Ans.**



Resultant - Couple

P6. Determine the resultant of the system shown.



Solution: This is a system of five parallel forces. For a parallel system,

Resultant force $R = \Sigma F \uparrow +ve$

$\therefore \quad = -20 + 50 - 50 + 10 + 10$

$\therefore \quad R = 0$ .... implies resultant force is zero.

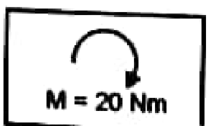
For parallel system if the resultant force is zero, it implies that the resultant is a couple.

Couple $M = \Sigma M_A^F \ \circlearrowleft +ve$

$= +(50 \times 2) - (50 \times 5) + (10 \times 6) + (10 \times 7)$

$= -20$ Nm Or Couple M = 20 Nm ↻

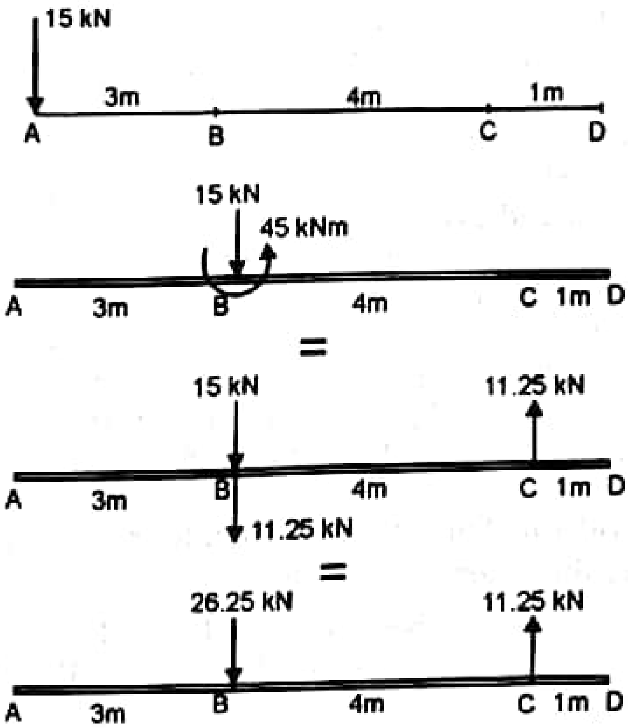
$\therefore$ The resultant is a couple M = 20 Nm ↻ as shown .... **Ans.**



Resultant - Couple

P7. Resolve 15 kN force acting at 'A' into two parallel components at B and C

(M.U. Dec 11)



Solution: To resolve the 15 kN force into two parallel component at B and C, we need to first shift the 15 kN force to either B or C.

To shift the 15 kN force to B, we need to add a couple. *(Refer properties of couple)*

Couple M $= 15 \times 3$

$= 45$ kNm ↺

(... force 15 kN is shifted by 3 m)

The couple of 45 kNm ↺ can be converted into two parallel forces at B and C equal in magnitude and apposite in sense.

Couple M = F × d

45 = F × 4 .... since points B and C are 4 m apart.

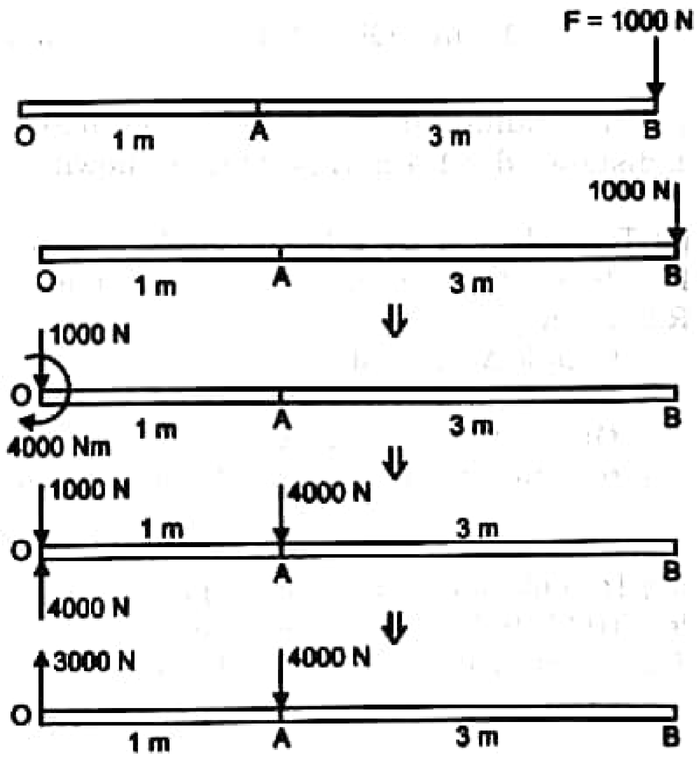
Or F = 11.25 kN

The two parallel forces are, Force F = 11.25 kN ↓ at B and F = 11.25 kN ↑ at C.

Adding forces at B we get the two parallel components of 15 kN force as 26.25 kN ↓ at B and 11.25 kN ↑ at C as shown. .... **Ans.**

P8. Resolve the force F = 1000 N acting at B into parallel component forces at O and A.

(VJTI Dec 16)



Solution: To resolve the 1000 N force into two parallel components at O and A, we need to first shift the 1000 N force to either O or A.

To shift the 1000 N force to O, we need to add a couple.

Couple M $= 1000 \times 4$

$= 4000$ Nm ↻

(... force 1000 N is shifted by 4 m)

The couple of 4000 Nm ↻ can be converted into two parallel forces at O and A equal in magnitude and apposite in sense.

Couple M = F × d

4000 = F × 1 .... since points O and A are 1 m apart.

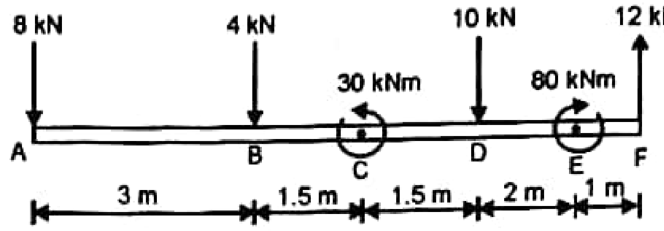
Or F = 4000 N

The two parallel forces are, Force F = 4000 N ↓ at A and F = 4000 N ↑ at O.

Adding forces at O we get the two parallel components of 1000 N force as 3000 N ↑ at O and 4000 N ↓ at A as shown. .... **Ans.**

P9. Figure shows a parallel system of four forces and two couples.

(i) Replace it by a single force and obtain its location from point A



(ii) Replace it by a force couple system at point A.

(iii) Replace it by a force couple system at point D.

(iv) Replace it by two parallel forces at B and D.

Solution: This is a system of four parallel forces and two couples.

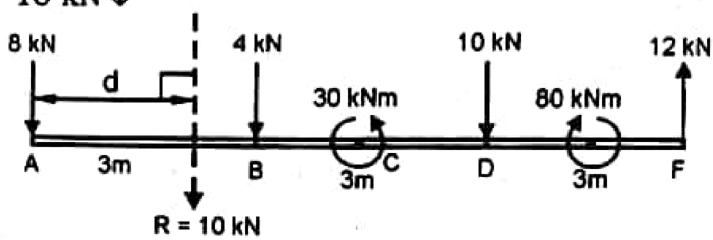
Resultant force $R = \Sigma F \uparrow +ve$

$\therefore \quad R = -8 - 4 - 10 + 12$

$= -10$ kN Or $R = 10$ kN ↓

Location of resultant force

Let us assume the resultant force is located at a perpendicular distance d to the right of A as shown.



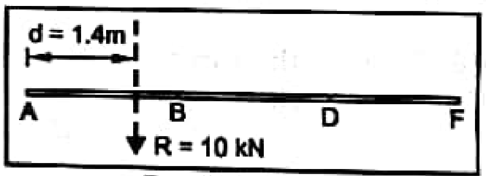
Using Varignon's theorem

$\Sigma M_A{}^F = M_A{}^R$ ↺ + ve

$-(4\times3)-(10\times6)+(12\times9)+30-80=-(10\times d)$

$\therefore \quad d = 1.4\,m$ Or d = 1.4 m .... right of A

$\therefore$ The resultant is R = 10 kN ↓ is located at a $\perp$ distance d = 1.4 m right of A as shown. ... **Ans.**



Resultant - Force

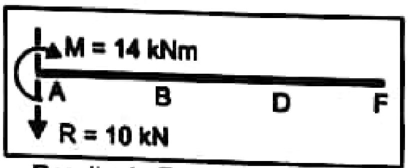
(ii) To replace it by a force couple system at point A, we need to shift resultant force R = 10 kN to A by introducing a couple M. The $\perp$ distance between point A and force R is 1.4 m

Couple M = F × d

= − (10 × 1.4) = − 14 kNm

Or M = 14 kNm ↻

The resultant force couple at A is as shown in figure.



Resultant - Force Couple at A

(iii) To replace it by a force couple system at point D, we need to shift the force R = 10 kN to D by introducing a couple M.

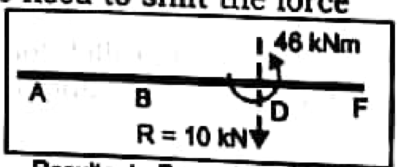
The $\perp$ distance between D and force R is 4.6 m

Couple M = F × d

= + (10 × 4.6) = 46 kNm ↺

Or M = 46 kNm ↺

The force couple system is as shown in figure.



