DJC

Solutions: Chapter 9

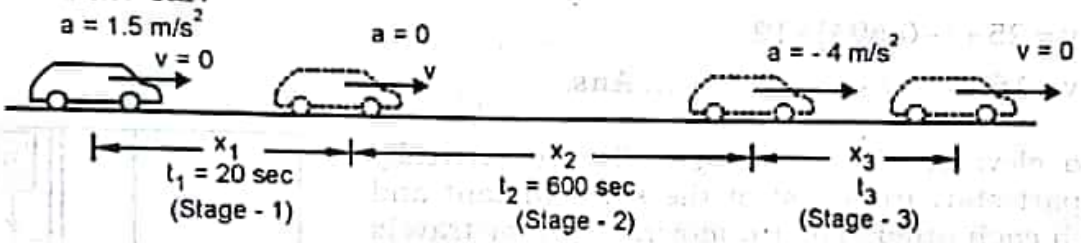
Introduction to Dynamics & Kinematics of Particles

Exercise 9.1– A

Rectilinear Motion : Uniform Velocity / Uniform Acceleration

P1. A car starts from rest and accelerates uniformly at 1.5 m/s² for 20 sec. It then maintains a constant speed for the next 10 minutes after which it decelerates uniformly at 4 m/s² and comes to a halt. Find the total distance travelled, total time of travel and average speed of the car.

Solution:



Kinematic Diagram

Motion of car – Rectilinear – 3 stages

Stage (1) Uniform Acceleration	Stage (2) Uniform velocity	Stage (3) Uniform Acceleration
$u = 0$	$v = 30\,m/s$	$u = 30\,m/s$
$v = v$	$s = x_2$	$v = 0$
$s = x_1$	$t = 600$ sec	$s = x_3$
$a = 1.5$	using $v = \frac{s}{t}$	$a = -4$
$t = 20$ sec	$30 = \frac{x_2}{600}$	$t = t_3$
using $v = u + at$	$\therefore$	using $v = u + at$
$v = 0 + 1.5 \times 20$	$x_2 = 18000\,m$	$0 = 30 + (-4) \times t_3$
$\therefore\ v = 30\ m/s$		$\therefore t_3 = -7.5$ sec
using $s = ut + \frac{1}{2}at^2$		using $s = ut + \frac{1}{2}at^2$
$x_1 = 0 + \frac{1}{2} \times 1.5 \times 20^2 = 300\ m$		$x_3 = 30 \times 7.5 + \frac{1}{2} \times (-4) \times 7.5^2 = 112.5\,m$

From above analysis of the entire motion, we get

a) Total distance travelled = $x = x_1 + x_2 + x_3 = 300 + 18000 + 112.5 = 18412.5$ m

b) Total time of travel = $t = t_1 + t_2 + t_3 = 20 + 600 + 7.5 = 627.5$ sec

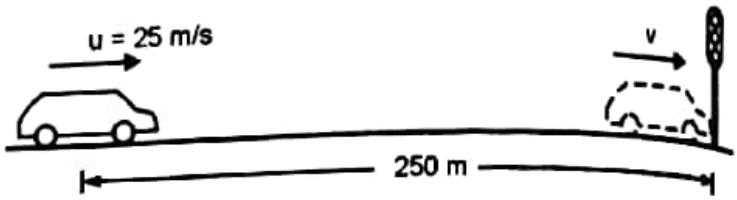
c) Avg. speed = $\frac{\text{Total Distance}}{\text{Total time}} = \frac{18412.5}{627.5} = 29.34\,m/s$ ..... **Ans.**

P2. A motorist is travelling at 90 kmph, when he observes a traffic signal 250 m ahead of him turns red. The traffic signal is timed to stay red for 12 sec. If the motorist wishes to pass the signal without stopping just as it turns green. Determine (i) the required uniform deceleration of the motor. (ii) The speed of motor as it passes the signal. **(M.U. Dec 13)**

Solution: Let a be the uniform deceleration and v be the speed of the car as it passes the signal.

Motion of car – Rectilinear – Uniform deceleration

$u = 25\ m/s,\ v = v,\ s = 250\,m,\ a = a,\ t = 12\ sec$



Using $s = ut + \frac{1}{2}at^2$

$$250 = 25 \times 12 + \frac{1}{2} \times a \times 12^2$$

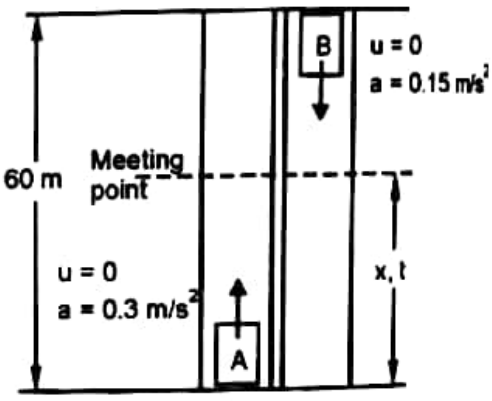
$\therefore \quad a = -0.694\ m/s^2$ ..... **Ans.**

Using $v = u + at$

$$v = 25 + (-0.694) \times 12$$

$\therefore \quad v = 16.67\ m/s$ ..... **Ans.**

P3. Two elevators in adjoining wells are vertically 60 m apart start from rest at the same instant and approach each other. The up moving elevator travels up with a uniform acceleration of 0.3 m/s². The down moving elevator travels down with an acceleration of 0.15 m/s². When do the elevators cross each other? What is the distance traveled by each of them at this instant.



Kinematic Diagram

Solution: Let the two elevators cross each other at a distance of x meters from the ground, t sec after the start. The kinematics diagram is shown.

<u>Up moving elevator A</u>

Motion–Rectilinear–Uniform acceleration

$u = 0$

$v = -\ *$

$s = x$

$a = 0.3\ m/s^2$

$t = t\ sec$

using $s = ut + \frac{1}{2}at^2$

$$x = 0 + \frac{1}{2} \times 0.3 \times t^2 \quad \text{..... (1)}$$

<u>Down moving elevator B</u>

Motion–Rectilinear–Uniform acceleration

$u = 0$

$v = -\ *$

$s = 60 - x$

$a = 0.15\ m/s^2$

$t = t\ sec$

using $s = ut + \frac{1}{2}at^2$

$$60 - x = 0 + \frac{1}{2} \times 0.15 \times t^2 \quad \text{..... (2)}$$

Equating x in equations (1) and (2) we get

$$0.15t^2 = 60 - 0.075t^2$$

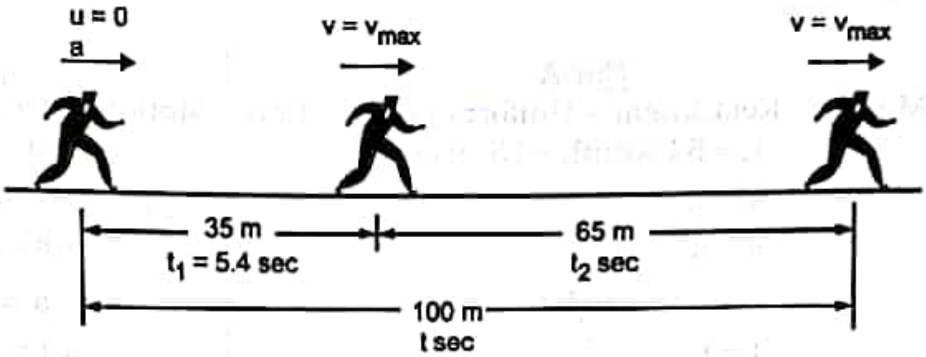
$$\therefore \quad t = 16.33 \text{ sec and } x = 40 \text{ m}$$

$\therefore$ elevator A travels 40 m up and B travels 20 m down before they cross ..... **Ans.**

- Note: *The sign – implies that we are not interested in finding that particular parameter.*

P4. A sprinter in a 100 m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m is 5.4 seconds, determine his time for the race. ***(M.U. May 14)***

Solution: Let the sprinter accelerate with an acceleration a and reach his maximum speed v_{max} in 5.4 sec after covering 35 m.



Motion of sprinter – Rectilinear – 2 stages

Stage (1) Uniform acceleration

$u = 0$

$v = v_{max}$

$s = 35\,m$

$a = a$

$t = 5.4$ sec

Using $s = ut + \frac{1}{2}at^2$

$$35 = 0 + \frac{1}{2} \times a \times 5.4^2$$

$\therefore \quad a = 2.4 \text{ m/s}^2$

Using $v = u + at$

$$v_{max} = 0 + 2.4 \times 5.4$$

$\therefore \quad v_{max} = 12.96 \text{ m/s}$

Stage (2) Uniform velocity

$v = v_{max} = 12.96$ m/s

$s = 65$ m

$t = t_2$

Using $v = \frac{s}{t}$

$$12.96 = \frac{65}{t_2}$$

$\therefore \quad t_2 = 5.015$ sec

$\therefore$ Total time spent for the race = 5.4 + 5.015 = 10.415 sec ..... **Ans.**

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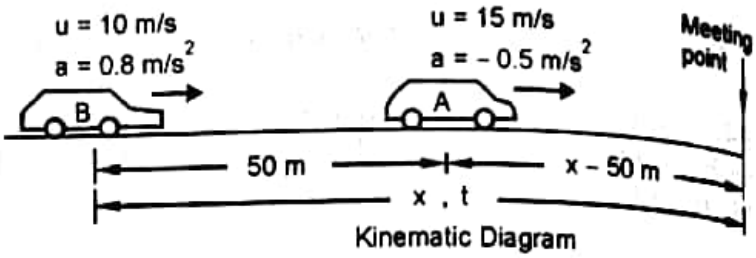
P5. Cars A and B travel on a straight highway. At t = 0, car A travelling at 54 kmph decelerates at 0.5 m/s², while car B, 50 m behind car A, travelling at 36 kmph accelerates at 0.8 m/s². Find the distances and time taken by the two cars before B overtakes A.

Solution: Let the two cars A and B take t sec to cross each other at a distance x meters from car B.



Kinematic Diagram

Car A

Motion - Rectilinear - Uniform acceleration

$u = 54 \text{ kmph} = 15 \text{ m/s}$

$v = -$

$s = x - 50$

$a = -0.5 \text{ m/s}^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$x - 50 = 15t + \frac{1}{2} \times (-0.5) \times t^2 \quad \text{..... (1)}$$

Car B

Motion - Rectilinear - Uniform acceleration

$u = 36 \text{ kmph} = 10 \text{ m/s}$

$v = -$

$s = x$

$a = 0.8 \text{ m/s}^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$x = 10t + \frac{1}{2} \times 0.8 \times t^2 \quad \text{..... (2)}$$

solving equations (1) and (2) we get

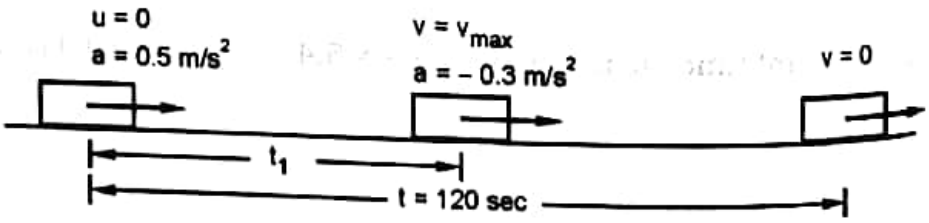
x = 206.2 m and t = 13.42 sec

∴ Car B travels 206.2 m and car A travels (206.2 – 50) = 156.2 m,

also Time taken to cross = 13.42 sec. ..... **Ans.**

P6. A particle starts from rest and accelerates at a constant rate of 0.5 m/s² for sometime. Thereafter it decelerates at a constant rate of 0.3 m/s² and comes to rest. If the particle was in motion for 2 minutes, find the maximum velocity acquired by it.

Solution: Let v_{max} be the maximum velocity acquired by the particle after t_1 sec.



Kinematic Diagram

Motion of Particle – Rectilinear – 2 stages

Stage (1)

Uniform acceleration

$u = 0$

$v = v_{max}$

$s = -$

$a = 0.5\ m/s^2$

$t = t_1$

using $v = u + at$

$v_{max} = 0 + 0.5 \times t_1$ ..... (1)

Stage (2)

Uniform acceleration

$u = v_{max}$

$v = 0$

$s = -$

$a = 0.3\ m/s^2$

$t = 120 - t_1$

using $v = u + at$

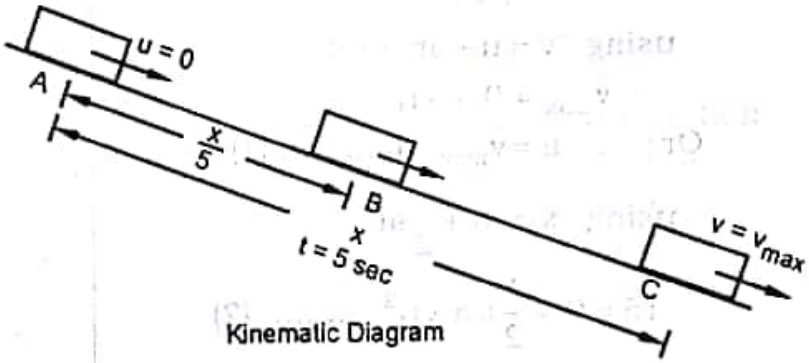
$0 = v_{max} + (-0.3) \times (120 - t_1)$ ..... (2)

Solving equations (1) and (2), we get

$t_1 = 45$ sec and $v_{max} = 22.5$ m/s ..... **Ans.**

P7. A block is released from rest from the top of an inclined plane and it takes 5 sec to reach the bottom of the plane. What is the time it takes to travel one fifth the distance from the top.

Solution: Let block take t_1 sec to travel one fifth of the entire distance x m.



Kinematic Diagram

Motion of block – Rectilinear – Uniform acceleration

Entire motion A to C

$u = 0$

$v = -$

$s = x$

$a = a$

$t = 5$ sec

using $s = ut + \frac{1}{2}at^2$

$x = 0 + \frac{1}{2} \times a \times 5^2$

$\therefore \quad x = 12.5a$ ..... (1)

One-fifth motion A to B

$u = 0$

$v = -$

$s = \frac{x}{5}$

$a = a$

$t = t_1$

using $s = ut + \frac{1}{2}at^2$

$\frac{x}{5} = 0 + \frac{1}{2} \times a \times t_1^2$

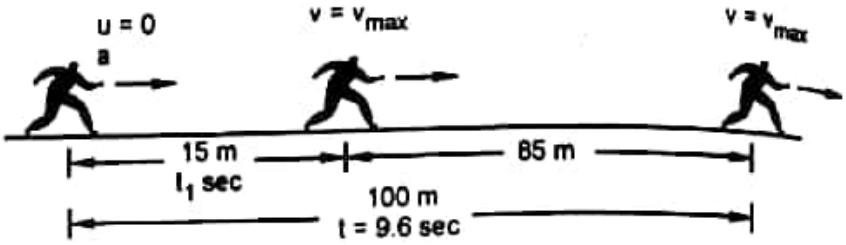
$\therefore \quad x = 2.5\,a\,t_1^2$ ..... (2)

Equating x in equations (1) and (2) we get

$12.5\,a = 2.5\,a\,t_1^2 \quad \therefore \quad t_1 = 2.236$ sec ..... **Ans.**

P8. An athlete running a 100 m run starts from rest and reaches his maximum speed in a distance of 15 m. He runs the remaining distance with that velocity and reaches the finish in 9.6 sec. Determine the initial acceleration and the maximum velocity acquired by the athlete.

Solution: Let the athlete accelerates with an acceleration a and reach his maximum speed v_{max} in t_1 sec starting from rest.



Kinematic Diagram

Motion of Athlete – Rectilinear – 2 stages

Stage (1) Uniform acceleration

$u = 0$

$v = v_{max}$

$s = 15\,m$

$a = a$

$t = t_1$

using $v = u + at$

$v_{max} = 0 + a\,t_1$

Or $a = v_{max} \times t_1$ ..... (1)

using $s = ut + \frac{1}{2}at^2$

$15 = 0 + \frac{1}{2} \times a \times t_1^2$ ..... (2)

From equations (1) and (2)

$30 = v_{max} \times t_1$ ..... (3)

Stage (2) Uniform velocity

$v = v_{max}$

$s = 85$ m

$t = 9.6 - t_1$

using $v = \frac{s}{t}$

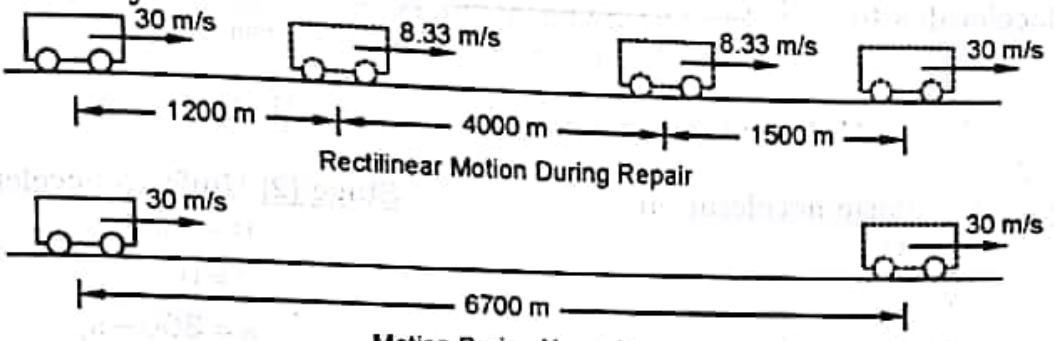
$v_{max} = \frac{85}{9.6 - t_1}$ ..... (4)

Solving equations (3) and (4) we get

$t_1 = 2.504$ sec , $v_{max} = 11.98$ m/s also $a = 4.785$ m/s² ..... **Ans.**

P9. Track repairs are going on a 4 km length of railway track. The maximum speed of the train is 108 kmph. The speed over the repair track is 30 kmph. If the train approaching the repair track decelerates uniformly from a full speed of 108 kmph to 30 kmph in a distance of 1200 m and after covering the repair track accelerates uniformly to full speed of 108 kmph in a distance of 1500 m, find the time lost due to reduction of the speed in the repair track.

Solution: Figure below shows kinematics diagrams for motion of train during repairs and during normalcy.



Rectilinear Motion During Repair

Motion During Normalcy
Kinematic Diagram

Motion of train during repair of tracks – Rectilinear – 3 stages

Stage (1)	Stage (2)	Stage (3)
Uniform acceleration	Uniform velocity	Uniform acceleration
$u = 30\ m/s$	$v = 8.33\ m/s$	$u = 8.33\ m/s$
$v = 8.33\ m/s$	$s = 4000\ m$	$v = 30\ m/s$
$s = 1200\ m$	$t = t_2$	$s = 1500\ m$
$a = a_1$	using $v = \frac{s}{t}$	$a = a_3$
$t = t_1$	$8.33 = \frac{4000}{t_2}$	$t = t_3$
using $v^2 = u^2 + 2as$	$\therefore\ t_2 = 480.19$ sec	using $v^2 = u^2 + 2as$
$8.33^2 = 30^2 + 2 \times a_1 \times 1200$		$30^2 = 8.33^2 + 2 \times a_3 \times 1500$
$\therefore\ a_1 = -0.3461\ m/s^2$		$\therefore\ a_3 = 0.2769\ m/s^2$
using $v = u + at$		using $v = u + at$
$8.33 = 30 + (-0.3461) \times t_1$		$30 = 8.33 + 0.2769 \times t_3$
$\therefore\ t_1 = 62.61$ sec		$\therefore\ t_3 = 78.27$ sec

Time taken by train during repairs = $t_1 + t_2 + t_3 = 62.61 + 480.19 + 78.27 = 621.07$ sec

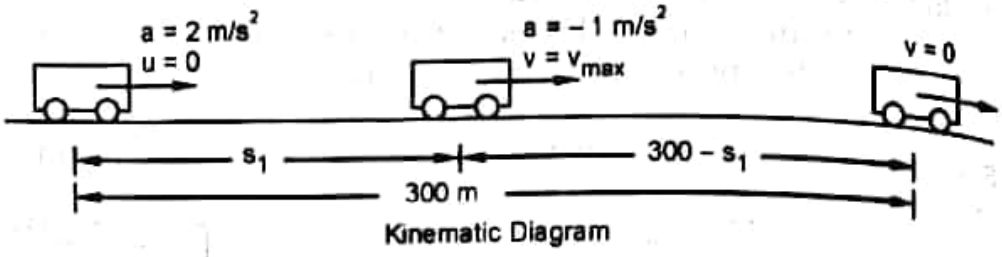
Motion of train during normalcy – Rectilinear Motion – Uniform Velocity

$v = 30\ m/s$	using $v = \frac{s}{t}$ $\quad \therefore\ 30 = \frac{6700}{t}$
$s = 6700\ m$	$\therefore\ t = 223.33$ sec
$t = t$ sec	

$\therefore$ Time lost = $621.07 - 223.33$ = 397.73 sec

P10. An automobile starts from rest and travels on a straight path at 2 m/s² for some time. After which it decelerates at 1 m/s², till it comes to a halt. If the distance covered is 300 m, find the maximum velocity of the automobile and the total time of travel.

Solution: Let the automobile travel distance s_1 and acquire a velocity v_{max} before it decelerates to a halt.



Kinematic Diagram

Motion of automobile – Rectilinear – 2 stages

Stage (1) Uniform acceleration

$u = 0$

$v = v_{max}$

$s = s_1$

$a = 2\ m/s^2$

$t = t_1$

using $v^2 = u^2 + 2as$

$v_{max}^2 = 0 + 2 \times 2 \times s_1$

Or $v_{max}^2 = 4s_1$ ..... (1)

Stage (2) Uniform acceleration

$u = v_{max}$

$v = 0$

$s = 300 - s_1$

$a = -1\ m/s^2$

$t = t_2$

using $v^2 = u^2 + 2as$

$0 = v_{max}^2 + 2 \times (-1) \times (300 - s_1)$

Or $v_{max}^2 = 600 - 2s_1$ ..... (2)

Solving equations (1) and (2) we get

$4s_1 = 600 - 2s_1$

$\therefore$ $s_1 = 100$ m also $v_{max} = 20$ m/s ..... **Ans.**

using $v = u + at$

$v_{max} = 0 + 2 \times t_1$

$20 = 2t_1$

$\therefore$ $t_1 = 10$ sec

using $v = u + at$

$0 = v_{max} + (-1) \times t_2$

$0 = 20 - t_2$

$\therefore$ $t_2 = 20$ sec

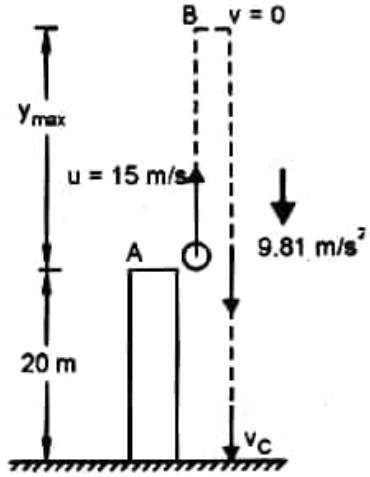
$\therefore$ Total time of travel = 10 + 20 = 30 sec ..... **Ans.**

Exercise 9.1– B

Motion Under Gravity (M U G)

P1. A stone is thrown vertically up from the top of the tower 20 m high with a speed of 15 m/s. Find (1) max. height reached by the stone above ground. (2) velocity with which the stone hits the ground. (3) total time of flight.

Solution: Let the stone travel a maximum height y_{max} above the tower and take t sec to complete the journey. Let the stone land on the ground at C with velocity v_C.



Kinematic Diagram

Motion of Stone (A to B) – MUG ↑ + ve

$u = 15\,m/s$

$v = 0$

$s = y_{max}$ ... s means level difference (+ ve since B is above A)

$a = -9.81\ m/s^2$

$t = -$

using $v^2 = u^2 + 2as$

$$0 = 15^2 + 2\times(-9.81)\times y_{max}$$

$\therefore \quad y_{max} = 11.468\,m$

∴ Maximum height reached by the stone above ground = 11.468 + 20 = 31.468 m ..... **Ans.**

Entire motion of stone (A to C) – MUG ↑+ve

$u = 15\,m/s$

$v = -v_C$

$s = -20\,m$ ... s means level difference (– ve since C is below A)

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$-20 = 15t + \frac{1}{2}\times(-9.81)t^2$$

or $\quad t = 4.062$ sec

..... total time of flight ..... **Ans.**

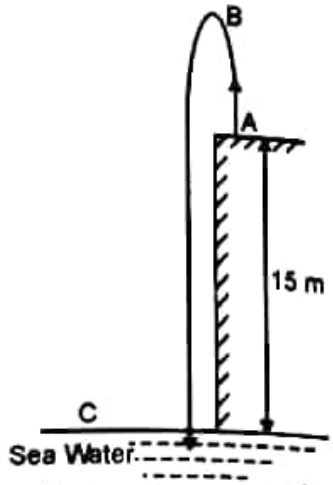
using $v = u + at$

$$-v_C = 15 + (-9.81)\times 4.062$$

$\therefore \quad v_C = 24.848\ m/s$

..... landing velocity ..... **Ans.**

P2. A ball is thrown vertically upwards with a velocity 9 m/s from the edge of a cliff 15 m above the sea level. What is the highest point above sea level reached? How long does it take to hit the water? With what velocity does it hit the water? ***(VJTI Nov 09)***



Solution: Let the ball travel a maximum height y_{max} upto point B above the cliff and take t sec to finally hit the sea water at C with a velocity v_C.

Also at peak B, the stone's velocity becomes zero.

Motion of Stone (A to B) – MUG ↑ + ve

$u = 9\,m/s$

$v = 0$

$s = y_{max}$ ... s means level difference

$a = -9.81\ m/s^2$

$t = -$

using $v^2 = u^2 + 2as$

$$0 = 9^2 + 2\times(-9.81)\times y_{max}$$

$\therefore \quad y_{max} = 4.128\,m$

$\therefore$ Maximum height reached by the ball above sea = 15 + 4.128 = 19.128 m ... **Ans.**

Motion of stone (A to C) – MUG ↑+ve

$u = 9\,m/s$

$v = -v_C$

$s = -15\,m$ ... s means level difference

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$-15 = 9t + \frac{1}{2}\times(-9.81)t^2$$

$\therefore \quad t = 2.892$ sec ..... **Ans.**

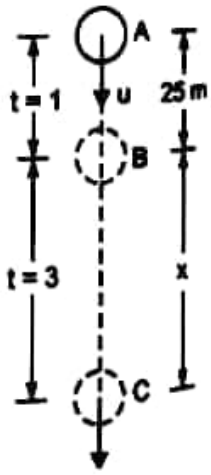
using $v = u + at$

$$-v_C = 9 + (-9.81)\times 2.892$$

$\therefore \quad v_C = 19.37\ m/s$ ..... **Ans.**

P3. A particle falling under gravity travels 25 m in a particular second. Find the distance travelled by it in next three seconds. ***(M.U Dec 16)***

Solution: Let the distance travelled in next 3 seconds be x metres.



Motion of particle (A – B): MUG ↓ + ve

$u = u$

$v = -$

$s = 25\,m$

$a = 9.81\ m/s^2$

$t = 1$ sec

Using $s = ut + \frac{1}{2}at^2$

$$25 = u\times 1 + \frac{1}{2}\times 9.81\times 1^2$$

$\therefore \quad u = 20.095\ m/s$

Motion of particle (A – C): MUG ↓ + ve

$u = 20.095$

$v = -$

$s = 25 + x$

$a = 9.81\ m/s^2$

$t = 4$ sec

Using $s = ut + \frac{1}{2}at^2$

$$25 + x = 20.095\times 4 + \frac{1}{2}\times 9.81\times 4^2$$

$\therefore \quad x = 133.86$ m ... **Ans.**

P4. A hot air balloon starts rising vertically up from the ground with an acceleration of 0.2 m/s². 12 seconds later the man sitting inside the balloon releases a stone. Find the time taken by the stone to hit the ground. ***(M.U Dec 15)***

Solution:

Motion of balloon
(Ground to height 'h')
Rectilinear – Uniform acceleration

$u = 0$
$v = v$
$s = h$
$a = 0.2\ m/s^2$
$t = 12$ sec

using $v = ut + at$
$v = 0 + 0.2 \times 12$
$\therefore \quad v = 2.4\ m/s$

using $s = ut + \frac{1}{2}at^2$
$h = 0 + \frac{1}{2} \times 0.2 \times 12^2$
$h = 14.4\ m$

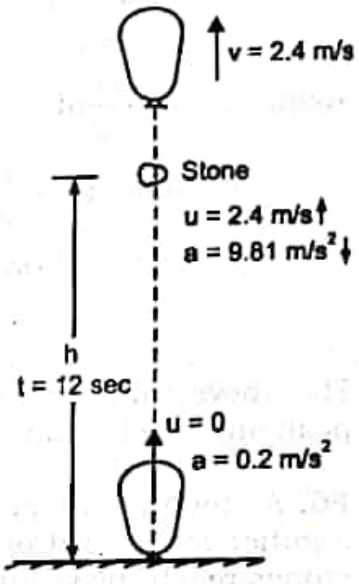
Motion of stone
(From height 'h' to ground)
MUG ↓ + ve
u = velocity of balloon
$= -2.4\ m/s$
$v = -$
$s = h = 14.4\ m$
$a = 9.81\ m/s^2$
$t = t$

using $s = ut + \frac{1}{2}at^2$

$14.4 = -2.4 \times t + \frac{1}{2} \times 9.81 \times t^2$

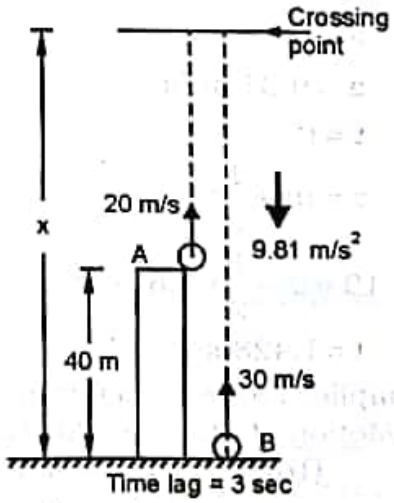
$\therefore\ 4.905t^2 - 2.4 \times t - 14.4 = 0$

Solving $t = 1.977$ sec. ... **Ans.**



P5. A stone is thrown vertically up from the top of the tower 40 m high with a velocity of 20 m/s. Three seconds later another stone is thrown vertically up from the ground with a velocity of 30 m/s. Calculate when and where the two stones will meet from the foot of the tower.

Solution: Let the two stones cross each other at a distance x from the ground, t sec after stone A was thrown. The stone B has a time lag of 3 sec. This means if stone A has spent t sec in motion, stone B has spent (t – 3) sec in motion.



Kinematic Diagram

Motion of Stone A – MUG ↑ + ve

$u = 20\,m/s$

$v = -$

$s = x - 40$

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$x - 40 = 20t + \frac{1}{2} \times (-9.81)t^2$

$x = 20t - 4.905t^2 + 40$ ..... (1)

Motion of stone B – MUG ↑+ve

$u = 30\,m/s$

$v = -$

$s = x$

$a = -9.81\ m/s^2$

$t = t - 3$

using $s = ut + \frac{1}{2}at^2$

$x = 30(t-3) + \frac{1}{2} \times (-9.81)(t-3)^2$ ..... (2)

Solving equations (1) and (2) we get

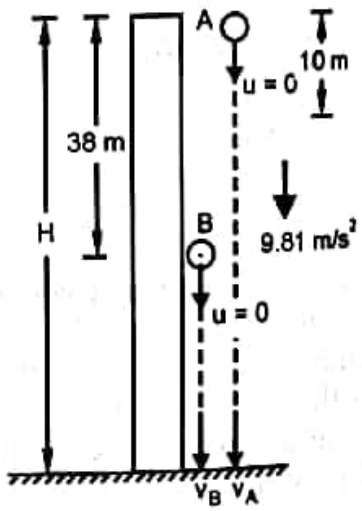
$t = 4.416$ sec and $x = 32.654\,m$

The above value of x indicates that the stone A was returning down after reaching its peak and meets stone B below the tower top. ..... **Ans.**

P6. A stone is dropped from the top of a tower. When it has fallen a distance of 10 m, another stone is dropped from a point 38 m below the top of the tower. If both the stones reach the ground at the same time calculate

(i) the height of the tower

(ii) the velocity of the stones when they reach the ground. ***(M. U. Dec 09)***



Kinematic Diagram

Solution: From the problem we note that both the stones did not start their journey at the same time. Let us first find the time lag of stone B.

Motion of Stone A for first 10 m – MUG ↓ + ve

$u = 0$

$v = -$

$s = 10\ m$

$a = 9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$10 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$\therefore \quad t = 1.428$ sec

This implies stone B was dropped 1.428 sec after stone A was dropped.

Motion of Stone A – MUG ↓ + ve

(Tower Top to Ground)

$u = 0$

$v = v_A$

$s = H$

Motion of stone B – MUG ↓ +ve

(38 m below Tower Top to Ground)

$u = 0$

$v = v_B$

$s = H - 38$

$a = 9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$H = 0 + \frac{1}{2} \times 9.81 \times t^2$ ..... (1)

$a = 9.81\ m/s^2$

$t = t - 1.428$

using $s = ut + \frac{1}{2}at^2$

$H - 38 = 0 + \frac{1}{2} \times 9.81 \times (t - 1.428)^2$ ..... (2)

Solving equations (1) and (2) we get

$t = 3.428$ sec and $H = 57.64$ m ..... **Ans.**

using $v = u + at$

$v_A = 0 + 9.81 \times 3.428$

$\therefore$ $v_A = 33.63$ m/sec ↓ ..... **Ans.**

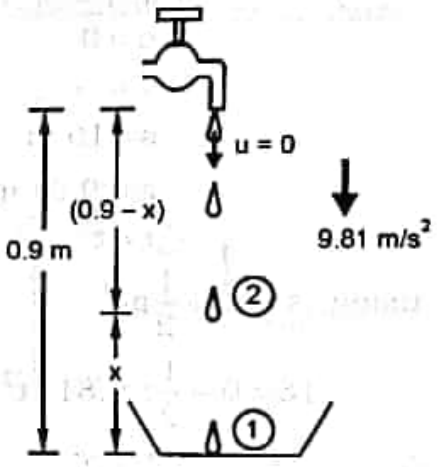
using $v = u + at$

$v_B = 0 + 9.81 \times (3.428 - 1.428)$

$\therefore$ $v_B = 19.62$ m/sec ↓ ..... **Ans.**

P7. Water drips from a tap at a rate of 5 drops per second. The tap is 900 mm above the basin. When one drop strikes the basin, how far is the next drop above the basin.

Solution: Drops fall at a rate of 5 drops per second. This means each drop falls after 1/5 = 0.2 sec. Therefore the time lag between two drops is 0.2 sec. Let drop 1 start from the tap and hit the basin. At this time drop 2 would be x metres above the basin since it starts 0.2 sec later. Let us analyse their motion.



Kinematic Diagram

DJC

<u>Drop 1 – MUG ↓ + ve</u>

$u = 0$

$v = -$

$s = 0.9$ m

$a = 9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$0.9 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$\therefore$ $t = 0.4283$ sec

$\therefore$ Drop 1 takes 0.4283 sec to reach the basin.

