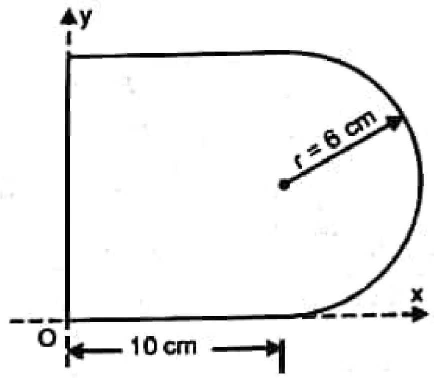
Solutions: Chapter 6

Centroid and Centre of Gravity

Exercise 6.1

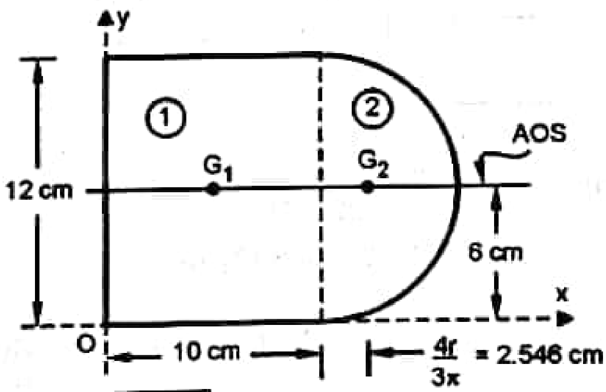
Centroid of Plane Areas

P2. Locate the centroid of the composite figure shown.



Solution: The given composite plane area can be made up by taking a rectangle of 10 cm × 12 cm and adding a semicircle of radius 6 cm. The plane area is symmetrical about an axis parallel to x axis. The centroid lies on the AOS.

$\therefore \quad \bar{Y} = 6$ cm

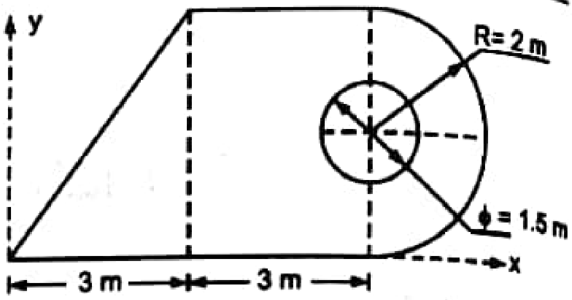


Part	Area A cm^2	x cm	A x cm^3
1. Rectangle	$10 \times 12 = 120$	5	600
2. Semicircle	$(\pi \times 6^2)/2 =$ 56.55	12.546	709.5
	$\Sigma A =$ 176.55		$\Sigma A x =$ 1309.5

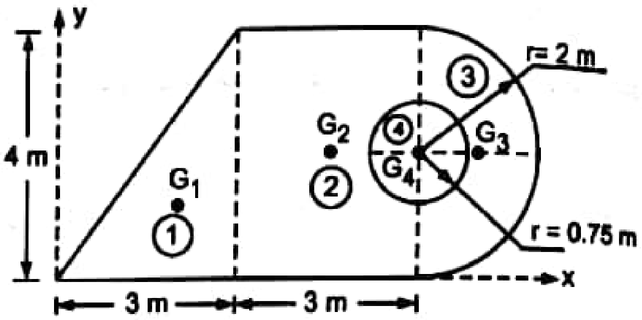
Using $\bar{X} = \frac{\Sigma A x}{\Sigma A} = \frac{1309.5}{176.55} = 7.417$ cm

$\therefore \quad \bar{X}, \bar{Y} = (7.417, 6)$ cm .... **Ans.**

P3. A circle of diameter 1.5 m is cut from a composite plate. Determine the Centroid of the remaining area of the plate. ***(M.U Dec 16)***



Solution: The given composite area can be obtained by taking rt. angled triangle (Part 1), adding a rectangle (Part 2), also adding a semicircle (Part 3) and subtracting a circle (Part 4) from it.

