Solutions: Chapter 12

Kinetics of Particles:

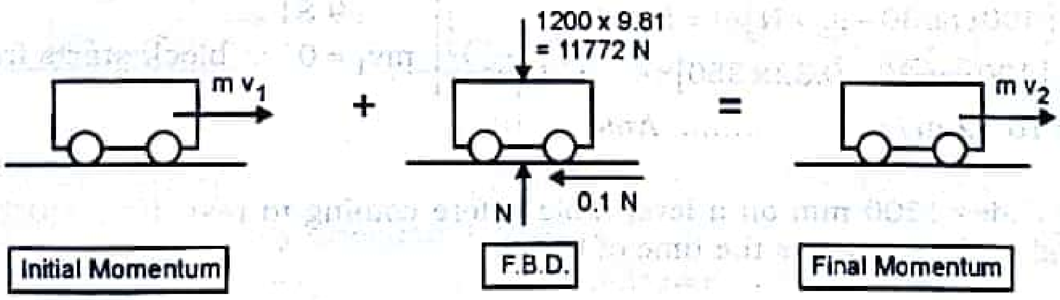
Impulse Momentum Method

Exercise 12.1

Kinetics using Impulse Momentum Equation

P1. A 1200 kg automobile is traveling at 90 kmph when brakes are fully applied, causing all four wheels to skid. If μ_k is 0.1, find the time required for the automobile to come to halt.

Solution: Applying Impulse Momentum Equation (I M E) to the automobile. Three figures of the automobile are drawn below. The L.H.S. and R.H.S. figures represent the initial and final momentum, while the centre figure represents the FBD and is drawn to calculate the impulse. Let the automobile take 't' seconds to come to rest.



Applying I M E in the y direction ↑ + ve

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

$$0 + [N - 11772] \times t = 0$$

$$\therefore \quad N = 11772 \text{ N}$$

Impulse = Force × Time

In y direction velocity is zero ∴ momentum is zero

Applying I M E in the x direction → + ve

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

$$1200 \times 25 - [0.1N] \times t = 0$$

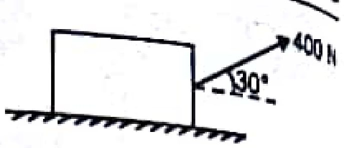
$$30000 - 0.1 \times 11772 \times t = 0$$

$$\therefore \quad t = 25.48 \text{ sec}$$

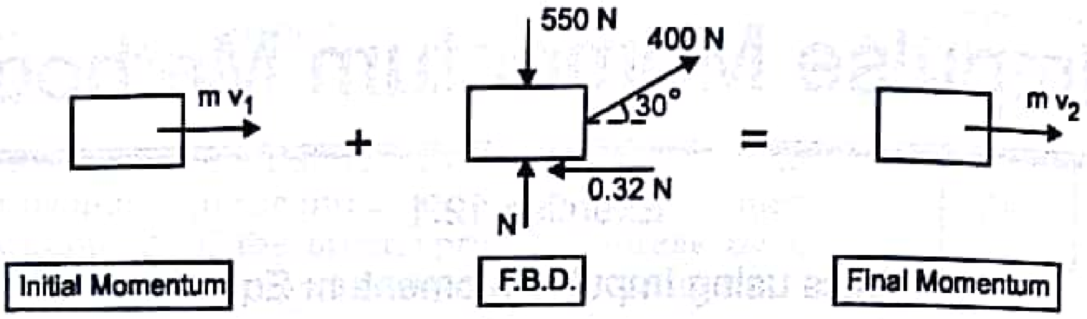
$mv_2 = 0$ because the car comes to rest.

.... **Ans.**

P2. The 550 N box rests on a horizontal plane for which the coefficient of kinetic friction, μ_k = 0.32. If the box is subjected to a 400 N towing force as shown, find the velocity of the box in 4 seconds starting from rest. ***(M.U May 15)***



Solution: Let v be the velocity of the block after 4 sec.



Applying I M E in the y direction ↑ + ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$0 + [N + 400 \sin 30 - 550] \times 4 = 0$$

or $N = 350$ N

In y direction velocity is zero ∴ mv_1 and mv_2 are zero.
t = 4 sec using Impulse = F×t

Applying I M E in the x direction → + ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$0 + [400 \cos 30 - \mu_s \times N] \times t = 56.06 \times v$$

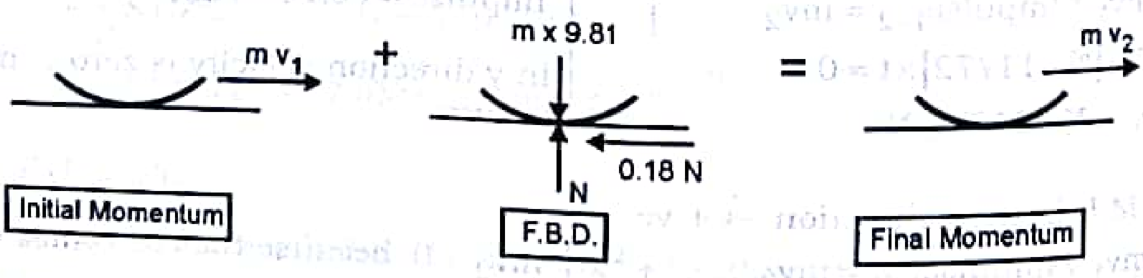
∴ $$0 + [400 \cos 30 - 0.32 \times 350] \times 4 = 56.06 v$$

or $v = 16.72$ m/s .... **Ans.**

$m = \dfrac{550}{9.81} = 56.06$ kg
$mv_1 = 0$ ∵ block starts from rest.

P3. A dish slides 1200 mm on a level table before coming to rest. If μ_k = 0.18 between the dish and table, what was the time of travel.

Solution: Applying Impulse Momentum Equation (I M E) to the dish in motion. Let the initial speed of the dish be v_1 and let it travel for t sec before coming to a halt.



Applying I M E in the y direction ↑ + ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$0 + [N - mg] \times t = 0$$

or $N = m \times 9.81$

Impulse = Force × Time
In y direction velocity is zero ∴ momentum is zero

Applying I M E in the x direction $\rightarrow$ + ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$mv_1 - [0.18N] \times t = 0$$

$$\therefore \quad mv_1 - 0.18 \times (m \times 9.81) \times t = 0$$

or $\quad v_1 = 1.7658\,t$ sec

Kinematics: Motion of dish – Rectilinear – Uniform acceleration

$u = 1.7658t$ m/s, $v = 0$, $s = 1.2$ m, $a = ?$, $t = ?$

Using $v^2 = u^2 + 2as$

$$0 = (1.7658\ t)^2 + 2 \times a \times 1.2$$

Or $\quad a = -1.3\ t^2\ m/s^2$

Using $v = u + at$

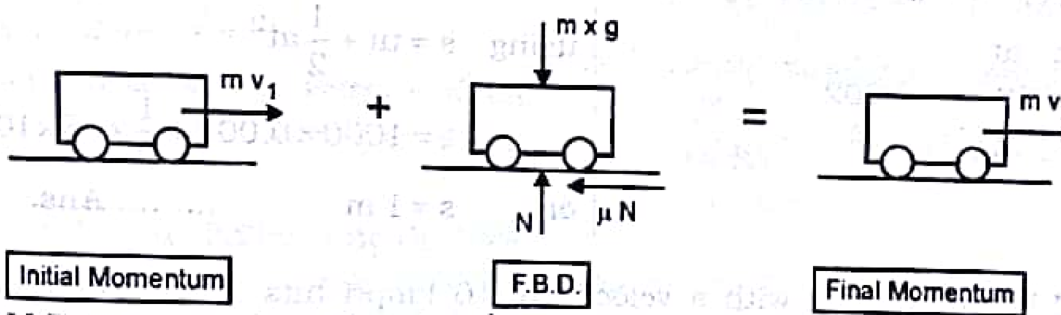
$$0 = 1.7658\ t - 1.3t^2 \times t$$

$$1.358 = t^2$$

Or $\quad t = 1.165$ sec .... **Ans.**

P4. A car at 72 kmph applies brakes and comes to rest in 8 sec. Find the minimum coefficient of friction between wheel and road.

Solution: Let μ be the coefficient of friction at the ground.



Applying I M E to the car in y direction ↑ + ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$0 + [N - mg] \times t = 0$$

or $\quad N = m \times g$

Impulse = F × t

In y direction velocity is zero ∴ momentum is zero

Applying I M E to the car in x direction $\rightarrow$ + ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$m \times 20 - [\mu \times N] \times t = 0$$

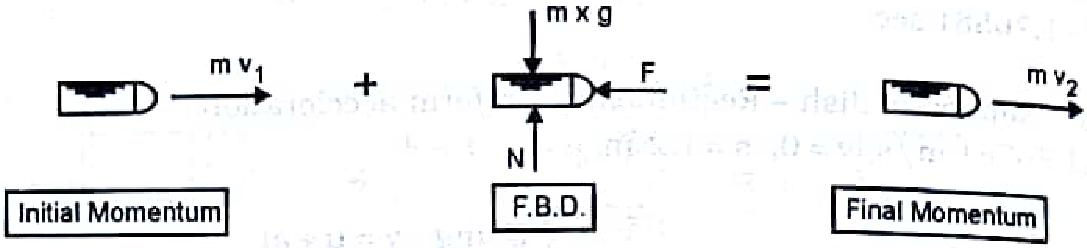
$$20m - \mu \times mg \times 8 = 0$$

$\therefore \quad \mu = 0.255$ .... **Ans.**

$mv_2 = 0 \quad \because$ the car comes to rest.

P5. A bullet of mass 1 gram has a velocity of 1000 m/s as it enters a fixed block of wood. It comes to rest in 2 milliseconds after entering the block. Determine the average force that acted on the bullet and the distance penetrated by it.

Solution: Let F be the average retarding force acting on the bullet.



Applying I M E to the bullet in x direction $\rightarrow$ + ve

$mv_1 + Impulse_{1-2} = mv_2$

$0.001 \times 1000 - [F \times 0.002] = 0$

$\therefore \quad F = 500\,N$ .... **Ans.**

$Impulse = F \times t$

$t = 0.002$ sec

$mv_2 = 0 \quad \because$ final velocity is zero.

Kinematics

Motion of bullet – Rectilinear – Uniform acceleration

$u = 1000$ m/s, $v = 0$, $s = ?$, $a = ?$, $t = 0.002$ sec

using $v = u + at$

$0 = 1000 + a \times 0.002$

or $a = -5 \times 10^5\ m/s^2$

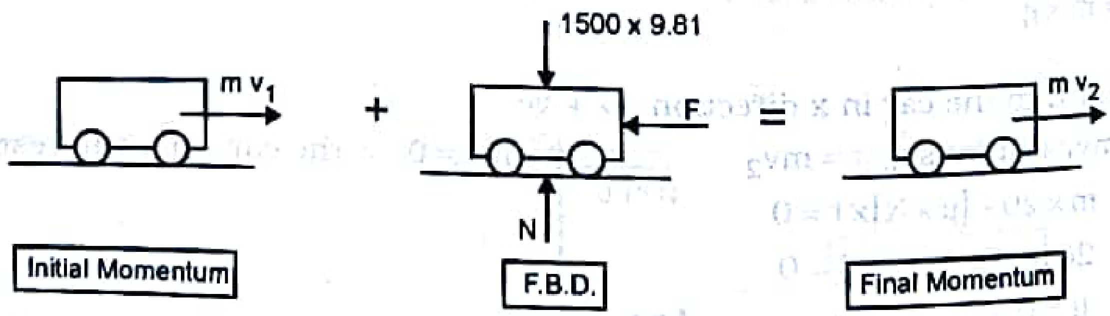
using $s = ut + \frac{1}{2}at^2$

$s = 1000 \times 0.002 + \frac{1}{2} \times -5 \times 10^5 \times 0.002^2$

or $s = 1\ m$ .... **Ans.**

P6. A 1500 kg car moving with a velocity of 10 kmph hits a compound wall and is brought to rest in 400 milliseconds. What is the average impulsive force exerted by the wall on the car bumper?

Solution: Let F be the force exerted by the wall on the car.



Applying I M E to the car in x direction $\rightarrow$ + ve

$mv_1 + Impulse_{1-2} = mv_2$

$1500 \times 2.778 - F \times 0.4 = 0$

$\therefore \quad F = 10417\ N$ .... **Ans.**

$Impulse = F \times t = -F \times 0.4$

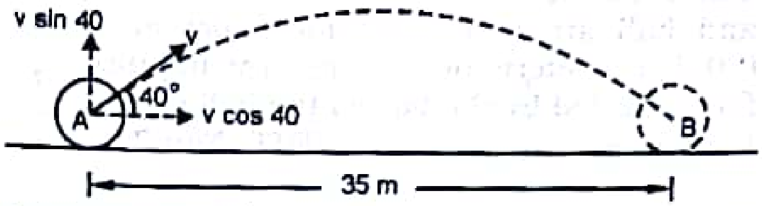
10 kmph = 2.778 m/s

400 milliseconds = 0.4 sec

$mv_2 = 0 \quad \because$ final velocity is zero.

P7. A 380 gm football is kicked by a player so that it leaves the ground at an angle of 40° with the horizontal and lands on the ground 35 m away. Determine the impulse given to the ball. Also find the impulsive force if the contact was for 0.3 sec.

Solution: Let the velocity of the ball be v just after the kick. The ball now performs projectile motion from A to B.



Projectile motion (A – B)

Horizontal Motion

$v = v\cos 40$

$s = 35$ m

$t = t$

Using $v = \dfrac{S}{t}$

$v\cos 40 = \dfrac{35}{t}$

or $v = \dfrac{35}{t\cos 40}$ .... (1)

Substitute $t = 2.447$ sec

We get $v = 18.67$ m/s ...... velocity of ball just after the kick

Vertical Motion ↑ + ve

$u = v\sin 40$

$v = -$

$s = 0$

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \dfrac{1}{2}at^2$

$0 = v\sin 40 \times t + \dfrac{1}{2}\times(-9.81)\times t^2$ ...... (2)

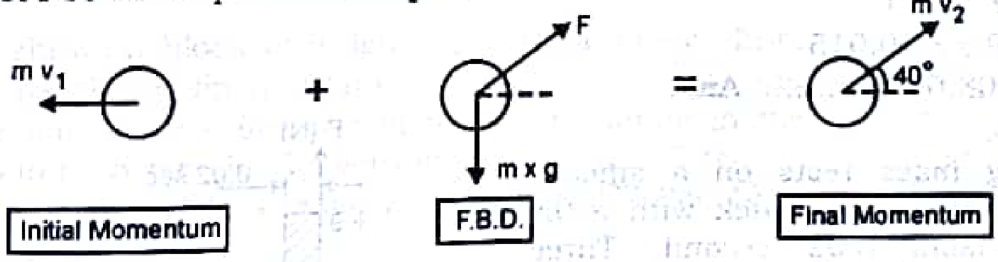
Substitute equation (1) and (2)

$0 = \dfrac{35}{t\cos 40}\sin 40 \times t - 4.905\times t^2$

$\therefore$ $t = 2.447$ sec

Applying I M E to the ball during the kick

Let F be the impulsive force imparted by the player on the ball. Due to this force acting for 0.3 sec, let I be the impulse developed,



$mv_1 + \text{Impulse}_{1-2} = mv_2$

$0 + I = 0.38 \times 18.67$

$\therefore$ $I = 7.095$ N sec .... **Ans.**

Initial velocity before the ball is kicked is zero

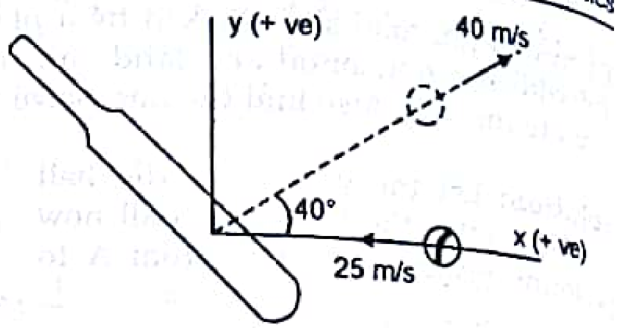
$\therefore$ $mv_1 = 0$

Now Impulse = $F \times t$

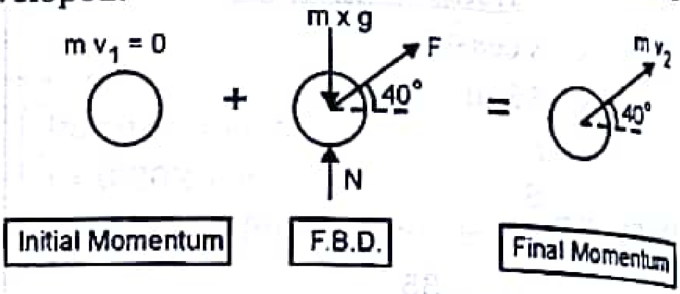
$7.095 = F \times 0.3$

Or $F = 23.65$ N .... **Ans.**

P8. A ball of mass 100 g is moving towards a bat with a velocity of 25 m/s as shown in figure. When hit by a bat, the ball attains a velocity of 40 m/s. If the bat and ball are in contact for a period of 0.015 s. Determine the average impulse force exerted by the bat on the ball during the impact. ***(VJTI Nov 09)***



Solution: Let F be the impulsive force imparted by the bat on the ball. Due to this force acting for 0.015 sec, let I be the impulse developed.