<u>Drop 2 – MUG ↓ +ve</u>

$u = 0$

$v = -$

$s = 0.9 - x$

$a = 9.81\ m/s^2$

$t = 0.4283 - 0.2 = 0.2283$ sec

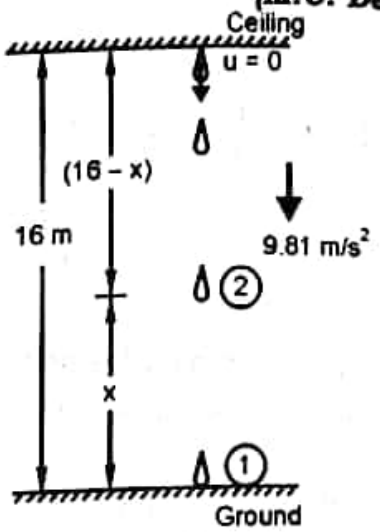
using $s = ut + \frac{1}{2}at^2$

$0.9 - x = 0 + \frac{1}{2} \times 9.81 \times 0.2283^2$

$\therefore$ $x = 0.6443$ m ..... **Ans.**

P8. Water leaks from a ceiling 16 m high, at the rate of 5 drops per second. Find the distance between first and second drop when the first drop has just touched the ground. **(M.U. Dec 07)**

Solution: Drops fall at a rate of 5 drops per second. This means each drop falls after 1/5 = 0.2 sec. Therefore the time lag between two drops is 0.2 sec. Let drop 1 start from the ceiling and hit the ground. At this time drop 2 would be x metres above the ceiling, since it starts 0.2 sec later. Let us analyse their motion.



Kinematic Diagram

Drop 1 – MUG ↓ + ve

$u = 0$

$v = -$

$s = 16$ m

$a = 9.81$ m/s²

$t = t$

using $s = ut + \frac{1}{2}at^2$

$16 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$\therefore$ t = 1.806 sec

$\therefore$ Drop 1 takes 1.806 sec to reach the ground

Drop 2 – MUG ↓ +ve

$u = 0$

$v = -$

$s = 16 - x$

$a = 9.81$ m/s²

$t = 1.806 - 0.2 = 1.606$ sec

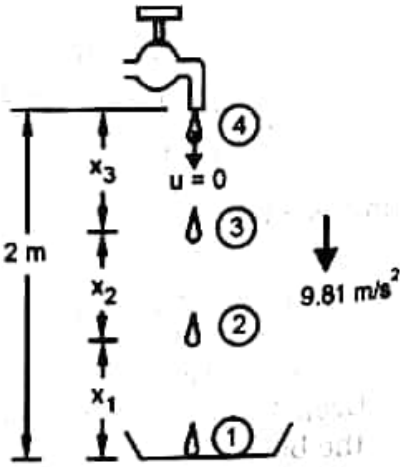
using $s = ut + \frac{1}{2}at^2$

$16 - x = 0 + \frac{1}{2} \times 9.81 \times 1.606^2$

$\therefore$ x = 3.349 m ..... **Ans.**

P9. Drops of water leak from a tap 2 m above the basin at regular intervals. As the first drop strikes the basin the fourth drop starts its fall. What is the spacing between the drops 1 and 2, drop 2 and 3, drops 3 and 4 at this instant.

Solution: Let us first find the time t taken by drop 1 to hit the basin. In this time 3 drops fall and the fourth drop is about to fall. Dividing the time t by 3 would give us the time lag (interval) between fall of two consecutive drops. Next we analyse motion of drop 2 and drop 3. Let x_1, x_2 and x_3 be the spacing between drop 1 and 2, drop 2 and 3, drop 3 and 4 respectively.



Kinematic Diagram

Drop 1 – MUG ↓ + ve

$u = 0$

$v = -$

$s = 2$ m

$a = 9.81$ m/s^2

$t = t$

using $s = ut + \frac{1}{2}at^2$

$2 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$\therefore$ $t = 0.6385$ sec

$\therefore$ Drop 1 takes 0.6385 sec to hit the ground.

$\therefore$ Time lag between drops

$= \frac{0.6385}{3} = 0.2128$ sec

Drop 2 – MUG ↓ + ve

$u = 0$

$v = -$

$s = 2 - x_1$

$a = 9.81$ m/s^2

$t = 0.6385 - 0.2128$

$= 0.4257$ sec

using $s = ut + \frac{1}{2}at^2$

$2 - x_1 = 0 + \frac{1}{2} \times 9.81 \times 0.4257^2$

$\therefore$ $x_1 = 1.111$ m ... **Ans.**

Drop 3 – MUG ↓ + ve

$u = 0$

$v = -$

$s = x_3$

$a = 9.81$ m/s^2

$t = 0.4257 - 0.2128$

$= 0.2129$ sec

using $s = ut + \frac{1}{2}at^2$

$x_3 = 0 + \frac{1}{2} \times 9.81 \times 0.2129^2$

$\therefore$ $x_3 = 0.2223$ m... **Ans.**

also spacing

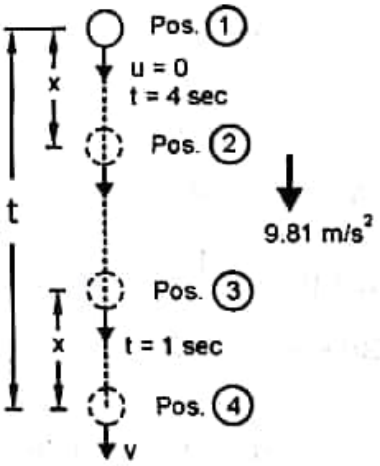
$x_2 = 2 - x_1 - x_3$

$= 2 - 1.111 - 0.2223$

$\therefore$ $x_2 = 0.6667$ m ... **Ans.**

P10. A stone released from rest falls freely under gravity. The distance covered by it in the last second of its motion equals the distance covered in the first four seconds of its motion. Find the time the stone was in motion.

Solution: Let x bet the distance covered in the first four seconds of motion. The kinematic diagram is shown.



Kinematic Diagram

Motion of stone – MUG ↓ + ve

Position (1) to (2)

$u = 0$

$v = -$

$s = x$

$a = 9.81$ m/s^2

$t = 4$ sec

using $s = ut + \frac{1}{2}at^2$

$x = 0 + \frac{1}{2} \times 9.81 \times 4^2$

$\therefore$ $x = 78.48$ m

Position (3) to (4)

$u = u$

$v = v$

$s = x = 78.48$ m

$a = 9.81$ m/s^2

$t = 1$ sec

using $s = ut + 1/2\ at^2$

$78.48 = u \times 1 + 1/2 \times 9.81 \times 1^2$

$\therefore$ $u = 73.575$ m/s

using $v = u + at$

$v = 73.575 + 9.81 \times 1$

$\therefore$ $v = 83.385$ m/s

Position (1) to (4)

$u = 0$

$v = 83.385$ m/s

$s = -$

$a = 9.81$ m/s^2

$t = t$ sec

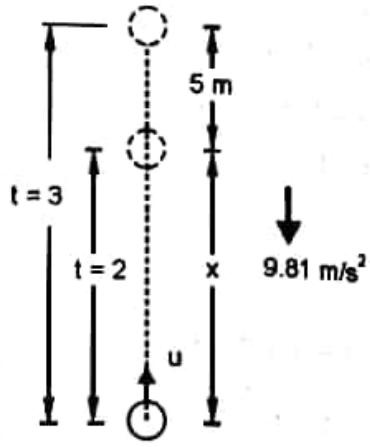
using $v = u + at$

$83.385 = 0 + 9.81 \times t$

or $t = 8.5$ sec ... **Ans.**

P11. A ball thrown vertically up was found to travel a distance of 5 m during 3rd second of its travel. What is the initial velocity of the ball? ***(NMIMS Feb 10)***

Solution: Let the stone projected with initial velocity u travel x metres in 2 sec. Therefore it travels x + 5 metres in 3 sec.



Kinematic Diagram

Motion of stone – M U G ↑ + ve

Motion for first 2 sec

$u = u$

$v = -$

$s = x$

$a = -9.81\ m/s^2$

$t = 2\,sec$

using $s = ut + \frac{1}{2}at^2$

$x = u \times 2 + \frac{1}{2} \times (-9.81) \times 2^2$

Or $2u - x = 19.62$ ..... (1)

Motion for first 3 sec

$u = u$

$v = -$

$s = x + 5$

$a = -9.81\ m/s^2$

$t = 3\,sec$

using $s = ut + \frac{1}{2}at^2$

$x + 5 = u \times 3 + \frac{1}{2} \times (-9.81) \times 3^2$

Or $3u - x = 49.145$ ..... (2)

Solving equations 1 and 2, we get

$u = 29.525\ m/s$ and $x = 39.43\ m$ ..... **Ans.**

Alternatively using the formula for distance travelled in the nth second

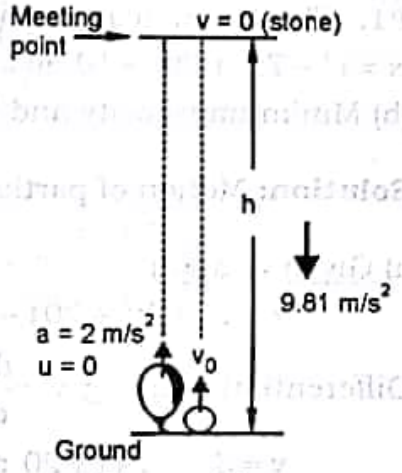
$$s^{n^{th}} = u + a \times n - \frac{a}{2}$$

$$\therefore \quad 5 = u + (-9.81) \times 3 - \frac{(-9.81)}{2}$$

or $u = 29.525\ m/s$ ..... **Ans.**

P12. A halogen gas filled balloon released from the ground, travels vertically up with a constant acceleration of 2 m/s². Three seconds later, a stone is projected vertically up from the ground to hit the ascending balloon. The stone just manages to touch the balloon at the peak of its path. Find the velocity with which the stone was projected and the height at which the stone touches the balloon.

Solution: Let the stone be projected with initial velocity v_0 to hit the balloon at some height h. Note that the velocity of stone becomes zero since this is the peak of path for the stone. Also note that stone starts 3 sec later, so it has a time lag of 3 sec. This means that if the balloon spends t sec in motion, the stone spends (t -3) sec in motion.



Kinematic Diagram

Motion of balloon
Rectilinear – Uniform acceleration

$u = 0$

$v = -$

$s = h$

$a = 2\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2} \times 2 \times t^2$

Or $h = t^2$ ..... (1)

Motion of stone
MUG ↑ + ve

$u = v_0$

$v = 0$

$s = h$

$a = -9.81\ m/s^2$

$t = t - 3$

using $v = u + at$

$0 = v_0 + (-9.81) \times (t - 3)$

$\therefore \quad v_0 = 9.81t - 29.43$ ..... (2)

using $s = ut + \frac{1}{2}at^2$

$h = v_0(t-3) + \frac{1}{2}(-9.81)(t-3)^2$ ..... (3)

Solving equations 1, 2 and 3 we get

$v_0 = 24.22$ m/s and $h = 29.9$ m ..... **Ans.**

Exercise 9.2

Rectilinear Motion : Variable Acceleration

P1. The rectilinear motion of a particle has its position defined by the relation $x = t^3 - 7t^2 + 20t - 10$ m. Determine, a) Position, velocity and acceleration at t = 2 sec. (b) Minimum velocity and the corresponding time.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given relation

$$x = t^3 - 7t^2 + 20t - 10 \text{ m} \quad \text{..... (1)}$$

Differentiating using $v = \frac{dx}{dt}$

$$v = 3t^2 - 14t + 20 \text{ m/s} \quad \text{..... (2)}$$

Differentiating using $a = \frac{dv}{dt}$

$$a = 6t - 14 \text{ m/s}^2 \quad \text{..... (3)}$$

Boundary Conditions
Nil

To find
a) At t = 2 sec : x = ?, v = ?, a = ?
b) v_{min} = ?, t = ?

Position, velocity and acceleration at t = 2 sec. Substitute t = 2 in equations 1, 2, and 3

$$x = 2^3 - 7 \times 2^2 + 20 \times 2 - 10$$

$\therefore \quad x_2 = 10 \text{ m}$ ..... **Ans.**

$$v = 3 \times 2^2 - 14 \times 2 + 20$$

$\therefore \quad v_2 = 4 \text{ m/s}$ ..... **Ans.**

$$a = 6 \times 2 - 14$$

$\therefore \quad a_2 = -2 \text{ m/s}^2$ ..... **Ans.**

b) for minimum or maximum velocity condition, acceleration of the particle should be zero.

$$a = 6t - 14 = 0$$

Or $\quad t = 2.33$ sec.

$\therefore$ Velocity is minimum at t = 2.33 sec ..... **Ans.**

Substituting t = 2.33 sec

$v_{min} = v_{2.33} = 3.67 \text{ m/s}$ ..... **Ans.**

P2. The motion of a particle moving in straight line is given by the expression $s = t^3 - 3t^2 + 2t + 5$ where 's' is the displacement in metres and 't' is time in seconds. Determine (i) velocity and acceleration after 4 sec. (ii) maximum or minimum velocity and corresponding displacement and (iii) time at which velocity is zero. ***(M. U. May 09)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given relation

$$s = t^3 - 3t^2 + 2t + 5 \text{ m} \quad \text{..... (1)}$$

Differentiating using $v = \frac{ds}{dt}$

$$\therefore \quad v = 3t^2 - 6t + 2 \text{ m/s} \quad \text{..... (2)}$$

Differentiating using $a = \frac{dv}{dt}$

$$\therefore \quad a = 6t - 6 \text{ m/s}^2 \quad \text{..... (3)}$$

Boundary Conditions

Nil

To find

a) At t = 4 sec : v = ?, a = ?

b) v_{max} = ?, x = ?

c) when v = 0 : t = ?

Velocity and acceleration at t = 4 sec.

$$v_4 = 3 \times 4^2 - 6 \times 4 + 2 = 26 \text{ m/s} \quad \text{..... Ans.}$$

$$a_4 = 6 \times 4 - 6 = 18 \text{ m/s}^2 \quad \text{..... Ans.}$$

b) For maximum or minimum velocity condition, acceleration of the particle should be zero.

$\therefore \quad a = 6t - 6 = 0$

Or $\quad t = 1$ sec.

$\therefore \quad v_1 = 3 \times 1^2 - 6 \times 1 + 2 = -1 \text{ m/s}$

$\therefore$ Velocity at t = 1 sec is minimum and $v_{min} = v_1 = -1 \text{ m/s}$ ..... **Ans.**

Also displacement at t = 1 sec, $s_1 = 1^3 - 3 \times 1^2 + 2 \times 1 + 5 = 5 \text{ m}$ ..... **Ans.**

2) For zero velocity, equating equation (2) to zero

$$v = 3t^2 - 6t + 2 = 0$$

Or $\quad t = 1.577$ sec or 0.422 sec

Velocity is zero at 0.422 sec and again at 1.577 sec ..... **Ans.**

P3. The position of a particle moving along a straight line is defined by the relation $x = t^3 - 9t^2 + 15t + 18$ where x is expressed in meters and t in seconds. Determine the time, position and acceleration of the particle when its velocity becomes zero. ***(VJTI Nov 09)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given relation

$$x = t^3 - 9t^2 + 15t + 18 \text{ m} \quad \text{..... (1)}$$

Differentiating using $v = \frac{dx}{dt}$

$$\therefore \quad v = 3t^2 - 18t + 15 \text{ m/s} \quad \text{..... (2)}$$

Boundary Conditions

Nil

To find

a) when v = 0 : t = ?, x = ?, a = ?

Differentiating using $a = \frac{dv}{dt}$ $\therefore$ $a = 6t - 18 \text{ m/s}^2$ ..... (3)

For velocity to be zero

$$v = 3t^2 - 18t + 15 = 0$$

$\therefore$ t = 1 sec Or t = 5 sec. ..... **Ans.**

Position at t = 1 sec and t = 5 sec

$$x_1 = 1^3 - 9 \times 1^2 + 15 \times 1 + 18 = 25 \text{ m}$$ ..... **Ans.**

$$x_5 = 5^3 - 9 \times 5^2 + 15 \times 5 + 18 = -7 \text{ m}$$ ..... **Ans.**

Acceleration at t = 1 sec and t = 5 sec

$$a_1 = 6 \times 1 - 18 = -12 \text{ m/s}^2$$ ..... **Ans.**

$$a_5 = 6 \times 5 - 18 = 12 \text{ m/s}^2$$ ..... **Ans.**

P4. The car at origin starts from the rest and moves in a straight line such that for a short time its velocity is defined by $v = \left(9t^2 + 2t\right)$ m/s, where t is in seconds. Determine its position and acceleration when t = 3 sec. ***(M.U. Dec 12)***

Solution: Motion of particle – Rectilinear – Variable acceleration

Given relation $v = 9t^2 + 2t \text{ m/s}$ ..... (1)

Boundary Conditions

at $t = 0, v = 0, x = 0$

Differentiating using $a = \frac{dv}{dt}$

$\therefore$ $a = 18t + 2 \text{ m/s}^2$ ..... (2)

Integrating using $v = \frac{dx}{dt}$

$\therefore$ $dx = \left(9t^2 + 2t\right) dt$

$\therefore$ $\int_{18}^{x} dx = \int_{0}^{t} 9t^2 + 2t \, dt$

$\therefore$ $x = 3t^3 + t^2 \text{ m}$ ..... (3)

Substituting t = 3 sec in equations (2) and (3), we get

$a = 56 \text{ m/s}^2$ and $x = 90 \text{ m}$

P5. Acceleration of a particle is directly proportional to the square of time t. When t = 0, particle is at x = 24 m. Knowing that at t = 6 sec, x = 96 m and v = 18 m/s, find expressions of x and v in terms of t. Also find velocity and position at t = 2 sec. ***(M.U. Dec 07)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given : acceleration is proportional to square of time

$\therefore \quad a = kt^2 \quad$ ..... (1)

Integrating using $a = \dfrac{dv}{dt}$

$\therefore \quad dv = kt^2\, dt$

$\therefore \quad \int_{18}^{v} dv = \int_{6}^{t} kt^2\, dt$

$\therefore \quad [v]_{18}^{v} = \left[k\dfrac{t^3}{3}\right]_{6}^{t}$

$\therefore \quad v - 18 = \dfrac{k}{3}\left[t^3 - 216\right]$ Or $v = \dfrac{k}{3}\left[t^3 - 216\right] + 18$ ..... (2)

Integrating using $v = \dfrac{dx}{dt}$

$$\int_{24}^{x} dx = \int_{0}^{t} \frac{k}{3}\left[t^3 - 216\right] + 18\ dt$$

$$[x]_{24}^{x} = \left[\frac{k}{3}\left(\frac{t^4}{4} - 216t\right) + 18t\right]_{0}^{t}$$

$\therefore \quad x - 24 = \dfrac{k}{3}\left(\dfrac{t^4}{4} - 216t\right) + 18t$ Or $x = \dfrac{k}{3}\left(\dfrac{t^4}{4} - 216t\right) + 18t + 24$ ..... (3)

Substituting x = 96 m and t = 6 sec

$$96 = \frac{k}{3}\left(\frac{6^4}{4} - 216 \times 6\right) + 18 \times 6 + 24$$

$\therefore \quad k = 0.111$ ..... **Ans.**

Substituting k = 0.111 in equation 2 and 3

$$v = \frac{0.111}{3}\left[t^3 - 216\right] + 18$$

$\therefore \quad v = 0.037t^3 + 10$ m/s ..... **Ans.**

Also $\quad x = \dfrac{0.111}{3}\left(\dfrac{t^4}{4} - 216t\right) + 18t + 24$

$\therefore \quad x = 9.25 \times 10^{-3} t^4 + 10t + 24$ m ..... **Ans.**

b) at t = 2 sec

$v_2 = 10.296$ m/s and $x_2 = 44.15$ m ..... **Ans.**

Boundary Conditions

at $t = 0, x = 24$ m

at $t = 6$ sec, $x = 96$ m and $v = 18$ m/s

To find

a) x = f (t), v = f(t)

b) at t = 2: v = ?, x = ?

P6. The acceleration of a particle is given by $a = -kx^{-3}$ m/s². Knowing at x = 2 m, v = 0 and at x = 0.5 m, v = 3 m/s. Determine a) the value of k b) the particle's velocity at x = 1 m.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given : Relation

$$a = -kx^{-3} \quad \text{..... (1)}$$

Boundary Conditions

at $x = 2, v = 0$

at $x = 0.5\,m, v = 3\,m/s$

To find

a) k

b) at x = 1 m : v = ?

Integrating using $a = \dfrac{vdv}{dx}$

$$\therefore \quad vdv = -kx^{-3}\,dx$$

$$\therefore \quad \int_0^v vdv = \int_2^x -kx^{-3}\,dx$$

$$\therefore \quad \left[\frac{v^2}{2}\right]_0^v = -k\left[\frac{x^{-2}}{-2}\right]_2^x \quad \text{or} \quad v^2 = k\left[x^{-2} - 0.25\right] \quad \text{..... (2)}$$

Substituting v = 3 m/s and x = 0.5 m

$$3^2 = k\left[0.5^{-2} - 0.25\right] \quad \therefore \quad k = 2.4 \quad \text{..... Ans.}$$

b) Substituting x = 1 m in equation 2

$$v^2 = 2.4\left[1^{-2} - 0.25\right] \quad \therefore \quad v_{x=1} = 1.342\ m/s \quad \text{..... Ans.}$$

P7. The acceleration of an oscillating particle is defined by the relation $a = -kx$ m/s². Determine a) the value of k such that v = 12 m/s at x = 2 m and v = 0 at x = 6 m b) velocity at x = 4 m c) maximum velocity.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = -kx\ m/s^2 \quad \text{..... (1)}$$

Boundary Conditions

at $x = 2, v = 12\ m/s$

at $x = 6\,m, v = 0$

To find

a) k

b) at x = 4 m : v = ?

c) v_{max}

Integrating using $a = \dfrac{vdv}{dx}$

$$\therefore \quad vdv = -kx\,dx$$

$$\therefore \quad \int_0^v vdv = \int_2^x -kx\,dx$$

$$\therefore \quad \left[\frac{v^2}{2}\right]_0^v = -k\left[\frac{x^2}{6}\right]_6^x \quad \text{or} \quad v^2 = -k\left[x^2 - 36\right] \quad \text{..... (2)}$$

Substituting v = 12 m/s and x = 2 m

$$12^2 = -k\left[2^2 - 36\right] \quad \therefore \quad k = 4.5 \quad \text{..... Ans.}$$

b) to find velocity at x = 4 m, substituting x = 4 m in equation 2

$$v^2 = -4.5\left[4^2 - 36\right] \quad \therefore \quad v_{x=4} = 9.487\ m/s \quad \text{..... Ans.}$$

c) for maximum or minimum velocity, acceleration should be zero
Equating acceleration equation 1 to zero

$$a = -kx = 0$$

$$\therefore \quad x = 0$$

Substituting x = 0 in equation 2

$$v^2 = -4.5[0-36] \quad \therefore \quad v_{max} = 12.728 \text{ m/s} \quad \text{..... Ans.}$$

P8. A particle performing rectilinear motion starts from rest from origin and has its acceleration defined by $a = 25 - v^2$ m/s². Determine the time spent and the particle's position, when v = 4 m/s.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 25 - v^2 \quad m/s^2 \quad \text{..... (1)}$$

Boundary Conditions

at $t = 0, v = 0, x = 0$

To find

at v = 4 m/s : t = ?, x = ?

Integrating using $a = \dfrac{dv}{dt}$

$$\therefore \quad dv = 25 - v^2\, dt$$

$$\therefore \quad \frac{dv}{25 - v^2} = dt$$

$$\therefore \quad \int_0^v \frac{dv}{25 - v^2} = \int_0^t dt$$

$$\therefore \quad \frac{1}{2\times 5}\left[\log_e\left(\frac{5+v}{5-v}\right)\right]_0^v = [t]_0^t \quad \therefore \quad \frac{1}{10}\left[\log_e\left(\frac{5+v}{5-v}\right) - \log_e\left(\frac{5}{5}\right)\right] = t$$

$$\text{or} \quad t = \frac{1}{10}\log_e\left(\frac{5+v}{5-v}\right) \quad \text{..... (2)}$$

Substituting v = 4 m/s in equation 2, we get $t = 0.2197$ sec ..... **Ans.**

Given $a = 25 - v^2 \quad m/s^2$

Integrating using $a = \dfrac{vdv}{dx}$

$$\therefore \quad vdv = (25 - v^2)dx$$

$$\therefore \quad \int_0^v \frac{vdv}{25 - v^2} = \int_0^x dx \quad \therefore \quad \frac{1}{-2}\int_0^v \frac{-2vdv}{25 - v^2} = x$$

$$\therefore \quad \frac{1}{-2}\left[\log_e(25 - v^2)\right]_0^v = x \quad \text{or} \quad x = -0.5\left[\log_e(25 - v^2) - \log_e 25\right]$$

$$\text{or} \quad x = -0.5\left[\log_e \frac{(25 - v^2)}{25}\right] \quad \text{..... (3)}$$

Substituting v = 4 m/s in equation 3, we get

$$x = 0.5108 \text{ m} \quad \text{..... Ans.}$$

P9. A particle moving in the + ve x direction has an acceleration $a = 100 - 4v^2$ m/s². Determine the time interval and displacement of a particle when speed changes from 1 m/s to 3 m/s. ***(M.U. Dec 12)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 100 - 4v^2 \ \text{m/s}^2 \quad \text{..... (1)}$$

Integrating using $a = \dfrac{dv}{dt}$

Boundary Conditions

at $t = t_1, v = 1$ m/s, $x = x_1$

at $t = t_2, v = 3$ m/s, $x = x_2$

$\therefore \quad dv = 100 - 4v^2\,dt$

$$\therefore \quad \frac{dv}{100 - 4v^2} = dt \qquad \therefore \quad \int_1^3 \frac{dv}{100 - 4v^2} = \int_{t_1}^{t_2} dt$$

$$\therefore \quad \frac{1}{2 \times 2 \times 10}\left[\log_e\left(\frac{10 + 2v}{10 - 2v}\right)\right]_1^3 = [t]_{t_1}^{t_2} \qquad \therefore \quad \frac{1}{40}\left[\log_e\left(\frac{16}{4}\right) - \log_e\left(\frac{12}{8}\right)\right] = t_2 - t_1$$

or $\quad \Delta t = 0.0245$ sec

$\therefore$ The time interval during which velocity changes from 1 m/s to 3 m/s is 0.0245 sec ..... **Ans.**

Given $\quad a = 100 - 4v^2 \ \text{m/s}^2$

Integrating using $\quad a = \dfrac{vdv}{dx}$

$$\therefore \quad vdv = \left(100 - 4v^2\right)dx$$

$$\therefore \quad \int_1^3 \frac{vdv}{100 - 4v^2} = \int_{x_1}^{x_2} dx \qquad \therefore \quad \frac{1}{-8}\int_1^3 \frac{-8v\,dv}{100 - 4v^2} = x_2 - x_1$$

$$\therefore \quad -\frac{1}{8}\left[\log_e\left(100 - 4v^2\right)\right]_1^3 = \Delta x \qquad \therefore \quad x = -\frac{1}{8}\left[\log_e 64 - \log_e 96\right] = \Delta x$$

or $\quad \Delta x = 0.05068$ m ..... (3)

$\therefore$ Displacement during which velocity changes from 1 m/s to 3 m/s is 0.05068 m. ..... **Ans.**

P10. The motion of a particle is defined by the relation $a = 0.8\,t$ m/s². It is found that at $x = 5$ m, $v = 12$ m/s when $t = 2$ sec. Find position & velocity at $t = 6$ sec. ***(KJS May 15)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 0.8t \ \text{m/s}^2 \quad \text{..... (1)}$$

Integrating using $a = \dfrac{dv}{dt}$

$\therefore \quad dv = 0.8t\,dt$

Boundary Conditions

at $t = 2$ sec, $x = 5$ m, $v = 12$ m/s

To find

at $t = 6$ sec ; $x = ?$, $v = ?$

$\therefore \quad \int_{12}^{v} dv = \int_{0}^{t} 0.8\,t\,dt \qquad \therefore \quad [v]_{12}^{v} = 0.8\left[\frac{t^2}{2}\right]_2^t$

$\therefore \quad v - 12 = 0.4\left[t^2 - 4\right]$

$\therefore \quad v = 0.4t^2 + 10.4 \quad \ldots\ldots (2)$

Integrating $\quad v = \frac{dx}{dt}$

$\therefore \quad dx = \left(0.4t^2 + 10.4\right)dt \qquad \therefore \quad \int_{5}^{x} dx = \int_{0}^{t} 0.4t^2 + 10.4\,dt$

$\therefore \quad [x]_5^x = \left[0.4\frac{t^3}{3} + 10.4\,t\right]_2^t \qquad \therefore \quad x - 5 = \left(0.133t^3 + 10.4t\right) - (21.87)$

or $\quad x = 0.133t^3 + 10.4t - 16.87 \quad \ldots\ldots (3)$

Substituting t = 6 sec in equation 2 and 3, we get

$v_6 = 24.8$ m/s and $x_6 = 74.26$ m ..... **Ans.**

P11. The velocity relation of a rectilinear moving particle is defined as $v = 4\,t^2 - 3t - 1$ m/s. At t = 0, x = – 4m. Determine

a) The time at which the particle reverses its sense of motion

b) At t = 3 sec. i) acceleration ii) position iii) displacement iv) distance traveled.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$v = 4t^2 - 3t - 1$ m/s ..... (1)

Differentiating using $a = \frac{dv}{dt}$

$\therefore \quad a = 8t - 3$ m/s^2 ..... (2)

Boundary Conditions

at $t = 0,\ x = -4$ m

To find

a) When v = 0: t = ?

b) At t = 3: a = ?, x = ?, Δx = ?, s = ?

Integrating using $v = \frac{dx}{dt}$

$\therefore \quad dx = \left(4t^2 - 3t - 1\right)dt$

$\therefore \quad \int_{-4}^{x} dx = \int_{0}^{t}\left(4t^2 - 3t - 1\right)dt \qquad \therefore \quad [x]_{-4}^{x} = \left[\frac{4t^3}{3} - \frac{3t^2}{2} - t\right]_0^t$

$\therefore \quad x + 4 = 1.333t^3 - 1.5t^2 - t$

Or $\quad x = 1.333\,t^3 - 1.5t^2 - t - 4 \quad \ldots\ldots (3)$

For reversing sense of motion, the particle's velocity should become zero. Therefore equate equation 1 to zero.

$v = 4t^2 - 3t - 1 = 0 \qquad \therefore \quad t = 1$ sec

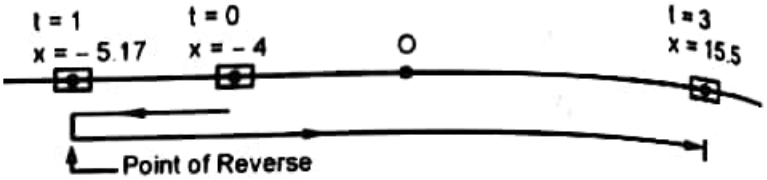
$\therefore$ Particle reverses its sense of motion at t = 1 sec ..... **Ans.**

b) Acceleration, Position, displacement, distance at t = 3 sec.

(i) Acceleration in 3 sec,

$$a_3 = 8 \times 3 - 3$$

$$\therefore \quad a_3 = 21 \text{ m/s}^2 \quad \text{..... Ans.}$$



(ii) Position in 3 sec,

$$x_3 = 1.333 \times 3^3 - 1.5 \times 3^2 - 3 - 4$$

$$\therefore \quad x_3 = 15.5 \text{ m} \quad \text{..... Ans.}$$

(iii) Displacement in 3 sec,

$$\Delta x_3 = x_3 - x_0$$

$$= 15.4 - (-4)$$

$$\therefore \quad \Delta x_3 = 19.5 \text{ m} \quad \text{..... Ans.}$$

(iv) Distance travelled is different from displacement since there is a reverse in sense of motion at t = 1 sec.

Distance in 3 sec = Distance (0 to 1 sec) + Distance (1 to 3 sec)
= (1.17) + (5.17 + 15.5)
= 21.84 m ..... **Ans.**

P12. A particle starting from rest moves in a straight line, whose acceleration is given by $a = 8 - 0.003s^2$, where a is in m/s^2 and s is in meters. Determine

i. Velocity of the particle when it has traveled 40m and

ii. Distance traveled by the particle when its comes to rest ***(VJTI May 06)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 8 - 0.003s^2 \text{ m/s}^2 \quad \text{..... (1)}$$

Boundary Conditions

at $t = 0, s = 0, v = 0$

To find

a) When s = 40 m: v = ?

b) When v = 0: s = ?

Integrating using $a = \dfrac{vdv}{ds}$

$$\therefore \quad vdv = 8 - 0.003s^2 ds$$

$$\therefore \quad \int_0^v vdv = \int_0^s \left(8 - 0.003s^2\right) ds \qquad \therefore \quad \left[\frac{v^2}{2}\right]_0^v = \left[8s - \frac{0.003s^3}{3}\right]_0^s$$

$$\text{or} \quad \frac{v^2}{2} = 8s - 0.001s^3 \quad \text{..... (2)}$$

For velocity at s = 40 m

$$\frac{v^2}{2} = 8 \times 40 - 0.001 \times 40^3$$

$$\therefore \quad v = 22.63 \text{ m/s} \quad \text{..... Ans.}$$

b) Distance travelled as the particle comes to rest.

Substitute v = 0 in equation 2

$$0 = 8s - 0.001s^3$$

$$\therefore \quad s = 89.44 \text{ m} \quad \text{..... Ans.}$$

P13. A point moves along a straight line such that its displacement is $s = 8t^2 + 3t$ where s is in m and t is in seconds. Find the displacement and velocity at t = 4 sec. Also find the distance traveled in the 10th second. ***(VJTI May 08)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$s = 8t^2 + 3t \text{ m} \quad \text{..... (1)}$$

Differentiating using $v = \frac{ds}{dt}$

$$\therefore \quad v = 16t + 3 \text{ m/s} \quad \text{..... (2)}$$

Differentiating using $a = \frac{dv}{dt}$

$$a = 16 \text{ m/s}^2 \quad \text{..... (3)}$$

Boundary Conditions

Nil

To find

a) At t = 4 sec : s = ?, v = ?

b) Distance travelled in 10th sec

Velocity and displacement at t = 4 sec

Put t = 4 sec in equation 1 and 2

$$s_4 = 8 \times 4^2 + 3 \times 4 = 140 \text{ m} \quad \text{..... Ans.}$$

$$v_4 = 16 \times 4 + 3 = 67 \text{ m/s} \quad \text{..... Ans.}$$

b) Distance travelled in 10th sec

$$= s_{10} - s_9$$

$$= 830 - 675 = 155 \text{ m} \quad \text{..... Ans.}$$

P14. The acceleration of the particle is defined by the relation $a = 25 - 3x^2$ mm/s². The particle starts with no initial velocity at the position x = 0. (a) Determine the velocity when x = 2mm (b) the position when velocity is again zero, (c) position where the velocity is maximum and the corresponding maximum velocity. ***(M.U Dec 10)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 25 - 3x^2 \text{ mm/s}^2 \quad \text{..... (1)}$$

Integrating using $a = \frac{vdv}{dx}$

$$\therefore \quad vdv = 25 - 3x^2 dx$$

$$\int_0^v vdv = \int_0^x 25 - 3x^2 dx$$

$$\left[\frac{v^2}{2}\right]_0^v = \left[25x - x^3\right]_0^x$$

$$\frac{v^2}{2} = 25x - x^3 \quad \text{..... (2)}$$

Boundary Conditions

At $t = 0, x = 0, v = 0$

To find

a) when x = 2 mm : v = ?

b) when v = 0 : x = ?

c) v_{max}, x = ?

