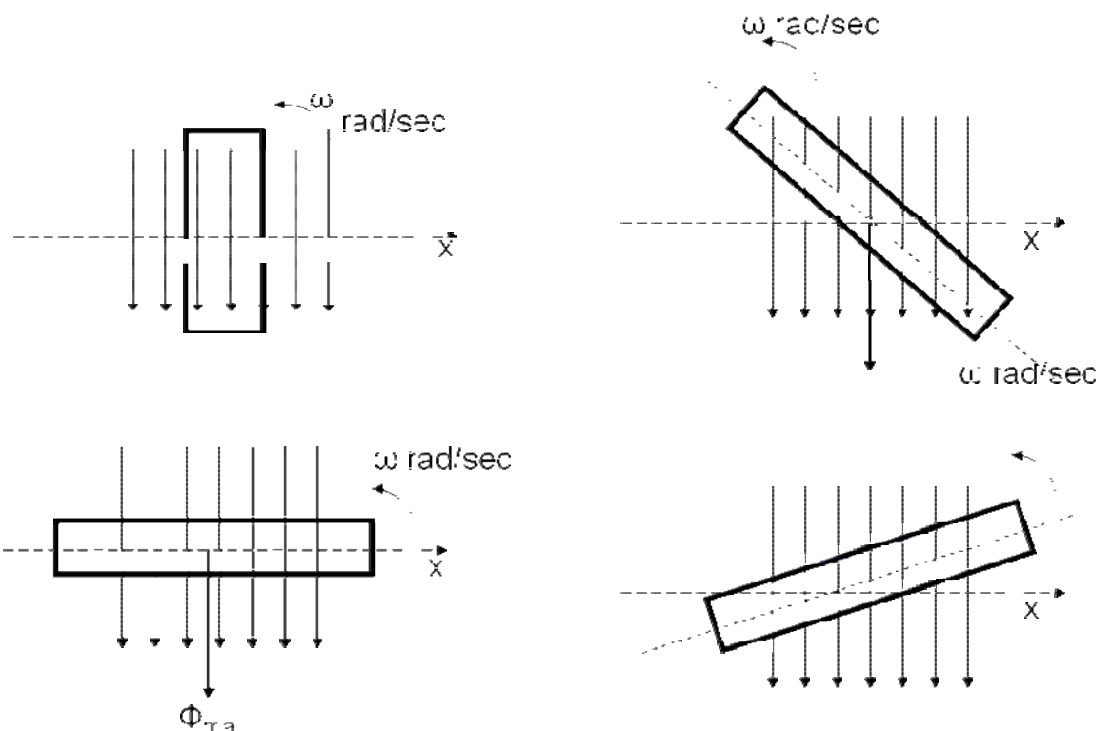


# **UNIT-II**

## **Alternating Quantities**

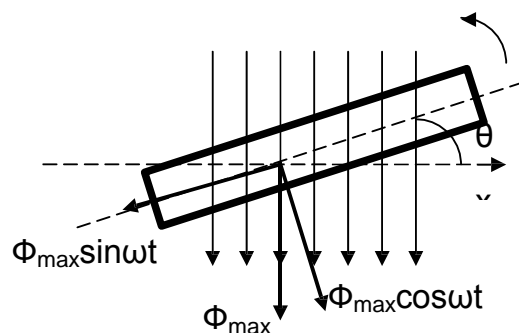
**Principle of AC voltage:** Consider a rectangular coil of  $N$  turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of  $\omega$  rad/sec



When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated. The generation of sinusoidal AC

Voltage can also be explained using mathematical equations. Consider a rectangular coil of  $N$  turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure. This flux can be divided into two components, one component acting along the plane of the coil  $\Phi_{\max} \sin \omega t$  and another component acting perpendicular to the plane of the coil  $\Phi_{\max} \cos \omega t$ .

$\omega$  rad/sec



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie  $\Phi_{\max} \cos \omega t$  induces an emf in the coil.