Resultant - Force Couple at D

(iv) To replace it by two parallel forces at B and D.

The force couple system at D is shown. The couple of 46 kNm can be replaced by two parallel forces at B and D, equal in magnitude and opposite in sense.

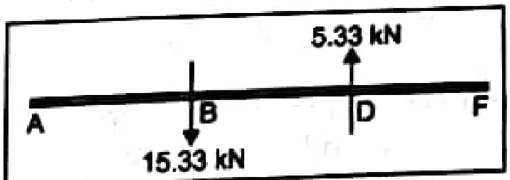
Couple M = F × d

46 = F × 3 .... since points B and D are 3 m apart.

Or F = 15.33 kN

∴ Force F = 15.33 kN ↓ at B and F = 15.33 kN ↑ at D can replace the couple of 46 kNm

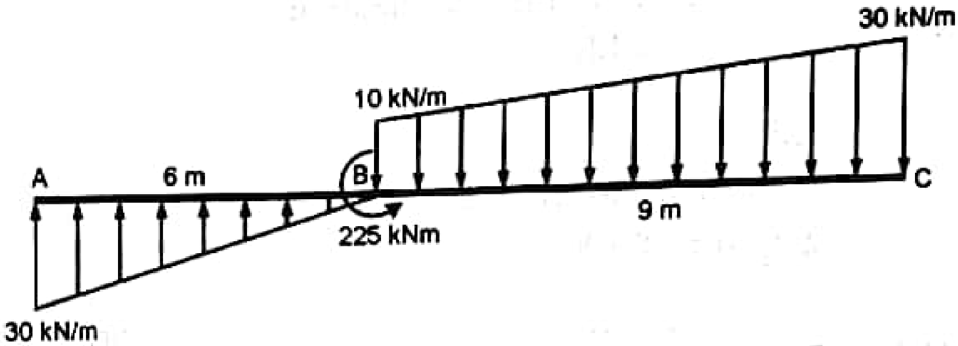
Adding forces at D i.e. $-10+15.33=5.33$ kN, we get the two parallel components as 15.33 kN ↓ at B and 5.33 kN ↑ at D as shown.



System replaced as two forces at B and D

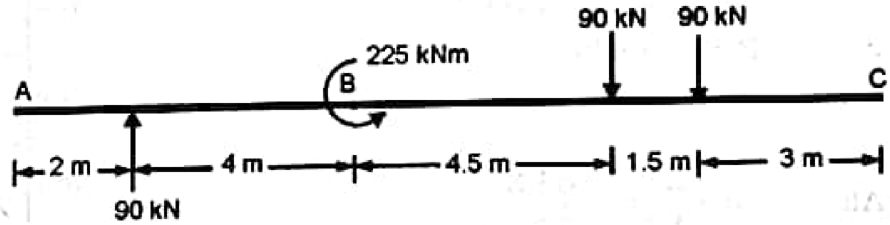
P10. A member ABC is loaded by distributed load and pure moment as shown in the figure. Find the (i) magnitude and (ii) position along AC of the resultant.

(M.U. Dec 13, KJS Dec 14)



[Solve this problem after referring 'Types of loads' from Chapter 3]

Solution: The u v l and trapezoidal loads are converted to point loads as shown.



This is a system of three parallel forces at one couple.

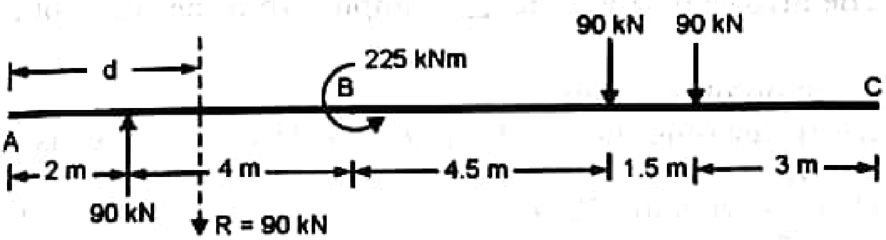
Resultant force R = ΣF_y ↑ + ve

∴ $R = 90-90-90$

∴ $R=-90$ kN or $R = 90$ kN ↓

Location of resultant force

Let us assume the resultant force is located at a perpendicular distance d to the right of A as shown.

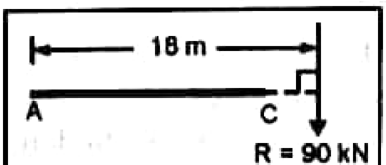


Using Varignon's theorem

$\Sigma M_A^F = M_A^R$ ↺ + ve

$+(90\times2)+225-(90\times10.5)-(90\times12)=-(90\times d)$

∴ $d=18\,m$ Or d = 18 m .... right of A



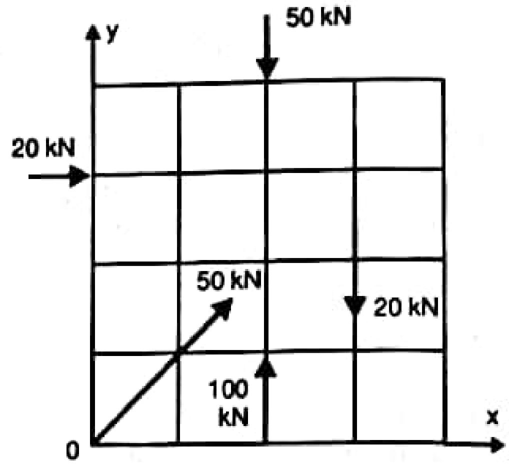
Resultant Force

∴ The resultant is R = 90 kN ↓ is located at a ⊥ dist. d = 18 m right of A as shown. **Ans.**

Exercise 2.3

Resultant of General Force System

P1. Determine the resultant of the force system shown. The side of each small square is 1 m. The overall size of the body is 4 m × 4 m.



Solution: This is a system of five general forces.
Using Method of Resolution

$\Sigma F_x \quad \rightarrow + ve$

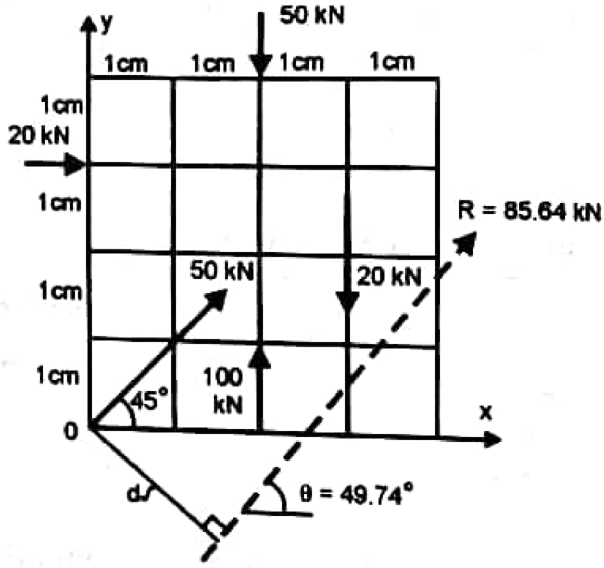
$= 20 + 50\cos 45 \quad = \quad 55.35 \text{ kN}$

$\therefore \quad \Sigma F_x = 55.35 \text{ kN} \rightarrow$

$\Sigma F_y \quad \uparrow + ve$

$= 50\sin 45 + 100 - 50 - 20 \; = \; 65.35 \text{ kN}$

$\therefore \quad \Sigma F_y = 65.35 \text{ kN} \uparrow$



Using $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}}$

$= \sqrt{55.35^2 + 65.35^2}$

$\therefore \quad R = 85.64 \text{ kN}$

.... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{65.35}{55.35}$

Or $\quad \theta = 49.74°$

.... Direction of resultant force

The arrows of ΣF_x and ΣF_y implies that the sense of resultant is first quadrant i.e. $\measuredangle$

.... Sense of resultant force

Location of resultant force
Let us assume the resultant force is located at a ⊥ distance d to the right of O as shown.
Using Varignon's Theorem

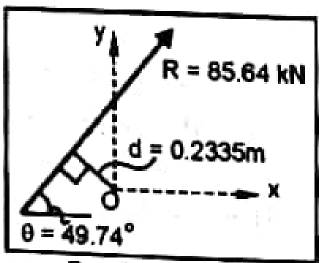
$\Sigma M_O^{\,F} = M_O^{\,R} \quad \circlearrowleft + ve$

$-(20 \times 3) - (50 \times 2) - (20 \times 3) + (100 \times 2) = 85.64 \times d$

$\therefore \quad d = -0.2335$

Or $\quad d = 0.2335$ m to the left of O

.... Location of resultant force

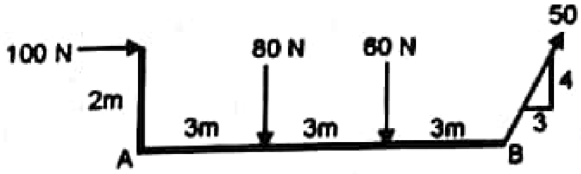


Resultant - Force

$\therefore$ The resultant force R = 85.64 kN at θ = 49.74° $\measuredangle$ acts at a ⊥ distance d = 0.2335 m left of O as shown. ... **Ans.**

P2. Determine the resultant of the given force system. Also find out where the resultant force will meet arm AB. Take A as the origin.

