Solutions: Chapter 7

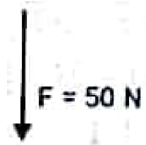
Space Forces

Exercise 7.1

Basic Operations

P1. A force of 50 N acts parallel to the y axis in the –ve direction. Put the force in vector form.

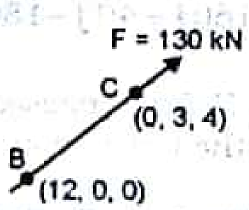
Solution: Given magnitude of force is 50 N, direction is parallel to y axis and its sense is – ve.



$\therefore \quad \bar{F} = -50\,\mathbf{j}$ N .... vector form. .... **Ans.**

P2. A 130 kN force acts at B (12, 0, 0) and passes through C (0, 3, 4). Put the force in vector form.

Solution: The force F =130 kN in vector form is



$$\therefore \quad \bar{F} = F.\hat{e}_{BC}$$

$$= 130\left[\frac{-12\mathbf{i}+3\mathbf{j}+4\mathbf{k}}{\sqrt{12^2+3^2+4^2}}\right]$$

$\bar{F} = -120\mathbf{i}+30\mathbf{j}+40\mathbf{k}$ kN .... **Ans.**

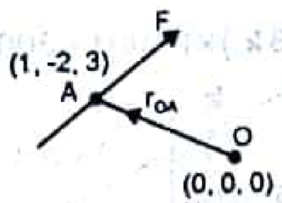
P3. A force $F = (3\,\mathbf{i} - 4\,\mathbf{j} + 12\,\mathbf{k})$ N acts at a point A (1, -2, 3) m. Find
a) moment of the force about origin.
b) moment of the force about point B (2, 1, 2) m. ***(M. U. May 13)***

Solution: The given force in vector form is $\bar{F} = 3\mathbf{i}-4\mathbf{j}+12\mathbf{k}$ N

a) Moment of that force F about origin

$$\bar{M}_O^F = \bar{r}_{OA} \times \bar{F}$$

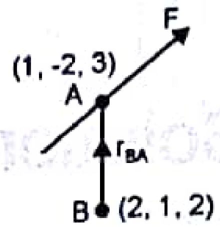
$$= (\mathbf{i}-2\mathbf{j}+3\mathbf{k}) \times (3\mathbf{i}-4\mathbf{j}+12\mathbf{k})$$



$$\bar{M}_O^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 3 \\ 3 & -4 & 12 \end{vmatrix}$$

$\therefore \quad \bar{M}_O^F = -12\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ Nm .... **Ans.**

b) Moment of that force F about point B



$$\overline{M}_B^F = \bar{r}_{BA} \times \overline{F}$$

$$= (-\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k})$$

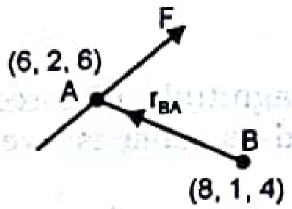
$$\overline{M}_B^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1 \\ 3 & -4 & 12 \end{vmatrix}$$

$\therefore \quad \overline{M}_B^F = -32\mathbf{i} + 15\mathbf{j} + 13\mathbf{k}$ Nm .... **Ans.**

P4. A force F = 80 **i** + 50 **j** − 60 **k** passes through a point A (6, 2, 6). Compute its moment about a point B (8, 1, 4) ***(M. U. Dec 12)***

Solution: The force F in vector form is $\overline{F} = 80\mathbf{i} + 50\mathbf{j} - 60\mathbf{k}$

Moment of the force F about point B



$$\overline{M}_B^F = \bar{r}_{BA} \times \overline{F}$$

$$= (-2\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \times (80\mathbf{i} + 50\mathbf{j} - 60\mathbf{k})$$

$$\overline{M}_B^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 2 \\ 80 & 50 & -60 \end{vmatrix}$$

$\therefore \quad \overline{M}_B^F = -160\mathbf{i} + 40\mathbf{j} - 180\mathbf{k}$ units .... **Ans.**

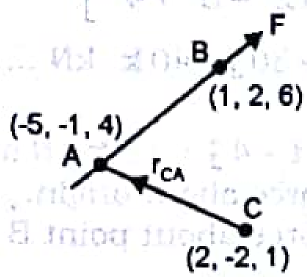
P5. A 700 N force passes through two points A (−5,−1,4) towards B (1,2,6) m. Find moment of the force about a point C (2,−2,1) m.

Solution: The force F =700 N in vector form is

$$\therefore \quad \overline{F} = F.\hat{e}_{AB}$$

$$= 700\left[\frac{6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}}{\sqrt{6^2 + 3^2 + 2^2}}\right]$$

$$\overline{F} = 600\mathbf{i} + 300\mathbf{j} + 200\mathbf{k} \text{ N}$$



Moment of the force F about point C

$$\overline{M}_C^F = \bar{r}_{CA} \times \overline{F}$$

$$= (-7\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \times (600\mathbf{i} + 300\mathbf{j} + 200\mathbf{k})$$

$$\overline{M}_C^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 1 & 3 \\ 600 & 300 & 200 \end{vmatrix}$$

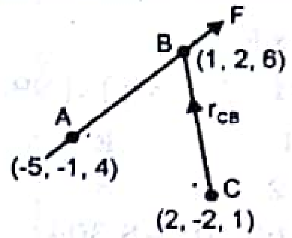
$\therefore \quad \overline{M}_C^F = -700\mathbf{i} + 3200\mathbf{j} - 2700\mathbf{k}$ Nm .... **Ans.**

Alternatively

$$\overline{M}_C^F = \overline{r}_{CB} \times \overline{F}$$

$$= (-\mathbf{i}+4\mathbf{j}+5\mathbf{k}) \times (600\mathbf{i}+300\mathbf{j}+200\mathbf{k})$$

$$\overline{M}_C^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 4 & 5 \\ 600 & 300 & 200 \end{vmatrix}$$



$\therefore \quad \overline{M}_C^F = -700\mathbf{i} + 3200\mathbf{j} - 2700\mathbf{k}$ Nm .... **Ans.**

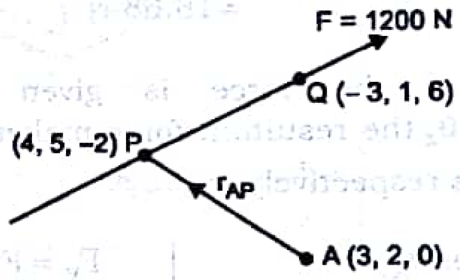
Note that the position vector $\overline{r}$, *extends from the moment centre to any point on the line of action of the force.*

P6. A force of 1200 N acts along PQ, P (4, 5, −2)m and Q (−3, 1, 6) m. Calculate its moment about a point A (3, 2, 0) m. ***(M.U. May 14)***

Solution: The force F =1200 N in vector form is

$$\therefore \quad \overline{F} = F.\hat{e}_{PQ}$$

$$= 1200\left[\frac{-7\mathbf{i}-4\mathbf{j}+8\mathbf{k}}{\sqrt{7^2+4^2+8^2}}\right]$$



$$\overline{F} = -739.6\,\mathbf{i} - 422.6\,\mathbf{j} + 845.2\,\mathbf{k}\ \text{N}$$

Moment of the force F about point A

$$\overline{M}_A^F = \overline{r}_{AP} \times \overline{F}$$

$$= (\mathbf{i}+3\mathbf{j}-2\mathbf{k}) \times (-739.6\mathbf{i}-422.6\mathbf{j}+845.2\mathbf{k})$$

$$\overline{M}_A^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -2 \\ -739.6 & -422.6 & 845.2 \end{vmatrix}$$

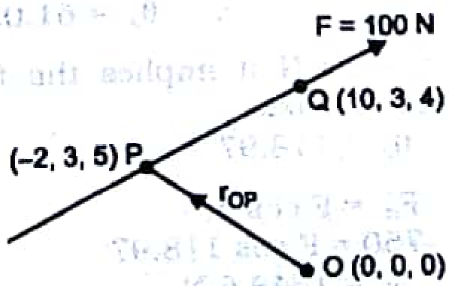
$\therefore \quad \overline{M}_A^F = 1690.4\,\mathbf{i} + 634\,\mathbf{j} + 1796.2\,\mathbf{k}$ Nm .... **Ans.**

P7. A force of 100 N acts at a point P (−2, 3, 5) m has its line of action passing through Q (10, 3, 4) m. Calculate moment of this force about origin (0, 0, 0). ***(M. U. Dec 14)***

Solution: The force F =100 N in vector form is

$$\therefore \quad \overline{F} = F.\hat{e}_{PQ}$$

$$= 100\left[\frac{12\mathbf{i}+0\mathbf{j}-\mathbf{k}}{\sqrt{12^2+1^2}}\right]$$



$$\therefore \quad \overline{F} = 99.65\,\mathbf{i} - 8.304\,\mathbf{k}\ \text{N}$$

Moment of the force F about point O

$$\overline{M}_O^F = \overline{r}_{OP} \times \overline{F}$$

$$= (-2\mathbf{i}+3\mathbf{j}+5\mathbf{k}) \times (99.65\mathbf{i}-8.304\mathbf{k})$$

$$\overline{M}_O^F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 5 \\ 99.65 & 0 & -8.304 \end{vmatrix}$$

$\therefore \quad \overline{M}_O^F = -24.91\mathbf{i} + 481.6\mathbf{j} - 298.9\mathbf{k}$ Nm .... **Ans.**

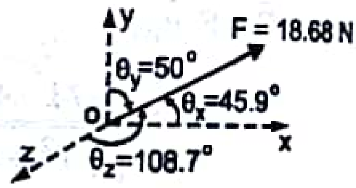
P8. Find the direction angles for the force given by $\overline{F} = 13\,\mathbf{i} + 12\,\mathbf{j} - 6\,\mathbf{k}$ N. *(NMIMS Dec 13)*

Solution: Given $\overline{F} = 13\,\mathbf{i} + 12\,\mathbf{j} - 6\,\mathbf{k}$ N

Magnitude of the force $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{13^2 + 12^2 + (-6)^2}$

$= 18.68$ N

Direction of the force is given by the angles θ_x, θ_y and θ_z the resultant force makes with the + ve x, y and z axes respectively.



$F_x = F\cos\theta_x$

$13 = 18.68\cos\theta_x$

Or $\theta_x = 45.9°$ .... **Ans.**

$F_y = F\cos\theta_y$

$12 = 18.68\cos\theta_y$

Or $\theta_y = 50°$ .... **Ans.**

$F_z = F\cos\theta_z$

$-6 = 18.68\cos\theta_z$

Or $\theta_z = 108.7°$ .... **Ans.**

P9. A force acts at the origin in a direction defined by the angles $\theta_y = 65°$, $\theta_z = 40°$. Knowing that the x-component of the force is –750 N, determine,
i) the other components ii) magnitude of the force iii) the value of θ_x. *(MU Dec 15)*

Solution: Using $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$$\cos^2\theta_x + \cos^2 65 + \cos^2 40 = 1$$

$\therefore \quad \cos^2\theta_x = 0.2346$

$\therefore \quad \cos\theta_x = \pm 0.4843$

$\therefore \quad \theta_x = 61.03°$ or $\theta_x = 118.97°$

Since $F_x = -750$ N it implies the force component is directed towards the negative direction of the x axis.

$\therefore \quad \theta_x = 118.97°$ .... **Ans.**

using $F_x = F\cos\theta_x$

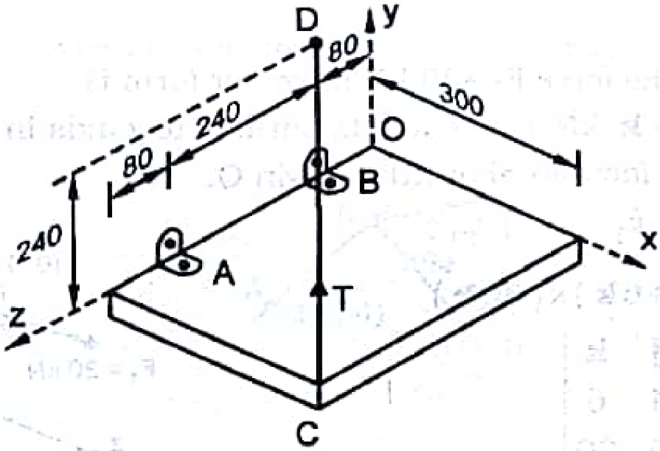
$-750 = F\cos 118.97$

$\therefore \quad F = 1548.6$ N .... **Ans.**

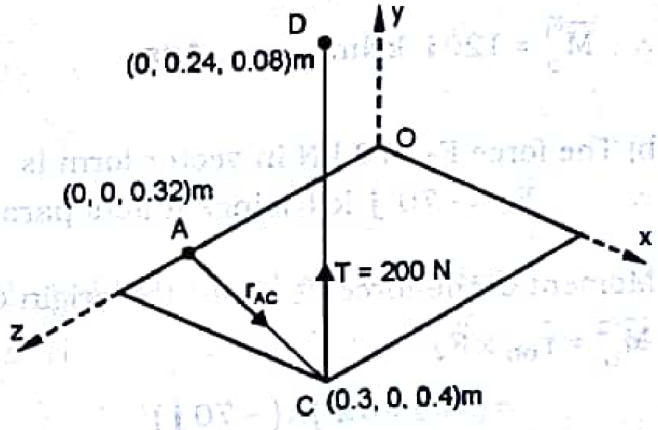
using $F_y = F \cos \theta_y$
$= 1548.6 \cos 65$
$\therefore\ F_y = 654.5\ \text{N}$ .... **Ans.**

using $F_z = F \cos \theta_z$
$= 1548.6 \cos 40$
$\therefore\ F_z = 1186.3\ \text{N}$ .... **Ans.**

P10. A rectangular plate is supported by brackets to the wall at A and B by wire CD as shown in figure. Knowing that tension in wire is 200 N determine the moment about point A, of the force exerted by wire on point C.
All dimensions are in mm.



