Solutions: Chapter 13

Kinematics of Rigid Bodies

Exercise 13.1

Rotation About Fixed Axis

P1. The tub of a washing machine is rotating at 60 rad/sec when the power is switched off. The tub makes 49 revolutions before coming to rest. Determine the constant angular deceleration of the tub and the time it takes to come to a halt.

Solution: Motion of Tub – Rotation about fixed axis – Uniform angular acceleration

$\omega_0 = 60$ rad/s

$\omega = 0$

$\alpha = \alpha$

$\theta = 49 \times 2\pi = 307.88$ rad .... since 1 rev $= 2\pi$ rad

$t = t$

Using $\omega^2 = \omega_0^{\,2} + 2\alpha\theta$

$0 = 60^2 + 2 \times \alpha \times 307.88$

or $\alpha = -5.846 \text{ r/s}^2$ .... **Ans.**

Using $\omega = \omega_0 + \alpha t$

$0 = 60 - 5.846 \times t$

or $t = 10.26$ sec .... **Ans.**

P2. A wheel rotating about a fixed axis at 15 rpm is uniformly accelerated for 60 sec during which it makes 30 revolutions. Find

a) angular velocity in rpm at the end of the interval.
b) time required to attain a speed of 30 rpm.

Solution: Motion of wheel – Rotation about fixed axis – Uniform angular acceleration

a) $\omega_0 = 15 \text{ RPM} = 15 \times \dfrac{2\pi}{60} = 1.571$ rad/s

$\omega = \omega$

$\alpha = \alpha$

$\theta = 30 \text{ rev} = 30 \times 2\pi = 188.49$ rad

$t = 60$ sec

Using $\theta = \omega_0 \times t + \dfrac{1}{2}\alpha t^2$

$188.49 = 1.571 \times 60 + \dfrac{1}{2} \times \alpha \times 60^2$

$\therefore \quad \alpha = 0.05215 \text{ r/s}^2$

Using $\omega = \omega_0 + \alpha t$

$\omega = 1.571 + 0.05215 \times 60$

$\therefore \quad \omega = 4.7$ r/s or $\omega = 4.7 \times \dfrac{60}{2\pi} = 44.88$ RPM .... **Ans.**

b) $\omega_O = 1.571\ \text{rad/s}$

$\omega = 30\ \text{RPM} = 3.1416\ \text{r/s}$

$\alpha = 0.05215\ \text{r/s}^2$

$\theta = -$

$t = t$

Using $\omega = \omega_O + \alpha t$

$3.1416 = 1.571 + 0.05215 \times t$

$\therefore$ $t = 30.1\ \text{sec}$ .... **Ans.**

P3. A point on the rim of a flywheel has a peripheral speed of 6 m/s at an instant which is decreasing at a rate of 30 m/s^2. If the magnitude of the total acceleration of the point at this instant is 50 m/s^2, find the diameter of the flywheel.

Solution: Let R be the radius of the flywheel. It is given that at an instant, v = 6 m/s, $a_t = -30\ \text{m/s}^2$, $a = 50\ \text{m/s}^2$

Using $a = \sqrt{a_n^2 + a_t^2}$ $\quad\therefore\quad$ $a_n = \sqrt{50^2 - (-30)^2} = 40\ \text{m/s}^2$

Using $a_n = \dfrac{v^2}{\rho}$ $\quad\therefore\quad$ $40 = \dfrac{6^2}{r}$ $\quad\therefore\quad$ $r = 0.9\ \text{m}$

$\therefore$ Diameter of fly wheel = 2 r = 1.8 m .... **Ans.**

P4. A 1 m diameter flywheel has an initial clockwise angular velocity of 5 rad/s and a constant angular acceleration of 1.5 rad/s². Determine the number of revolutions it must make and the time required to acquire a clockwise angular velocity of 30 rad/s. Also find the magnitude of linear velocity and linear acceleration of a point on the rim of the flywheel at t = 0.

Solution: Motion of Flywheel – Rotation about fixed axis – Uniform angular acceleration

$\omega_O = 5\ \text{rad/s}$, $\omega = 30\ \text{r/s}$, $\alpha = 1.5\ \text{r/s}^2$, $\theta = \theta$, $t = t$

Using $\omega = \omega_O + \alpha t$

$30 = 5 + 1.5 \times t$

or $t = 16.67\ \text{sec}$ .... **Ans.**

Using $\theta = \omega_O \times t + \dfrac{1}{2}\alpha t^2$

$\theta = 5 \times 16.67 + \dfrac{1}{2} \times 1.5 \times 16.67^2$

$\therefore$ $\theta = 291.68\ \text{rad}$

No. of revolutions $N = \dfrac{\theta}{2\pi} = 46.42$ ... **Ans.**

Linear velocity and linear acceleration 'a' of a point on the rim of fly wheel

Using $v = r\omega$ $\quad\therefore\quad$ $v = 0.5 \times 5 = 2.5\ \text{m/s}$ .... **Ans.**

Using $a_n = \dfrac{v^2}{\rho} = \dfrac{2.5^2}{0.5} = 12.5\ \text{m/s}^2$ also $a_t = r\alpha = 0.5 \times 1.5 = 0.75\ \text{m/s}^2$

Using $a = \sqrt{a_n^2 + a_t^2} = \sqrt{12.5^2 + 0.75^2}$ $\quad\therefore\quad$ $a = 12.52\ \text{m/s}^2$ .... **Ans.**

P5. A wheel is attached to the shaft of an electric motor of the rated speed of 1740 rpm. When power is turned on, the unit attains the rated speed in 5 sec and when the power is turned off, the unit comes to rest in 90 sec. Assuming uniformly accelerated motion, determine the number of revolutions the unit turns (i) to attain the rated speed (ii) to come to rest. ***(M.U May 15)***

Solution: Motion of Wheel – From start to rated speed.
The wheel performs rotation motion with uniform angular acceleration.
$\omega_0 = 0$, $\omega = 1740$ rpm $= 182.21$ rad/s, $\alpha = ?$, $\theta = ?$, $t = 5$ sec.

Using $\omega = \omega_0 + \alpha t$

$$182.21 = 0 + \alpha \times 5$$

$$\therefore \quad \alpha = 36.44 \text{ rad/s}^2$$

Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 0 + \tfrac{1}{2} \times 36.44 \times 5^2$$

$$= 455.5 \text{ rad}$$

or Revolutions $N = \dfrac{455.5}{2\pi} = 72.5$ ... **Ans.**

Motion of Wheel – From rated speed to stop.
The wheel performs rotation motion with uniform angular acceleration.
$\omega_0 = 1740$ rpm $= 182.21$ rad/s, $\omega = 0$, $\alpha = ?$, $\theta = ?$, $t = 90$ sec.

Using $\omega = \omega_0 + \alpha t$

$$0 = 182.21 + \alpha \times 90$$

$$\therefore \quad \alpha = -2.024 \text{ rad/s}^2$$

Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$= 182.21 \times 90 + \tfrac{1}{2} \times (-2.024) \times 90^2$$

$$= 8199.4 \text{ rad}$$

or Revolutions $N = \dfrac{8199.4}{2\pi} = 1305$ ... **Ans.**

P6. A concrete mixer drum is being rotated. If the concrete mixer is designed to attain a speed of 6 rad/s uniformly in 30 sec, starting from rest and then maintain this speed, determine the number of revolutions undergone by the drum at t = 300 sec.

Solution: Motion of Mixer – Rotation about fixed axis – 2 stages

Stage (1)
Uniform Angular Acceleration
$\omega_0 = 0$, $\omega = 6$ r/s, $\alpha = \alpha$, $\theta = \theta_1$, $t = 30$ sec

Using $\omega = \omega_0 + \alpha t$

$$6 = 0 + \alpha \times 30$$

$$\therefore \quad \alpha = 0.2 \text{ r/s}^2$$

Using $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2 \quad \therefore \quad \theta_1 = 0 + \dfrac{1}{2} \times 0.2 \times 30^2$

or $\theta_1 = 90$ rad

Stage (2)
Uniform Angular Velocity
$\omega = 6$ rad/s, $\theta = \theta_2$, $t = 270$ sec

Using $\omega = \dfrac{\theta}{t}$

$$6 = \frac{\theta}{270}$$

$$\therefore \quad \theta_2 = 1620 \text{ rad}$$

Total angle turned in 300 sec $= \theta_1 + \theta_2 = 1710$ rad

$\therefore$ No. of revolutions made $= \dfrac{\theta}{2\pi} = 272.15$ .... **Ans.**

P7. A windmill fan during a certain interval of time has an angular acceleration defined by a relation $\alpha = 18\,e^{-0.3t}$ rad/s². The blades of the fan describes a circle of radius 2.5 m. If at t = 0, ω = 0, determine at t = 5 sec a) angular velocity of the fan b) revolutions undergone by the fan. c) Speed of the tip of fan blade.

Solution: Motion of Windmill fan – Rotation about fixed axis – variable angular acceleration

$$\alpha = 18\,e^{-0.3t}\ r/s^2 \qquad \text{......... (1)}$$

Integrating using $\alpha = \dfrac{d\omega}{dt}$ $\quad\therefore\quad d\omega = 18\,e^{-0.3t}\,dt$

$$\therefore \int_0^\omega d\omega = \int_0^t 18\,e^{-0.3t}\,dt \qquad \text{......... Knowing at } t = 0,\ \omega = 0$$

$$\therefore [\omega]_0^\omega = 18\left[\frac{e^{-0.3t}}{-0.3}\right]_0^t \quad \therefore \quad \omega = -60\left[e^{-0.3t} - 1\right] r/s \qquad \text{......... (2)}$$

Integrating using $\omega = \dfrac{d\theta}{dt}$ $\quad\therefore\quad d\theta = -60\left[e^{-0.3t} - 1\right]dt$

$$\therefore \int_0^\theta d\theta = \int_0^t -60\left[e^{-0.3t} - 1\right]dt \qquad \text{......... Knowing at } t = 0,\ \theta = 0$$

$$\therefore [\theta]_0^\theta = -60\left[\frac{e^{-0.3t}}{-0.3} - t\right]_0^t \quad \therefore \quad \theta = -60\left[-3.33e^{-0.3t} - t + 3.33\right] \text{rad} \qquad \text{......... (3)}$$

Substitute t = 5 in relation (2) and (3) we get

$$\omega = 46.61\ r/s \quad \text{and} \quad \theta = 144.78\ \text{rad} = 23.04\ \text{revolution} \ \text{.........} \ \textbf{Ans.}$$

Speed of tip of blade

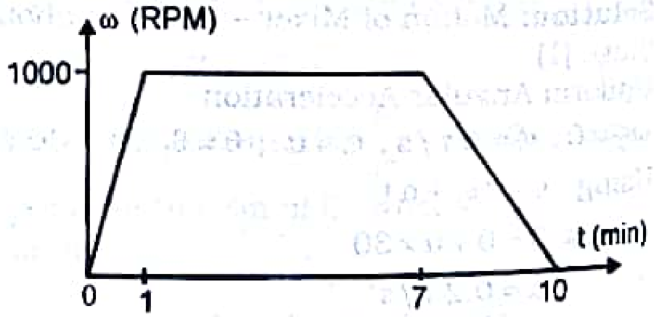
Using $v = r\omega \quad = 2.5 \times 46.61 \quad = 116.52\ m/s$ .... **Ans.**

P8. The variation of angular speed with time of a fan is shown. Find

a) number of revolutions undergone by the fan during a 10 minutes interval.

b) the angular acceleration and angular deceleration during this time interval.

c) the magnitude of velocity and acceleration of a point on the tip of the fan at t = 9 minutes, knowing that the fan described a circle of 1200 mm diameter.