Velocity when x = 2 mm

$$\frac{v^2}{2} = 25 \times 2 - 2^3 \qquad \therefore \qquad v = 9.165 \text{ mm/s}$$ ..... **Ans.**

b) Position when velocity is zero

$$0 = 25x - x^3 \qquad \text{or} \qquad x = \pm 5 \text{ mm}$$ ..... **Ans.**

c) For maximum velocity, the particle's acceleration should be zero

$$a = 25 - 3x^2 = 0 \qquad \text{or} \qquad x = \pm 2.886 \text{ mm}$$ ..... **Ans.**

$\therefore$ Maximum velocity is at x = + 2.886 mm and v_{max} = 9.809 mm/s ..... **Ans.**

P15. Acceleration of particle is given by $a = 90 - 6x^2$ cm/s^2 where x is in cm. If particle starts with zero initial velocity from origin determine velocity when x = 5 cm, position where velocity is again zero, position where velocity is again maximum. ***(KJS Nov 15)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 90 - 6x^2 \text{ cm/s}^2 \qquad \text{..... (1)}$$

Boundary Conditions

At $t = 0, x = 0, v = 0$

Integrating using $a = \frac{vdv}{dx}$

$$\therefore \qquad vdv = 90 - 6x^2 dx$$

$$\therefore \qquad \int_0^v vdv = \int_0^x 90 - 6x^2\, dx$$

$$\therefore \qquad \left[\frac{v^2}{2}\right]_0^v = \left[90x - 2x^3\right]_0^x$$

$$\therefore \qquad \frac{v^2}{2} = 90x - 2x^3 \qquad \text{..... (2)}$$

Velocity when x = 5 cm

$$\frac{v^2}{2} = 90 \times 5 - 2 \times 5^3 \qquad \therefore \qquad v = 20 \text{ cm/s}$$ ..... **Ans.**

b) Position where velocity is again zero

putting v = 0 in equation (2)

$$0 = 90x - 2x^3 \qquad \text{or} \qquad x = \pm 6.708 \text{ cm}$$ ..... **Ans.**

c) For maximum velocity, the particle's acceleration should be zero

$$a = 90 - 6x^2 = 0 \qquad \text{or} \qquad x = \pm 3.873 \text{ cm}$$ ..... **Ans.**

$\therefore$ Maximum velocity is at x = + 3.873 cm and v_{max} = 21.56 cm/s ..... **Ans.**

P16. A particle performing rectilinear motion has its acceleration given by $a = (2 - 5\,t)$ m/s². At t = 4 sec its velocity is 15 m/s and at t = 8 sec its position is 60 m. Find at t = 0, the particles position, velocity and acceleration.

Solution: Motion of particle – Rectilinear – Variable acceleration

Given Relation

$$a = 2 - 5t \text{ m/s}^2 \qquad \text{..... (1)}$$

Boundary Conditions

At $t = 4$ sec, $v = 15$ m/s

At $t = 8$ sec, $x = 60$ m

To find

a) at t = 0 : x = ? , v = ? , a = ?

b) when v = 0 : t = ?

Integrating using $a = \dfrac{dv}{dt}$

$$\therefore \quad dv = 2 - 5t\,dt$$

$$\therefore \quad \int_{15}^{v} dv = \int_{4}^{t} 2 - 5t\,dt$$

$$\therefore \quad [v]_{15}^{v} = \left[2t - 2.5t^2\right]_{4}^{t}$$

$$\therefore \quad v - 15 = \left(2t - 2.5t^2\right) - (-32) \qquad \therefore \quad v = -2.5t^2 + 2t + 47 \text{ m/s} \qquad \text{..... (2)}$$

Integrating using $v = \dfrac{dx}{dt}$

$$\therefore \quad dx = -2.5t^2 + 2t + 47\,dt \qquad \therefore \quad \int_{60}^{x} dx = \int_{8}^{t} -2.5t^2 + 2t + 47\,dt$$

$$\therefore \quad [x]_{60}^{x} = \left[-2.5\frac{t^3}{3} + \frac{2t^2}{2} + 47t\right]_{8}^{t} \qquad \therefore \quad x - 60 = \left[-2.5\frac{t^3}{3} + t^2 + 47t\right] - (13.33)$$

$$\therefore \quad x = -0.833t^3 + t^2 + 32t + 46.67 \text{ m} \qquad \text{..... (3)}$$

To find position, velocity and acceleration at t = 0

Substitute t = 0 in equation 1, 2 and 3

$$a_0 = 2 \text{ m/s}^2, \quad v_0 = 47 \text{ m/s}, \quad x_0 = 46.67 \text{ m} \qquad \text{..... } \textbf{Ans.}$$

P17. A stone is projected from the ground vertically up with an initial velocity of 20 m/s. a) Knowing that the air resistance causes an additional deceleration of $0.01v^2$ m/s², determine the maximum height reached by stone above the ground.
(Hint: $a = -9.81 - 0.01v^2$ m/s²)
b) solve for maximum height if air resistance is neglected.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = -9.81 - 0.01v^2 \text{ m/s}^2 \qquad \text{..... (1)}$$

Boundary Conditions

At $t = 4$ sec, $v = 15$ m/s

At $t = 8$ sec, $x = 60$ m

To find

i) at t = 0: x = ?, v = ?, a = ?

ii) when v = 0: t = ?

Integrating using $a = \dfrac{vdv}{dx}$

$$\therefore \quad vdv = \left(-9.81 - 0.01v^2\right) dx$$

$$\therefore \quad \int_{20}^{v} \frac{v\,dv}{-9.81-0.01v^2} = \int_{0}^{x} dx$$

$$\therefore \quad \frac{1}{-0.02}\int_{20}^{v} \frac{-0.02\,v\,dv}{-9.81-0.01v^2} = 2 \qquad \therefore \quad -50\left[\log_e\left(-9.81-0.01v^2\right)\right]_{20}^{v} = x$$

$$\therefore \quad -50\left[\log_e\left(-9.81-0.01v^2\right)-\log_e(-13.81)\right] = x \qquad \therefore \quad -50\log_e\left(\frac{-9.81-0.01v^2}{-13.81}\right) = x \quad \ldots (2)$$

At maximum height, velocity becomes zero

$$\therefore \quad -50\log_e\left(\frac{-9.81}{-13.81}\right) = h_{max} \qquad \text{Or} \qquad h_{max} = 17.1 \text{ m} \qquad \ldots\ldots \textbf{Ans.}$$

b) If air resistance is neglected, the motion of stone becomes MUG where u = 20 m/s, v = 0, s = h_{max}, a = −9.81 m/s².

Using $v^2 = u^2 + 2as$

$$0 = 20^2 + 2\times(-9.81)\times h_{max}$$

$$\therefore \quad h_{max} = 20.387 \text{ m} \qquad \ldots\ldots \textbf{Ans.}$$

P18. A particle starting from the origin with an initial velocity u and travelling in a straight path has an accelerating of (5t + 6) m/s². The position of the particle at t = 2 sec is 10 m. Calculate the particles initial velocity u and its position at t = 4 sec.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = 5t + 6 \text{ m/s}^2 \qquad \ldots\ldots (1)$$

Boundary Conditions

At t = 0 sec, x = 0, v = u m/s

At t = 2 sec, x = 10 m

To find

a) u = ?

b) at t = 4 sec; x = ?

Integrating using $a = \frac{dv}{dt}$

$$\therefore \quad dv = 5t + 6\,dt$$

$$\therefore \quad \int_{u}^{v} dv = \int_{0}^{t} 5t + 6\,dt$$

$$\therefore \quad [v]_{u}^{v} = \left[2.5t^2 + 6t\right]_{0}^{t}$$

$$v - u = 2.5t^2 + 6t$$

$$\therefore \quad v = 2.5t^2 + 6t + u \text{ m/s} \qquad \ldots\ldots (2)$$

Integrating the above equation 2 using $v = \frac{dx}{dt}$

$$\therefore \quad dx = 2.5t^2 + 6t + u\,dt \qquad \therefore \quad \int_{0}^{x} dx = \int_{0}^{t} 2.5t^2 + 6t + u\,dt$$

$$\therefore \quad x = \frac{2.5t^3}{3} + \frac{6t^2}{2} + ut \qquad \ldots\ldots (3)$$

Substituting boundary condition at t = 2 sec, x = 10 m in equation 3, we get

$$10 = \frac{2.5\times 2^3}{3} + 3\times 2^2 + u\times 2 \qquad \therefore \qquad u = -4.33 \text{ m/s} \qquad \text{..... Ans.}$$

b) To find position at t = 4 sec

$$x_4 = \frac{2.5\times 4^3}{3} + \frac{6\times 4^2}{2} + (-4.33)\times 4 \qquad \therefore \qquad x_4 = 84 \text{ m} \qquad \text{..... Ans.}$$

P19. The acceleration of a vehicle at any instant is given by the expression $a = \left(\frac{10}{v^2+4}\right)$ m/s². If the vehicle starts from rest, find its velocity when its displacement is 125 m. Also find its displacement when its velocity is 10 m/s.

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = \frac{10}{v^2+4} \text{ m/s}^2 \qquad \text{..... (1)}$$

Integrating using $a = \frac{vdv}{dx}$

$$\therefore \qquad vdv = \frac{10}{v^2+4}dx$$

$$\therefore \qquad \int_0^v \frac{v\left(v^2+4\right)}{10}dv = \int_0^x dx \qquad \therefore \qquad \frac{1}{10}\left(\frac{v^4}{4}+\frac{4v^2}{2}\right) = x \text{ ..... (2)}$$

Boundary Conditions

At $t = 0, v = 0, x = 0$

To find

a) when x = 125 m: v = ?

b) when v = 10 m/s: x = ?

To find velocity at x = 125 m, substitute x = 125 m in equation 2

$$\frac{1}{10}\left(\frac{v^4}{4}+2v^2\right) = 125$$

Solving, we get v = 8.174 m/s ..... **Ans.**

b) To find displacement when v = 10 m/s, substitute v = 10 m/s in equation 2

$$\frac{1}{10}\left(\frac{10^4}{4}+\frac{4\times 10^2}{2}\right) = x \qquad \therefore \qquad x = 270 \text{ m} \qquad \text{..... Ans.}$$

P20. A particle moves in a straight line with acceleration $a = \frac{-8}{s^2}$ m/s². When t = 1sec, s = 4m and v = 2 m/s. Determine acceleration where t = 3 sec. ***(SPCE Mar 11)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given Relation

$$a = \frac{-8}{s^2} \text{ m/s}^2 \qquad \text{..... (1)}$$

Integrating using $a = \frac{vdv}{ds}$

$$\therefore \qquad vdv = \frac{-8}{s^2}ds$$

Boundary Conditions

At $t = 1\text{sec}, s = 4\text{ m}, v = 2\text{ m/s}$

To find

a) at t = 3 sec : a = ?

$$\therefore \quad \int_2^v v\,dv = \int_4^s \frac{-8}{s^2}\,ds$$

$$\therefore \quad \left[\frac{v^2}{2}\right]_2^v = \left[\frac{8}{s}\right]_4^s \qquad \therefore \quad \frac{1}{2}\left[v^2 - 4\right] = 8\left[\frac{1}{s} - \frac{1}{4}\right]$$

$$\therefore \quad v^2 = 16\left[\frac{1}{s} - \frac{1}{4}\right] + 4 \qquad \text{Or} \qquad v = \frac{4}{\sqrt{s}} \quad \text{..... (2)}$$

Integrating using $v = \dfrac{ds}{dt}$

$$\therefore \quad \frac{ds}{dt} = \frac{4}{\sqrt{s}} \qquad \text{Or} \qquad \sqrt{s}\,ds = 4\,dt$$

$$\therefore \quad \int_4^s \sqrt{s}\,ds = \int_1^t 4\,dt \qquad \therefore \quad \left[\frac{2s^{3/2}}{3}\right]_4^s = 4[t]_1^t \qquad \therefore \quad \frac{2}{3}\left[s^{3/2} - 4^{3/2}\right] = 4(t-1) \quad \text{..... (3)}$$

Let us first find value of s at t = 3 sec, by substituting t = 3 sec in equation 3

$$\frac{2}{3}\left[s^{3/2} - 4^{3/2}\right] = 4(3-1) \qquad \therefore \quad s_3 = 7.368\,\text{m} \qquad \text{..... } \textbf{Ans.}$$

Substitute the above value in equation 1

$$a = \frac{-8}{7.368^2} \qquad \therefore \quad a_3 = -0.147\ \text{m/s}^2 \qquad \text{..... } \textbf{Ans.}$$

P21. Acceleration of a particle moving along a straight line is represented by the relation $a = 30 - 4.5x^2\ \text{m/s}^2$. The particle starts with zero initial velocity at x = 0. Determine (a) The velocity when x = 3 m, (b) the position when the velocity is again zero (c) the position when the velocity is maximum. ***(M.U. Dec 14)***

Solution: Motion of particle – Rectilinear – Variable acceleration

a) Given : Relation

$$a = 30 - 4.5\,x^2\ \text{m/s}^2 \quad \text{..... (1)}$$

Boundary Conditions

At $t = 0,\ v = 0,\ x = 0$

Integrating using $a = \dfrac{vdv}{dx}$

$$\therefore \quad vdv = 30 - 4.5\,x^2\,dx$$

$$\therefore \quad \int_0^v vdv = \int_0^x 30 - 4.5\,x^2\,dx \qquad \therefore \quad \left[\frac{v^2}{2}\right]_0^v = \left[30x - 1.5x^3\right]_0^x$$

$$\text{or} \quad \frac{v^2}{2} = 30x - 1.5\,x^3 \quad \text{..... (2)}$$

Velocity when x = 3 m Substitute x = 3 m in equation (2)

$$\frac{v^2}{2} = 30\times3 - 1.5\times3^3 \qquad \therefore \quad v = 9.95\ \text{m/s} \qquad \text{..... } \textbf{Ans.}$$

b) Position when the velocity is again zero

$$0 = \left[30x - 1.5x^3\right] \quad \text{or} \quad x = \pm 4.472 \text{ m} \quad \text{..... Ans.}$$

c) For minimum velocity, the particle's acceleration should be zero.

$$a = 30 - 4.5x^2 = 0 \quad \text{or} \quad x = \pm 2.582 \text{ m} \quad \text{..... Ans.}$$

$\therefore$ Maximum velocity is at x = + 2.582 m ..... **Ans.**

P22. A particle starts from rest from origin and its acceleration is given by, $a = \dfrac{k}{(x+4)^2}$ m/s². Knowing that v = 4 m/s when x = 8 m, find

(i) Value of k and (ii) Position when v = 4.5 m/s. ***(M.U Dec 16)***

Solution: Motion of Particle – Rectilinear –variable acceleration

Given Relation

$$a = \frac{k}{(x+4)^2} \text{ m/s}^2 \quad \text{... (1)}$$

Boundary Conditions
At t = 0, v = 0, x = 0
At x = 8 m, v = 4 m/s

Integrating Using $a = \dfrac{v\,dv}{dx}$

$$\therefore \quad v\,dv = \frac{k}{(x+4)^2}dx$$

$$\therefore \quad \int_0^v v\,dv = \int_0^x \frac{k}{(x+4)^2}dx \qquad \therefore \quad \frac{v^2}{x} = k\left[\frac{-1}{(x+4)}\right]_0^x$$

$$\therefore \quad \frac{v^2}{x} = -k\left[\frac{1}{(x+4)} - \frac{1}{4}\right] \quad \text{... (2)}$$

To find value of k, substitute the second boundary condition, at x = 8 m, v = 4 m/s in equation (2)

$$\frac{4^2}{2} = -k\left[\frac{1}{(8+4)} - \frac{1}{4}\right] \qquad \therefore \quad k = 48 \qquad \text{... Ans.}$$

b) To find position when v = 4.5 m/s
substitute v = 4.5 m/s in equation (2)

$$\frac{4.5^2}{2} = -48\left[\frac{1}{(x+4)} - \frac{1}{4}\right] \qquad \therefore \quad x = 21.6 \text{ m} \qquad \text{... Ans.}$$

Exercise 9.3

Curvilinear Motion

P1. A curvilinear motion is defined by $y = 8t^3 - 6t$ m and $a_x = 4t$ m/s². At t = 1 sec, $v_x = 6$ m/s. Calculate magnitude of velocity and acceleration at t = 3 sec.

Solution: Motion of particle – Curvilinear : x – y system

x direction

Given $a_x = 4t \text{ m/s}^2$ ..... (1)

At t = 1 sec; $v_x = 6$ m/s

Integrating using $a_x = \dfrac{dv_x}{dt}$

$\therefore \quad dv_x = 4t\,dt$

$\therefore \quad \int_6^{v_x} dv_x = \int_1^t 4t\,dt$

$\therefore \quad [v_x]_6^{v_x} = \left[2t^2\right]_1^t$

or $\quad v_x - 6 = 2t^2 + 4 \text{ m/s}$ ..... (2)

Substitute t = 3 sec in equation 2 and 3

At t = 3 sec

$v_x = 22 \text{ m/s}$ and $a_x = 12 \text{ m/s}^2$

y direction

Given $y = 8t^3 - 6t$ m ..... (1)

Differentiating using $v_y = \dfrac{dy}{dt}$

$\therefore \quad v_y = 24t^2 - 6 \text{ m/s}$ ..... (2)

Differentiating using $a_y = \dfrac{dv_y}{dt}$

$\therefore \quad a_y = 48t \text{ m/s}^2$ ..... (3)

Substitute t = 3 sec in equation 2 and 3

At t = 3 sec

$v_y = 210 \text{ m/s}$

$a_y = 144 \text{ m/s}^2$

Total velocity $v = \sqrt{v_x^2 + v_y^2}$ $\therefore$ $v = \sqrt{22^2 + 210^2}$ or $v_3 = 211.15$ m/s ..... **Ans.**

Total acceleration $a = \sqrt{a_x^2 + a_y^2}$ $\therefore$ $a = \sqrt{12^2 + 144^2}$ or $a_3 = 144.5 \text{ m/s}^2$ ..... **Ans.**

P2. A particle starts from rest from the position (- 5, - 2, 4) m. Its acceleration is defined by $\bar{a} = 3t\,i + 12t^2 j + 5\,k$ m/s². Find the particle's position, displacement, velocity and acceleration at t = 2 sec.

Solution: Motion of particle – Curvilinear : x – y – z system

x direction

Given $a_x = 3t \text{ m/s}^2$ ..... (1)

At t = 0: x = – 5 m, $v_x = 0$

Integrating

using $a_x = \dfrac{dv_x}{dt}$

$\therefore \quad dv_x = 3t\,dt$

$\therefore \quad \int_0^{v_x} dv_x = \int_0^t 3t\,dt$

or $\quad v_x = 1.5t^2 \text{ m/s}$ ..... (2)

y direction

Given $a_y = 12t^2 \text{ m/s}^2$ ... (1)

At t = 0: y = – 2 m, $v_y = 0$

Integrating

using $a_y = \dfrac{dv_y}{dt}$

$\therefore \quad dv_y = 12t^2\,dt$

$\therefore \quad \int_0^{v_y} dv_y = \int_0^t 12t^2\,dt$

or $\quad v_y = 4t^3 \text{ m/s}$ ..... (2)

z direction

Given $a_z = 5 \text{ m/s}^2$ ... (1)

At t = 0: z = 4 m, $v_z = 0$

Integrating

using $a_z = \dfrac{dv_z}{dt}$

$\therefore \quad dv_z = 5\,dt$

$\therefore \quad \int_0^{v_z} dv_z = \int_0^t 5\,dt$

or $\quad v_z = 5t \text{ m/s}$ ..... (2)

Integrating

using $v_x = \frac{dx}{dt}$

$\therefore \quad dx = 1.5t^2\,dt$

$\therefore \quad \int_{-5}^{x} dx = \int_0^t 1.5t^2\,dt$

or $\quad x + 5 = 0.5t^3$ ..... (3)

At t = 2 sec

$x = -1$ m

$v_x = 6$ m/s

$a_x = 6$ m/s^2

Integrating

using $v_y = \frac{dy}{dt}$

$\therefore \quad dy = 4t^3\,dt$

$\therefore \quad \int_{-2}^{y} dy = \int_0^t 4t^3\,dt$

or $\quad y + 2 = t^4$ ..... (3)

At t = 2 sec

$y = 14$ m

$v_y = 32$ m/s

$a_y = 48$ m/s^2

Integrating

using $v_z = \frac{dz}{dt}$

$\therefore \quad dz = 5t\,dt$

$\therefore \quad \int_4^z dz = \int_0^t 5t\,dt$

or $\quad z - 4 = 2.5t^2$ ..... (3)

At t = 2 sec

$z = 14$ m

$v_z = 10$ m/s

$a_z = 5$ m/s^2

At t = 2 sec

Position = $r = \sqrt{x^2 + y^2 + z^2} = \sqrt{-1^2 + 14^2 + 14^2} = 19.82$ m ..... **Ans.**

Velocity = $v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{6^2 + 32^2 + 10^2} = 34.06$ m/s ..... **Ans.**

Acceleration = $a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{6^2 + 48^2 + 5^2} = 48.63$ m/s^2 ..... **Ans.**

Displacement = $\Delta r = r_2 - r_0 = (-i + 14j + 14k) - (-5i - 2j + 4k)$

$= 4i + 16j + 10k$ m

$\therefore \quad \Delta r = \sqrt{4^2 + 16^2 + 10^2} = 19.29$ m ..... **Ans.**

P3. A particle starting from rest at the position (5, 6, 2) m accelerates at $\bar{a} = 6t\,\bar{i} - 24t^2\,\bar{j} + 10\,\bar{k}$ m/s². Determine the acceleration, velocity, position and displacement of the particle at the end of 2 seconds. ***(M. U. Dec 09)***

Solution: Motion of particle – Curvilinear : x – y – z system

<u>x direction</u>

Given $a_x = 6t$ m/s^2 ..... (1)

At t = 0: x = 5 m, $v_x = 0$

Integrating

using $a_x = \frac{dv_x}{dt}$

$\therefore \quad dv_x = 6t\,dt$

$\therefore \quad \int_0^{v_x} dv_x = \int_0^t 6t\,dt$

or $\quad v_x = 3t^2$ m/s ..... (2)

<u>y direction</u>

Given $a_y = -24t^2$ m/s^2 ...(1)

At t = 0: y = 6 m, $v_y = 0$

Integrating

using $a_y = \frac{dv_y}{dt}$

$\therefore \quad dv_y = -24t^2\,dt$

$\therefore \quad \int_0^{v_y} dv_y = \int_0^t -24t^2\,dt$

or $\quad v_y = -8t^3$ m/s ..... (2)

<u>z direction</u>

Given $a_z = 10$ m/s^2 ... (1)

At t = 0: z = 2 m, $v_z = 0$

Integrating

using $a_z = \frac{dv_z}{dt}$

$\therefore \quad dv_z = 10\,dt$

$\therefore \quad \int_0^{v_z} dv_z = \int_0^t 10\,dt$

or $\quad v_z = 10t$ m/s ..... (2)

Integrating

using $v_x = \frac{dx}{dt}$

$\therefore \quad dx = 3t^2\,dt$

$\therefore \quad \int_5^x dx = \int_0^t 3t^2\,dt$

or $\quad x - 5 = t^3 \quad$ ..... (3)

At t = 2 sec

$x = 13$ m

$v_x = 12$ m/s

$a_x = 12$ m/s^2

Integrating

using $v_y = \frac{dy}{dt}$

$\therefore \quad dy = -8t^3\,dt$

$\therefore \quad \int_6^y dy = \int_0^t -8t^3\,dt$

or $\quad y - 6 = -2t^4 \quad$ ..... (3)

At t = 2 sec

$y = -26$ m

$v_y = -64$ m/s

$a_y = -96$ m/s^2

Integrating

using $v_z = \frac{dz}{dt}$

$\therefore \quad dz = 10t\,dt$

$\therefore \quad \int_2^z dz = \int_0^t 10t\,dt$

or $\quad z - 2 = 5t^2 \quad$ ..... (3)

At t = 2 sec

$z = 22$ m

$v_z = 20$ m/s

$a_z = 10$ m/s^2

At t = 2 sec

Velocity = $v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{12^2 + (-64)^2 + 20^2} = 68.11$ m/s ..... **Ans.**

Acceleration = $a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{12^2 + 96^2 + 10^2} = 97.26$ m/s^2 ..... **Ans.**

Position = $r = \sqrt{x^2 + y^2 + z^2} = \sqrt{13^2 + (-26)^2 + 22^2} = 36.45$ m ..... **Ans.**

Displacement = $\Delta r = r_2 - r_0 = (13i - 26j + 22k) - (5i + 6j + 2k)$

$= 8i - 32j + 20k$ m

$\therefore \quad \Delta r = \sqrt{8^2 + (-32)^2 + 20^2} = 38.57$ m ..... **Ans.**

P4. A particle performing curvilinear motion has velocity components given as $v_x = 32\,t - 4$ m/s and $v_y = 4$ m/s. At t = 3 sec it occupied the position (5, 12) m. Determine the equation of the path traced by the particle.

Solution: Motion of particle – Curvilinear: x – y system

x direction

Given $\quad v_x = 32t - 4$ m/s ..... (1)

At t = 3 sec ; x = 5 m

Integrating using $v_x = \frac{dx}{dt}$

$\therefore \quad dx = 32t - 4\,dt$

$\therefore \quad \int_5^x dx = \int_3^t 32t - 4\,dt$

$\therefore \quad x - 5 = (16t^2 - 4t) - (32 \times 3 - 4 \times 3)$

or $\quad x = 16t^2 - 4t - 127$ m ..... (2)

y direction

Given $\quad v_y = 4$ m/s ..... (1)

At t = 3 sec ; y = 12 m

Integrating using $v_y = \frac{dy}{dt}$

$\therefore \quad dy = 4\,dt$

$\therefore \quad \int_{12}^y dy = \int_3^t 4\,dt$

$\therefore \quad y - 12 = 4t - 12$

or $\quad y = 4t$ m ..... (2)

Eliminating t from equation no (2) on both sides we get the equation of path as

$x = y^2 - y - 127$ ..... **Ans.**

P5. If $x = 1 - t$ and $y = t^2$ where x and y are in metres and 't' is in seconds, determine x and y components of velocity and acceleration. Also write equation of the path.

(M. U. May 09)

Solution: Motion of particle – Curvilinear: x – y system

x direction

Given $x = 1 - t$ m ..... (1)

Differentiating using $v_x = \frac{dx}{dt}$

$\therefore$ $v_x = -1$ m/s ..... (2) ..... **Ans.**

Differentiating using $a_x = \frac{dv_x}{dt}$

$\therefore$ $a_x = 0$ ..... (3) ..... **Ans.**

y direction

Given $y = t^2$ m ..... (1)

Differentiating using $v_y = \frac{dy}{dt}$

$\therefore$ $v_y = 2t$ m/s ..... (2) ..... **Ans.**

Differentiating using $a_y = \frac{dv_y}{dt}$

$\therefore$ $a_y = 2$ m/s^2 ..... (3) ..... **Ans.**

Eliminating t from equation (1) on both sides we get the equation of path as

$$y = (1 - x)^2$$ ..... **Ans.**

P6. A particle moves along a curved path defined by $y = {}^1/_8\, x^2$. At any instant its x coordinate is given by $x = 2t^2 - 4t$. Determine its velocity and acceleration at $x = 8$ m.

Solution: Motion of particle – Curvilinear: x – y system

x direction

Given $x = 2t^2 - 4t$ m ..... (1)

Differentiating using $v_x = \frac{dx}{dt}$

$\therefore$ $v_x = 4t - 4$ m/s ..... (2)

Differentiating using $a_x = \frac{dv_x}{dt}$

$\therefore$ $a_x = 4$ m/s^2 ..... (3)

Substitute x = 8 m equation (1)

$8 = 2t^2 - 4t$

$\therefore$ $2t^2 - 4t - 8 = 0$

or $t = 3.236$ sec

Substitute t = 3.236 sec in equation (2) and (3), we get

$v_x = 8.944$ m/s

and $a_x = 4$ m/s^2

y direction

Given $y = \frac{1}{8}x^2$ m .....Path Equation.

$= \frac{1}{8} \times (2t^2 - 4t)^2$

$\therefore$ $y = \frac{1}{2}t^4 - 2t^3 + 2t^2$ m ..... (1)

Differentiating using $v_y = \frac{dy}{dt}$

$\therefore$ $v_y = 2t^3 - 6t^2 + 4t$ m/s ..... (2)

Differentiating using $a_y = \frac{dv_y}{dt}$

$\therefore$ $a_y = 6t^2 - 12t + 4$ m/s^2 ..... (3)

Now, at x = 8 m, t = 3.236 sec

Substitute t = 3.236 sec in equation (2) and (3), we get

$v_y = 18$ m/s

$a_y = 28$ m/s^2

$\therefore$ At x = 8 m, t = 3.236 sec

Total Velocity = $v = \sqrt{v_x^2 + v_y^2} = \sqrt{8.944^2 + 18^2} = 20$ m/s ..... **Ans.**

Total Acceleration = $a = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 28^2} = 28.28$ m/s^2 ..... **Ans.**

P7. In a curvilinear motion particle P moves along the fixed path $9y = x^2$, where x and y are expressed in centimetres. At any instant t, the x coordinate of P is given by $x = t^2 - 14t$. Determine y component of velocity and acceleration of P when t = 15 sec.

(KJS Nov 15)

Solution: Motion of particle – Curvilinear: x – y system

x direction

Given $x = t^2 - 14t$ cm ..... (1)

Differentiating using $v_x = \frac{dx}{dt}$

$\therefore \quad v_x = 2t - 14$ cm/s ..... (2)

Differentiating using $a_x = \frac{dv_x}{dt}$

$\therefore \quad a_x = 2$ cm/s^2 ..... (3)

At t = 15 sec

$v_x = 16$ cm/s , $a_x = 2$ cm/s^2

y direction

Given $y = \frac{x^2}{9} = \frac{(t^2 - 14t)^2}{9}$

$\therefore \; y = \frac{1}{9}\left[t^4 - 28t^3 + 196t^2\right]$ cm ..... (1)

Differentiating using $v_y = \frac{dy}{dt}$

$\therefore \; v_y = \frac{1}{9}\left[4t^3 - 84t^2 + 392t\right]$ cm/s ..... (2)

Differentiating using $a_y = \frac{dv_y}{dt}$

$\therefore a_y = \frac{1}{9}\left[12t^2 - 168t + 392\right]$ cm/s^2 ..... (3)

At t = 15 sec

$v_y = 53.33$ cm/s , $a_y = 63.56$ cm/s^2

At t = 15 sec,

The y component of velocity $v_y = 53.33$ cm/s ..... **Ans.**

Total acceleration $a = \sqrt{a_x^2 + a_y^2} = \sqrt{2^2 + 63.56^2} = 63.59$ cm/s^2 ..... **Ans.**

DJC

P8. A particle travels along the path defined by the parabola $y = 0.5\ x^2$. If the x component of velocity is $v_x = 5\ t$ m/s, determine the distance of particle from the origin O and the magnitude of acceleration when t = 1 sec. At t = 0, x = 0 and v = 0.

(M.U May 11)

Solution: Motion of particle – Curvilinear: x – y system

<u>x direction</u>

Given $v_x = 5t$ m/s ..... (1)

Also at $t = 0$, $x = 0$, $v_x = 0$

Integrating using $v_x = \dfrac{dx}{dt}$

$\therefore \quad dx = 5t\ dt$

$\therefore \quad \int_0^x dx = \int_0^t 5t\ dt$

$\therefore \quad x = 2.5\ t^2$ ..... (2)

Differentiating equation (1)

Using $a_x = \dfrac{dv_x}{dt}$

$\therefore \quad a_x = 5\ m/s^2$ ..... (3)

At t = 1 sec

$x = 2.5$ m and $a_x = 5\ m/s^2$

<u>y direction</u>

Given $y = 0.5x^2$ m .....Path Equation.

Substituting equation (2) from x direction, we get

$$y = 0.5 \times (2.5t^2)^2$$

or $y = 3.125\ t^4$ m ..... (1)

Differentiating using $v_y = \dfrac{dy}{dt}$

$\therefore \quad v_y = 12.5\ t^3$ m/s ..... (2)

Differentiating using $a_y = \dfrac{dv_y}{dt}$

$\therefore \quad a_y = 37.5t^2\ m/s^2$ ..... (3)

At t = 1 sec

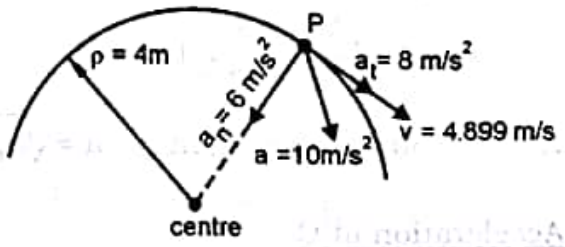
$y = 3.125$ m and $a_y = 37.5\ m/s^2$

$\therefore$ At t = 1 sec

Position of r from origin = $\sqrt{x^2 + y^2} = \sqrt{2.5^2 + 3.125^2} = 4$ m ..... **Ans.**

Total Acceleration = $a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.83\ m/s^2$ ..... **Ans.**

P9. A particle P moves in a circular path of 4 m radius. At an instant the speed is increasing at a rate of 8 m/s² and its total acceleration is 10 m/s². Determine the particle's speed at this instant.



Solution: Motion of particle – Curvilinear: N – T system

Given Tangential acceleration $a_t = 8\ m/s^2$

Total acceleration $a = 10\ m/s^2$

Total acceleration = $a = \sqrt{a_n^2 + a_t^2}$

$\therefore \quad a_n = \sqrt{a^2 - a_t^2} = \sqrt{10^2 - 8^2}$

$\therefore \quad a_n = 6\ m/s^2$

but $a_n = \dfrac{v^2}{\rho}$ $\therefore$ $6 = \dfrac{v^2}{4}$ $\therefore$ $v = 4.899$ m/s ..... **Ans.**

P10. A racing car traveling on a circular curve ABC of 400 m radius increases its speed uniformly from 72 kmph at A to 108 kmph at C over a distance of 300 m along the curve. What was the total acceleration of the car when it was at B, 250 m from A. Also find the total acceleration of the car when it was at A and at C.

Solution: Motion of car – Curvilinear: N – T system

Motion (A to C)

$u = 20\,m/s$

$v = 30\,m/s$

$s = 300\,m$

$a_t = a_t$

$t = -$

Using $v^2 = u^2 + 2as$

$30^2 = 20^2 + 2 \times a_t \times 300$

$\therefore\ a_t = 0.833\ m/s^2$

Motion (A to B)

$u = 20\,m/s$

$v = v$

$s = 250\,m$

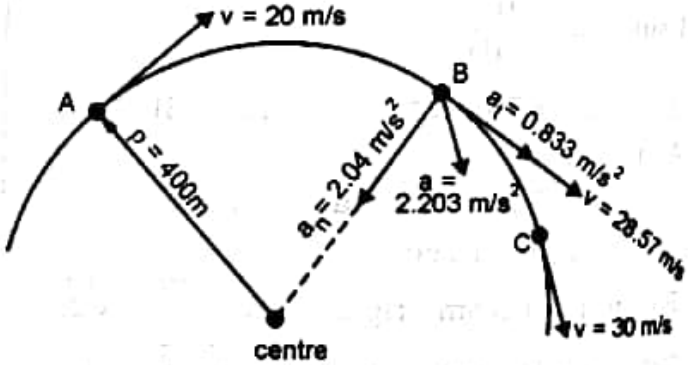
$a_t = 0.833\ m/s^2$

$t = -$

Using $v^2 = u^2 + 2as$

$v^2 = 20^2 + 2 \times 0.833 \times 250$

$\therefore\ v = 28.57\ m/s$ ...velocity at B



Acceleration at B

$a_t = 0.833\ m/s^2$

$a_n = \dfrac{v^2}{\rho} = \dfrac{28.57^2}{400} = 2.04\ m/s^2$

$\therefore$ Total acceleration

$= a = \sqrt{a_n^2 + a_t^2}$

$= \sqrt{2.04^2 + 0.833^2} = 2.203\ m/s^2$ ..... **Ans.**

Acceleration at A

$a_t = 0.833\ m/s^2$

$a_n = \dfrac{v^2}{\rho} = \dfrac{20^2}{400} = 1\ m/s^2$

$\therefore$ Total acceleration = $a = \sqrt{a_n^2 + a_t^2} = \sqrt{1^2 + 0.833^2} = 1.301\ m/s^2$ ..... **Ans.**

Acceleration at C

$a_t = 0.833\ m/s^2$

$a_n = \dfrac{v^2}{\rho} = \dfrac{30^2}{400} = 2.25\ m/s^2$

$\therefore$ Total acceleration = $a = \sqrt{a_n^2 + a_t^2} = \sqrt{2.25^2 + 0.833^2} = 2.4\ m/s^2$ ..... **Ans.**

P11. A train enters a curve of radius 500 m with a speed of 60 kmph. Determine magnitude of total deceleration at the instant the brakes are applied so that the train stops by covering a distance of 400 m along the curve. Also determine the time required by the train to come to rest.