Applying I M E to the ball in x direction $\rightarrow +$ ve

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

$$-0.1 \times 25 + I_x = 0.1 \times 40 \cos 40$$

$$\therefore \quad I_x = 5.564 \text{ N sec}$$

Applying I M E to the ball in y direction $\uparrow +$ ve

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

$$0 + I_y = 0.1 \times 40 \sin 40$$

$$\therefore \quad I_y = 2.571 \text{ N sec}$$

Total Impulse $I = \sqrt{5.564^2 + 2.571^2}$

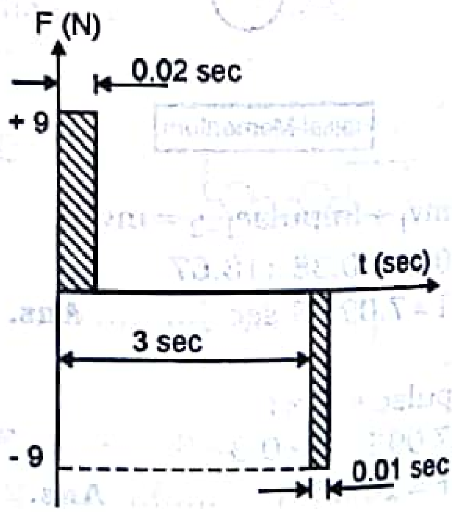
$$\therefore \quad I = 6.129 \text{ N sec}$$

Now Impulse $= F \times t$

$$6.129 = F \times 0.015$$

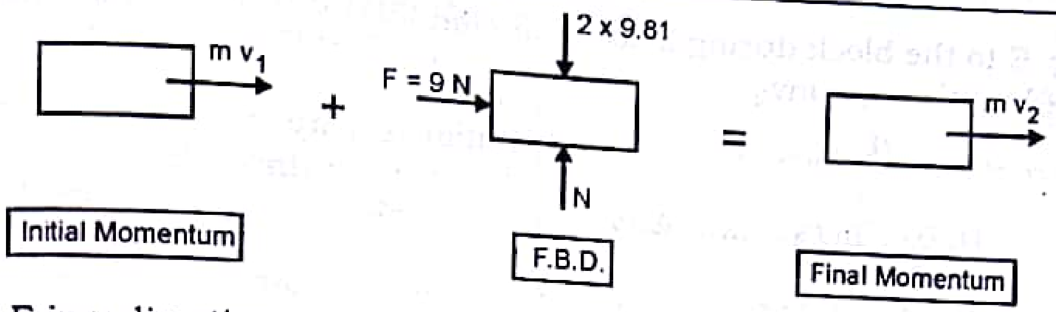
or $\quad F = 408.63$ N .... **Ans.**

P9. A 2 kg mass rests on a smooth horizontal plane. It is struck with a 9N blow that lasts 0.02 second. Three seconds after the start of the first blow a second blow of 9N is delivered. This lasts for 0.01 second. What is the speed of the body after 4 seconds? Refer figure. ***(VJTI May 08)***



Solution: *First Blow*

Let the 2 kg mass acquire a speed v_1 after being hit by a blow of 9 N acting to the right and lasting for 0.02 sec.



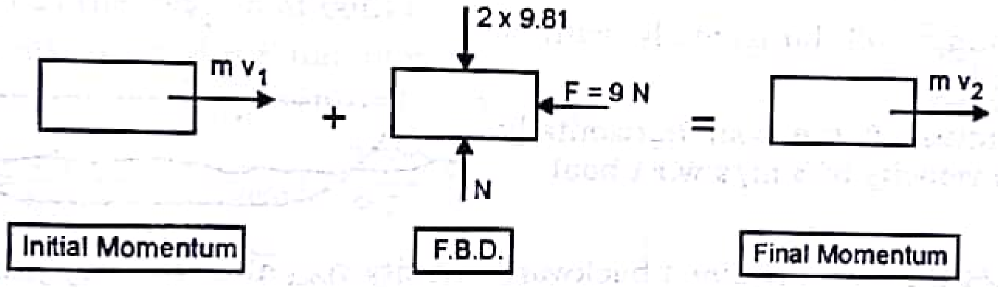
Applying I M E in x direction $\rightarrow +$ ve

$$mv_1 + Impulse_{1-2} = mv_2$$

$$0 + 9 \times 0.02 = 2 \times v_1 \qquad \therefore \qquad v_1 = 0.09 \text{ m/s}$$

Second blow

The second blow of 9 N acts to the left. Let v_2 be the new velocity at the end of the blow lasting for 0.01 sec. Also note that for a period of 2.98 sec between the first and second blow, the 2 kg mass moves with constant speed of 0.09 m/s, since no unbalanced force act on it.

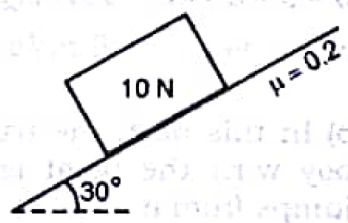


Applying I M E in x direction $\rightarrow +$ ve

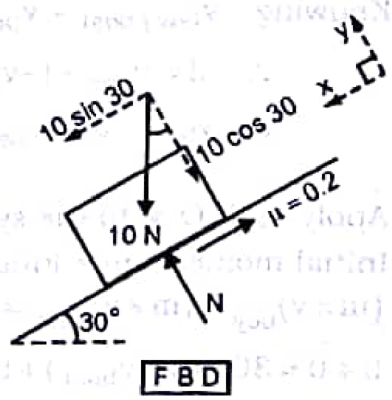
$$mv_1 + Impulse_{1-2} = mv_2$$

$$2 \times 0.09 - 9 \times 0.01 = 2 \times v_2 \qquad \therefore \qquad v_2 = 0.045 \text{ m/s} \rightarrow$$ .... **Ans.**

P10. Figure shows a block of weight 10 N sliding down from rest on a rough inclined plane. Taking $\mu = 0.2$ and $\theta = 30°$, calculate (i) The impulse of the forces acting in the interval t = 0 to t = 5 sec (ii) The velocity at the end of 5 sec (iii) The distance covered by the block. ***(VJTI Dec 13)***



Solution: Impulse is the product of force and time for which force acts. The forces acting on the 10 N block are shown in the FBD.



$$F = 10\sin 30 - 0.2N$$

$$\therefore \quad F = 10\sin 30 - 0.2 \times (10\cos 30)$$

$$\therefore \quad F = 3.268 \text{ N}$$

Net force acting on the block in the x direction

Now Impulse $= F \times t$

$= 3.268 \times 5$

$= 16.34$ Nsec .... **Ans.**

Applying I M E to the block during 5 sec of motion

$mv_1 + Impulse_{1-2} = mv_2$

$0 + 16.34 = \frac{10}{9.81} \times v$

$\therefore \quad v = 16.029\ m/s$ .... **Ans.**

Initial velocity is zero

$\therefore \quad mv_1 = 0$

Kinematics of block
Motion – rectilinear – uniform acceleration
$u = 0, \quad v = 16.029\ m/s, \quad s = ?, \quad a = ?, \quad t = 5\ sec$

Using $v = u + at$

$16.029 = 0 + a \times 5$

$\therefore \quad a = 3.2058\ m/s^2$

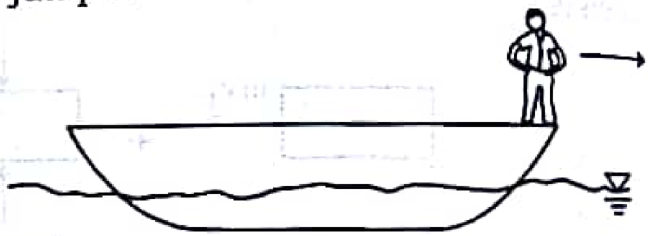
Using $s = ut + \frac{1}{2}at^2 \quad \therefore \quad s = 0 + \frac{1}{2} \times 3.0258 \times 5^2$

$\therefore \quad s = 40.07\ m$

P11. A 30 kg boy stands stationary on a 50 kg boat. If the boy jumps off the boat, determine the velocity of the boat just after the jump if

a) The boy jumps off horizontally with a velocity of 3 m/s.

b) The boy jumps off the boat horizontally with a relative velocity of 3 m/s w.r.t boat.



Solution: a) Let the boat acquire a backward velocity v_{boat} after the boy jumps from it.

Applying COM to the system of boy and boat in x direction $\rightarrow +ve$

Initial momentum = Final momentum

$(m \times v)_{boy} + (m \times v)_{boat} = (m \times v)_{boy} + (m \times v)_{boat}$

$0 + 0 = 30 \times 3 + 50(-v_{boat})$

$\therefore \quad v_{boat} = 1.8\ m/s \leftarrow$ .... **Ans.**

Initial momentum is zero, since initially both boy and boat are at rest

b) In this part, the true velocity of the boy is not given, instead the relative velocity of boy w.r.t the bloat is given. Also assuming the boat moves backward after the boy jumps from it.

Knowing $v_{boy/boat} = v_{boy} - v_{boat}$ ...... using Relative Motion Equation

$\therefore \quad 3 = v_{boy} - (-v_{boat})$

or $v_{boy} = 3 - v_{boat}$ ... true velocity of boy

Applying C O M to the system of boy and boat in the x direction $\rightarrow +ve$

Initial momentum = Final momentum

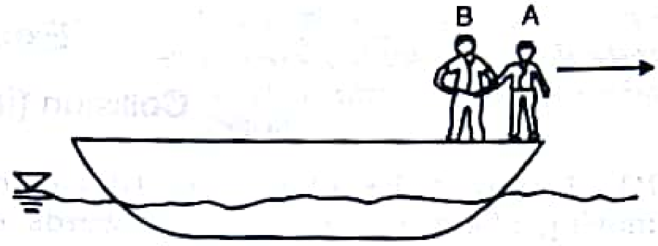
$(m \times v)_{boy} + (m \times v)_{boat} = (m \times v)_{boy} + (m \times v)_{boat}$

$0 + 0 = 30 \times (3 - v_{boat}) + 50 \times (-v_{boat})$

$\therefore \quad v_{boat} = 1.125\ m/s \leftarrow$ .... **Ans.**

Initial momentum is zero, since initially both boy and boat are at rest

P12. Two men A (500 N) and B (700 N) lined up at one end of the boat (3000 N) and dived horizontally off the boat in succession with a velocity of 4 m/s relative to the boat. Find the velocity of the boat after the second man B had dived. Neglect water resistance.



Solution: In this problem the true velocity of the men is not given, instead their relative velocity w.r.t boat is given. Also assuming the boat moves backwards as the men jump from it.

Knowing $v_{man/boat} = v_{man} - v_{boat}$.

$$4 = v_{man} - (-v_{boat})$$

Or $v_{man} = 4 - v_{boat}$ .... true velocity of man.

This means that every time a man jump from the boat, his true speed = $(4 - v_{boat})$

Let the first man A jump.

Applying COM to the system of both men A, B and boat in x direction $\rightarrow +ve$

Initial momentum = Final momentum

$$(m \times v)_A + (m \times v)_{boat} = (m \times v)_A + (m \times v)_{boat}$$

$$0 + 0 = \frac{500}{9.81} \times (4 - v_{boat}) + \frac{3700}{9.81} \times (-v_{boat})$$

$\therefore$ $v_{boat} = 0.476 \text{ m/s} \leftarrow$

Initial momentum is zero, since both men and boat are at rest

It means that just after the first man jumps, the boat acquires a backward speed of 0.476 m/s .

Now let the second man B jump from the boat moving backward with 0.476 m/s.

Applying COM to the system of man B and boat in x direction $\rightarrow +ve$

Initial momentum = Final momentum

$$(m \times v)_{boat} = (m \times v)_B + (m \times v)_{boat}$$

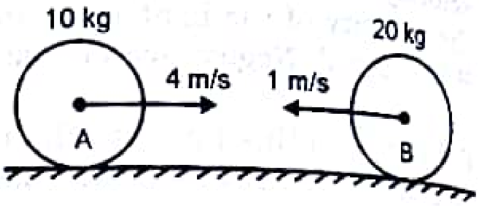
$$\frac{3700}{9.81} \times (-0.476) = \frac{700}{9.81} \times (4 - v_{boat}) + \frac{3000}{9.81} \times (-v_{boat})$$

$$-1761.2 = 2800 - 3700 \times v_{boat}$$

$\therefore$ $v_{boat} = 1.232 \text{ m/s} \leftarrow$ .... **Ans.**

Exercise 12.2

Collision (Impact) of Bodies



P1. Two particles of masses 10 kg and 20 kg are moving along a straight line towards each other at velocities of 4 m/s amd 1 m/s respectively as shown. If e = 0.6, determine the velocities of the particles immediately after collision. Also find the loss of kinetic energy. *(VJTI Dec 13, KJS Nov 15)*

Solution: This is a case of direct impact.

Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$10 \times 4 + 20 \times (-1) = 10 \times v_A' + 20 \times v_B'$$

$$\therefore \quad v_A' + 2v_B' = 2 \quad \text{......... (1)}$$

Applying C O R equation → + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$\therefore \quad v_B' - v_A' = 0.6(4 - (-1))$$

$$\therefore \quad -v_A' + v_B' = 3 \quad \text{......... (2)}$$

Solving equations (1) and (2)

$$v_A' = -1.33 \text{ m/s} \quad \therefore \quad v_A' = 1.33 \text{ m/s} \leftarrow$$

and $\quad v_B' = 1.67 \text{ m/s} \quad \therefore \quad v_B' = 1.67 \text{ m/s} \rightarrow$ .... **Ans.**

Kinetic Energy before impact

$$K.E_1 = \frac{1}{2} \times 10 \times 4^2 + \frac{1}{2} \times 20 \times 1^2 = 90 \text{ J}$$

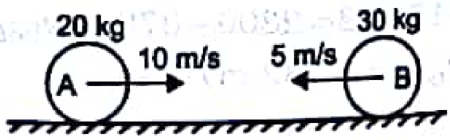
Kinetic Energy after impact

$$K.E_2 = \frac{1}{2} \times 10 \times 1.33^2 + \frac{1}{2} \times 20 \times 1.67^2 = 36.73 \text{ J}$$

Loss in K.E. = $K.E_1 - K.E_2 = 90 - 36.73 = 53.27$ J .... **Ans.**

P2. Two balls with masses 20 kg and 30 kg are moving towards each other with velocities 10 m/s and 5 m/s respectively. If after impact the ball having mass 30 kg reverse its direction of motion and moves with velocity of 6 m/s, then determine the coefficient of restitution between the two balls. *(M.U Dec 15)*

Solution: This is a case of direct impact.



Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$20 \times 10 + 30 \times (-5) = 20 v_A' + 30 \times 6$$ .... Given $v_B' = 6$ m/s →

$$\therefore \quad v_A' = -6.5 \text{ m/s} \quad \text{or} \quad v_A' = 6.5 \text{ m/s} \leftarrow$$

Applying C O R equation → + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$6 - (-6.5) = e(10 - (-5)) \quad \therefore \quad e = 0.833$$ .... **Ans.**

P3. Ball A of mass 0.6 kg moving to the right with a velocity of 4 m/s has a direct central impact with ball B of mass 0.3 kg moving to left with a velocity of 1 m/s. If after impact the velocity of ball B is observed to be 5 m/s to the right, determine the coefficient of restitution between the two balls.

Solution: This is a case of direct impact.

Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$0.6 \times 4 + 0.3 \times (-1) = 0.6 v_A' + 0.3 \times 5$$

$$\therefore \quad v_A' = 1 \text{ m/s} \rightarrow$$

Applying C O R equation → + ve

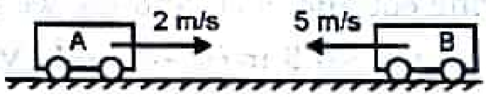
$$v_B' - v_A' = e(v_A - v_B)$$

$$5 - 1 = e(4 - (-1))$$

$$\therefore \quad e = 0.8 \quad \text{......... Ans.}$$

P4. A railway wagon weighing 400 kN and moving to the right with a velocity of 2 m/s, collides with another wagon weighing 200 kN and moving with a velocity of 5 m/s in the opposite direction. If the two wagons move together after impact, find the magnitude and direction of their common velocity. Also find the percentage loss of kinetic energy of the system.

Solution: This is a case of direct impact. Also it is given that both the wagons have a common velocity v after impact. This is a case of plastic impact.



Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$400 \times 2 + 200 \times (-5) = 400 \times v + 200 \times v$$

$$\therefore \quad v = -0.333 \text{ m/s}$$

$$\text{Or} \quad v = 0.333 \text{ m/s} \leftarrow \quad \text{......... Ans.}$$

Note that the multiplying factor, converting weight in kN to mass in kg cancels out.

Kinetic Energy before impact

$$K.E_1 = \frac{1}{2} \times \frac{400 \times 10^3}{9.81} \times 2^2 + \frac{1}{2} \times \frac{200 \times 10^3}{9.81} \times 5^2 = 336391 \text{ J}$$

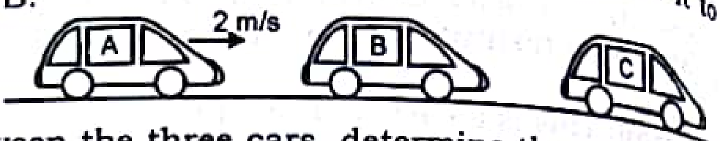
Kinetic Energy after impact

$$K.E_2 = \frac{1}{2} \times \frac{(400 + 200) \times 10^3}{9.81} \times 0.333^2 = 3391 \text{ J}$$

$$\% \text{ loss in K.E.} = \frac{K.E_1 - K.E_2}{K.E_1} \times 100 = \frac{336391 - 3391}{336391} \times 100 = 98.99 \quad \text{......... Ans.}$$

P5. Three identical cars A, B and C are ready for dispatch from a factory. At the instant the cars have their brakes released. An accidental forward push to car A causes it to move with a velocity of 2 m/s and hit car B.

(a) Determine the speed of cars A and B just after the impact.



(b) If a series of collision takes place between the three cars, determine the velocities of the three cars after all collisions are over.

Take e = 0.7 between the bumpers.

Solution: a) First collision between car A and car B.

This is a case of direct impact. Applying C O M equation → + ve

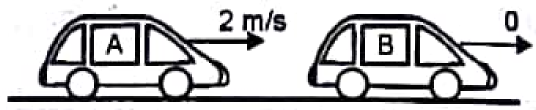
Masses are identical, so they cancel.

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$2 + 0 = v_A' + v_B'$$

$$\therefore \quad v_A' + v_B' = 2 \qquad \text{......... (1)}$$

Applying C O R equation → + ve



$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = 0.7(2 - 0)$$

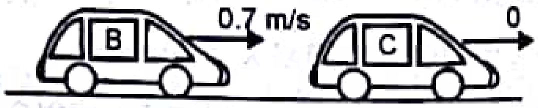
$$\therefore \quad -v_A' + v_B' = 1.4 \qquad \text{......... (2)}$$

Solving equations (1) and (2), we get

$$v_A' = 0.3 \text{ m/s} \rightarrow \quad \text{and} \quad v_B' = 1.7 \text{ m/s} \rightarrow \qquad \text{......... } \textbf{Ans.}$$

b) Car B moving with 0.7 m/s now collides with car C which was initially at rest.

Applying C O M equation → + ve



$$m_B v_B + m_C v_C = m_B v_B' + m_C v_C'$$

$$1.7 + 0 = v_B' + v_C'$$

$$\therefore \quad v_B' + v_C' = 1.7 \qquad \text{......... (3)}$$

Applying C O R equation → + ve

$$v_C' - v_B' = e(v_B - v_C)$$

$$v_C' - v_B' = 0.7(1.7 - 0)$$

$$\therefore \quad -v_B' + v_C' = 1.19 \qquad \text{......... (4)}$$

Note: v_A' *implies velocity of A after impact and* v_B' *implies velocity of B after impact*

Solving equations (3) and (4), we get $v_B' = 0.255 \text{ m/s} \rightarrow$ and $v_C' = 1.445 \text{ m/s} \rightarrow$

At the end of 2nd collision we note that car A has a speed of 0.3 m/s →, while car B is moving at 0.255 m/s →. This results in 3rd collision between car A and car B.

Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$0.3 + 0.255 = v_A' + v_B' \qquad \therefore \qquad v_A' + v_B' = 0.555 \quad \text{......... (5)}$$

Applying C O R equation → + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = 0.7(0.3 - 0.255) \qquad \therefore \qquad -v_A' + v_B' = 0.0315 \quad \text{......... (6)}$$

Solving equations (5) and (6), we get

$$v_A' = 0.2617 \text{ m/s} \rightarrow \quad \text{and} \quad v_B' = 0.293 \text{ m/s} \rightarrow$$

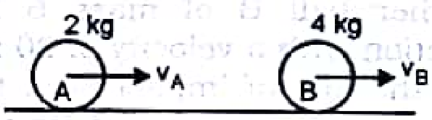
No more collisions take place since $v_A' < v_B' < v_C'$, the three cars now travel with

$$v_A' = 0.2617 \text{ m/s} \rightarrow, v_B' = 0.293 \text{ m/s} \rightarrow \quad \text{and} \quad v_C' = 1.445 \text{ m/s} \rightarrow \text{......... } \mathbf{Ans.}$$

P6. A ball of mass 2kg impinges on a ball of mass 4 kg which is moving in the same direction as the first. The coefficient of restitution is ¾ and the first ball is reduced to rest after the impact. Find the ratio between velocities of balls before the impact.

(VJTI Nov 10)

Solution: This is a case of direct impact between the two balls A and B.



Also given $e = \dfrac{3}{4}$ and $v_A' = 0$

Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$2 \times v_A + 4 \times v_B = 0 + 4 \times v_B'$$

$$\therefore \quad 2v_A + 4v_B = 4v_B' \quad \text{......... (1)}$$

Applying C O R equation → + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$\therefore \quad v_B' - 0 = \frac{3}{4}(v_A - v_B) \quad \text{......... (2)}$$

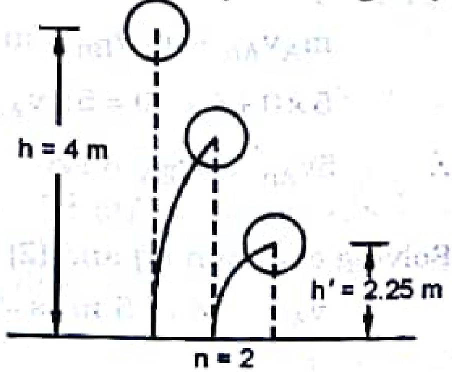
Substituting equation (2) in (1)

$$2v_A + 4v_B = 4 \times \frac{3}{4}(v_A - v_B) \qquad \therefore \qquad v_A = 7v_B \quad \text{......... } \mathbf{Ans.}$$

P7. A ball is dropped on to a smooth horizontal floor from a height of 4 m. On the second bounce it attains a height of 2.25 m. What is the coefficient of restitution between the ball and the floor?

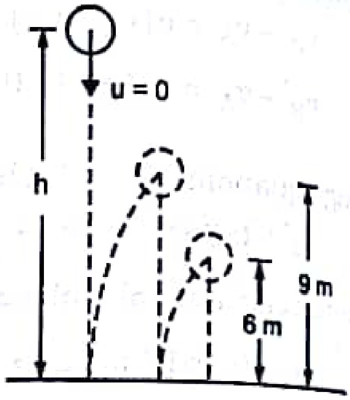
(M.U May 11)

Solution: For a ball dropped from a height h on a smooth floor, rising to a height of h′ after n bounces, we have a relation $e = \left(\dfrac{h'}{h}\right)^{\frac{1}{2n}}$



$$\therefore \qquad e = \left(\frac{2.25}{4}\right)^{\frac{1}{2\times 2}} \quad \text{or} \quad e = 0.866 \quad \text{......... } \mathbf{Ans.}$$

P8. A glass ball is dropped on to a smooth horizontal floor from which it bounces to a height of 9 m. On the second bounce it rises to a height of 6 m. From what height the ball was dropped and what is the coefficient of restitution between the glass and the floor. ***(M.U Dec 16)***



Solution: For a ball dropped from a height h on a smooth floor, rising to a height of h' after n bounces, we have a relation $e = \left(\frac{h'}{h}\right)^{\frac{1}{2n}}$

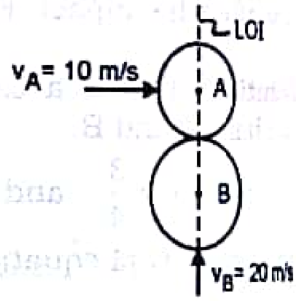
Case (1) : $h = 9$ m, $h' = 6$ m, $n = 1$

$\therefore \quad e = \left(\frac{6}{9}\right)^{\frac{1}{2\times1}} \qquad \therefore \quad e = 0.8165$ ... **Ans.**

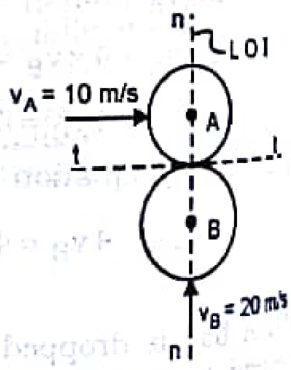
Case (2) : $h = ?$, $h' = 9$ m, $n = 1$, $e = 0.8165$

$\therefore \quad 0.8165 = \left(\frac{9}{h}\right)^{\frac{1}{2\times1}} \qquad \therefore \quad h = 13.5$ m ... **Ans.**

P9. A smooth spherical ball A of mass 5 kg is moving in a horizontal plane from left to right with a velocity of 10 m/s. Another ball B of mass 6 kg traveling in a perpendicular direction with a velocity of 20 m/s collides with A in such a way that the line of impact is in the direction of motion of ball B. Assuming e = 0.7, determine the velocities of balls A and B after impact. ***(M. U. Dec 09)***



Solution: This is a case of oblique impact. The line joining the centres forms the Line of Impact (L O I). Let the line of impact be the n direction and a perpendicular to it be the t direction.



Resolving the initial velocities along n and t directions

$v_{An} = 0, \ v_{At} = 10 \text{ m/s} \rightarrow$

$v_{Bn} = 20 \text{ m/s} \uparrow, \ v_{Bt} = 0$

Working in n direction

Applying C O M equation $\uparrow$ + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$5\times0 + 6\times20 = 5\times v_{An}' + 6\times v_{Bn}'$$

$$\therefore \quad 5v_{An}' + 6v_{Bn}' = 120 \quad \text{......... (1)}$$

Applying C O R equation $\uparrow$ + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$v_{Bn}' - v_{An}' = 0.7(0-20)$$

$$\therefore \quad -v_{An}' + v_{Bn}' = -14 \quad \text{......... (2)}$$

Solving equation (1) and (2)

$$v_{An}' = 18.545 \text{ m/s} = 18.545 \text{ m/s} \uparrow \quad \text{and} \quad v_{Bn}' = 4.545 \text{ m/s} = 4.545 \text{ m/s} \uparrow$$

Working in t direction: since velocities don't change in t direction

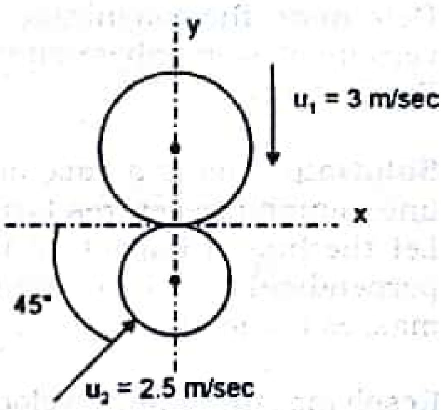
$v_{At}' = v_{At} = 10 \text{ m/s} \rightarrow$ and $v_{Bt}' = v_{Bt} = 0$

Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{18.545^2 + 10^2} = 21.07 \text{ m/s}$

at an angle $\alpha' = \tan^{-1}\left(\frac{v_{An}'}{v_{At}'}\right) = \tan^{-1}\left(\frac{18.545}{10}\right) = 61.66°$ .... **Ans.**

Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{4.545^2 + 0} = 4.545 \text{ m/s} \uparrow$ **Ans.**

P10. The 9 kg ball moving down at 3 m/sec strikes the 5.5kg ball moving at 2.5 m/sec as shown. The coefficient of restitution e = 0.8. Find the speeds v_1 and v_2 after the impact. ***(VJTI May 08)***



Solution: This is a case of oblique impact. The line joining the centres forms the line of impact. Let the line of impact be the n direction and a perpendicular to it be the t direction. Let A be the upper ball and B be the lower ball.

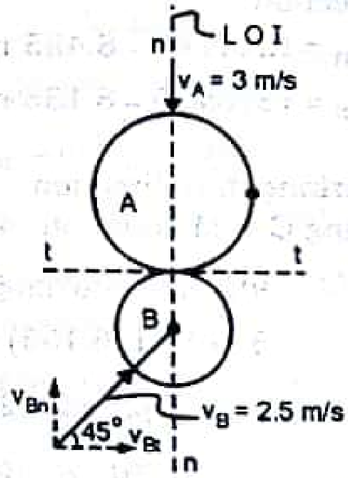
Let v_A' and v_B' be the velocities of A and B after impact.