Solution: Motion of Fan – Rotation about fixed axis – 3 stages

Stage (1)	Stage (2)	Stage (3)
Uniform Angular Acceleration	Uniform Angular Velocity	Uniform Angular Acceleration
$\omega_0 = 0$	$\omega = 104.72$ rad/s	$\omega_0 = 104.72$ rad/s
	$\theta = \theta_2$	

$\omega = 1000$ RPM $= 104.72$ rad/s $\alpha = \alpha_1$ $\theta = \theta_1$ $t = 1\,min = 60$ sec Using $\omega = \omega_0 + \alpha t$ $104.72 = 0 + \alpha_1 \times 60$ $\therefore \alpha_1 = 1.745\ r/s^2$ Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\theta_1 = 0 + \frac{1}{2} \times 1.745 \times 60^2$ $\therefore \theta_1 = 3141.6$ rad	$t = 6\,min = 360$ sec Using $\omega = \frac{\theta}{t}$ $104.72 = \frac{\theta_2}{360}$ $\therefore \theta_2 = 37699.2$ rad	$\omega = 0$ $\alpha = \alpha_3$ $\theta = \theta_3$ $t = 3\,min = 180$ sec Using $\omega = \omega_0 + \alpha t$ $0 = 104.72 + \alpha_3 \times 180$ $\therefore \alpha_3 = -0.5818\ r/s^2$ Using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\theta_3 = 104.72 \times 180$ $+ \frac{1}{2} \times (-0.5818) \times 180^2$ $\therefore \theta_3 = 9424.4$ rad

$\therefore$ Total angle turned $\theta = \theta_1 + \theta_2 + \theta_3 = 50265.2$ rad

$\therefore$ No. of revolutions made $= \frac{\theta}{2\pi} = 8000$ .... **Ans.**

Angular Acceleration $= \alpha_1 = 1.745\ r/s^2$ .... **Ans.**

Angular Deceleration $= \alpha_3 = 0.5818\ r/s^2$ .... **Ans.**

From graph at t = 9 minutes, ω = 333.3 RPM = 34.906 r/s
At the tip of blade, r = 0.6 m

$v = r\omega = 0.6 \times 34.906 = 20.94\ m/s$ .... **Ans.**

$a_n = r\omega^2 = 0.6 \times 34.906^2 = 731\ m/s^2$

$a_t = r\alpha = 0.6 \times 0.5818 = 0.35\ m/s^2$

$\therefore$ Total acceleration $a = \sqrt{a_n^2 + a_t^2} = \sqrt{731^2 + 0.35^2} = 731\ m/s^2$ .... **Ans.**

P9. The angular displacement of the rotating wheel is defined by the relation $\theta = \frac{1}{4} t^3 + 2 t^2 + 18$ rad. Determine the angular velocity and angular acceleration of the wheel at t = 5 sec.

Solution: Motion of Wheel – Rotation about fixed axis – Variable angular acceleration

$\theta = \frac{1}{4}t^3 + 2t^2 + 18$ rad	$\omega = \frac{d\theta}{dt} = \frac{3}{4}t^2 + 4t\ r/s$	$\alpha = \frac{d\omega}{dt} = \frac{3}{2}t + 4\ r/s^2$

At t = 5 sec

$\omega = 38.75\ r/s,\ \alpha = 11.5\ r/s^2$ .... **Ans.**

P10. The angular acceleration of a rotating rod is given by the relation $\alpha = 9.81 \cos\theta - 2.22$ rad/s². The rod starts from rest at $\theta = 0$. Find

a) the angular velocity and angular acceleration of the rod at $\theta = 30°$.

b) the maximum angular velocity and the corresponding angle θ.

Solution: Motion of rod – Rotation about fixed axis – Variable angular acceleration

$$\alpha = 9.81\cos\theta - 2.22 \;\; r/s^2 \quad \text{.........} (1)$$

Integrating using $\alpha = \dfrac{\omega.d\omega}{d\theta}$

$$\therefore \quad \omega.d\omega = 9.81\cos\theta - 2.22 d\theta$$

$$\therefore \quad \int_0^{\omega} \omega.d\omega = \int_0^{\theta} 9.81\cos\theta - 2.22 d\theta \qquad \text{knowing at } t = 0, \theta = 0, \omega = 0$$

$$\left[\frac{\omega}{2}\right]_0^{\omega} = [9.81\sin\theta - 2.22\,\theta]_0^{\theta} \quad \therefore \quad \frac{\omega^2}{2} = 9.81\sin\theta - 2.22\theta \quad \text{.........} (2)$$

To find ω and α at $\theta = 30°$

$$\alpha = 9.81 \times \cos 30 - 2.22 = 6.276 \;\; r/s^2$$ **... Ans.**

$$\frac{\omega^2}{2} = 9.81 \times \sin 30 - 2.22 \times \left(\frac{\pi}{6}\right) \quad \therefore \quad \omega = 2.736 \;\; r/s$$ **... Ans.**

For maximum angular velocity, angular acceleration is zero.

$\therefore$ Equating $\alpha = 0$

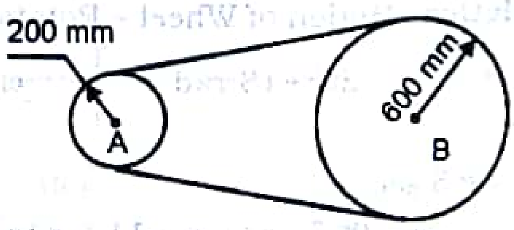
$$9.81\cos\theta - 2.22 = 0$$

$\therefore$ at $\theta = 76.72°$ .... angle at which ω is max. **... Ans.**

Substitute $\theta = 76.72°$ in equation (2) to get ω_{max}

$$\frac{\omega_{max}{}^2}{2} = 9.81 \times \sin 76.92 - 2.22 \times \left(76.92 \times \frac{\pi}{180}\right) \quad \therefore \quad \omega_{max} = 3.626 \;\; r/s$$ **... Ans.**

P11. A belt is wrapped over two pulleys transmitting the motion without slipping. If the angular velocity of the driver pulley A is increased uniformly from 2 rad/s to 16 rad/s in 4 sec, determine



a) the acceleration of the straight position of the belt

b) the magnitude of total acceleration of a point on the rim of pulley B at t = 4 sec.

c) the number of revolutions turned by the two pulleys at t = 4 sec.

Solution: Motion of Pulley A – Rotation about fixed axis – Uniform angular acceleration

$\omega_0 = 2$ rad/s $\omega = 16$ r/s $\alpha = \alpha_A$ $\theta = \theta_A$ $t = 4$ sec	Using $\omega = \omega_0 + \alpha t$ $16 = 2 + \alpha_A \times 4$ or $\alpha_A = 3.5\ r/s^2$ ... **Ans.**	Using $\theta = \omega_0 \times t + \frac{1}{2}\alpha t^2$ $\theta_A = 2\times 4 + \frac{1}{2}\times 3.5 \times 4^2$ $\therefore$ $\theta_A = 36$ rad Revolutions $N_A = \frac{\theta_A}{2\pi} = 5.729$ ... **Ans.**

a) Acceleration of straight portion of belt

$a = r\alpha$ .... using special case 1 of rotation

$= 0.2 \times 3.5 = 0.7\ m/s^2$ .... **Ans.**

b) Since pulley B is connected to pulley A, by a belt. Using special case (2) of rotation

$r_A\omega_A = r_B\omega_B$ $\quad\therefore\quad$ $0.2\times 16 = 0.6\times\omega_B$ $\quad$ or $\quad$ $\omega_B = 5.33\ r/s$

also $\quad r_A\alpha_A = r_B\alpha_B$ $\quad\therefore\quad$ $0.2\times 3.5 = 0.6\times\alpha_B$ $\quad$ or $\quad$ $\alpha_B = 11.67\ r/s^2$

also $\quad r_A\theta_A = r_B\theta_B$ $\quad\therefore\quad$ $0.2\times 36 = 0.6\times\theta_B$ $\quad$ or $\quad$ $\theta_B = 12$ rad

For a point on the rim of pulley B

$a_n = r\omega^2 = 0.6\times 5.33^2 = 17.04\ m/s^2$

$a_t = r\alpha = 0.6\times 1.167^2 = 0.7\ m/s^2$

$\therefore$ Total acceleration $a = \sqrt{a_n^2 + a_t^2} = \sqrt{17.04^2 + 0.7^2} = 17.05\,m/s^2$ .... **Ans.**

also No. of revolutions $N_B = \frac{\theta_B}{2\pi} = \frac{12}{2\pi} = 1.91$ .... **Ans.**

P12. Find the angular velocity in rad/s for

a) the second hand, the minute hand and hour hand of a watch,

b) the earth about its own axis.

Solution: Second hand covers 1 revolution i.e. 2π radians in 60 sec

$\therefore\quad \omega = \frac{\theta}{t} = \frac{2\pi}{60} = 0.1047\ r/s$ .... **Ans.**

Minute hand covers 1 revolution in 1 hr = 3600 sec

$\therefore\quad \omega = \frac{\theta}{t} = \frac{2\pi}{3600} = 1.745\times 10^{-3}\ r/s$ .... **Ans.**

Hour hand covers 1 revolution in 12 hrs = 43200 sec

$\therefore\quad \omega = \frac{\theta}{t} = \frac{2\pi}{43200} = 1.454\times 10^{-4}\ r/s$ .... **Ans.**

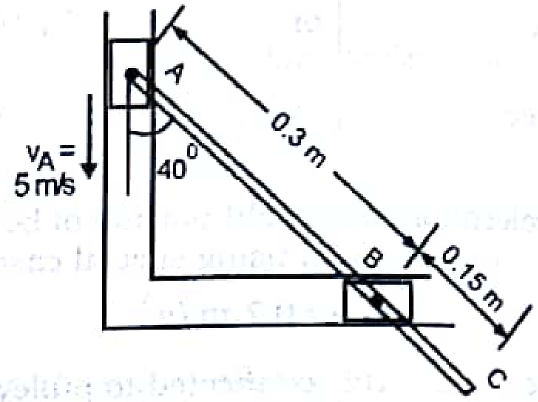
Earth covers 1 revolution in 24 hrs = 86400 sec

$\therefore\quad \omega = \frac{\theta}{t} = \frac{2\pi}{86400} = 7.27\times 10^{-5}\ r/s$ .... **Ans.**

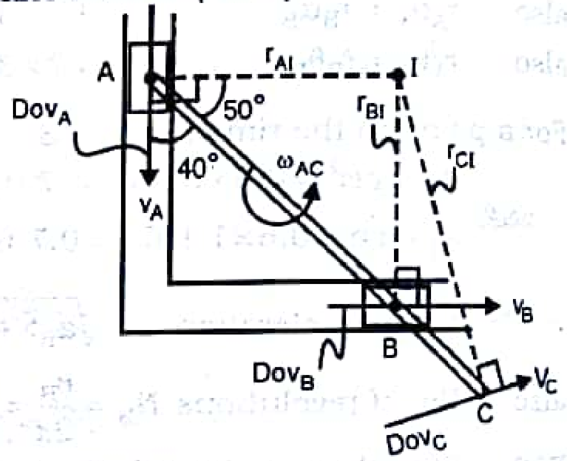
Exercise 13.2

General Plane Motion (G P M)

P1. The rod ABC is guided by two blocks A and B which move in channels as shown. At the given instant, velocity of block A is 5 m/s downwards. Determine
a) the angular velocity of rod ABC
b) velocities of block B and end C of rod.



Solution: The system consists of three bodies in motion. Blocks A and B perform translation motion, while rod AC performs General Plane Motion (GPM).



GPM of rod AC

Using Instantaneous Centre Method

We need two points on the GPM body whose Direction of Velocity (DOV) are known.

DOV of end A i.e. DOV_A is vertical at A since end A is connected to block A which translates vertically.

DOV of end B i.e. DOV_B is horizontal at B since end B is connected to block B which translates horizontally.

To locate the instantaneous centre of rotation I of rod AC, draw perpendiculars to DOV_A and DOV_B and get the point of intersection I as shown in figure.

Using $v = r\omega$

$$v_A = r_{AI} \times \omega_{AC}$$

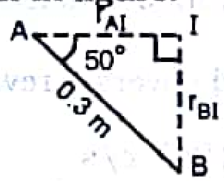
$$5 = 0.1928 \times \omega_{AC}$$

$\therefore$ $\omega_{AC} = 25.93$ rad/s ↺ .... **Ans.**

also $v_B = r_{BI} \times \omega_{AC}$

$$= 0.2298 \times 25.93$$

$\therefore$ $v_B = 5.959$ m/s → .... **Ans.**



From ΔABI

$$r_{AI} = 0.3 \cos 50 = 0.1928 \text{ m}$$

$$r_{BI} = 0.3 \sin 50 = 0.2298 \text{ m}$$

To find velocity of end C of rod, join C to I to get radius r_{CI}

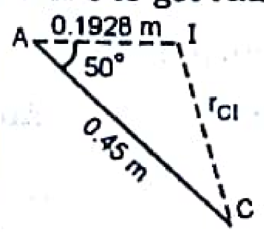
Draw a $\perp$ to r_{CI} to get the DOV_C.

Using $v = r\omega$

$$v_C = r_{CI} \times \omega_{AC}$$

$$= 0.3579 \times 25.93$$

$\therefore$ $v_C = 9.282$ m/s ... **Ans.**



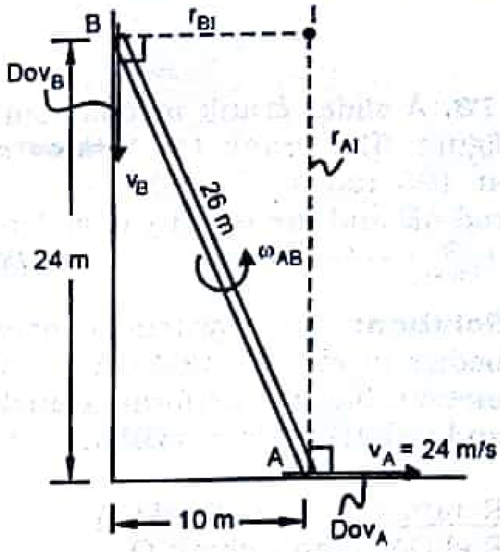
From ΔACI

Using cosine Rule

$$r_{CI} = \sqrt{0.1928^2 + 0.45^2 - 2 \times 0.1928 \times 0.45 \cos 50}$$

$$= 0.3579 \text{ m}$$

P2. A rod AB 26 m long leans against a vertical wall. The end 'A' on the floor is drawn away from the wall at a rate of 24 m/s. When the end 'A' of the rod is 10 m from the wall, determine the velocity of the end 'B' sliding down vertically and the angular velocity of the rod AB. ***(M. U. May 09)***



Solution: The system consists of a single rod AB which performs General Plane Motion (GPM).