Solution: Motion of train – Curvilinear: N – T system

Motion (A to B)

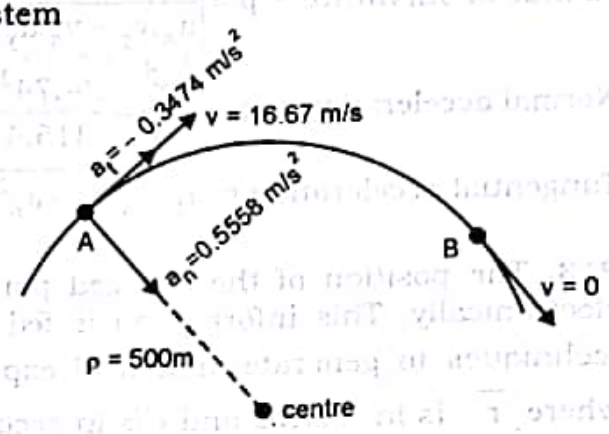
$u = 16.67\ m/s$

$v = 0$

$s = 400\ m$

$a_t = a_t$

$t = t$



Using $v^2 = u^2 + 2a_t s$

$0 = 16.67^2 + 2 \times a_t \times 400$

$\therefore \quad a_t = -0.3474\ m/s^2$

Using $v = u + a_t t$

$0 = 16.67 - 0.3474 \times t \quad \therefore \quad t = 48$ sec ..... **Ans.**

Acceleration at A (since brakes are applied at A)

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{500} = 0.5558\ m/s^2$$

and $\quad a_t = -0.3474\ m/s^2$

$\therefore$ Total acceleration = $a = \sqrt{a_n^2 + a_t^2} = \sqrt{0.5558^2 + 0.3474^2} = 0.655\ m/s^2$ ..... **Ans.**

P12. The motion of a particle is defined by the position vector $\bar{r} = 6\,t\,i + 4\,t^2\,j$ where r is in metres and t is in seconds. At the instant when t = 3 sec, find (i) Tangential and Normal components of accelerations (ii) Radius of curvature. ***(M.U May 11)***

Solution: Motion of particle – Curvilinear: x – y system

<u>x direction</u>

Given $\quad x = 6t\ m \quad$ ..... (1)

Differentiating using $v_x = \dfrac{dx}{dt}$

$\therefore \quad v_x = 6\ m/s \quad$ ..... (2)

Differentiating using $a_x = \dfrac{dv_x}{dt}$

$\therefore \quad a_x = 0 \quad$ ..... (3)

At t = 3 sec

$v_x = 6\ m/s$ and $a_x = 0$

<u>y direction</u>

Given $\quad y = 4t^2\ m \quad$ ..... (1)

Differentiating using $v_y = \dfrac{dy}{dt}$

$\therefore \quad v_y = 8t\ m/s \quad$ ..... (2)

Differentiating using $a_y = \dfrac{dv_y}{dt}$

$\therefore \quad a_y = 8\ m/s^2 \quad$ ..... (3)

At t = 3 sec

$v_y = 24\ m/s$ and $a_y = 8\ m/s^2$

$\therefore$ At t = 3 sec ;

Total Velocity = $v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 24^2} = 24.74 \text{ m/s}$

Total Acceleration = $a = \sqrt{a_x^2 + a_y^2} = \sqrt{0 + 8^2} = 8 \text{ m/s}^2$

Radius of curvature = $\rho = \left|\dfrac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\dfrac{24.73^3}{0 - 6\times 8}\right| = 315.4 \text{ m}$ ..... **Ans.**

Normal acceleration = $a_n = \dfrac{v^2}{\rho} = \dfrac{24.74^2}{315.4} = 1.941 \text{ m/s}^2$ ..... **Ans.**

Tangential acceleration = $a_t = \sqrt{a^2 - a_n^2} = \sqrt{8^2 - 1.941^2} = 7.761 \text{ m/s}^2$ ..... **Ans.**

P13. The position of the charged particle moving in a horizontal plane is measured electronically. This information is fed into a computer which employs a curve fitting techniques to generate analytical expression for its position given by $\bar{r} = t^3\,\bar{i} + t^4\,\bar{j}$ where $\bar{r}$ is in metres and t is in seconds. For t = 1 sec, determine (i) the acceleration of the particle in rectangular components (ii) its normal and tangential acceleration and (iii) the radius of curvature of the path. ***(M.U May 08)***

Solution: Motion of particle – Curvilinear: x – y system

x direction	y direction
Given $x = t^3$ m ..... (1)	Given $y = t^4$ m ..... (1)
Differentiating using $v_x = \dfrac{dx}{dt}$	Differentiating using $v_y = \dfrac{dy}{dt}$
$\therefore$ $v_x = 3t^2$ m/s ..... (2)	$\therefore$ $v_y = 4t^3$ m/s ..... (2)
Differentiating using $a_x = \dfrac{dv_x}{dt}$	Differentiating using $a_y = \dfrac{dv_y}{dt}$
$\therefore$ $a_x = 6t$ m/s^2 ..... (3)	$\therefore$ $a_y = 12t^2$ m/s^2 ..... (3)
At t = 1 sec	At t = 1 sec
$v_x = 3$ m/s and $a_x = 6$ m/s^2	$v_y = 4$ m/s and $a_y = 12$ m/s^2

$\therefore$ At t = 1 sec ;

Total Velocity = $v = \sqrt{v_x^2 + v_y^2} = \sqrt{3^2 + 4^2} = 5 \text{ m/s}$

Total Acceleration = $a = \sqrt{a_x^2 + a_y^2} = \sqrt{6^2 + 12^2} = 13.42 \text{ m/s}^2$

Acceleration in rectangular component = $\bar{a} = 6i + 12j \text{ m/s}^2$ ..... **Ans.**

Radius of curvature = $\rho = \left|\dfrac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\dfrac{5^3}{6\times 4 - 3\times 12}\right| = 10.417 \text{ m}$ ..... **Ans.**

Normal acceleration = $a_n = \dfrac{v^2}{\rho} = \dfrac{5^2}{10.417} = 2.4 \text{ m/s}^2$ ..... **Ans.**

Tangential acceleration = $a_t = \sqrt{a^2 - a_n^2} = \sqrt{13.42^2 - 2.4^2} = 13.2 \text{ m/s}^2$ ..... **Ans.**

P14. A particle travels along a parabolic shaped track $y = 10 + 0.4\,x^2$ with a constant speed of 6 m/s. At x = 3 m, find a) acceleration. b) components of velocity

Solution: Motion of particle – Curvilinear: N – T system – Uniform Speed

a) Since speed is uniform, $a_t = 0$

Radius of curvature of path at x = 3 m

$$\rho = \left|\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\right| = \left|\frac{\left[1+2.4^2\right]^{3/2}}{0.8}\right|$$

$$= 21.97 \text{ m}$$

Equation of path

$$y = 10 + 0.4x^2$$

$$\frac{dy}{dx} = 0.8x, \quad \frac{d^2y}{dx^2} = 0.8$$

At x = 3 m

$$\frac{dy}{dx} = 2.4, \quad \frac{d^2y}{dx^2} = 0.8$$

Normal acceleration = $a_n = \frac{v^2}{\rho} = \frac{6^2}{21.97} = 1.638 \text{ m/s}^2$

Tangential acceleration = $a_t = 0$

$\therefore$ Total Acceleration = $a = \sqrt{a_n^2 + a_t^2} = 1.638 \text{ m/s}^2$ ..... **Ans.**

b) Components of velocity

Velocity is always tangent to the path and makes an angle θ with the x axis

here $\tan\theta = \frac{dy}{dx} = 2.4$ or $\theta = 67.38°$

$\therefore$ $v_x = v\cos\theta = 6\cos 67.38 = 2.307 \text{ m/s}$ ..... **Ans.**

also $v_y = v\sin\theta = 6\sin 67.38 = 5.538 \text{ m/s}$ ..... **Ans.**

P15. A point moves along a path $y = \frac{x^2}{3}$ with a constant speed of 8 m/s. What are the x and y components of its velocity when x = 3 m ? What is the acceleration of the point at this instant? ***(M.U. Dec 12, May 14, KJS May 15)***

Solution: Motion of particle – Curvilinear: N – T system – Uniform Speed

a) Since speed is constant at v = 8 m/s, $a_t = 0$

Radius of curvature of path at x = 3 m

$$\rho = \left|\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\right| = \left|\frac{\left[1+2^2\right]^{3/2}}{\frac{2}{3}}\right|$$

$$= 16.77 \text{ m}$$

Equation of path

$$y = \frac{x^2}{3} \quad \therefore \quad \frac{dy}{dx} = \frac{2}{3}x, \quad \frac{d^2y}{dx^2} = \frac{2}{3}$$

At x = 3 m

$$\frac{dy}{dx} = 2, \quad \frac{d^2y}{dx^2} = \frac{2}{3}$$

Normal acceleration = $a_n = \frac{v^2}{\rho} = \frac{8^2}{16.77} = 3.816 \text{ m/s}^2$

Tangential acceleration = $a_t = 0$

$\therefore$ Total Acceleration = $a = \sqrt{a_n^2 + a_t^2} = 3.816\ m/s^2$ ..... **Ans.**

b) Components of velocity

Velocity is always tangent to the path and makes an angle θ with the x axis

here $\tan\theta = \frac{dy}{dx} = 2$ or $\theta = 63.43°$

$\therefore$ $v_x = v\cos\theta = 8\cos 63.43 = 3.578\ m/s$ ..... **Ans.**

also $v_y = v\sin\theta = 8\sin 63.43 = 7.155\ m/s$ ..... **Ans.**

P16. A particle travels on a circular path, whose distance travelled is defined by $s = (0.5t^3 + 3t)m$. If the total acceleration is 10 m/s², at t = 2 sec, find its radius of curvature. ***(MU Dec 15)***

Solution: Motion of particle – Curvilinear: N – T system – variable tangential acceleration.

Given $s = 0.5t^3 + 3t\ m$

$$v = \frac{ds}{dt} = 1.5t^2 + 3\ m/s$$

$$a_t = \frac{dv}{dt} = 3t\ \ m/s^2$$

At $t = 2$ sec, $v = 9\ m/s$, $a_t = 6 m/s^2$

Total acceleration = $a = \sqrt{a_n^2 + a_t^2}$

$\therefore$ $10 = \sqrt{a_n^2 + 6^2}$

$\therefore$ $a_n = 8\ m/s^2$

but $a_n = \frac{v^2}{\rho}$ $\therefore$ $8 = \frac{9^2}{\rho}$ $\therefore$ $\rho = 10.125\ m$ ..... **Ans.**

P17. A particle travels along a cubic curve y = 0.2x³. At a position y = 5.4 m its speed was 5 m/s and decreasing at a rate of 0.8 m/s². Find a) At this instant the particle's total acceleration and velocity components along x and y directions. b) time when the particle comes to rest.

Solution: Motion – Curvilinear: N – T system – Uniform Tangential Acceleration

a) Given v = 5 m/s and $a_t = -0.8\ m/s^2$

Radius of curvature of path at y = 5.4 m

$$\rho = \left| \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{\left[1+5.4^2\right]^{3/2}}{3.6} \right|$$

$= 46$ m

Equation of path

$y = 0.2x^3$

$\therefore \quad \frac{dy}{dx} = 0.6x^2, \quad \frac{d^2y}{dx^2} = 1.2x$

At y = 5.4 m, we find x = 3 m

$\therefore \quad \frac{dy}{dx} = 5.4, \quad \frac{d^2y}{dx^2} = 3.6$

Normal acceleration = $a_n = \frac{v^2}{\rho} = \frac{5^2}{46} = 0.543 \text{ m/s}^2$

Tangential acceleration = $a_t = -0.8 \text{ m/s}^2$

$\therefore$ Total Acceleration = $a = \sqrt{a_n^2 + a_t^2} = 0.967 \text{ m/s}^2$ ..... **Ans.**

b) Components of velocity

Velocity is always tangent to the path and makes an angle θ with the x axis

here $\tan\theta = \frac{dy}{dx} = 5.4 \quad \therefore \quad \theta = 79.5°$

$\therefore \quad v_x = v\cos\theta = 5\cos 79.5 = 0.91 \text{ m/s}$ ..... **Ans.**

also $\quad v_y = v\sin\theta = 5\sin 79.5 = 4.916 \text{ m/s}$ ..... **Ans.**

c) $u = 5$ m/s, $v = 0$, $s = -$, $a_t = -0.8$ m/s², $t = ?$

using $\quad v = u + a_t.t$

$0 = 5 + (-0.8) \times t$

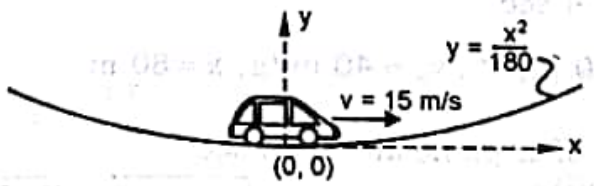
$\therefore \quad t = 6.25$ sec ..... time when particle comes to rest. ..... **Ans.**

P18. A car travels along a depression in a road, the equation of depression being $x^2 = 180y$. The speed of the car is constant and equal to 54 km/hr. Find the acceleration when the car is at the deepest point in the depression. What is the radius of curvature of the depression? ***(VJTI May 06)***

Solution: Motion of car – Curvilinear: N – T system – Uniform Speed

a) Given v = 54 km/hr = 15 m/s

Since speed is uniform, $a_t = 0$



Radius of curvature at the deepest point i.e (0, 0)

$$\rho = \left| \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{\left[1+0^2\right]^{3/2}}{\frac{1}{90}} \right|$$

$= 90$ m ..... **Ans.**

Equation of path $y = \frac{x^2}{180}$

$\frac{dy}{dx} = \frac{2x}{180} = \frac{x}{90}, \quad \frac{d^2y}{dx^2} = \frac{1}{90}$

At x = 0 m

$\frac{dy}{dx} = 0, \quad \frac{d^2y}{dx^2} = \frac{1}{90}$

Normal acceleration = $a_n = \frac{v^2}{\rho} = \frac{15^2}{90} = 2.5\ m/s^2$

Tangential acceleration = $a_t = 0$

$\therefore$ Total Acceleration = $a = \sqrt{a_n^2 + a_t^2} = 2.5\ m/s^2$ ..... **Ans.**

P19. An airplane travels along a path such that its acceleration is given by $\bar{a} = 10\,i + 6t\,j\ m/s^2$. If the plane starts from rest from the origin, determine at t = 4 sec. a) speed of the airplane b) radius of curvature of the path c) position of the airplane.

Solution: Motion of airplane – Curvilinear: x – y system

x direction

Given $a_x = 10\,m/s^2$ ..... (1)

At t = 0; x = 0, $v_x = 0$

Integrating using $a_x = \frac{dv_x}{dt}$

$\therefore \quad dv_x = 10\,dt$

$\therefore \quad \int_0^{v_x} dv_x = \int_0^t 10\,dt$

$\therefore \quad v_x = 10t\ m/s$ ..... (2)

Again integrating using $v_x = \frac{dx}{dt}$

$\therefore \quad dx = 10t\,dt$

$\therefore \quad \int_0^x dx = \int_0^t 10t\,dt$

$\therefore \quad x = 5t^2\ m$ ..... (3)

At t = 4 sec

$a_x = 10\ m/s^2,\ v_x = 40\ m/s,\ x = 80\ m$

y direction

Given $a_y = 6t\ m/s^2$ ..... (1)

At t = 0; y = 0, $v_y = 0$

Integrating using $a_y = \frac{dv_y}{dt}$

$\therefore \quad dv_y = 6t\,dt$

$\therefore \quad \int_0^{v_y} dv_y = \int_0^t 6t\,dt$

$\therefore \quad v_y = 3t^2\ m/s$ ..... (2)

Again integrating using $v_y = \frac{dy}{dt}$

$\therefore \quad dy = 3t^2\,dt$

$\therefore \quad \int_0^y dy = \int_0^t 3t^2\,dt$

$\therefore \quad y = t^3\ m$ ..... (3)

At t = 4 sec

$a_y = 24\ m/s^2,\ v_y = 48\ m/s,\ y = 64\ m$

Speed of airplane at t = 4 sec

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 48^2} = 62.48\ m/s$$ ..... **Ans.**

Radius of curvature = $\rho = \left|\frac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\frac{62.48^3}{10\times48 - 40\times24}\right| = 508.14\ m$ ..... **Ans.**

Position of airplane = $\bar{r} = 80\,i + 64\,j\ m$ $\therefore$ $r = \sqrt{80^2 + 64^2} = 102.45\ m$ ..... **Ans.**

P20. A rocket follows a path such that its acceleration is given by $\bar{a} = (4i + tj)\,m/s^2$. At $\bar{r} = 0$, it starts from rest. At t = 10 sec. determine-

(i) Speed of the rocket
(ii) Radius of curvature of its path
(iii) Magnitude of normal and tangential components of acceleration.
(iv) Position of the rocket.

(M.U. Dec 08)

Solution: Motion of rocket – Curvilinear: x – y system

<u>x direction</u>

Given $a_x = 4\,m/s^2$ ..... (1)

At t = 0; x = 0, $v_x = 0$

Integrating using $a_x = \dfrac{dv_x}{dt}$

$\therefore \quad dv_x = 4\,dt$

$\therefore \quad \int_0^{v_x} dv_x = \int_0^t 4\,dt$

$\therefore \quad v_x = 4t \;\; m/s$ ..... (2)

Again integrating using $v_x = \dfrac{dx}{dt}$

$\therefore \quad dx = 4t\,dt$

$\int_0^x dx = \int_0^t 4t\,dt$

$\therefore \quad x = 2t^2 \;\; m$ ..... (3)

At t = 10 sec

$a_x = 4\ m/s^2,\ v_x = 40\ m/s,\ x = 200\ m$

<u>y direction</u>

Given $a_y = t\ m/s^2$ ..... (1)

At t = 0; y = 0, $v_y = 0$

Integrating using $a_y = \dfrac{dv_y}{dt}$

$\therefore \quad dv_y = t\,dt \quad \therefore \quad \int_0^{v_y} dv_y = \int_0^t t\,dt$

$\therefore \quad v_y = \dfrac{t^2}{2} \;\; m/s$ ..... (2)

Again integrating using $v_y = \dfrac{dy}{dt}$

$\therefore \quad dy = \dfrac{t^2}{2}dt \quad \therefore \quad \int_0^y dy = \int_0^t \dfrac{t^2}{2}dt$

$\therefore \quad y = \dfrac{t^3}{6} \;\; m$ ..... (3)

At t = 10 sec

$a_y = 10\ m/s^2,\ v_y = 50\ m/s,\ y = 166.67\ m$

At t = 10 sec

Speed of rocket $v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 50^2} = 64.03\ m/s$ ..... **Ans.**

Radius of curvature = $\rho = \left|\dfrac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\dfrac{64.03^3}{4\times 50 - 40\times 10}\right| = 1312.56\ m$ ..... **Ans.**

Normal acceleration = $a_n = \dfrac{v^2}{\rho} = \dfrac{64.03^2}{1312.56} = 3.123\ m/s^2$ ..... **Ans.**

Tangential acceleration = $a_t = \sqrt{a^2 - a_n^2} = \sqrt{10.77^2 - 3.123^2} = 10.307\ m/s^2$ ..... **Ans.**

here $a = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 10^2} = 10.77\ m/s^2$

Position of rocket $\bar{r} = xi + yj = 200i + 166.67j \;\therefore\; r = \sqrt{200^2 + 166.67^2} = 260.34\ m$ ... **Ans**

P21. A particle moves in the x–y plane with acceleration components $a_x = -5\ m/s^2$ and $a_y = 2\ m/s^2$. If at t = 0, its velocity is 10 m/s directed at 36.87° with the + ve x-axis, find the radius of curvature at t = 8 sec and the corresponding normal and tangential components of acceleration.

Solution: Motion of rocket – Curvilinear: x – y system

<u>x direction</u>

Given $a_x = -5\,m/s^2$ ..... (1)

At t = 0; x = 0, $v_x = 10\cos 36.87 = 8\,m/s$

Integrating using $a_x = \dfrac{dv_x}{dt}$

$\therefore \quad dv_x = -5\,dt \quad \therefore \quad \int_8^{v_x} dv_x = \int_0^t -5\,dt$

$\therefore \quad v_x - 8 = -5t$

or $\quad v_x = -5t + 8$ ..... (2)

At t = 8 sec

$a_x = -5\ m/s^2,\ v_x = -32\ m/s$

<u>y direction</u>

Given $a_y = 2\,m/s^2$ ..... (1)

At t = 0; y = 0, $v_y = 10\sin 36.87 = 6\,m/s$

Integrating using $a_y = \dfrac{dv_y}{dt}$

$\therefore \quad dv_y = 2\,dt \quad \therefore \quad \int_6^{v_y} dv_y = \int_0^t 2\,dt$

$\therefore \quad v_y - 6 = 2t$

or $\quad v_y = 2t + 6$ ..... (2)

At t = 8 sec

$a_y = 2\ m/s^2,\ v_y = 22\ m/s$

At t = 8 sec,

Total velocity $v = \sqrt{v_x^2 + v_y^2} = \sqrt{32^2 + 22^2} = 38.83\ m/s$

Total acceleration = $a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 2^2} = 5.385\ m/s^2$

Radius of curvature = $\rho = \left|\dfrac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\dfrac{38.83^3}{-5\times 22 - (-32)\times 2}\right| = 1272.7\ m$ ..... **Ans.**

Normal acceleration = $a_n = \dfrac{v^2}{\rho} = \dfrac{38.83^2}{1272.7} = 1.185\ m/s^2$ ..... **Ans.**

Tangential acceleration = $a_t = \sqrt{a^2 - a_n^2} = \sqrt{5.385^2 - 1.185^2} = 5.253\ m/s^2$ ..... **Ans.**

P22. A particle is moving in x–y plane and its position is defined by $\bar{r} = \left(\frac{3}{2}t^2\right)i + \left(\frac{2}{3}t^3\right)j$. Find radius of curvature when t = 2 sec. ***(M.U. Dec 07)***

Solution: Motion of particle – Curvilinear: x – y system

x direction

Given $x = \frac{3}{2}t^2$ ..... (1)

Differentiating using $v_x = \frac{dx}{dt}$

$\therefore \quad v_x = 3t$ ..... (2)

Differentiating using $a_x = \frac{dv_x}{dt}$

$\therefore \quad a_x = 3 \ m/s^2$ ..... (3)

At t = 2 sec

$v_x = 6 \ m/s, \ a_x = 3 \ m/s^2$

y direction

Given $y = \frac{2}{3}t^3$ ..... (1)

Differentiating using $v_y = \frac{dy}{dt}$

$\therefore \quad v_y = 2t^2$ ..... (2)

Differentiating using $a_y = \frac{dv_y}{dt}$

$\therefore \quad a_y = 4t \ m/s^2$ ..... (3)

At t = 2 sec

$v_y = 8 \ m/s, \ a_y = 8 \ m/s^2$

At t = 2 sec,

Total velocity $v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 8^2} = 10 \ m/s$ ..... **Ans.**

Radius of curvature = $\rho = \left|\frac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\frac{10^3}{3\times8 - 6\times8}\right| = 41.67 \ m$ ..... **Ans.**

P23. a) A particle moves in x – y plane and its position is given by $\bar{r} = (6t)\ i + (3t - 4t^2)\ j$ m. Find the radius of curvature of its path and a_n and a_t components of acceleration when it crosses the x axis again.

Solution: Motion of particle – Curvilinear: x – y system

x direction

Given $x = 6t$ ..... (1)

Differentiating using $v_x = \frac{dx}{dt}$

$\therefore \quad v_x = 6 \ m/s$ ..... (2)

Differentiating using $a_x = \frac{dv_x}{dt}$

$\therefore \quad a_x = 0$ ..... (3)

y direction

Given $y = 3t - 4t^2$ ..... (1)

Differentiating using $v_y = \frac{dy}{dt}$

$\therefore \quad v_y = 3 - 8t \ m/s$ ..... (2)

Differentiating using $a_y = \frac{dv_y}{dt}$

$\therefore \quad a_y = -8 \ m/s^2$ ..... (3)

The particle crosses the x axis implies that y = 0 at that instant.

$\therefore \quad y = 3t - 4t^2 = 0 \quad$ or $\quad t = 0.75$ sec

At t = 0.75 sec

$v_x = 6 \ m/s, \ a_x = 0$ and $v_y = 3 - 8\times0.75 = -3 \ m/s, \ a_y = -8 \ m/s^2$

Total velocity $v = \sqrt{v_x^2 + v_y^2} = \sqrt{6^2 + 3^2} = 6.708 \text{ m/s}$

Total acceleration $a = \sqrt{a_x^2 + a_y^2} = \sqrt{0 + 8^2} = 8 \text{ m/s}^2$ ..... **Ans.**

Radius of curvature = $\rho = \left|\dfrac{v^3}{a_x.v_y - v_x.a_y}\right| = \left|\dfrac{6.708^3}{0 - 6\times(-8)}\right| = 6.289 \text{ m}$ ..... **Ans.**

Normal acceleration = $a_n = \dfrac{v^2}{\rho} = \dfrac{6.708^2}{6.289} = 7.155 \text{ m/s}^2$ ..... **Ans.**

Tangential acceleration = $a_t = \sqrt{a^2 - a_n^2} = \sqrt{8^2 - 7.155^2} = 3.579 \text{ m/s}^2$ ..... **Ans.**

P24. A particle moves along a track which has a parabolic shape with a constant speed of 10 m/s. The curve is given by $y = 5 + 0.3x^2$. Find the components of velocity and normal acceleration when x = 2 m. ***(M. U. Dec 14)***

Solution: Motion of particle – Curvilinear: N – T system – Uniform Speed

a) Since speed is uniform, $a_t = 0$

Radius of curvature of path at x = 2 m

$$\rho = \left|\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}\right| = \left|\frac{\left[1 + 1.2^2\right]^{3/2}}{0.6}\right|$$

$= 6.352$ m

Equation of path

$y = 5 + 0.3x^2$

$\dfrac{dy}{dx} = 0.6x, \quad \dfrac{d^2y}{dx^2} = 0.6$

At x = 2 m

$\dfrac{dy}{dx} = 1.2, \quad \dfrac{d^2y}{dx^2} = 0.6$

Normal acceleration = $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{6.352} = 15.74 \text{ m/s}^2$

b) Components of velocity

Velocity is always tangent to the path and makes an angle θ with the x axis

here $\tan\theta = \dfrac{dy}{dx} = 1.2$ or $\theta = 50.19°$

∴ $v_x = v\cos\theta = 10\cos 50.19 = 6.402 \text{ m/s}$ ..... **Ans.**

also $v_y = v\sin\theta = 10\sin 50.19 = 7.682 \text{ m/s}$ ..... **Ans.**

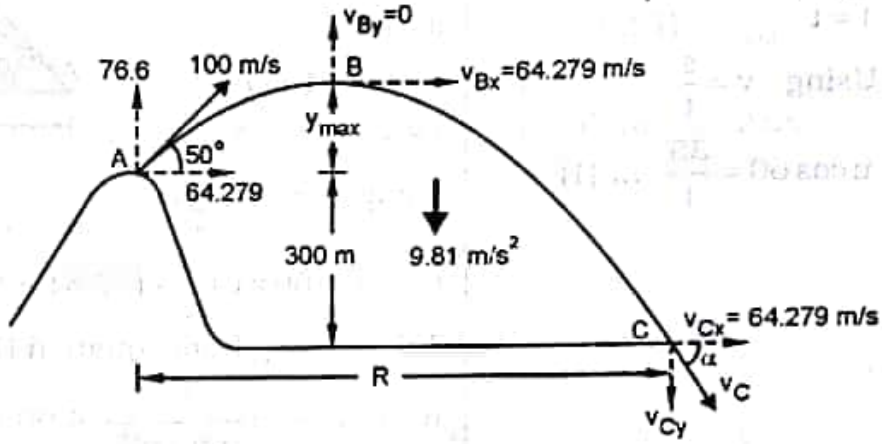
DJC

Exercise 9.4

Projectile Motion

P1. A gunman fires a bullet with a velocity of 100 m/s, 50° upwards from the top of a hill 300 m high to hit a bird. The bullet misses its target and finally lands on the ground. Calculate (a) the maximum height reached by the bullet above the ground (b) total time of flight (c) horizontal range of the bullet (d) velocity with which the bullet hits the ground. ***(M.U. May 14)***

Solution:



Projectile Motion (A – B)

Vertical Motion ↑ + ve

$u = 76.6\,m/s$

$v = 0$

$s = y_{max}$

$a = -9.81\ m/s^2$

$t = -$

using $v^2 = u^2 + 2as$

$$0 = 76.6^2 + 2\times(-9.81)\times y_{max} \quad \therefore \quad y_{max} = 299.1\ m$$

∴ The maximum height reached by the bullet above ground = 299.1 + 300 = 599.1 m ..... **Ans.**

Projectile Motion (A – C)

Horizontal Motion

$v = 64.279\,m/s$

$s = R$

$t = t$

using $v = \dfrac{s}{t}$

$$64.279 = \frac{R}{t} \quad ..... (1)$$

Substituting t = 18.8597 sec, we get

R = 1212.3 m ..... **Ans.**

Vertical Motion ↑ + ve

$u = 76.6\,m/s$

$v = -v_{cy}\ m/s$

$s = -300\ m$

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

$$-300 = 76.6t + \frac{1}{2}\times(-9.81)\times t^2$$

$\therefore \quad t = 18.8597$ sec

using $v = u + at$

$$-v_{cy} = 76.6 + (-9.81)\times 18.8597$$

$\therefore \quad v_{cy} = 108.4\ m/s$

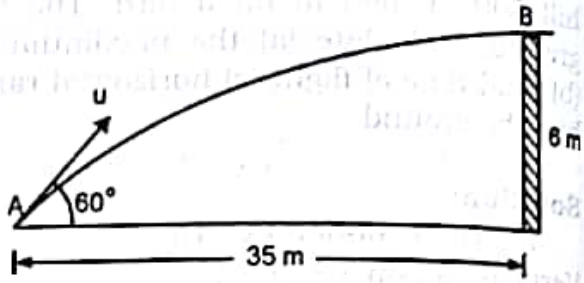
Landing velocity at C = $v_c = \sqrt{v_{cx}^2 + v_{cy}^2} = \sqrt{64.279^2 + 108.4^2} = 126\ m/s$

Also $\tan\alpha = \dfrac{v_{cy}}{v_{cx}} = \dfrac{108.4}{64.279} \quad \therefore \quad \alpha = 59.33°$ ↘

∴ Landing velocity $v_C = 126\ m/s$ at $\alpha = 59.33°$ ↘ ..... **Ans.**

P2. Ball is thrown from horizontal level, such that it clears a wall 6 m high, situated at a horizontal distance of 35 m. If the angle of projection is 60° with respect to the horizontal, what should be the minimum velocity of projection. ***(M.U. Dec 13)***

Solution: Let u be the minimum velocity with which the ball be projected, so as to clear the 6 m high wall. Projectile Motion (A – B)



Horizontal motion

$v = u\cos 60$

$s = 35\,m$

$t = t$

Using $v = \frac{s}{t}$

$u\cos 60 = \frac{35}{t}$ ... (1)

Vertical motion

$u = u\sin 60$

$v = -$

$s = 6\,m$

$a = -9.81\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

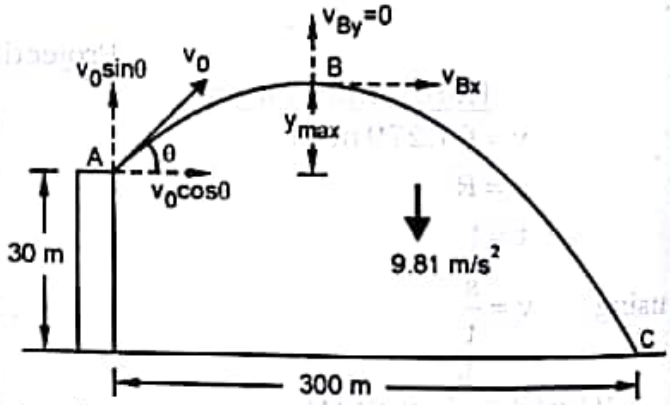
$6 = u\sin 60 \times t + \frac{1}{2}\times(-9.81)\times t^2$ ..... (2)

Substituting from equation (1) in (2) we get

$$6 = u\sin 60 \times \frac{35}{u\cos 60} - 4.905 \times \left(\frac{35}{u\cos 60}\right)^2$$

on solving we get u = 20.977 m/s ..... **Ans.**

P3. A ball is projected upwards from the top of a tower 30 m high and strikes the ground after 8 sec at a point 300 m from the foot of the tower. Determine the velocity of projection and also the maximum height attained by the ball above the ground.



Solution: Let the ball be projected with an initial velocity v_0 from point A as shown.

Projectile Motion (A – C)

Horizontal Motion

$v = v_0\cos\theta$

$s = 300\ m$

$t = 8\ sec$

using $v = \frac{s}{t}$

$v_0\cos\theta = \frac{300}{8}$

or $v_0\cos\theta = 37.5$ ..... (1)

Vertical Motion ↑ + ve

$u = v_0\sin\theta$

$v = -$

$s = -30\,m$

$a = -9.81\ m/s^2$

$t = 8\,sec$

using $s = ut + \frac{1}{2}at^2$

$-30 = v_0\sin\theta \times 8 + \frac{1}{2}\times(-9.81)\times 8^2$

$\therefore\ v_0\sin\theta = 35.49$ ..... (2)

∴ Solving equations (1) and (2) $v_0 = 51.63\ m/s$ and $\theta = 43.42°\ \measuredangle$ ... **Ans.**

Projectile Motion (A – B)

Horizontal Motion

Vertical Motion ↑ + ve

$u = v_0 \sin\theta = 35.49 \text{ m/s}$

$v = 0$

$s = y_{max}$

$a = -9.81 \text{ m/s}^2$

$t = -$

using $v^2 = u^2 + 2as$

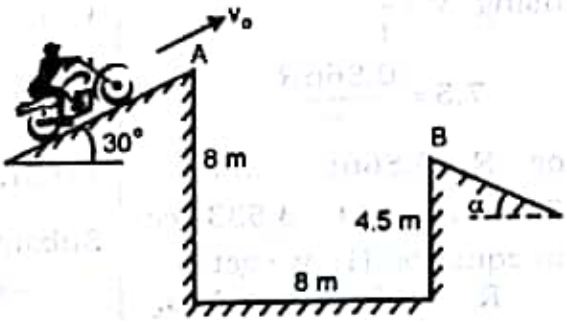
$0 = 35.49^2 + 2\times(-9.81)\times y_{max}$

$\therefore \quad y_{max} = 64.197 \text{ m}$

$\therefore$ The maximum height attained above ground = 64. 197 + 30 = 94.197 m ..... **Ans.**

P4. A stunt motorcyclist has to clear a ditch. Find the minimum speed it should have at A to clear the ditch. Also find his speed at B and the angle θ of ramp which should be provided.