Angular Frequency ( $\omega$ )

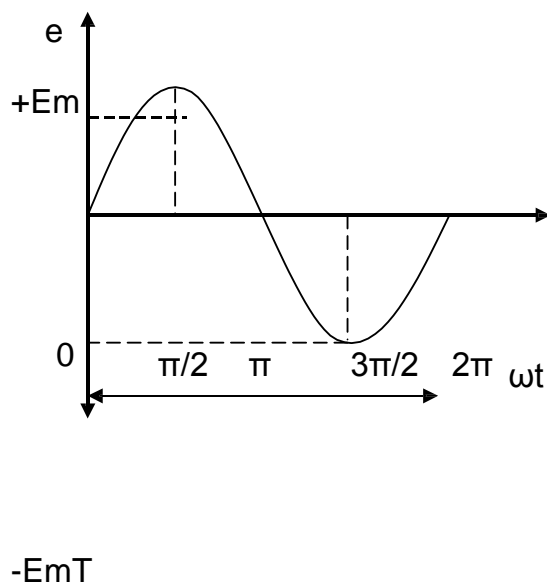
Angular frequency is defined as the number of radians covered in one second (ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

#### Advantages of AC system over DC system

1. AC voltages can be efficiently stepped up/down using transformer
2. AC motors are cheaper and simpler in construction than DC motors
3. Switchgear for AC system is simpler than DC system

#### Definition of Alternating Quantity



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time.

Important terms associated with an alternating quantity are defined below.

#### Amplitude

It is the maximum value attained by an alternating quantity. Also called as maximum or peak value

#### Time Period (T)

It is the Time Taken in seconds to complete one cycle of an alternating quantity

#### Instantaneous Value

It is the value of the quantity at any instant

#### Frequency (f)

It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec. The relationship between frequency and time period can be derived as follows.

Time taken to complete f cycles = 1 second

Time taken to complete 1 cycle = 1/f second

$$T = 1/f$$

#### Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

Average value =  $\frac{\text{Area under one cycle}}{\text{Base}}$

\_\_\_\_\_

Base

$$V_{av} = \frac{1}{2\pi} \int_0^{2\pi} v d(\alpha)$$

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

Average value =  $\frac{\text{Area under one half cycle}}{\text{Base}}$

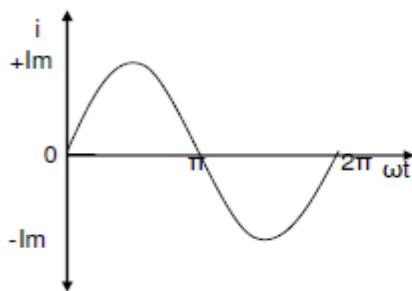
Base

### RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v^2 d(\omega t)}$$

RMS value of a sinusoidal current



$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

### Form Factor

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMSValue}{AverageValue}$$

### Peak Factor or Crest Factor

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

$$PF = \frac{MaximumValue}{RMSValue}$$

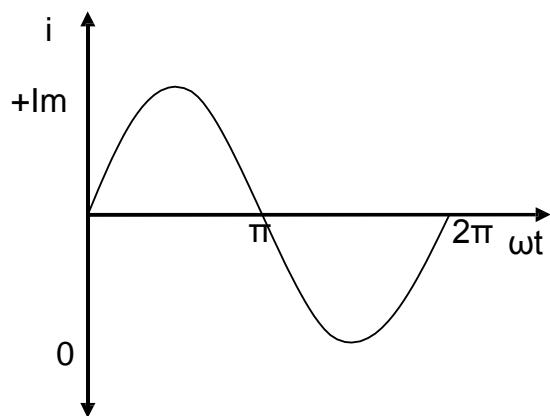
### Phasor Representation

An alternating quantity can be represented using

- (i) Waveform
- (ii) Equations
- (iii) Phasor

A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor**. A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

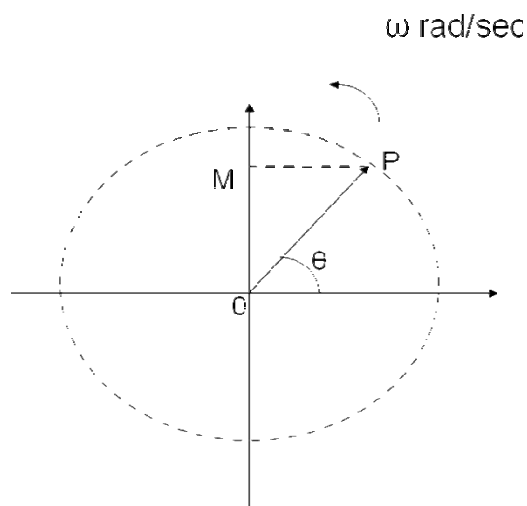
The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.