GPM of rod AB

Using instantaneous Centre Method.

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end A i.e. DOV_A lies along the horizontal floor.

DOV of end B i.e. DOV_B lies along the vertical wall.

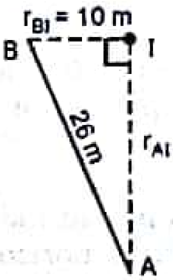
To locate the instantaneous centre of rotation I of rod AB, draw perpendiculars to DOV_A and DOV_B and get the point of intersection I as shown in figure.

Using $v = r\omega$

$$v_A = r_{AI} \times \omega_{AB}$$

$$24 = 24 \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = 1 \text{ rad/s} \circlearrowleft \text{ ... } \textbf{Ans.}$$



From ΔABI

$r_{AI} = 24$ m

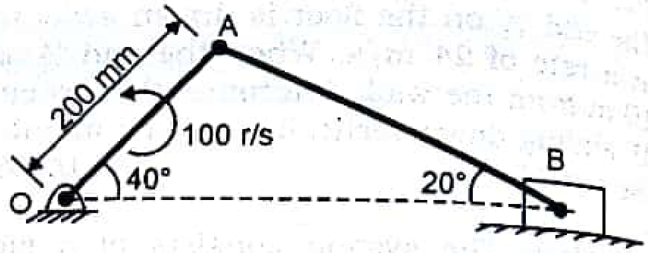
also $\quad v_B = r_{BI} \times \omega_{AB}$

$$= 10 \times 1$$

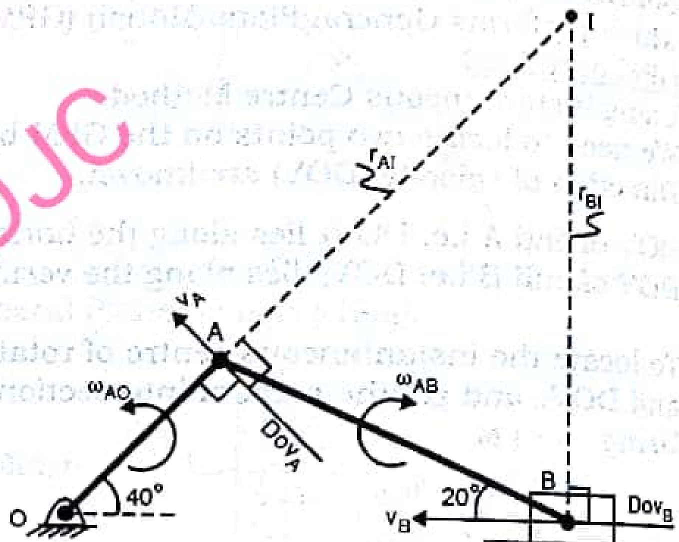
$$\therefore \quad v_B = 10 \text{ m/s} \downarrow \text{ ... } \textbf{Ans.}$$

P3. A slider crank mechanism is shown in figure. The crank OA rotates anticlockwise at 100 rad/s. Find the angular velocity of rod AB and the velocity of slider at B.

(M. U. Dec 09)



Solution: The system consists of three bodies in motion. Rod OA performs rotation motion, block B performs translation motion and rod AB performs GPM.



<u>Rotation Motion of rod OA</u>

Rod OA rotates about O

$\therefore \quad v_A = r_{AO} \times \omega_{OA}$

$\therefore \quad v_A = 0.2 \times 100$

$\therefore \quad v_A = 20 \text{ m/s}$ ↖ .... **Ans.**

<u>GPM of rod OB</u>

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end A i.e. DOV_A is ⊥ to rod AO at A.

DOV of end B i.e. DOV_B is horizontal at B since end B is connected to block B which translates horizontally.

To locate the instantaneous centre of rotation I of rod AB, draw perpendiculars to DOV_A and DOV_B and get the point of intersection I as shown in figure.

Using $v = r\omega$

$v_A = r_{AI} \times \omega_{AB}$

$20 = 0.4611 \times \omega_{AB}$

$\therefore \quad \omega_{AB} = 43.37 \text{ rad/s}$ ↺

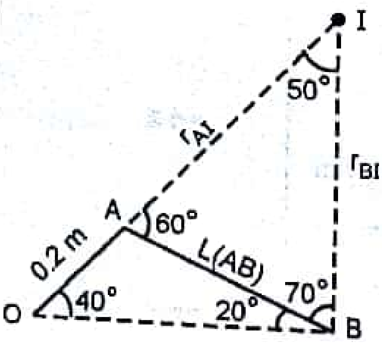
.... **Ans.**

also $\quad v_B = r_{BI} \times \omega_{AB}$

$= 0.4249 \times 43.37$

$\therefore \quad v_B = 18.43 \text{ m/s} \leftarrow$

.... **Ans.**



From ΔOAB

$$\frac{0.2}{\sin 20} = \frac{L(AB)}{\sin 40}$$

$\therefore \quad L(AB) = 0.3759 \text{ m}$

From ΔABI

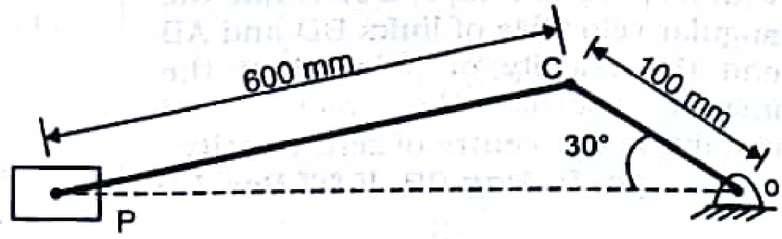
$$\frac{0.3759}{\sin 50} = \frac{r_{AI}}{\sin 70} = \frac{r_{BI}}{\sin 60}$$

$\therefore \quad r_{AI} = 0.4611 \text{ m}$

and $\quad r_{BI} = 0.4249 \text{ m}$

P4. In a slider crank mechanism as shown in figure the crank is rotating at as constant speed of 120 rev/min. The connecting rod is 600 mm long and the crank is 100 mm long. For an angle of 30°, determine the absolute velocity of the crosshead P.

(VJTI May 08)



Solution: The system consists of three bodies in motion. Crank OC performs Rotation motion, block P performs Translation motion and rod CP performs GPM.

<u>Rotation Motion of Crank OC</u>

Crank OC rotates about O

$\therefore v_C = r_{CO} \times \omega_{OC}$

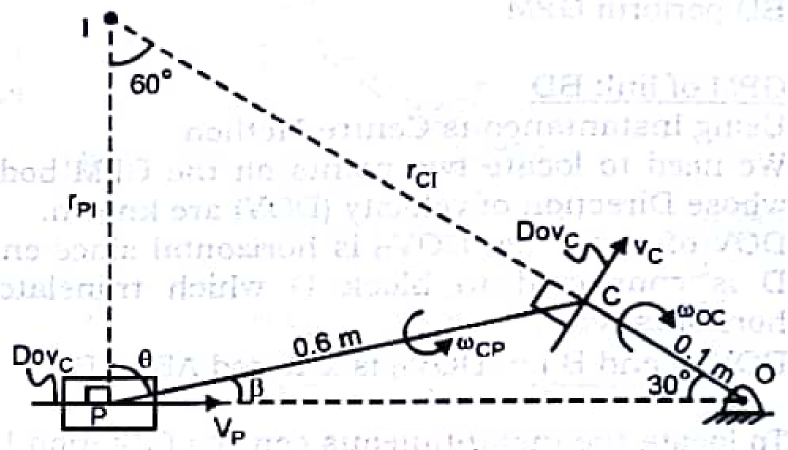
$\therefore v_C = 0.1 \times 12.566$

$\therefore v_C = 1.257$ m/s .... **Ans.**

120 RPM

$= 120 \times \dfrac{2\pi}{60}$

$= 12.566$ r/s



<u>GPM of Rod CP</u>

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end C i.e. DOV_C is ⊥ to rod OC at C.

DOV of end P i.e. DOV_P is horizontal at P since end P is connected to crosshead P which translates horizontally.

To locate the instantaneous centre of rotation I of rod CP, draw perpendiculars to DOV_C and DOV_P and get the point of intersection I as shown in figure.

From ΔCPI

$$\frac{0.6}{\sin 30} = \frac{0.1}{\sin\beta} \quad \therefore \beta = 4.8°$$

From ΔCPI

$\angle\theta = 90 - 4.8 = 85.2°$

$\angle I = 60°$,

$\angle C = 180 - 85.2 - 60 = 34.8°$

Using sine Rule

$$\frac{0.6}{\sin 60} = \frac{r_{CI}}{\sin 85.2} = \frac{r_{PI}}{\sin 34.8}$$

$\therefore \quad r_{CI} = 0.69$ m and $r_{PI} = 0.395$ m

Using $v = r\omega$

$v_C = r_{CI} \times \omega_{CP}$

$1.257 = 0.69 \times \omega_{CP}$

$\therefore \quad \omega_{CP} = 1.822$ rad/s

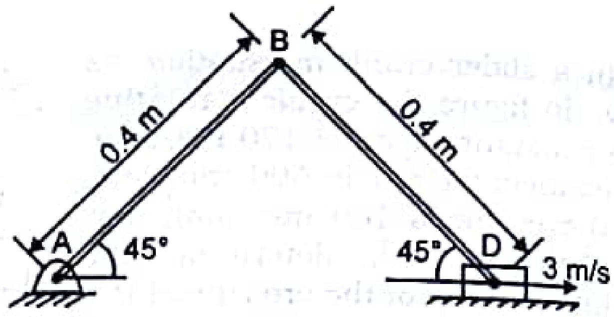
also $\quad v_P = r_{PI} \times \omega_{CP}$

$= 0.395 \times 1.822$

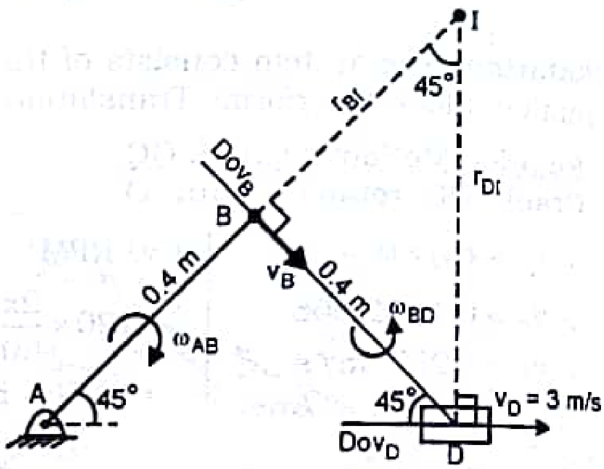
$\therefore \quad v_P = 0.7196$ m/s → .... **Ans.**

P5. Block 'D' shown in figure moves with a speed of 3 m/s. Determine the angular velocities of links BD and AB and the velocity of point B at the instant shown. Use method of instantaneous centre of zero velocity.

(M. U. May 09, VJTI Dec 13)



Solution: The system consists of three bodies in motion. Link AB performs Rotation Motion, block D performs Translation Motion and link BD perform GPM.



<u>GPM of link BD</u>

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end D i.e. DOV_D is horizontal since end D is connected to block D which translates horizontally.

DOV of end B i.e. DOV_B is $\perp$ to rod AB at B.

To locate the instantaneous centre of rotation I of rod BD, draw perpendiculars to DOV_B and DOV_D and get the point of intersection I as shown in figure.