(VJTI May 09)



Solution: Projectile Motion (A – B)

Horizontal Motion

$v = v_0 \cos 30$

$s = 8 \text{ m}$

$t = t$

using $\quad v = \dfrac{s}{t}$

$v_0 \cos 30 = \dfrac{8}{t}$

or $\quad v_0 = \dfrac{8}{t\times\cos 30}$ ... (1)

Substituting $t = 1.286$ sec in equation (1)

$v_0 = 7.18 \text{ m/s}$ ... **Ans.**

Vertical Motion ↑ + ve

$u = v_0 \sin 30$

$v = -v_{By}$

$s = -3.5 \text{ m}$

$a = -9.81 \text{ m/s}^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$-3.5 = v_0 \sin 30\times t + \dfrac{1}{2}\times(-9.81)\times t^2$ ..... (2)

Substituting equation (1) in (2)

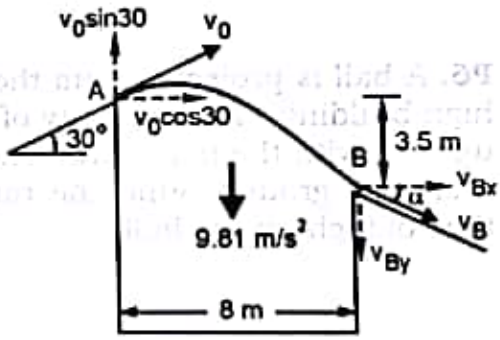
$-3.5 = \dfrac{8}{t\cos 30}\times\sin 30\times t - 4.905t^2$

$\therefore \quad t = 1.286$ sec

using $v = u + at$

$-v_{By} = 7.18\sin 30 + (-9.81)\times 1.286$

$\therefore \quad v_{By} = 9.02 \text{ m/s}$

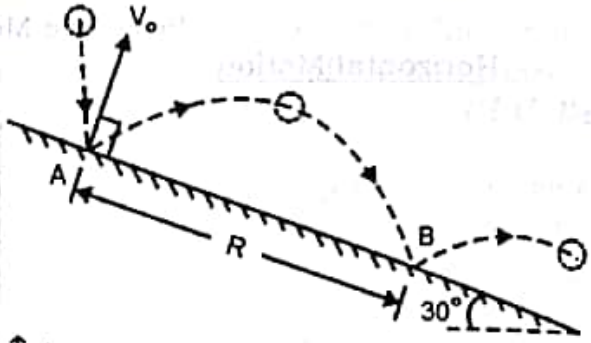


Landing velocity at B = $v_B = \sqrt{v_{Bx}^2 + v_{By}^2} = \sqrt{(7.18\cos 30)^2 + 9.02^2} = 10.96 \text{ m/s}$

Also $\quad \tan\alpha = \dfrac{v_{By}}{v_{Bx}} = \dfrac{9.02}{6.218} \quad \therefore \quad \alpha = 55.42°$ ↘

$\therefore$ Landing velocity $v_B = 10.96 \text{ m/s}, \alpha = 55.42°$ ↘ ..... **Ans.**

P5. A ball dropped vertically on an inclined plane at A rebounds with a velocity v_o perpendicular to the plane and lands again on the plane at B. Knowing v_o = 15 m/s, determine the range R.



Solution: Projectile Motion (A – B)

Horizontal Motion

$v = 7.5$ m/s

$s = 0.866$ R

$t = t$

using $v = \frac{s}{t}$

$7.5 = \frac{0.866R}{t}$

or $R = 0.866t$ ... (1)

Substituting $t = 3.533$ sec in equation (1), we get

R = 30.6 m ..... **Ans.**

Vertical Motion ↑ + ve

$u = 13$ m/s

$v = -$

$s = -0.5R$

$a = -9.81$ m/s^2

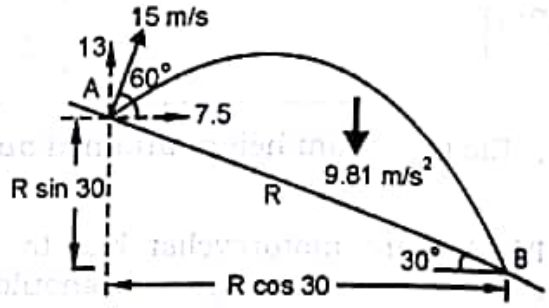
$t = t$

using $s = ut + \frac{1}{2}at^2$

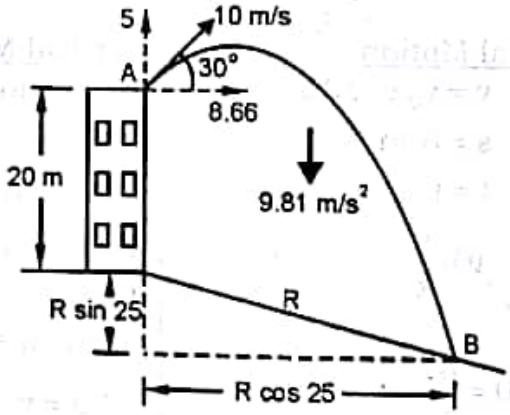
$-0.5R = 13t + \frac{1}{2}\times(-9.81)\times t^2$ ..... (2)

Substituting equation (1) in (2)

$-0.5\times0.866t = 13t - 4.905t^2$ ∴ $t = 3.533$ sec



P6. A ball is projected from the top of a 20 m high building with a velocity of 10 m/s at 30° upward with the horizontal. The ball lands on a sloping ground. Find the range R and the time of flight of the ball.



Solution: Projectile Motion (A – B)

Horizontal Motion

$v = 10\cos 30 = 8.66$ m/s

$s = R\cos 65$

$t = t$

using $v = \frac{s}{t}$

$8.66 = \frac{R\cos 25}{t}$

or $R = 9.555t$ ..... (1)

Substituting $t = 3.14$ sec obtained from vertical motion, we get

R = 30 m ..... **Ans.**

Vertical Motion ↑ + ve

$u = 10\sin 30 = 5$ m/s

$v = -$

$s = -(20 + R\sin 25)$

$a = -9.81$ m/s^2

$t = t$

using $s = ut + ½ at^2$

$-(20 + R\sin 25) = 5t + \frac{1}{2}\times(-9.81)\times t^2$ ... (2)

Substituting equation (1) in (2)

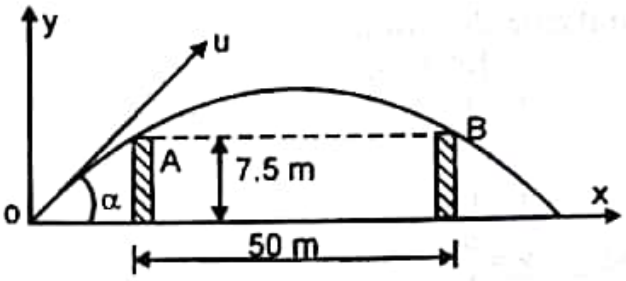
$-(20 + 9.555t\sin 25) = 5t - 4.905t^2$

$-4.905t^2 + 9.038t + 20 = 0$

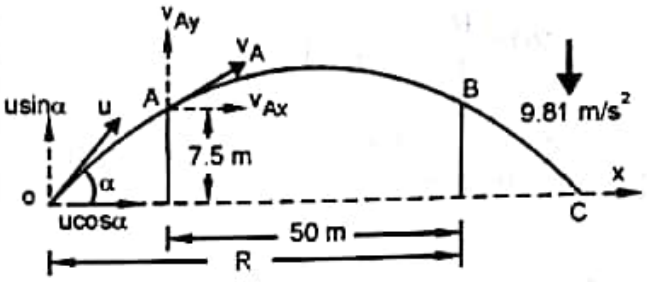
∴ $t = 3.14$ sec ..... **Ans.**

P7. An object is projected so that it just clears two obstacles each 7.5 m high which are situated 50 m from each other. If the time of passing between two obstacles is 2.5 seconds, calculate the complete range of projection and initial velocity of projection.

(M.U May 08, Dec 15, VJTI Dec 13)



Solution: Let us first work with projectile motion from A to B. Let v_A be the velocity of object at A.



Projectile Motion (A – B)

Horizontal Motion

$v = v_{Ax}$

$s = 50$ m

$t = 2.5$ sec

using $v = \dfrac{s}{t}$

$$v_{Ax} = \frac{50}{2.5} = 20 \text{ m/s}$$

Vertical Motion ↑ + ve

$u = v_{Ay}$

$v = -$

$s = 0$ ∵ vertical level difference between A and B is zero

$a = -9.81 \text{ m/s}^2$

$t = 2.5$ sec

using $s = ut + \dfrac{1}{2}at^2$

$$0 = v_{Ay} \times 2.5 + \frac{1}{2} \times (-9.81) \times 2.5^2$$

$\therefore \quad v_{Ay} = 12.26$ m/s

Let us now work with projectile motion between O and A

Projectile Motion (O – A)

Horizontal Motion

Vertical Motion ↑ + ve

$u = u\sin\alpha$

$v = v_{Ay} = 12.26$ m/s

$s = 7.5$ m

$a = -9.81 \text{ m/s}^2$

$t = -$

using $v^2 = u^2 + 2as$

$$12.26^2 = (u\sin\alpha)^2 + 2 \times (-9.81) \times 7.5$$

$\therefore \quad u\sin\alpha = 17.249$ ..... (1)

Since horizontal component of velocity remains constant

$u\cos\alpha = v_{Ax} = 20$ m/s ..... (2)

Solving equations (1) and (2), we get $u = 26.41$ m/s and $\alpha = 40.78°$ ∠ ..... **Ans.**

To find complete range R, let us work with projectile motion between O to C.

Horizontal Motion

$v = u\cos\alpha = 20$ m/s

$s = R$

$t = t$

using $v = \frac{s}{t}$

$20 = \frac{R}{t}$ ..... (3)

Substituting t = 3.517 sec obtained from vertical motion, we get

R = 70.33 m ..... **Ans.**

Vertical Motion ↑ + ve

$u = u\sin\alpha = 17.249$ m/s

$v = -$

$s = 0$

$a = -9.81$ m/s^2

$t = t$

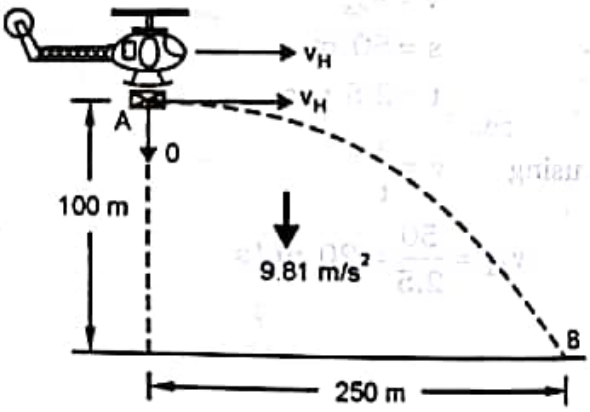
using $s = ut + \frac{1}{2}at^2$

$0 = 17.249 \times t + \frac{1}{2} \times (-9.81) \times t^2$

$\therefore$ $t = 3.517$ sec

P8. A helicopter moving horizontally at an altitude of 100 m drops a packet. The packet landed on the ground 250 m away from the point on the ground directly below the point of drop. What is the velocity of the helicopter?

Solution: Let v_H bet the velocity of the helicopter. As the packet is dropped from the helicopter, it acquires a horizontal velocity v_H of the helicopter. The vertical component of initial velocity is zero as shown.



Projectile Motion (A – B)

Horizontal Motion

$v = v_H$

$s = 250$ m

$t = t$

using $v = \frac{s}{t}$

$v_H = \frac{250}{t}$ ..... (1)

Substituting t = 4.515 sec obtained from vertical motion, we get

$v_H = 55.37$ m/s ..... **Ans.**

Vertical Motion ↓ + ve

$u = 0$

$v = -$

$s = 100$ m

$a = +9.81$ m/s^2

$t = t$

Note: We have taken ↓ sense as positive

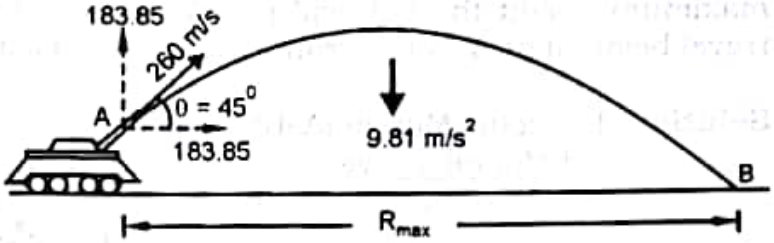
using $s = ut + \frac{1}{2}at^2$

$100 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$\therefore$ $t = 4.515$ sec

P9. An artillery gun can fire a shell with a velocity of 260 m/s. How much maximum it can penetrate into the enemy's territory as it targets the enemy from the border?

Solution: Knowing for maximum horizontal range, the angle of projection θ should be 45°.



Projectile Motion (A – B)

Horizontal Motion

$v = 183.85$ m/s

$s = R_{max}$

$t = t$

using $v = \frac{s}{t}$

$183.85 = \frac{R_{max}}{t}$ ..... (1)

Substituting t = 37.48 sec obtained from vertical motion, we get

$R_{max} = 6891$ m ..... **Ans.**

Vertical Motion ↑ + ve

$u = 183.85$ m/s

$v = -$

$s = 0$ ∵ vertical level difference between A and B is zero

$a = -9.81$ m/s²

$t = t$

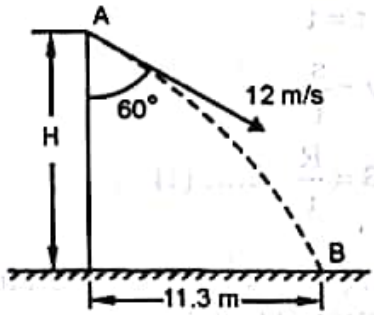
using $s = ut + \frac{1}{2}at^2$

$0 = 183.85 \times t + \frac{1}{2} \times (-9.81) \times t^2$

∴ $t = 37.48$ sec

P10. A ball thrown with speed of 12 m/s at an angle of 60° with a building strikes the ground 11.3 m horizontally from the foot of the building as shown. Determine the height of the building.

(M.U Dec 12)



Solution:

Projectile Motion (A – B)

Horizontal Motion

$v = 12 \sin 60 = 10.39$ m/s

$s = 11.3$ m

$t = t$

using $v = \frac{s}{t}$

$10.39 = \frac{11.3}{t}$

∴ $t = 1.087$ sec

Vertical Motion ↓ + ve

$u = 12 \cos 60 = 6$ m/s

$v = -$

$s = H$

$a = +9.81$ m/s²

Note: We have taken ↓ sense as positive

$t = 1.087$ sec ..... obtained from H.M.

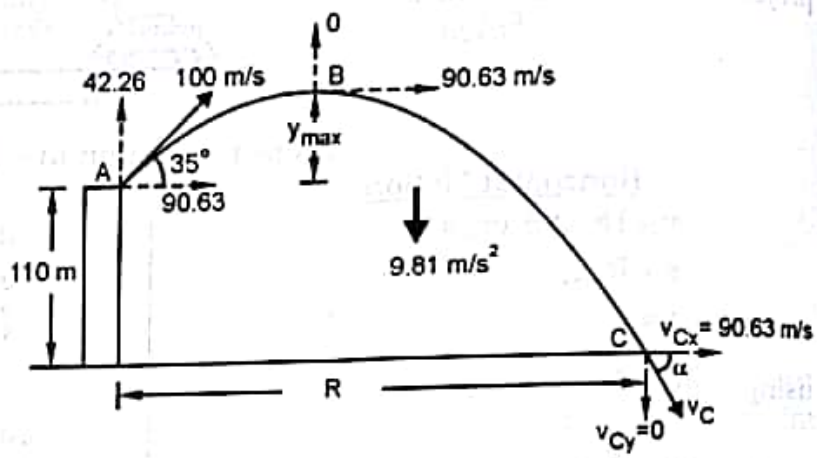
using $s = ut + \frac{1}{2}at^2$

$H = 6 \times 1.087 + \frac{1}{2} \times 9.81 \times 1.087^2$

∴ $H = 12.32$ m ..... **Ans.**

P11. A ball is projected from the top of a tower 110 m height with the velocity of 100 m/s at an angle of 25° to the horizontal. Neglecting the air resistance, find (1) The maximum height the ball will rise from the ground. (2) The horizontal distance it will travel before it strikes the ground. (3) The velocity with which it strikes the ground. ***(M.U May 11)***

Solution: Projectile Motion (A–B)



Vertical Motion ↑ + ve

$u = 42.26$ m/s

$v = 0$

$s = y_{max}$

$a = -9.81$ m/s²

$t = -$

using $v^2 = u^2 + 2as$

$0 = 42.26^2 + 2\times(-9.81)\times y_{max}$

$\therefore \quad y_{max} = 91.02$ m ..... **Ans.**

$\therefore$ Max. Height reached above ground = 110 + 91.02 = 201.02 m

Projectile Motion (A – C)

Horizontal Motion

$v = 90.63$ m/s

$s = R$

$t = t$

using $v = \dfrac{s}{t}$

$90.63 = \dfrac{R}{t}$ ..... (1)

Substituting t = 10.71 sec obtained from vertical motion, we get

$\therefore \quad R = 970.65$ m ..... **Ans.**

Vertical Motion ↑ + ve

$u = 42.26$ m/s

$v = -v_{Cy}$

$s = -110$ m

$a = -9.81$ m/s²

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

$-110 = 42.26t - 4.905t^2$

$\therefore \quad t = 10.71$ sec

using $v = u + at$

$-v_{Cy} = 42.26 + (-9.81)\times 10.71$

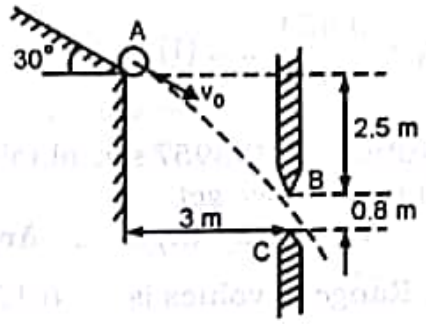
$\therefore \quad v_{Cy} = 62.8$ m/s ..... **Ans.**

Landing velocity at C = $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = \sqrt{90.63^2 + 62.8^2} = 110.26$ m/s

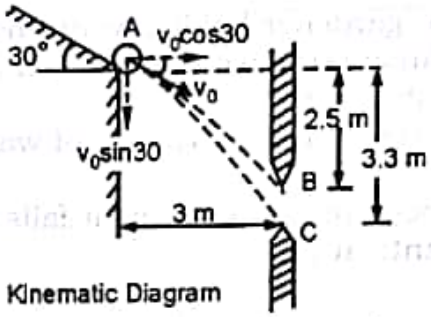
Also $\tan\alpha = \dfrac{v_{Cy}}{v_{Cx}} = \dfrac{62.8}{90.63} \quad \therefore \quad \alpha = 34.72°$ ↘

$\therefore$ Landing velocity $v_C = 110.26$ m/s, $\alpha = 34.72°$ ↘ ..... **Ans.**

P12. A ball bearing slides down a 30° plane and leaves the plane with a certain velocity v_0. Find for what range of value of v_0, the ball bearing can pass through a 0.8 m wide slot.



Solution: Range of values of v_0 implies minimum and maximum values of v_0. If v_0 is minimum the ball bearings will pass through the lower edge of the slot. If v_0 is maximum, the ball bearings will pass through the upper edge of the slot.



Kinematic Diagram

<u>To find minimum v_0.</u>

Projectile Motion (A – C)

<u>Horizontal Motion</u>

$v = v_0 \cos 30$

$s = 3\,m$

$t = t$

using $v = \dfrac{s}{t}$

$$v_0 \cos 30 = \frac{3}{t}$$

or $$v_0 = \frac{3.464}{t} \quad \text{..... (1)}$$

<u>Vertical Motion ↓ + ve</u>

$u = v_0 \sin 30$

$v = -$

$s = 3.3\ m$

$a = 9.81\ m/s^2$

$t = t$

Note: We have taken ↓ sense as positive

using $s = ut + \dfrac{1}{2}at^2$

$$3.3 = v_0 \sin 30 \times t + \frac{1}{2} \times 9.81 \times t^2 \quad \text{..... (2)}$$

Substituting from equation (1)

$$3.3 = \frac{3.464}{t} \times \sin 30 \times t + 4.905 \times t^2$$

$\therefore \quad t = 0.5654$ sec

Substituting t = 0.5654 sec obtained from vertical motion, we get

$\therefore \quad v_0 = 6.127\ m/s$ ..... **Ans.**

<u>To find maximum v_0.</u>

Projectile Motion (A – B)

<u>Horizontal Motion</u>

$v = v_0 \cos 30$

$s = 3\,m$

$t = t$

using $v = \dfrac{s}{t}$

$$v_0 \cos 30 = \frac{3}{t}$$

<u>Vertical Motion ↓ + ve</u>

$u = v_0 \sin 30$

$v = -$

$s = 2.5\ m$

$a = 9.81\ m/s^2$

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

or $v_0 = \frac{3.464}{t}$ ..... (1)

Substituting $t = 0.3957$ sec obtained from vertical motion, we get

$\therefore$ $v_0 = 8.754$ m/s ..... **Ans.**

$$2.5 = v_0 \sin 30 \times t + \frac{1}{2} \times 9.81 \times t^2$$

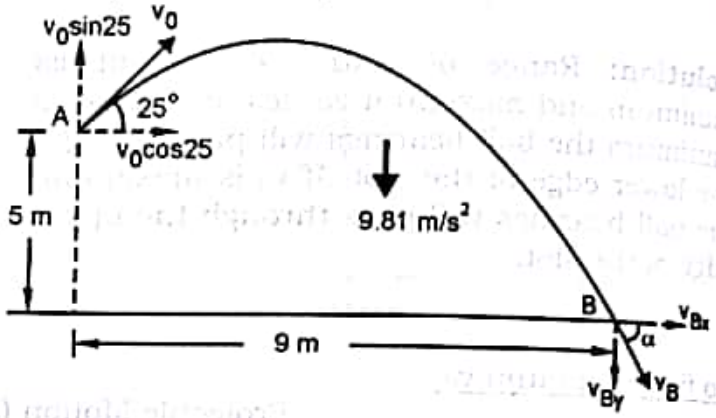
Substituting from equation (1)

$$2.5 = \frac{3.464}{t} \times \sin 30 \times t + 4.905 \times t^2$$

$\therefore$ $t = 0.3957$ sec

$\therefore$ Range of values is $6.127 \text{ m/s} \le v_0 \le 8.754 \text{ m/s}$ ..... **Ans.**

P13. A gardener holds a water hose at A causing the water to fall on the plants at B. find
(a) the discharge velocity v_o of water at A
(b) velocity of the water as it falls on the plants at B.



Solution: Projectile Motion (A – B)

Horizontal Motion

$v = v_0 \cos 25$

$s = 9$ m

$t = t$

using $v = \frac{s}{t}$

$$v_0 \cos 25 = \frac{9}{t}$$

Or $v_0 = \frac{9.93}{t}$ ..... (1)

Substituting t = 1.369 sec obtained from vertical motion, we get

$\therefore$ $v_0 = 7.252$ m/s ..... **Ans.**

Vertical Motion ↑ + ve

$u = v_0 \sin 25$

$v = -v_{By}$

$s = -5$ m

$a = -9.81$ m/s²

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$-5 = v_0 \sin 25 \times t + \frac{1}{2} \times (-9.81) \times t^2$$

Substituting from equation (1)

$$-5 = \frac{9.93}{t} \sin 25 \times t - 4.905 \times t^2$$

$\therefore$ $t = 1.369$ sec

using $v = u + at$

$$-v_{By} = 7.252 \sin 25 + (-9.81) \times 1.369$$

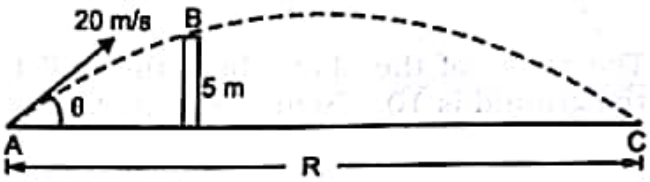
$\therefore$ $v_{By} = 10.365$ m/s

Landing velocity at B = $v_B = \sqrt{v_{Bx}^2 + v_{By}^2} = \sqrt{(7.252 \cos 25)^2 + (10.365)^2} = 12.27$ m/s

Also $\tan \alpha = \frac{v_{By}}{v_{Bx}} = \frac{10.365}{6.572}$ $\therefore$ $\alpha = 57.62°$ ↘ ..... **Ans.**

P14. A shot is fired with a bullet with an initial velocity of 20 m/s from a point 10 m infront of a vertical wall 5 m high. Find the angle of projection with the horizontal to enable the shot to just clear the wall. Also find the range of shot where the bullet falls on the ground. ***(M.U Dec 16)***

Solution: Projectile Motion (A – B)



Horizontal motion

$v = 20\cos\theta$

$s = 10$ m

$t = t$

using $v = \frac{s}{t}$

$20\cos\theta = \frac{10}{t}$

$\therefore\ t = \frac{10}{20\cos\theta}$ ... (1)

Vertical motion ↑ +ve

$u = 20\sin\theta$

$v = -$

$s = 5$ m

$a = -9.81\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

$5 = (20\sin\theta)\,t + \frac{1}{2}(-9.81)\,t^2$ ...(2)

Substituting equation (1) in (2)

$$\therefore 5 = 20\sin\theta \times \frac{10}{20\cos\theta} - 4.905 \times \left(\frac{10}{20\cos\theta}\right)^2$$

$$\therefore 5 = 10\tan\theta - 1.226 \times \sec^2\theta$$

$$\therefore 5 = 10\tan\theta - 1.226\left(1 + \tan^2\theta\right)$$

$$\therefore 1.226\tan^2\theta - 10\tan\theta - 6.226 = 0$$

$$\therefore \tan\theta = 7.477 \quad \text{or} \quad \tan\theta = 0.679$$

$$\therefore\ \theta = 82.35° \quad \text{or} \quad \theta = 34.18°$$

Projectile Motion (A – C) for $\theta = 82.35°$

Horizontal motion

$v = 20\cos 82.35$

$s = R$

$t = t$

using $v = \frac{s}{t}$

$20\cos 82.35 = \frac{R}{t}$

$\therefore\ 20\cos 82.35 = \frac{R}{4.041}$ $\quad\therefore\ R = 10.759$ m

Vertical motion ↑ +ve

$u = 20\sin 82.35$

$v = -$

$s = 0$

$a = -9.81\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

$0 = (20\sin 82.35)\,t + \frac{1}{2} \times (-9.81) \times t^2$

$\therefore\ t = 4.041$ sec

Projectile Motion (A – C) for $\theta = 34.18°$

Horizontal motion

$v = 20\cos 34.18$ m/s

$s = R$

$t = t$

using $v = \frac{s}{t}$

Vertical motion ↑ +ve

$u = 20\sin 34.18$ m/s

$v = -$

$s = 0$

$a = -9.81\ m/s^2$

$t = t$

$$20\cos 34.18 = \frac{R}{t}$$

$$\therefore\ 20\cos 34.18 = \frac{R}{2.2907}$$

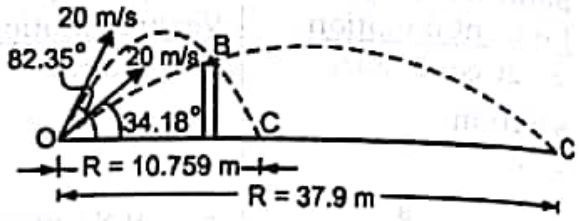
$$\therefore\ R = 37.9 \text{ m}$$

Using $s = ut + \frac{1}{2}at^2$

$$0 = (20\sin 34.18)t + \frac{1}{2}\times(-9.81)\times t^2$$

$$\therefore\ t = 2.2907 \text{ sec}$$

The range of the shot where the bullet falls on the ground is $10.759 \text{ m} \le R \le 37.9 \text{ m}$ ... **Ans.**



P15. A fighter plane flying horizontally at 324 kmph at an altitude of 3500 m plans to strike an enemy train moving with a constant velocity of 90 km/hr. Find the distance the plane should be behind the train just as it drops the bomb.

Solution: In this problem, the target i.e train is moving. We will first find the relative velocity of the plane w.r.to train

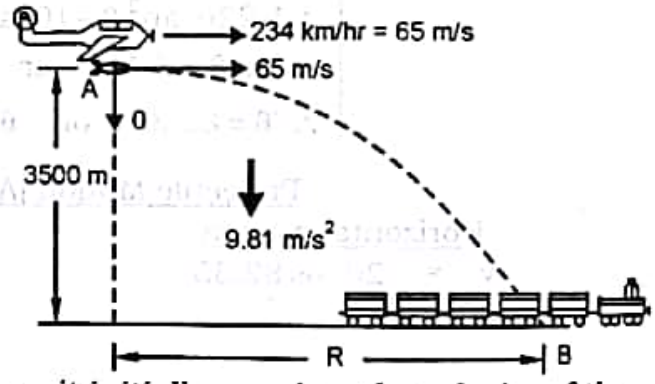
Velocity of plane = 324 kmh →

Velocity of train = 90 kmph →

∴ Velocity of plane w.r.t. train

= 324 − 90 = 234 kmph = 65 m/s →

Now we may take the velocity of train to be 65 m/s and the train at rest.



As the bomb is dropped from the moving plane, it initially acquires the velocity of the plane. Let R be the distance, the plane needs to be behind the train to hit the target.

Projectile Motion (A – B)

Horizontal Motion

$v = 65$ m/s

$s = R$

$t = t$

using $v = \frac{s}{t}$

$$65 = \frac{R}{t} \quad \text{..... (1)}$$

Substituting $t = 26.71$ sec obtained from vertical motion, we get

$\therefore\ R = 1736.2\text{m}$ ..... **Ans.**

Vertical Motion ↓ + ve

$u = 0$

$v = -$

$s = 3500$ m

$a = 9.81$ m/s²

$t = t$

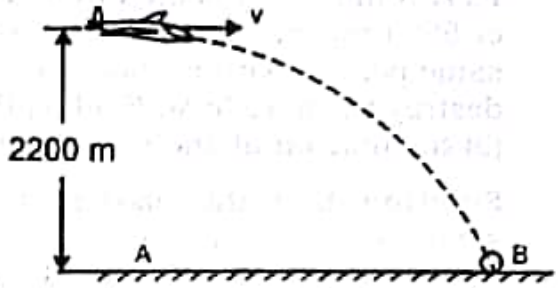
Note we have taken ↓ sense as + ve

using $s = ut + \frac{1}{2}at^2$

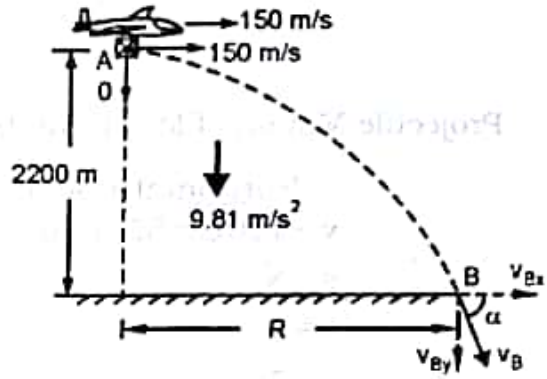
$$3500 = 0 + \frac{1}{2}\times 9.81\times t^2$$

$\therefore\ t = 26.71$ sec

P16. An aero plane is flying in horizontal direction with a velocity of 540 km/hr and at height of 2200m. When it is vertically above the point A on the ground, an object is dropped from it. The object strikes the ground at point B. Calculate the distance AB (ignore air resistance). Also find velocity at B and time taken to reach B. ***(M.U Dec 10)***



Solution: Let R be the distance AB. As the object leaves the plane, it initially acquires the velocity of the plane which is horizontal 540 kmph = 150 m/s. The initial vertical component of velocity is zero.



Projectile Motion (A – B)

Horizontal Motion

$v = 150\ m/s$

$s = R$

$t = t$

using $v = \dfrac{s}{t}$

$150 = \dfrac{R}{t}$ ..... (1)

Substituting $t = 21.178$ sec obtained from vertical motion, we get

$\therefore$ $R = 3176.7\,m$ ..... **Ans.**

Vertical Motion ↓ + ve

$u = 0$

$v = v_{By}$

$s = 2200\ m$

$a = +9.81\ m/s^2$

$t = t$

Note we have taken ↓ sense as + ve

using $s = ut + \dfrac{1}{2}at^2$

$2200 = 0 + \dfrac{1}{2} \times 9.81 \times t^2$

$\therefore$ $t = 21.178$ sec ..... **Ans**

using $v = u + at$

$v_{By} = 0 + 9.81 \times 21.178$

$\therefore$ $v_{By} = 207.76\ m/s$

Landing velocity at B

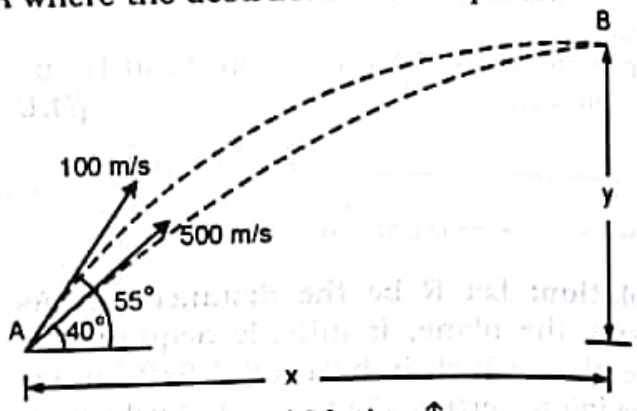
$v_{Bx} = 150\,m/s \rightarrow,\ v_{By} = 207.76\ m/s \downarrow$

$\therefore$ $v_B = \sqrt{{v_{Bx}}^2 + {v_{By}}^2} = \sqrt{150^2 + 207.76^2} = 256.2\ m/s$ ..... **Ans.**

Also $\tan\alpha = \dfrac{v_{By}}{v_{Bx}} = \dfrac{207.76}{150}$ $\therefore$ $\alpha = 54.17°$ ↘ ..... **Ans.**

P17. A missile M is fired from a certain position A with a velocity of 100 m/s at an angle of 55° upwards with the horizontal. After some time an antimissile N is fired from the same point A with a velocity of 500 m/s at an angle of 40° upwards to the horizontal to destroy the missile M. Find (a) the time of flight of missile M before it is destroyed. (b) the horizontal and vertical distance from A where the destruction takes place.

Solution: Both the missiles start from the same point A and meet at B. Let x and y be the location of point B from A. Let missile M take t_1 sec and missile N take t_2 sec to meet at B.



Projectile Motion of Missile M (A – B)

Horizontal Motion

$v = 100\cos 55$ m/s

$s = x$

$t = t_1$

using $v = \dfrac{s}{t}$

$100\cos 55 = \dfrac{x}{t_1} \quad \therefore \quad t_1 = \dfrac{x}{100\cos 55}$ ... (1)

Vertical Motion ↑ + ve

$u = 100\sin 55$ m/s

$v = -$

$s = y$

$a = -9.81$ m/s^2

$t = t_1$

using $s = ut + \dfrac{1}{2}at^2$

$y = 100\sin 55 \times t_1 + \dfrac{1}{2}\times(-9.81)\times t_1^2$ ..... (2)

Substituting equation (1) in (2), we get

$$y = 100\sin 55 \times \frac{x}{100\cos 55} - 4.905 \times \left(\frac{x}{100\cos 55}\right)^2$$

$$\therefore \quad y = 1.428x - 1.491\times 10^{-3}x^2 \quad \text{..... (A)}$$

Projectile Motion of Anti-missile N (A – B)

Horizontal Motion

$v = 500\cos 40$ m/s

$s = x$

$t = t_2$

using $v = \dfrac{s}{t}$

$500\cos 40 = \dfrac{x}{t_2}$

or $t_2 = \dfrac{x}{500\cos 40}$ ..... (3)

Vertical Motion ↑ + ve

$u = 500\sin 40$ m/s

$v = -$

$s = y$

$a = -9.81$ m/s^2

$t = t_2$

using $s = ut + \dfrac{1}{2}at^2$

$y = 500\sin 40 \times t_2 + \dfrac{1}{2}\times(-9.81)\times t_2^2$ ..... (4)

Substituting equation (3) in (4), we get

$$y = 500\sin 40 \times \frac{x}{500\cos 40} - 4.905 \times \left(\frac{x}{500\cos 40}\right)^2$$

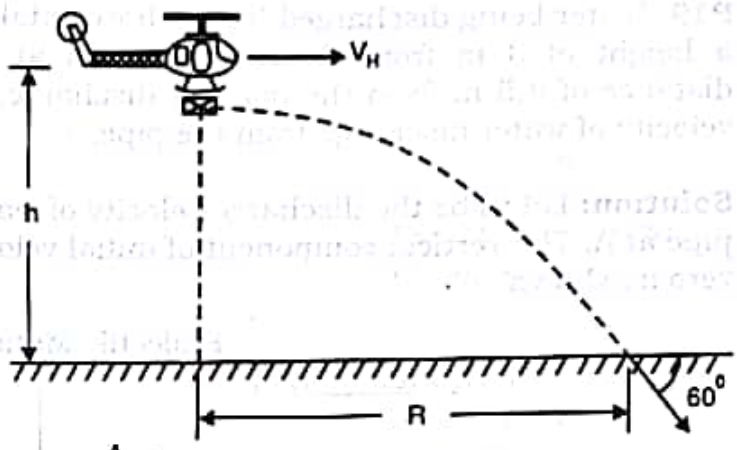
$\therefore \quad y = 0.8391x - 3.343\times10^{-5}x^2$ ..... (B)

Equating equations (A) and (B) and solving, we get x = 404 m and y = 333.56 m **Ans.**

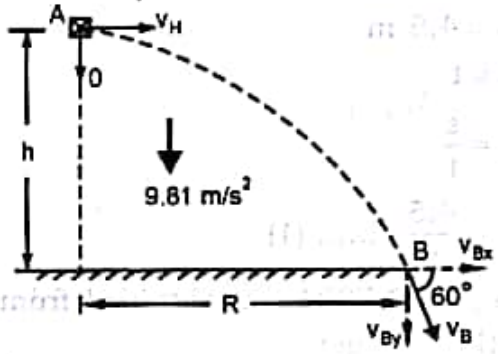
Substituting x = 404 in equation (1) we get t_1 = 7.043 sec ... time to destroy missile M. **... Ans.**

P18. A box dropped from a helicopter moving horizontally with a constant velocity v_H, takes 12 sec to reach the ground and strikes the ground at an angle of 60°. Determine

a) the altitude h of the helicopter
b) the horizontal distance R traveled by the box
c) the velocity v_H of the helicopter.