(SPCE Dec 10)



Solution: This is a system of four general forces.
Using Method of Resolution

$\Sigma F_x \quad \rightarrow +\text{ve}$

$= 100 + 50\cos 53.13 \quad = \quad 130\text{N}$

$\therefore \quad \Sigma F_x = 130\text{N} \rightarrow$

$\Sigma F_y \quad \uparrow +\text{ve}$

$= -80 - 60 + 50\sin 53.13 \quad = \quad -100\text{ N}$

$\therefore \quad \Sigma F_y = 100\text{N} \downarrow$

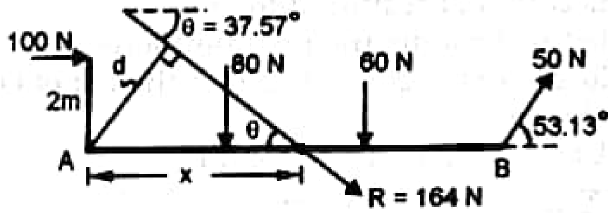
Using $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}} \quad = \sqrt{130^2 + 100^2}$

$\therefore \quad R = 164\text{N}$ .... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{100}{130}$ Or $\theta = 37.57°$ .... Direction of resultant force

The arrows of ΣF_x and ΣF_y implies that the sense of resultant is fourth quadrant i.e. ↘ .... Sense of resultant force

Location of resultant force
Let us assume the resultant force is located at a ⊥ distance d to the right of A as shown.



Using Varignon's Theorem

$\Sigma M_A^{\,F} = M_A^{\,R} \quad ↻ +\text{ve}$

$-(100\times2) - (80\times3) - (60\times6) + (50\sin 53.13\times9) = -(164\times d)$

$\therefore \quad d = 2.683\text{ m}$

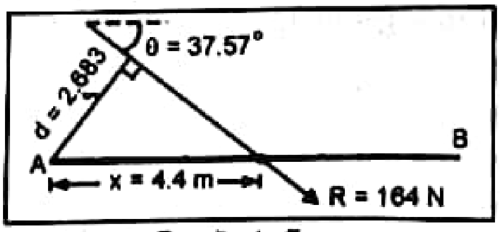
Or $\quad d = 2.683$ m to the right of A .... location of resultant force

Let the resultant force R cut the arm AB at a distance x from A.

From geometry $\sin\theta = \dfrac{d}{x}$

$\therefore \quad \sin 37.57 = \dfrac{2.683}{x}$

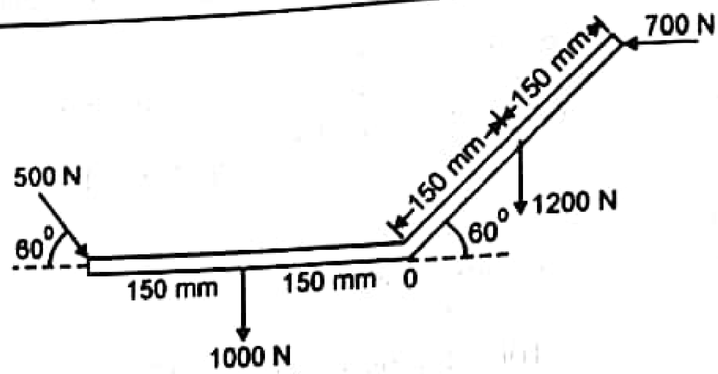
Or $\quad x = 4.4\text{ m}$



Resultant - Force

∴ The resultant is a force R = 164 N at θ = 37.57° ↘ is located at x = 4.4 m on arm AB as shown ... **Ans.**

P3. a) A system of forces acting on a bell crank lever is as shown. Determine the magnitude, direction and the point of application of the resultant w.r.t 'O'.
b) Also find the location of the resultant on the horizontal arm of the lever. ***(M.U. Dec 08, May 14)***



Solution: This is a system of four general forces.
Using Method of Resolution

$\Sigma F_x \rightarrow +ve$

$= 500\cos 60 - 700 = -450\,N \qquad \therefore \quad \Sigma F_x = 450\,N \leftarrow$

$\Sigma F_y \uparrow +ve$

$= -500\sin 60 - 1000 - 1200 = -2633\ N \qquad \therefore \quad \Sigma F_y = 2633\,N \downarrow$

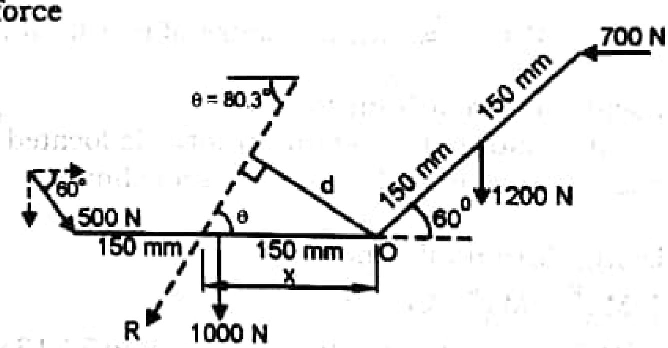
Using $R = \sqrt{\Sigma {F_x}^2 + \Sigma {F_y}^2} = \sqrt{450^2 + 2633^2}$

$\therefore \quad R = 2671\,N$ .... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{2633}{450}$ Or $\theta = 80.3°$......... Direction of resultant force

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the third quadrant i.e. ↙ .... Sense of resultant force

Location of resultant force
Let us assume the resultant force R is located at a ⊥ distance d to the left of O as shown.
Using Varignon's Theorem
$\Sigma M_O{}^F = M_O{}^R$ ↺ +ve



$(500\sin 60 \times 300) + (1000 \times 150) - (1200 \times 150\cos 60) + (700 \times 300\sin 60) = 2671 \times d$

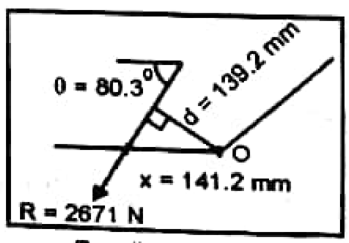
$\therefore \quad d = 139.2$ mm

Or $\quad d = 139.2$ mm to the left of O .... Location of resultant force

Let the resultant force R cut the horizontal arm AB at a distance x from O.

From geometry $\sin\theta = \dfrac{d}{x} \quad \therefore \sin 80.3 = \dfrac{139.2}{x}$

Or $\quad x = 141.2$ mm

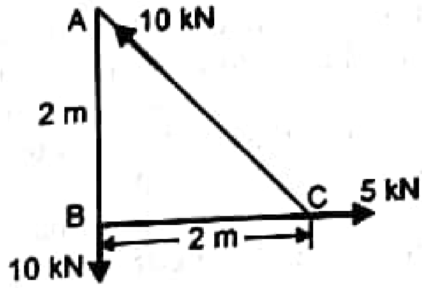


Resultant - Force

$\therefore$ The resultant is a force R = 2671 N at θ = 80.3° ↙ is located at a ⊥ distance d = 139.2 mm to left of O and cut the horizontal arm AB at x = 141.2 mm from O as shown ... **Ans.**

P4. Find the resultant of the system of coplanar forces shown in figure.

(VJTI May 08)



Solution: This is a system of three general forces.
Using Method of Resolution

$\Sigma F_x \;\; \rightarrow + ve$

$= 5 - 10\cos 45$

$= -2.071$ kN

$\therefore \;\; \Sigma F_x = 2.071\,\text{kN} \leftarrow$

$\Sigma F_y \;\; \uparrow + ve$

$= -10 + 10\sin 45$

$= -2.929\,\text{kN}$

$\therefore \;\; \Sigma F_y = 2.929\,\text{kN} \downarrow$

Using $R = \sqrt{\Sigma {F_x}^2 + \Sigma {F_y}^2} \quad = \sqrt{2.071^2 + 2.929^2}$

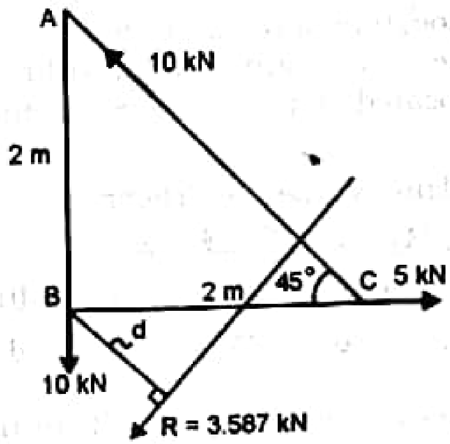
$\therefore \quad R = 3.587\,\text{kN}$ .... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{2.929}{2.071}$

Or $\quad \theta = 54.74°$ .... Direction of resultant force

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the third quadrant i.e. ↙ .... Sense of resultant force

Location of resultant force
Let us assume the resultant force R is located at a ⊥ distance d to the right of B as shown.