Solution: The tension T = 200 N in vector force is



$\therefore\ \bar{T} = T.\hat{e}_{CD}$

$$= 200\left[\frac{-0.3\mathbf{i}+0.24\mathbf{j}-0.32\mathbf{k}}{\sqrt{0.3^2+0.24^2+0.32^2}}\right]$$

$$\bar{T} = -120\ \mathbf{i}+96\mathbf{j}-128\mathbf{k}\ \text{N}$$

Moment of the force T about point A.

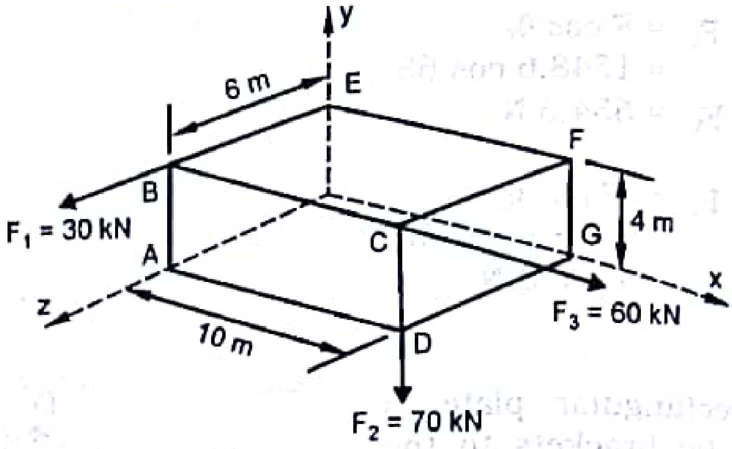
$$\bar{M}_A^T = \bar{r}_{AC}\times\bar{T}$$

$$= (0.3\mathbf{i}+0.08\mathbf{k})\times(-120\mathbf{i}+96\mathbf{j}-128\mathbf{k})$$

$$\bar{M}_A^T = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0 & 0.08 \\ -120 & 96 & -128 \end{vmatrix}$$

$\therefore\ \bar{M}_A^T = -7.68\ \mathbf{i}+28.8\mathbf{j}+28.8\mathbf{k}$ Nm .... **Ans.**

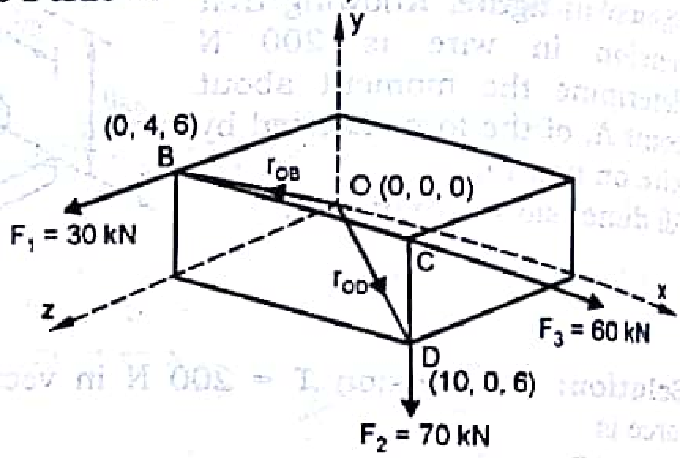
P11. Three forces act on a rectangular box as shown in figure. Determine the moment of each force about the origin.



Solution: a) The force F_1 =30 kN in vector form is

$\therefore \quad \bar{F}_1 = 30\ \mathbf{k}$ kN (since it acts parallel to z axis in + ve sense)

Moment of the force F_1 about the origin O.



$$\bar{M}_O^{F_1} = \bar{r}_{OB} \times \bar{F}_1$$

$$= (4\mathbf{j} + 6\mathbf{k}) \times (30\mathbf{k})$$

$$\bar{M}_O^{F_1} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 6 \\ 0 & 0 & 30 \end{vmatrix}$$

$\therefore \quad \bar{M}_O^{F_1} = 120\mathbf{i}$ kNm .... **Ans.**

b) The force F_2 =70 kN in vector form is

$\therefore \quad \bar{F}_2 = -70\ \mathbf{j}$ kN (since it acts parallel to y axis in – ve sense)

Moment of the force F_2 about the origin O.

$$\bar{M}_O^{F_2} = \bar{r}_{OD} \times \bar{F}_2$$

$$= (10\mathbf{i} + 6\mathbf{k}) \times (-70\mathbf{j})$$

$$\therefore \quad \bar{M}_O^{F_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 0 & 6 \\ 0 & -70 & 0 \end{vmatrix}$$

$\therefore \quad \bar{M}_O^{F_2} = 420\mathbf{i} - 700\mathbf{k}$ Nm .... **Ans.**

c) The force F_3 =60 kN in vector form is

$\therefore \quad \bar{F}_3 = 60\ \mathbf{i}$ kN (since it acts parallel to x axis in + ve sense)

Moment of the force F_3 about the origin O.

$$\bar{M}_O^{F_3} = \bar{r}_{OB} \times \bar{F}_3$$

$$= (4\mathbf{j} + 6\mathbf{k}) \times (60\mathbf{i})$$

$$\bar{M}_O^{F_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 6 \\ 60 & 0 & 0 \end{vmatrix}$$

Note that force F_3 passes through B and C. Therefore r_{OB} or r_{OC} may be taken as position vectors.

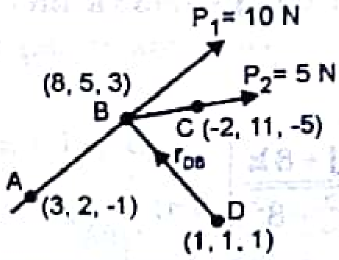
$\therefore \quad \bar{M}_O^{F_3} = 360\mathbf{j} - 240\mathbf{k}$ kNm .... **Ans.**

Exercise 7.2

Resultant of Space Force System

P1. A force P_1 = 10 N in magnitude acts along direction AB whose co-ordinates of points A and B are (3, 2, −1) and (8, 5, 3) respectively. Another force P_2 = 5 N in magnitude acts along BC where C has co-ordinates (−2, 11, −5). Determine a) The resultant of P_1 and P_2. b) The moment of the resultant about a point D (1, 1, 1).

Solution: a) This is a concurrent space force system consisting of two forces P_1 and P_2 meeting at B.



$$\bar{P}_1 = P_1 . \hat{e}_{AB}$$

$$= 10\left[\frac{5\mathbf{i}+3\mathbf{j}+4\mathbf{k}}{\sqrt{5^2+3^2+4^2}}\right]$$

$$\therefore \quad \bar{P}_1 = 7.07\,\mathbf{i}+4.24\,\mathbf{j}+5.65\,\mathbf{k} \text{ N}$$

$$\bar{P}_2 = P_2 . \hat{e}_{BC}$$

$$= 5\left[\frac{-10\mathbf{i}+6\mathbf{j}-8\mathbf{k}}{\sqrt{10^2+6^2+8^2}}\right]$$

$$\therefore \quad \bar{P}_2 = -3.535\,\mathbf{i}+2.12\,\mathbf{j}-2.828\,\mathbf{k} \text{ N}$$

The resultant force $\bar{R} = \bar{P}_1 + \bar{P}_2$

$$= (7.07\,\mathbf{i}+4.24\,\mathbf{j}+5.65\,\mathbf{k}) + (-3.535\,\mathbf{i}+2.12\,\mathbf{j}-2.828\,\mathbf{k})$$

$$\bar{R} = 3.535\,\mathbf{i}+6.36\,\mathbf{j}+2.822\,\mathbf{k} \text{ N}$$ .... **Ans.**

b) Moment of the force R about the point D.

$$\bar{M}_D^R = \bar{r}_{DB} \times \bar{R}$$

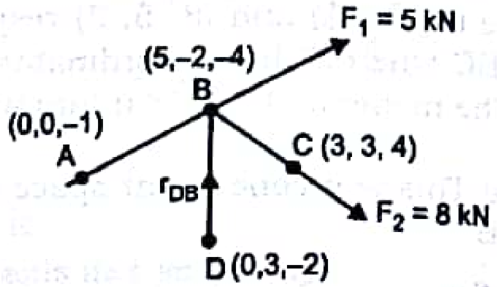
$$= (7\mathbf{i}+4\mathbf{j}+2\mathbf{k}) \times (3.535\,\mathbf{i}+6.36\,\mathbf{j}+2.822\,\mathbf{k})$$

$$\bar{M}_D^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 4 & 2 \\ 3.535 & 6.36 & 2.822 \end{vmatrix}$$

$$\therefore \quad \bar{M}_D^R = -1.432\,\mathbf{i}-12.68\,\mathbf{j}+30.38\,\mathbf{k} \text{ Nm}$$ .... **Ans.**

P2. A force 5 kN is acting along AB where A (0, 0, -1) m and B (5, -2, -4) m. Another force 8 kN is acting along BC where C (3, 3, 4) m. Find resultant of two forces and find moment of resultant force about a point D (0, 3, -2) m. **(MU Dec 2015)**

Solution: This is a concurrent space force system consisting of two forces $F_1 = 5$ kN and $F_2 = 8$ kN meeting at B.



$$\therefore \quad \overline{F_1} = F_1 . \hat{e}_{AB}$$

$$= 5\left[\frac{5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}}{\sqrt{5^2 + 2^2 + 3^2}}\right]$$

$$\therefore \quad \overline{F_1} = 4.055\,\mathbf{i} - 1.622\,\mathbf{j} - 2.433\,\mathbf{k} \text{ kN}$$

$$\overline{F_2} = F_2 . \hat{e}_{BC}$$

$$= 8\left[\frac{-2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}}{\sqrt{2^2 + 5^2 + 8^2}}\right]$$

$$\therefore \quad \overline{F_2} = -1.659\,\mathbf{i} + 4.148\,\mathbf{j} + 6.636\,\mathbf{k} \text{ kN}$$

The resultant force $\overline{R} = \overline{F_1} + \overline{F_2}$

$$= (4.055\,\mathbf{i} - 1.622\,\mathbf{j} - 2.433\,\mathbf{k}) + (-1.659\,\mathbf{i} + 4.148\,\mathbf{j} + 6.636\,\mathbf{k})$$

$$\therefore \quad \overline{R} = 2.396\,\mathbf{i} + 2.526\,\mathbf{j} + 4.203\,\mathbf{k} \text{ kN}$$ .... **Ans.**

Now, Moment of resultant R about point D.

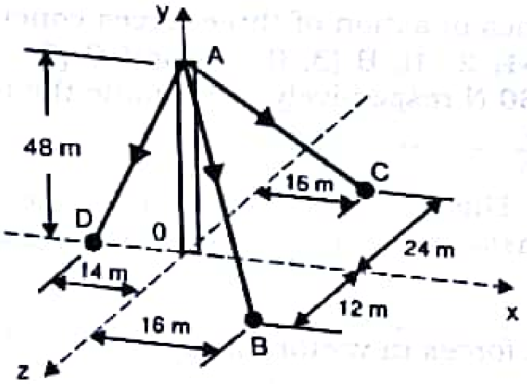
$$\overline{M}_D^R = \overline{r}_{DB} \times \overline{R}$$

$$= (5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}) \times (2.396\,\mathbf{i} + 2.526\,\mathbf{j} + 4.203\,\mathbf{k})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -5 & -2 \\ 2.396 & 2.526 & 4.203 \end{vmatrix}$$

$$\therefore \quad \overline{M}_D^R = -15.96\,\mathbf{i} - 25.81\,\mathbf{j} + 24.61\,\mathbf{k} \text{ kNm}$$ .... **Ans.**

P3. Knowing that the tension in AC = 20 kN, determine the required values of T_{AB} and T_{AD} so that the resultant of the three forces applied at A is vertical. Also find the resultant.