Solution: As the box leaves the helicopter its initial velocity is equal to the velocity of helicopter. The vertical component of initial velocity at A is zero.



Projectile Motion (A – B)

Horizontal Motion

$v = v_H$

$s = R$

$t = 12$ sec

using $v = \frac{s}{t}$

$v_H = \frac{R}{12}$ ..... (1)

Vertical Motion ↓ + ve

Note we have taken ↓ sense as + ve

$u = 0$

$v = v_{By}$

$s = h$

$a = 9.81\ m/s^2$

$t = 12$ sec

using $s = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2}\times 9.81 \times 12^2$

$\therefore \quad h = 706.32$ m ..... **Ans.**

using $v = u + at$

$v_{By} = 0 + 9.81 \times 12$

$\therefore \quad v_{By} = 117.72$ m/s

Knowing landing velocity angle α is given by

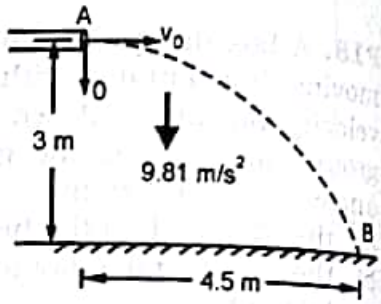
$$\tan\alpha = \frac{v_{By}}{v_{Bx}} \quad \therefore \quad \tan 60 = \frac{117.72}{v_{Bx}} \quad \therefore \quad v_{Bx} = 67.96 \text{ m/s}$$

Knowing horizontal component of velocity does not change, we have

$v_H = v_{Bx} = 67.96$ m/s ..... **Ans.**

Substituting $v_H = 67.96$ m/s in equation (1), we get R = 815.59 m ..... **Ans.**

P19. Water being discharged from a horizontal pipe fixed at a height of 3 m from the ground, falls at a horizontal distance of 4.5 m from the point of discharge. What is the velocity of water discharge from the pipe.



Solution: Let v_0 be the discharge velocity of water from the pipe at A. The vertical component of initial velocity at A is zero as shown.

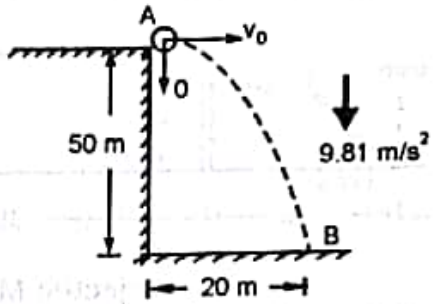
Projectile Motion (A – B)

Horizontal Motion	Vertical Motion ↓ + ve
$v = v_0$	$u = 0$
$s = 4.5$ m	$v = -$
$t = t$	$s = 3$ m
using $v = \frac{s}{t}$	$a = 9.81$ m/s²
$v_0 = \frac{4.5}{t}$ ..... (1)	$t = t$
Substituting $t = 0.7821$ sec obtained from vertical motion, we get	using $s = ut + \frac{1}{2}at^2$
$v_0 = 5.754$ m/s ..... **Ans.**	$3 = 0 + \frac{1}{2} \times 9.81 \times t^2$
	$\therefore \quad t = 0.7821$ sec

Note we have taken ↓ sense as + ve

P20. A ball thrown horizontally from the top of a 50 m high building hits the horizontal ground 20 m from the base of the building. What is the initial velocity of the ball? ***(VJTI May 08)***

Solution: As the ball is thrown horizontally, the vertical component of initial velocity at A is zero.



Projectile Motion (A – B)

Horizontal Motion

$v = v_0$

$s = 20$ m

$t = t$

using $v = \frac{s}{t}$

$v_0 = \frac{20}{t}$ ..... (1)

Substituting $t = 3.193$ sec obtained from vertical motion, we get

$v_0 = 6.264$ m/s ..... **Ans.**

Vertical Motion ↓ + ve

$u = 0$

$v = -$

$s = 50$ m

$a = 9.81$ m/s²

$t = t$

Note we have taken ↓ sense as + ve

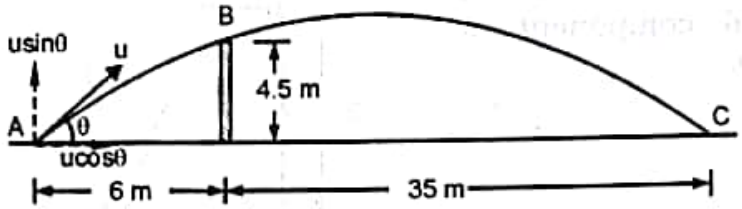
using $s = ut + \frac{1}{2}at^2$

$50 = 0 + \frac{1}{2} \times 9.81 \times t^2$

$\therefore$ $t = 3.193$ sec

P21. Find the initial velocity and the corresponding angle of projection of a projectile such that when projected from the ground it just clears a wall 4.5 m high at a horizontal distance of 6 m and finally lands on the ground at a distance of 35 m beyond the wall.

Solution:



Projectile Motion (A – B)

Horizontal Motion

$v = u\cos\theta$

$s = 6\text{ m}$

$t = t_1$

using $v = \dfrac{s}{t}$

$$u\cos\theta = \frac{6}{t_1} \quad \text{..... (1)}$$

Vertical Motion ↑ + ve

$u = u\sin\theta$

$v = -$

$s = 4.5\text{ m}$

$a = -9.81\text{ m/s}^2$

$t = t_1$

using $s = ut + \dfrac{1}{2}at^2$

$$4.5 = u\sin\theta \times t_1 + \frac{1}{2}\times(-9.81)\times t_1^2 \quad \text{..... (2)}$$

Eliminating t_1 from equation (1) in (2), we get

$$4.5 = u\sin\theta \times \frac{6}{u\cos\theta} - 4.905\times\left(\frac{6}{u\cos\theta}\right)^2 \quad \therefore \quad 4.5 = 6\tan\theta - \frac{176.58}{u^2\cos^2\theta} \quad \text{..... (A)}$$

Projectile Motion (A – C)

Horizontal Motion

$v = u\cos\theta$

$s = 41\text{ m}$

$t = t_2$

using $v = \dfrac{s}{t}$

$$u\cos\theta = \frac{41}{t_2} \quad \text{..... (3)}$$

Vertical Motion ↑ + ve

$u = u\sin\theta$

$v = -$

$s = 0$

$a = -9.81\text{ m/s}^2$

$t = t_2$

using $s = ut + \dfrac{1}{2}at^2$

$$0 = u\sin\theta \times t_2 + \frac{1}{2}\times(-9.81)\times t_2^2 \quad \text{..... (4)}$$

Eliminating t_2 from equation (3) in (4), we get

$$0 = u\sin\theta \times \frac{41}{u\cos\theta} - 4.905\times\left(\frac{41}{u\cos\theta}\right)^2 \quad \therefore \quad 0 = 41\tan\theta - \frac{8245.3}{u^2\cos^2\theta} \quad \text{..... (B)}$$

Solving equations (A) and (B), we get $u = 20.14\text{ m/s}$ and $\theta = 41.3°$ ..... **Ans.**

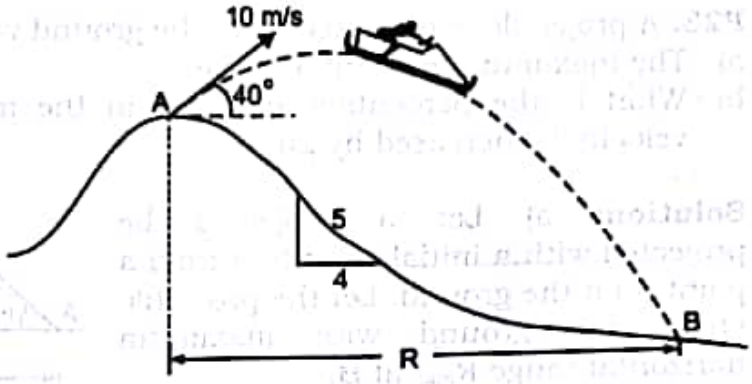
P22. The snowmobile is travelling at 10 m/s as shown in figure when it leaves embankment at A. Determine,

a) The time of flight from A to B.

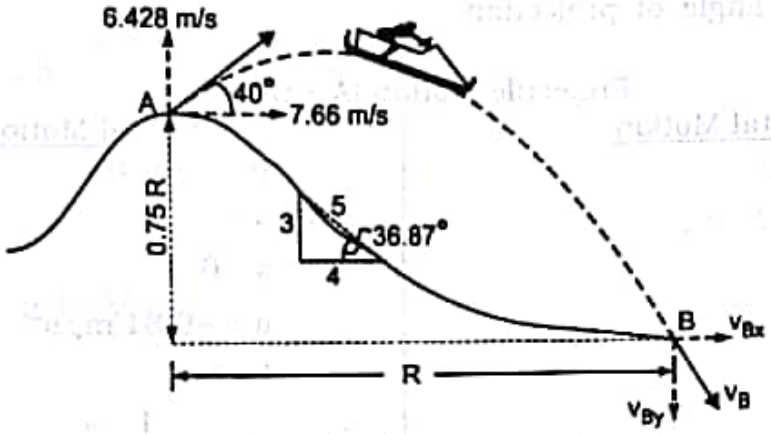
b) The speed at which it strikes the ground at B.

c) Range 'R'.

(VJTI Dec14)



Solution:



Projectile Motion (A – B)

Horizontal Motion

$v = 7.66$ m/s

$s = R$

$t = t$

using $v = \dfrac{s}{t}$

$7.66 = \dfrac{R}{t}$ ..... (1)

Vertical Motion ↑ + ve

$u = 6.428$ m/s

$v = -v_{By}$

$s = -0.75R$

$a = -9.81$ m/s^2

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

$-0.75R = 6.428 + \dfrac{1}{2} \times (-9.81) \times t^2$ ..... (2)

Solving equations (1) and (2), we get

$t = 2.482$ sec and $R = 19.01$ m ..... **Ans.**

Landing speed at B

$\therefore \quad v_B = \sqrt{v_{Bx}^2 + v_{By}^2}$

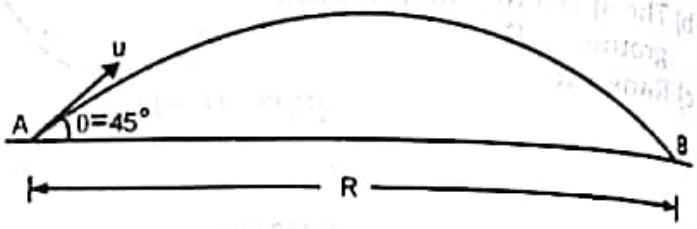
$= \sqrt{7.66^2 + 17.92^2}$

or $\quad v_B = 19.49$ m/s ..... **Ans.**

$v_{Bx} = 7.66$ m/s $\because$ horizontal component of velocity remains constant.

P23. A projectile is projected from the ground with a velocity of 25 m/s. Find
a) The maximum horizontal range.
b) What is the percentage increase in the maximum horizontal range, if its initial velocity is increased by 20%.

Solution: a) Let a projectile be projected with a initial velocity u from a point A on the ground. Let the projectile strike the ground with maximum horizontal range R_{max} at B.
Knowing for R_{max}, angle of projection $\theta = 45°$



Projectile Motion (A – B)

Horizontal Motion

$v = u\cos 45$

$s = R_{max}$

$t = t$

using $v = \frac{s}{t}$

$u\cos 45 = \frac{R_{max}}{t}$

or $t = \frac{R_{max}}{u\cos 45}$ ..... (1)

Vertical Motion ↑ + ve

$u = u\sin 45$

$v = -$

$s = 0$

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$0 = u\sin 45 \times t + \frac{1}{2} \times (-9.81) \times t^2$

Substituting from eqn. (1) in (2), we get

$0 = u\sin 45 \times \frac{R_{max}}{u\cos 45} - 4.905 \times \left(\frac{R_{max}}{u\cos 45}\right)^2$

$0 = u\sin 45 - 4.905 \times \frac{R_{max}}{u\cos 45}$

$\therefore \quad R_{max} = 0.10194\,u^2$ ..... (A)

Given u = 25 m/s

$\therefore \quad R_{max} = 0.10194 \times 25^2 = 63.71$ m ..... **Ans.**

b) When velocity is increased by 20%, u′ = 1.2 × 25 = 30 m/s

Now $R_{max} = 0.10194 \times 30^2 = 91.74$ m

Increase in Max. Range = $\frac{R'_{max} - R_{max}}{R_{max}} \times 100 = \frac{91.74 - 63.71}{63.71} \times 100 = 44\%$ ..... **Ans.**

P24. A gunman standing on the ground fires his gun to hit a bird flying at an altitude of 20 m from the ground. The angle of projection being 60° upwards with horizontal. If the bullet hits the bird 2.5 sec after firing, find
a) The velocity of the bullet as it left the gun.
b) The bird is killed instantly. Find the time it takes to reach the ground.
Neglect the height of gunman.

Solution: a) Projectile Motion (A – B)

<u>Vertical Motion ↑ + ve</u>

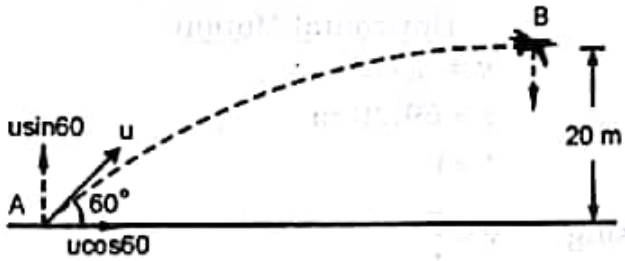
$u = u\sin 60 \text{ m/s}$

$v = -$

$s = 20 \text{ m}$

$a = -9.81 \text{ m/s}^2$

$t = 2.5 \text{ sec}$



using $s = ut + \frac{1}{2}at^2$

$$20 = u\sin 60 \times 2.5 + \frac{1}{2} \times (-9.81) \times 2.5^2$$

$\therefore \quad u = 23.4 \text{ m/s}$ ..... **Ans.**

b) As per the bird is killed instantly, it falls vertically down performing motion under gravity

Motion of bird – M U G ↓+ve

$u = 0$

$v = -$

$s = 20 \text{ m}$

$a = +9.81 \text{ m/s}^2$

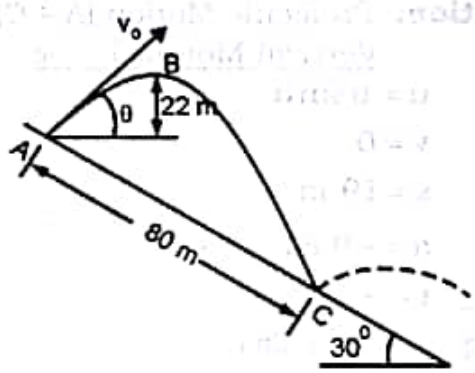
$t = t$

using $s = ut + \frac{1}{2}at^2$

$$20 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$\therefore \quad t = 2.019 \text{ sec}$ ..... **Ans.**

P25. A ball is projected on an incline from A with velocity v_o at an angle θ as shown. The ball lands at C, 80 m away from A. If the maximum height covered above A is 22 m find v_o and θ.



Solution: Projectile Motion (A – B)

<u>Vertical Motion ↑ + ve</u>

$u = v_0 \sin\theta$

$v = 0$

$s = 22 \text{ m}$

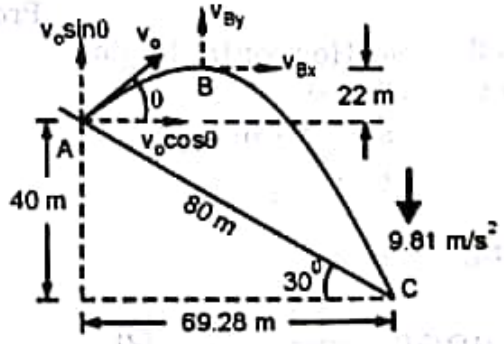
$a = -9.81 \text{ m/s}^2$

$t = -$

using $v^2 = u^2 + 2as$

$$0 = (v_0 \sin\theta)^2 + 2(-9.81) \times 22$$

$\therefore \quad v_0 \sin\theta = 20.776$ .....(1)



Projectile Motion (A – C)

Horizontal Motion

$v = v_0 \cos\theta$

$s = 69.28$ m

$t = t$

using $v = \dfrac{s}{t}$

$v_0 \cos\theta = \dfrac{69.28}{t}$ ..... (2)

Vertical Motion ↑ + ve

$u = v_0 \sin\theta = 20.776$ m/s

$v = -$

$s = -40$ m

$a = -9.81$ m/s^2

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

$-40 = 20.776 \times t + \dfrac{1}{2} \times (-9.81) \times t^2$

$\therefore$ $t = 5.673$ sec

Substituting t = 5.673 sec, we get

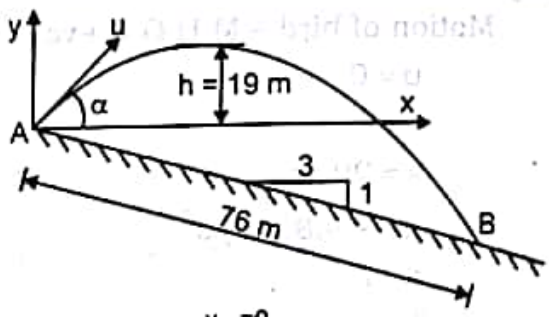
$v_0 \cos\theta = 12.21$ ..... (3)

Solving equations (1) and (3), we get

$\therefore$ $v_0 = 24.1$ m/s and $\theta = 59.56°$ ..... **Ans.**

P26. A ball rebounds at A and strikes the incline plane at point B at a distance 76 m as shown in figure. If the ball rises to maximum height h = 19 m above the point of projection, compute the initial velocity and the angle of projection α.

(M. U. May 09)



Solution: Projectile Motion (A – C)

Vertical Motion ↑ + ve

$u = u\sin\alpha$

$v = 0$

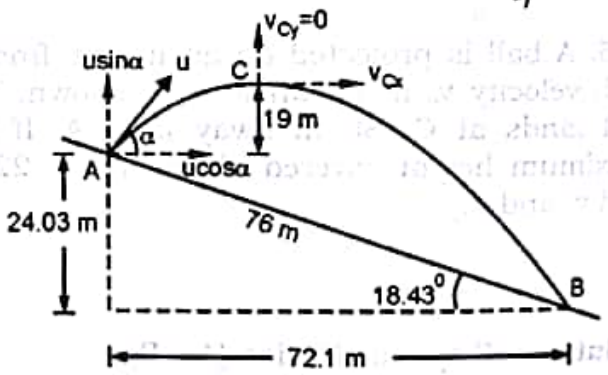
$s = 19$ m

$a = -9.81$ m/s^2

$t = -$

using $v^2 = u^2 + 2as$

$0 = (u\sin\alpha)^2 + 2(-9.81) \times 19$ $\therefore$ $u\sin\alpha = 19.31$ .....(1)



Projectile Motion (A – B)

Horizontal Motion

$v = u\cos\alpha$

$s = 72.1$ m

$t = t$

using $v = \dfrac{s}{t}$

$u\cos\alpha = \dfrac{72.1}{t}$ ..... (2)

Vertical Motion ↑ + ve

$u = u\sin\alpha$

$v = -$

$s = -24.03$ m

$a = -9.81$ m/s^2

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

Substituting $t = 4.93$ sec, we get

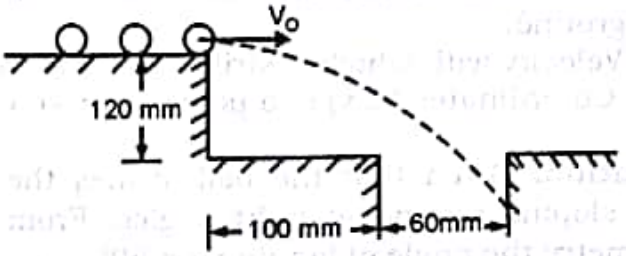
$u\cos\alpha = 14.625$ ..... (3)

$$-24.03 = 19.31 \times t + \frac{1}{2} \times (-9.81) \times t^2$$

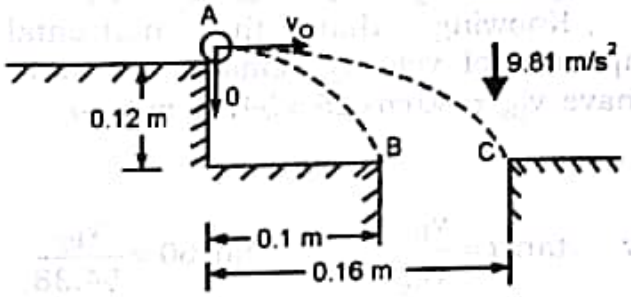
$\therefore \quad t = 4.93$ sec

Solving equations (1) and (3), we get $u = 24.22$ m/s and $\alpha = 52.86°$ ..... **Ans.**

P27. Ball bearings leave the horizontal trough with a horizontal velocity v_0 to fall through a gap of 60 mm. calculate the permissible range of velocity v_0 that will enable the balls to enter the gap.



Solution: Range of values of v_0 implies minimum and maximum values of v_0. If v_0 is minimum the ball bearings will pass close to the left edge of the opening, while for v_0 maximum, the ball bearings pass close to the right edge of the opening. As the ball leave horizontally the vertical component of velocity is zero at the start of projectile motion.



Kinetic Diagram

Projectile Motion (A – B)

Horizontal Motion

$v = v_0$

$s = 0.1$ m

$t = t$

using $v = \frac{s}{t}$

$v_0 = \frac{0.1}{t}$ ..... (1)

Substituting $t = 0.1564$ sec,

minimum $v_0 = 0.639$ m/s ..... **Ans.**

Vertical Motion ↓ + ve

$u = 0$

$v = -$

$s = 0.12$ m

$a = 9.81$ m/s²

$t = t$

Note we have taken ↓ sense as + ve

using $s = ut + \frac{1}{2}at^2$

$0.12 = 0 + \frac{1}{2} \times 9.81 \times t^2 \quad \therefore \quad t = 0.1564$ sec

Projectile Motion (A – C)

Horizontal Motion

$v = v_0$

$s = 0.16$ m

$t = t$

using $v = \frac{s}{t}$

$v_0 = \frac{0.16}{t}$ ..... (2)

Substituting $t = 0.1564$ sec, we get

maximum $v_0 = 1.023$ m/s ..... **Ans.**

Vertical Motion ↓ + ve

$u = 0$

$v = -$

$s = 0.12$ m

$a = 9.81$ m/s²

$t = t$

Note we have taken ↓ sense as + ve

using $s = ut + \frac{1}{2}at^2$

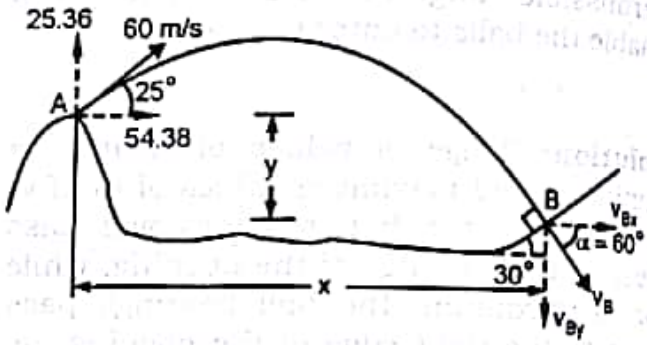
$0.12 = 0 + \frac{1}{2} \times 9.81 \times t^2 \quad \therefore \quad t = 0.1564$ sec

P28. A ball is thrown upward from a high cliff as shown. with a velocity of 60 m/s at The ball strikes the inclined ground at right angles. If inclination of ground is 30° as shown, determine

(i) Time after which the ball strikes the ground.
(ii) Velocity with which it strikes the ground.
(iii) Co-ordinates (x, y) of a point of strike w.r.t. point of projection.

Solution: Given that the ball strikes the 30° sloping ground at right angles. From geometry the angle of landing $\alpha = 60°$.

Knowing that the horizontal component of velocity remains constant, we have $v_{Bx} = 60\cos 25 = 54.38 \text{ m/s} \rightarrow$



Now $\tan\alpha = \dfrac{v_{By}}{v_{Bx}}$ $\therefore$ $\tan 60 = \dfrac{v_{By}}{54.38}$ $\therefore$ $v_{By} = 94.19 \text{ m/s} \downarrow$

Landing velocity at B

$\therefore$ $v_B = \sqrt{v_{Bx}^2 + v_{By}^2} = \sqrt{54.38^2 + 94.19^2} = 108.76 \text{ m/s}$ at $\alpha = 60° \searrow$ ..... **Ans.**

Projectile Motion (A – B)

Horizontal Motion

$v = 54.38 \text{ m/s}$

$s = x$

$t = t$

using $v = \dfrac{s}{t}$

$54.38 = \dfrac{x}{t}$ ..... (1)

Substituting $t = 12.186$ sec, we get

$x = 662.67$ m ..... **Ans.**

Vertical Motion ↑ + ve

$u = 25.36$

$v = -v_{By} = -94.19 \text{ m/s}$

$s = -y$

$a = -9.81 \text{ m/s}^2$

$t = t$

using $v = u + at$

$-94.19 = 25.36 - 9.81 \times t$

$\therefore$ $t = 12.186$ sec ..... **Ans.**

using $s = ut + \dfrac{1}{2}at^2$

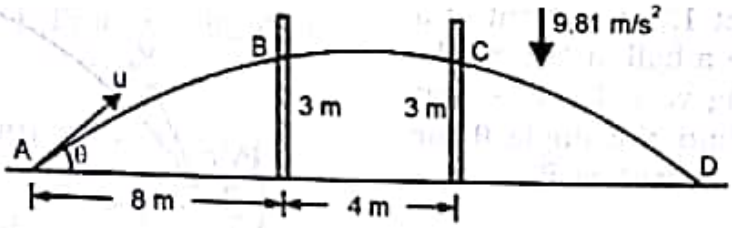
$-y = 25.36 \times 12.186 + \dfrac{1}{2} \times (-9.81) \times 12.186^2$

$\therefore$ $y = 419.4$ m ..... **Ans.**

$\therefore$ Co-ordinates (x, y) of point of landing = (662.67, 419.4) m ..... **Ans.**

P29. A jet of water discharging from a nozzle hits a vertical screen placed at a distance of 8m from the nozzle at a height of 3m. When the screen is shifted by 4m further away from nozzle, the jet hits the screen again at the same point. Find the angle of projection and velocity of projection of the jet at the nozzle. ***(VJTI May 06)***

Solution:



Projectile Motion (A – B)

Horizontal Motion	Vertical Motion ↑ + ve
$v = u\cos\theta$	$u = u\sin\theta$
$s = 8$ m	$v = -$
$t = t_1$	$s = 3$ m
using $v = \frac{s}{t}$	$a = -9.81$ m/s²
$u\cos\theta = \frac{8}{t_1}$ ..... (1)	$t = t_1$
	using $s = ut + \frac{1}{2}at^2$
	$3 = u\sin\theta \times t_1 + \frac{1}{2}\times(-9.81)\times t_1^2$ ..... (2)

Substituting t_1 from equation (1) and (2), we get

$$3 = u\sin\theta \times \frac{8}{u\cos\theta} - 4.905 \times \left(\frac{8}{u\cos\theta}\right)^2$$

$$\therefore \quad 3 = 8\tan\theta - \frac{313.92}{u^2\cos^2\theta} \quad \text{..... (A)}$$

Projectile Motion (A – C)

Horizontal Motion	Vertical Motion ↑ + ve
$v = u\cos\theta$	$u = u\sin\theta$
$s = 12$ m	$v = -$
$t = t_2$	$s = 3$ m
using $v = \frac{s}{t}$	$a = -9.81$ m/s²
$u\cos\theta = \frac{12}{t_2}$ ..... (3)	$t = t_2$
	using $s = ut + \frac{1}{2}at^2$
	$3 = u\sin\theta \times t_2 + \frac{1}{2}\times(-9.81)\times t_2^2$ ..... (4)

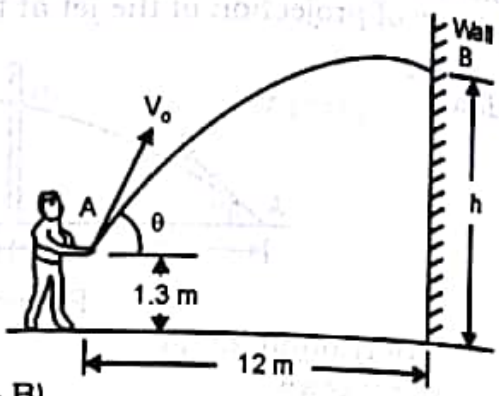
eliminating t_2 from equation (3) and (4), we get

$$\therefore \quad 3 = 12\tan\theta - \frac{706.32}{u^2\cos^2\theta} \quad \text{..... (B)}$$

Solving equations (A) and (B), we get

$u = 14.77\ m/s$ and $\theta = 32°$ ..... **Ans.**

P30. A boy standing at 12 m in front of a wall attempts to strike a ball at the height h on the wall. Taking v_o = 15 m/s and knowing h = 7.5 m find the angle θ for which the ball strikes the wall at B.



Solution:

Projectile Motion (A – B)

Horizontal Motion	Vertical Motion ↑ + ve
$v = 15\cos\theta$	$u = 15\sin\theta$
$s = 12\ m$	$v = -$
$t = t$	$s = 6.2\,m$
using $v = \frac{s}{t}$	$a = -9.81\ m/s^2$
	$t = t$
$15\cos\theta = \frac{12}{t}$ ..... (1)	using $s = ut + \frac{1}{2}at^2$
	$6.2 = 15\sin\theta \times t + \frac{1}{2}\times(-9.81)\times t^2$ ..... (2)

Eliminating t from equation (1) and (2), we get

$$6.2 = 15\sin\theta \times \frac{12}{15\cos\theta} - 4.905 \times \left(\frac{12}{15\cos\theta}\right)^2$$

$$6.2 = 12\tan\theta - 3.139\sec^2\theta$$

$$6.2 = 12\tan\theta - 3.139\left(1 + \tan^2\theta\right)$$

$$3.139\tan^2\theta - 12\tan\theta + 9.339 = 0$$

Solving the equation, we get

$\tan\theta = 2.735$ and $\tan\theta = 1.0877$

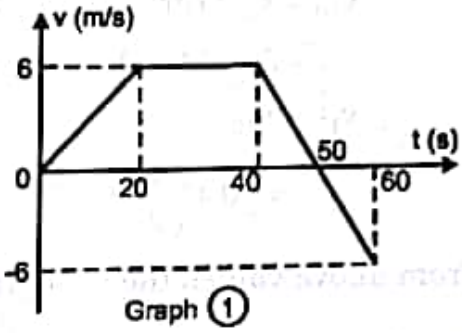
$\therefore$ $\theta = 69.92°$ and $\theta = 47.4°$

Hence the ball can be projected at two values of angle θ viz $\theta = 69.92°$ and $\theta = 47.4°$ to strike the wall at B. ..... **Ans.**

Exercise 9.5

Graphical Solutions

P1. Refer graph (1). Find acceleration at t =10, 30, 45 and 50 sec.



Graph ①

Solution: Acceleration Calculations

From v – t graph, acceleration can be obtained using

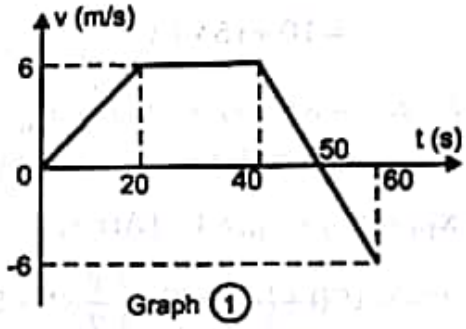
a = slope of v – t curve

$$a_{(0-20)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{20-0} = 0.3 \text{ m/s}^2 \quad \text{..... Ans.}$$

$$a_{(20-40)} = \frac{y_2 - y_1}{x_2 - x_1} = 0 \quad \text{..... since for (20 – 40) sec, graph is horizontal ..... Ans.}$$

$$a_{(40-60)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{50-40} = -0.6 \text{ m/s}^2 \quad \text{..... Ans.}$$

P2. From graph (1) find position at t = 20, 40, 50 and 60 sec knowing x_o = 0.



Graph ①

Solution: From v – t graph, position can be calculated using $x_f = x_i$ + area under v – t curve

$$\therefore \quad x_{20} = x_0 + (\text{area under v} - \text{t curve})_{0-20}$$

$$= 0 + \left(\frac{1}{2} \times 20 \times 6\right) = 60 \text{ m} \quad \text{..... Ans.}$$

$$\therefore \quad x_{40} = x_{20} + (\text{area under v} - \text{t curve})_{20-40}$$

$$= 60 + (20 \times 6) = 180 \text{ m} \quad \text{..... Ans.}$$

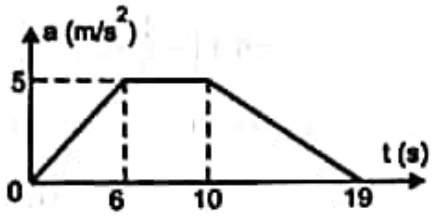
$$\therefore \quad x_{50} = x_{40} + (\text{area under v} - \text{t curve})_{40-50}$$

$$= 180 + \left(\frac{1}{2} \times 10 \times 6\right) = 210 \text{ m} \quad \text{..... Ans.}$$

$$\therefore \quad x_{60} = x_{50} + (\text{area under v} - \text{t curve})_{50-60}$$

$$= 210 + \left(-\frac{1}{2} \times 10 \times 6\right) = 180 \text{ m} \quad \text{..... Ans.}$$

P3. From graph (2) of a particle in rectilinear motion shown, plot x – t and v – t curves knowing x_o = 10 m, v_o = 5 m/s.



Graph ②

Solution: Velocity Calculations

From a – t graph, velocity can be obtained by using.