Using Varignon's Theorem

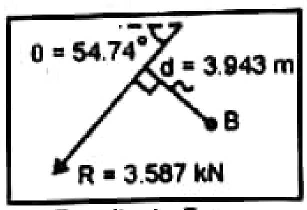
$\Sigma {M_B}^F = {M_B}^R \;\; \circlearrowleft + ve$

$+(10\sin 45 \times 2) = -(3.587 \times d)$

$\therefore \quad d = -3.943$ m

Or $\quad d = 3.943$ m to the left of B .... Location of resultant force

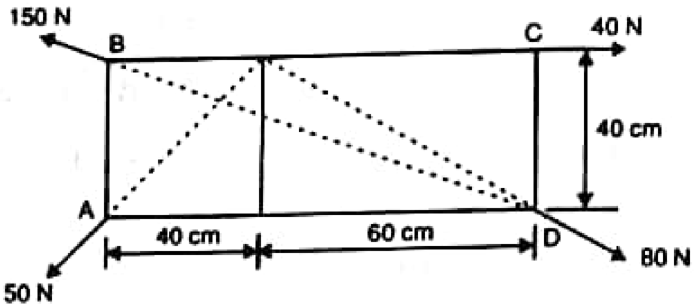
$\therefore$ The resultant is a force R = 3.587 kN at θ = 54.74° ↙ is located at a ⊥ distance d = 3.943 m to left of B as shown .... **Ans.**



Resultant - Force

P5. a) A block ABCD of 100 cm × 40 cm dimensions is acted upon by four forces as shown. Calculate the resultant and then state its position with reference to A.

b) Also find the location x where the resultant force cuts the base AD.



Solution: Using Method of Resolution

$\Sigma F_x \quad \rightarrow + ve$

$$= -150\cos 21.8 - 50\cos 45 + 80\cos 33.69 + 40 \quad = \quad -68.06\text{N}$$

$\therefore \quad \Sigma F_x = 68.06\text{N} \leftarrow$

$\Sigma F_y \quad \uparrow + ve$

$$= 150\sin 21.8 - 50\sin 45 - 80\sin 33.69 \quad = \quad -24.03 \text{ N}$$

$\therefore \quad \Sigma F_y = 24.03\text{N} \downarrow$

Using $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{68.06^2 + 24.03^2}$

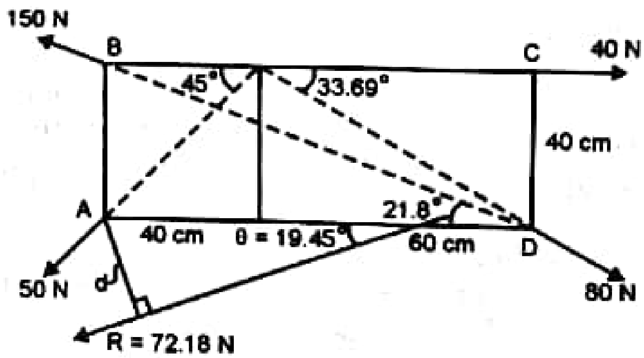
$\therefore \quad R = 72.18\text{N}$

.... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{24.03}{68.06}$

Or $\theta = 19.45°$ .... Direction of resultant force

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the third quadrant i.e. ↙ .... sense of resultant force



Location of resultant force

Let us assume the resultant force R is located at a ⊥ distance d to the right of A.

Using Varignon's Theorem

$\Sigma M_A^{\,F} = M_A^{\,R} \quad ↻ + ve$

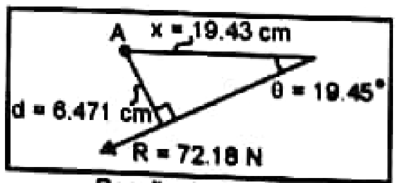
$$+(150\sin 21.8 \times 100) - (40 \times 40) - (80\sin 33.69 \times 100) = -(72.18 \times d)$$

$\therefore \quad d = 6.471$ cm or $d = 6.471$ cm to the right of A ... Location of resultant force

Let the resultant force R cut the base AD at a distance x from A as shown.

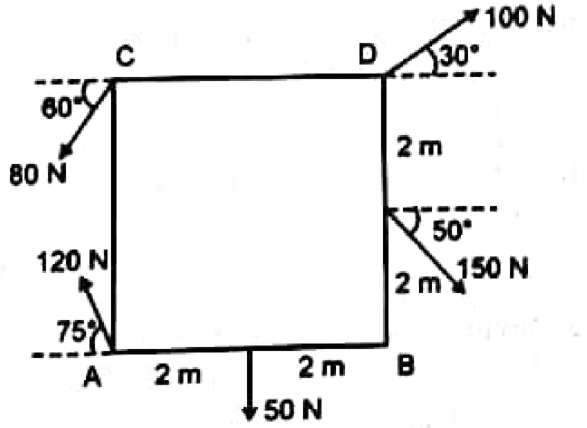
From geometry $\sin\theta = \dfrac{d}{x} \quad \therefore \quad \sin 19.45 = \dfrac{6.471}{x}$ Or $x = 19.43$ cm

$\therefore$ The resultant force is R = 72.18 N at θ = 19.45° ↙ is located at a ⊥ distance d = 6.471 cm to right of A and cuts the base AD at distance x = 19.43 cm from A as shown. .... **Ans.**



Resultant - Force

P6. Determine the resultant of the system of forces shown in figure. Locate the point where the resultant cuts the base AB. ***(M.U. Dec 09)***



Solution: This is a system of five general forces.
Using Method of Resolution

$\Sigma F_x \rightarrow +ve$

$= 100\cos 30 + 150\cos 50 - 120\cos 75 - 80\cos 60$

$= 111.96 N$

$\therefore \ \Sigma F_x = 111.96 \text{ N} \rightarrow$

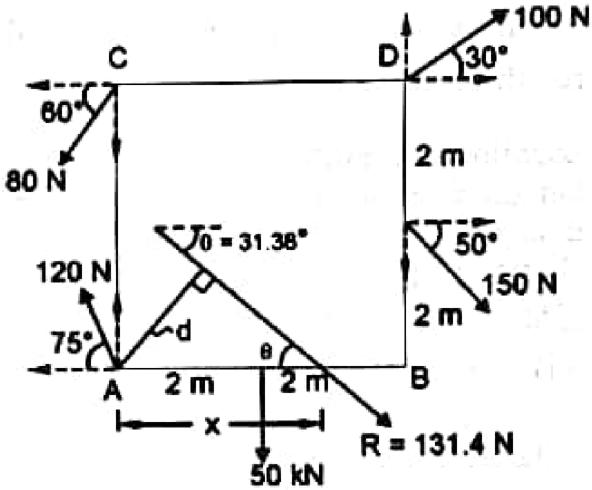
$\Sigma F_y \uparrow +ve$

$= 100\sin 30 - 150\sin 50 - 80\sin 60 + 120\sin 75 - 50 = -68.28 \text{ N}$

$\therefore \ \Sigma F_y = 68.28 \text{ N} \downarrow$

Using $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}} = \sqrt{111.96^2 + 68.28^2}$ $\therefore$ $R = 131.14 N$.:. Magnitude

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{68.28}{111.96}$ Or $\theta = 31.38°$ .... Direction of resultant force



The arrows of ΣF_x and ΣF_y indicate that the of resultant force is located in 4th quad. i.e. ↘ .... Sense of resultant force

Location of resultant force
Let us assume the resultant force R is located at a ⊥ distance d to the right of A as shown.

Using Varignon's Theorem

$\Sigma M_A^{\ F} = M_A^{\ R}$ ↺ +ve

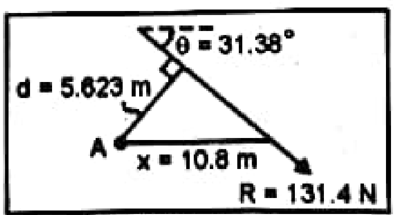
$$+(80\cos 60 \times 4) - (100\cos 30 \times 4) + (100\sin 30 \times 4)$$
$$-(150\cos 50 \times 2) - (150\sin 50 \times 4) - (50 \times 2) = -(131.4 \times d)$$

$\therefore$ d = 5.623 m Or d = 5.623 m to the right of A ... Location of resultant force

Let the resultant force R cut the base AB at a distance x from A as shown.

From geometry $\sin\theta = \dfrac{d}{x}$ $\therefore$ $\sin 31.38 = \dfrac{5.623}{x}$

Or x = 10.8 m



Resultant - Force

$\therefore$ The resultant force is R = 131.4 N at θ = 31.38° ↘ is located at a ⊥ distance d = 5.623 m to right of A and cuts the base AB at distance x = 10.8 m from A as shown

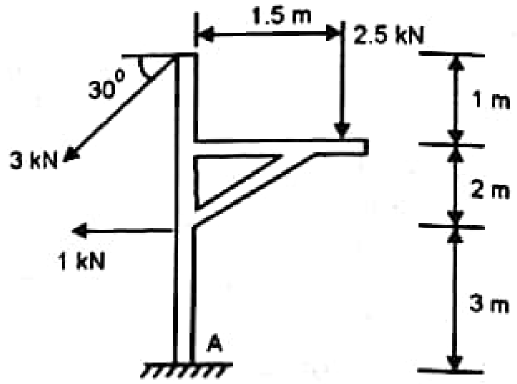
.... **Ans.**

P7. Three forces 1 kN, 3 kN and 2.5 kN act on a vertical pole 6 m high.

a. Find the magnitude, direction and position of resultant w.r.t A

b. The position where the resultant cuts the pole from the base

c. Reduce it to a force couple system at A.