Solution: This is a concurrent space force system consisting of three forces T_{AC}, T_{AB} and T_{AD} meeting at A. Coordinates of different points are A (0, 48, 0) m, B (16, 0, 12) m, C (16, 0, − 24) m and D (− 14, 0, 0) m

$$\bar{T}_{AC} = T_{AC}\,.\,\hat{e}_{AC}$$

$$= 20\left[\frac{16\mathbf{i}-48\mathbf{j}-24\mathbf{k}}{\sqrt{16^2+48^2+24^2}}\right]$$

$$\therefore\ \bar{T}_{AC} = 5.714\,\mathbf{i} - 17.143\,\mathbf{j} - 8.57\,\mathbf{k}\ \text{kN}$$

$$\bar{T}_{AB} = T_{AB}\,.\,\hat{e}_{AB}$$

$$= \bar{T}_{AB}\left[\frac{16\mathbf{i}-48\mathbf{j}+12\mathbf{k}}{\sqrt{16^2+48^2+12^2}}\right]$$

$$\therefore\ \bar{T}_{AB} = \bar{T}_{AB}\,(0.3077\,\mathbf{i} - 0.923\,\mathbf{j} + 0.2307\,\mathbf{k}\,)\ \text{kN}$$

$$\bar{T}_{AD} = T_{AD}\,.\,\hat{e}_{AD}$$

$$= \bar{T}_{AD}\left[\frac{-14\mathbf{i}-48\mathbf{j}}{\sqrt{14^2+48^2}}\right]$$

$$\therefore\ \bar{T}_{AD} = \bar{T}_{AD}\,(-0.28\,\mathbf{i} - 0.96\,\mathbf{j})\ \text{kN}$$

The resultant force $\bar{R} = \bar{T}_{AC} + \bar{T}_{AB} + \bar{T}_{AD}$

Also since the resultant force is vertical, i.e. along y axis, implies that $\Sigma F_y = R$, $\Sigma F_x = 0$ and $\Sigma F_z = 0$,

Using $\Sigma F_x = 0$

$$5.714 + 0.3077\,T_{AB} - 0.28\,T_{AD} = 0 \quad \ldots\ldots\ldots (1)$$

Using $\Sigma F_z = 0$

$$-8.57 + 0.2307\,T_{AB} = 0 \quad \ldots\ldots\ldots (2)$$

Solving equations (1) and (2), we get

$T_{AB} = 37.15$ kN and $T_{AD} = 61.23$ kN .... **Ans.**

Using $\Sigma F_y = R$

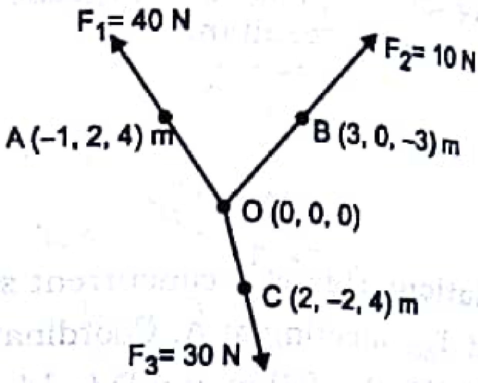
$$-17.143 - 0.923\,T_{AB} - 0.96\,T_{AD} = R$$

$$\therefore\ R = -17.143 - 0.923(37.15) - 0.96(61.23)$$

$$\therefore\ R = -110.2\ \text{kN} \quad \text{Or} \quad \bar{R} = -110.2\,\mathbf{j}\ \text{kN}$$ .... **Ans.**

P4. The lines of action of three forces concurrent at origin 'O' pass respectively through points A (-1, 2, 4), B (3, 0, −3) and C (2, −2, 4) m. The magnitude of forces are 40 N, 10 N and 30 N respectively. Determine the magnitude and direction of their resultant.

(M.U. May 14)



Solution: This is a concurrent space force system consisting of three forces meeting at the origin.

Putting the forces in vector form.

$\overline{F}_1 = F_1.\hat{e}_{OA}$

$= 40\left[\dfrac{-i+2j+4k}{\sqrt{1+4+16}}\right]$

$= -8.729\,i + 17.457\,j + 34.915\,k$ N .... (1)

$\overline{F}_2 = F_2.\hat{e}_{OB}$

$= 10\left[\dfrac{3i+0j-3k}{\sqrt{9+9}}\right]$

$= 7.071\,i - 7.071\,k$ N .... (2)

$\overline{F}_3 = F_3.\hat{e}_{OC}$

$= 30\left[\dfrac{2i-2j+4k}{\sqrt{4+4+16}}\right]$

$= 12.247\,i - 12.247\,j + 24.495\,k$ N .... (3)

$\therefore$ The resultant force $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$

$= (-8.729i + 17.457\,j + 34.915k) + (7.07i - 7.071k)$
$+ (12.247i - 12.247\,j + 24.495k)$

$\therefore$ $\overline{R} = 10.589i + 5.27\,j + 52.339k$ N .... **Ans.**

$\therefore$ Resultant of the concurrent system is a force $\overline{R} = 10.589i + 5.27\,j + 52.339k$ N acting at origin O.

$\therefore$ Magnitude of resultant $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$\therefore$ $R = \sqrt{10.589^2 + 5.27^2 + 52.339^2}$

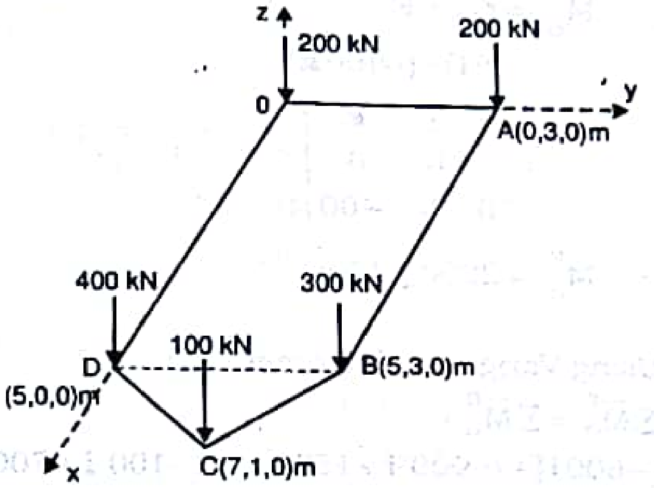
or $R = 53.66$ N

Direction: $R_x = R\cos\theta_x$ $\therefore$ $10.589 = 53.66\cos\theta_x$ or $\theta_x = 78.62°$

$R_y = R\cos\theta_y$ $\therefore$ $5.27 = 53.66\cos\theta_y$ or $\theta_y = 84.36°$

$R_z = R\cos\theta_z$ $\therefore$ $52.339 = 53.66\cos\theta_z$ or $\theta_z = 12.73°$

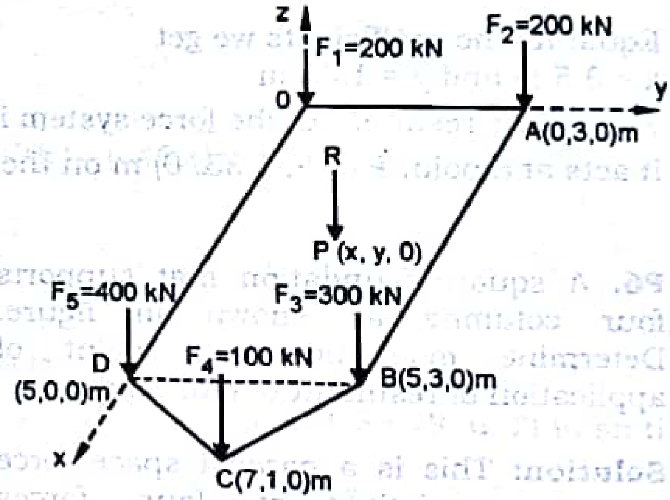
P5. A plate foundation is subjected to five vertical forces as shown. Replace these five forces by means of a single vertical force and find the point of replacement.



Solution: This is a parallel space force system consisting of five forces. Let $F_1 = 200$ kN, $F_2 = 200$ kN, $F_3 = 300$ kN, $F_4 = 100$ kN and $F_5 = 400$ kN. Note that all five forces act parallel to z-axis in – ve sense.

Putting the forces in vector form.

$$\bar{F}_1 = -200\,\mathbf{k} \text{ kN}$$
$$\bar{F}_2 = -200\,\mathbf{k} \text{ kN}$$
$$\bar{F}_3 = -300\,\mathbf{k} \text{ kN}$$
$$\bar{F}_4 = -100\,\mathbf{k} \text{ kN}$$
$$\bar{F}_5 = -400\,\mathbf{k} \text{ kN}$$



The resultant of the force system is

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 + \bar{F}_5$$
$$= (-200\,\mathbf{k}) + (-200\,\mathbf{k}) + (-300\,\mathbf{k}) + (-100\,\mathbf{k}) + (-400\,\mathbf{k})$$
$$\therefore \quad \bar{R} = -1200\,\mathbf{k} \text{ kN}$$

Let the resultant force act at a point P (x, y, 0) on the x – y plane.
Taking moments of all the forces about the origin.

$\bar{M}_o^{F_1} = 0$ since force F_1 acts at O.

$$\bar{M}_o^{F_2} = \bar{r}_{OA} \times \bar{F}_2$$
$$= (3\,\mathbf{j}) \times (-200\,\mathbf{k})$$
$$= -600\,\mathbf{i} \text{ kNm}$$

$$\bar{M}_o^{F_3} = \bar{r}_{OB} \times \bar{F}_3$$
$$= (5\,\mathbf{i} + 3\,\mathbf{j}) \times (-300\,\mathbf{k})$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 3 & 0 \\ 0 & 0 & -300 \end{vmatrix}$$
$$\therefore \quad \bar{M}_o^{F_3} = -900\,\mathbf{i} + 1500\,\mathbf{j} \text{ kNm}$$

$$\bar{M}_o^{F_4} = \bar{r}_{OC} \times \bar{F}_4$$
$$= (7\,\mathbf{i} + \mathbf{j}) \times (-100\,\mathbf{k})$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 1 & 0 \\ 0 & 0 & -100 \end{vmatrix}$$
$$\therefore \quad \bar{M}_o^{F_4} = -100\,\mathbf{i} + 700\,\mathbf{j} \text{ kNm}$$

$$\overline{M}_O^{F_5} = \bar{r}_{OD} \times \overline{F}_5$$
$$= (5\mathbf{i}) \times (-400\mathbf{k})$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 0 \\ 0 & 0 & -400 \end{vmatrix}$$
$$\therefore \quad \overline{M}_O^{F_5} = 2000\mathbf{j} \text{ kNm}$$

$$\overline{M}_O^R = \bar{r}_{OP} \times \overline{R}$$
$$= (x\mathbf{i} + y\mathbf{j}) \times (-1200\mathbf{k})$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ 0 & 0 & -1200 \end{vmatrix}$$
$$\therefore \quad \overline{M}_O^R = (-1200y)\,\mathbf{i} + (1200x)\,\mathbf{j} \text{ kNm}$$

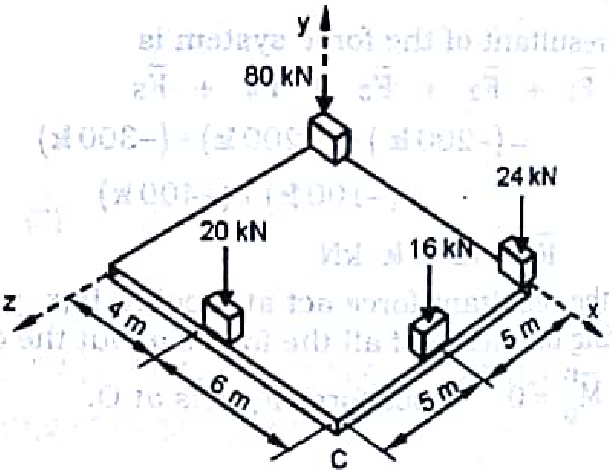
Using Varignon's Theorem

$$\Sigma \overline{M}_O^F = \Sigma \overline{M}_O^R$$
$$(-600\mathbf{i}) + (-900\,\mathbf{i} + 1500\,\mathbf{j}) + (-100\,\mathbf{i} + 700\,\mathbf{j}) + (2000\,\mathbf{j}) = (-1200y)\,\mathbf{i} + (1200x)\,\mathbf{j}$$
$$-1600\mathbf{i} + 4200\mathbf{j} = (-1200y)\,\mathbf{i} + (1200x)\,\mathbf{j}$$

Equating the coefficients we get
$x = 3.5$ m and $y = 1.33$ m

$\therefore$ The resultant of the force system is $\overline{R} = -1200\mathbf{k}$ kN,
It acts at a point P (3.5, 1.33, 0) m on the plate foundation .... **Ans.**

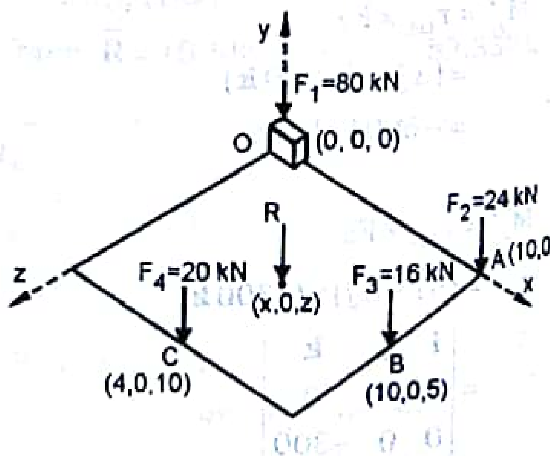
P6. A square foundation mat supports four columns as shown in figure. Determine magnitude and point of application of resultant of four loads.



Solution: This is a parallel space force system consisting of four forces $F_1 = 80$ kN, $F_2 = 24$ kN, $F_3 = 16$ kN and $F_4 = 20$ kN. Note that all six forces act parallel to y-axis in – ve sense.