Resolving the initial velocities along n and t directions

$v_{An} = 3 \text{ m/s} \downarrow, \quad v_{At} = 0$

$v_{Bn} = 2.5\sin 45 = 1.768 \text{ m/s} \uparrow,$

$v_{Bt} = 2.5\cos 45 = 1.768 \text{ m/s} \rightarrow$



Working in n direction

using C O M equation $\uparrow$ + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$9 \times (-3) + 5.5 \times 1.768 = 9 \times v_{An}' + 5.5 \times v_{Bn}'$$

$\therefore \quad 9v_{An}' + 5.5\, v_{Bn}' = -17.276$ .... (1)

using C O R equation $\uparrow$ + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$v_{Bn}' - v_{An}' = 0.8(-3 - 1.768)$$

$\therefore \quad -v_{An}' + v_{Bn}' = -3.814$ .... (2)

Solving equation (1) and (2) we get

$v_{An}' = 0.255 \text{ m/s} = 0.255 \text{ m/s} \uparrow$ and $v_{Bn}' = -3.559 \text{ m/s} = 3.559 \text{ m/s} \downarrow$

Working in t direction: since velocities don't change in t direction

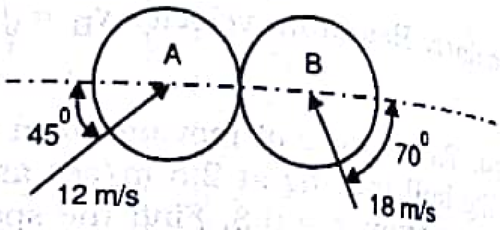
$v_{At}' = v_{At} = 0$ and $v_{Bt}' = v_{Bt} = 1.768 \text{ m/s} \rightarrow$

Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{0.255^2 + 0} = 0.255 \text{ m/s} \uparrow$

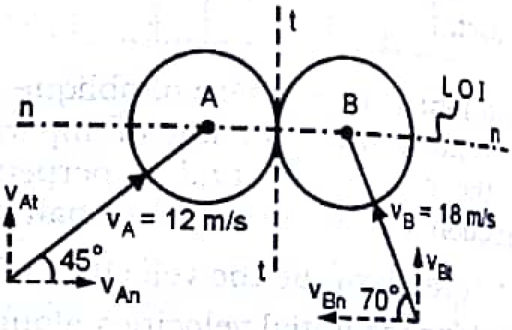
Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{3.559^2 + 1.768^2} = 3.974 \text{ m/s}$

at an angle $\beta' = \tan^{-1}\left(\dfrac{v_{Bn}'}{v_{Bt}'}\right) = \tan^{-1}\left(\dfrac{3.559}{1.768}\right) = 63.58°$ .... **Ans.**

P11. Two identical spheres A and B approach each other with the velocities as shown. Determine the magnitude and direction of the velocity of each sphere after impact. Take e = 0.7



Solution: This is a case of oblique impact. The line joining the centres forms the line of impact. Let the line of impact be the n direction and a perpendicular to it be the t direction. Note that masses are identical.



Resolving the initial velocities along n and t directions

$v_{An} = 12\cos 45 = 8.485 \text{ m/s} \rightarrow$, $v_{At} = 12\sin 45 = 8.485 \text{ m/s} \uparrow$

$v_{Bn} = 18\cos 70 = 6.156 \text{ m/s} \leftarrow$, $v_{Bt} = 16.91 \text{ m/s} \uparrow$

Working in n direction

using C O M equation $\rightarrow$ + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$8.485 + (-6.156) = v_{An}' + v_{Bn}'$$

$$\therefore \quad v_{An}' + v_{Bn}' = 2.329 \quad \text{......... (1)}$$

using C O R equation $\rightarrow$ + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$v_{Bn}' - v_{An}' = 0.7(8.485 - (-6.156))$$

$$\therefore \quad -v_{An}' + v_{Bn}' = 10.249 \quad \text{......... (2)}$$

Solving equation (1) and (2), we get

$v_{An}' = -3.96 \text{ m/s} = 3.96 \text{ m/s} \leftarrow$ and $v_{Bn}' = 6.289 \text{ m/s} = 6.289 \text{ m/s} \rightarrow$

Working in t direction: since velocities don't change in t direction, we have

$v_{At}' = v_{At} = 8.485 \text{ m/s} \uparrow$ and $v_{Bt}' = v_{Bt} = 16.91 \text{ m/s} \uparrow$

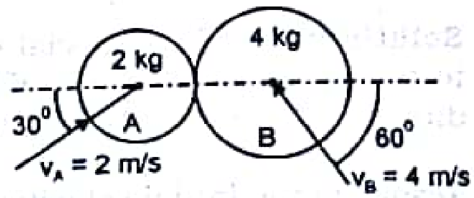
Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{3.96^2 + 8.485^2} = 9.36 \text{ m/s}$

at an angle $\alpha' = \tan^{-1}\left(\dfrac{v_{At}'}{v_{An}'}\right) = \tan^{-1}\left(\dfrac{8.485}{3.96}\right) = 64.98°$ .... **Ans.**

Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{6.289^2 + 16.91^2} = 18.04\ m/s$

at an angle $\beta' = \tan^{-1}\left(\frac{v_{Bt}'}{v_{Bn}'}\right) = \tan^{-1}\left(\frac{16.91}{6.289}\right) = 69.6°$ .... **Ans.**

P12. Two smooth spheres A and B having a mass of 2 kg and 4 kg respectively collide with initial velocities as shown in figure. If the coefficient of restitution for the spheres is $e = 0.8$, determine the velocities of each sphere after collision. ***(M.U. May 08)***



Solution: This is a case of oblique impact. The line joining the centres forms the line of impact. Let the line of impact be the n direction and a perpendicular to it be the t direction.

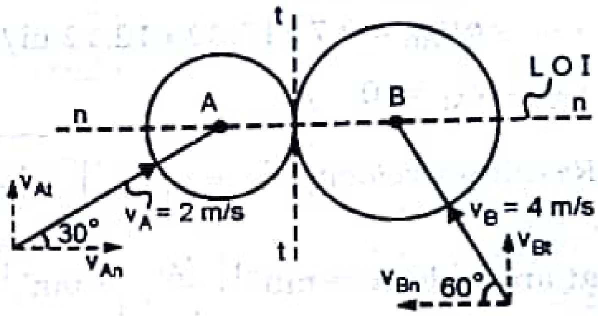
Resolving the initial velocities along n and t directions

$v_{An} = 2\cos 30 = 1.732\ m/s \rightarrow$,

$v_{At} = 2\sin 30 = 1\ m/s \uparrow$

$v_{Bn} = 4\cos 60 = 2\ m/s \leftarrow$,

$v_{Bt} = 4\sin 60 = 3.464\ m/s \uparrow$



Working in n direction
using C O M equation → + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$2 \times 1.732 + 4 \times (-2) = 2v_{An}' + 4v_{Bn}'$$

$$\therefore \quad 2v_{An}' + 4v_{Bn}' = -4.536 \quad \text{......... (1)}$$

using C O R equation → + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$v_{Bn}' - v_{An}' = 0.8(1.732 - (-2))$$

$$\therefore \quad -v_{An}' + v_{Bn}' = 2.986 \quad \text{......... (2)}$$

Solving equation (1) and (2), we get

$\therefore \quad v_{An}' = -2.747\ m/s = 2.747\ m/s \leftarrow$ and $v_{Bn}' = 0.239\ m/s = 0.239\ m/s \rightarrow$

Working in t direction: since velocities don't change in t direction, we have

$v_{At}' = v_{At} = 1\ m/s \uparrow$ and $v_{Bt}' = v_{Bt} = 3.464\ m/s \uparrow$

Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{2.747^2 + 1^2} = 2.923\ m/s$

at an angle $\alpha' = \tan^{-1}\left(\frac{v_{At}'}{v_{An}'}\right) = \tan^{-1}\left(\frac{1}{2.747}\right) = 20°$ .... **Ans.**

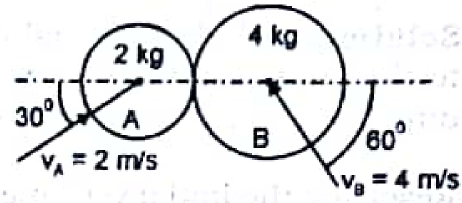
Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{0.239^2 + 3.464^2} = 3.472\ m/s$

at an angle $\beta' = \tan^{-1}\left(\frac{v_{Bt}'}{v_{Bn}'}\right) = \tan^{-1}\left(\frac{3.464}{0.239}\right) = 86.05°$ .... **Ans.**

Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{6.289^2 + 16.91^2} = 18.04\ m/s$

at an angle $\beta' = \tan^{-1}\left(\frac{v_{Bt}'}{v_{Bn}'}\right) = \tan^{-1}\left(\frac{16.91}{6.289}\right) = 69.6°$ .... **Ans.**

P12. Two smooth spheres A and B having a mass of 2 kg and 4 kg respectively collide with initial velocities as shown in figure. If the coefficient of restitution for the spheres is e = 0.8, determine the velocities of each sphere after collision. *(M.U. May 08)*



Solution: This is a case of oblique impact. The line joining the centres forms the line of impact. Let the line of impact be the n direction and a perpendicular to it be the t direction.

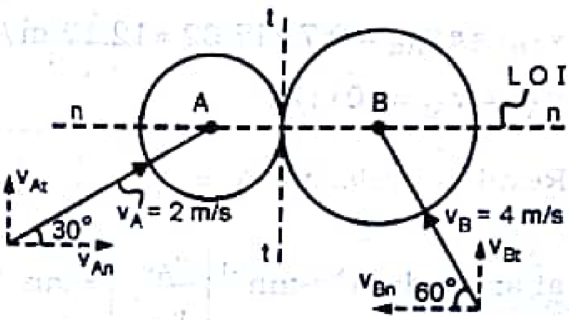
Resolving the initial velocities along n and t directions

$v_{An} = 2\cos 30 = 1.732\ m/s \rightarrow$,

$v_{At} = 2\sin 30 = 1\ m/s \uparrow$

$v_{Bn} = 4\cos 60 = 2\ m/s \leftarrow$,

$v_{Bt} = 4\sin 60 = 3.464\ m/s \uparrow$



Working in n direction

using C O M equation → + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$2\times1.732 + 4\times(-2) = 2v_{An}' + 4v_{Bn}'$$

$$\therefore \quad 2v_{An}' + 4v_{Bn}' = -4.536 \quad .... (1)$$

using C O R equation → + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$v_{Bn}' - v_{An}' = 0.8(1.732 - (-2))$$

$$\therefore \quad -v_{An}' + v_{Bn}' = 2.986 \quad .... (2)$$

Solving equation (1) and (2), we get

$\therefore \quad v_{An}' = -2.747\ m/s = 2.747\ m/s \leftarrow$ and $v_{Bn}' = 0.239\ m/s = 0.239\ m/s \rightarrow$

Working in t direction: since velocities don't change in t direction, we have

$v_{At}' = v_{At} = 1\ m/s \uparrow$ and $v_{Bt}' = v_{Bt} = 3.464\ m/s \uparrow$

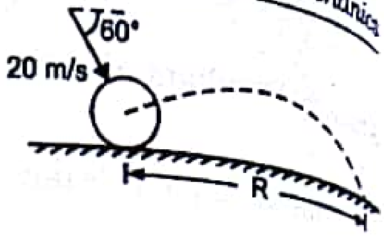
Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{2.747^2 + 1^2} = 2.923\ m/s$

at an angle $\alpha' = \tan^{-1}\left(\frac{v_{At}'}{v_{An}'}\right) = \tan^{-1}\left(\frac{1}{2.747}\right) = 20°$ .... **Ans.**

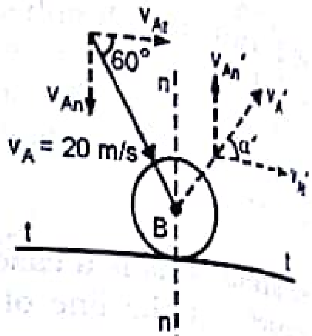
Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{0.239^2 + 3.464^2} = 3.472\ m/s$

at an angle $\beta' = \tan^{-1}\left(\frac{v_{Bt}'}{v_{Bn}'}\right) = \tan^{-1}\left(\frac{3.464}{0.239}\right) = 86.05°$ .... **Ans.**

P13. a) A bowler bowls a ball which strikes the ground at an angle of 60° with a speed of 20 m/s as shown. Find the velocity of the ball just after it strikes the ground. Assume smooth surfaces and e = 0.7
b) Find at what horizontal distance R, the ball hits the ground again.



Solution: This is a special case of oblique impact. The line ⊥ to the ground is the line of impact (L O I). The L O I is the n direction of impact and a ⊥ to it is the t direction.



Resolving the initial velocities along n and t directions
$v_{An} = 20\sin 60 = 17.32 \text{ m/s} \downarrow$, $v_{At} = 20\cos 60 = 10 \text{ m/s} \rightarrow$

Since it is a special oblique impact

$$v_{An}' = e v_{An} = 0.7 \times 17.32 = 12.12 \text{ m/s} \uparrow$$

$$v_{At}' = v_{At} = 10 \text{ m/s} \rightarrow$$

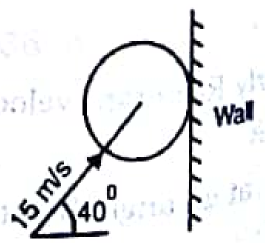
∴ Resultant velocity $v_A' = \sqrt{\left(v_{An}'\right)^2 + \left(v_{At}'\right)^2} = \sqrt{12.12^2 + 10^2} = 15.716 \text{ m/s}$

at an angle $\alpha' = \tan^{-1}\left(\dfrac{v_{An}'}{v_{At}'}\right) = \tan^{-1}\left(\dfrac{12.12}{10}\right) = 50.47° \measuredangle$ .... **Ans.**

b) Projectile Motion of ball after Impact

Horizontal Motion	Vertical Motion ↑ + ve
$v = 10 \text{ m/s}$	$u = 12.12 \text{ m/s}$
$s = R$	$v = -$
$t = t$	$s = 0$
Using $v = \dfrac{s}{t}$	$a = -9.81 \text{ m/s}^2$
	$t = t$
$10 = \dfrac{R}{t}$	Using $s = ut + \dfrac{1}{2}at^2$ ∴ $0 = 12.12t + \dfrac{1}{2} \times (-9.81) \times t^2$
Subsituting t = 2.471 sec	or $t = 2.471$ sec
$10 = \dfrac{R}{2.471}$	
or $R = 24.709$ m ... **Ans.**	

P14. A ball is thrown against a frictionless wall. Its velocity before striking the wall is shown.
If e = 0.8, find the velocity after impact.



Solution: This is a special case of oblique impact. The line ⊥ to the wall is the line of impact (L O I). The L O I is the n direction of impact and a ⊥ to it is the t direction.
Resolving the initial velocities along n and t directions

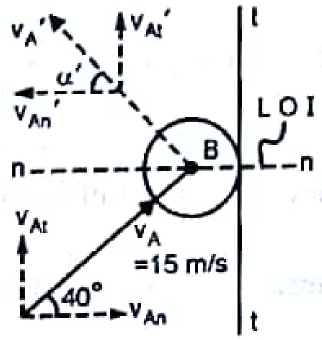
$v_{An} = 15\cos 40 = 11.49\ m/s \rightarrow$,

$v_{At} = 15\sin 40 = 9.642\ m/s \uparrow$

Since it is a special oblique impact

$$v_{An}' = e v_{An} = 0.8 \times 11.49 = 9.192\ m/s \leftarrow$$

$$v_{At}' = v_{At} = 9.642\ m/s \uparrow$$

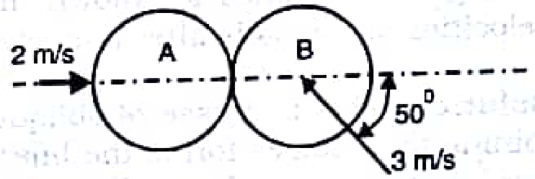


Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2}$

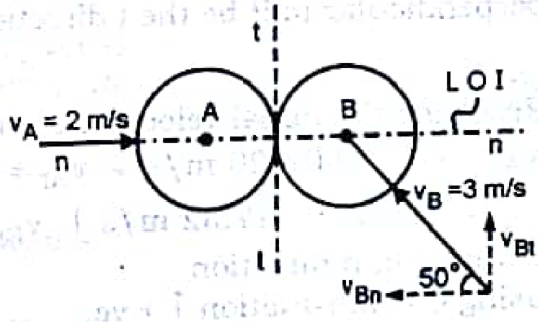
$$= \sqrt{9.192^2 + 9.642^2} = 13.32\ m/s$$

at an angle $\alpha' = \tan^{-1}\left(\frac{v_{At}'}{v_{An}'}\right) = \tan^{-1}\left(\frac{9.642}{9.192}\right) = 46.37°$ .... **Ans.**

P15. Two identical billiard balls collide with velocities as shown. Find the velocities of the balls after impact. Also find the percentage loss of kinetic energy. Take e = 0.9.