Solution: a) This is a system of 3 general forces. Using Method of Resolution

$$\Sigma F_x \quad \rightarrow +\text{ve}$$

$$= -3\cos 30 - 1 = -3.598\,\text{kN} \qquad \therefore \qquad \Sigma F_x = 3.598\ \text{kN} \leftarrow$$

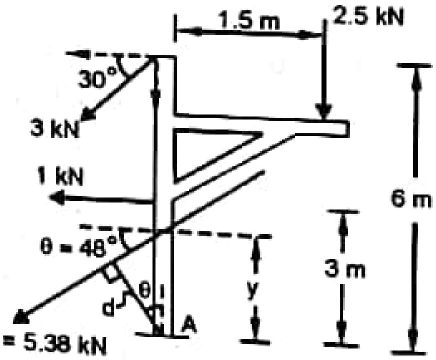
$$\Sigma F_y \quad \uparrow +\text{ve}$$

$$= -3\sin 30 - 2.5 = -4\ \text{kN} \quad \therefore \qquad \Sigma F_y = 4\ \text{kN} \downarrow$$

Using $R = \sqrt{\Sigma F_x{}^2 + \Sigma F_y{}^2} = \sqrt{3.598^2 + 4^2} \quad \therefore \quad R = 5.38\,\text{kN}$ ... Magnitude of resultant

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{4}{3.598}$ Or $\theta = 48°$ .... Direction of resultant force

$\therefore$ The arrows of ΣF_x and ΣF_y indicate that the of resultant force is located in 3rd quadrant i.e. ↙



Location of resultant force

Let us assume the resultant force R is located at a ⊥ distance d to the left of A as shown.

Using Varignon's Theorem

$$\Sigma M_A{}^F = M_A{}^R \quad ↺ +\text{ve}$$

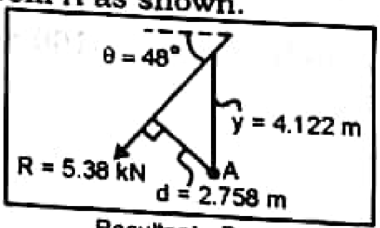
$$+(1\times 3) + (3\cos 30 \times 6) - (2.5\times 1.5) = +(5.38\times d)$$

$\therefore$ d = 2.758 m Or d = 2.758 m to the left of A ... Location of resultant force

b) Let the resultant force R cut the pole at a vertical distance y from A as shown.

From geometry $\cos\theta = \dfrac{d}{y} \quad \therefore \quad \cos 48 = \dfrac{2.758}{y}$

Or y = 4.122 m



Resultant - Force

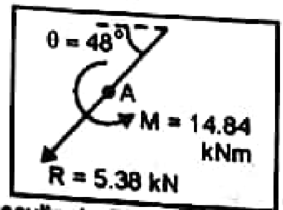
$\therefore$ The resultant force is R = 5.38 kN at θ = 48° ↙ is located at a ⊥ distance d = 2.758 m to left of A and cuts the pole at vertical distance y = 4.122 m from A as shown ... **Ans.**

c) To replace the system by a force couple at point A, we need to shift the resultant force R = 5.38 kN to A by introducing a couple M. The ⊥ distance between point A and force R is 2.758 m

Couple M = F × d

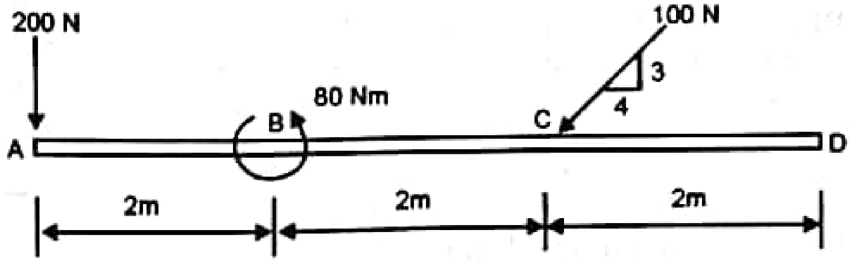
= + (5.38 × 2.758) = 14.84 kNm Or M = 14.84 kNm ↺

The resultant force couple at A is shown in figure. .... **Ans.**



Resultant - Force Couple at A

P8. Resolve the system of forces shown into a force and couple at point 'A'. ***(M. U. Dec 07)***



Solution: This is a system of two forces and a couple.

Using Method of Resolution

$\Sigma F_x \quad \rightarrow +ve$

$= -100\cos 36.87 \quad = \quad -80\,N$

$\therefore \quad \Sigma F_x = 80\,N \leftarrow$

$\Sigma F_y \quad \uparrow +ve$

$= -200 - 100\sin 36.87 \quad = \quad -260\ N$

$\therefore \quad \Sigma F_y = 260\ N \downarrow$

Using $R = \sqrt{\Sigma F_x{}^2 + \Sigma F_y{}^2} \quad = \sqrt{80^2 + 260^2}$

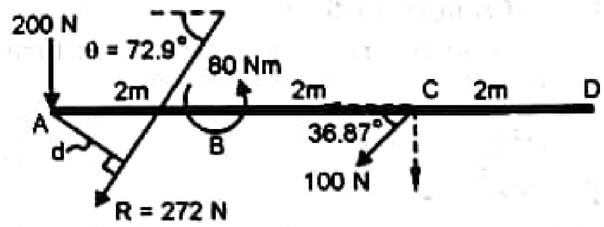
$\therefore \quad R = 272\,N$ .... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{260}{80}$ Or $\theta = 72.9°$......... Direction of resultant force

$\therefore$ The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the 3rd quadrant i.e. ↙ .... Sense of resultant force

Location of resultant force

Let us assume the resultant force R is located at a ⊥ distance d to the right of A as shown.



Using Varignon's Theorem

$\Sigma M_A{}^F = M_A{}^R \quad \circlearrowleft +ve$

$+80 - (100\sin 36.87 \times 4) = -(272 \times d)$

$\therefore \quad d = 0.588\ m$

Or $\quad d = 0.588$ m to the right of A

.... Location of resultant force

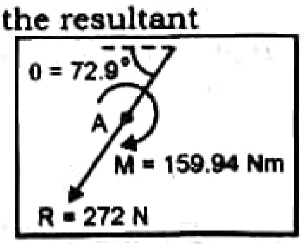
To replace the system by a force couple at point A, we need to shift the resultant force R = 272 N to A by introducing a couple M.

The ⊥ distance between point A and force R is 0.588 m

Couple $M = F \times d$

$= -(272 \times 0.588) \quad = \quad -159.94\ Nm$

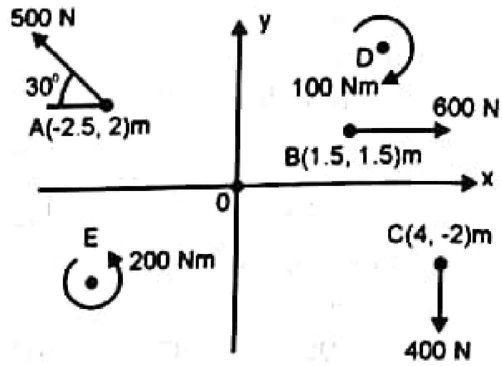
Or $\quad M = 159.94\ Nm \circlearrowright$



Resultant - Force couple at A

The resultant force couple at A is shown. .... **Ans.**

P9. Replace the force system shown by a single force acting at the origin and couple.



Solution: This is a system of three general forces and two couples.
Using method of resolution

$\Sigma F_x \;\; \rightarrow +ve$

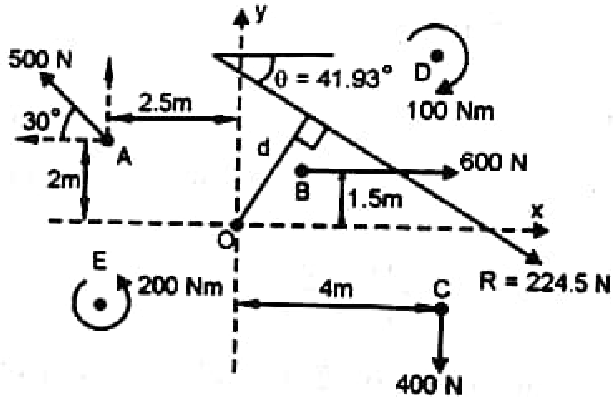
$= -500\cos 30 + 600 \;=\; 167 \text{ N}$

$\therefore \quad \Sigma F_x = 167 \text{ N} \rightarrow$

$\Sigma F_y \;\; \uparrow +ve$

$= 500\sin 30 - 400 \;=\; -150 \text{ N}$

$\therefore \quad \Sigma F_y = 150 \text{ N} \downarrow$



Using $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}} = \sqrt{167^2 + 150^2}$

$\therefore \quad R = 224.5\text{N}$ .... Magnitude

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{150}{167}$

Or $\quad \theta = 41.93°$ .... Direction

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the 4th quadrant. i.e. ↘ .... Sense of resultant force

Location of resultant force
Let us assume the resultant force R is located at a ⊥ distance d to the right of origin O as shown.