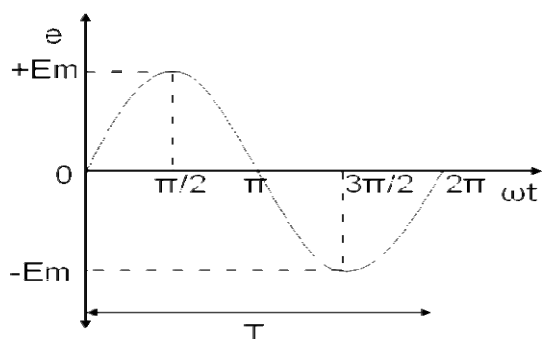
-Im

$$i = I_m \sin \omega t$$

Draw a line  $OP$  of length equal to  $I_m$ . This line  $OP$  rotates in the anticlockwise direction with a uniform angular velocity  $\omega$  rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of  $OP$  on the  $y$ -axis is given by  $OM = OP \sin \theta = I_m \sin \omega t$ . Hence the line  $OP$  is the phasor representation of the sinusoidal current



## Phase



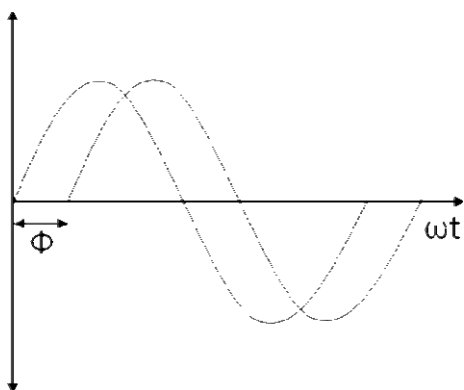
Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference

Phase of  $+E_m$  is  $\pi/2$  rad or  $T/4$

sec Phase of  $-E_m$  is  $3\pi/2$  rad or

$3T/4$  sec

## Phase Difference



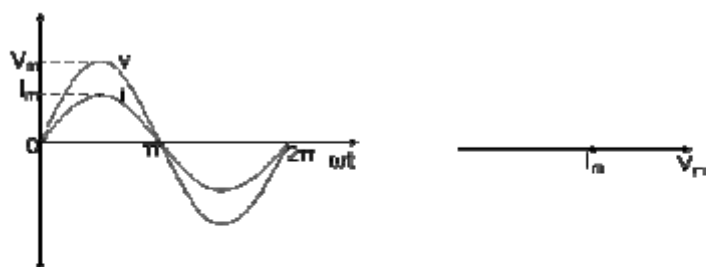
When two alternating quantities of the same frequency have different zero points,



they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

### In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.

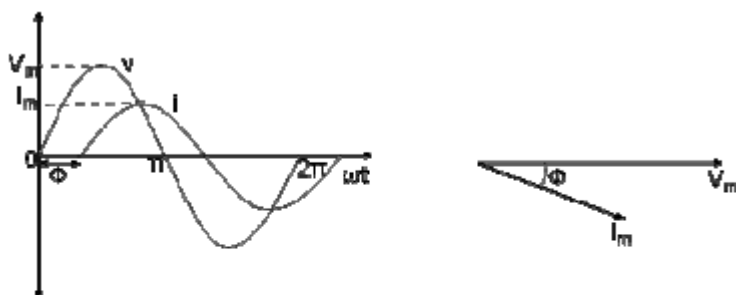


$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

### Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.



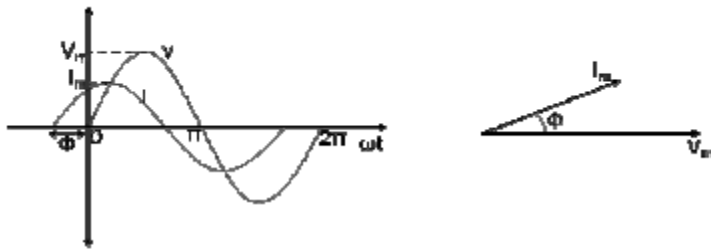
$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \Phi)$$

### Leading

In the figure shown, the zero point of the current waveform is before the zero point of

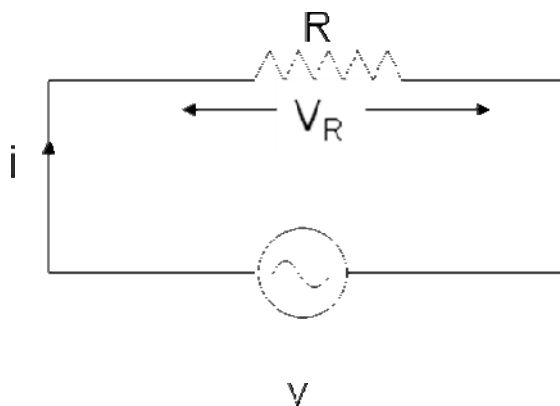
the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.