Solution: This is a case of oblique impact. The line joining the centres forms the L O I. Let the L O I be the n direction and a ⊥ to it be the t direction.
Resolving the initial velocities along n and t directions



$v_{An} = 2\ m/s \rightarrow$, $v_{At} = 0$

$v_{Bn} = 3\cos 50 = 1.928\ m/s \leftarrow$,

$v_{Bt} = 3\sin 50 = 2.298\ m/s \uparrow$

Working in n direction
Applying C O M equation → + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$2 + (-1.928) = v_{An}' + v_{Bn}'$$

$$\therefore \quad v_{An}' + v_{Bn}' = 0.072 \quad \text{......... (1)}$$

Applying C O R equation → + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$v_{Bn}' - v_{An}' = 0.9(2 - (-1.928))$$

$$\therefore \quad -v_{An}' + v_{Bn}' = 3.535 \quad \text{......... (2)}$$

Solving equation (1) and (2), we get

$v_{An}' = -1.73\ m/s = 1.73\ m/s \leftarrow$ and $v_{Bn}' = 1.8\ m/s = 1.8\ m/s \rightarrow$

Working in t direction:
Since velocities don't change in t direction

$v_{At}' = v_{At} = 0$ and $v_{Bt}' = v_{Bt} = 2.298\ m/s \uparrow$

Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{1.73^2 + 0} = 1.73 \text{ m/s} \leftarrow$ .... **Ans.**

Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{1.8^2 + 2.298^2} = 2.919 \text{ m/s}$

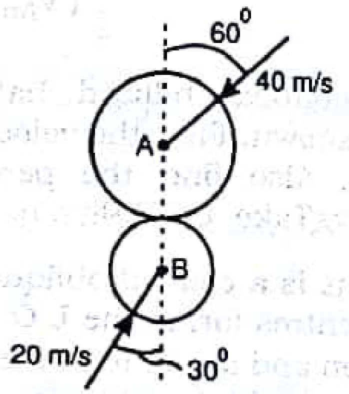
at an angle $\beta' = \tan^{-1}\left(\dfrac{v_{Bn}'}{v_{Bt}'}\right) = \tan^{-1}\left(\dfrac{2.298}{1.8}\right) = 51.93° \measuredangle$ .... **Ans.**

Kinetic Energy before impact = $K.E_1 = \frac{1}{2} \times m \times 2^2 + \frac{1}{2} \times m \times 3^2 = 6.5 \text{m Joules}$

Kinetic Energy after impact = $K.E_2 = \frac{1}{2} \times m \times 1.73^2 + \frac{1}{2} \times m \times 2.919^2 = 5.757 \text{m Joules}$

% loss in K.E. = $\dfrac{K.E_1 - K.E_2}{K.E_1} \times 100 = \dfrac{6.5\text{ m} - 5.757\text{ m}}{6.5\text{ m}} \times 100 = 11.43$ .... **Ans.**

P16. Two smooth balls A (3 kg) and B (2 kg) strike with given velocities as shown. If e = 0.7 find the velocities of the balls after impact.

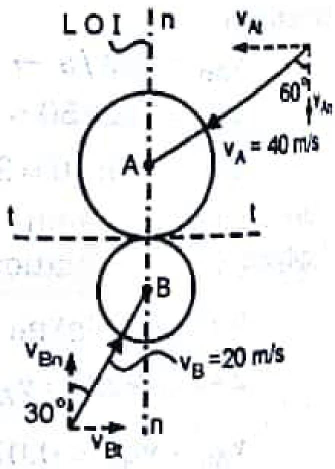


Solution: This is a case of oblique impact. The line joining the centres forms the line of impact. Let the line of impact be the n direction of impact and a perpendicular to it be the t direction.

Resolving the initial velocities along n and t directions

$v_{An} = 40\cos 60 = 20 \text{ m/s} \downarrow$, $v_{At} = 40 \sin 60 = 34.64 \text{ m/s} \leftarrow$

$v_{Bn} = 20\cos 30 = 17.32 \text{ m/s} \uparrow$, $v_{Bt} = 20\sin 30 = 10 \text{ m/s} \rightarrow$



Working in n direction

using C O M equation $\uparrow$ + ve

$$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$$

$$3 \times (-20) + 2 \times (17.32) = 3 v_{An}' + 2 v_{Bn}'$$

$$\therefore \quad 3 v_{An}' + 2 v_{Bn}' = -25.36 \quad \text{......... (1)}$$

using C O R equation $\uparrow$ + ve

$$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$$

$$\therefore \quad v_{Bn}' - v_{An}' = 0.7(-20 - (-17.32))$$

$$\therefore \quad -v_{An}' + v_{Bn}' = -26.12 \quad \text{......... (2)}$$

Solving equation (1) and (2), we get

$v_{An}' = 5.376 \text{ m/s} = 5.376 \text{ m/s} \uparrow$ and $v_{Bn}' = -20.74 \text{ m/s} = 20.74 \text{ m/s} \downarrow$

Working in t direction: since velocities don't change in t direction

$v_{At}' = v_{At} = 34.64\ m/s \leftarrow$ and $v_{Bt}' = v_{Bt} = 10\ m/s \rightarrow$

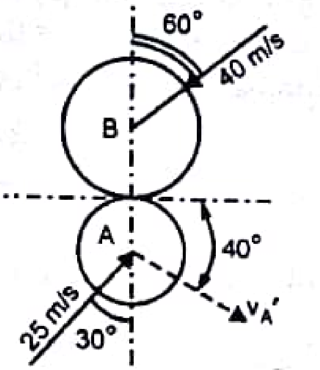
Resultant velocity $v_A' = \sqrt{(v_{An}')^2 + (v_{At}')^2} = \sqrt{5.376^2 + 34.64^2} = 35.05\ m/s$

at an angle $\alpha' = \tan^{-1}\left(\frac{v_{An}'}{v_{At}'}\right) = \tan^{-1}\left(\frac{5.376}{34.64}\right) = 8.82°$ .... **Ans.**

Similarly Resultant velocity $v_B' = \sqrt{(v_{Bn}')^2 + (v_{Bt}')^2} = \sqrt{20.74^2 + 10^2} = 23.02\ m/s$

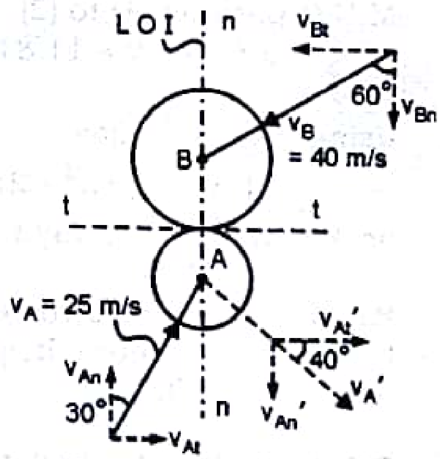
at an angle $\beta' = \tan^{-1}\left(\frac{v_{Bn}'}{v_{Bt}'}\right) = \tan^{-1}\left(\frac{20.74}{10}\right) = 64.26°$ .... **Ans.**

P17. Two smooth balls A (3 kg) and B (4 kg) are moving with velocities as shown before they collide. After collision ball A rebounds in a direction at 40° counterclockwise with the horizontal axis. Determine the coefficient of restitution between the balls.



Solution: This is a case of oblique impact. The line joining the centres forms the line of impact. Let the line of impact be the n direction and a perpendicular to it be the t direction.

In this problem the direction of velocity of A after impact is known. Resolving velocities along n and t directions, we have,



$v_{An} = 25\cos 30 = 21.65\ m/s \uparrow,\ v_{At} = 25\sin 30 = 12.5\ m/s \rightarrow$

$v_{Bn} = 40\cos 60 = 20\ m/s \downarrow,\ v_{Bt} = 40\sin 60 = 34.64\ m/s \leftarrow$

$v_{An}' = v_A'\sin 40 = 0.643 v_A' \downarrow,\ v_{At}' = v_A'\cos 40 = 0.766 v_A' \rightarrow$

Working in t direction:
since velocities don't change in t direction

$v_{At}' = v_{At}$

$\therefore\quad 0.766 v_{At}' = 12.5 \quad \therefore \quad v_{At}' = 16.317\ m/s$

also $\quad v_{An}' = 0.643\ v_A' = 0.643 \times 16.317 = 10.488\ m/s \downarrow$

Working in n direction
using C O M equation ↑ + ve

$m_A v_{An} + m_B v_{Bn} = m_A v_{An}' + m_B v_{Bn}'$

$3 \times 21.65 + 4 \times (-20) = 3(-10.488) + 4 v_{Bn}'$

$\therefore \quad v_{Bn}' = 4.104\ m/s$

using C O R equation ↑ + ve

$v_{Bn}' - v_{An}' = e(v_{An} - v_{Bn})$

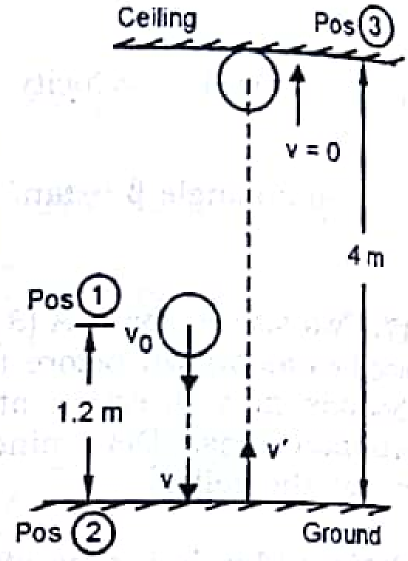
$\therefore\ 4.104 - (-10.488) = e(21.65 - (-20))$

or $\quad e = 0.35$ .... **Ans.**

Exercise 12.3

Combination Problems

P1. A ball is thrown vertically downwards with a velocity v_0 from a height of 1.2 m so that it hits the ground and just touches the ceiling after impact. If the ceiling is 4 m high from the ground find v_0. Take $e = 0.75$.



Solution: Let the ball be thrown from position (1) with an unknown velocity v_0. Let it strike the ground (position 2) with velocity v and rebound with a velocity v'. The ball finally just touches the ceiling at position (3). This means its velocity as it reaches the ceiling is zero.

M U G position (2) to (3) ↑ + ve

$u = v'$, $v = 0$, $s = 4$ m, $a = -9.81$ m/s², $t = -$

using $v^2 = u^2 + 2as$

$$0 = (v')^2 + 2 \times -9.81 \times 4$$

or $\quad v' = 8.859 \text{ m/s}$

Impact at position (2) is between ball and ground. This is special case of direct impact.

$\therefore \quad v' = ev \quad$ or $\quad 8.859 = 0.75 \times v \quad$ or $\quad v = 11.812 \text{ m/s}$

M U G position (1) to (2) ↓ + ve

$u = v_0$, $v = 11.812$ m/s, $s = 1.2$ m, $a = 9.81$ m/s², $t = -$

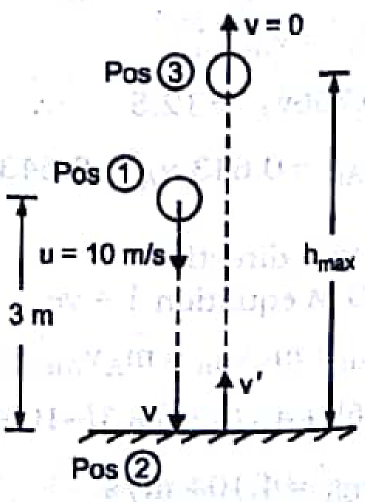
using $v^2 = u^2 + 2as$

$$11.812^2 = v_0^2 + 2 \times 9.81 \times 1.2$$

or $\quad v_0 = 10.769 \text{ m/s}$ .... **Ans.**

P2. If a ball is thrown vertically down with a velocity of 10 m/s from a height of 3 m. Find the maximum height it can reach after hitting the floor, if the coefficient of restitution is 0.7

(M.U. Dec 14)



Solution: Let the ball be thrown from Pos (1) with initial velocity u = 10 m/s. Let it strike the ground Pos (2) with velocity v and rebound with velocity v'. The ball finally reaches its maximum height h_{max} at Pos (3), where its velocity becomes zero.

MUG position (1) to position (2) ↓ +ve

$u = 10$ m/s, $v = v$, $s = 3$ m, $a = 9.81$ m/s², $t = -$

Using $v^2 = u^2 + 2as$

$$v^2 = 10^2 + 2 \times 9.81 \times 3$$

$\therefore \quad v = 12.604 \text{ m/s} \downarrow$ ...... **Ans.**

Impact at position (2) is between ball and ground. This is a special case of direct impact.

$\therefore \quad v' = e\,v$

or $\quad v' = 0.7 \times 12.604$

or $\quad v' = 8.823\ m/s \uparrow$

MUG position (2) to position (3) $\uparrow$ +ve

$u = v' = 8.823\ m/s \uparrow,\ v = 0,\ s = h_{max},\ a = -9.81\ m/s^2,\ t = -$

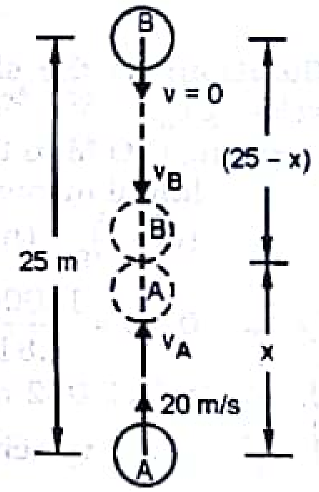
Using $v^2 = u^2 + 2as$

$$0 = 8.823^2 + 2 \times (-9.81) \times h_{max}$$

$\therefore \quad h_{max} = 3.967\ m$ ...... **Ans.**

P3. A body A of mass 2 kg is projected upwards from the surface of the ground at t = 0 with a velocity of 20 m/s. At the same time another body B of mass 2 kg is dropped along the same line from a height of 25 m. If they collide elastically, find the velocities of body A and B just after collision. ***(M. U. Dec 12, KJS May 15)***

Solution: Let the two bodies collide at some position x from the ground. Let v_A and v_B be the velocities just before impact.