Using Varignon's Theorem

$\Sigma M_O^{\,F} = M_O^{\,R} \;\; \circlearrowleft +ve$

$-(600\times1.5) - (400\times4) + (500\cos 30\times2) - (500\sin 30\times2.5) + 200 - 100 = -(224.5\times d)$

$\therefore \quad d = 9.617 \text{ m} \quad$ Or $\quad d = 9.617$ m to the right of O ... Location of resultant force

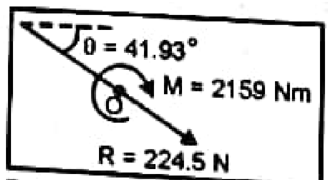
To replace the system by a force couple at point O, we need to shift the resultant force R = 224.5 N to Origin O by introducing a couple M.
The ⊥ distance between point O and force R is 9.617 m

Couple M = F × d

= – (224.5 × 9.617) = – 2159 Nm

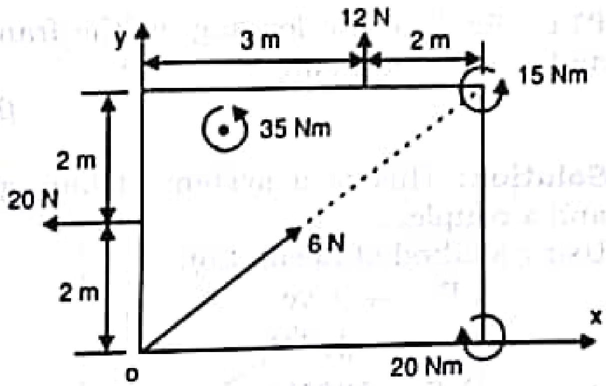
Or M = 2159 Nm ↻

The resultant force couple at origin O is shown ... **Ans.**



Resultant - Force couple at O

P10. Replace the system of forces and couples by a single force and locate the point on the x-axis through which the line of action of the resultant passes. ***(M.U Dec 12)***



Solution: This is a system of three general forces and two couples.
Using Method of Resolution

$\Sigma F_x \rightarrow +ve$

$= -20 + 6\cos 38.66 = -15.315$ N

$\therefore \quad \Sigma F_x = 15.315\,\text{N} \leftarrow$

$\Sigma F_y \uparrow +ve$

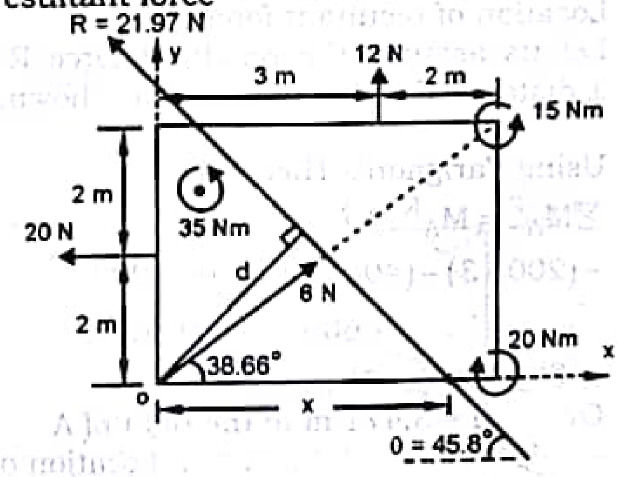
$= 12 + 6\sin 38.66 = 15.748$ N

$\therefore \quad \Sigma F_y = 15.748$ N $\uparrow$

Using $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{15.315^2 + 15.748^2}$ $\therefore$ $R = 21.97\,\text{N}$ .... Magnitude

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{15.748}{15.315}$ Or $\theta = 45.8°$ .... Direction of resultant force

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the 2nd quadrant. i.e. ↖ ...... Sense of resultant force



Location of resultant force
Let us assume the resultant force R is located at a ⊥ distance d to the right of origin O as shown.

Using Varignon's Theorem

$\Sigma M_O^F = M_O^R$ ↺ + ve

$+(20\times2)+(12\times3)+35+15-20 = +(21.97\times d)$

$\therefore$ d = 4.825 m Or d = 4.825 m

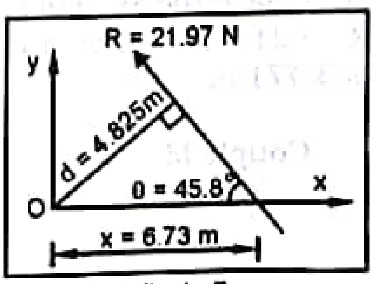
to the right of O ... Location of resultant force

Let the resultant R cut the x axis at a distance x from origin O.

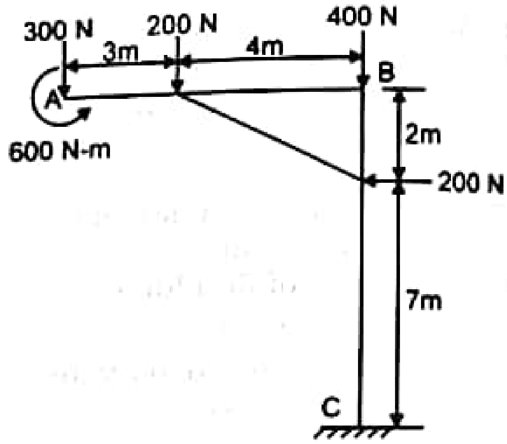
From geometry $\sin\theta = \dfrac{d}{x}$ $\therefore$ $\sin 45.8 = \dfrac{4.825}{x}$

Or x = 6.73 m

$\therefore$ The resultant force R = 21.97 N at $\theta = 45.8°$ ↖, it located at a ⊥ distance d = 4.825 to right of O and cuts the x axis at a distance x = 6.73 m from O as shown. .... **Ans.**



Resultant - Force



P11. Replace the loading on the frame by a force and moment at point A.

(M.U. May 09)

Solution: This is a system of four general forces and a couple.

Using Method of Resolution

$\Sigma F_x \quad \rightarrow + ve$

$= -200 N$

$\therefore \quad \Sigma F_x = 200 N \leftarrow$

$\Sigma F_y \quad \uparrow + ve$

$= -300 - 200 - 400 \quad = -900 \text{ N}$

$\therefore \quad \Sigma F_y = 900 \text{ N} \downarrow$

Using $R = \sqrt{\Sigma F_x{}^2 + \Sigma F_y{}^2} \quad = \sqrt{200^2 + 900^2}$

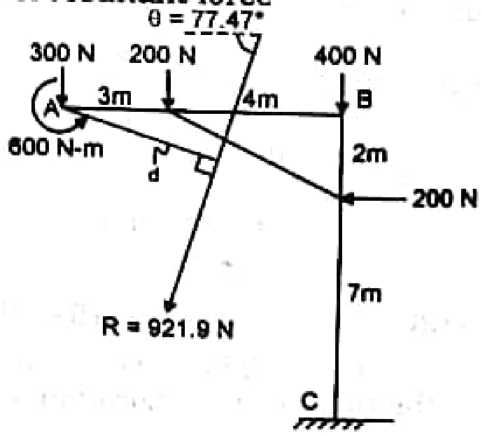
$\therefore \quad R = 921.9 N$ .... Magnitude of resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{900}{200}$ Or $\theta = 77.47°$ .... Direction of resultant force

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the 3rd quadrant. i.e. ↙ .... Sense of resultant force

Location of resultant force

Let us assume the resultant force R is located at a ⊥ distance d to the right of A as shown.



Using Varignon's Theorem

$\Sigma M_A{}^F = M_A{}^R \quad \circlearrowleft + ve$

$-(200 \times 3) - (400 \times 7) - (200 \times 2)$
$+ 600 = -(921.9 \times d)$

$\therefore \quad d = 3.471 \text{ m}$

Or $\quad d = 3.471$ m to the right of A

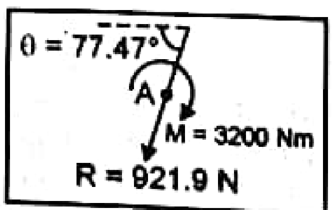
.... Location of resultant force

To replace the system by a force couple at point A, we need to shift the resultant force R = 921.9 N to A by introducing a couple M. The ⊥ distance between point A and force R is 3.471 m

Couple M $= F \times d$
$= -(921.9 \times 3.471)$
$= -3200$ Nm

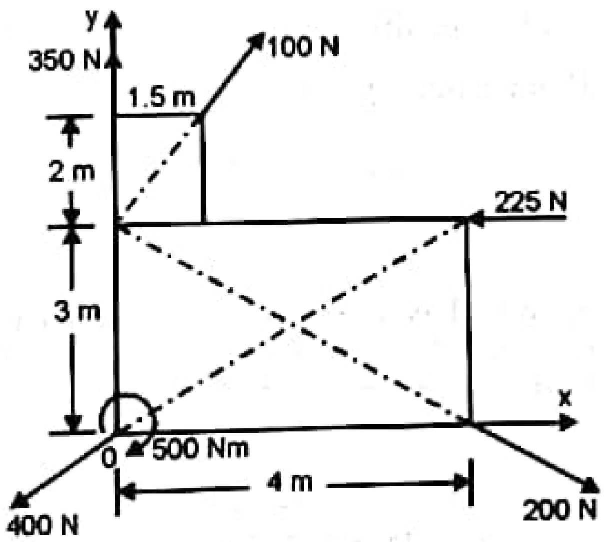
Or $\quad M = 3200$ Nm ↻

The resultant force couple at A is shown .... **Ans.**



Resultant - Force couple at A

P12. Determine the resultant of the given coplanar system of forces and a couple. Also locate the resultant on the x axis w.r.t. the origin.
b) Reduce the system to a force couple system at O.



Solution: This is a system of five general forces and a couple.