Putting the forces in vector form.

$$\overline{F}_1 = -80\,\mathbf{j} \text{ kN}$$
$$\overline{F}_2 = -24\,\mathbf{j} \text{ kN}$$
$$\overline{F}_3 = -16\,\mathbf{j} \text{ kN}$$
$$\overline{F}_4 = -20\,\mathbf{j} \text{ kN}$$



Let $\overline{R}$ be the resultant of the force system

$$\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4$$
$$= (-80\,\mathbf{j}) + (-24\,\mathbf{j}) + (-16\,\mathbf{j}) + (-20\,\mathbf{j})$$

Or $\quad \overline{R} = -140\,\mathbf{j}$

Let the resultant force R be located at a point P (x, 0, z) in the x-z plane.

Taking moments of all the forces about the origin.

$\overline{M}_o^{F_1} = 0$ Since force F_1 passes through O.

$\overline{M}_o^{F_2} = \bar{r}_{OA} \times \overline{F}_2$
$= (10\mathbf{i}) \times (-24\mathbf{j})$
$= -240\mathbf{k}$ kNm

$\overline{M}_o^{F_3} = \bar{r}_{OB} \times \overline{F}_3$
$= (10\mathbf{i} + 5\mathbf{k}) \times (-16\mathbf{j})$
$= 80\mathbf{i} - 160\mathbf{k}$ kNm

$\overline{M}_o^{F_4} = \bar{r}_{OC} \times \overline{F}_4$
$= (4\mathbf{i} + 10\mathbf{k}) \times (-20\mathbf{j})$
$= 200\mathbf{i} + 80\mathbf{k}$ kNm

$\overline{M}_o^{R} = \bar{r}_{OP} \times \overline{R}$
$= (x\mathbf{i} + z\mathbf{k}) \times (-140\mathbf{j})$
$= (140z)\mathbf{i} - (140x)\mathbf{k}$ kNm

Using Varinon's Theorem

$$\Sigma \overline{M}_o^F = \Sigma \overline{M}_o^R$$

$$\overline{M}_o^{F_1} + \overline{M}_o^{F_2} + \overline{M}_o^{F_3} + \overline{M}_o^{F_4} = \overline{M}_o^{R}$$

$$0 + (-240\mathbf{k}) + (80\mathbf{i} - 160\mathbf{k}) + (200\mathbf{i} + 80\mathbf{k}) = (140z)\mathbf{i} - (140x)\mathbf{k}$$

$$280\mathbf{i} - 480\mathbf{k} = (140z)\mathbf{i} - (140x)\mathbf{k}$$

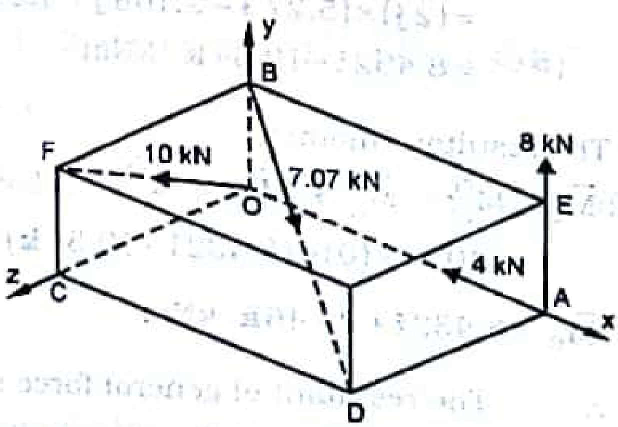
Equating the coefficients, we get

$280 = 140\,z \qquad \therefore \qquad z = 2$ m

also $-480 = -140\,x \qquad \therefore \qquad x = 3.428$ m

$\therefore$ The resultant of the force system is $\overline{R} = -140\mathbf{j}$. It acts at P (3.428, 0, 2) m on the square foundation. .... **Ans.**

P7. A rectangular parallelepiped carries four forces as shown in the figure. Reduce the force system to a resultant force applied at the origin and moment around the origin.
OA = 5 m, OB = 2 m, OC = 4 m. ***(M. U Dec 12)***



Solution: This is a General space force system consisting of four forces $F_1 = 8$ kN, $F_2 = 4$ kN, $F_3 = 7.07$ kN and $F_4 = 10$ kN.

Putting the forces in vector form.

$\overline{F}_1 = 8\mathbf{j}$ kN .... | Since the force parallel along the y axis in the + ve sense.

$\overline{F}_2 = -4\mathbf{i}$ kN .... | Since the force acts along the x axis in the – ve sense.

$$\bar{F}_3 = F_3 \cdot \hat{e}_{BD}$$

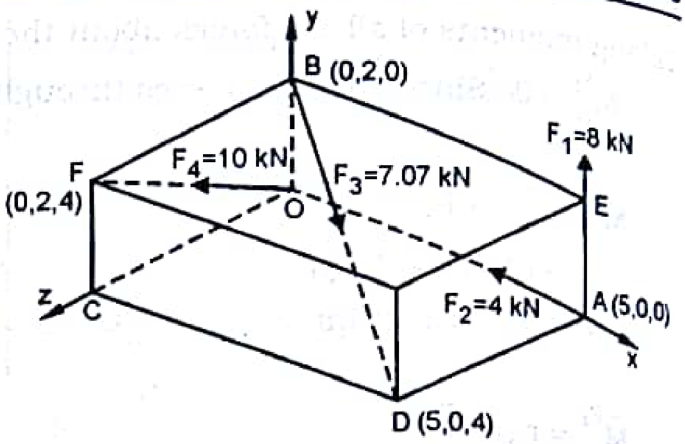
$$= 7.07\left[\frac{5\mathbf{i}-2\mathbf{j}+4\mathbf{k}}{\sqrt{5^2+2^2+4^2}}\right]$$

$$\therefore \quad \bar{F}_3 = 5.27\,\mathbf{i} - 2.108\,\mathbf{j} + 4.216\,\mathbf{k} \text{ kN}$$

$$\bar{F}_4 = F_4 \cdot \hat{e}_{DF}$$

$$= 10\left[\frac{2\mathbf{j}+4\mathbf{k}}{\sqrt{2^2+4^2}}\right]$$

$$\therefore \quad \bar{F}_4 = 4.472\,\mathbf{j} + 8.944\,\mathbf{k} \text{ kN}$$



The Resultant force $\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$

$$= (8\,\mathbf{j}) + (-4\,\mathbf{i}) + (5.27\,\mathbf{i} - 2.108\,\mathbf{j} + 4.216\,\mathbf{k}) + (4.472\,\mathbf{j} + 8.944\,\mathbf{k})$$

Or $\bar{R} = 1.27\,\mathbf{i} + 10.364\,\mathbf{j} + 13.16\,\mathbf{k}$ kN

Since the resultant of the general system is required at the origin, taking moments of all the forces at the origin.

$$\bar{M}_O^{F_1} = \bar{r}_{OA} \times \bar{F}_1$$

$$= (5\,\mathbf{i}) \times (8\,\mathbf{j})$$

$$= 40\,\mathbf{k} \text{ kNm}$$

$$\bar{M}_O^{F_2} = 0$$

Since the force F_2 passes through the moment centre O.

$$\bar{M}_O^{F_3} = \bar{r}_{OB} \times \bar{F}_3$$

$$= (2\,\mathbf{j}) \times (5.27\,\mathbf{i} - 2.108\,\mathbf{j} + 4.216\,\mathbf{k})$$

$$= 8.432\,\mathbf{i} - 10.54\,\mathbf{k} \text{ kNm}$$

$$\bar{M}_O^{F_4} = 0$$

Since the force F_4 passes through the moment centre O.

The resultant moment

$$\bar{M}_O = \bar{M}_O^{F_1} + \bar{M}_O^{F_2} + \bar{M}_O^{F_3} + \bar{M}_O^{F_4}$$

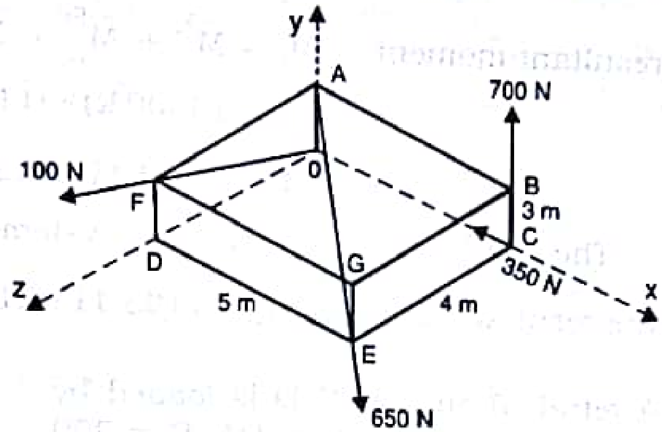
$$= (40\,\mathbf{k}) + (0) + (8.432\,\mathbf{i} - 10.54\,\mathbf{k}) + (0)$$

$$\bar{M}_O = 8.432\,\mathbf{i} + 29.46\,\mathbf{k} \text{ kNm}$$

$\therefore$ The resultant of general force system is $\bar{R} = 1.27\,\mathbf{i} + 10.364\,\mathbf{j} + 13.16\,\mathbf{k}$ kN

and the resultant moment $\bar{M}_O = 8.432\,\mathbf{i} + 29.46\,\mathbf{k}$ kNm .... **Ans.**

P8. Figure shows a rectangular parallelepiped subjected to four forces in the direction shown. Reduce them to a resultant force at the origin and a moment.



Solution: This is a General space force system consisting of four forces $F_1 = 700\,N$, $F_2 = 350\,N$, $F_3 = 650\,N$ and $F_4 = 100\,N$.

Putting the forces in vector form.

$\bar{F}_1 = 700\,\mathbf{j}$ N .... Since the force acts parallel to y axis in the + ve sense.

$\bar{F}_2 = -350\,\mathbf{i}$ N .... Since the force acts along the x axis in the – ve sense.

$\bar{F}_3 = F_3 \cdot \hat{e}_{AE}$

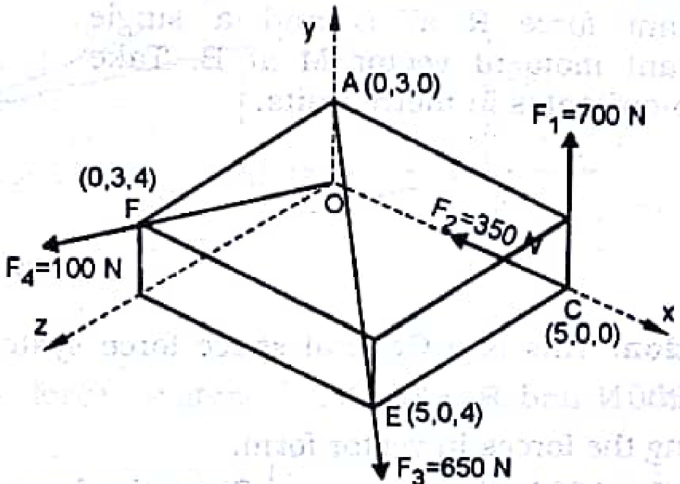
$$= 650\left[\frac{5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}}{\sqrt{5^2 + 3^2 + 4^2}}\right]$$

$\therefore \quad \bar{F}_3 = 459.6\mathbf{i} - 275.8\mathbf{j} + 367.7\mathbf{k}$ N

$\bar{F}_4 = F_4 \cdot \hat{e}_{OF}$

$$= 100\left[\frac{3\mathbf{j} + 4\mathbf{k}}{\sqrt{3^2 + 4^2}}\right]$$

$\therefore \quad \bar{F}_4 = 60\mathbf{j} + 80\mathbf{k}$ N



The Resultant force $\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$

$$= (700\,\mathbf{j}) + (-350\,\mathbf{i}) + (459.6\mathbf{i} - 275.8\mathbf{j} + 367.7\mathbf{k}) + (60\,\mathbf{j} + 80\,\mathbf{k})$$

Or $\quad \bar{R} = 109.6\mathbf{i} + 484.2\mathbf{j} + 447.7\mathbf{k}$ N

Since the resultant of the general system is required at the origin, taking moments of all the forces at the origin.

$\bar{M}_o^{F_1} = \bar{r}_{OC} \times \bar{F}_1$

$= (5\mathbf{i}) \times (700\,\mathbf{j}) = 3500\,\mathbf{k}$ Nm

$\bar{M}_o^{F_2} = 0$ .... Since the force F_2 passes through the moment centre O.

$\bar{M}_o^{F_3} = \bar{r}_{OA} \times \bar{F}_3$

$= (3\,\mathbf{j}) \times (459.6\,\mathbf{i} - 275.8\,\mathbf{j} + 367.7\mathbf{k}) = 1103.1\,\mathbf{i} - 1378.8\mathbf{k}$ Nm

$\bar{M}_o^{F_4} = 0$ Since the force F_4 passes through the moment centre O.