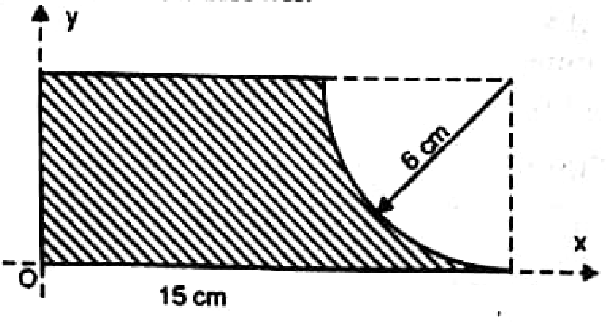


Part	Area A m^2	Coordinates x m	y m	Ax m^3	Ay m^3
1. Rt. Triangle	$\left(\frac{1}{2}\times 3\times 4\right)=6$	2	1.333	12	8
2. Rectangle	$3\times 4=12$	4.5	2	54	24
3. Semi-circle	$\frac{\pi\times 2^2}{2}=6.283$	6.849	2	43.03	12.57
4. Circle	$-\pi\times 0.75^2=-1.767$	6	2	- 10.6	- 3.534
	$\Sigma A=22.516$			$\Sigma Ax=$ 98.43	$\Sigma Ay=$ 41.036

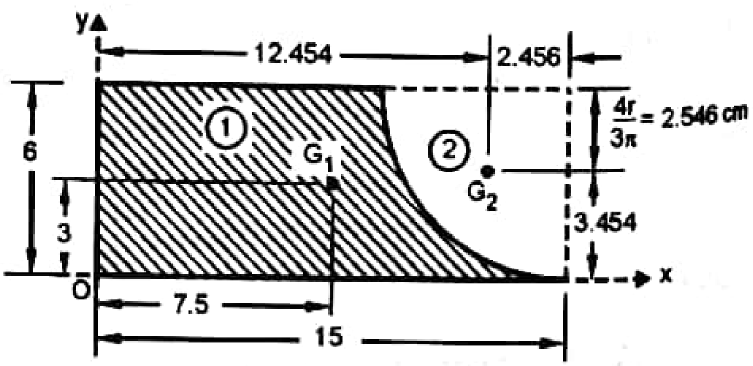
Using $\overline{X}=\frac{\Sigma Ax}{\Sigma A}=\frac{98.43}{22.516}=4.371$ m and $\overline{Y}=\frac{\Sigma Ay}{\Sigma A}=\frac{41.036}{22.516}=1.822$ m

$\therefore \quad \overline{X},\overline{Y}=(4.371, 1.822)$ m

P4. Determine the centroid of the shaded area shown.



Solution: The given composite plane area can be obtained by taking a rectangle (Part 1) and subtracting a quarter circle (Part 2) from it.



All dimensions are in cm

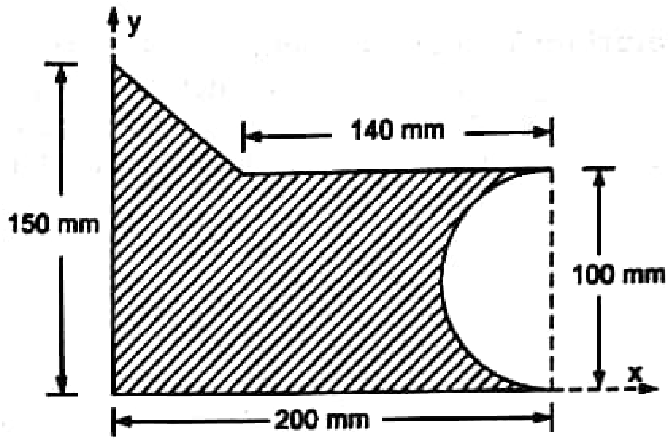
Part	Area A cm^2	Co-ordinates		A x cm^3	A y cm^3
		x cm	y cm		
1. Rectangle	15 × 6 = 90	7.5	3	675	270
2. Quarter Circle	$-(\pi \times 6^2)/4 =$ − 28.27	12.454	3.454	− 352.1	− 97.66
	$\Sigma A =$ 61.73			$\Sigma A x =$ 322.9	$\Sigma A y =$ 172.34