Using $v = r\omega$

$$v_D = r_{DI} \times \omega_{BD}$$

$$3 = 0.5657 \times \omega_{BD}$$

$\therefore \quad \omega_{BD} = 5.303 \text{ rad/s}$ ↺ .... **Ans.**

also $\quad v_B = r_{BI} \times \omega_{BD}$

$$= 0.4 \times 5.303$$

$\therefore \quad v_B = 2.121 \text{ m/s}$ ↘ .... **Ans.**

In ΔBDI

$\angle D = 45°, \angle I = 45°$ and $\angle B = 90°$

Using sine Rule

$$\frac{0.4}{\sin 45} = \frac{r_{BI}}{\sin 45} = \frac{r_{DI}}{\sin 90}$$

$\therefore \quad r_{BI} = 0.4 \text{ m}$

and $\quad r_{DI} = 0.5657 \text{ m}$

<u>Rotation Motion of link AB</u>

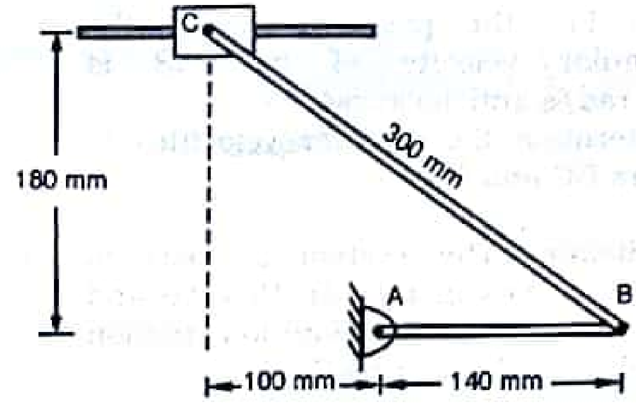
Link AB rotates about A

$\therefore \quad$ velocity of end B = $v_B = r_{BA} \times \omega_{AB}$

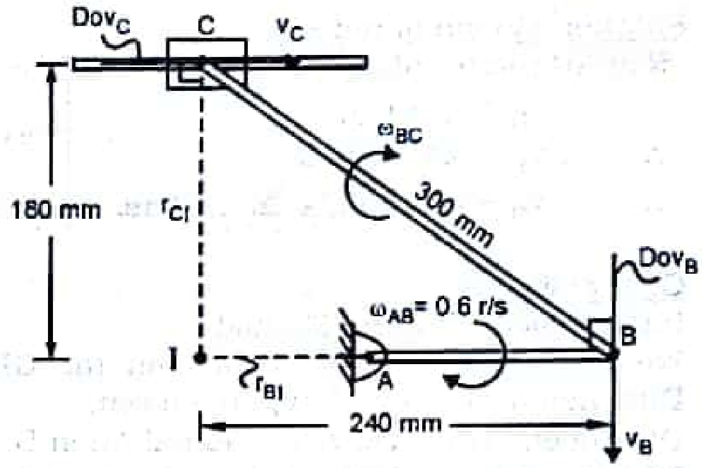
$\therefore \quad 2.121 = 0.4 \times \omega_{AB}$

or $\quad \omega_{AB} = 5.303 \text{ r/s}$ ↻ .... **Ans.**

P6. In figure collar C slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine angular velocity of BC and velocity of collar C. ***(M.U. Dec 13)***



Solution: The system consists of three bodies in motion. Rod AB performs rotation motion, block C performs translation motion and rod BC performs GPM.



<u>Rotation Motion of rod AB</u>

Rod AB rotates about A

$\therefore \quad v_B = r_{BA} \times \omega_{AB}$

$= 0.14 \times 0.6$

$\therefore \quad v_B = 0.084 \text{ m/s} \downarrow$

GPM of rod BC

Using Instantaneous Centre Method

We need to locate two point on the GPM body whose Direction of velocity (Dov) are known.

Dov of end B i.e. Dov_B is $\perp$ to rod AB at B.

Dov of end C i.e. Dov_C is horizontal at C since C is connected to collar C which translates horizontally.

To locate the instantaneous centre of rotation I of rod BC, draw perpendicular to Dov_B and Dov_C and get the point of intersection I as shown in figure.

Using $v = r\omega$

$v_B = r_{BI} \times \omega_{BC}$

$0.084 = 0.24 \times \omega_{BC}$

$\therefore \quad \omega_{BC} = 0.35 \text{ r/s} \circlearrowleft$ .... **Ans.**

From the figure

$r_{BI} = 240 \text{ mm} = 0.24 \text{ m}$

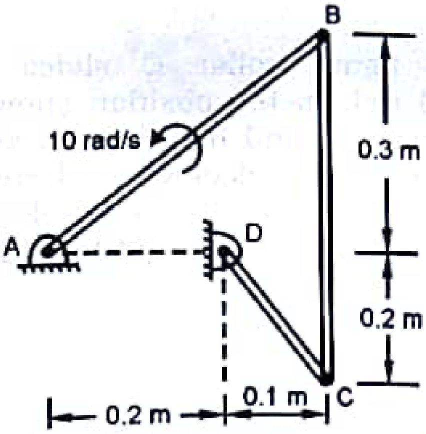
$r_{CI} = 180 \text{ mm} = 0.18 \text{ m}$

also $\quad v_C = r_{CI} \times \omega_{BC}$

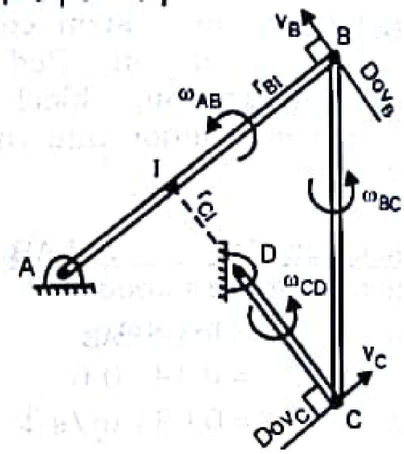
$= 0.18 \times 0.35$

$\therefore \quad v_C = 0.063 \text{ m/s} \rightarrow$ .... **Ans.**

P7. For the position shown, the angular velocity of bar AB is 10 rad/s anticlockwise.
Determine the angular velocities of bars BC and CD.



Solution: The system consists of three bodies in motion. Rod AB and rod CD performs Rotation motion and rod BC performs GPM.



Rotation Motion of rod AB

Rod AB rotates about A

$\therefore \quad v_B = r_{BA} \times \omega_{AB}$

$\therefore \quad v_B = 0.4243 \times 10$

$\therefore \quad v_B = 4.243$ m/s ... **Ans.**

From geometry

$r_{BA} = \sqrt{0.3^2 + 0.3^2}$

$= 0.4243$ m

GPM of Rod BC

Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end B i.e. DOV_B is $\perp$ to rod AB at B.

DOV of end C i.e. DOV_C is $\perp$ to rod CD at C.

To locate the instantaneous centre of rotation I of rod BC, draw perpendiculars to DOV_B and DOV_C and get their point of intersection I as shown in figure.

Using $v = r\omega$

$v_B = r_{BI} \times \omega_{BC}$

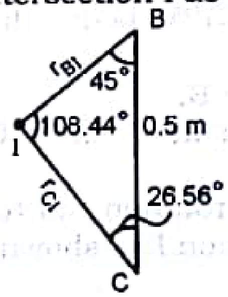
$4.243 = 0.2357 \times \omega_{BC}$

$\therefore \quad \omega_{BC} = 18$ rad/s .... **Ans.**

also $\quad v_C = r_{CI} \times \omega_{BC}$

$= 0.3727 \times 18$

$\therefore \; v_C = 6.708$ m/s



In ΔBCI

$\angle C = 26.56°, \angle B = 45°, \angle I = 108.44°$

Using sine Rule

$$\frac{0.5}{\sin 108.44} = \frac{r_{BI}}{\sin 26.56} = \frac{r_{CI}}{\sin 45}$$

$\therefore \quad r_{BI} = 0.2357$ m

and $\quad r_{CI} = 0.3727$ m

Rotation Motion of rod CD

Rod CD rotates about D

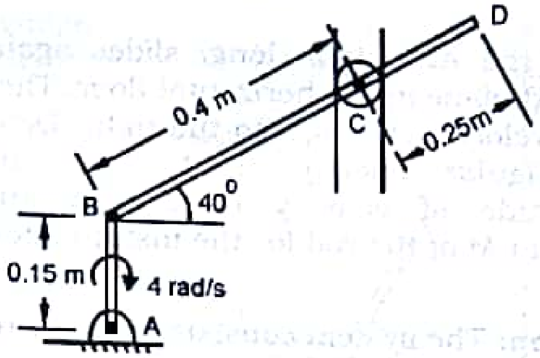
$\therefore \quad v_C = r_{CD} \times \omega_{CD}$

$\therefore \quad 6.708 = 0.2236 \times \omega_{CD}$

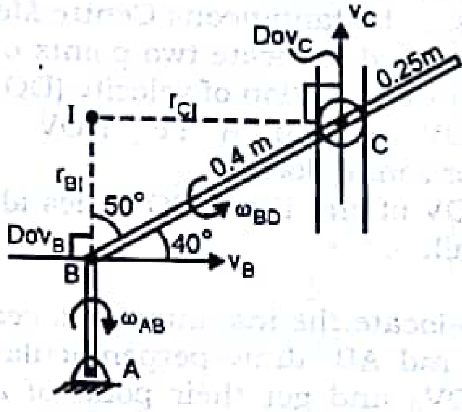
or $\quad \omega_{CD} = 30$ r/s .... **Ans.**

P8. Rod BCD is pinned to rod AB at B and has a slider at C which slides freely in the vertical slot. At the instant shown, the angular velocity of rod AB is 4 rad/s clockwise. Determine

a) angular velocity of rod BD
b) velocity of slider C
c) velocity of end D of the rod BD



Solution: The system consists of three bodies in motion. Rod AB performs Rotation motion, slider C translates vertically and rod BD performs GPM.



Rotation Motion of rod AB

Rod AB rotates about A

$\therefore \quad v_B = r_{BA} \times \omega_{AB}$

$\therefore \quad v_B = 0.15 \times 4$

$\therefore \quad v_B = 0.6 \text{ m/s} \rightarrow$

GPM of Rod BD

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end B i.e. DOV_B is ⊥ to rod AB at A.

DOV of end C i.e. DOV_C is vertical since it is connected to slider C which translates vertically.

To locate the instantaneous centre of rotation I of rod BD, draw perpendiculars to DOV_B and DOV_C and get their point of intersection I as shown in figure.

Using $\quad v = r\omega$

$v_B = r_{BI} \times \omega_{BD}$

$0.6 = 0.2571 \times \omega_{BD}$

$\therefore \quad \omega_{BD} = 2.334 \text{ rad/s} \circlearrowleft$ .... **Ans.**

also $\quad v_C = r_{CI} \times \omega_{BD}$

$= 0.3064 \times 2.334$

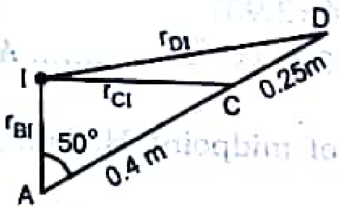
$\therefore \quad v_C = 0.715 \text{ m/s} \uparrow$ .... **Ans.**

To find velocity of point D, join D to I to get radius r_{DI} = 0.5232 m

Using $\quad v_D = r_{DI} \times \omega_{BD}$

$v_D = 0.5232 \times 2.334$

$\therefore \quad v_D = 1.22 \text{ m/s}$ .... **Ans.**



From ΔBCI

$r_{BI} = 0.4\cos 50 = 0.2571 \text{ m}$

$r_{CI} = 0.4\sin 50 = 0.3064 \text{ m}$

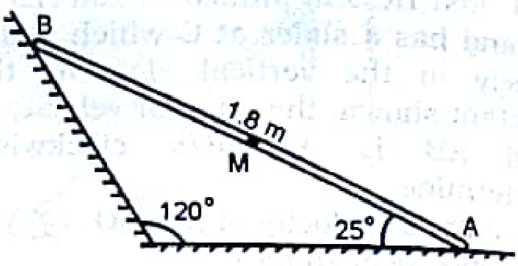
From ΔBDI

Using Cosine Rule

$$r_{DI} = \sqrt{0.2571^2 + 0.65^2 - 2 \times 0.2571 \times 0.65\cos 50}$$

$= 0.5232 \text{ m}$

P9. A rod AB 1.8 m long, slides against an inclined plane and a horizontal floor. The end A has a velocity of 5 m/s to the right. Determine the angular velocity of the rod and the magnitude of velocity of end B and the midpoint M of the rod for the instant shown.



Solution: The system consists of single rod AB, which performs GPM.

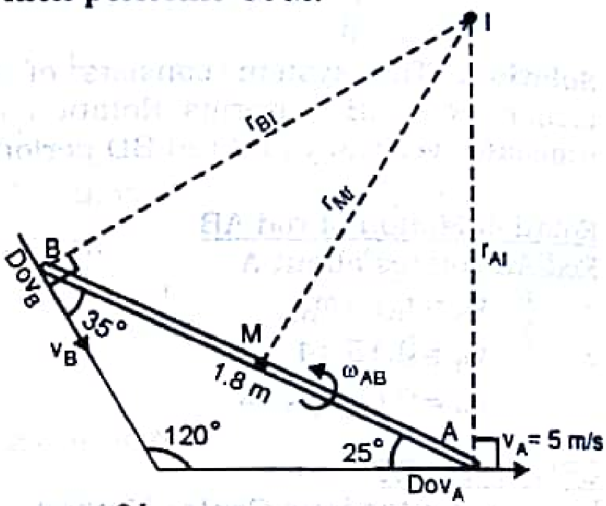
<u>GPM of Rod AB</u>

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.

DOV of end A i.e. DOV_A lies along the horizontal floor.

DOV of end B i.e. DOV_B lies along the inclined wall.



To locate the instantaneous centre of rotation I of rod AB, draw perpendiculars to DOV_A and DOV_B and get their point of intersection I as shown in figure.