$v_f = v_i$ + (area under a – t curve)

$$v_6 = v_0 + (\text{area under a} - \text{t curve})_{0\text{-}6}$$

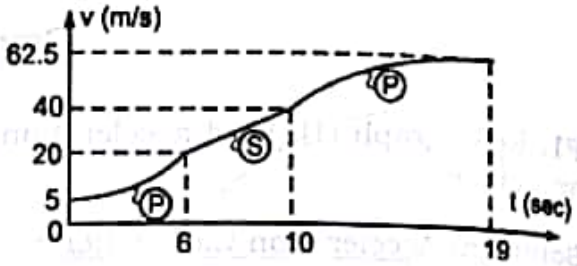
$$= 5 + \left(\frac{1}{2} \times 6 \times 5\right) = 20 \text{ m/s}$$

$$v_{10} = v_6 + (\text{area under a} - \text{t curve})_{6\text{-}10}$$

$$= 20 + (4 \times 5) = 40 \text{ m/s}$$

$$v_{19} = v_{10} + (\text{area under a} - \text{t curve})_{10\text{-}19}$$

$$= 40 + \left(\frac{1}{2} \times 9 \times 5\right) = 62.5 \text{ m/s}$$



From above values the v – t graph is drawn as shown. ..... **Ans.**

<u>Position calculations</u>

From a – t graph, position can be calculated using

$x_f = x_i + v_i \times t +$ (Area under a – t curve) $\times (t - t_G)$

..... here $t = t_f - t_i$

$$\therefore \quad x_6 = x_0 + v_0 \times t + (\text{Area})_{0\text{-}6} \times (t - t_G)$$

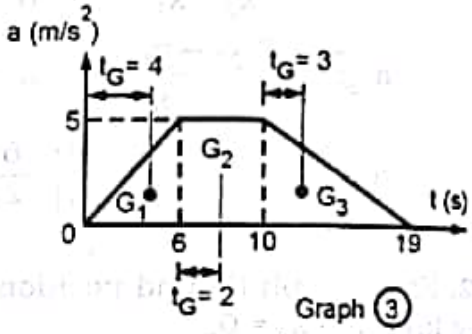
$$= 10 + (5 \times 6) + \left(\frac{1}{2} \times 6 \times 5\right)(6 - 4) = 70 \text{ m}$$

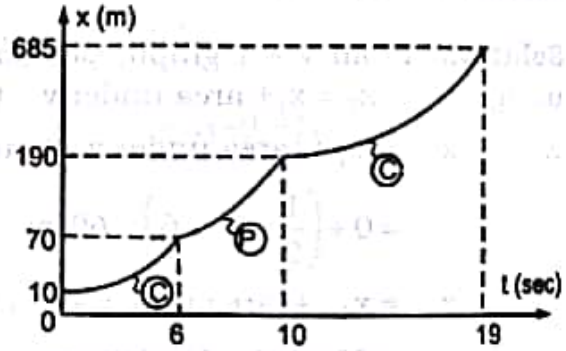
$$\therefore \quad x_{10} = x_6 + v_6 \times t + (\text{Area})_{6\text{-}10} \times (t - t_G)$$

$$= 70 + (20 \times 4) + (4 \times 5)(4 - 2) = 190 \text{ m}$$

$$x_{19} = x_{10} + v_{10} \times t + (\text{Area})_{10\text{-}19} \times (t - t_G)$$

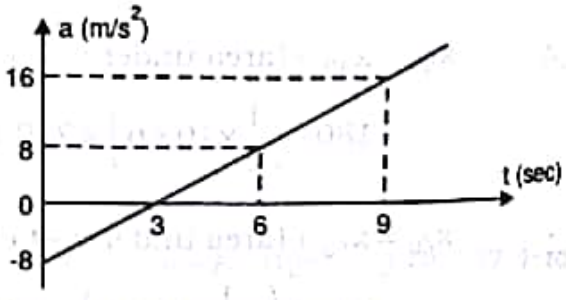
$$= 190 + (40 \times 9) + \left(\frac{1}{2} \times 9 \times 5\right)(9 - 3) = 685 \text{ m}$$





From above values the x – t graph is drawn as shown. ..... **Ans.**

P4. Figure shows a – t curve for a particle performing rectilinear motion. Knowing that at t = 0, v = 4 m/s and x = 20 m, find graphically the velocity and position of the particle at t = 9 sec.



Solution: <u>Velocity Calculations</u>

From a – t graph, velocity can be obtained by using $v_f = v_i + (\text{area under a} - \text{t curve})$

$$v_3 = v_0 + (\text{area under a} - \text{t curve})_{0\text{-}3}$$

$$= 4 + \left(-\frac{1}{2} \times 3 \times 8\right) = -8 \text{ m/s}$$

$$v_9 = v_3 + (\text{area under a} - \text{t curve})_{3\text{-}9}$$

$$= -8 + \left(\frac{1}{2} \times 6 \times 16\right) = 40 \text{ m/s} \text{ ..... } \textbf{Ans.}$$

Position calculations

From a – t graph, position can be calculated using

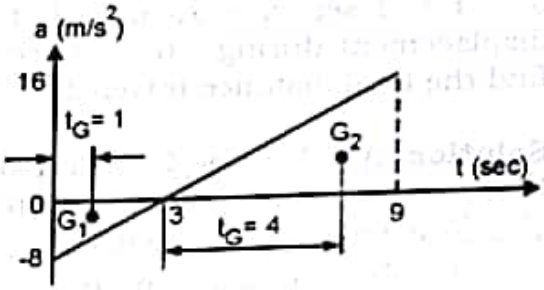
$x_f = x_i + v_i \times t +$ (Area under a – t curve) $\times (t - t_G)$ ..... here $t = t_f - t_i$

$x_3 = x_0 + v_0 \times t + (\text{Area})_{0-3} \times (t - t_G)$

$= 20 + (4 \times 3) + \left(-\frac{1}{2} \times 3 \times 8\right)(3-1) = 8$ m

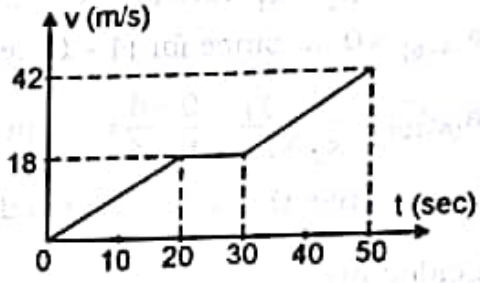
$x_9 = x_3 + v_3 \times t + (\text{Area})_{3-9} \times (t - t_G)$

$= 8 + (-8) \times 6 + \left(\frac{1}{2} \times 6 \times 16\right)(6-4) = 56$ m ..... **Ans**



P5. The race car starts from rest and travels along a straight road until it reaches a speed of 42 m/s in 50 seconds as shown by v–t graph. Determine the distance travelled by race car in 50 sec. Draw x–t and a–t graph.

(M.U May 08, KJS May 15)



Solution: Acceleration Calculations

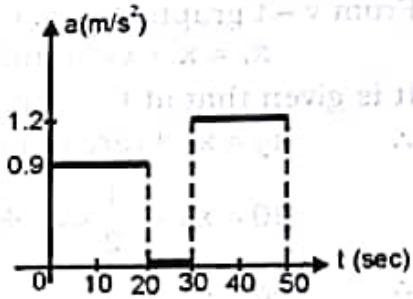
From v – t graph, acceleration can be obtained by using

a = slope of v – t curve

$a_{(0-20)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18-0}{20-0} = 0.9$ m/s²

$a_{(20-30)} = 0$ ... since for (20 – 30) sec, graph is horz.

$a_{(30-50)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{42-18}{50-30} = 1.2$ m/s²



From above values the a – t graph is drawn as shown. ..... **Ans.**

Position calculations

From v – t graph, position can be calculated using

$x_f = x_i +$ area under v – t curve

$x_{20} = x_0 + (\text{area under v – t curve})_{0-20}$

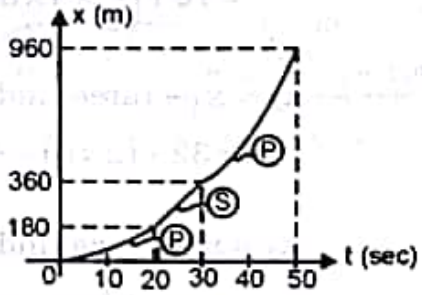
$= 0 + \left(\frac{1}{2} \times 20 \times 18\right) = 180$ m

$x_{30} = x_{20} + (\text{area under v – t curve})_{20-30}$

$= 180 + (10 \times 18) = 360$ m

$x_{50} = x_{30} + (\text{area under v – t curve})_{30-50}$

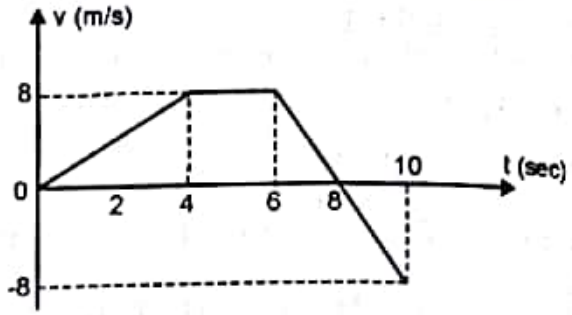
$= 360 + \left(\frac{1}{2} \times (18 + 42) \times 20\right) = 960$ m



∴ The distance travelled by the car in50 sec is 960 m. ..... **Ans.**

From the above values the x – t graph is drawn as shown. ..... **Ans.**

P6. For a particle performing rectilinear motion, the v – t diagram is shown. Draw a – t and x – t diagrams for the motion if at t = 2 sec, x = 20 m. What is the displacement during 6 – 10 sec ? Also find the total distance traveled.



Solution: Acceleration Calculations

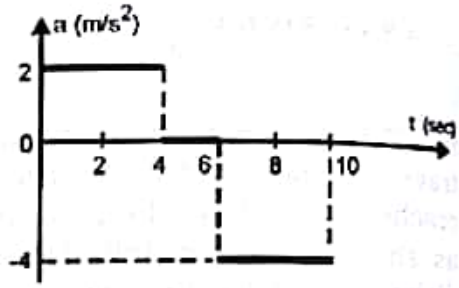
From v – t graph, acceleration can be obtained by using

$$a = \text{slope of } v - t \text{ curve}$$

$$a_{(0-4)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{4-0} = 2 \text{ m/s}^2$$

$a_{(4-6)} = 0$ .... since for (4 – 6) sec, graph is horz.

$$a_{(6-10)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-8}{8-6} = -4 \text{ m/s}^2$$



From above values the a – t graph is drawn as shown. ..... **Ans.**

Position calculations

From v – t graph, position can be calculated using

$$x_f = x_i + \text{area under } v - t \text{ curve}$$

It is given that at t = 2 sec, x = 20 m also from graph, v_2 = 4 m/s

$\therefore \quad x_2 = x_0 + (\text{area under } v - t \text{ curve})_{0-2}$

$$20 = x_0 + \left(\frac{1}{2} \times 2 \times 4\right)$$

$\therefore \quad x_0 = 16 \text{ m}$

$$x_4 = x_0 + (\text{area under } v - t \text{ curve})_{0-4}$$

$$= 16 + \left(\frac{1}{2} \times 4 \times 8\right) = 32 \text{ m}$$

$$x_6 = x_4 + (\text{area under } v - t \text{ curve})_{4-6}$$

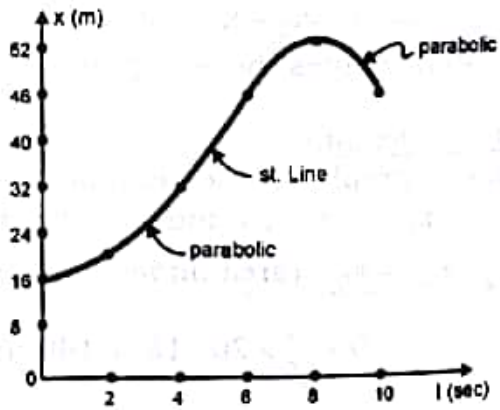
$$= 32 + (2 \times 8) = 48 \text{ m}$$

$$x_8 = x_6 + (\text{area under } v - t \text{ curve})_{6-8}$$

$$= 48 + \left(\frac{1}{2} \times 2 \times 8\right) = 56 \text{ m}$$

$$x_{10} = x_8 + (\text{area under } v - t \text{ curve})_{8-10}$$

$$= 56 + \left(-\frac{1}{2} \times 2 \times 8\right) = 48 \text{ m}$$



Displacement 6 – 10 sec can be obtained from x – t graph,

$$= x_{10} - x_6 = 48 - 48 = 0$$ ..... **Ans.**

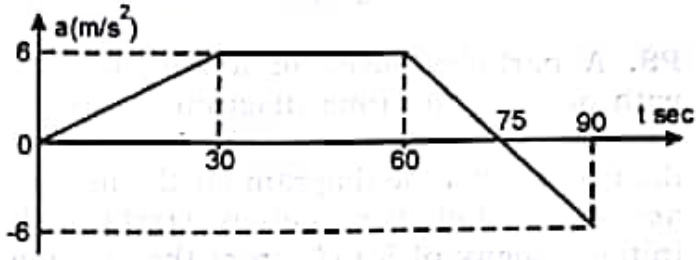
Distance 0 – 10 sec

= total area under v – t curve

$$=\left(\frac{1}{2}\times4\times8\right)+(2\times8)+\left(\frac{1}{2}\times2\times8\right)+\left(\frac{1}{2}\times2\times8\right)\quad=48\text{ m}$$ ..... **Ans.**

The x – t graph is plotted as shown. ..... **Ans.**

P7. Figure shows an a – t graph for a particle moving along the x axis. Draw v – t and x – t graphs and find the speed and position of the particle at t = 90 sec. Also find the maximum speed attained by the particle. Take $x_0 = 0$ and $v_0 = 0$.



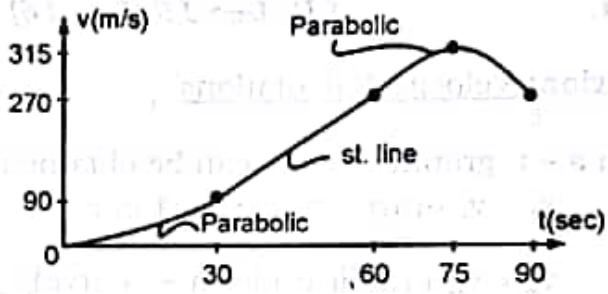
Solution: Velocity Calculations

From a – t graph, velocity can be obtained by using.

$v_f = v_i + (\text{area under a} - \text{t curve})$

$v_{30} = v_0 + (\text{area under a} - \text{t curve})_{0-30}$

$$=0+\left(\frac{1}{2}\times30\times6\right)=90\text{ m/s}$$



$v_{60} = v_{30} + (\text{area under a} - \text{t curve})_{30-60} \quad = 90 + (30\times6) = 270\text{ m/s}$

$v_{75} = v_{60} + (\text{area under a} - \text{t curve})_{60-75} \quad = 270 + \left(\frac{1}{2}\times15\times6\right) = 315\text{ m/s}$

$v_{90} = v_{75} + (\text{area under a} - \text{t curve})_{75-90} \quad = 315 + \left(-\frac{1}{2}\times15\times6\right) = 270\text{ m/s}$

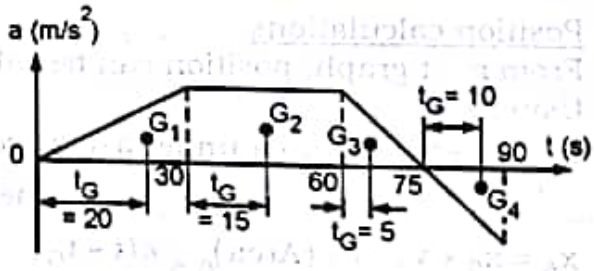
∴ Max. Speed of the particle is 315 m/s ..... **Ans.**

Position calculations

From a–t graph, position can be calculated using

$x_f = x_i + v_i \times t +$ (Area under a – t curve) $\times(t - t_G)$

..... here $t = t_f - t_i$



$x_{30} = x_0 + v_0 \times t + (\text{Area})_{0-20} \times (t - t_G)$

$$=0+0+\left(\frac{1}{2}\times30\times6\right)\times(30-20)\quad=900\text{ m}$$

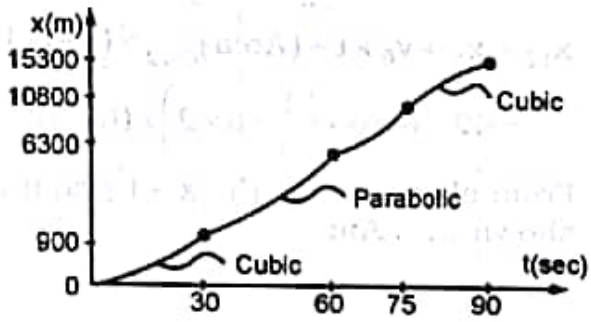
$x_{60} = x_{30} + v_{30} \times t + (\text{Area})_{30-60} \times (t - t_G)$

$$=900+90\times30+(30\times6)\times(30-15)=6300\text{ m}$$

$x_{75} = x_{60} + v_{60} \times t + (\text{Area})_{60-75} \times (t - t_G)$

$$=6300+270\times15+\left(\frac{1}{2}\times15\times6\right)\times(15-5)$$

$$=10800\text{ m}$$



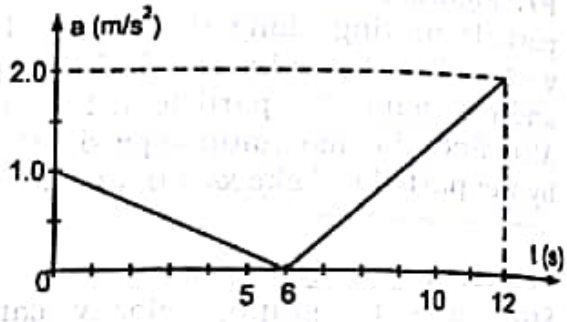
$$x_{90} = x_{75} + v_{75} \times t + (\text{Area})_{75\text{-}90} \times (t - t_G)$$

$$= 10800 + 315 \times 15 + \left(-\frac{1}{2} \times 15 \times 6\right) \times (15 - 10)$$

$$= 15300 \text{ m}$$

The v – t and x – t graphs are plotted as shown. ..... **Ans.**

P8. A particle moves in a straight line with acceleration-time diagram shown in figure. Construct velocity-time and displacement-time diagram for the motion assuming that the motion starts with initial velocity of 5 m/s from the starting point. ***(M.U. Dec 15, Dec 16)***



Solution: Velocity Calculations

From a – t graph, velocity can be obtained by using.

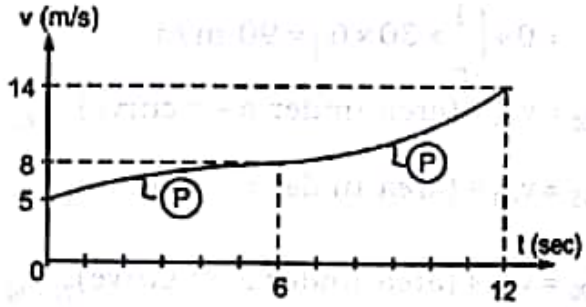
$$v_f = v_i + (\text{area under a} - \text{t curve}) \quad \text{..... given } x_0 = 0,\ v_0 = 5 \text{ m/s}$$

$$v_6 = v_0 + (\text{area under a} - \text{t curve})_{0\text{-}6}$$

$$= 5 + \left(\frac{1}{2} \times 6 \times 1\right) = 8 \text{ m/s}$$

$$v_{12} = v_6 + (\text{area under a} - \text{t curve})_{6\text{-}12}$$

$$= 8 + \left(\frac{1}{2} \times 6 \times 2\right) = 14 \text{ m/s}$$



From above values the v – t graph is plotted as shown ..... **Ans.**

Position calculations

From a – t graph, position can be calculated, Using,

$$x_f = x_i + v_i \times t + (\text{Area under a-t curve}) \times (t - t_G)$$

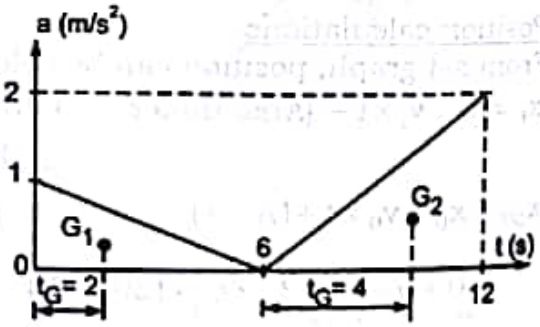
..... here $t = t_f - t_i$

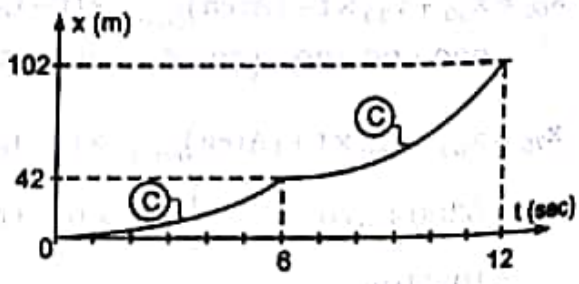
$$x_6 = x_0 + v_0 \times t + (\text{Area})_{0\text{-}6} \times (t - t_G)$$

$$= 0 + 5 \times 6 + \left(\frac{1}{2} \times 6 \times 1\right) \times (6 - 2) = 42 \text{m}$$

$$x_{12} = x_6 + v_6 \times t + (\text{Area})_{6\text{-}12} \times (t - t_G)$$

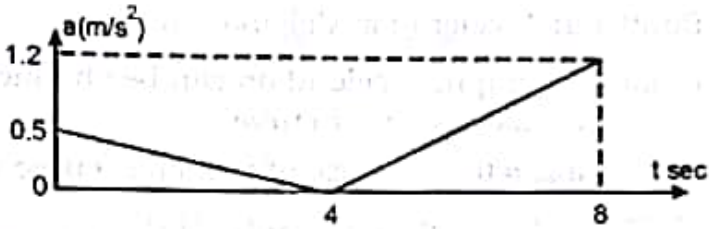
$$= 42 + 8 \times 6 + \left(\frac{1}{2} \times 6 \times 2\right) \times (6 - 4) = 102 \text{m}$$





From above values the x – t graph is plotted as shown ..... **Ans.**

P9. Figure shows an a – t graph for a rectilinear moving particle. Construct v – t and x – t graphs for the motion. At t = 0 the particle is at x = 0 and its velocity is v = 3 m/s.



Solution: <u>Velocity Calculations</u>

From a – t graph, velocity can be obtained by using.

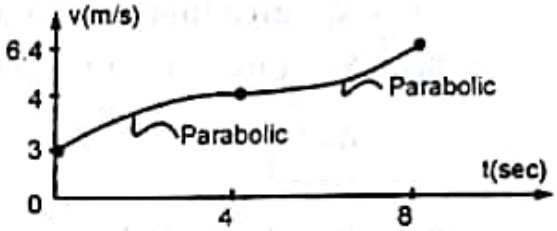
$$v_f = v_i + (\text{area under a} - \text{t curve})$$

$$v_4 = v_0 + (\text{area under a} - \text{t curve})_{0-4}$$

$$= 3 + \left(\frac{1}{2} \times 4 \times 0.5\right) = 4 \text{ m/s}$$

$$v_8 = v_4 + (\text{area under a} - \text{t curve})_{4-8}$$

$$= 4 + \left(\frac{1}{2} \times 4 \times 1.2\right) = 6.4 \text{ m/s}$$



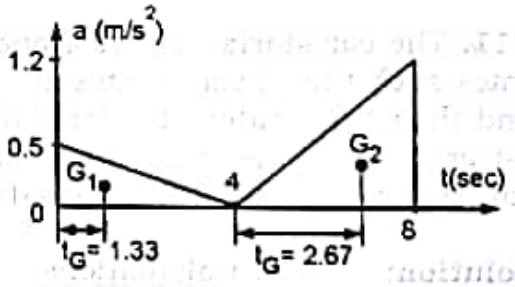
From above values the v – t graph is plotted as shown ..... **Ans.**

<u>Position calculations</u>

From a – t graph, position can be calculated using

$x_f = x_i + v_i \times t +$ (Area under a – t curve) $\times (t - t_G)$

..... here $t = t_f - t_i$



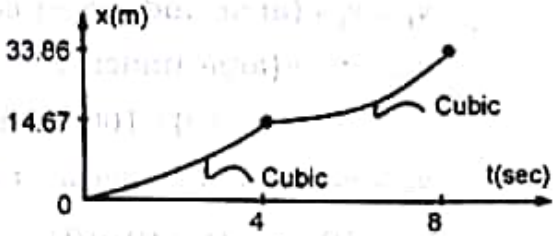
$$x_4 = x_0 + v_0 \times t + (\text{Area})_{0-4} \times (t - t_G)$$

$$= 0 + 3 \times 4 + \left(\frac{1}{2} \times 4 \times 0.5\right) \times (4 - 1.33) = 14.67 \text{ m}$$

$$x_8 = x_4 + v_4 \times t + (\text{Area})_{4-8} \times (t - t_G)$$

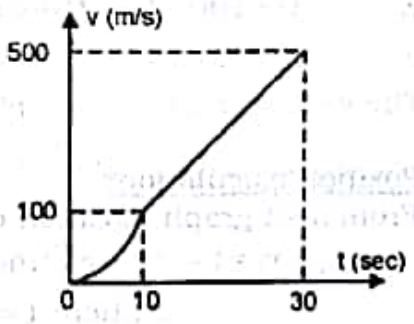
$$= 14.67 + 4 \times 4 + \left(\frac{1}{2} \times 4 \times 1.2\right) \times (4 - 2.67)$$

$$= 33.86 \text{ m}$$



From above values the x – t graph is plotted as shown. ..... **Ans.**

P10. A car moves along a straight road such that its velocity is described by the graph shown in figure. For the first 10 seconds the velocity variation is parabolic and between 10 seconds to 30 seconds the variation is linear. Construct the x – t and a – t graphs for the time period $0 \le t \le 30$ sec (Hint: area under parabolic curve of base a and height $h = \frac{1}{3}ah$) ***(M.U Dec 10)***



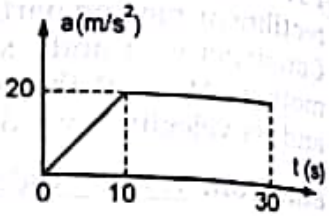
Solution: Acceleration Calculations

From v – t graph, acceleration can be obtained by using

$a = \text{slope of } v - t \text{ curve}$

$a_{(0)} = 0$ ..... slope of parabolic curve at vertex is zero

$$a_{(10-30)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{500-100}{30-10} = 20 \text{ m/s}^2$$



From above values the a – t graph is plotted as shown. ..... **Ans.**

Position calculations

From v – t graph, position can be calculated using

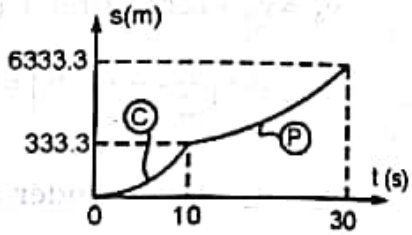
$x_f = x_i + \text{area under } v - t \text{ curve}$

$$x_{10} = x_0 + (\text{area under } v - t \text{ curve})_{0-10}$$

$$= 0 + \left(\frac{1}{3} \times 10 \times 100\right) = 333.3 \text{ m}$$

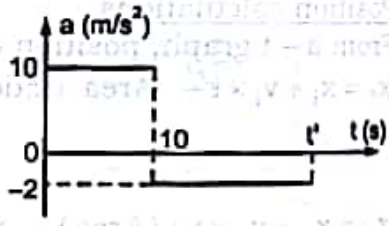
$$x_{30} = x_{10} + (\text{area under } v - t \text{ curve})_{10-30}$$

$$= 333.3 + \left(\frac{1}{2}(100+500) \times 20\right) = 6333.3 \text{ m}$$



From above values the x – t graph is plotted as shown. ..... **Ans.**

P11. The car starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 sec. and then decelerates at a constant rate. Draw the v-t and x-t graphs and determine the time t' needed to stop the car. How far has the car travelled? ***(M. U. May 09)***



Solution: Velocity Calculations

From a – t graph, velocity can be obtained by using.

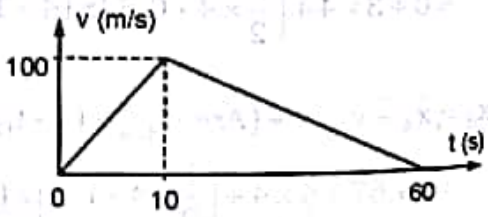
$v_f = v_i + (\text{area under } a - t \text{ curve})$

$$v_{10} = v_0 + (\text{area under } a - t \text{ curve})_{0-10}$$

$$= 0 + (10 \times 10) = 100 \text{ m/s}$$

$$v_{t'} = v_{10} + (\text{area under } a - t \text{ curve})_{10-t'}$$

$$= 100 + [-(t' - 10) \times 2]$$

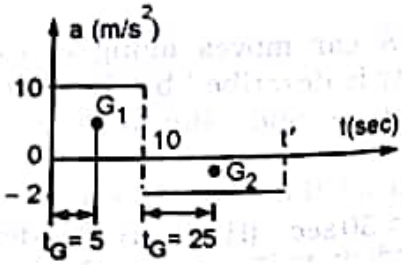


But $v_{t'} = 0$ ..... since at t', the car stops

$\therefore \quad 0 = 100 - (t' - 10) \times 2$

$\therefore \quad t' = 60$ sec ... **Ans.**

The v – t graph has been plotted as shown. ... **Ans.**



Position calculations

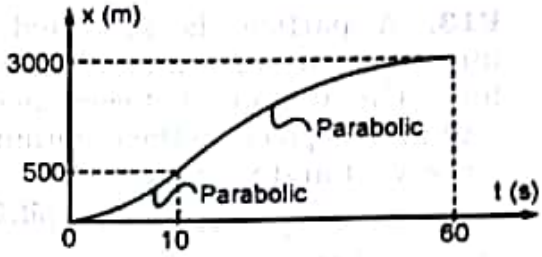
From a – t graph, position can be calculated using

$x_f = x_i + v_i \times t + + (\text{Area under } a - t \text{ curve}) \times (t - t_G)$

..... here $t = t_f - t_i$

$$x_{10} = x_0 + v_0 \times t + (\text{Area})_{0-10} \times (t - t_G)$$
$$= 0 + 0 + (10 \times 10) \times (10 - 5) = 500\,\text{m}$$

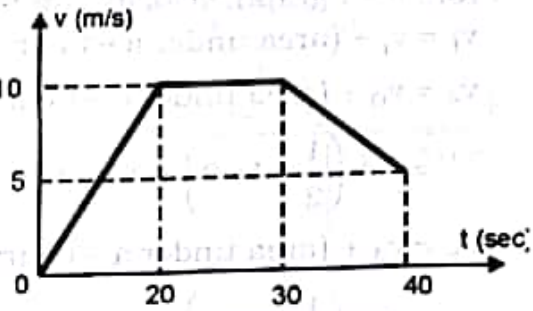
$$x_{60} = x_{10} + v_{10} \times t + (\text{Area})_{10-60} \times (t - t_G)$$
$$= 500 + 100 \times 50 + (-50 \times 2) \times (50 - 25) = 3000\,\text{m}$$



The x – t graph has been plotted as shown ..... **Ans.**

P12. Velocity time graph for a particle moving along a straight line is shown. Draw displacement-time and acceleration-time graphs. Also find the maximum displacement of the particle.

(M. U. May 11)



Solution: <u>Acceleration Calculations</u>

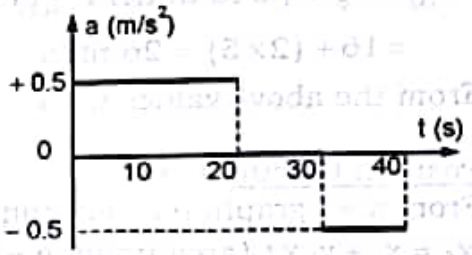
From v – t graph, acceleration can be obtained by using

$$a = \text{slope of } v - t \text{ curve}$$

$$a_{(0-20)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{20 - 0} = 0.5\ \text{m/s}^2$$

$a_{(20-30)} = 0$ .... since the graph is horz.

$$a_{(30-40)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 10}{40 - 30} = -0.5\ \text{m/s}^2$$



From above values the a – t graph is plotted as shown. ..... **Ans.**

<u>Position calculations</u> [Taking $x_0 = 0$, $v_0 = 0$]

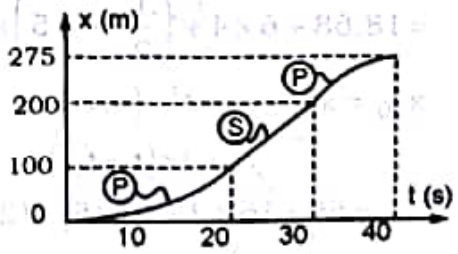
From v – t graph, position can be calculated using

$$x_f = x_i + (\text{area under } v - t \text{ curve})$$

$$x_{20} = x_0 + (\text{area under } v - t \text{ curve})_{0-20}$$
$$= 0 + \left(\frac{1}{2} \times 20 \times 10\right) = 100\ \text{m}$$

$$x_{30} = x_{20} + (\text{area under } v - t \text{ curve})_{20-30}$$
$$= 100 + (10 \times 10) = 200\ \text{m}$$

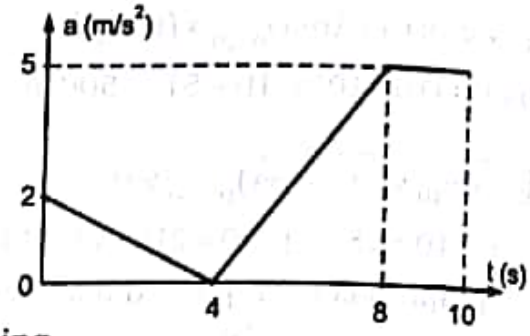
$$x_{40} = x_{30} + (\text{area under } v - t \text{ curve})_{30-40}$$
$$= 200 + \left(\frac{1}{2} \times (10 + 5) \times 10\right) = 275\ \text{m}$$



The x – t graph, has been plotted as shown ..... **Ans.**

P13. A particle is projected with an initial velocity of 2 m/s along a straight line. The relation between acceleration and time is given in the diagram. Draw v – t and x – t diagrams.