Motion of A

M U G $\uparrow$ + ve

$u = 20\ m/s$

$v = v_A$

$s = x$

$a = -9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$x = 20t - 4.905t^2 \ \dots (1)$$

Motion of B

M U G $\downarrow$ + ve

$u = 0$

$v = v_B$

$s = 25 - x$

$a = 9.81\ m/s^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$25 - x = 0 + 4.905t^2 \ \dots (2)$$

Equating x we get

$$20t - 4.905t^2 = 25 - 4.905t^2$$

$\therefore \quad t = 1.25$ sec also $x = 17.33\ m$

using $v = u + at$

$v_A = 20 - 9.81 \times 1.25$

or $\quad v_A = 7.737\ m/s$

using $v = u + at$

$v_B = 0 + 9.81 \times 1.25$

or $\quad v_B = 12.26\ m/s$

Impact at meeting point

A direct impact takes place at the meeting point

Applying C O M equation $\uparrow$ + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$7.737 + (-12.26) = v_A' + v_B'$$

masses are identical, so they cancel.

or $v_A' + v_B' = -4.525$ .... (1)

Applying C O R equation ↑ + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = 1(7.737 - (-12.26))$$

e = 1 since collision is elastic.

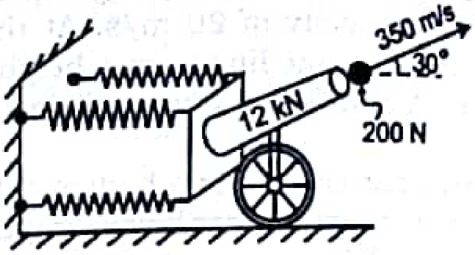
or $-v_A' + v_B' = 20$ .... (2)

Substituting equation (2) in (1)

$v_A' = -12.26$ m/s or $v_A' = 12.26$ m/s ↓

and $v_B' = 7.737$ m/s or $v_B' = 7.737$ m/s ↑ .... **Ans.**

P4. A shell of weight 200 N is fired from a gun with a velocity of 350 m/s. The gun and its carriage have a total weight of 12 kN. Find the stiffness of each of the three springs which is required to bring the gun to a halt within 300 mm of the spring compression. ***(VJTI Dec 14)***



Solution: As the shell (s) is fired, the gun (g) recoils in the backward direction with velocity v_g.

Applying C O M to the system of gun and shell → + ve

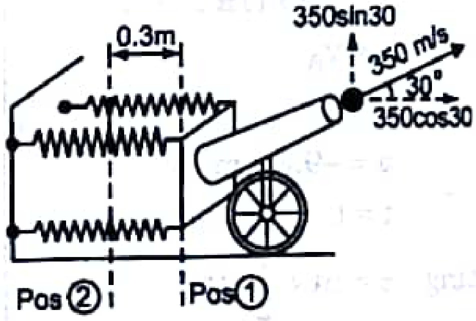
Initial momentum = Final momentum

$$(m \times v)_g + (m \times v)_s = (m \times v)_g + (m \times v)_s$$

$$0 + 0 = \frac{12000}{9.81} \times (-v_g) - \frac{200}{9.81} \times (350 \cos 30)$$

∴ $v_g = 5.052$ m/s ←

.... velocity of gun just after firing the shell.



The gun at position (1), moves backwards and compresses the spring to position (2) by 300 mm

Applying W E P (1) – (2)

$$T_1 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times \frac{12000}{9.81} \times 5.052^2 = 15610 \text{ J}$$

$T_2 = 0$ .... since the gun comes to rest

$$U_{1-2} = \text{by spring} = \frac{1}{2} k \left(x_1^2 - x_2^2\right)$$ .... here $x_1 = 0$ and $x_2 = 300$ mm $= 0.3$ m

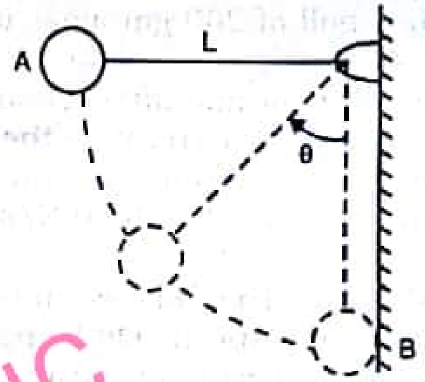
$$= \left[\frac{1}{2} \times k \times \left(0 - 0.3^2\right)\right] \times 3 = -0.135k \text{ J}$$ .... since there are 3 springs

Using $T_1 + \Sigma U_{1-2} = T_2$

$$15610 + (-0.135k) = 0$$

∴ $k = 115630$ N/m $= 115.63$ kN/m .... **Ans.**

P5. A sphere tied to a string of length L is released from rest from the horizontal position at A. The sphere swings as a pendulum and strikes a vertical wall at B. If e = 0.7, find the angle θ defining its total rebound.



Solution: The sphere starts from rest at position (1). Let v be the velocity of the sphere as it strikes the wall at position (2).

Applying W E P (1) – (2)

$$T_1 = 0$$

$$T_2 = \frac{1}{2} \times mv^2$$

U_{1-2} = by weight = + m g h .... + ve ∵ displacement is downwards.

$$= m \times 9.81 \times L \quad \text{Joules}$$

Using $T_1 + \Sigma U_{1-2} = T_2$

$$0 + m \times 9.81 \times L = \frac{1}{2} \times m v^2$$

∴ $v = \sqrt{19.62 L}$ m/s .... velocity of sphere as it strikes the wall. .... **Ans.**

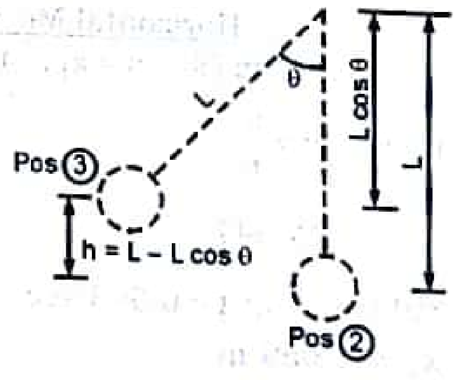
The ball on striking the wall at position (2) rebounds back. Let v′ be the velocity of the ball after impact with wall.

It is a special direct impact

Knowing $v' = e\,v$

$$v' = 0.7 \times \sqrt{19.62\,L} = 3.1\sqrt{L} \text{ m/s} \leftarrow$$

After rebounding the ball rises to position (3) making an angle θ with the vertical. At highest position (3), the ball momentarily comes to rest.



Applying W E P (2) – (3)

$$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times m \times \left(3.1\sqrt{L}\right)^2 = 4.805\,mL \quad \text{Joules}$$

$T_3 = 0$ ∵ at the highest position velocity becomes zero.

U_{2-3} = by weight = – m g h

(– ve ∵ displacement is upwards.)

$$= -m \times 9.81 \times (L - L\cos\theta) \quad \text{Joules}$$

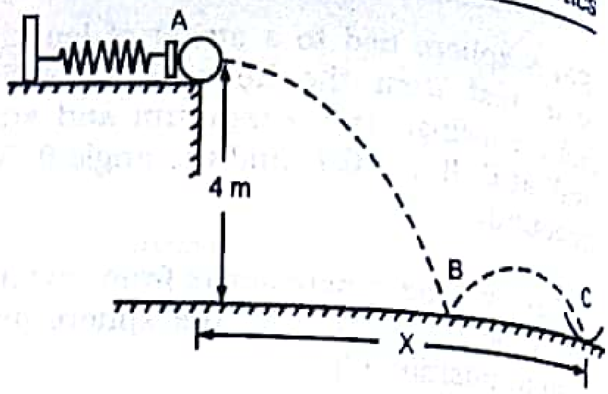
Using $T_2 + \Sigma U_{2-3} = T_3$

$$4.805\,mL - m \times 9.81 \times L(1 - \cos\theta) = 0$$

$$4.805 - 9.81(1 - \cos\theta) = 0$$

or $\theta = 59.32°$ .... **Ans.**

P6. A ball of 200 gm mass is propelled from A by a spring mechanism. The ball falls on a smooth floor and after rebounding at B falls in the cup at C. Determine the location x of the cup. The spring is initially compressed 100 mm. Take k = 3000 N/m and e = 0.75



Solution: The ball is initially kept pressed against the spring. On being released it shoots horizontally with a velocity v.

Applying W E P at position A

$$T_1 = 0$$

$$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 0.2 \times v^2 = 0.1 v^2 \text{ J}$$

$$U_{1-2} = \text{by spring } = \frac{1}{2} k\left(x_1^2 - x_2^2\right) = \frac{1}{2} \times 3000\left(0.1^2 - 0\right) = 15 \text{ J}$$

$x_1 = 100\,\text{mm} = 0.1\,\text{m}$

$x_2 = 0 \because$ the spring becomes free.

Using $T_1 + \Sigma U_{1-2} = T_2$

$$0 + 15 = 0.1\, v^2$$

or $v = 12.247$ m/s .... velocity of ball just after release at A.

The ball leaves from A, horizontally with a speed of 12.247 m/s and lands at B in a projectile motion. Let x_1 bet the horizontal distance covered.

Projectile motion (A – B)

<u>Horizontal Motion</u>

$v = 12.247$ m/s, $s = x_1$, $t = t$

Using $v = \frac{s}{t}$

$$12.247 = \frac{x_1}{t}$$

Substituting $t = 0.903$ sec

$x_1 = 11.059$ m

<u>Vertical Motion ↓ + ve</u>

$u = 0$

$v = v_{By}$

$s = 4$ m

$a = 9.81$

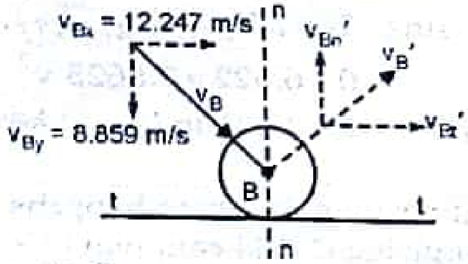
$t = t$

Using $s = ut + \frac{1}{2}at^2 \therefore 4 = 0 + \frac{1}{2} \times 9.81 \times t^2$

or $t = 0.903$ sec

Using $v = u + at$

$v_{By} = 0 + 9.81 \times 0.903 = 8.859$ m/s



Impact at position B

It is a special case of oblique impact.

Taking n and t axes as shown.

$$v_{Bn}' = e\,v_{Bn} = e\,v_{By} = 0.75 \times 8.859 = 6.644 \text{ m/s} \uparrow$$

$$v_{Bt}' = v_{Bx} = 12.242 \text{ m/s} \rightarrow$$

After rebounding at B, the ball performs projectile motion up to C,

Let x_2 be the horizontal distance between B and C.

Projectile motion B to C

Horizontal Motion	Vertical Motion $\uparrow$ + ve
$v = 12.242$ m/s	$u = 6.644$ m/s
$s = x_2$	$v = -$
$t = t$	$s = 0$
Using $v = \dfrac{s}{t}$	$a = -9.81$ m/s^2
$12.242 = \dfrac{x_2}{1.354}$	$t = t$
or $x_2 = 16.582$ m	Using $s = ut + \frac{1}{2}at^2$ $\therefore$ $0 = 6.644t + \frac{1}{2} \times (-9.81) \times t^2$
	or $t = 1.354$ sec

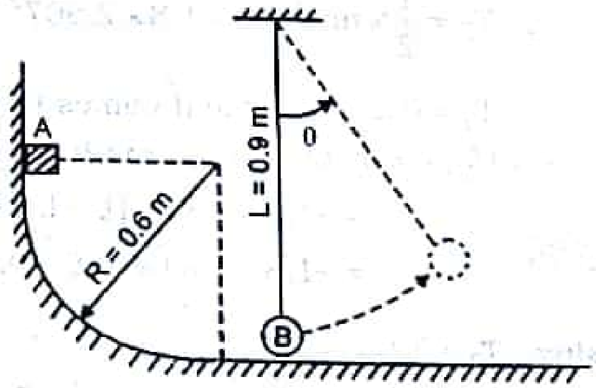
Location x of the cup $x = x_1 + x_2$

$= 11.059 + 16.582 = 27.641$ m .... **Ans.**

P7. Block A of mass 1.125 kg is released from rest in the position shown and slides without friction until it strikes the ball B of mass 1.8 kg of a simple pendulum. Knowing that coefficient of restitution between A and B is 0.9, determine

a) the velocity of B immediately after impact

b) the maximum angular displacement of the pendulum. ***(M.U Dec 08)***



Solution: The block is initially at rest at A (position 1) Let it acquire a velocity v at position (2) just before it strikes the pendulum B.

Applying W E P pos (1) – pos (2)

$T_1 = 0$ .... Since the block starts from rest.

$$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 1.125 \times v^2 = 0.5625 v^2 \text{ J}$$

U_{1-2} = by weight $= +mgh$ .... + ve $\because$ Displacement is downwards

$= 1.125 \times 9.81 \times 0.6$

$= 6.622$ J

Using $T_1 + \Sigma U_{1-2} = T_2$

$$0 + 6.622 = 0.5625\ v^2$$

or $v = 3.431\ m/s$

At position (2), there happens a direct impact between block A and pendulum B.

Applying C O M equation $\rightarrow$ + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$1.125 \times 3.431 + 0 = 1.125 v_A' + 1.8 \times v_B'$$

$$\therefore \quad 1.125 v_A' + 1.8 v_B' = 3.86 \quad \dots (1)$$

Applying C O R equation $\rightarrow$ + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = 0.9(3.431 - 0)$$

$$\therefore \quad -v_A' + v_B' = 3.088 \quad \dots (2)$$

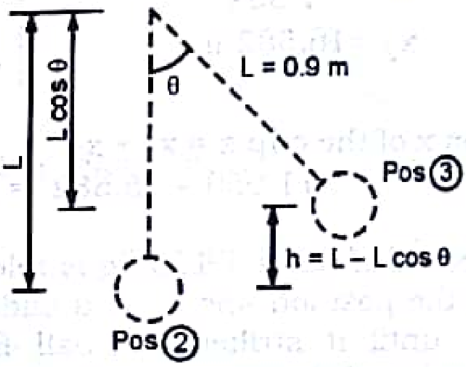
Solving equation (1) and (2) we get

$$v_A' = -0.5806\ m/s = 0.5806\ m/s \leftarrow$$

$$v_B' = 2.507\ m/s = 2.507\ m/s \rightarrow$$

$\therefore$ Velocity of B just after impact is $v_B' = 2.507\ m/s \rightarrow$ .... **Ans.**

The pendulum now rises from position (2) to position (3), describing an angle θ. At peak position (3), it momentarily stops making its velocity zero.



Applying W E P pos (2) – pos (3)

$$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 1.8 \times 2.507^2 = 5.656\ J$$

$T_3 = 0$ .... Since it comes to rest.

U_{2-3} = by weight $= -mgh$ .... – ve $\because$ Displacement is upwards

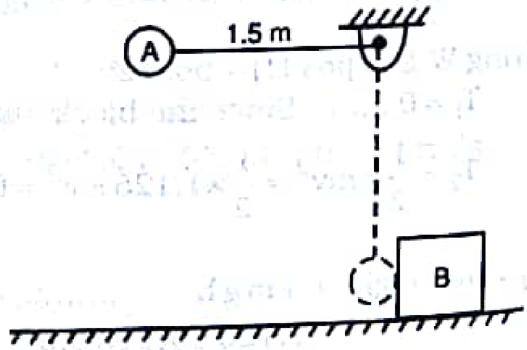
$$= -1.8 \times 9.81 \times (L - L\cos\theta)$$

$$= -1.8 \times 9.81 \times (0.9 - 0.9\cos\theta) = -15.89(1 - \cos\theta)$$

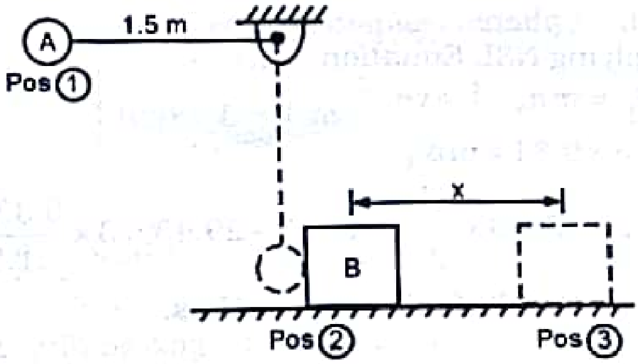
Using $T_2 + \Sigma U_{2-3} = T_3$

$$5.656 + [-15.89(1 - \cos\theta)] = 0 \quad \text{or} \quad \theta = 49.9°$$ .... **Ans.**

P8. A 3 kg sphere is released from rest and strikes a 5 kg block kept on a horizontal floor. If e = 0.7 between the block and sphere and μ_k = 0.3 between the block and ground, determine (a) the distance traveled by the block before it comes to rest.
(b) the tension in the cable just after impact.