$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \Phi)$$

## AC circuit with a pure resistance



Consider an AC circuit with a pure resistance  $R$  as shown in the figure. The alternating voltage  $v$  is given by  $v = V_m \sin \omega t$

----- (1)

The current flowing in the circuit is  $i$ . The voltage across the resistor is given as  $V_R$  which is the same as  $v$ .

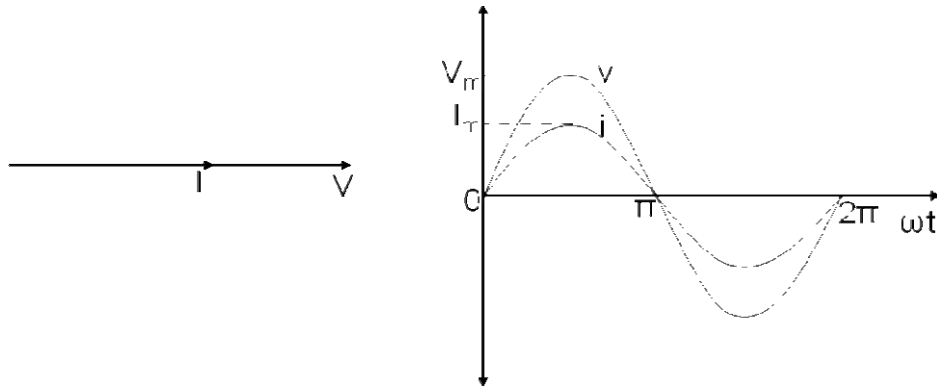
Using ohms law, we can write the following relations

$$i = I_m \sin \omega t \quad \text{----- (2)}$$

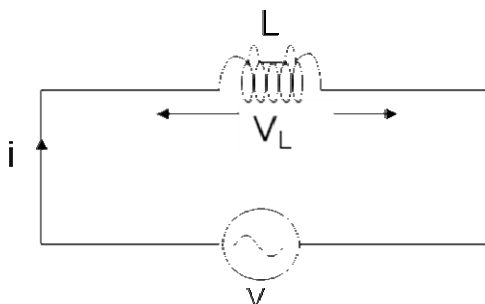
Where  $I_m = \frac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be

drawn as below.



## AC circuit with a pure inductance



Consider an AC circuit with a pure inductance  $L$  as shown in the figure. The alternating voltage  $v$  is given by

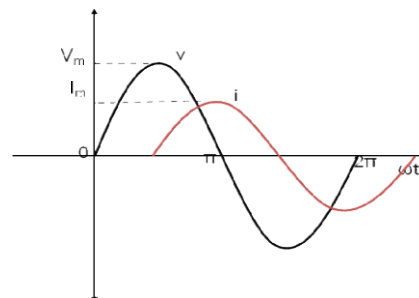
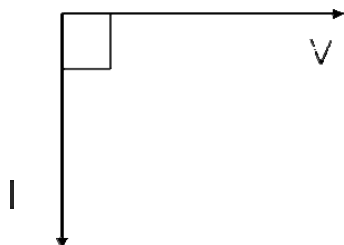
$$v = V_m \sin \omega t$$

----- (1)

The current flowing in the circuit is  $i$ . The voltage across the inductor is given as  $V_L$  which is the same as  $v$ .

$$i = I_m \sin(\omega t - \pi/2) \quad \text{----- (2)}$$

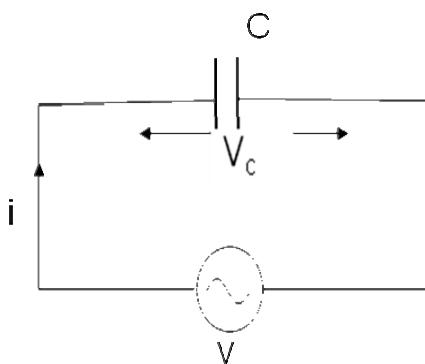
From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by  $90^\circ$ . Hence the voltage and current waveforms and phasors can be drawn as below.



The inductive reactance  $X_L$  is given as

$$\omega L = 2\pi fL$$

### AC circuit with a pure capacitance



Consider an AC circuit with a pure capacitance  $C$  as shown in the figure. The alternating voltage  $v$  is given by