The resultant moment $\overline{M}_O = \overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3} + \overline{M}_O^{F_4}$

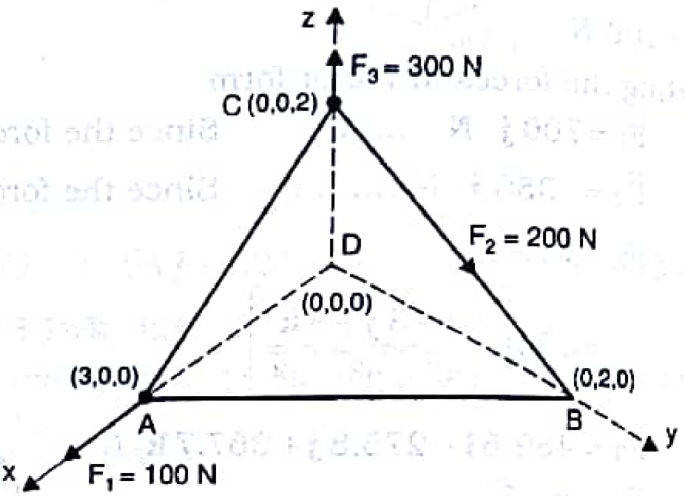
$$= (3500\,\mathbf{k}) + (1103.1\mathbf{i} - 1378.8\,\mathbf{k})$$

$$\overline{M}_O = 1103.1\mathbf{i} + 2121.2\mathbf{k} \text{ Nm}$$

$\therefore$ The resultant of General force system is $\overline{R} = 109.6\mathbf{i} + 484.2\mathbf{j} + 447.7\mathbf{k}$ N

and the resultant moment $\overline{M}_O = 1103.1\mathbf{i} + 2121.2\mathbf{k}$ Nm .... **Ans.**

P9. A tetrahedron A B C D is loaded by forces F_1 = 100 N at A along DA, F_2 = 200 N at B along CB and F_3 = 300 N at C along DC as shown in the figure. Replace the three force system by a single resultant force R at B and a single resultant moment vector M at B. Take the co-ordinates in metre units.



Solution: This is a General space force system consisting of three forces $F_1 = 100$N, $F_2 = 200$N and $F_3 = 300$N.

Putting the forces in vector form.

$\overline{F}_1 = 100\mathbf{i}$ N .... | Since the force acts along the x axis in the + ve sense.

$$\overline{F}_2 = F_2 . \hat{e}_{CB}$$

$$= 200\left[\frac{2\mathbf{j} - 2\mathbf{k}}{\sqrt{2^2 + 2^2}}\right]$$

$$\therefore \overline{F}_2 = 141.42\mathbf{j} - 141.42\mathbf{k} \text{ N}$$

$\overline{F}_3 = 300\,\mathbf{k}$ N .... | Since the force acts along the z axis in the + ve sense.

The Resultant force $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$

$$= (100\mathbf{i}) + (141.42\mathbf{j} - 141.42\mathbf{k}) + (300\,\mathbf{k})$$

Or $\overline{R} = 100\mathbf{i} + 141.42\mathbf{j} + 158.58\mathbf{k}$ N

Since the resultant of the general system is required at point B, taking moments of all the forces at B.

$\overline{M}_B^{F_1} = \overline{r}_{BA} \times \overline{F}_1$ $= (3\mathbf{i} - 2\mathbf{j}) \times (100\mathbf{i})$ $= 200\mathbf{k}$ Nm	$\overline{M}_B^{F_2} = 0$ Since the force F_2 passes through the moment centre B.	$\overline{M}_O^{F_3} = \overline{r}_{BC} \times \overline{F}_3$ $= (-2\mathbf{j} + 2\mathbf{k}) \times (300\mathbf{k})$ $= -600\,\mathbf{i}$ Nm

The resultant moment

$$\overline{M}_B = \overline{M}_O^{F_1} + \overline{M}_O^{F_2} + \overline{M}_O^{F_3}$$

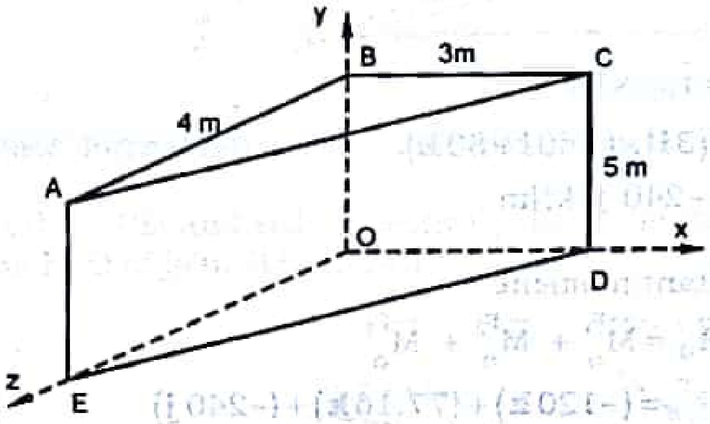
$$= (200\,\mathbf{k}) + (-600\,\mathbf{i})$$

$$\overline{M}_B = -600\,\mathbf{i} + 200\,\mathbf{k}\ \text{Nm}$$

$\therefore$ The resultant of General force system is $\overline{R} = 100\,\mathbf{i} + 141.42\,\mathbf{j} + 158.58\,\mathbf{k}$ N

and The resultant moment $\overline{M}_B = -600\,\mathbf{i} + 200\,\mathbf{k}$ Nm .... **Ans.**

P10. The following forces act on the block shown in figure. F_1 = 40 kN at point C along CD, F_2 = 30 kN at point D along DB and F_3 = 100 kN at point D along DE. Find the resultant force and resultant moment of these forces acting at O. ***(SPCE Nov 12)***



Solution: This is a concurrent space force system consisting of three forces $F_1 = 40\,\text{kN}$, $F_2 = 30\,\text{kN}$ and $F_3 = 100\,\text{kN}$.

Putting the forces in vector form.

$\overline{F}_1 = -40\,\mathbf{j}$ kN .... | Since the force acts parallel to the y axis in the – ve sense.

$$\overline{F}_2 = F_2 \cdot \hat{e}_{DB}$$

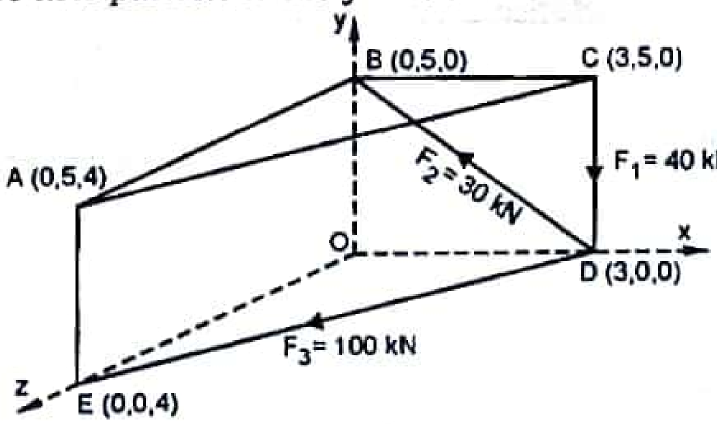
$$= 30\left[\frac{-3\,\mathbf{i} + 5\,\mathbf{j}}{\sqrt{3^2 + 5^2}}\right]$$

$$\therefore\quad \overline{F}_2 = -15.43\,\mathbf{i} + 25.72\,\mathbf{j}\ \text{kN}$$

$$\overline{F}_3 = F_3 \cdot \hat{e}_{DE}$$

$$= 100\left[\frac{-3\,\mathbf{i} + 4\,\mathbf{k}}{\sqrt{3^2 + 4^2}}\right]$$

$$\therefore\quad \overline{F}_3 = -60\,\mathbf{i} + 80\,\mathbf{k}\ \text{kN}$$



The Resultant force $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3$

$$= (-40\,\mathbf{j}) + (-15.43\,\mathbf{i} + 25.72\,\mathbf{j}) + (-60\,\mathbf{i} + 80\,\mathbf{k})$$

Or $\quad \overline{R} = -75.43\,\mathbf{i} - 14.28\,\mathbf{j} + 80\,\mathbf{k}$ kN

Since the resultant of the concurrent system is required at the origin, taking moments of all the forces at origin.

$$\overline{M}_{O}^{F_1} = \bar{r}_{OD} \times \overline{F}_1$$
$$= (3\mathbf{i}) \times (-40\,\mathbf{j})$$
$$= -120\mathbf{k} \;\; \text{kNm}$$

$$\overline{M}_{O}^{F_2} = \bar{r}_{OD} \times \overline{F}_2$$
$$= (3\mathbf{i}) \times (-15.43\,\mathbf{i} + 25.72\,\mathbf{j})$$
$$= 77.16\,\mathbf{k} \;\; \text{kNm}$$

$$\overline{M}_{O}^{F_3} = \bar{r}_{OD} \times \overline{F}_3$$
$$= (3\mathbf{i}) \times (-60\mathbf{i} + 80\mathbf{k})$$
$$= -240\,\mathbf{j} \;\; \text{kNm}$$

The resultant moment

$$\overline{M}_O = \overline{M}_{O}^{F_1} + \overline{M}_{O}^{F_2} + \overline{M}_{O}^{F_3}$$
$$= (-120\mathbf{k}) + (77.16\mathbf{k}) + (-240\,\mathbf{j})$$
$$\overline{M}_O = -240\,\mathbf{j} - 42.84\,\mathbf{k} \;\; \text{kNm}$$

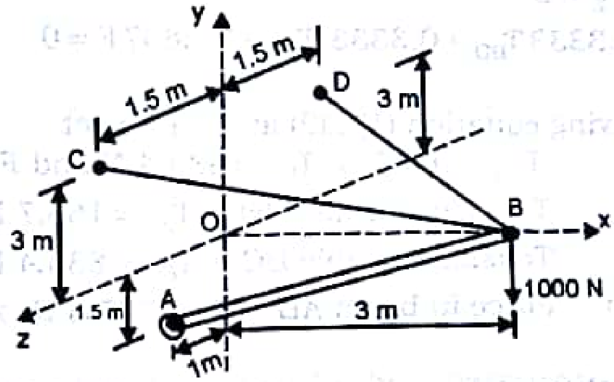
$\therefore$ The resultant of the concurrent system at origin is $\overline{R} = -75.43\,\mathbf{i} - 14.28\,\mathbf{j} + 80\mathbf{k}$ kN

and the resultant moment $\overline{M}_O = -240\mathbf{i} - 42.84\,\mathbf{k}$ kNm .... **Ans.**

Exercise 7.3

Equilibrium of Space Force System

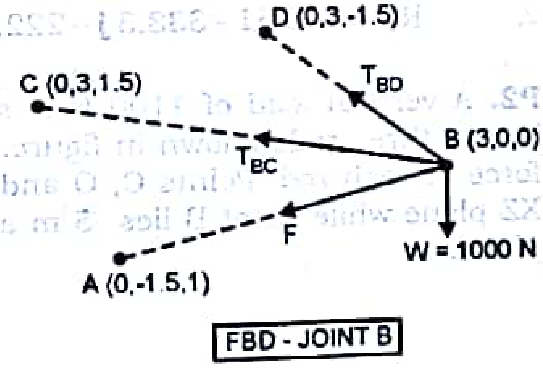
P1. A boom AB supports a load of 1000 N as shown. Neglect weight of the boom. Determine tension in each cable and the reaction at A.



Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at B.

Let T_{BD}, T_{BC} be the tension in the cables BD and BC respectively. Let F be the force in the boom AB and W be the load. The FBD of joint B is shown.

Putting the forces in vector form,



$$\bar{T}_{BD} = T_{BD}\cdot\hat{e}_{BD}$$

$$= T_{BD}\left[\frac{-3\mathbf{i}+3\mathbf{j}-1.5\mathbf{k}}{\sqrt{3^2+3^2+1.5^2}}\right]$$

$$\therefore \quad \bar{T}_{BD} = T_{BD}(-0.6667\mathbf{i}+0.6667\mathbf{j}-0.3333\mathbf{k})\ \text{N}$$

$$\bar{T}_{BC} = T_{BC}\cdot\hat{e}_{BC}$$

$$= T_{BC}\left[\frac{-3\mathbf{i}+3\mathbf{j}+1.5\mathbf{k}}{\sqrt{3^2+3^2+1.5^2}}\right]$$

$$\therefore \quad \bar{T}_{BC} = T_{BC}(-0.6667\mathbf{i}+0.6667\mathbf{j}+0.3333\mathbf{k})\ \text{N}$$

$$\bar{F} = F.\hat{e}_{BA}$$

$$= F\left[\frac{-3\mathbf{i}-1.5\mathbf{j}+\mathbf{k}}{\sqrt{3^2+1.5^2+1^2}}\right]$$

$$\therefore \quad \bar{F} = F(-0.8571\mathbf{i}-0.4286\mathbf{j}+0.2857\mathbf{k})\ \text{N}$$

$\bar{W} = -1000\,\mathbf{j}$ N .... Since the force acts parallel to the y axis in the – ve sense.