Using $\overline{X} = \dfrac{\Sigma Ax}{\Sigma A} = \dfrac{322.9}{61.73} = 5.23$ cm and $\overline{Y} = \dfrac{\Sigma Ay}{\Sigma A} = \dfrac{172.34}{61.73} = 2.792$ cm

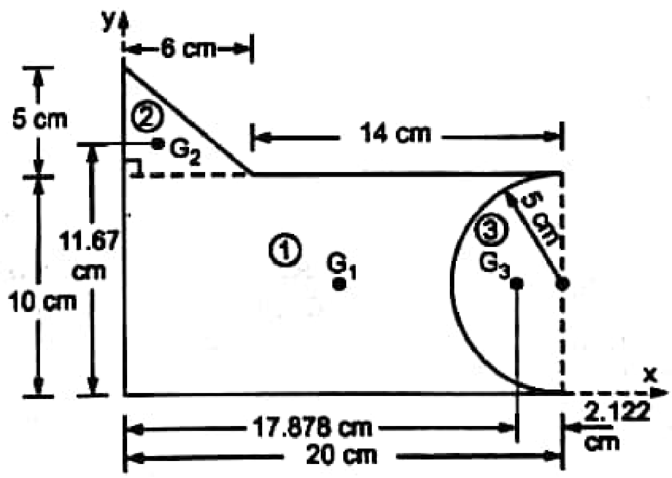
$\therefore\ \overline{X}, \overline{Y} = (5.23, 2.792)$ cm .... **Ans.**

P5. Find Centroid of the shaded area.

(M.U. Dec 14)



Solution: The shaded composite figure can be obtained by taking rectangle (Part 1), adding a triangle (Part 2) and subtracting a semi circle (Part 3). Working in cm units.

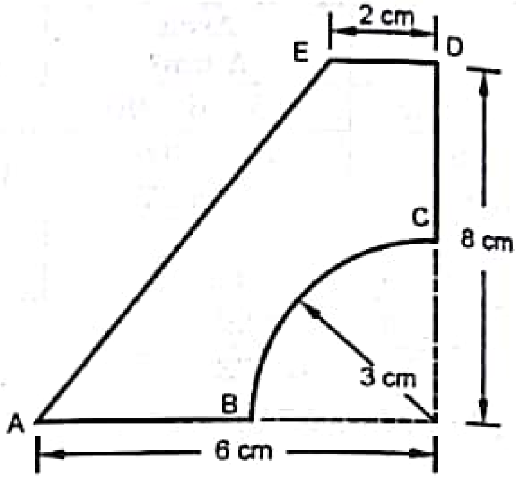


Part	Area A cm^2	Coordinates		Ax cm^3	Ay cm^3
		x cm	y cm		
1. Rectangle	200	10	5	2000	1000
2. Rt. Triangle	15	2	11.67	30	175
3. Semi-circle	− 39.27	17.878	5	− 702.1	− 196.3
	$\Sigma A =$ 175.73			$\Sigma AX =$ 1327.9	$\Sigma AY =$ 978.7