$$= V_m \sin \omega t$$

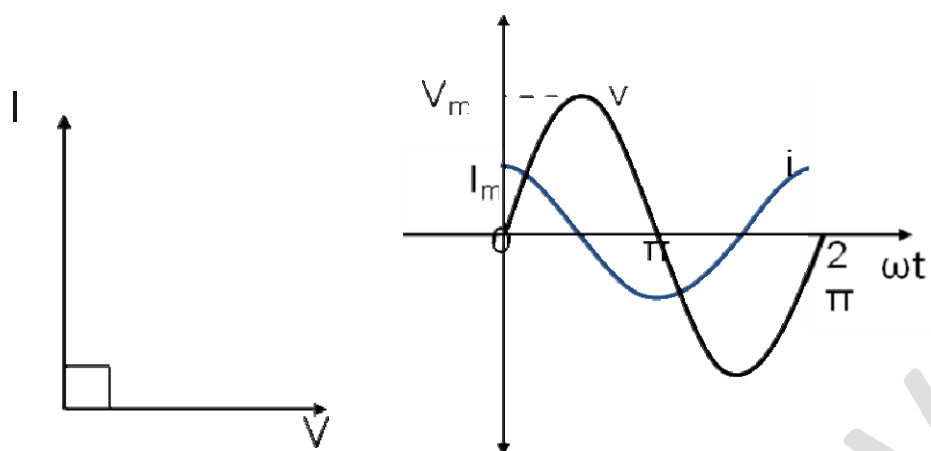
----- (1)

The current flowing in the circuit.

$$i = I_m \sin(\omega t + \pi/2)$$

----- (2)

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by  $90^\circ$ . Hence the voltage and current waveforms and phasors can be drawn as below.



## Capacitive reactance

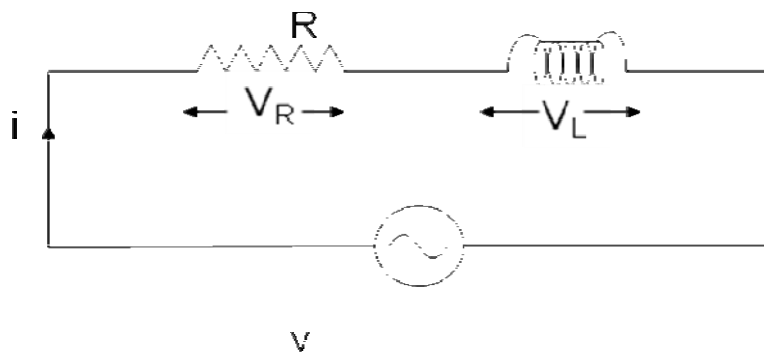
The capacitive reactance  $X_C$  is given as

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I_m = \frac{V_m}{X_C}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms ( $\Omega$ )

## R-L Series circuit



Consider an AC circuit with a resistance  $R$  and an inductance  $L$  connected in series as shown in the figure. The alternating voltage  $v$  is given by

$$v = V_m \sin \omega t$$

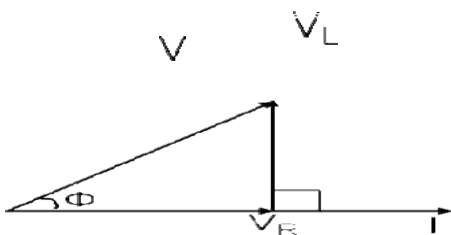
The current flowing in the circuit is  $i$ . The voltage across the resistor is  $V_R$  and that across the inductor is  $V_L$ .

$V_R = IR$  is in phase with  $I$

$V_L = IX_L$  leads current by 90 degrees

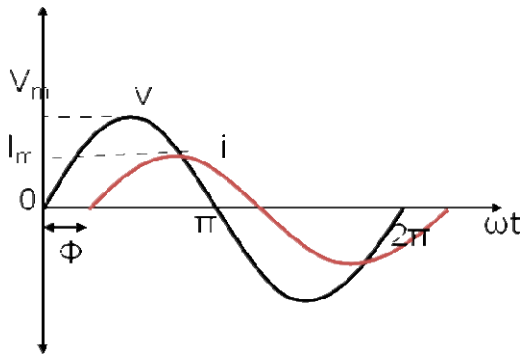
With the above information, the phasor diagram can be drawn as shown.





The current  $I$  is taken as the reference phasor. The voltage  $V_R$  is in phase with  $I$  and the voltage  $V_L$  leads the current by  $90^\circ$ . The resultant voltage  $V$  can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle  $\Phi$  or in other words the current lags behind the voltage by an angle  $\Phi$ .

The waveform and equations for an RL series circuit can be drawn as below.



