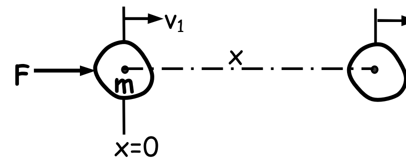
WORK – ENERGY PRINCIPLE

WORK – ENERGY EQUATION :

Consider a body of mass 'm', moving with an initial velocity 'v' and acted upon by a force 'F' as shown. Then, according to Newton's law -
Magnitude of the net unbalanced force = mass × acceleration of the system.

ie., $F = m\dfrac{dv}{dt} = m\dfrac{dv}{ds}\dfrac{ds}{dt} = mv\dfrac{dv}{ds}$



Hence - $\boxed{\int_0^x F.ds = \int_{v_1}^{v_2} mv.dv}$

Therefore, $\mathbf{F.x = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}$

Hence, according to Work-Energy principle, the net work done by all external forces on a system is equal to the change in kinetic energy of the system.

ENERGY BALANCE EQUATION :

The general Energy Balance equation is written as follows :

Initial Energy (E_1) + Gain of Energy = Final Energy (E_2) + Loss of Energy

Initial Energy & Final Energy →
- Potential Energy of the mass
- Kinetic Energy of the mass
- Energy stored if the spring

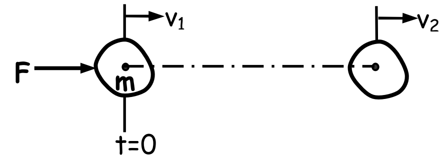
Gain of Energy → Work done by all external forces in the direction of motion.

Loss of Energy → Work done by all external forces resisting the motion.

IMPULSE – MOMENTUM PRINCIPLE :

We have - $F = m\dfrac{dv}{dt}$. Hence - $\boxed{\int_0^t F.dt = \int_{v_1}^{v_2} m.dv}$

ie. $F.t = mv_2 - mv_1$



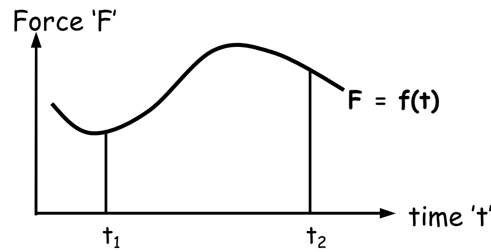
According to Impulse – Momentum principle, the impulse acting on a system is equal to the change in momentum of the system.

Note :

If a system is acted upon by a force that varies with respect to time, as shown, then

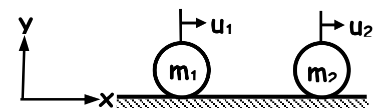
$\int_{t_2}^{t_1} F.dt$ = Area under F vs t plot

= change in momentum of the system.



CONSERVATION OF LINEAR MOMENTUM:

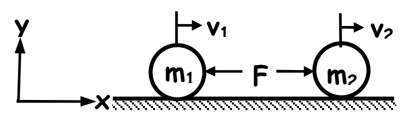
Consider a mass 'm_1' moving with velocity 'u_1' colliding with another mass 'm_2' moving with a velocity 'u_2'. At the instant of collision, both masses experience equal and opposite impulsive forces, due to which their velocities change to 'v_1' and 'v_2', as shown. Let 'Δt' be the time of contact between the two masses.



Applying Impulse – Momentum equation to both masses:

For m_1 :- $- F.t = m_1v_1 - m_1u_1$ → (i)

For m_2 :- $+ F.t = m_2v_2 - m_2u_2$ → (ii)



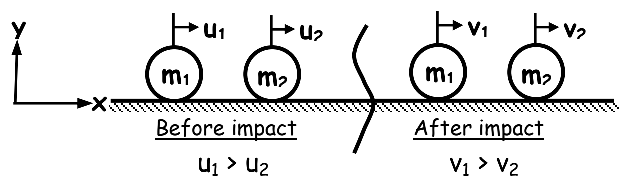
Combining the two equations, we get - $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_1$.

Hence, when two masses interact, their total initial momentum is equal to total final momentum 'or' their total momentum is conserved.

COLLISION OF ELASTIC BODIES :

(I) DIRECT CENTRAL COLLISION :

Consider a mass ‘m_1’ moving with velocity ‘u_1’ colliding with another mass ‘m_2’ moving with a velocity ‘u_2’. Let ‘v_1’ and ‘v_2’ be their final velocities after collision.



a) When two masses collide, their total momentum is conserved. Hence –

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_1} \quad \rightarrow \quad \text{eq.(i)}$$

b) Law of collision : The relative velocity after impact is directly proportional to the relative velocity before impact.

ie, $\mathbf{v_{21}} \;\alpha\; \mathbf{u_{21}}$

$$\mathbf{v_2 - v_1 = -e\,(u_2 - u_1)} \quad \rightarrow \quad \text{eq.(ii)}$$

where e = coefficient of restitution. For perfectly elastic bodies: $e = 1$, for perfectly plastic bodies : $e = 0$ & for semi-elastic bodies : $0 < e < 1$.

Note : Assumptions made in collision theory :

a) The colliding masses are considered to be perfectly smooth.
b) The masses are assumed to undergo central collision, ie, the force of impact, at the instant of collision, passes through the individual mass centers of the colliding masses.

(II) OBLIQUE COLLISION :

Case (i) :

Consider a ball thrown with an initial velocity 'u' at an angle 'α' and rebounding with velocity 'v' at angle 'β', as shown.

a) The component of velocity remains constant along the plane.

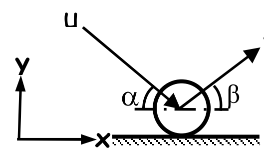
ie, $U_x = V_x$ → eq.(i)

ie, $u.\cos\alpha = v.\cos\beta$

b) Velocity of rebound in Y-direction is equal to 'e' times velocity of approach in Y-direction.

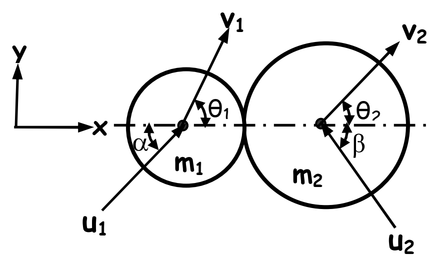
ie, $V_y = e.U_y$ → eq.(ii)

ie, $v.\sin\beta = e.\ u\sin\alpha$



Case (ii) :

Consider a mass 'm_1' moving with velocity 'u_1' colliding with another mass 'm_2' moving with velocity 'u_2' as shown in the figure.
Let 'v_1' and 'v_2' be their velocities after impact.



The velocity components along Y – direction remain unchanged. Hence –

$U_{1Y} = V_{1Y}$ → (i)

$U_{2Y} = V_{2Y}$ → (ii)

The change in velocity and momentum takes place only in X-direction. Hence –

$m_1U_{1X} + m_2U_{2X} = m_1V_{1X} + m_2V_{2X}$ → (iii)

$V_{2X} - V_{1X} = -e\ (U_{2X} - U_{1X})$ → (iv)