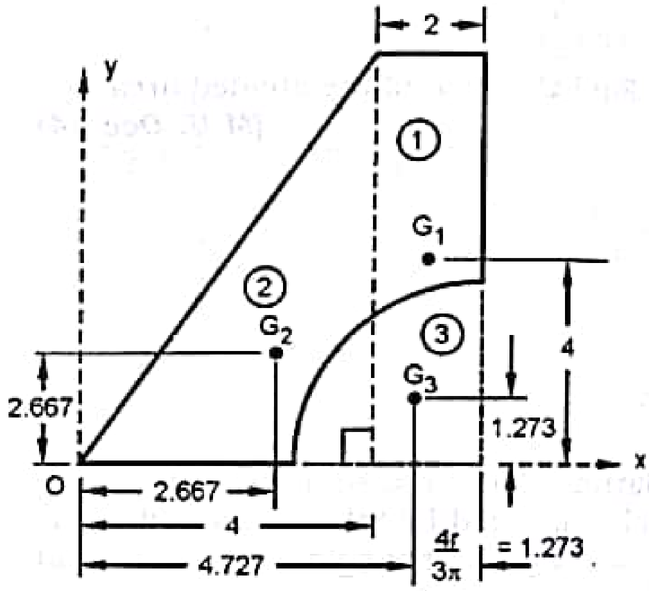
Using $\overline{X} = \dfrac{\Sigma Ax}{\Sigma A} = \dfrac{1327.9}{175.73} = 7.556$ cm and $\overline{Y} = \dfrac{\Sigma Ay}{\Sigma A} = \dfrac{978.7}{175.73} = 5.569$ cm

$\therefore\ \overline{X}, \overline{Y} = (7.556, 5.569)$ cm

P6. Determine centroid of plane area ABCDE w.r.t. A.



Solution: Taking the origin at A. The given composite area can be obtained by taking a rectangle (Part 1), adding a rt. angle triangle (Part 2) and subtracting a quarter circle (Part 3) from it.



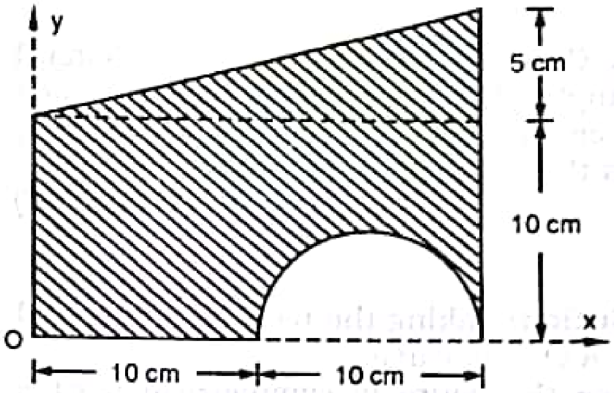
All dimensions are in cm

Part	Area A cm²	Co-ordinates		Ax cm³	Ay cm³
		x cm	y cm		
1. Rectangle	2 × 8 = 16	5	4	80	64
2. Rt. Triangle	$(\frac{1}{2} \times 4 \times 8) = 16$	2.667	2.667	42.67	42.67
3. Quarter Circle	$-(\pi \times 3^2)/4 = -7.068$	4.727	1.273	– 33.41	– 9
	Σ A = 24.932			Σ A x = 89.26	Σ A y = 97.67

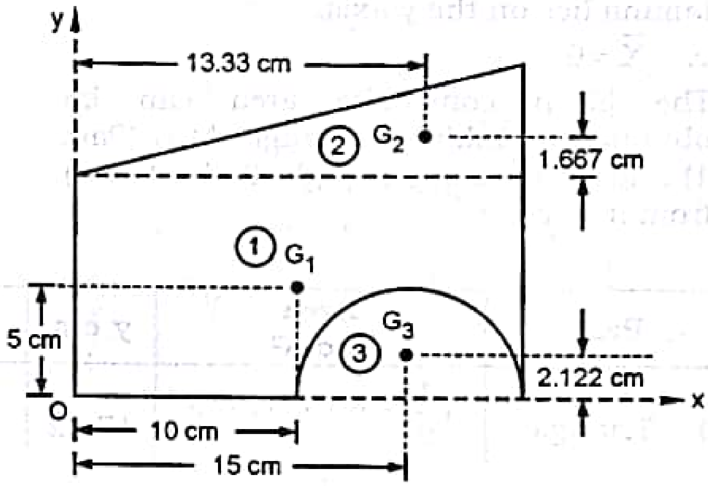
Using $\overline{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{89.26}{24.932} = 3.58$ cm and $\overline{Y} = \frac{\Sigma Ay}{\Sigma A} = \frac{97.67}{24.932} = 3.917$ cm