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t - \Phi)$$

From the phasor diagram, the expressions for the resultant voltage  $V$  and the angle  $\Phi$  can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V_R = IR$$

$$V_L = IX_L$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I \sqrt{R^2 + X_L^2}$$

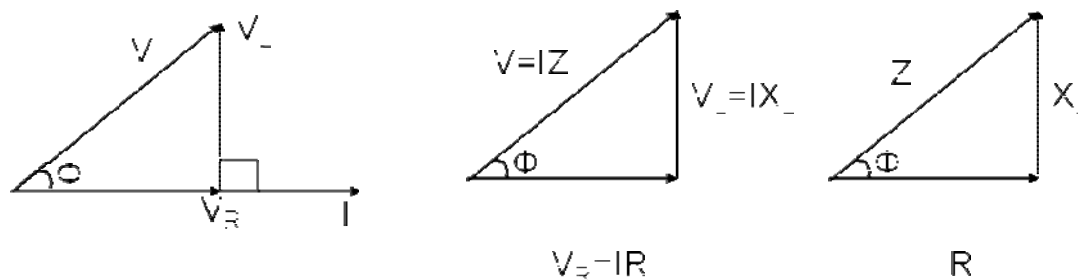
$$V = IZ$$

Where impedance  $Z = \sqrt{R^2 + X_L^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms ( $\Omega$ ).

## Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



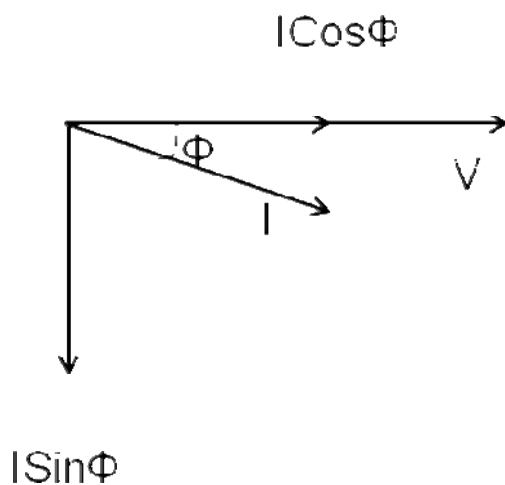
The impedance triangle is right angled triangle with  $R$  and  $X_L$  as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is  $\Phi$ .

## Power

In an AC circuit, the various powers can be classified as

1. Real or Active power
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the circuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.



From the phasor diagram of an RL series circuit, the current can be divided into two components. One component along the voltage  $I \cos \Phi$ , that is called as the active component of current and another component perpendicular to the voltage  $I \sin \Phi$  that is called as the reactive component of current.

### Real Power

The power due to the active component of current is called as the active power or real power. It is denoted by  $P$ .

$$P = V \times I \cos \Phi = I^2 R$$

Real power is the power that does useful power. It is the power that is consumed by the resistance. The unit for real power is Watt (W).

### Reactive Power

The power due to the reactive component of current is called as the reactive power. It is denoted by  $Q$ .

$$Q = V \times I \sin \Phi = I^2 X_L$$

Reactive power does not do any useful work. It is the circulating power in the L and C components. The unit for reactive power is Volt Amperes Reactive (VAR).

### Apparent Power

The apparent power is the total power in the circuit. It is denoted by  $S$ .

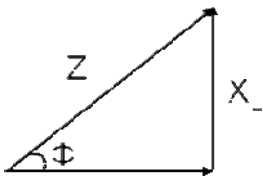
$$S = V \times I = I^2 Z$$

$$S = \sqrt{P^2 + Q^2}$$

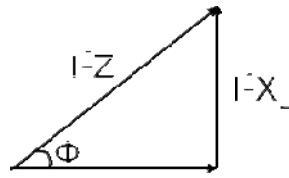
The unit for apparent power is Volt Amperes (VA).

### Power Triangle

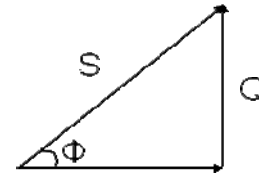
From the impedance triangle, another triangle called the power triangle can be derived as shown.



R



IR



P

The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is  $\Phi$ . The power triangle enables us to calculate the following things.

1. Apparent power  $S = \sqrt{P^2 + Q^2}$

2. Power Factor  $\cos\Phi = \frac{P}{S} = \frac{\text{Real Power}}{\text{Apparent Power}}$

The power Factor in an AC circuit can be calculated by any one of the following methods

- ❖ Cosine of angle between V and I
- ❖ Resistance/Impedance  $R/Z$
- ❖ Real Power/Apparent Power  $P/S$