Solution: The 3 kg sphere A is initially held at rest in the horizontal position (1). Let it acquire a velocity v just before it strikes the block B at rest in position (2).



Applying W E P pos (1) – pos (2)

$T_1 = 0$ .... Since the sphere is at rest.

$$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 3 \times v^2 = 1.5 v^2 \text{ J}$$

$$U_{1-2} = \text{by weight} = +mgh = 3 \times 9.81 \times 1.5 = 44.145 \text{ J}$$

+ ve ∵ Displacement is downwards, h = 1.5 m

Using $T_1 + \Sigma U_{1-2} = T_2$

$$0 + 44.145 = 1.5\, v^2$$

or $v = 5.125 \text{ m/s}$

At position (2), there happens a direct impact between sphere and block.

Applying C O M equation → + ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 \times 5.425 + 0 = 3 v_A' + 5 \times v_B'$$

$$\therefore \quad 3 v_A' + 5 v_B' = 16.275 \text{ .... (1)}$$

Applying C O R equation → + ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = 0.7(5.425 - 0)$$

$$\therefore \quad -v_A' + v_B' = 3.797 \text{ .... (2)}$$

Solving equation (1) and (2) we get

$$v_A' = -0.339 \text{ m/s} = 0.339 \text{ m/s} \leftarrow \quad \text{and} \quad v_B' = 3.458 \text{ m/s} = 3.458 \text{ m/s} \rightarrow$$

Block B now starts moving to the right from position (2) and travels a distance x before it comes to rest at position (3)

Applying W E P pos (2) – pos (3)

$$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 5 \times 3.458^2 = 29.89 \text{ J}$$

$T_3 = 0$ .... Since it comes to rest.

$$U_{2-3} = \text{by friction} = -\mu_k . N . s = -0.3 \times (5 \times 9.81) \times x = -14.715x \text{ Joules}$$

Using $T_2 + \Sigma U_{2-3} = T_3$

$$29.89 + [-14.715x] = 0 \quad \text{or} \quad x = 2.032 \text{ m}$$ .... **Ans.**

To find tension in the cable just after impact, let us apply NSL to sphere A performing curvilinear motion.

NSL – Sphere

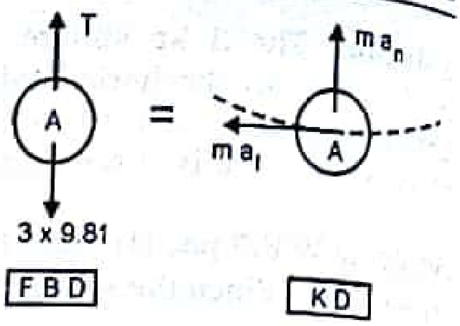
Applying NSL Equation

$\Sigma F_y = m a_y \quad \uparrow +ve$

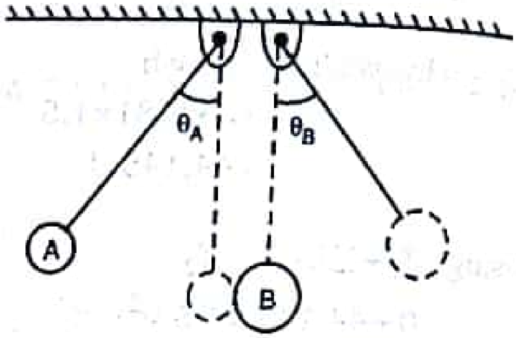
$T - 3 \times 9.81 = m a_n$

$T - 29.43 = 3 \times \dfrac{v^2}{\rho} \quad \therefore \quad T - 29.43 = 3 \times \dfrac{0.339^2}{1.5}$

$\therefore \quad T = 29.66 \text{ N}$ .... **Ans.**



P9. A 2 kg sphere A is released from rest when $\theta_A = 50°$ and strikes a 4 kg sphere B which is at rest. If e = 0.8 between the two spheres, determine the values of θ_A and θ_B corresponding to the highest position to which the spheres will rise after impact. Take length of both the pendulums to be 1 m.



Solution: Sphere A starts from position (1) and strikes sphere B at position (2).

Applying W E P pos (1) – pos (2) to sphere A

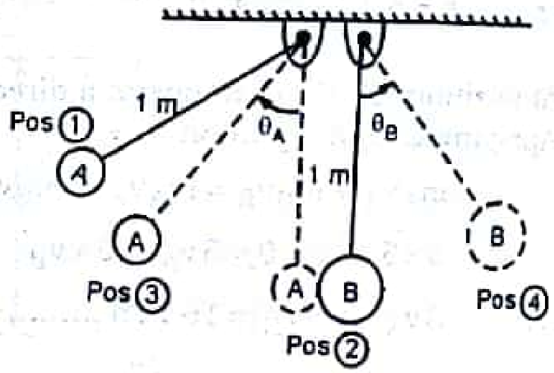
$T_1 = 0$ .... Since the sphere is at rest.

$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 2 \times v^2 = v^2 \text{ J}$

U_{1-2} = by weight $= +mgh$

$= 2 \times 9.81 \times (L - L\cos\theta)$

$= 19.62 \times 1(1 - \cos 50) \quad = \quad 7 \text{ J}$



Using $T_1 + \Sigma U_{1-2} = T_2$

$0 + 7 = v^2 \qquad \text{or} \qquad v = 2.647 \text{ m/s}$

At position (2), there takes place a direct impact between sphere A and B.

Applying C O M equation $\rightarrow$ + ve

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$2 \times 2.647 + 0 = 2v_A' + 4v_B'$

$2v_A' + 4v_B' = 5.295$ .... (1)

Applying C O R equation $\rightarrow$ + ve

$v_B' - v_A' = e(v_A - v_B)$

$v_B' - v_A' = 0.8(2.647 - 0)$

$\therefore \quad -v_A' + v_B' = 2.118$ .... (2)

Solving equation (1) and (2) we get

$v_A' = -0.5295 \text{ m/s} = 0.5295 \text{ m/s} \leftarrow$ and $v_B' = 1.588 \text{ m/s} = 1.588 \text{ m/s} \rightarrow$

Sphere A travels backwards and describes a new angle θ to reach its highest position (3)

Applying W E P pos (2) – pos (3) to sphere A

$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 2 \times 0.5295^2 = 0.2804 \text{ J}$

$T_3 = 0$ .... Since sphere A comes to rest.

U_{2-3} = by weight $= -mgh$ — –ve because displacement is upwards.

$= -2 \times 9.81 \times (L - L\cos\theta_A)$

$= -19.62(1 - \cos\theta_A)$ — here $L = 1$ m

Using $T_2 + \Sigma U_{2-3} = T_3$

$0.2804 - 19.62[1 - \cos\theta_A] = 0$ or $\theta_A = 9.698°$ .... **Ans.**

Sphere B is now set in motion and swings describing angle θ_B at position (4)

Applying W E P pos (2) – pos (4) to sphere B

$T_2 = \frac{1}{2} \times mv^2 = \frac{1}{2} \times 4 \times 1.588^2 = 5.04$ J

$T_4 = 0$ .... Since sphere B comes to rest.

U_{2-4} = by weight $= -mgh$ .... – ve since displacement is upwards

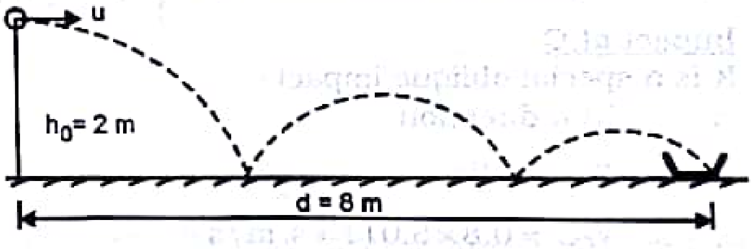
$= -m \times g \times (L - L\cos\theta_B) = -4 \times 9.81 \times 1(1 - \cos\theta_B)$

$= -39.24 \times (1 - \cos\theta_B)$

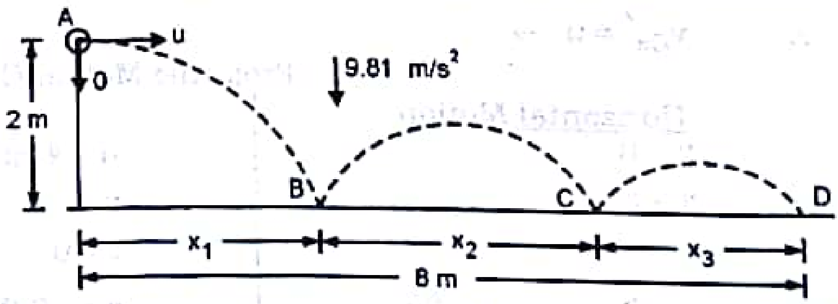
Using $T_2 + \Sigma U_{2-4} = T_4$

$5.04 + [-39.24\ (1 - \cos\theta_B)] = 0$ or $\theta_B = 29.36°$ .... **Ans.**

P10. A small steel ball is to be projected horizontally such that it bounces twice on the surface and lands into a cup placed at a distance of 8m as shown. If the coefficient of restitution for each impact is 0.8, determine the velocity of projection 'u' of the ball. ***(M.U Dec 10)***



Solution: Projectile Motion A to B



<u>Horizontal Motion</u>

$v = u,\ s = x_1,\ t = t_1$

Using $v = \frac{s}{t}$ $\therefore$ $u = \frac{x_1}{t_1}$

$\therefore$ $u = \frac{x_1}{0.6385}$

or $x_1 = 0.6385u$

<u>Vertical Motion ↓ + ve</u>

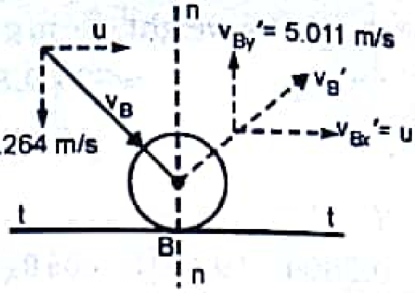
$u = 0,\ v = v_{By},\ s = 2\text{m},\ a = 9.81\ \text{m/s}^2,\ t = t_1$

Using $s = ut + \frac{1}{2}at^2$ $\therefore$ $2 = 0 + \frac{1}{2} \times 9.81 \times t_1^2$

or $t_1 = 0.6385$ sec

Using $v = u + at$

$v_{By} = 0 + 9.81 \times 0.6385$ or $v_{By} = 6.264$ m/s



<u>Impact at B</u>

It is a special oblique impact

In n direction

$$v_{By}' = e\, v_{By}$$

$$\therefore \quad v_{By}' = 0.8 \times 6.364 = 5.011 \text{ m/s} \uparrow$$

Also in t direction, velocities do not change $\quad \therefore \quad v_{Bx}' = u \rightarrow$

Projectile Motion B to C

<u>Horizontal Motion</u>

$v = u,\ s = x_2,\ t = t_2$

Using $v = \dfrac{s}{t}$

$$u = \frac{x_2}{t_2}$$

$$\therefore \quad u = \frac{x_2}{1.0216}$$

or $\quad x_2 = 1.0216u$

<u>Vertical Motion ↑ + ve</u>

$u = 0,\ v = v_{Cy},\ s = 0,\ a = -9.81 \text{ m/s}^2,\ t = t_2$

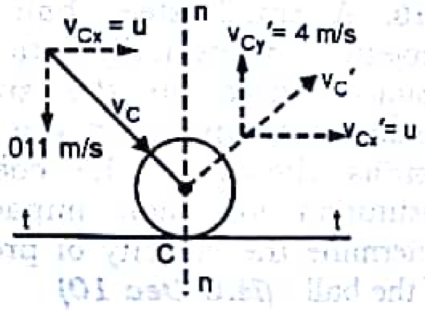
Using $\quad s = ut + \dfrac{1}{2}at^2 \quad \therefore \ 0 = 5.011t - \dfrac{1}{2} \times 9.81 \times t_2^2$

or $\quad t_2 = 1.0216$ sec

Using $\quad v = u + at$

$$-v_{Cy} = 5.011 - 9.81 \times 1.0216$$

$$\therefore \quad v_{Cy} = 5.011 \text{ m/s}$$



<u>Impact at C</u>

It is a special oblique impact

$\therefore$ In n direction

$$v_{Cy}' = e\, v_{Cy}$$

$$\therefore \quad v_{Cy}' = 0.8 \times 5.011 = 4 \text{ m/s} \uparrow$$

Also in t direction, velocities do not change

$$\therefore \quad v_{Cx}' = u \rightarrow$$

Projectile Motion C to D

<u>Horizontal Motion</u>

$v = u$

$s = x_3$

$t = t_3$

Using $\quad v = \dfrac{s}{t} \quad \therefore \quad u = \dfrac{x_3}{t_3}$

$$\therefore \quad u = \frac{x_3}{0.8155}$$

or $\quad x_3 = 0.8155u$

<u>Vertical Motion ↑ + ve</u>

$u = 4$ m/s

$v = -$

$s = 0$

$a = -9.81 \text{ m/s}^2$

$t = t_3$

Using $\quad s = ut + \dfrac{1}{2}at^2 \quad \therefore \quad 0 = 4 \times t + \dfrac{1}{2} \times (-9.81) \times t_3^2$

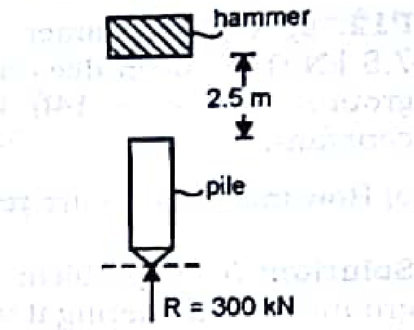
$t_3 = 0.8155$ sec

Now $\quad d = x_1 + x_2 + x_3$

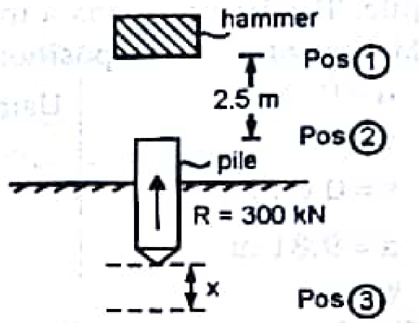
$$8 = 0.6385u + 1.0216u + 0.8155u$$

or $\quad u = 3.23$ m/s .... **Ans.**

P11. A pile of 400 kg mass is being driven into ground with the help of a hammer of mass 1000 kg. Hammer falls through a constant height of 2.5 m. Assuming plastic impact between hammer and pile, find the number of blows required to drive the pile by 1 m when the resistance offered by the ground to penetration is 300 kN. ***(M.U May 11)***



Solution: This problem relates to driving of pile in the ground by hammering it with a hammer.
Let v be the velocity of the hammer just before striking the pile. The hammer has a free fall under gravity.
Motion of hammer position (1) – position (2) – M U G ↓ +ve



$u = 0$	Using $v^2 = u^2 + 2as$
$v = v$	$v^2 = 0 + 2 \times 9.81 \times 2.5$
$s = 2.5$ m	$v = 7$ m/s
$a = 9.81$ m/s^2	
$t = -$	

The hammer has a direct impact with the pile at rest at position (2). Since Impact is plastic i.e. e = 0, both hammer and pile move with a common speed v' after impact.
Applying C O M ↓ +ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$1000 \times 7 + 0 = 1000 v' + 400 v'$$

$\therefore$ $v' = 5$ m/s velocity of hammer and pile just after impact.