Applying COE

$\Sigma F_x = 0$

$-0.6667\,T_{BD} - 0.6667\,T_{BC} - 0.8571F = 0$ ....(1)

$\Sigma F_Y = 0$

$0.6667\,T_{BD} + 0.6667\,T_{BC} - 0.4286F - 1000 = 0$ .... (2)

$\Sigma F_Z = 0$

$-0.3333\,T_{BD} + 0.3333\,T_{BC} + 0.2857F = 0$ .... (3)

Solving equation (1), (2) and (3) we get

$T_{BD} = 166.7$ N, $T_{BC} = 833.4$ N and $F = -777.8$ N

∴ Tension in cable BD = $T_{BD} = 166.7$ N .... **Ans.**

Tension in cable BC = $T_{BC} = 833.4$ N .... **Ans.**

also Force in boom AB = $F = -777.8$ N

Reaction at A

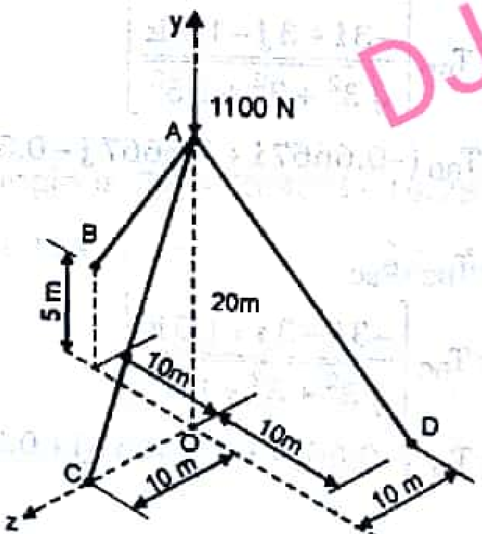
Since the boom AB is a rod with a ball and socket joint at A, the force F in the boom is equal to the reaction by the joint at A.

∴ $\overline{R}_A = \overline{F}$

$= -777.8(-0.8571\mathbf{i} - 0.4286\mathbf{j} + 0.2857\mathbf{k})$ N

∴ $\overline{R}_A = 666.6\mathbf{i} + 333.3\mathbf{j} - 222.2\mathbf{k}$ N .... **Ans.**

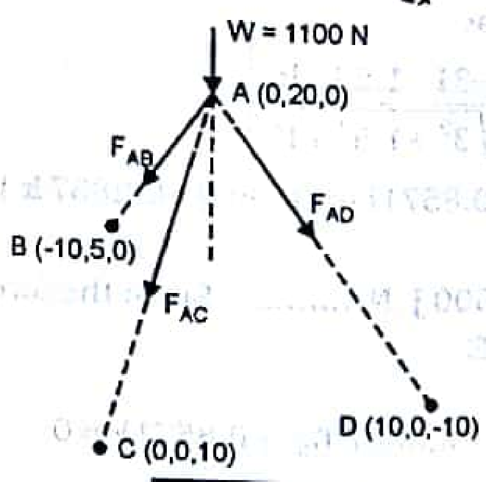
P2. A vertical load of 1100 N is supported by the three rods shown in figure. Find the force in each rod. Points C, O and D are in XZ plane while point B lies 5 m above this plane.



Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at A.

Let F_{AB}, F_{AC} and F_{AD} be the forces in rods AB, AC and AD respectively and W be the load. The FBD of joint A is shown.

Putting the forces in vector form,



FBD - JOINT A

$$\overline{F}_{AB} = F_{AB}\,.\hat{e}_{AB}$$

$$= F_{AB}\left[\frac{-10\mathbf{i} - 15\mathbf{j}}{\sqrt{10^2 + 15^2}}\right]$$

∴ $\overline{F}_{AB} = F_{AB}(-0.5547\mathbf{i} - 0.832\mathbf{j})$ N

$$\bar{F}_{AC} = F_{AC} \cdot \hat{e}_{AC}$$

$$= F_{AC}\left[\frac{-20\mathbf{j}+10\mathbf{k}}{\sqrt{20^2+10^2}}\right]$$

$$\therefore \quad \bar{F}_{AC} = F_{AC}(-0.894\mathbf{j}+0.447\mathbf{k}) \text{ N}$$

$$\bar{F}_{AD} = F_{AD} \cdot \hat{e}_{AD}$$

$$= F_{AD}\left[\frac{10\mathbf{i}-20\mathbf{j}-10\mathbf{k}}{\sqrt{10^2+20^2+10^2}}\right]$$

$$\therefore \quad \bar{F}_{AD} = F_{AD}(0.4082\mathbf{i}-0.8165\mathbf{j}-0.4082\mathbf{k}) \text{ N}$$

$\bar{W} = -1100\mathbf{j}$ N .... Since the force acts along the y axis in the – ve sense.

Applying COE

$\Sigma F_X = 0$

$-0.5547 F_{AB} + 0.4082 F_{AD} = 0$ .... (1)

$\Sigma F_Y = 0$

$-0.832 F_{AB} - 0.894 F_{AC} - 0.8165 F_{AD} - 1100 = 0$ .... (2)

$\Sigma F_Z = 0$

$0.447 F_{AC} - 0.4082 F_{AD} = 0$ .... (3)

Solving equation (1) , (2) and (3) we get

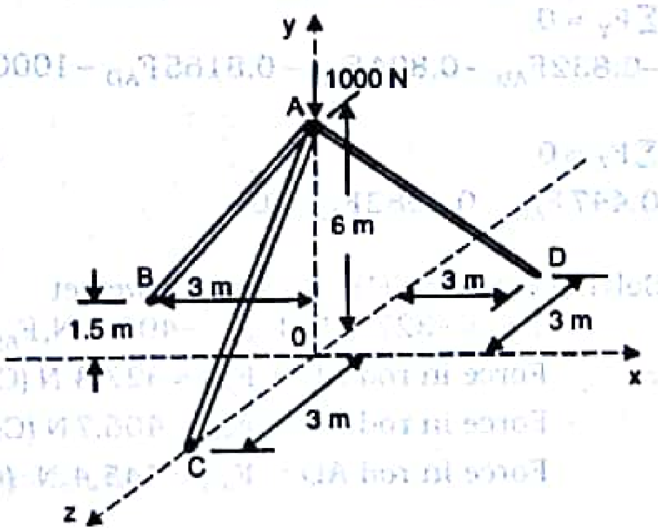
$F_{AB} = -360.5$ N, $F_{AC} = -447.4$ N, $F_{AD} = -489.9$ N

$\therefore$ Force in rod AB = $F_{AB} = 360.5$ N (Compressive) .... **Ans.**

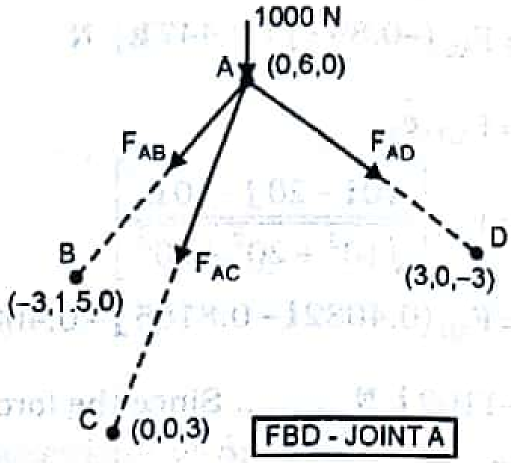
Force in rod AC = $F_{AC} = 447.4$ N (Compressive) .... **Ans.**

Force in rod AD = $F_{AD} = 489.9$ N (Compressive) .... **Ans.**

P3. A vertical load of 1000 N is supported by three bars as shown. Find the force in each bar. Point C, O and D are in the x-z plane while B is 1.5 m above this plane.



Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at A.
Let F_{AB}, F_{AC} and F_{AD} be the forces in rods AB, AC and AD respectively and W be the load. The FBD of joint A is shown.



FBD - JOINT A

Putting the forces in vector form,

$$\overline{F}_{AB} = F_{AB} \cdot \hat{e}_{AB}$$

$$= F_{AB}\left[\frac{-3\mathbf{i} - 4.5\mathbf{j}}{\sqrt{3^2 + 4.5^2}}\right]$$

$\therefore$ $$\overline{F}_{AB} = F_{AB}(-0.5547\mathbf{i} - 0.832\mathbf{j}) \text{ N}$$

$$\overline{F}_{AC} = F_{AC} \cdot \hat{e}_{AC}$$

$$= F_{AC}\left[\frac{-6\mathbf{j} + 3\mathbf{k}}{\sqrt{6^2 + 3^2}}\right]$$

$\therefore$ $$\overline{F}_{AC} = F_{AC}(-0.894\mathbf{j} + 0.447\mathbf{k}) \text{ N}$$

$$\overline{F}_{AD} = F_{AD} \cdot \hat{e}_{AD}$$

$$= F_{AD}\left[\frac{3\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}}{\sqrt{3^2 + 6^2 + 3^2}}\right]$$

$\therefore$ $$\overline{F}_{AD} = F_{AD}(0.4082\mathbf{i} - 0.8165\mathbf{j} - 0.4082\mathbf{k}) \text{ N}$$

$\overline{W} = -1000\,\mathbf{j}$ N .... Since the force acts along the y axis in the – ve sense.

Applying COE

$\Sigma F_X = 0$

$-0.5547 F_{AB} + 0.4082 F_{AD} = 0$ .... (1)

$\Sigma F_Y = 0$

$-0.832 F_{AB} - 0.894 F_{AC} - 0.8165 F_{AD} - 1000 = 0$ .... (2)

$\Sigma F_Z = 0$

$0.447 F_{AC} - 0.4082 F_{AD} = 0$ .... (3)

Solving equation (1), (2) and (3) we get

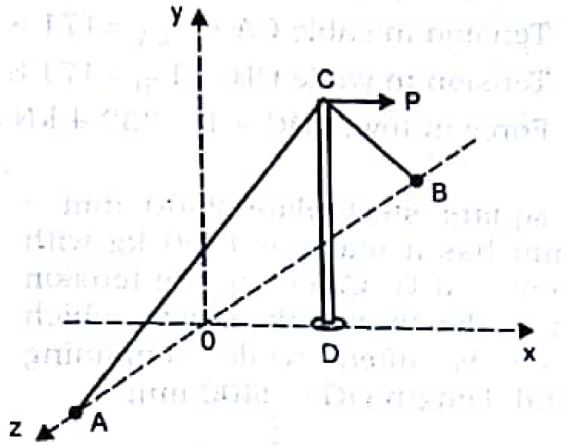
$F_{AB} = -327.8$ N, $F_{AC} = -406.7$ N, $F_{AD} = -445.4$ N

$\therefore$ Force in rod AB = $F_{AB} = 327.8$ N (Compressive) .... **Ans.**

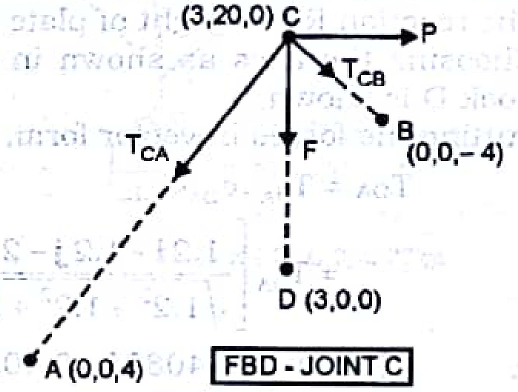
Force in rod AC = $F_{AC} = 406.7$ N (Compressive) .... **Ans.**

Force in rod AD = $F_{AD} = 445.4$ N (Compressive) .... **Ans.**

P4. A vertical tower DC shown is subjected to a horizontal force P = 50 kN at its top and is anchored by two similar guy wires BC and AC. Calculate
Tension in the guy wires.
Thrust in the tower pole.
Co-ordinates of the points are as below,
O (0, 0, 0), B (0, 0, – 4),
D (3, 0, 0), A (0, 0, 4),
C (3, 20, 0)



Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at C.
Let T_{CA}, T_{CB} be the tension in the cables CA and CB respectively. Let F be the force (thrust) in the tower DC. The FBD of joint C is shown.



Putting the forces in vector form,

$$\bar{T}_{CA} = T_{CA}\,.\hat{e}_{CA}$$

$$= T_{CA}\left[\frac{-3\mathbf{i}-20\,\mathbf{j}+4\,\mathbf{k}}{\sqrt{3^2+20^2+4^2}}\right]$$

$\therefore$ $$\bar{T}_{CA} = T_{CA}\,(-0.1455\mathbf{i}-0.9701\,\mathbf{j}+0.194\,\mathbf{k})\ \text{kN}$$

$$\bar{T}_{CB} = T_{CB}\,.\hat{e}_{CB}$$

$$= T_{CB}\left[\frac{-3\mathbf{i}-20\,\mathbf{j}-4\,\mathbf{k}}{\sqrt{3^2+20^2+4^2}}\right]$$

$\therefore$ $$\bar{T}_{CB} = T_{CB}\,(-0.1455\mathbf{i}-0.9701\,\mathbf{j}-0.194\,\mathbf{k})\ \text{kN}$$

$\bar{F} = -F\,\mathbf{j}$ kN .... Since the force acts parallel to the y axis in the – ve sense.

$\bar{P} = 50\,\mathbf{i}$ kN .... Since the force acts parallel to the x axis in the + ve sense.