Phasor algebra in a RL series circuit

$$\underline{V} = V + j0 = V\angle 0^\circ$$

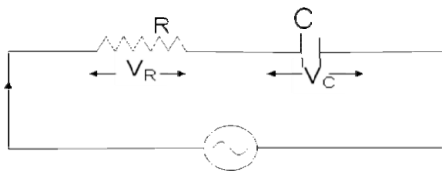
$$\underline{Z} = R + jX_L = Z\angle\Phi$$

$$\underline{I} = \frac{\underline{V}}{\underline{Z}} = \frac{V}{Z} \angle -\Phi$$

$$\angle -\Phi$$

$$\bar{S} = VI^* = P + jQ$$

### R-C Series circuit



i

v

Consider an AC circuit with a resistance  $R$  and a capacitance  $C$  connected in series as shown in the figure. The alternating voltage  $v$  is given by

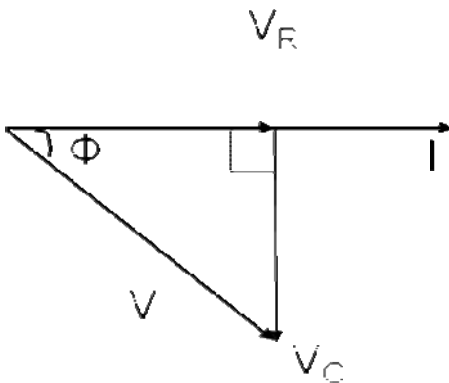
$$v = V_m \sin \omega t$$

The current flowing in the circuit is  $i$ . The voltage across the resistor is  $V_R$  and that across the capacitor is  $V_C$ .

$V_R = IR$  is in phase with  $I$

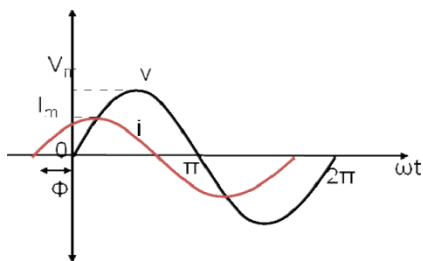
$V_C = IX_C$  lags behind the current by  $90^\circ$  degrees

With the above information, the phasor diagram can be drawn as shown.



The current  $I$  is taken as the reference phasor. The voltage  $V_R$  is in phase with  $I$  and the voltage  $V_C$  lags behind the current by  $90^\circ$ . The resultant voltage  $V$  can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle  $\Phi$  or in other words the current leads the voltage by an angle  $\Phi$ .

The waveform and equations for an RC series circuit can be drawn as below.



$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t + \Phi)$$

From the phasor diagram, the expressions for the resultant voltage  $V$  and the angle  $\Phi$

can be derived as follows.

$$V = \sqrt{V_R^2 + V_C^2}$$

$$V_R = IR$$

$$V_C = IX_C$$

$$V = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I \sqrt{R^2 + X_C^2}$$

$$V = IZ$$

Where impedance  $Z = \sqrt{R^2 + X_C^2}$

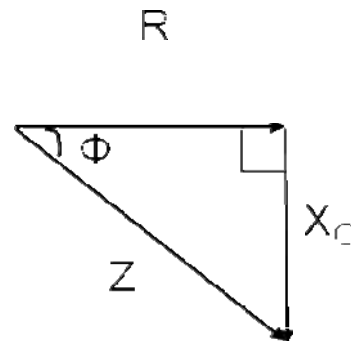
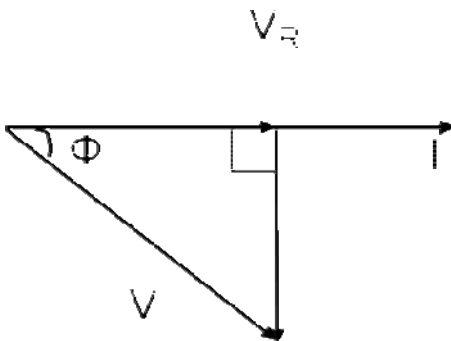
Average power

$$P = VI \cos \phi$$

Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

**Impedance Triangle**

We can derive a triangle called the impedance triangle from the phasor diagram of an RC series circuit as shown