Both pile and hammer start with initial velocity of 5 m/s and travel down distance x to position (3). The ground offers resistance R to this motion.
Applying W E P pos (2) – pos (3)

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 1400 \times 5^2 = 17500 \text{ Joules}$$

$$T_3 = 0$$

U_{2-3} = 1) by weight $= + mgh$

$$= 1400 \times 9.81 \times x \quad = \quad 13734\,x \text{ Joules}$$

2) by force $R = F \times s$

$$= -300 \times 10^3 \times x \text{ Joules}$$

Using $T_2 + \Sigma U_{2-3} = T_3$

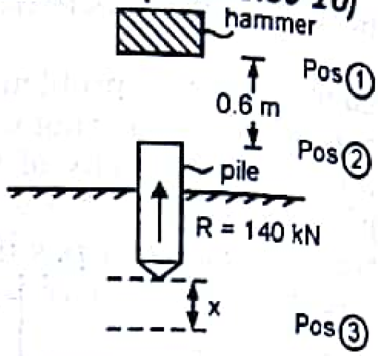
$$17500 + \left[13734\,x - 3 \times 10^5 x\right] = 0$$

or $x = 0.06113$ m .... **Ans.**

If one blow drives the pile by x = 0.06113 m, then no of blows to drive the pile by 1 m = $\frac{1}{0.06113} = 16.36$ □ 17 $\therefore$ 17 blows are required .... **Ans.**

P12. a) A pile hammer, weighing 15 kN drops from height of 600 mm on a pile of 7.5 kN. How deep does a single blow of hammer drive the pile if the resistance of the ground to pile is 140 kN? (Assume plastic impact and the ground resistance is constant)

b) How many blows are required to drive the pile 1 m in the ground. ***(VJTI Nov 10)***



Solution: This problem relates to driving of a pile in the ground by hammering it with a hammer.
Let v be the velocity of the hammer just before striking the pile. The hammer has a free fall under gravity.
Motion of hammer position (1) – position (2) – M U G ↓ +ve

$u = 0$	Using $v^2 = u^2 + 2as$
$v = v$	$v^2 = 0 + 2 \times 9.81 \times 0.6$
$s = 0.6$ m	$v = 3.43$ m/s
$a = 9.81$ m/s^2	
$t = -$	

The hammer has a direct impact with the pile at rest at position (2). Since Impact is plastic i.e. e = 0, both hammer and pile move with a common speed v' after impact.
Applying C O M ↓ +ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$\frac{15000}{9.81} \times 3.43 + 0 = \frac{15000}{9.81} v' + \frac{7500}{9.81} v'$$

∴ $v' = 2.2867$ m/s .... velocity of hammer and pile just after impact.

Both pile and hammer travel together with an initial speed of $v' = 2.2867$ m/s through ground for a distance x and finally comes to rest at pos (3). The ground offers resistance R = 140 kN to the pile.
Applying W E P pos (2) – pos (3)

$$T_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times \left(\frac{15000 + 7500}{9.81}\right) \times 2.2867^2 = 5996.4 \text{ Joules}$$

$$T_3 = 0$$

U_{2-3} = 1) by weight $= +mgh = (15000 + 7500) \times x = 22500x$ Joules

2) by force $R = F \times s = -140 \times 10^3 \times x$ Joules

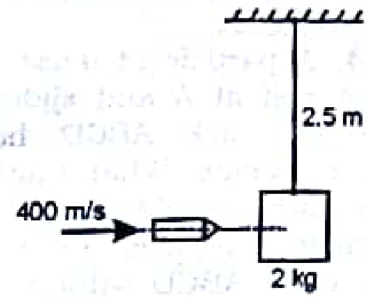
Using $T_2 + \Sigma U_{2-3} = T_3$

$$5996.4 + \left[22500x - 140 \times 10^3 x\right] = 0$$

or $x = 0.05103$ m .... **Ans.**

∴ If one blow drives by the pile by x = 0.05103 m, then no of blows to drive the pile by 1 m = $\frac{1}{0.05103} = 19.59 \approx 20$ ∴ 20 blows are required .... **Ans.**

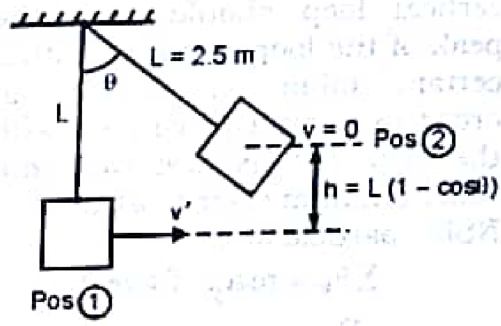
P13. A bullet of mass 30 gm moving horizontally with 400 m/s strikes a 2 kg wooden block suspended by a string 2.5 m long. To what maximum angle with vertical will the block and embedded bullet swing.



Solution: The wooden block is initially at rest at position (1). On being hit by a bullet, it gains energy and swings as a pendulum undergoing a maximum angular displacement of θ at position (2). Here at the peak, its velocity becomes zero.

Impact at position (1)

It is a case of Direct Impact. The bullet gets embedded in the block and together they move with a common velocity v'.



Applying C O M $\rightarrow$ +ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$0.03 \times 400 + 0 = 0.03 \times v' + 2 \times v'$$

$v' = 5.911\,m/s$ ... speed of block just after impact

Applying W E P pos (1) – pos (2) to the block

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2} \times 2.03 \times 5.911^2 = 35.468\ J$$

$T_2 = 0$ .... since at highest position it comes to rest

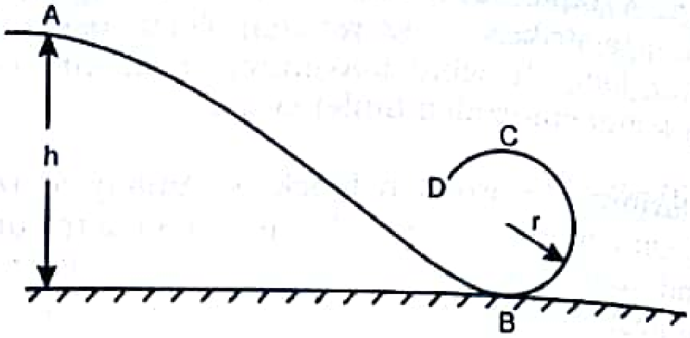
$$U_{1-2} = \text{U by weight} = -mgh$$
$$= -2.03 \times 9.81 \times L(1 - \cos\theta)$$
$$= -2.03 \times 9.81 \times 2.5(1 - \cos\theta)$$
$$= -49.785(1 - \cos\theta)$$

Using $T_1 + \Sigma U_{1-2} = T_2$

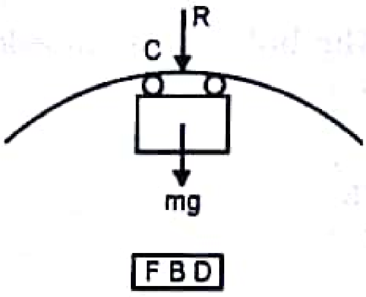
$$35.468 - 49.785(1 - \cos\theta) = 0$$

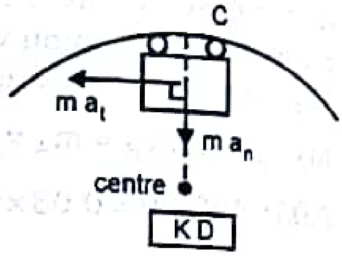
or $\theta = 73.286°$ .... **Ans**

P14. A particle of mass 'm' starts from rest at A and slides down a smooth track ABCD held in a vertical plane. What should be the minimum height h so that the particle completes its journey on the track ABCD without falling off the track at C. ***(NMIMS May 09)***



Solution: Any particle negotiating (traveling) on a vertical loop should pass the peak of the loop (point C) with a certain minimum velocity in order to remain in contact with the loop. Let us first work out this minimum velocity at C.





NSL – particle at C

$$\Sigma F_y = m a_y \quad \uparrow +ve$$

$$-R - mg = -m a_n$$

$\therefore \quad 0 - mg = -m a_n$ | For minimum velocity condition, the roller coaster tends to loose contact with the loop at C, making R = 0

$\therefore \quad a_n = g$

or $\quad \dfrac{v^2}{r} = g \quad$ or $\quad v = \sqrt{gr}$ .... velocity at C

Now applying W E P pos (A) – pos (C)

$$T_1 = 0$$

$$T_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times m \times \left(\sqrt{gr}\right)^2 = \frac{mgr}{2}$$

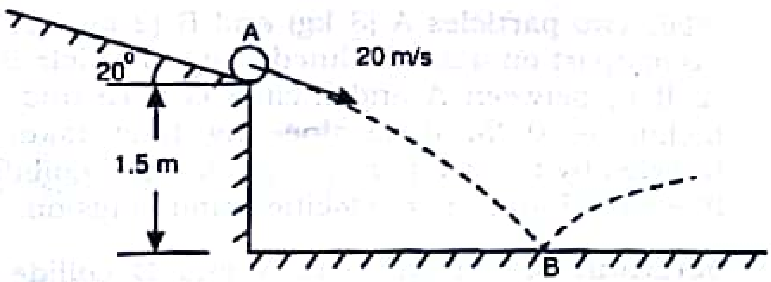
$$U_{1-2} = \text{by weight} = +mgh$$

$$= +mg(h - 2r)$$

Using $\quad T_1 + \Sigma U_{1-2} = T_2$

$$0 + mg(h - 2r) = \frac{mgr}{2}$$ .... h = level difference = h – 2r

or $\quad h = 2.5r$ .... **Ans.**



P15. A ball slides down on a smooth inclined surface and strikes the ground at B. Determine the magnitude of velocity and direction of the ball after impact. Take e = 0.6

Solution: The ball has a projectile motion from A to B and then a special oblique impact with the ground at B. Only working with vertical motion of projectile

Projectile Motion (A to B)

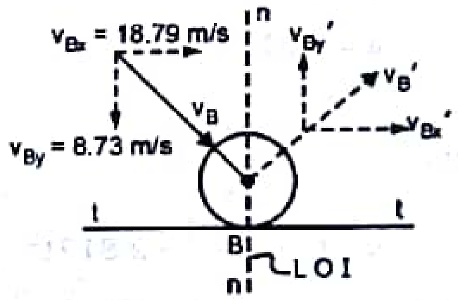
Vertical Motion ↓ + ve

$u = 20 \sin 20$	using $v^2 = u^2 + 2as$
$v = v_{By}$	$v_{By}{}^2 = (20 \sin 20)^2 + 2 \times 9.81 \times 1.5$
$s = 1.5\,m$	$\therefore \quad v_{By} = 8.73\ m/s \downarrow$
$a = 9.81\ m/s^2$	
$t = -$	

Knowing horizontal component of velocity does not change

$v_{Bx} = 20 \cos 20 \;= 18.79\,m/s \;\rightarrow$

Impact at B



It is a case of special oblique impact.

The n and t direction of impact have been shown

In n direction $\quad v_{By}' = e\, v_{By}$

$$v_{By}' = 0.6 \times 8.73 = 5.238\ m/s \uparrow$$

In t direction, velocity does not change

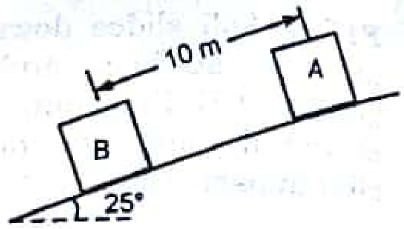
$$v_{Bx}' = v_{Bx} = 18.79\ m/s$$

Resultant velocity $\quad v_B' = \sqrt{5.238^2 + 18.79^2} = 19.506\ m/s$

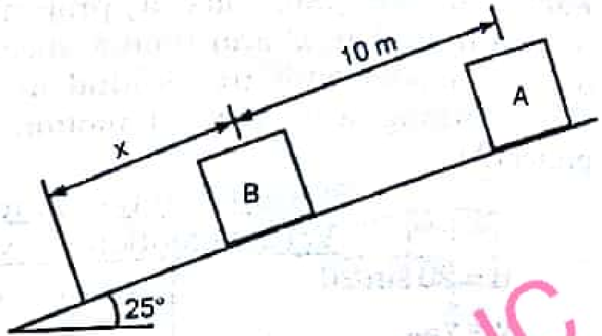
$$\tan\theta' = \frac{v_{By}'}{v_{Bx}'} = \frac{5.238}{18.79} = \quad \therefore \quad \theta' = 15.78°$$

$\therefore$ The velocity of the ball after impact is $v_B' = 19.506\ m/s$ at $\theta' = 15.78°$ ∠ ... **Ans.**

P16. Two particles A (3 kg) and B (2 kg) are initially at rest 10 m apart on a 25° inclined plane. Particle B being ahead of A. If μ_k between A and incline is 0.15 and between B and incline is 0.25, determine the time taken and distance traveled by the two particles before they collide.
If e = 0.75 find their velocities after collision.



Solution: Let the particle A and B collide at some distance x from B, t sec after being released. Let their velocities before collision be v_A and v_B respectively.
Using standard relation for acceleration of a block sliding down a rough inclined slope on its own. We have,



$$a = g(\sin\theta - \mu_k \cos\theta)$$

$$\therefore \quad a_A = 9.81(\sin 25 - 0.15\cos 25) = 2.812 \text{ m/s}^2$$

Also $a_B = 9.81(\sin 25 - 0.25\cos 25) = 1.923 \text{ m/s}^2$

Motion of A

Rectilinear – Uniform Acceleration

$u = 0$

$v = v_A$

$s = x + 10$

$a = 2.812 \text{ m/s}^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$x + 10 = 0 + \frac{1}{2} \times 2.812t^2 \quad \text{......... (1)}$$

Motion of B

Rectilinear – Uniform Acceleration

$u = 0$

$v = v_B$

$s = x$

$a = 1.923 \text{ m/s}^2$

$t = t$

using $s = ut + \frac{1}{2}at^2$

$$x = 0 + \frac{1}{2} \times 1.923t^2 \quad \text{......... (2)}$$

Solving equations (1) and (2), we get

$x = 21.63$ m, $t = 4.743$ sec .... **Ans.**

$\therefore$ B travels 21.63 m and A travels 31.63 m before they collide. .... **Ans.**

using $v = u + at$

$v_A = 0 + 2.812 \times 4.743$

$\therefore \quad v_A = 13.34 \text{ m/s}$

using $v = u + at$

$v_B = 0 + 1.923 \times 4.743$

$\therefore \quad v_B = 9.12 \text{ m/s}$

Collision of particles A and B : This is a case of direct impact.

Applying C O M equation □ +ve

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$3 \times 13.34 + 2 \times 9.12 = 3v_A' + 2v_B'$$

$$3v_A' + 2v_B' = 58.26 \quad \text{......... (1)}$$

Applying C O R equation □ +ve

$$v_B' - v_A' = e(v_A - v_B)$$

$$v_B' - v_A' = 0.75(13.34 - 9.12)$$

$$\therefore \quad -v_A' + v_B' = 3.165 \quad \text{......... (2)}$$

Solving, we get $v_A' = 10.386 \text{ m/s}$ and $v_B' = 13.55 \text{ m/s}$ .... **Ans.**

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