Applying COE

$\Sigma F_x = 0$

$-0.1455\,T_{CA} - 0.1455\,T_{CB} + 50 = 0$ .... (1)

$\Sigma F_y = 0$

$-0.9701\,T_{CA} - 0.9701\,T_{CB} - F = 0$ .... (2)

$\Sigma F_z = 0$

$0.194\,T_{CA} - 0.194\,T_{CB} = 0$ .... (3)

Solving equation (1), (2) and (3) we get

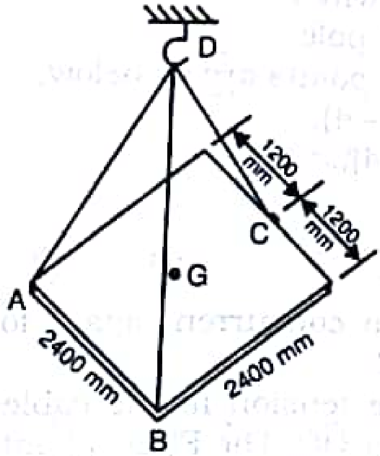
$T_{CA} = 171.8$ kN, $T_{CB} = 171.8$ kN, $F = -333.4$ kN

∴ Tension in cable CA = $T_{CA} = 171.8$ kN .... **Ans.**

Tension in cable CB = $T_{CB} = 171.8$ kN .... **Ans.**

also Force in tower DC = F = 333.4 kN (Compression) .... **Ans.**

P5. A square steel plate 2400 mm × 2400 mm has a mass of 1800 kg with mass centre at G. Calculate the tension in each of the three cables with which the plate is lifted while remaining horizontal. Length DG = 2400 mm



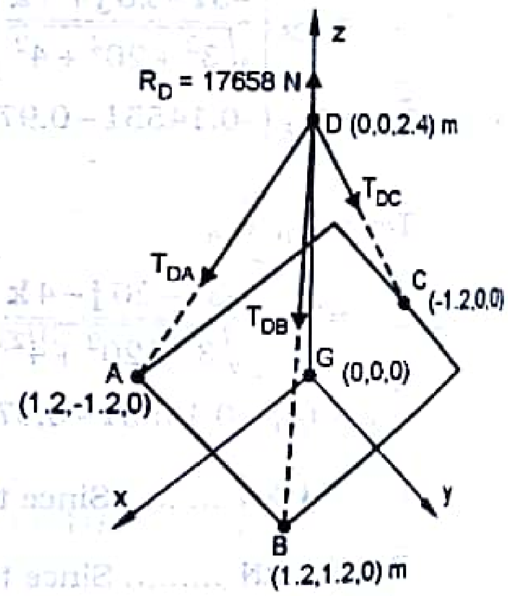
Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at D.

Let T_{DA}, T_{DB}, T_{DC} be the tension in the cables DA, DB and DC respectively. Also let R_D be the reaction at the hook D.

The reaction R_D = weight of plate = 1800 × 9.81 = 17658 N.

Choosing the axes as shown in figure with origin at G. The FBD of the forces at the hook D is shown.

Putting the forces in vector form,



FBD - HOOK D

$$\overline{T}_{DA} = T_{DA} \cdot \hat{e}_{DA}$$

$$= T_{DA}\left[\frac{1.2\mathbf{i} - 1.2\mathbf{j} - 2.4\mathbf{k}}{\sqrt{1.2^2 + 1.2^2 + 2.4^2}}\right]$$

$$\therefore \quad \overline{T}_{DA} = T_{DA}\,(0.4082\mathbf{i} - 0.4082\mathbf{j} - 0.8165\mathbf{k})\ \text{N}$$

$$\overline{T}_{DB} = T_{DB} \cdot \hat{e}_{DB}$$

$$= T_{DB}\left[\frac{1.2\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k}}{\sqrt{1.2^2 + 1.2^2 + 2.4^2}}\right]$$

$$\therefore \quad \overline{T}_{DB} = T_{DB}\,(0.4082\mathbf{i} + 0.4082\mathbf{j} - 0.8165\mathbf{k})\ \text{N}$$

$$\overline{T}_{DC} = T_{DC} \cdot \hat{e}_{DC}$$

$$= T_{DC}\left[\frac{-1.2\mathbf{i} - 2.4\mathbf{k}}{\sqrt{1.2^2 + 2.4^2}}\right]$$

$$\therefore \quad \overline{T}_{DC} = T_{DC}\,(-0.4472\mathbf{i} - 0.8944\mathbf{k})\ \text{N}$$

$\overline{R}_D = 17658\,\mathbf{k}$ N .... Since the force acts along the z axis in the + ve sense.

Applying COE

$\Sigma F_x = 0$

$0.4082\,T_{DA} + 0.4082\,T_{DB} - 0.4472\,T_{DC} = 0$ .... (1)

$\Sigma F_Y = 0$

$$-0.4082\,T_{DA} + 0.4082\,T_{DB} = 0 \quad \text{......... (2)}$$

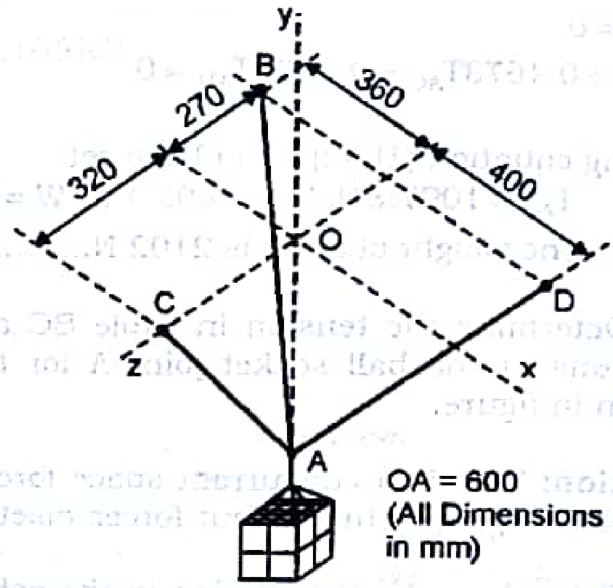
$\Sigma F_Z = 0$

$$-0.8165\,T_{DA} - 0.8165\,T_{DB} - 0.8944\,T_{DC} + 17658 = 0 \quad \text{......... (3)}$$

Solving equation (1), (2) and (3) we get

$T_{DA} = 5407$ N, $T_{DB} = 5407$ N, $T_{DC} = 9870$ N .... **Ans.**

P6. A crate is supported by three cables as shown. Determine the weight of the crate, if the tension in the cable AB is 750 N.



OA = 600
(All Dimensions are in mm)

Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at A.

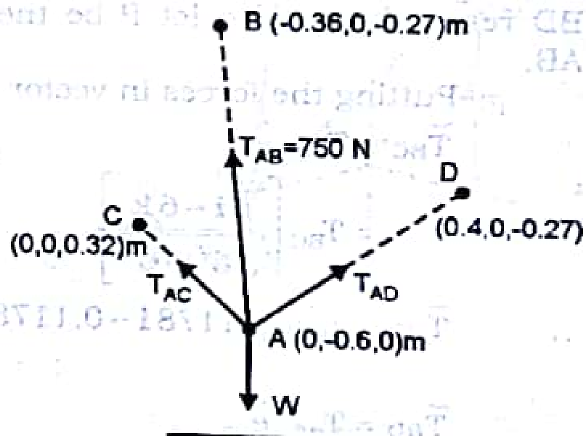
Let T_{AB}, T_{AC}, T_{AD} be the tension in the cables AB, AC and AD respectively. Also let W be the unknown weight.

Putting the forces in vector form,

$$\overline{T}_{AB} = T_{AB}\,.\,\hat{e}_{AB}$$

$$= 750\left[\frac{-0.36\mathbf{i} + 0.6\mathbf{j} - 0.27\mathbf{k}}{\sqrt{0.36^2 + 0.6^2 + 0.27^2}}\right]$$

$$\therefore \quad \overline{T}_{AB} = -360\mathbf{i} + 600\mathbf{j} - 270\mathbf{k} \text{ N}$$



FBD - JOINT A

$$\overline{T}_{AC} = T_{AC}\,.\,\hat{e}_{AC}$$

$$= T_{AC}\left[\frac{0.6\mathbf{j} + 0.32\mathbf{k}}{\sqrt{0.6^2 + 0.32^2}}\right]$$

$$\therefore \quad \overline{T}_{AC} = T_{AC}\,(0.8762\mathbf{j} + 0.4673\mathbf{k}) \text{ N}$$

$$\overline{T}_{AD} = T_{AD}\,.\,\hat{e}_{AD}$$

$$= T_{AD}\left[\frac{0.4\mathbf{i} + 0.6\mathbf{j} - 0.27\mathbf{j}}{\sqrt{0.4^2 + 0.6^2 + 0.27^2}}\right]$$

$$\therefore \quad \overline{T}_{AD} = T_{AD}\,(0.5194\mathbf{i} + 0.7792\mathbf{j} - 0.3506\mathbf{k}) \text{ N}$$

$\overline{W} = -W\mathbf{j}$ N .... Since the force acts along the y axis in the – ve sense.

Applying COE

$\Sigma F_X = 0$

$-360 + 0.5194T_{AD} = 0$ .... (1)

$\Sigma F_Y = 0$

$600 + 0.8762T_{AC} + 0.7792T_{AD} - W = 0$ .... (2)

$\Sigma F_Z = 0$

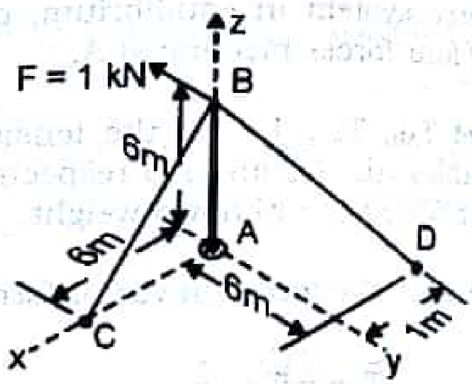
$-270 + 0.4673T_{AC} - 0.3506T_{AD} = 0$ .... (3)

Solving equation (1), (2) and (3) we get

$T_{AC} = 1097.8\text{ N},\ T_{AD} = 693.1\text{ N},\ W = 2102\text{ N}$

$\therefore$ The weight of crate is 2102 N......... **Ans.**

P7. Determine the tension in cable BC and BD and reactions at the ball socket joint A for the mast as shown in figure.



Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at B.

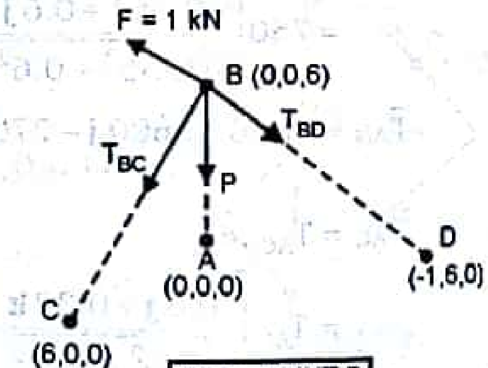
Let T_{BC} and T_{BD} be the tension in the cables BC and BD respectively. Also let P be the force in the mast AB.

Putting the forces in vector form,

$\bar{T}_{BC} = T_{BC} \cdot \hat{e}_{BC}$

$= T_{BC}\left[\dfrac{6\mathbf{i} - 6\mathbf{k}}{\sqrt{6^2 + 6^2}}\right]$

$\therefore \quad \bar{T}_{BC} = T_{BC}\,(0.1178\mathbf{i} - 0.1178\mathbf{k})\ \text{kN}$



FBD - JOINT B

$\bar{T}_{BD} = T_{BD} \cdot \hat{e}_{BD}$

$= T_{BD}\left[\dfrac{-\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}}{\sqrt{1^2 + 6^2 + 6^2}}\right]$

$\therefore \quad \bar{T}_{BD} = T_{BD}\,(-0.117\mathbf{i} + 0.7022\mathbf{j} - 0.7022\mathbf{k})\ \text{kN}$

$\bar{P} = -P\mathbf{k}\ \text{kN}$ .... Since the force acts along the z axis in the – ve sense.

$\bar{F} = -1\mathbf{j}\ \text{kN}$ .... Since the force acts parallel to the y axis in the – ve sense.

Applying COE

$\Sigma F_X = 0$

$0.1178T_{BC} - 0.117T_{BD} = 0$ .... (1)

$\sum F_Y = 0$

$0.7022T_{BD} - 1 = 0$ .... (2)

$\sum F_Z = 0$

$-0.1178T_{BC} - 0.7022T_{BD} - P = 0$ .... (3)

Solving equation (1), (2) and (3) we get

$$T_{BC} = 1.414\,\text{kN},\ T_{BD} = 1.424\ \text{kN},\ P = -1.166\,\text{kN}$$

∴ Tension in cable BC = $T_{BC} = 1.414$ kN .... **Ans.**

Tension in cable BD = $T_{BD} = 1.424$ kN .... **Ans.**

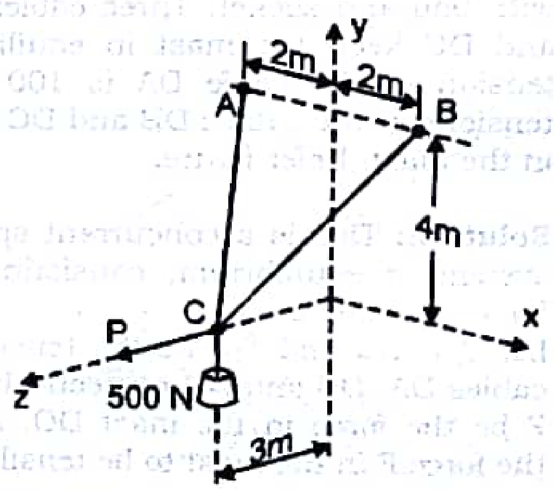
also Reaction at A

The reaction at ball and socket joint A = force in rod AB

∴ $R_A = -P\mathbf{k} = -(-1.166)\mathbf{k}$

∴ $R_A = 1.166\mathbf{k}$ kN .... **Ans.**

P8. A load of 500 N is held in equilibrium by means of two strings CA and CB and by a force P as shown in figure. Determine tensions in strings and magnitude of P.