$\therefore \quad \overline{X}, \overline{Y} = (3.58, 3.917)$ cm .... **Ans.**

P7. Find centroid of the Shaded area. ***(MU Dec 12)***



Solution: The given composite area can be obtained by taking a rectangle (Part 1), adding a rt. angled triangle (Part 2) and subtracting a semicircle (Part 3) from it



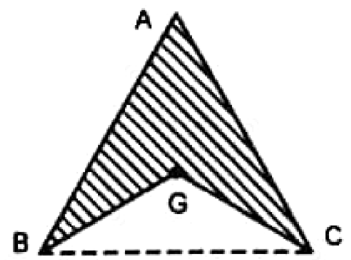
Part	Area A cm²	Co-ordinates x cm	Co-ordinates y cm	A x cm³	A y cm³
1. Rectangle	20 × 10 = 200	10	5	2000	1000
2. Rt. angled triangle	$(\frac{1}{2} \times 20 \times 5) = 50$	13.33	11.667	666.5	583.35
3. Semicircle	$-(\pi \times 5^2)/2$ $= -39.27$	15	2.122	− 589	− 83.33
	Σ A = 210.73			Σ A x = 2077.5	Σ A y = 1500

Using $\overline{X} = \dfrac{\Sigma A x}{\Sigma A} = \dfrac{2077.5}{210.73} = 9.858$ cm and $\overline{Y} = \dfrac{\Sigma A y}{\Sigma A} = \dfrac{1500}{210.73} = 7.118$ cm

$\therefore\ \overline{X}, \overline{Y} = (9.858\ ,\ 7.118)$ cm .... **Ans.**

P8. G is the centroid of an equilateral triangle ABC with base BC of side 60 cm. If GBC is cut from the lamina, find the centroid of the remaining area.

(VJTI May 06)

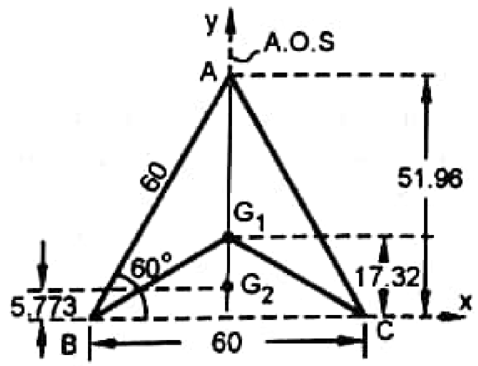


Solution: Taking the base as x axis and the A.O.S as y axis.

Since the figure is symmetrical w.r.t y axis, centroid of the shaded plane lamina lies on the y axis.

$\therefore \quad \overline{X} = 0$

The given composite area can be obtained by taking a triangle ABC (Part 1) and subtracting triangle GBC (Part 2) from it.



All dimensions are in mm

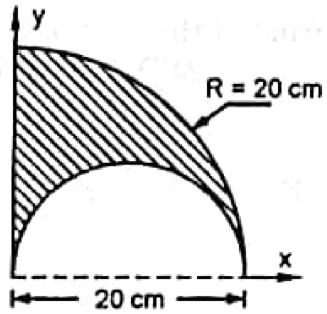
Part	Area A cm²	y cm	A y cm³
1. Triangle	$(\frac{1}{2} \times 60 \times 51.96)$ = 1558.8	17.32	26998
2. Triangle	$-(\frac{1}{2} \times 60 \times 17.32)$ = – 519.6	5.773	– 3000
	Σ A = 1039.2		Σ A x = 23998

$$\text{Using } \overline{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{23998}{1039.2} = 23.09 \text{ cm}$$