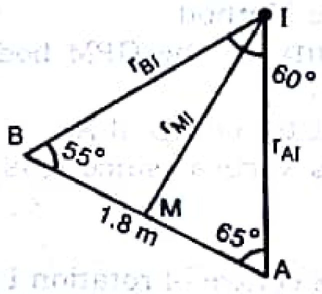
Using $v = r\omega$

$$v_A = r_{AI} \times \omega_{AB}$$

$$5 = 1.703 \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = 2.937 \text{ rad/s} \circlearrowleft$$

.... **Ans.**



From ΔABI

Using sine Rule

$$\frac{1.8}{\sin 60} = \frac{r_{BI}}{\sin 65} = \frac{r_{AI}}{\sin 55}$$

$$\therefore \quad r_{BI} = 1.884 \text{ m}$$

and $r_{AI} = 1.703 \text{ m}$

also $v_B = r_{BI} \times \omega_{AB}$

$$= 1.884 \times 2.937$$

$$\therefore \quad v_B = 5.533 \text{ m/s}$$

.... **Ans.**

To find velocity of midpoint M, join M to I and find length r_{MI}

Using $v_M = r_{MI} \times \omega_{AB}$

$$v_M = 1.5539 \times 2.937$$

$$\therefore \quad v_M = 4.564 \text{ m/s}$$

.... **Ans.**

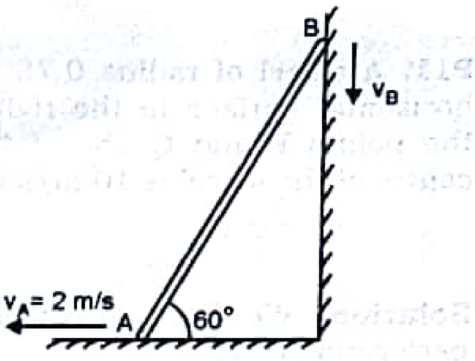
From ΔAMI

Using Cosine Rule

$$r_{MI} = \sqrt{0.9^2 + 1.703^2 - 2 \times 0.9 \times 1.703 \cos 65}$$

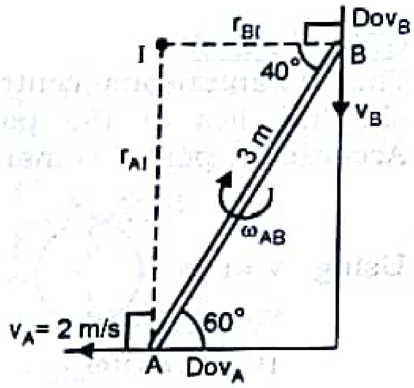
$$= 1.5539 \text{ m}$$

P10. A slender rod AB of length 3 m which remains always in a same vertical plane as its ends A and B are constrained to remain in contact with a horizontal floor and a vertical wall as shown. Determine the velocity at point B using instantaneous centre method. ***(VJTI Dec 11)***



Solution: The system consists of single rod AB, which performs GPM.

GPM of rod AB
Using Instantaneous Centre Method
We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.
DOV of end A i.e. DOV_A lies along the horizontal floor.
DOV of end B i.e. DOV_B lies along the vertical wall.



To locate the instantaneous centre of rotation I of rod AB, draw perpendiculars to DOV_A and DOV_B and get their point of intersection I as shown in figure.

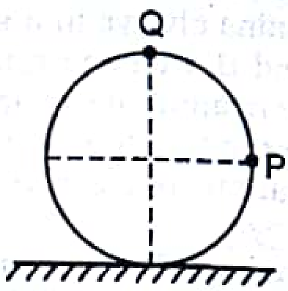
Using $v = r\omega$

$$v_A = r_{AI} \times \omega_{AB}$$
$$2 = 2.598 \times \omega_{AB}$$
$\therefore$ $$\omega_{AB} = 0.7698 \text{ rad/s} \circlearrowright$$ .... **Ans.**

From ΔABI
$$r_{AI} = 3\sin 60 = 2.598 \text{ m}$$
$$r_{BI} = 3\cos 60 = 1.5 \text{ m}$$

also $$v_B = r_{BI} \times \omega_{AB}$$
$$= 1.5 \times 0.7698$$
$\therefore$ $$v_B = 1.1547 \text{ m/s} \downarrow$$ .... **Ans.**

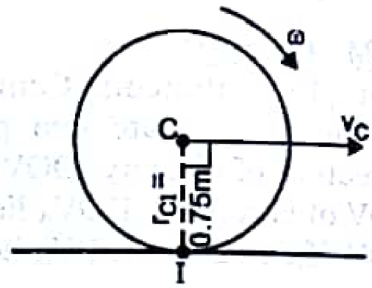
P11. A wheel of radius 0.75 m rolls without slipping on a horizontal surface to the right. Determine the velocities of the points P and Q shown in figure when the velocity of centre of the wheel is 10 m/s towards right. ***(M. U. Dec 09)***



Solution: We have a wheel which rolls on the ground performing GPM.

GPM of Wheel

The instantaneous centre I, of any body rolling without slipping, lies at the point of contact with the ground. Accordingly point I is marked as shown.



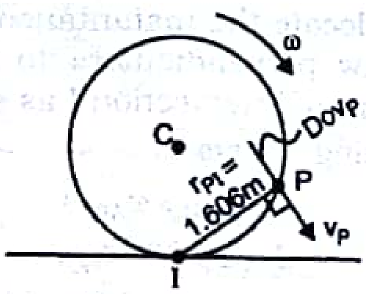
(a)

Using $v = r\omega$

$v_C = r_{CI} \times \omega_{wheel}$ .... refer fig (a)

$10 = 0.75 \times \omega_{wheel}$

$\therefore \quad \omega_{wheel} = 13.33$ rad/s ↻



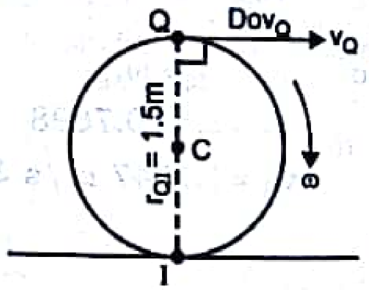
(b)

To find velocity of point P, join P and I to get length r_{PI}

Now $v_P = r_{PI} \times \omega_{wheel}$ .... refer fig (b)

$= 1.606 \times 13.33$

$\therefore \quad v_P = 14.142$ m/s ↘ .... **Ans.**



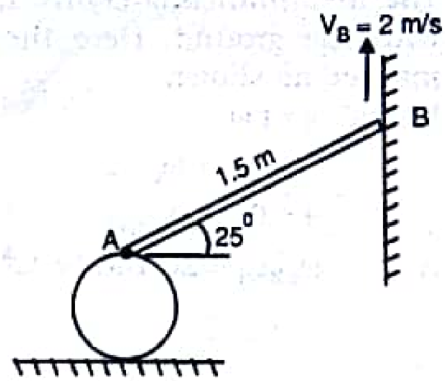
(c)

also $v_Q = r_{QI} \times \omega_{wheel}$ .... refer fig (c)

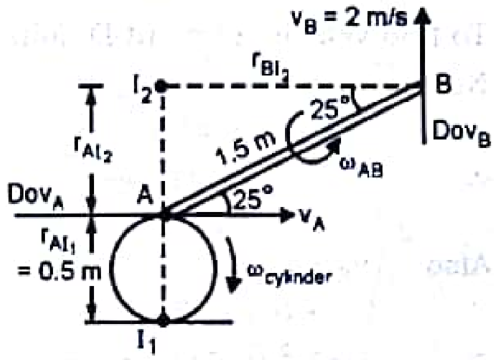
$= 1.5 \times 13.33$

$\therefore \quad v_Q = 20$ m/s → .... **Ans.**

P12. One end of rod AB is pinned to the cylinder of diameter 0.5 m while the other end slides vertically up the wall with a uniform speed of 2 m/s. For the instant, when the end A is vertically over the centre of the cylinder, find the angular velocity of the cylinder, assuming it to roll without slip.



Solution: The system consists of two bodies in motion viz. cylinder and rod AB. Both the bodies perform GPM.



GPM of rod AB

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.

DOV of end B i.e. DOV_B is along the vertical wall.

DOV of end A i.e. DOV_A is horizontal since point A is common for the cylinder and the rod. The I_1 for the cylinder A lies at the point of contact with the ground. If point A is joined to I_1, the radius r_{AI_1} is vertical.

A ⊥ drawn to the radius r_{AI1} gives us the DOV_A.

Using $v = r\omega$

$$v_B = r_{BI_2} \times \omega_{AB}$$

$$2 = 1.3595 \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = 1.471 \text{ rad/s} \circlearrowleft$$

From ΔABI_2

$$r_{AI_2} = 1.5 \sin 25 = 0.6339 \text{ m}$$

$$r_{BI_2} = 1.5 \cos 25 = 1.3595 \text{ m}$$

also $\quad v_A = r_{AI_2} \times \omega_{AB}$

$$= 0.6339 \times 1.471$$

$$\therefore \quad v_A = 0.9326 \text{ m/s} \rightarrow$$

GPM of cylinder

Using Instantaneous Centre Method

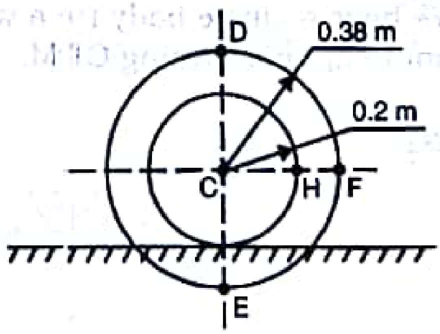
$$v_A = r_{AI_1} \times \omega_{cylinder}$$

$$0.9326 = 0.5 \times \omega_{cylinder}$$

$$\therefore \quad \omega_{cylinder} = 1.865 \text{ rad/s} \circlearrowright \quad \text{......... } \textbf{Ans.}$$

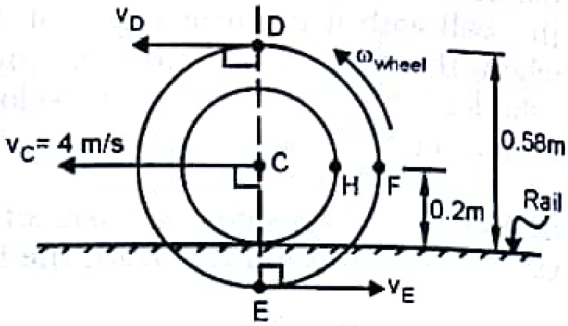
$$r_{AI_1} = 0.5 \text{ m}$$

P13. A flanged wheel rolls to the left on a horizontal rail as shown. The velocity of the wheel's centre is 4 m/s. Find velocities of points D, E, F and H on the wheel.



Solution: The system has a single flanged wheel which rolls on the rail performing GPM.

The instantaneous centre I, of a rolling body without slip, lies at the point of contact with the ground. Here the contact of wheel is with the rail. Accordingly point I is marked as shown.



Using $v = r\omega$

$$v_C = r_{CI} \times \omega_{wheel}$$

$$4 = 0.2 \times \omega_{wheel}$$

$\therefore$ $\omega_{wheel} = 20 \text{ rad/s} \circlearrowleft$ .... **Ans.**

To find velocity of point D, join D and I to get length r_{DI}

$r_{DI} = 0.38 + 0.2 = 0.58 \text{ m}$

$r_{EI} = 0.38 - 0.2 = 0.18 \text{ m}$

Now $v_D = r_{DI} \times \omega_{wheel}$

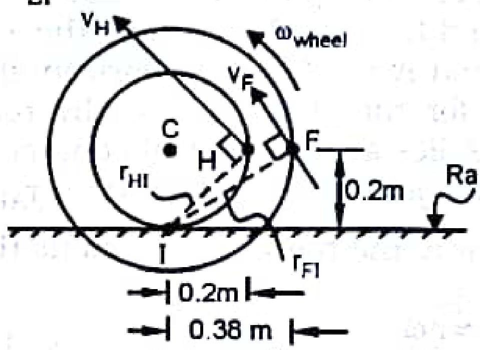
$= 0.58 \times 20$

$\therefore$ $v_D = 11.6 \text{ m/s} \leftarrow$ .... **Ans.**

Also $v_E = r_{EI} \times \omega_{wheel}$

$= 0.18 \times 20$

$\therefore$ $v_E = 3.6 \text{ m/s} \rightarrow$ .... **Ans.**



To find velocity of point H, join H and I to get length r_{HI}

$r_{HI} = \sqrt{0.2^2 + 0.2^2} = 0.2828 \text{ m}$

$r_{FI} = \sqrt{0.2^2 + 0.38^2} = 0.4294 \text{ m}$

Now $v_H = r_{HI} \times \omega_{wheel}$

$= 0.2828 \times 20$

$\therefore$ $v_H = 5.657 \text{ m/s}$ ↘ .... **Ans.**

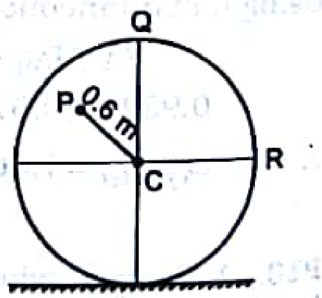
Also $v_F = r_{FI} \times \omega_{wheel}$

$= 0.4294 \times 20$

$\therefore$ $v_F = 8.588 \text{ m/s}$ ↘ .... **Ans.**

P14. A wheel of 2 m diameter rolls without slipping on a flat surface. The centre of the wheel is moving with a velocity of 4 m/s towards the right. Determine the angular velocity of the wheel and velocity of points P, Q and R on the wheel.