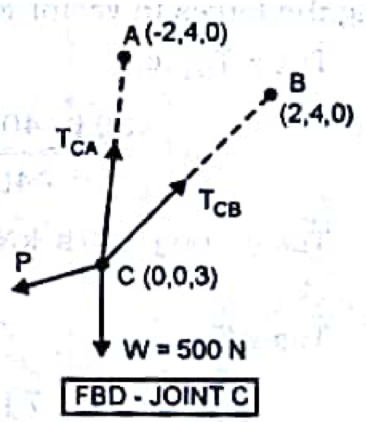
Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at C. Let T_{CA} and T_{CB} be the tension in the cables CA and CB respectively.

Putting the forces in vector form,

$$\bar{T}_{CA} = T_{CA}\cdot\hat{e}_{CA}$$

$$= T_{CA}\left[\frac{-2\mathbf{i}+4\mathbf{j}-3\mathbf{k}}{\sqrt{2^2+4^2+3^2}}\right]$$

∴ $\bar{T}_{CA} = T_{CA}(-0.3714\mathbf{i}+0.7428\mathbf{j}-0.5571\mathbf{k})$ N



$$\bar{T}_{CB} = T_{CB}\cdot\hat{e}_{CB}$$

$$= T_{CB}\left[\frac{2\mathbf{i}+4\mathbf{j}-3\mathbf{k}}{\sqrt{2^2+4^2+3^2}}\right]$$

∴ $\bar{T}_{CB} = T_{CB}(0.3714\mathbf{i}+0.7428\mathbf{j}-0.5571\mathbf{k})$ N

$\overline{W} = -500\,\mathbf{j}$ N .... Since the force acts parallel to the y axis in the – ve sense.

$\overline{P} = P\,\mathbf{k}$ N .... Since the force acts along the z axis in the + ve sense.

Applying COE

$\Sigma F_X = 0$

$-0.3714T_{CA} + 0.3714T_{CB} = 0$ .... (1)

$\Sigma F_Y = 0$

$0.7428T_{CA} + 0.7428T_{CB} - 500 = 0$ .... (2)

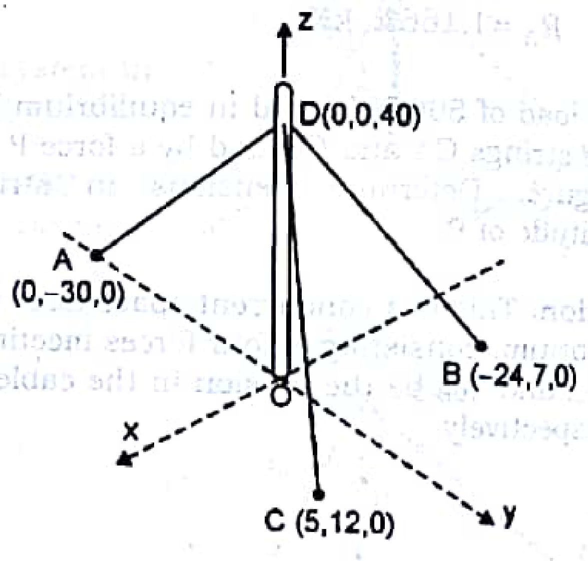
$\Sigma F_Z = 0$

$-0.5571T_{CA} - 0.5571T_{CB} + P = 0$ .... (3)

Solving equation (1), (2) and (3) we get

$T_{CA} = 336.56$ N, $T_{CB} = 336.56$ N, $P = 375$ N

P9. A vertical mast OD is having base 'O' with ball and socket. Three cables DA, DB and DC keep the mast in equilibrium. If tension in the cable DA is 100 kN, find tensions in the cables DB and DC and force in the mast. Refer figure.



Solution: This is a concurrent space force system in equilibrium, consisting of four forces meeting at D.

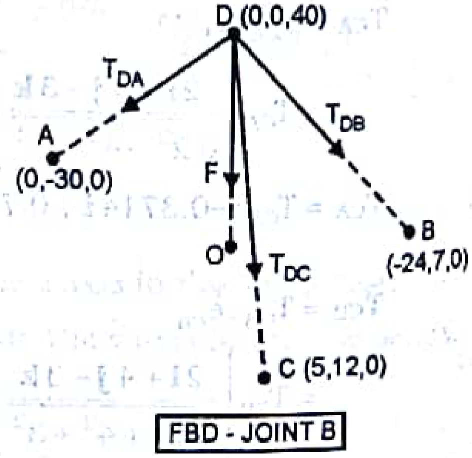
Let T_{DA}, T_{DB} and T_{DC} be the tension in the cables DA, DB and DC respectively. Also let F be the force in the mast DO. Assuming the force F in the mast to be tensile.

Putting the forces in vector form,

$$\overline{T}_{DA} = T_{DA}\,.\hat{e}_{DA}$$

$$= T_{DA}\left[\frac{-30\,\mathbf{j} - 40\,\mathbf{k}}{\sqrt{30^2 + 40^2}}\right]$$

$\therefore$ $\overline{T}_{DA} = -60\,\mathbf{j} - 80\,\mathbf{k}$ kN ...... since $T_{DA} = 100$ kN



$$\overline{T}_{DB} = T_{DB}\,.\hat{e}_{DB}$$

$$= T_{DB}\left[\frac{-24\,\mathbf{i} + 7\,\mathbf{j} - 40\,\mathbf{k}}{\sqrt{24^2 + 7^2 + 40^2}}\right]$$

$\therefore$ $\overline{T}_{DB} = T_{DB}\,(-0.5088\,\mathbf{i} + 0.1484\,\mathbf{j} - 0.848\,\mathbf{k})$ kN

$$\bar{T}_{DC} = T_{DC} \cdot \hat{e}_{DC}$$

$$= T_{DC}\left[\frac{5\mathbf{i}+12\mathbf{j}-40\mathbf{k}}{\sqrt{5^2+12^2+40^2}}\right]$$

$\therefore$ $$\bar{T}_{DC} = T_{DC}(0.1189\mathbf{i}+0.2853\mathbf{j}-0.951\mathbf{k})\ \text{kN}$$

$\bar{F} = -F\mathbf{k}$ kN .... Since the force acts parallel to the z axis in the – ve sense.

Applying COE

$\Sigma F_X = 0$

$-0.5088T_{DB} + 0.1189T_{DC} = 0$ ... (1)

$\Sigma F_Y = 0$

$-60 + 0.1484T_{DB} + 0.2853T_{DC} = 0$ ... (2)

$\Sigma F_Z = 0$

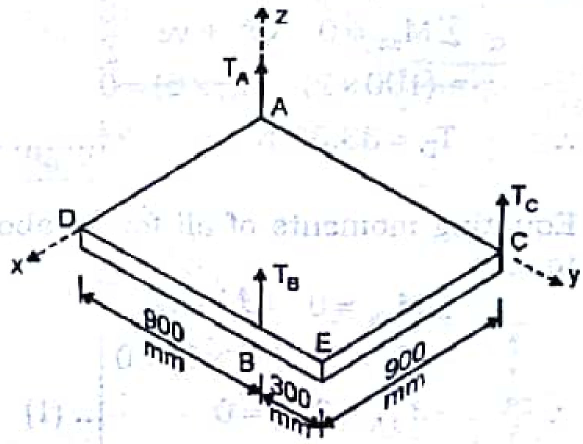
$-80 - 0.848T_{DB} - 0.951T_{DC} - F = 0$ ... (3)

Solving equation (1), (2) and (3) we get

$T_{DB} = 43.82$ kN, $T_{DC} = 187.51$ kN .... **Ans.**

and $F = -295.48$ kN = 295.48 kN (C) .... **Ans.**

P10. Plate ACED 10 mm thick weighs 7600 kg/m³. It is held in horizontal plane by three wires at A, B and C. find tensions in the wires.



Solution: This is a parallel space force system in equilibrium, consisting of four forces T_A, T_B, T_C and weight W of the plate.

Weight of the plate

$W = m \times g = (\rho \times v) \times g$

$= 7600 \times (1.2 \times 0.9 \times 0.01) \times 9.81$

$W = 805.2$ N acts through G.

Applying COE

Equating moments of all forces about yy axis to zero

$\Sigma M_{yy} = 0$ ↺ + ve

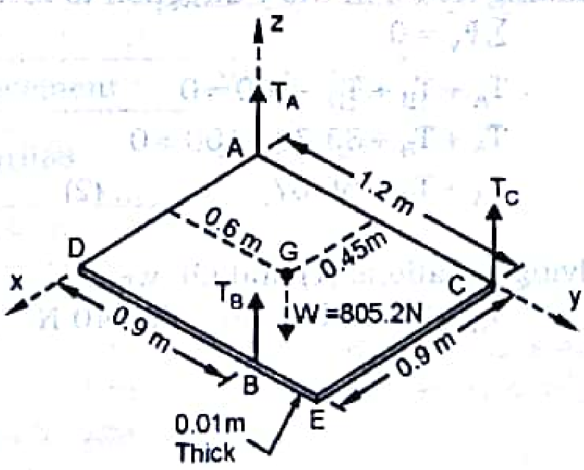
$+(805.2 \times 0.45) - (T_B \times 0.9) = 0$

$\therefore$ $T_B = 402.6$ N .... **Ans.**

Equating moments of all forces about xx axis to zero

$\Sigma M_{xx} = 0$ ↺ + ve

$+(T_B \times 0.9) + (T_C \times 1.2) - (805.2 \times 0.6) = 0$



FBD - PLATE

Substituting $T_B = 402.6$ N , we get

$T_C = 100.65$ N .... **Ans.**

Equating all forces in the z direction to zero

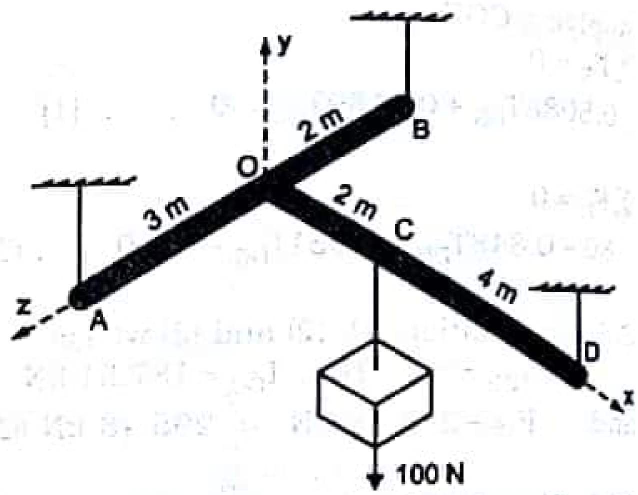
$$\Sigma F_Z = 0$$

$$T_A + T_B + T_C - W = 0$$

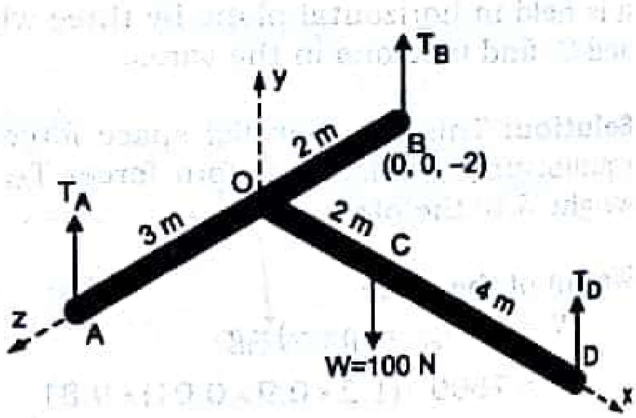
$$T_A + 402.6 + 100.65 - 805.2 = 0$$

$\therefore \quad T_A = 301.95$ N .... **Ans.**

P11. A T-shaped rod is suspended using three cable as shown. Neglecting the weight the weight of the rods, find the tension in each cable. ***(M.U Dec 16)***



Solution: This is a parallel space force system in equilibrium, consisting of four tension forces T_A, T_B, T_D and weight W = 100 N.



Applying COE

Equating moments of all forces about z-z axis to zero

$$\Sigma M_{zz} = 0 \quad \circlearrowleft + ve$$

$$-(100 \times 2) - (T_D \times 6) = 0$$

$\therefore \quad T_D = 33.33$ N .... **Ans.**

Equating moments of all forces about xx axis to zero

$$\Sigma M_{xx} = 0 \quad \circlearrowleft + ve$$

$$-(T_A \times 3) + (T_B \times 2) = 0$$

$\therefore \quad -3\,T_A + 2\,T_B = 0 \quad \ldots (1)$

Equating forces in the y direction to zero

$$\Sigma F_y = 0$$

$$T_A + T_B + T_D - 100 = 0$$

$\therefore \quad T_A + T_B + 33.33 - 100 = 0$

or $\quad T_A + T_B = 66.67 \quad \ldots (2)$

Solving equations (1) and (2), we get

$T_A = 26.67$ N and $T_B = 40$ N .... **Ans.**

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