Using Method of Resolution

$\Sigma F_x \;\; \rightarrow + ve$

$= 200\cos 36.87 + 100\cos 53.13 - 400\cos 36.87 - 225$

$= -325\,N$

$\therefore \;\; \Sigma F_x = 325\,N \leftarrow$

$\Sigma F_y \;\; \uparrow + ve$

$= -200\sin 36.87 - 400\sin 36.87 + 100\sin 53.13 + 350$

$= 70\ N$

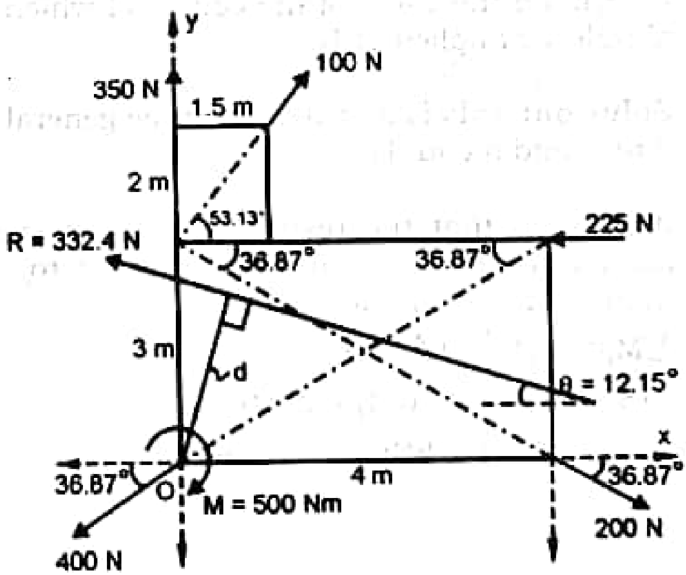
$\therefore \;\; \Sigma F_y = 70\ N \uparrow$

Using $R = \sqrt{\Sigma F_x^{\,2} + \Sigma F_y^{\,2}} = \sqrt{325^2 + 70^2}$ $\therefore R = 332.4\,N$ ...... Magnitude of resultant.

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{70}{325}$

Or $\theta = 12.15°$ .... Direction

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the 2nd quadrant i.e. ↖ .... sense of resultant force



Location of resultant force

Let us assume the resultant force R is located at a ⊥ distance d to the right of O as shown.

Using Varignon's Theorem

$\Sigma M_O^{\,F} = M_O^{\,R}$ ↺ + ve

$$-(200\sin 36.87\times 4) - (100\cos 53.13\times 5) + (100\sin 53.13\times 1.5) + (225\times 3) - 500 = +(332.4\times d)$$

$\therefore \;\; d = -1.459\ m$

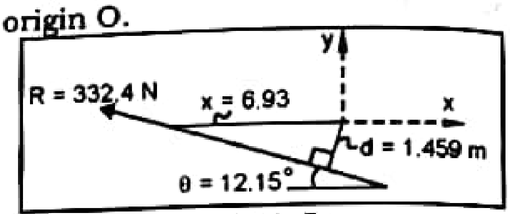
Or $\;\; d = 1.459\ m$ to the left of origin O .... Location of resultant force

Let the resultant R cut the x axis at a distance x from origin O.

From geometry $\sin\theta = \dfrac{d}{x}$

$$\sin 12.15 = \frac{1.459}{x}$$

Or $x = 6.93$ m .... **Ans.**



Resultant - Force

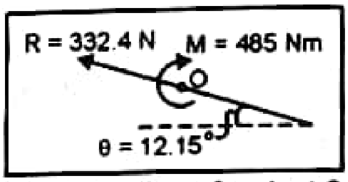
b) To replace the system by a force couple at point O, we need to shift the resultant force R = 332.4 N to origin O by introducing a couple M. The ⊥ distance between point O and force R is 1.459 m

Couple M $= F \times d$

$= -(332.4 \times 1.459)$

$= -485$ Nm

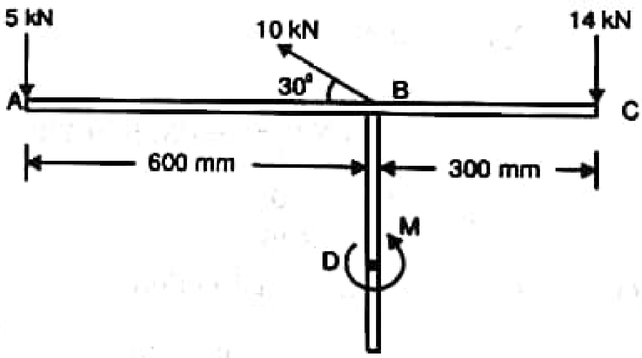
Or M = 485 Nm ↻



Resultant - Force Couple at O

The resultant force couple at origin O is shown ... **Ans.**

P13. A bracket is subjected to a coplanar force system as shown consisting of three forces and a couple. If the resultant force of the system is to pass through B, determine the value of the couple M which should be applied at D.



Solution: This is a system of three general forces and a couple.

It is given that the resultant of the system passes through B. This implies that the moment of the resultant force at B is zero.

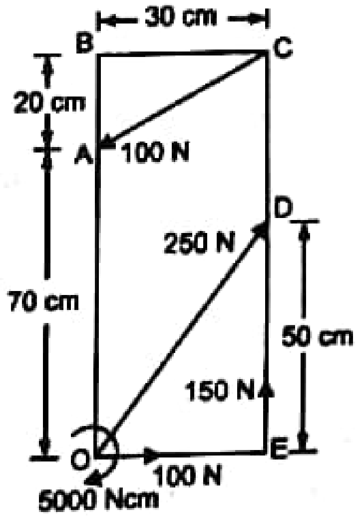
Using Varignon's Theorem

$\Sigma M_B^F = M_B^R$ ↺ + ve

$+(5\times0.6)-(14\times0.3)+M=0$

∴ M = 1.2 kNm .... **Ans.**

P14. For given system find resultant and its point of application with respect to point O on the x axis (x intercept). ***(M.U. Dec 14)***



Solution: This is a system of four general forces and one couple.

Using Method of Resolution

$\Sigma F_x \quad \rightarrow +ve$

$= 100 + 250\cos 59.04 - 100\cos 33.69$

$= 145.4\ N$

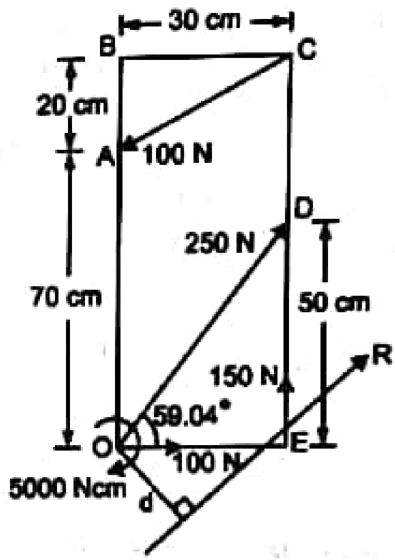
$\therefore \quad \Sigma F_x = 145.4\ N \rightarrow$

$\Sigma F_y \quad \uparrow +ve$

$= 150 + 250\sin 59.04 - 100\sin 33.69$

$= 308.9\ N$

$\therefore \quad \Sigma F_y = 308.9\ N \uparrow$



Using $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{145.4^2 + 308.9^2}$

$\therefore \quad R = 341.4\ N$ .... Magnitude of Resultant force

Also $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{308.9}{145.4}$

or $\theta = 64.79°$ .... Direction of Resultant force

The arrows of ΣF_x and ΣF_y indicate that the resultant force lies in the 1st quadrant i.e. $\measuredangle$

Location of resultant force.
Let us assume the resultant force is located at a $\perp$ distance d to the right of O.

Using Varignon's theorem

$\Sigma M_o^F = M_o^R \quad \circlearrowleft +ve$

$+(100\cos 33.69 \times 70) + (150 \times 30) - 5000 = 341.4 \times d$

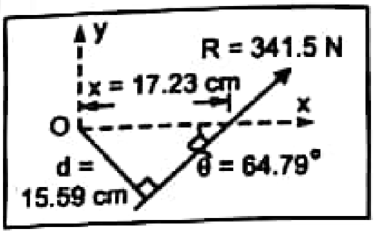
$\therefore \quad d = 15.59\ cm$ .... right of O.

Let the resultant R cut the x axis at a point distant x from the origin O.

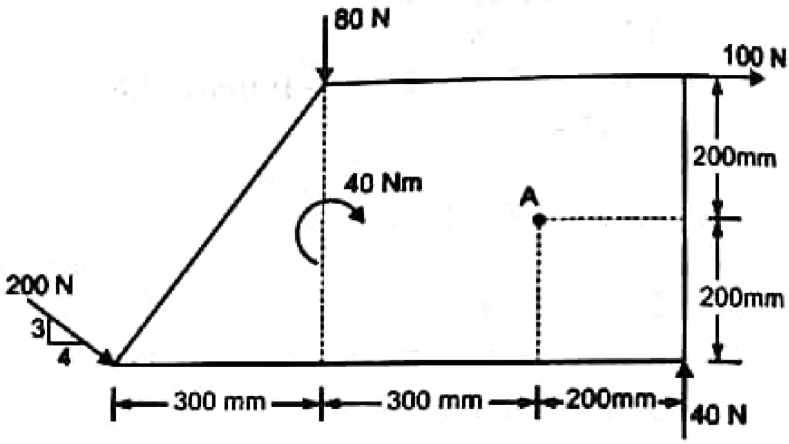
From geometry $\sin\theta = \frac{d}{x}$

$\therefore$ $\sin 64.79 = \frac{15.59}{x}$

Or $x = 17.23$ cm ...... **Ans.**



P15. Four forces and a couple are acting on a plate as shown in figure. Determine the resultant force and locate it with respect to point A.
(MU Dec 15)



Solution: This is a system of four general forces and a couple.
Using Method of Resolution