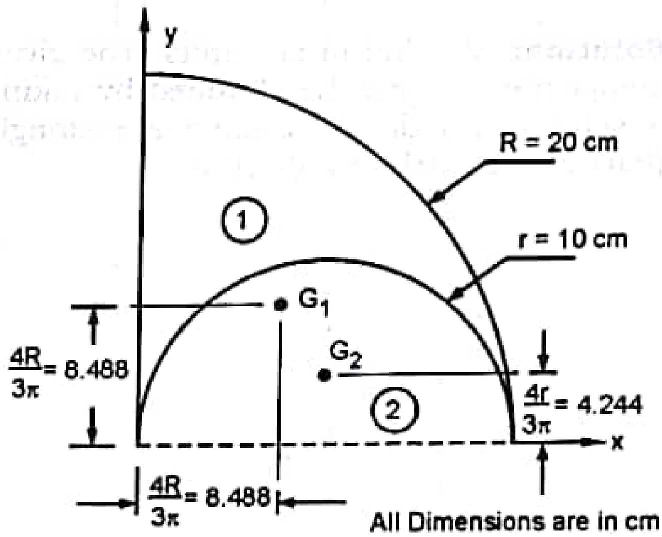
$\therefore \quad \overline{X}, \overline{Y} = (0,\ 23.09)$ cm .... **Ans.**

P9. Find centroid of the shaded area.

(M.U. May 13)



Solution: The given composite plane area can be obtained by taking a quarter circle (Part 1) and subtracting a semicircle (Part 2) from it.



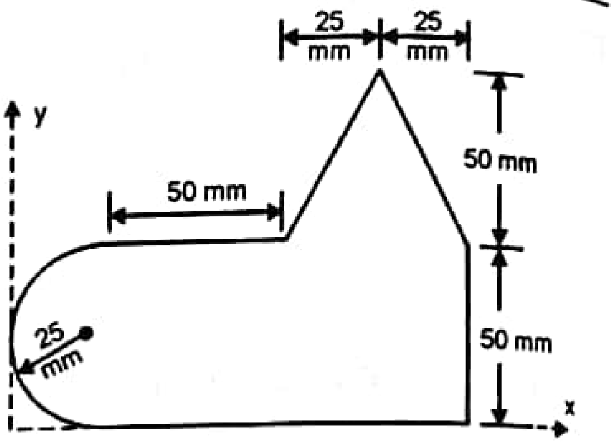
Part	Area A cm^2	Co-ordinates		Ax cm^3	Ay cm^3
		x cm	y cm		
1. Quarter Circle	$\frac{\pi\times 20^2}{4}=314.16$	8.488	8.488	2666.6	2666.6
2. Semi Circle	$\frac{\pi\times 10^2}{2}=-157.08$	10	4.244	− 1570.5	− 666.6
	ΣA = 157.08			ΣAx = 1095.8	ΣAy = 2000

Using $\overline{X}=\frac{\Sigma Ax}{\Sigma A}=\frac{1095.8}{157.08}=6.976$ cm and $\overline{Y}=\frac{\Sigma Ay}{\Sigma A}=\frac{2000}{157.08}=12.73$ cm

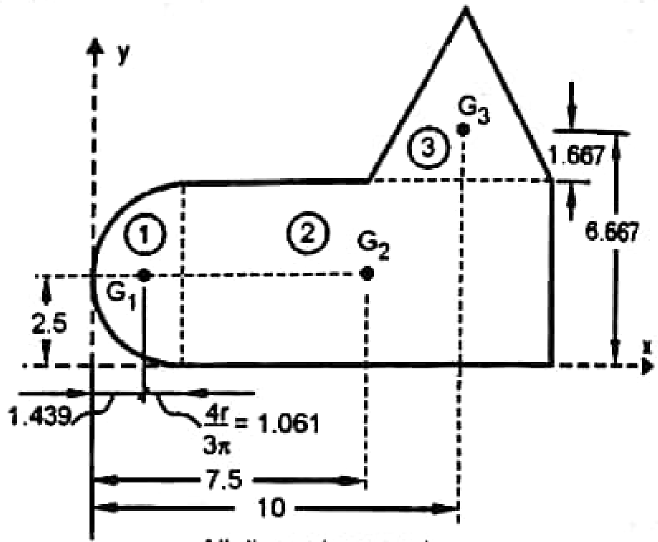
$\therefore\ \overline{X},\overline{Y}=(6.976,\ 12.73)$ cm .... **Ans.**

P10. Locate the centroid of the section.