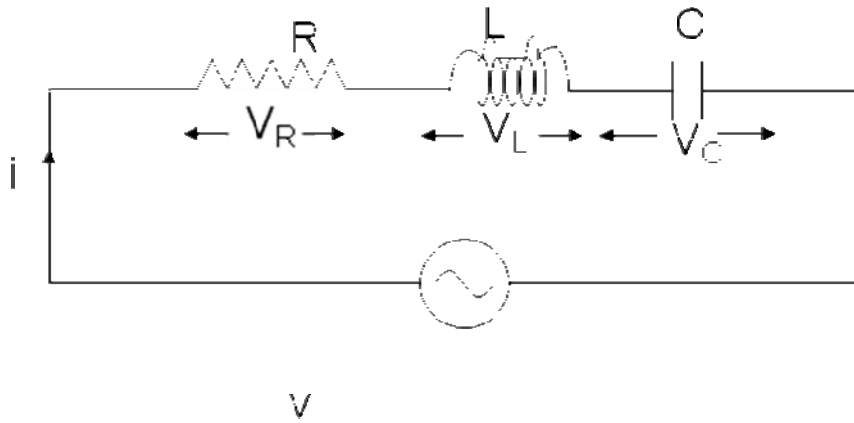
Phasor algebra for RC series circuit

$$\underline{V} = V + j0 = V\angle 0^\circ$$

$$\underline{Z} = R - jX_C = Z\angle -\Phi$$

$$\underline{I} = \frac{\underline{V}}{\underline{Z}} = \frac{V}{Z} \angle +\Phi$$

## R-L-C Series circuit



Consider an AC circuit with a resistance  $R$ , an inductance  $L$  and a capacitance  $C$  connected in series as shown in the figure. The alternating voltage  $v$  is given by

$$v = V_m \sin \omega t$$

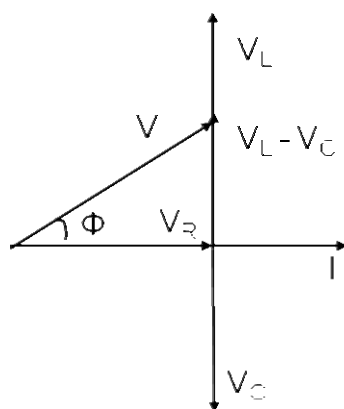
The current flowing in the circuit is  $i$ . The voltage across the resistor is  $V_R$ , the voltage across the inductor is  $V_L$  and that across the capacitor is  $V_C$ .

$V_R = IR$  is in phase with  $I$

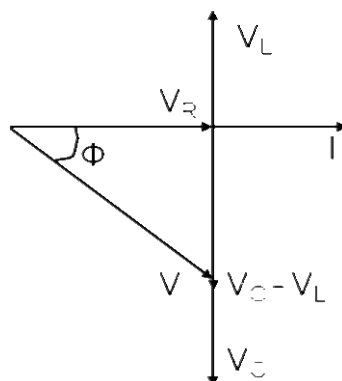
$V_L = IX_L$  leads the current by 90 degrees

$V_C = IX_C$  lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current  $I$  is taken as the reference phasor. The voltage  $V_R$  is in phase with  $I$ , the voltage  $V_L$  leads the current by  $90^\circ$  and the voltage  $V_C$  lags behind the current by  $90^\circ$ . There are two cases that can occur  $V_L > V_C$  and  $V_L < V_C$  depending on the values of  $X_L$  and  $X_C$ . And hence there are two possible phasor diagrams. The phasor  $V_L - V_C$  or  $V_C - V_L$  is drawn and then the resultant voltage  $V$  is drawn.



$$V_L > V_C$$



$$V_L < V_C$$

From the phasor diagram we observe that when  $V_L > V_C$ , the voltage leads the current by an angle  $\Phi$  or in other words the current lags behind the voltage by an angle  $\Phi$ . When  $V_L < V_C$ , the voltage lags behind the current by an angle  $\Phi$  or in other words the current leads the voltage by an angle  $\Phi$ .

From the phasor diagram, the expressions for the resultant voltage  $V$  and the angle  $\Phi$  can be derived as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

Where impedance  $Z = \sqrt{R^2 + (X_L - X_C)^2}$

From the expression for phase angle, we can derive the following three cases

Case (i): When  $X_L > X_C$

The phase angle  $\Phi$  is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When  $X_L < X_C$

The phase angle  $\Phi$  is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When  $X_L = X_C$

The phase angle  $\Phi = 0$  and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

The voltage and the current can be represented by the following equations. The angle  $\Phi$  is positive or negative depending on the circuit elements.

$$V = V_m \sin \omega t$$

$$I = I_m \sin(\omega t \pm \Phi)$$

Average power

$$P = VI \cos \phi$$

$$P = (IZ) \times I \times \frac{R}{Z}$$

$$P = I^2 R$$

Hence the power in an RLC series circuit is consumed only in the resistance. The inductance and the capacitance do not consume any power.

Phasor algebra for RLC series circuit

$$\underline{V} = V + j0 = V \angle 0^\circ$$

$$\underline{Z} = R + j(X_L - X_C) = Z \angle \Phi$$

$$\underline{I} = \frac{\underline{V}}{\underline{Z}} = \frac{V}{Z} \angle -\Phi$$