$\Sigma F_x \rightarrow +ve$

$= 200\cos 36.87 + 100$

$= 260\,N$

$\therefore$ $\Sigma F_x = 260\,N \rightarrow$

$\Sigma F_y \uparrow +ve$

$= -200\sin 36.87 - 80 + 40$

$= -160$ N

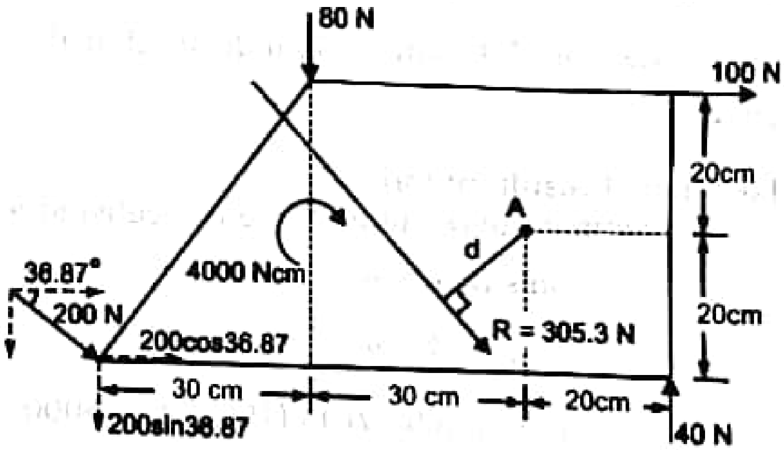
$\therefore$ $\Sigma F_y = 160$ N $\downarrow$

Using $R = \sqrt{\Sigma F_x{}^2 + \Sigma F_y{}^2} = \sqrt{260^2 + 160^2}$ $\therefore$ $R = 305.3\,N$ ...... Magnitude of resultant.

Also $\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{160}{260}$

Or $\theta = 31.61°$ .... Direction

The arrows of ΣF_x and ΣF_y indicate that the of resultant force lies in the 4th quadrant i.e. ↘ .... sense of resultant force



Location of resultant force
Let us assume the resultant force R is located at a ⊥ distance d to the left of A as shown.

Using Varignon's Theorem

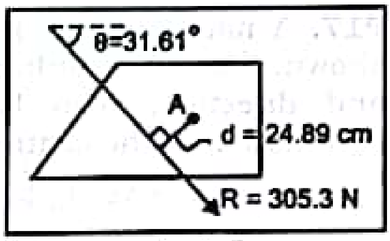
$\Sigma M_A{}^F = M_A{}^R$ ↺ + ve

$+(200\sin 36.87\times 60)+(200\cos 36.87\times 20)$

$+(80\times 30)+(40\times 20)-(100\times 20)-4000 = +(305.3\times d)$

$\therefore$ d = 24.89 cm ... to the left of origin A.

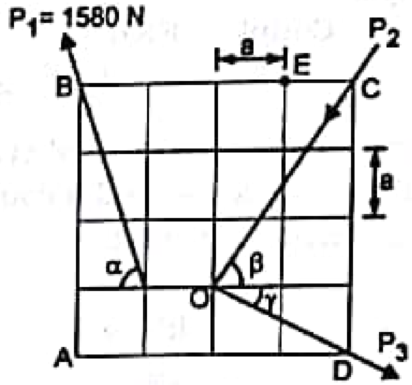
.... Location of resultant force



Resultant - Force

P16. A square lamina is subjected to a force of P_1 = 1580 N as shown in figure. Calculate values of forces P_2 and P_3 such that the resultant of system of forces will be a horizontal force at point E.

(KJS Nov 15)

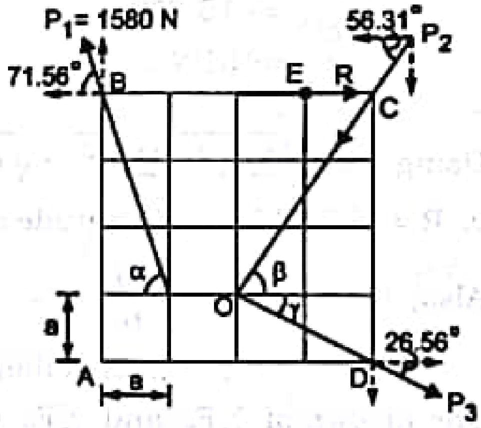


Solution: From geometry

$\tan\alpha = \dfrac{3a}{a}$ $\therefore$ $\alpha = 71.56°$

$\tan\beta = \dfrac{3a}{2a}$ $\therefore$ $\beta = 56.31°$

$\tan\gamma = \dfrac{a}{2a}$ $\therefore$ $\gamma = 26.56°$



This is a system of three general forces.
Given the resultant force 'R' is horizontal.
This implies $\Sigma F_x = R$ and $\Sigma F_y = 0$

Using Method of Resolution

ΣF_y ↑ + ve

$= P_1\sin\alpha - P_2\sin\beta - P_3\sin\gamma = 0$

$\therefore$ $1580\sin 71.56 - P_2\sin 56.31 - P_3\sin 26.56 = 0$

$\therefore$ $0.832P_2 + 0.4471P_3 = 1498.9$ .... (1)

Also given, the resultant is horizontal and passing through E.
Using Varignon's Theorem

$\Sigma M_E{}^F = M_E{}^R$ ↺ + ve

$-(1580\sin 71.56\times 4a)+(P_3\cos 26.56\times 4a) = 0$ | $M_C{}^R = 0$ since resultant passes through E

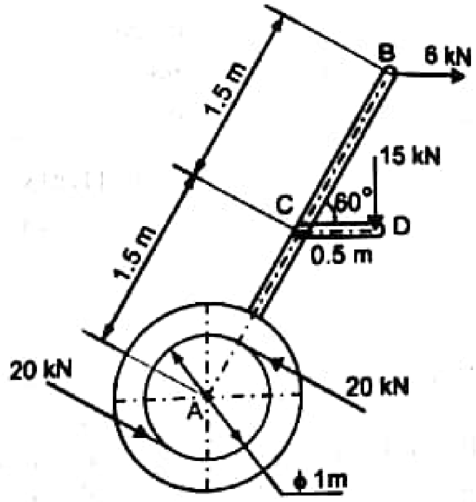
$\therefore$ $P_3 = 1675.7$ N ... **Ans.**

Substituting value of P_3 on equation (1), we get

$P_2 = 901$ N ... **Ans.**

P17. A machine part is subjected to forces as shown. Find the resultant of force in magnitude and direction. Also locate the point where resultant cuts the centre line of the bar AB.

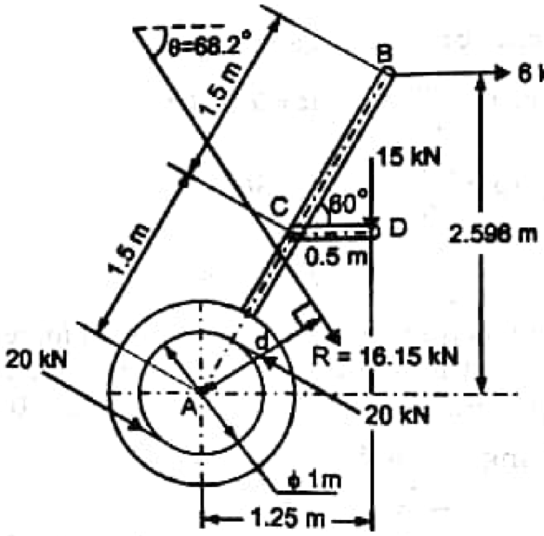
(M.U Dec 16)



Solution: This is a system of four general forces. The two 20 kN forces are parallel and opposite in sense. They can be reduced to a couple.

$$\text{Couple} = F \times d$$
$$= 20 \times d = 20 \text{ kNm} \circlearrowleft$$

So now we have a general system of two forces 6 kN →, 15 kN ↓ and a couple of 20 kNm $\circlearrowleft$



$\Sigma F_x \quad \rightarrow + ve$

$= 6$

$= 6 \text{ kN} \rightarrow$

$\Sigma F_y \quad \uparrow + ve$

$= -15 \text{ kN}$

$= 15 \text{ kN} \downarrow$

Using $R = \sqrt{\Sigma {F_x}^2 + \Sigma {F_y}^2} = \sqrt{6^2 + 15^2}$

$\therefore$ $R = 16.15$ kN ... Magnitude of Resultant.

Also, $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x} = \dfrac{15}{6}$

or $\theta = 68.2°$ ... Direction of Resultant

The arrows of ΣF_x and ΣF_y indicate that the resultant force lies in the 4th quadrant

i.e ↘ ... Sense of Resultant Force

Location of resultant force.

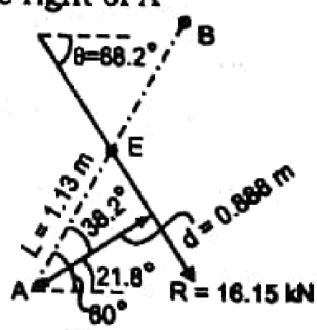
Let us assume the resultant force is located at a ⊥ distance d to the right of A

Using Varignon's theorem.

$$\Sigma {M_A}^F = {M_A}^R \quad \circlearrowleft + ve$$

$$-(6 \times 2.598) - (15 \times 1.25) + 20 = -(16.15 \times d)$$

$$\therefore \quad d = 0.888 \text{ m}$$



Resultant - Force

Let the resultant cut the centre line of the bar AB at a point E, distance L from A, From geometry

$\cos 38.2 = \dfrac{0.888}{L}$ Or $L = 1.13$ m ... Location of Resultant force

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