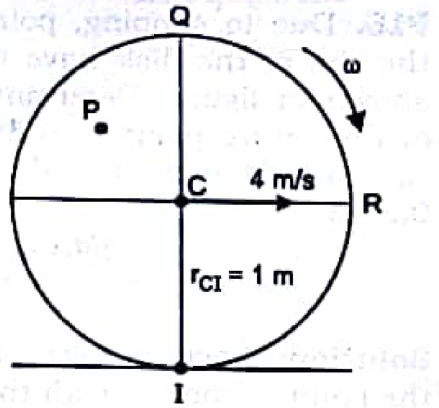
(M. U. Dec 14)



Solution: We have a single body i. e a wheel which rolls on the ground without slip, performing G PM.

GPM of Wheel

The instantaneous centre I, of any body rolling without slipping, lies at the point of contact with the ground. Accordingly point I is marked as shown.



Knowing $v_C = 4$ m/s →, Let us first find angular velocity of the wheel.

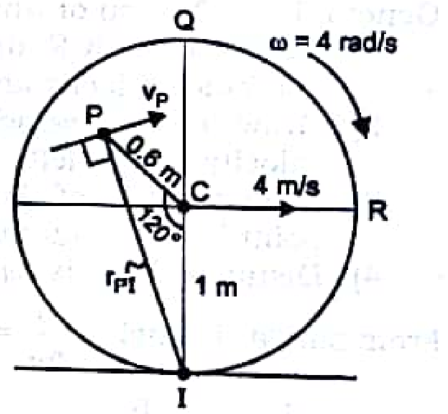
Using $v = r\omega$

$$v_C = r_{CI} \times \omega_{wheel}$$

$$4 = 1 \times \omega_{wheel}$$

$\therefore \quad \omega_{wheel} = 4$ rad/s ↻ .... **Ans.**

To find velocity of point P, join P and I to get length r_{PI}



$$r_{PI}^2 = 0.6^2 + 1^2 - 2 \times 0.6 \times 1 \times \cos 120$$

$$r_{PI} = 1.4 \text{ m}$$

Now $\quad v_P = r_{PI} \times \omega_{wheel}$

$$= 1.4 \times 4$$

$\therefore \quad v_P = 5.6$ m/s ∠ .... **Ans.**

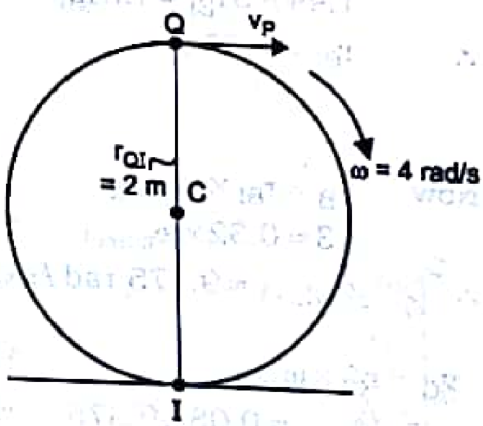
To find velocity of point Q, Join Q and I to get length r_{QI}

From geometry, $r_{QI} = 2$ m

Now $\quad v_Q = r_{QI} \times \omega_{wheel}$

$$= 2 \times 4$$

$\therefore \quad v_Q = 8$ m/s → .... **Ans.**



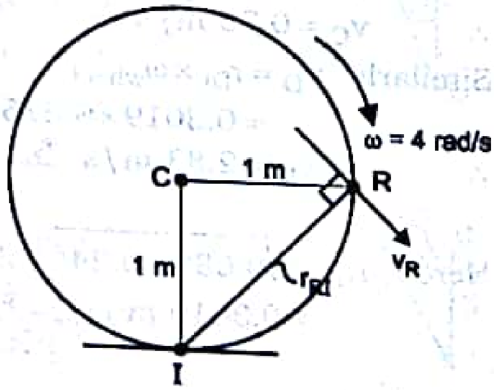
To find velocity of point R, Join R and I to get length r_{RI}

From geometry, $r_{RI} = \sqrt{1^2 + 1^2} = 1.414$ m

Now $\quad v_R = r_{RI} \times \omega_{wheel}$

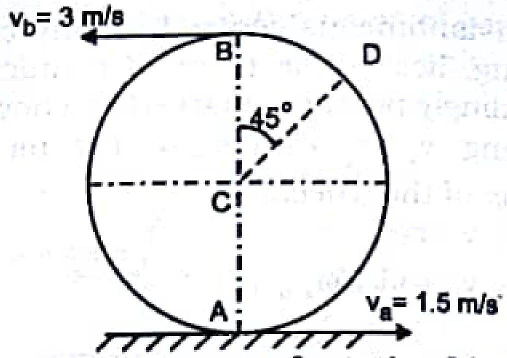
$$= 1.414 \times 4$$

$\therefore \quad v_R = 5.656$ m/s ↘ .... **Ans.**



P15. Due to slipping, points A and B on the rim of the disk have the velocities as shown in figure. Determine the velocities of the centre point C and point D on the rim at this instant. Take radius of disk 0.24 m.

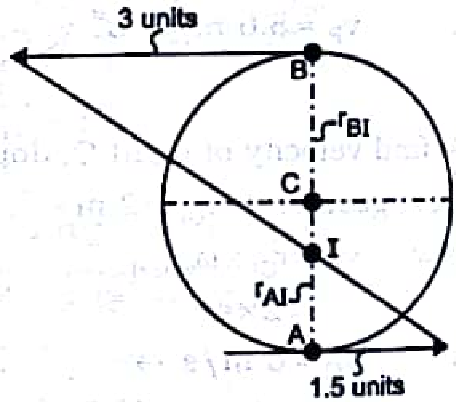
(M.U. May 14, Dec 15)



Solution: Since the wheel rolls with slip, the instantaneous centre of rotation I is not at the point of contact with the ground.

General Plane Motion of wheel

1) To locate the ICR, draw a velocity vector of length 3 units from B pointing to the left (since velocity acts to right).
2) Draw another velocity vector of 1.5 units from A pointing to the right (since velocity acts to left).
3) Joint the tips of the two arrows and let it intersect the vertical diameter. This point is the instantaneous centre I of the wheel.
4) Distance B to I is r_{BI} and A to I is r_{AI}.



From similar triangles $\frac{3}{r_{BI}} = \frac{1.5}{r_{AI}}$

$\therefore \quad \frac{3}{r_{BI}} = \frac{1.5}{0.48 - r_{BI}}$

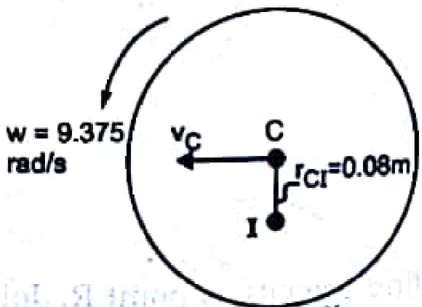
$1.44 - 3r_{BI} = 1.5r_{BI}$

$\therefore \quad r_{BI} = 0.32 \text{ m}$

Now $\quad v_B = r_{BI} \times \omega_{wheel}$

$3 = 0.32 \times \omega_{wheel}$

$\therefore \quad \omega_{wheel} = 9.375 \text{ rad/s} \circlearrowleft$



$v_C = r_{CI} \times \omega_{wheel}$ | here $r_{CI} = 0.24 - 0.16$

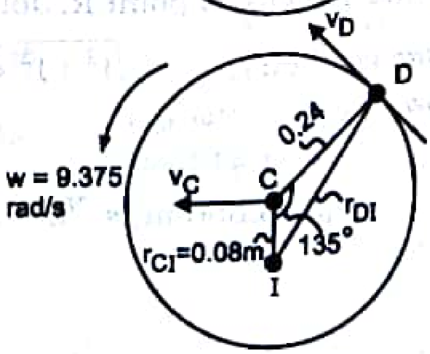
$= 0.08 \times 9.375$ | $= 0.08$ m

$\therefore \quad v_C = 0.75 \text{ m/s} \leftarrow$ .... **Ans.**

Similarly $\quad v_D = r_{DI} \times \omega_{wheel}$

$= 0.3019 \times 9.375$

$\therefore \quad v_D = 2.83 \text{ m/s}$ .... **Ans.**

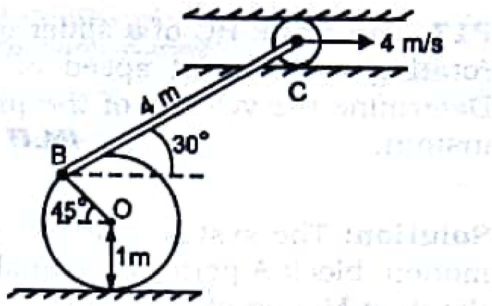


Here $\quad r_{DI} = \sqrt{0.08^2 + 0.24^2 - 2 \times 0.08 \times 0.24 \times \cos 135}$

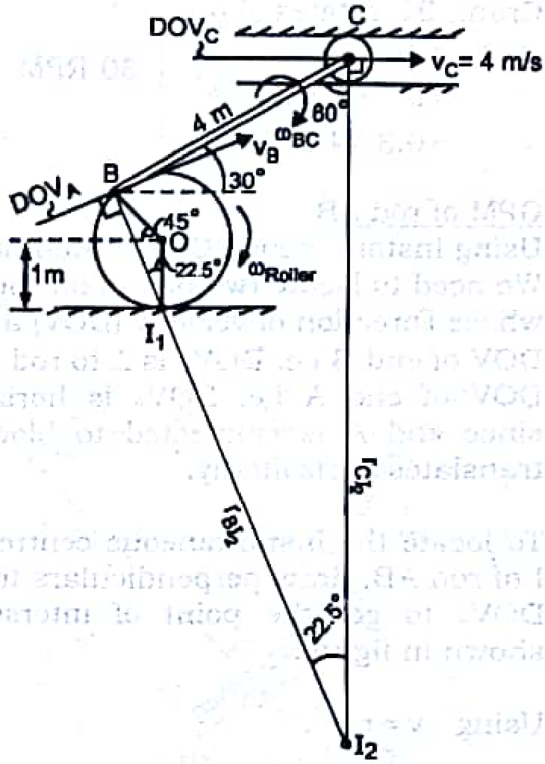
$= 0.3019$ m

P16. A bar BC slides at C in a collar at 4 m/sec. The other end B is pinned on a roller. Find angular velocity of bar BC and the roller.

(KJS Nov 15)



Solution: The system consists of three bodies in motion. Rod BC and roller, both performs GPM, while the slider C performs translation motion. Note that the instantaneous centre of rotation of roller is I_1 and is at point of contact with the ground.



GPM of Rod BC

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end C i.e. DOV_C is horizontal since it is connected to slider C which translates horizontally.

DOV of end B i.e DOV_B is ⊥ to the radius r_{BI}. Here I_1 is the instantaneous centre of the roller.

To locate the instantaneous centre of rotation I_2 of rod BC, draw perpendiculars to DOV_B and DOV_C and get their point of intersection I_2 as shown in figure.

Using $v = r\omega$

$$v_C = r_{CI_2} \times \omega_{BC}$$

$$4 = 10.36 \times \omega_{BC}$$

$\therefore\ \omega_{BC} = 0.386$ rad/s ↺ .... **Ans.**

also $v_B = r_{BI_2} \times \omega_{BC}$

$$= 9.052 \times 0.386$$

$\therefore\ v_B = 3.494$ m/s ∠ .... **Ans.**

GPM of roller

Using $v = r \times \omega$

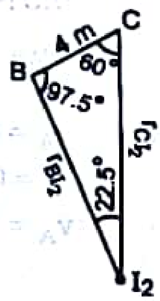
$$v_B = r_{BI_1} \times \omega_{roller}$$

$$3.494 = 1.848 \times \omega_{roller}$$

$\therefore\ \omega_{roller} = 1.89$ rad/s ↺ .... **Ans.**

From ΔBCI_2

Using sin rule



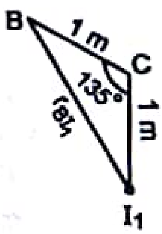
$$\frac{4}{\sin 22.5} = \frac{r_{BI_2}}{\sin 60} = \frac{r_{CI_2}}{\sin 97.5}$$

$$r_{BI_2} = 9.052 \text{ m}$$

and $r_{CI_2} = 10.36$ m

From ΔBCI_1

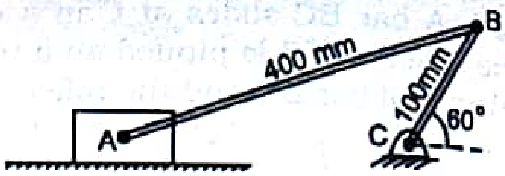
Using Cosine Rule



$$r_{BI_1} = \sqrt{1^2 + 1^2 - 2 \times 1 \times 1 \cos 135}$$

$$= 1.848 \text{ m}$$

P17. The crank BC of a slider crank mechanism is rotating at constant speed of 30 rpm clockwise. Determine the velocity of the piston A at the given instant. ***(M.U Dec 15)***



Solution: The system consists of three bodies in motion. Crank BC performs Rotation motion, block A performs Translation motion and rod AB performs GPM.