(M.U Dec 14)



Solution: Velocity Calculations

From a – t graph, velocity can be obtained by using

$v_f = v_i$ + (area under a – t curve)

$v_4 = v_0$ + (area under a – t curve)$_{0-4}$

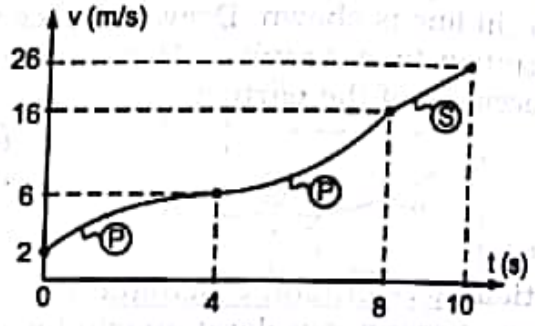
$$= 2 + \left(\frac{1}{2} \times 4 \times 2\right) = 6 \text{ m/s}$$

$v_8 = v_4$ + (area under a – t curve)$_{4-8}$

$$= 6 + \left(\frac{1}{2} \times 4 \times 5\right) = 16 \text{ m/s}$$

$v_{10} = v_8$ + (area under a – t curve)$_{8-10}$

$$= 16 + (2 \times 5) = 26 \text{ m/s}$$



From the above values the v – t curve is drawn as shown. ..... **Ans.**

Position Calculations

From a – t graph, position can be obtained by using

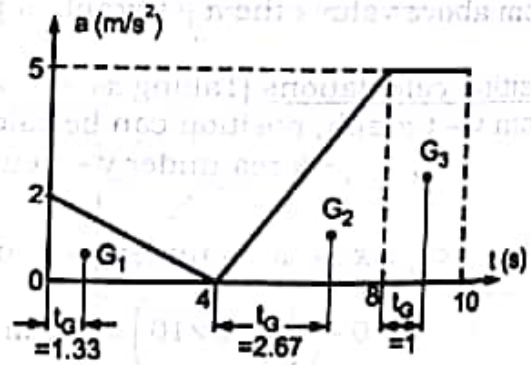
$x_f = x_i + v_i \times t$ (area under a – t curve)$(t - t_G)$ ..... here $t = t_f - t_i$

$x_4 = x_0 + v_0 \times t$ + (area under a – t curve)$_{0-4}$ $\times (t - t_G)$

$$= 0 + 2 \times 4 + \left(\frac{1}{2} \times 4 \times 2\right) \times (4 - 1.33)$$

$$= 18.68 \text{ m}$$

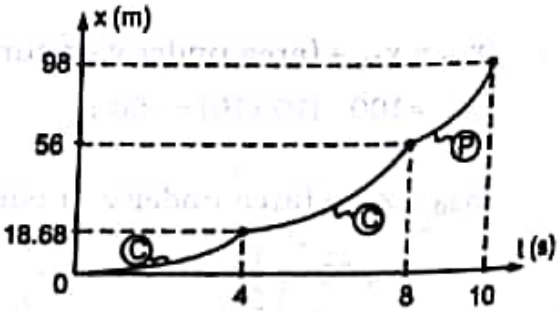
$x_8 = x_4 + v_4 \times t$ + (area under a – t curve)$_{4-8}$ $\times (t - t_G)$



$$= 18.68 + 6 \times 4 + \left(\frac{1}{2} \times 4 \times 5\right) \times (4 - 2.67) = 56 \text{ m}$$

$x_{10} = x_8 + v_8 \times t$ + (area under a – t curve)$_{8-10}$ $\times (t - t_G)$

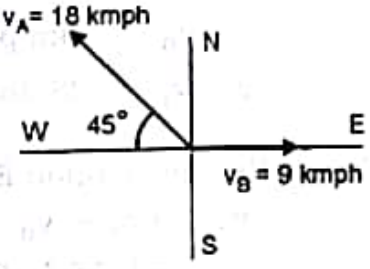
$$= 56 + 16 \times 2 + (2 \times 5) \times (2 - 1) = 98 \text{ m}$$



Exercise 9.6

Relative Motion

P1. A ship A travels in the north making an angle of 45° to the West with a velocity of 18 km/hr and ship B travels in the East with a velocity of 9 km/hr. find the relative velocity of B w.r.t. ship A. ***(M.U. Dec 13)***



Solution: From the figure drawn, we have

$$\bar{v}_A = -12.728\,\mathbf{i} + 12.728\,\mathbf{j} \text{ kmph}$$

$$\bar{v}_B = 9\,\mathbf{i} \text{ kmph}$$

Using Relative Motion Equation

$$\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A$$

$$= (9\mathbf{i}) - (-12.728\,\mathbf{i} + 12.728\,\mathbf{j})$$

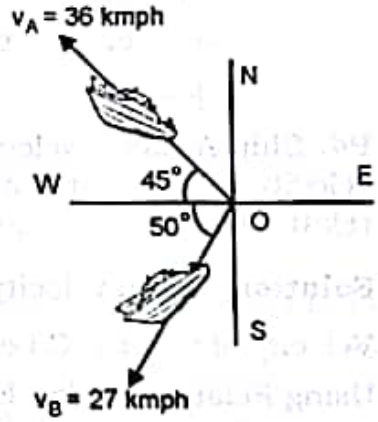
$$\therefore \quad \bar{v}_{B/A} = 21.728\,\mathbf{i} - 12.728\,\mathbf{j} \text{ kmph}$$

or $\quad v_{B/A} = 25.18 \text{ kmph}, \theta = 30.36°$ ↘ ..... **Ans.**

P2. Two ships A and B leave a port at the same time. A travels at 36 kmph in the North-West direction while B travels at 50° south of west at 27 kmph.

a) Determine the relative velocity of B w.r.t A.

b) After how much time will the two ships be 120 km apart.



Solution: $v_A = 36 \text{ kmph}, \theta = 45°$ ↖

$$\therefore \quad v_A = -25.45\,\mathbf{i} + 25.25\,\mathbf{j} \text{ kmph}$$

$$v_B = 27 \text{ kmph}, \theta = 50° \swarrow$$

$$\therefore \quad \bar{v}_B = -17.35\,\mathbf{i} - 20.68\,\mathbf{j} \text{ kmph}$$

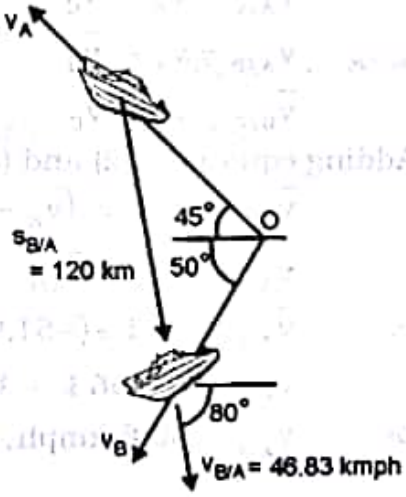
Using Relative Motion Equation for B w.r.to A

$$\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A$$

$$= (-17.35\,\mathbf{i} - 20.68\,\mathbf{j}) - (-25.45\,\mathbf{i} + 24.45\,\mathbf{j})$$

$$= 8.1\,\mathbf{i} - 46.13\,\mathbf{j} \text{ kmph}$$

or $\quad \bar{v}_{B/A} = 46.83 \text{ kmph}, \theta = 80°$ ↘ ..... **Ans.**

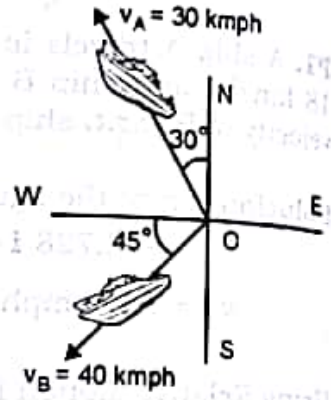


Let after t hours the two ships be apart by 120 km. At this instant position of B w.r.t A will be $s_{B/A} = 120$ km

Using $\quad v = \dfrac{s}{t} \quad \therefore \quad v_{B/A} = \dfrac{s_{B/A}}{t}$

$\therefore \quad 46.83 = \dfrac{120}{t}$ or $t = 2.562$ hrs. ..... **Ans.**

P3. a) Two ships move from a port at the same time. Ship 'A' has velocity of 30 km/hr and is moving North 30° West, while ship 'B' is moving in South-West direction with a velocity of 40 km/hr. Determine relative velocity of 'A' w.r.t. 'B'. ***(M. U. May 09)***
b) Also find the gap between the ships 2 hrs. later.



Solution: $v_A = 30$ kmph, $\theta = 60°$ ↖

$\therefore \quad \bar{v}_A = -15\,\mathbf{i} + 25.92\,\mathbf{j}$ kmph

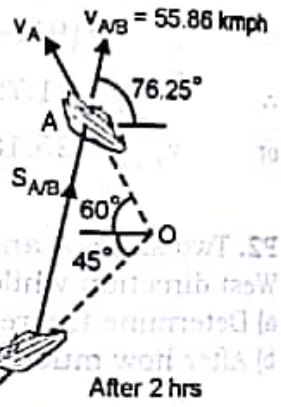
$v_B = 40$ kmph, $\theta = 45°$ ↙

$\therefore \quad \bar{v}_B = -28.28\,\mathbf{i} - 28.28\,\mathbf{j}$ kmph

Using Relative Motion Equation for A w.r.to B

$$\bar{v}_{A/B} = \bar{v}_A - \bar{v}_B$$
$$= (-15\,\mathbf{i} + 25.98\,\mathbf{j}) - (-28.28\,\mathbf{i} - 28.28\,\mathbf{j})$$
$$= 13.28\,\mathbf{i} + 54.26\,\mathbf{j} \text{ kmph}$$

or $\quad v_{A/B} = 55.86$ kmph, $\theta = 76.25°$ ↗ ..... **Ans.**



Let after 2 hrs, the two ships be s kilometres apart. At this instant position of A w.r.t. B will $s_{A/B}$

Using $\quad v = \dfrac{s}{t} \quad \therefore \quad v_{A/B} = \dfrac{s_{A/B}}{t} \quad \therefore \quad 55.86 = \dfrac{s_{A/B}}{2}$

or $\quad s_{A/B} = 111.72$ km ..... **Ans.**

P4. Ship A has a velocity of 45 kmph due east relative to ship B, which in turn has a velocity of 60 kmph at 30° South of West relative to ship C. Find the velocity of A relative to C.

Solution: Given velocity of A w.r.t. B i.e. $\bar{v}_{A/B} = 45$ kmph → $\therefore \quad \bar{v}_{A/B} = 45\,\mathbf{i}$ kmph →

Velocity of B w.r.t. C i.e. $\bar{v}_{B/C} = 60$ kmph, $\theta = 30°$ ↙ $\therefore \quad \bar{v}_{B/C} = -51.96\,\mathbf{i} - 30\,\mathbf{j}$ kmph

Using Relative Motion Equation, we have

$$\bar{v}_{A/C} = \bar{v}_A - \bar{v}_C \quad \text{..... (1)}$$
$$\bar{v}_{A/B} = \bar{v}_A - \bar{v}_B \quad \text{..... (2)}$$
$$\bar{v}_{B/C} = \bar{v}_B - \bar{v}_C \quad \text{..... (3)}$$

Adding equations (2) and (3), we get

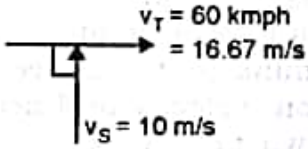
$$\bar{v}_{A/B} + \bar{v}_{B/C} = (\bar{v}_A - \bar{v}_B) + (\bar{v}_B - \bar{v}_C) = \bar{v}_A - \bar{v}_C$$
$$\bar{v}_{A/B} + \bar{v}_{B/C} = \bar{v}_{A/C}$$
$$\therefore \quad \bar{v}_{A/C} = 45\,\mathbf{i} + (-51.96\,\mathbf{i} - 30\,\mathbf{j})$$
$$\therefore \quad \bar{v}_{A/C} = -6.96\,\mathbf{i} - 30\,\mathbf{j}$$

or $\quad v_{A/C} = 30.8$ kmph, $\theta = 76.94°$ ↙ ..... **Ans.**

P5. a) A train running at 60 kmph to the right is struck by a stone thrown at right angles to the train with a speed of 10 m/s. Find the velocity and direction with which the stone appears to strike the train, to a person sitting at the window in the train.
b) Solve if velocity of train is 36 kmph and velocity of stone = 18 kmph. ***(VJTI Nov 10)***

Solution: Given $v_T = 16.67\,m/s \rightarrow \quad \therefore \quad \bar{v}_T = 16.67\,\mathbf{i}\ m/s$

$v_S = 10\ m/s \uparrow \quad \therefore \quad \bar{v}_S = 10\,\mathbf{j}\ m/s$



Using Relative Motion Equation for stone w.r.t. train

$$\bar{v}_{S/T} = \bar{v}_S - \bar{v}_T$$

$$\bar{v}_{S/T} = (10\,\mathbf{j}) - (16.67\,\mathbf{i})$$

or $\quad \bar{v}_{S/T} = -16.67\,\mathbf{i} + 10\,\mathbf{j}\ m/s$

$$v_{S/T} = \sqrt{16.67^2 + 10^2} = 19.44\ m/s$$

$$\tan\theta = \frac{10}{16.67} \quad \therefore \quad \theta = 30.96^\circ \nwarrow$$

..... 2nd Quad. since $\mathbf{i}$ is - ve and $\mathbf{j}$ is + ve

$\therefore \quad \bar{v}_{S/T} = 19.44\ m/s,\ \theta = 30.96^\circ \nwarrow$ ..... **Ans.**

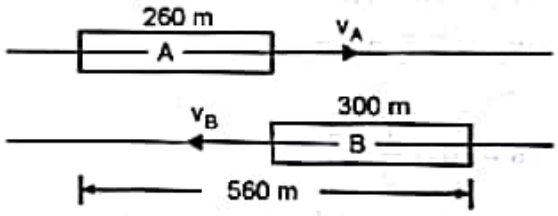
P6. Trains A and B are traveling on parallel tracks in opposite directions. Velocity of A is thrice the velocity of B. The trains take 20 sec to cross each other. Determine the velocities of the trains knowing that train A is 260 m long and train B is 300 m long.

Solution: Velocity of train A = $v_A \rightarrow$

or $\bar{v}_A = v_A\,\mathbf{i}\ m/s$

Velocity of train B = $v_B \leftarrow$

or $\bar{v}_B = -\,v_B\,\mathbf{i}\ m/s$



Using Relative Motion Equation for A w.r.t. B

$$\bar{v}_{A/B} = \bar{v}_A - \bar{v}_B$$

$$= v_A\,\mathbf{i} - (-v_B\,\mathbf{i})$$

$$= 3v_B\,\mathbf{i} + v_B\,\mathbf{i} \qquad \text{.... since } v_A = 3v_B$$

$$= 4v_B\,\mathbf{i}$$

$$\therefore\ v_{A/B} = 4v_B \rightarrow$$

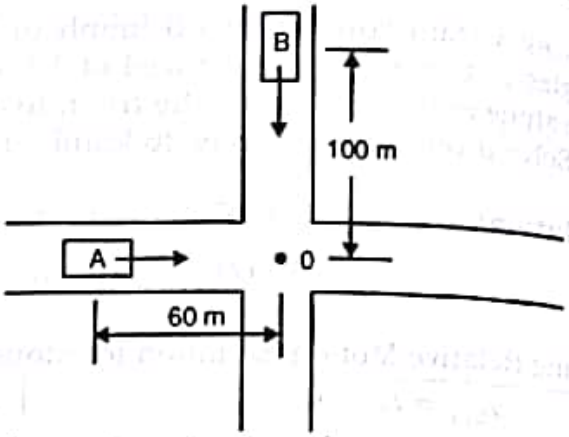
Note: $\bar{v}_{A/B}$ implies velocity of A w.r.t. B. It also implies that B is stationary w.r.t. A.
If B is stationary w.r.t. A, Train A needs to travel 560 m in 20 sec for crossing train B.

$\therefore \quad s_{A/B} = 560\ m$

Using $v = \dfrac{s}{t} \quad \therefore \quad v_{A/B} = \dfrac{s_{A/B}}{t} \quad \therefore \quad 4v_B = \dfrac{560}{20}$ or $v_B = 7\ m/s$ ... **Ans.**

also $\quad v_A = 3\,v_B = 21\ m/s$ ..... **Ans.**

P7. At t = 0 the location of cars A and B are shown. The cars travel towards the intersection. Car A has a speed of 10 m/s and a deceleration of 1.5 m/s². Car B starts from the rest and accelerates at 2 m/s². Determine at t = 3 sec the relative position, velocity and acceleration of car B w.r.t car A.



Solution: Motion of Car A

Rectilinear – Uniform Acceleration

$u = 10$ m/s

$v = v_A$

$s = s_A$

$a = -1.5$ m/s²

$t = 3$ sec

Using $v = u + at$

$v_A = 10 - 1.5 \times 3$

$\therefore \quad v_A = 5.5$ m/s

Also $s = ut + \frac{1}{2}at^2$

$\therefore \quad s_A = 10 \times 3 + \frac{1}{2} \times (-1.5) \times 3^2$

$\therefore \quad s_A = 23.25$ m

At t = 3 sec

$a_A = -1.5$ m/s² →

or $\bar{a}_A = -1.5\,\mathbf{i}$ m/s²

also $v_A = 5.5$ m/s →

or $\bar{v}_A = 5.5\,\mathbf{i}$ m/s

Location of Car A from O = 60 – 23.25
= 36.75 m

i.e Car A is located at 36.75 m left of O

$\therefore \quad \bar{r}_A = -36.75\,\mathbf{i}$ m

Motion of Car B

Rectilinear – Uniform Acceleration

$u = 0$

$v = v_B$

$s = s_B$

$a = 2$ m/s²

$t = 3$ sec

Using $v = u + at$

$v_B = 0 + 2 \times 3$

$\therefore \quad v_B = 6$ m/s

Also $s = ut + \frac{1}{2}at^2$

$\therefore \quad s_B = 0 + \frac{1}{2} \times 2 \times 3^2$

$\therefore \quad s_B = 9$ m

At t = 3 sec

$a_B = 2$ m/s² ↓

or $\bar{a}_B = -2\,\mathbf{j}$ m/s²

also $v_B = 6$ m/s ↓

or $\bar{v}_B = -6\,\mathbf{j}$ m/s

Location of Car B from O = 100 – 9
= 91 m

i.e Car B is located at 91 m above O

$\therefore \quad \bar{r}_B = 91\,\mathbf{j}$ m

Using Relative Motion Equation for B w.r.to A

$$\bar{a}_{B/A} = \bar{a}_B - \bar{a}_A$$
$$= (-2\,\mathbf{j}) - (-1.5\,\mathbf{i})$$
$$= 1.5\,\mathbf{i} - 2\,\mathbf{j}\ \text{m/s}^2$$

or $\bar{a}_{B/A} = 2.5\ \text{m/s}^2,\ \theta = 53.13°$ ↘ ..... **Ans.**

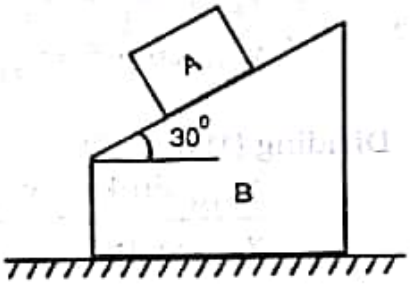
$$\bar{v}_{B/A} = \bar{v}_B - \bar{v}_A$$
$$= (-6\,\mathbf{j}) - (5.5\,\mathbf{i})$$
$$= -5.5\,\mathbf{i} - 6\,\mathbf{j}\ \text{m/s}$$

or $\bar{v}_{B/A} = 8.14\ \text{m/s},\ \theta = 47.49°$ ↙ ..... **Ans.**

$$\bar{r}_{B/A} = \bar{r}_B - \bar{r}_A$$
$$= (91\,\mathbf{j}) - (-36.75\,\mathbf{i})$$
$$= 36.75\,\mathbf{i} + 91\,\mathbf{j}\ \text{m/s}$$

or $\bar{r}_{B/A} = 98.14\ \text{m},\ \theta = 68°$ ↗ ..... **Ans.**

P8. Wedge B travels to the left with acceleration of 2 m/s², while block A travels up the slope of the wedge with an acceleration of 1 m/s² relative to the wedge. Determine the true velocity of the block at t = 5 sec after starting from rest.



Solution:

Given $a_B = 2\ \text{m/s}^2$ ← $\therefore\ \bar{a}_B = -2\,\mathbf{i}\ \text{m/s}^2$

$a_{A/B} = 1\ \text{m/s}^2,\ \theta = 30°$ ↗ $\therefore\ \bar{a}_{A/B} = 0.866\,\mathbf{i} + 0.5\,\mathbf{j}\ \text{m/s}^2$

Using Relative Motion Equation for A w.r.t. B

$$\bar{a}_{A/B} = \bar{a}_A - \bar{a}_B$$
$$0.866\,\mathbf{i} + 0.5\,\mathbf{j} = \bar{a}_A - (-2\mathbf{i})$$
$$\bar{a}_A = -1.134\,\mathbf{i} + 0.5\,\mathbf{j}\ \text{m/s}^2$$

or $a_A = 1.239\ \text{m/s}^2,\ \theta = 23.77°$ ↖

$$a_A = \sqrt{1.134^2 + 0.5^2} = 1.239\ \text{m/s}^2$$

$$\tan\theta = \frac{0.5}{1.134} \quad \therefore\ \theta = 23.77°$$ ↖

..... 2nd Quad. since **i** is – ve and **j** is + ve

Motion of block A – Rectilinear – Uniform Acceleration

$u = 0,\ v = v_A\ \text{m/s},\ s = -,\ a = 1.239\ \text{m/s}^2,\ t = 5\ \text{sec}$

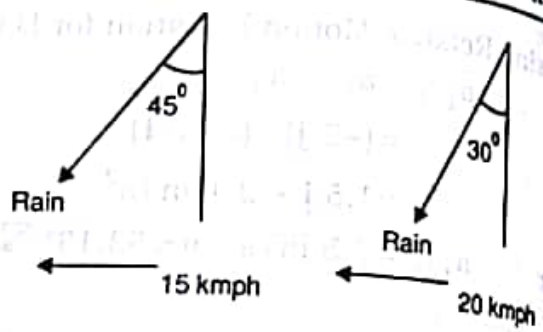
Using $v = u + at$

$$v_A = 0 + 1.239 \times 5$$

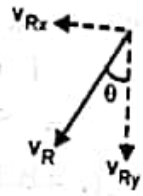
or $v_A = 6.197\ \text{m/s}$

$\therefore$ The velocity of block A = 6.197 m/s, $\theta = 23.77°$ ↖ ..... **Ans.**

P9. A man walking on a straight path at 15 kmph finds that the rain is falling at an angle of 45° to the vertical. As the man increases his speed to 20 kmph, he finds the rain now falling at 30° with the vertical. What is the true velocity of the rain. Refer Figure.



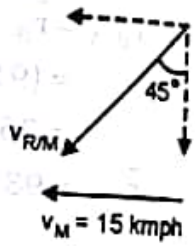
Solution: Let v_R be the true velocity of the rain falling at some angle θ with the vertical as shown. Let v_{Rx} and v_{Ry} be the components along x and y directions.



Situation (1): When man walks at 15 kmph
Applying Relative Motion Equation for
Man w.r.t. Rain in x direction, taking ← + ve

$$v_{R/M} = v_R - v_M$$

$$v_{R/M} \sin 45 = v_{Rx} - 15 \qquad \text{..... (1)}$$



Applying Relative Motion Equation for
Man w.r.t. Rain in y direction, taking ↓ + ve

$$v_{R/M} = v_R - v_M$$

$$v_{R/M} \cos 45 = v_{Ry} - 0 \qquad \text{..... (2)}$$

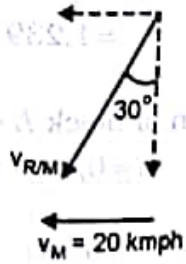
Dividing (1) and (2)

$$\frac{v_{R/M} \sin 45}{v_{R/M} \cos 45} = \frac{v_{Rx} - 15}{v_{Ry}} \quad \text{or} \quad \tan 45 = \frac{v_{Rx} - 15}{v_{Ry}} \qquad \text{..... (A)}$$

Situation (2): When man walks at 20 kmph.
Since the true velocity of rain remains unaffected with the change in speed of man, equation (A) obtained above can be used to get equation (B) as below.

Replacing angle 45° with 30° and speed of man 15 kmph with 20 kmph, we have

$$\tan 30 = \frac{v_{Rx} - 20}{v_{Ry}} \qquad \text{..... (B)}$$



Solving equations (A) and (B), we get

$$v_{Rx} = 26.83 \text{ kmph} \quad \text{and} \quad v_{Ry} = 11.83 \text{ kmph}$$

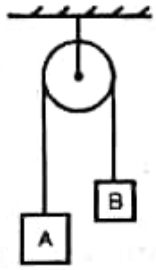
or $$v_R = \sqrt{v_{Rx}^2 + v_{Ry}^2} = \sqrt{26.83^2 + 11.83^2} = 29.32 \text{ kmph}$$

and $$\tan\theta = \frac{v_{Rx}}{v_{Ry}} = \frac{26.83}{11.83} \quad \therefore \quad \theta = 66.2° \text{ ↙} \qquad \text{..... Ans.}$$

Exercise 9.7

Dependent Motion

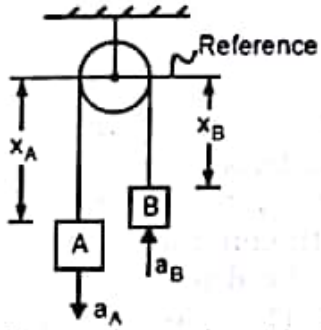
P1. If block A moves down with an acceleration of 2.5 m/s², determine the corresponding acceleration of block B.



Solution: Blocks A and B are two particles connected by a common string forming a dependent system.

Using CSLM

Taking a fixed reference as shown. Let x_A and x_B be the positions of A and B respectively from the reference. The total length L of the string in terms of variable position x_A and x_B is



$$L = x_A + x_B \pm \text{constants} \quad \text{..... (1)}$$

As blocks move, x_A would increase, while x_B would decrease. Accordingly correcting equation (1) we have

$$L = x_A + (-x_B) \pm \text{constants} \quad \text{..... (2)}$$

Note: String portion whose length do not change as the particles move, come under constants. Example string portion wrapped over the pulley

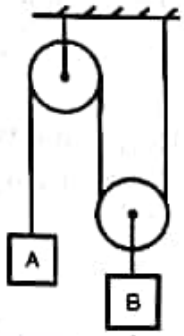
Differentiating twice w.r.t. time, we get

$$0 = a_A - a_B \qquad \text{Or} \qquad a_A = a_B \quad \text{..... (3)}$$

Given $a_A = 2.5\ m/s^2 \downarrow$, substituting this value in equation (3), we get

$$a_B = 2.5\ m/s^2 \qquad \text{Or} \qquad a_B = 2.5\ m/s^2 \uparrow \quad \text{..... } \textbf{Ans.}$$

P2. Knowing that the block A is moving up with a velocity v_A and acceleration a_A, determine the corresponding velocity and acceleration of block B.

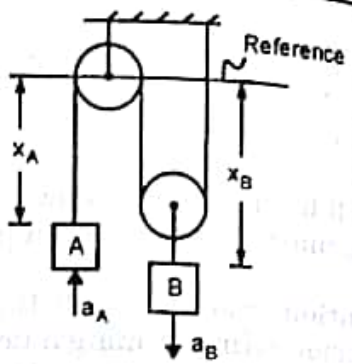


Solution: Blocks A and B are two particles connected by a common string forming a dependent motion system.

Using CSLM

Taking a fixed reference as shown. Let x_A and x_B be the positions of A and B respectively from the reference. The total length L of the string in terms of variable position x_A and x_B is

$$L = x_A + 2x_B \pm \text{constants} \quad \text{..... (1)}$$



As blocks move, x_A would decrease, while x_B would increase. Accordingly correcting equation (1) we have

$$L = (-x_A) + 2x_B \pm \text{constants} \quad \text{..... (2)}$$

Differentiating w.r.t. time, we get

$$0 = -v_A + 2v_B$$

$$\therefore \quad v_B = \frac{v_A}{2} \quad \text{..... (3) ..... Ans.}$$

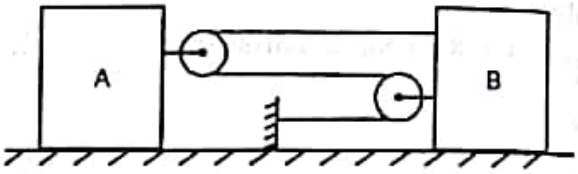
Differentiating equation (3) w.r.t. time, we get

$$a_B = \frac{a_A}{2} \quad \text{..... (4) ..... Ans.}$$

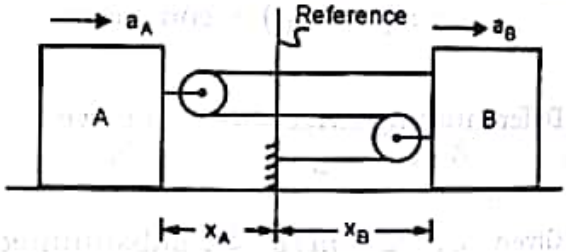
P3. Blocks A and B are connected by an inextensible string as shown. If block B moves to the right, starting from rest with constant acceleration of 3 m/s².

a) Find the acceleration of block A.

b) The velocity of block A at t = 8 sec.



Solution: Blocks A and B are two particles connected by a common string forming a dependent system.



Using CSLM

Taking a fixed reference as shown. Let x_A and x_B be the positions of A and B respectively from the reference. The total length L of the string in terms of variable positions x_A and x_B is

$$L = 2x_A + 3x_B \pm \text{constants} \quad \text{..... (1)}$$

As blocks move, x_A would decrease, while x_B would increase. Accordingly correcting equation (1) we have

$$L = (-2x_A) + 3x_B \pm \text{constants} \quad \text{..... (2)}$$

Differentiating twice w.r.t. time, we get

$$0 = -2a_A + 3a_B$$

$$\therefore \quad a_A = \frac{3}{2}a_B \quad \text{..... (3)}$$

since $a_B = 3\ m/s^2$, we get $a_A = 4.5\ m/s^2 \rightarrow$ ..... **Ans.**

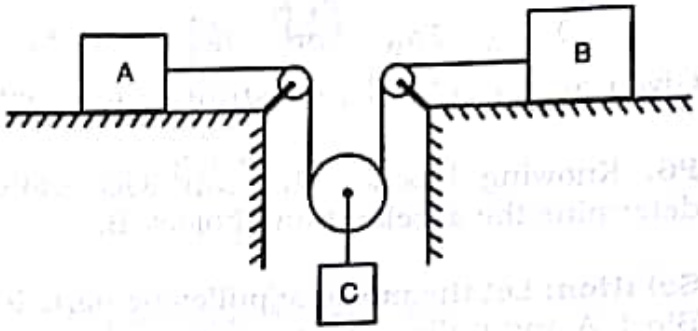
Motion of block A – Rectilinear – Uniform Acceleration

Using $v = u + at$

$$v = 0 + 4.5 \times 8 = 36\ m/s$$

or $v_A = 36\ m/s \rightarrow$ ..... **Ans.**

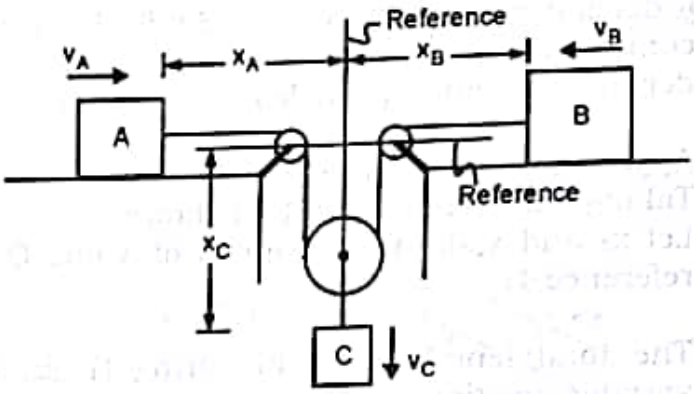
P4. Knowing velocity of A is 3 m/s to the right and that of B is 4 m/s to the left, determine the velocity of block C.



Solution: Blocks A, B and C are three particles connected by a common string forming a dependent motion system.

Using CSLM

Taking a horizontal and a vertical fixed reference as shown. Let x_A, x_B and x_C be the positions of A, B and C respectively from the reference. The total length L of the string in terms of variable position x_A, x_B and x_C is



$$L = x_A + x_B + 2x_C \pm \text{constants} \quad .... (1)$$

As blocks move, x_A and x_B would decrease, while x_C would increase.

Accordingly correcting equation (1), we have

$$L = (-x_A) + (-x_B) + 2x_C \pm \text{constants} \quad ... (2)$$

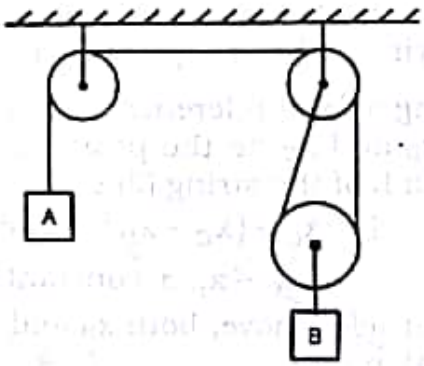
Differentiating w.r.t. time, we get

$$0 = -v_A - v_B + 2v_C \qquad \therefore \qquad v_C = \frac{v_A + v_B}{2} \quad ... (3)$$

Given $v_A = 3$ m/s →, $v_B = 4$ m/s ← substituting in equation (3), we get

$$v_C = \frac{3+4}{2} = 3.5 \text{ m/s} \quad \text{or} \quad v_C = 3.5 \text{ m/s} \downarrow \quad ..... \textbf{Ans.}$$

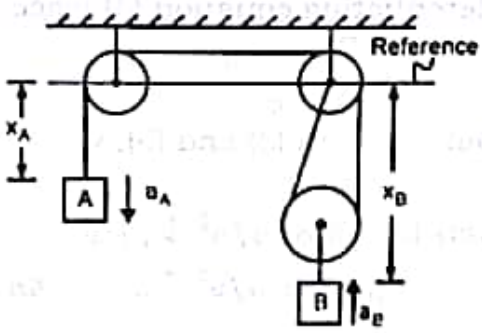
P5. Knowing acceleration of block B = 3 m/s² ↑, determine the acceleration of block A.



Solution: Blocks A and B are two particles connected by a common string forming a dependent motion system.

Using CSLM

Taking a fixed horizontal reference as shown. Let x_A and x_B be the positions of A and B respectively from the reference. The total length L of the string in terms of variable positions x_A and x_B is



$$L = x_A + 2x_B \pm \text{constants} \quad ..... (1)$$

As blocks move, x_A would increase, while x_B would decrease. Accordingly correcting equation (1) we have

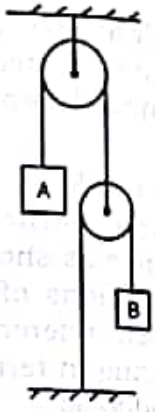
$$L = x_A + (-2x_B) \pm \text{constants} \quad ..... (2)$$

Differentiating twice w.r.t. time, we get

$0 = a_A - 2a_B$ or $a_A = 2\,a_B$ ..... (3)

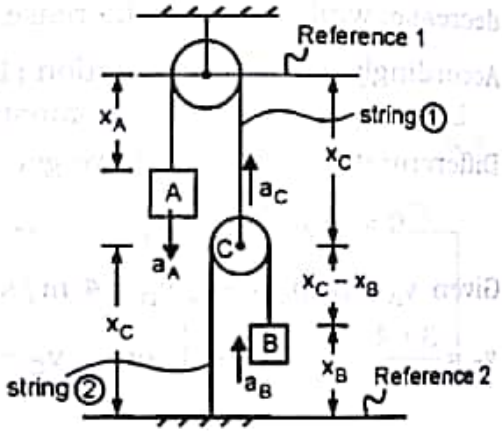
Given $a_B = 3\ m/s^2 \uparrow$, substituting in equation (3), we get $a_A = 6\,m/s^2 \downarrow$ ..... **Ans.**

P6. Knowing block A has an acceleration of $0.8\ m/s^2 \downarrow$, determine the acceleration of block B.



Solution: Let the movable pulley be particle C.
Block A and pulley C are connected by a common string forming a dependent motion system. Similarly block B and pulley C are connected by another string forming a different pair of dependent motion particles.

Applying CSLM to system of A and C.
Taking a fixed reference 1 as shown.
Let x_A and x_C be the positions of A and C respectively from the reference 1.



The total length L of the string (1) in terms of variable positions x_A and x_C is

$L = x_A + x_C \pm$ constants ..... (1)

As particles move, x_A would increase, while x_C would decrease. Accordingly correcting equation (1), we have

$L = x_A + (-x_C) \pm$ constants ..... (2)

Differentiating twice w.r.t. time, we get

$0 = a_A - a_C$

or $a_A = a_C$ ..... (3)

Applying CSLM to system of B and C.

Taking a fixed reference 2 as shown.
Let x_B and x_C be the positions of B and C respectively from the reference 2. The total length L of the string (2) in terms of variable positions x_B and x_C is

$L = x_C + (x_C - x_B) \pm$ constants

$\therefore$ $L = 2x_C - x_B \pm$ constants ..... (4)

As particles move, both x_B and x_C would increase,. Accordingly no change is required in equation (4).
Differentiating equation (4) twice w.r.t. time, we get

$0 = 2a_C - a_B$

or $a_B = 2\,a_C$ ..... (5)

From equation (3) and (5), we have

$a_B = 2\,a_A$ ..... (6)

Given $a_A = 0.8\ m/s^2 \downarrow$, substituting this value in equation (5), we get

$\therefore$ $a_B = 1.6\,m/s^2 \uparrow$ ..... **Ans.**

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