Rotation Motion of crank BC

Crank BC rotates about C.

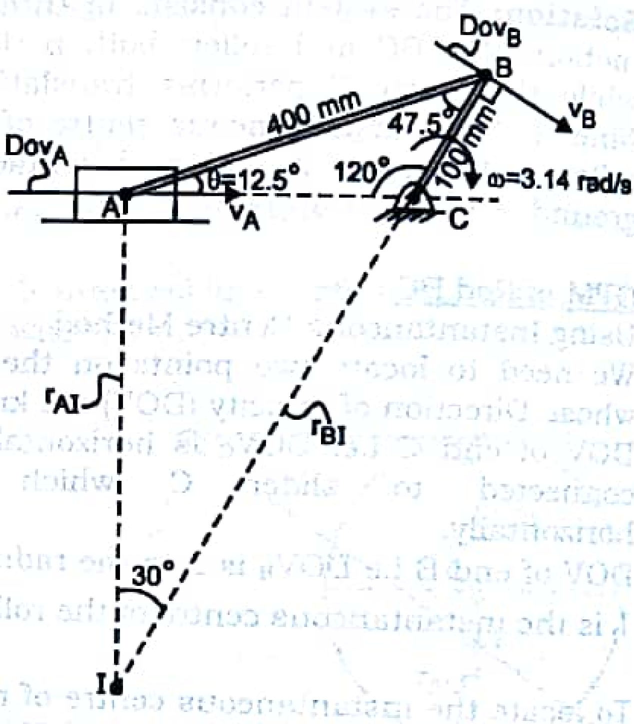
$\therefore \quad v_B = r_{BC} \times \omega_{BC}$

$= 0.1 \times 3.14$

$= 0.3.14 \text{ m/s}$

$30 \text{ RPM} = 30 \times \dfrac{2\pi}{60}$

$= 3.14 \text{ r/s}$



<u>GPM of rod AB</u>

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end B i.e. DOV_B is ⊥ to rod BC at B.

DOV of end A i.e. DOV_A is horizontal at A since end A is connected to block A which translates horizontally.

To locate the instantaneous centre of rotation I of rod AB, draw perpendiculars to DOV_B and DOV_A to get the point of intersection I as shown in figure.

Using $v = r\omega$

$v_B = r_{BI} \times \omega_{AB}$

$0.314 = 0.781 \times \omega_{AB}$

$\therefore \quad \omega_{AB} = 0.402 \text{ rad/s}$ ↺

also $\quad v_A = r_{AI} \times \omega_{AB}$

$= 0.589 \times 0.402$

$\therefore \quad v_A = 0.237 \text{ m/s} \rightarrow$ .... **Ans.**

From Δ ABC

$$\frac{100}{\sin\theta} = \frac{400}{\sin 120} \quad \therefore \theta = 12.5°$$

From Δ ABI

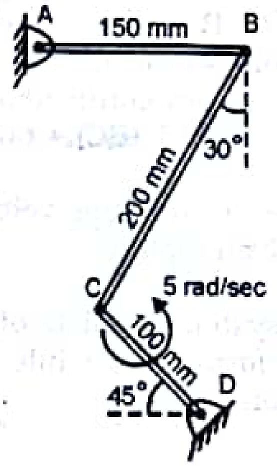
$$\frac{400}{\sin 30} = \frac{r_{BI}}{\sin 102.5} = \frac{r_{AI}}{\sin 47.5}$$

$\therefore \quad r_{BI} = 781 \text{ mm} = 0.781 \text{ m}$

and $r_{AI} = 589.8 \text{ mm} = 0.589 \text{ m}$

P18. If the link CD is rotating at 5 rad/sec anticlockwise, determine the angular velocity of link AB at the instant shown.

(M. U. Dec 11)



Solution: The system consists of three bodies. Rods AB and CD perform rotation about fixed axis, while rod BC performs GPM.

Rotation Motion of rod CD

Rod CD rotates about D

Using $v = r\omega$

$$v_C = r_{CD} \times \omega_{CD}$$
$$= 0.1 \times 5$$
$$\therefore \quad v_C = 0.5 \text{ m/s}$$

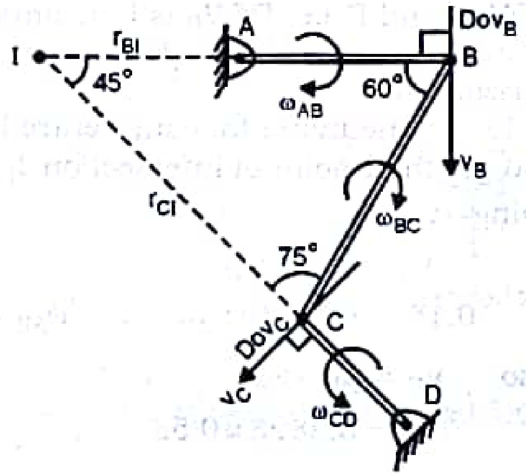
GPM of rod BC

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.

DOV of end C i.e. DOV_C is ⊥ to rod CD at C.

DOV of end B i.e. DOV_B is ⊥ to rod AB at B.



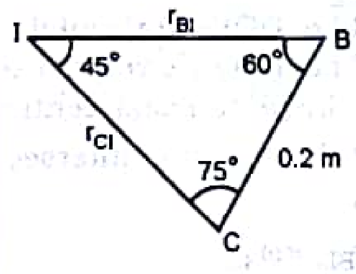
To locate the instantaneous centre I of rod BC, draw perpendiculars to DOV_C and DOV_B and get their point of intersection I as shown in figure.

Using $v = r\omega$

$$v_C = r_{CI} \times \omega_{BC}$$
$$0.5 = 0.2449 \times \omega_{BC}$$
$$\therefore \quad \omega_{BC} = 2.04 \text{ r/s}$$

also

$$v_B = r_{BI} \times \omega_{BC}$$
$$= 0.2732 \times 2.04$$
$$\therefore \quad v_B = 0.5577 \text{ m/s}$$



From ΔBCI

Using Sine Rule

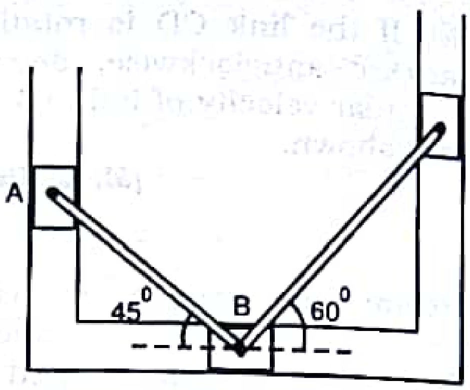
$$\frac{0.2}{\sin 45} = \frac{r_{BI}}{\sin 75} = \frac{r_{CI}}{\sin 60}$$
$$\therefore \quad r_{BI} = 0.2732 \text{ m}$$
and $r_{CI} = 0.2449$ m

Rotation Motion of rod AB

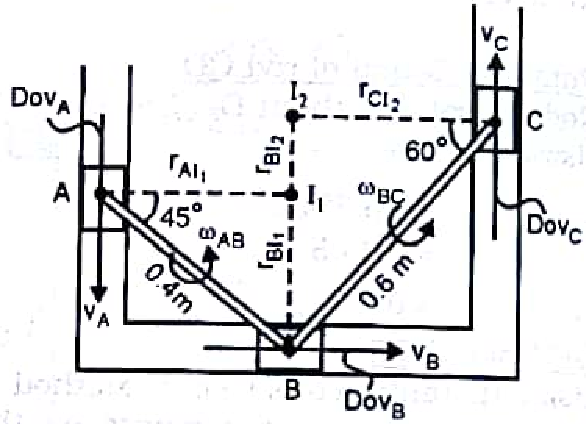
Rod AB rotates about A.

$$\therefore \quad v_B = r_{BA} \times \omega_{AB}$$
$$0.5577 = 0.15 \times \omega_{AB}$$
$$\therefore \quad \omega_{AB} = 3.718 \text{ r/s}$$ .... **Ans.**

P19. Blocks A, B and C slide in fixed slots as shown. The blocks form a mechanism, being interconnected by pin-connected links AB and BC. L (AB) = 400 mm and L (BC) = 600 mm. At the given instant, block A has a velocity of 0.15 m/s downwards. Determine the velocities of blocks B and C for the given instant.



Solution: The system consists of three bodies. Rods AB and BC perform GPM while blocks A, B and C are in translation.



GPM of rod AB

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.

DOV of end A i.e. DOV_A is vertical since it is connected to block A which translates vertically.

DOV of end B i.e. DOV_B is horizontal since it is connected to block B which translates horizontally.

To locate the instantaneous centre I of rod AB, draw perpendiculars to DOV_A and DOV_B and get their point of intersection I_1 as shown in figure.

Using $v = r\omega$

$$v_A = r_{AI_1} \times \omega_{AB}$$

$$0.15 = 0.2828 \times \omega_{AB} \quad \therefore \quad \omega_{AB} = 0.53 \ r/s \circlearrowright$$

also $v_B = r_{BI_1} \times \omega_{AB}$

$$= 0.2828 \times 0.53 \quad \therefore \quad v_B = 0.15 \ m/s \rightarrow$$

.... **Ans.**

From ΔABI_1

$$r_{AI_1} = 0.4\cos 45 = 0.2828 \text{ m}$$

$$r_{BI_1} = 0.4\sin 45 = 0.2828 \text{ m}$$

GPM of rod BC

DOV of end B i.e. DOV_B horizontal.

DOV of end C i.e. DOV_C is vertical since block C which translates vertically.

To locate the instantaneous centre I_2 of rod BC, draw perpendiculars to DOV_B and DOV_C and get their point of intersection I_2 as shown in figure.

Using $v = r\omega$

$$v_B = r_{BI_2} \times \omega_{BC}$$

$$0.15 = 0.5196 \times \omega_{BC}$$

$$\therefore \quad \omega_{BC} = 2.886 \ r/s \circlearrowright$$

From ΔBCI_2

$$r_{BI_2} = 0.6\sin 60 = 0.5196 \text{ m}$$

$$r_{CI_2} = 0.6\cos 60 = 0.3 \text{ m}$$

also $v_C = r_{CI_2} \times \omega_{BC}$

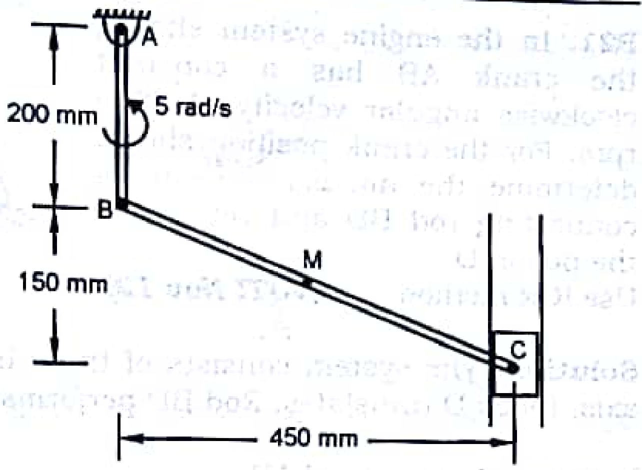
$$= 0.3 \times 2.886$$

$$\therefore \quad v_C = 0.0866 \ m/s \uparrow$$

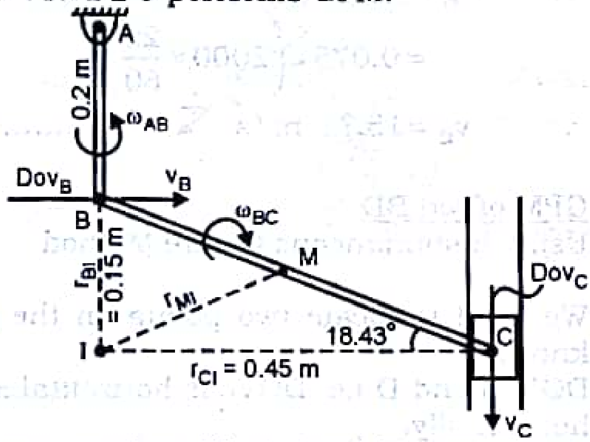
.... **Ans.**

P20. In the mechanism shown the angular velocity of link AB is 5 rad/s anticlockwise. At the instant shown, determine the angular velocity of link BC, velocity of piston C and velocity of midpoint M of link BC.

(M.U May 14)



Solution: The system consists of three bodies. Rod AB performs rotation motion about fixed axis, block C performs translation motion and Rod BC performs GPM.



Rotation Motion of rod AB

Rod AB rotates about A

$\therefore \quad v_B = r_{BA} \times \omega_{AB}$

$= 0.2 \times 5$

$\therefore \quad v_B = 1 \ \text{m/s} \rightarrow$

GPM of rod BC

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.

DOV of end C i.e. DOV_C is vertical since it is connected to block C which translates vertically.

DOV of end B i.e. DOV_B is $\perp$ to rod AB at point B (i.e. horizontal).

To locate the instantaneous centre I of rod BC, draw perpendiculars to DOV_B and DOV_C and get their point of intersection I as shown in figure.

Using $\quad v = r\omega$

$v_B = r_{BI} \times \omega_{BC}$

$1 = 0.15 \times \omega_{BC}$

$\therefore \quad \omega_{BC} = 6.667 \ \text{r/s} \circlearrowleft$ .... **Ans.**

also $\quad v_C = r_{CI} \times \omega_{BC}$

$= 0.45 \times 6.667$

$\therefore \quad v_C = 3 \ \text{m/s} \downarrow$ .... **Ans.**

To find velocity of midpoint M, join M to I and get r_{MI}

$\therefore \quad v_M = r_{MI} \times \omega_{BC}$

$= 0.237 \times 6.667$

$\therefore \quad v_M = 1.58 \ \text{m/s}$ .... **Ans.**

From Figure

$r_{BI} = 0.15$ m

$r_{CI} = 0.45$ m

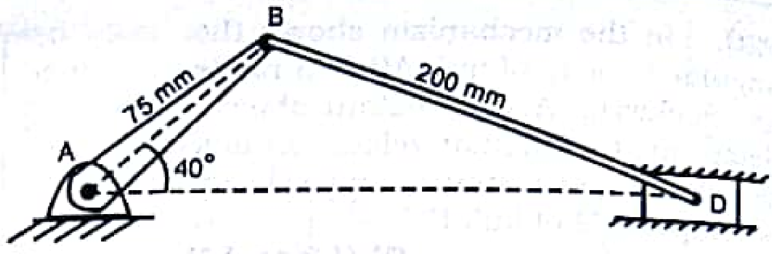
In ΔMCI,

$$MC = \frac{BC}{2} = \frac{0.474}{2} = 0.237 \text{ m}$$

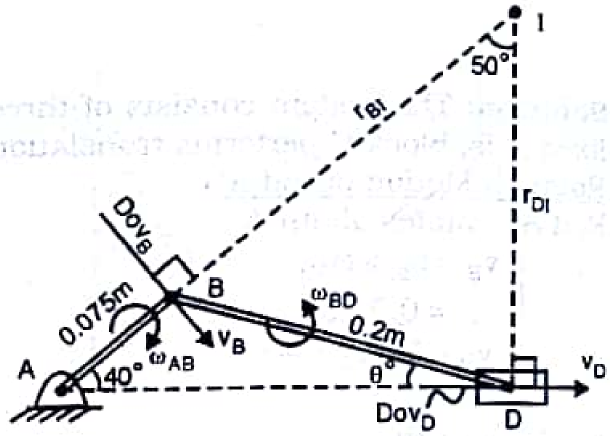
$$r_{MI} = \sqrt{0.45^2 + 0.237^2 - 2 \times 0.45 \times 0.237 \cos 18.43}$$

$= 0.237$ m

P21. In the engine system shown, the crank AB has a constant clockwise angular velocity of 2000 rpm. For the crank position shown determine the angular velocity of connecting rod BD and velocity of the piston D.
Use ICR method. ***(VJTI Nov 12)***



Solution: The system consists of three bodies. Rod AB performs rotation about fixed axis. Block D translates. Rod BD performs GPM.



Rotation Motion of rod AB

Rod AB rotates about A

$$\therefore \quad v_B = r_{AB} \times \omega_{AB}$$

$$= 0.075 \times \left(2000 \times \frac{2\pi}{60}\right)$$

$$\therefore \quad v_B = 15.71 \text{ m/s}$$ .... **Ans.**

GPM of rod BD

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.
DOV of end D i.e. DOV_D is horizontal since it is connected to block D, which translates horizontally.
DOV of end B i.e. DOV_B is ⊥ to rod AB at point B.
To locate the instantaneous centre I of rod BD, draw perpendiculars to DOV_B and DOV_D and get their point of intersection I as shown in figure.

Using $v = r\omega$

$$v_B = r_{BI} \times \omega_{BD}$$

$$15.71 = 0.2534 \times \omega_{BD}$$

$$\therefore \quad \omega_{BD} = 62 \text{ r/s}$$

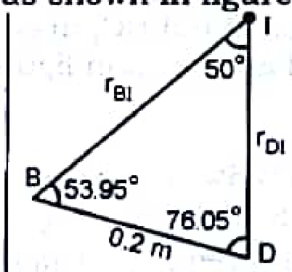
.... **Ans.**

also $v_D = r_{DI} \times \omega_{BD}$

$$= 0.211 \times 62$$

$$\therefore \quad v_D = 13.08 \text{ m/s} \rightarrow$$

.... **Ans.**



From Δ ABD

Using Sine Rule

$$\frac{0.075}{\sin\theta} = \frac{0.2}{\sin 40}$$

$$\therefore \quad \theta = 13.95°$$

In Δ BDI,

Using Sine Rule

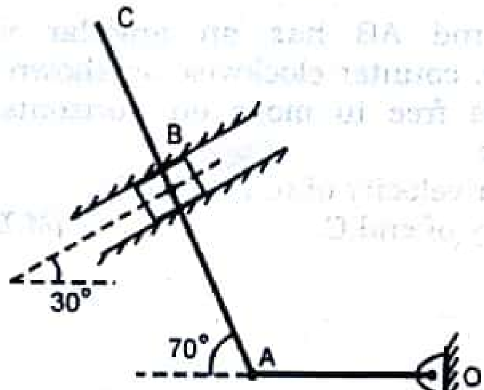
$$\frac{r_{BI}}{\sin 76.05} = \frac{r_{DI}}{\sin 53.95} = \frac{0.2}{\sin 50}$$

$$\therefore \quad r_{BI} = 0.2534 \text{ m}$$

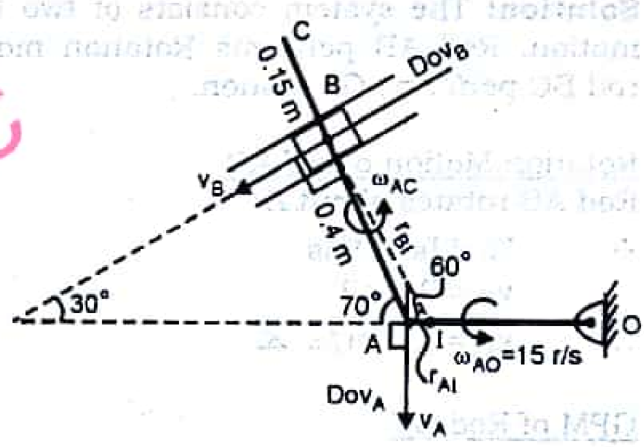
and $r_{DI} = 0.211$ m

P22. Locate the instantaneous center of rotation for the link ABC and determine velocity of points B & C. Angular velocity of rod OA is 15 rad/sec counter clock wise. Length of OA is 200 mm, AB is 400 mm and BC is 150 mm.

(M.U. Dec 10)



Solution: The system consist of three bodies. Rod AO rotates about fixed axis. Block B translates. Rod AC performs GPM.



<u>Rotation Motion of rod OA</u>

Rod OA rotates about O

$\therefore \quad v_A = r_{AO} \times \omega_{AO}$

$= 0.2 \times 15$

$\therefore \quad v_A = 3 \text{ m/s} \downarrow$

<u>GPM of rod AC</u>

Using Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) is known.

DOV of end A i.e. DOV_A is $\perp$ to rod AO at A.

DOV of end B i.e. DOV_B is along the track of the block B.

To locate the instantaneous centre I of rod AC, draw perpendiculars to DOV_A and DOV_B and get their point of intersection I as shown in figure.

Using $v = r\omega$

$v_A = r_{AI} \times \omega_{AC}$

$3 = 0.0802 \times \omega_{AC}$

$\therefore \quad \omega_{AC} = 37.4 \text{ r/s}$ ↺ .... **Ans.**

also $\quad v_B = r_{BI} \times \omega_{AC}$

$= 0.434 \times 37.4$

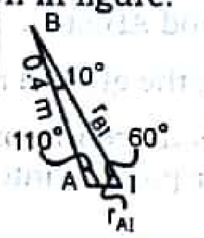
$\therefore \quad v_B = 16.23 \text{ m/s}$ ↗ .... **Ans.**

To find velocity of end C, join C to I and get r_{CI}

$\therefore \quad v_C = r_{CI} \times \omega_{AC}$

$= 0.5823 \times 37.4$

$\therefore \quad v_C = 21.77 \text{ m/s}$ ↗ .... **Ans.**



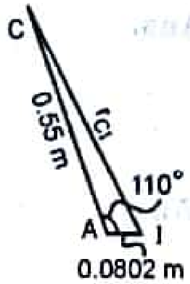
From ΔABI

Using Sine Rule

$$\frac{0.4}{\sin 60} = \frac{r_{BI}}{\sin 110} = \frac{r_{AI}}{\sin 10}$$

$\therefore \quad r_{BI} = 0.434 \text{ m}$

and $\quad r_{AI} = 0.0802 \text{ m}$



Using Cosine rule

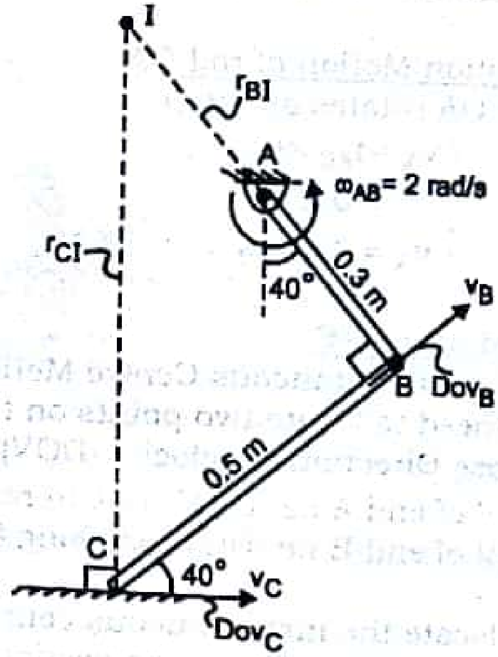
$$r_{CI} = \sqrt{0.0802^2 + 0.55^2 - 2 \times 0.0802 \times 0.55 \cos 110}$$

$= 0.5823 \text{ m}$

✻✣✻✣✻

P23. A rod AB has an angular velocity of 2 rad/sec, counter clockwise as shown. End C of rod BC is free to move on horizontal surface. Determine
(i) Angular velocity of rod BC and
(ii) Velocity of end C. ***(M.U Dec 16)***

Solution: The system consists of two bodies in motion. Rod AB performs Rotation motion and rod BC performs GP Motion.



Rotation Motion of rod AB

Rod AB rotates about A

$\therefore \quad v_B = r_{BA} \times \omega_{AB}$

$v_B = 0.3 \times 2$

$\therefore \quad v_B = 0.6 \text{ m/s}$

GPM of Rod BC

Instantaneous Centre Method

We need to locate two points on the GPM body whose Direction of velocity (DOV) are known.

DOV of end B i.e. DOV_B is $\perp$ to rod AB at B.

DOV of end C i.e. DOV_C is along the ground at C.

To locate the instantaneous centre of rotation I of rod BC, draw perpendiculars to DOV_B and DOV_C and get their point of intersection I as shown in figure.

Using $v = r\omega$

$v_B = r_{BI} \times \omega_{BC}$

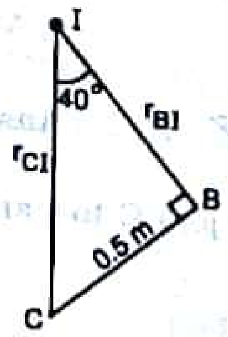
$0.6 = 0.5959 \times \omega_{BC}$

$\therefore \quad \omega_{BC} = 1 \text{ rad/s}$ ↺ ... **Ans.**

also $v_C = r_{CI} \times \omega_{BC}$

$= 0.7786 \times 1$

$\therefore \quad v_C = 0.7786 \text{ m/s} \rightarrow$ ... **Ans.**



In ΔBCI

$\sin 40 = \frac{0.5}{r_{CI}}$

$\therefore r_{CI} = 0.7786$ m

$\cos 40 = \frac{r_{BI}}{0.7786}$

$\therefore \quad r_{BI} = 0.5959$ m

++*