(SPCE Mar 11)



Solution: Working in cm units. The given composite area can be obtained by taking a semicircle (Part 1), adding a rectangle (Part 2) and a triangle (Part 3).



All dimensions are in cm

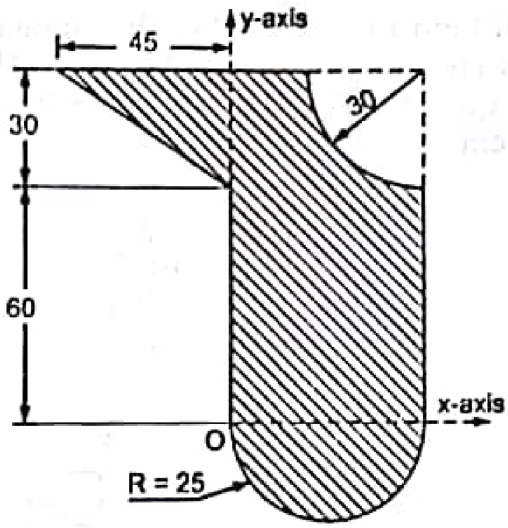
Part	Area A cm²	Co-ordinates		A x cm³	A y cm³
		x cm	y cm		
1. Semicircle	$(\pi \times 2.5^2)/2 = 9.817$	1.439	2.5	14.13	24.54
2. Rectangle	$10 \times 5 = 50$	7.5	2.5	375	125
3. Triangle	$(\frac{1}{2} \times 5 \times 5) = 12.5$	10	6.667	125	83.34
	Σ A = 72.317			Σ A x = 514.13	Σ A y = 232.88

Using $\overline{X} = \dfrac{\Sigma Ax}{\Sigma A} = \dfrac{514.13}{72.317} = 7.109$ cm and $\overline{Y} = \dfrac{\Sigma Ay}{\Sigma A} = \dfrac{232.88}{72.317} = 3.22$ cm

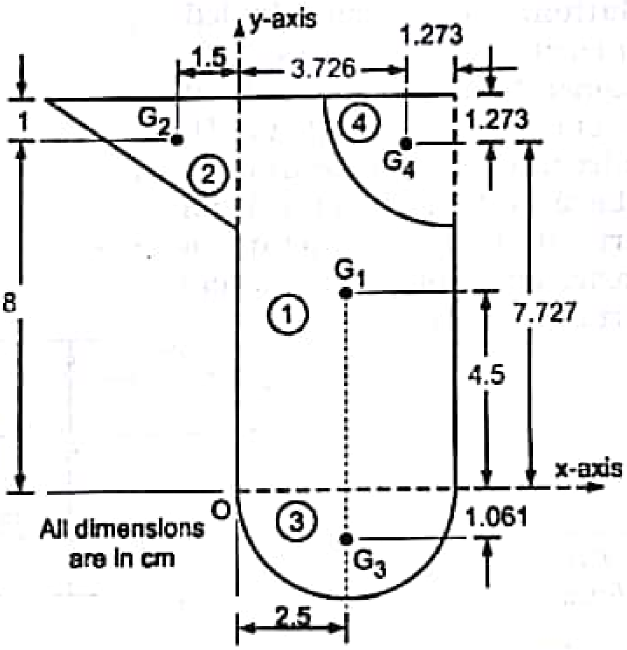
$\therefore \quad \overline{X}, \overline{Y} = (7.109, 3.22)$ cm .... **Ans.**

P11. Determine the centroid of the shaded area shown.
All dimensions are in mm.

(M.U Dec 15)



Solution: The given shaded composite area can be obtained by taking a rectangle (Part 1), adding a rt. angle triangle (Part 2), also adding a semi-circle (Part 3) and subtracting a quarter circle (Part 4) from it. Working in cm units.

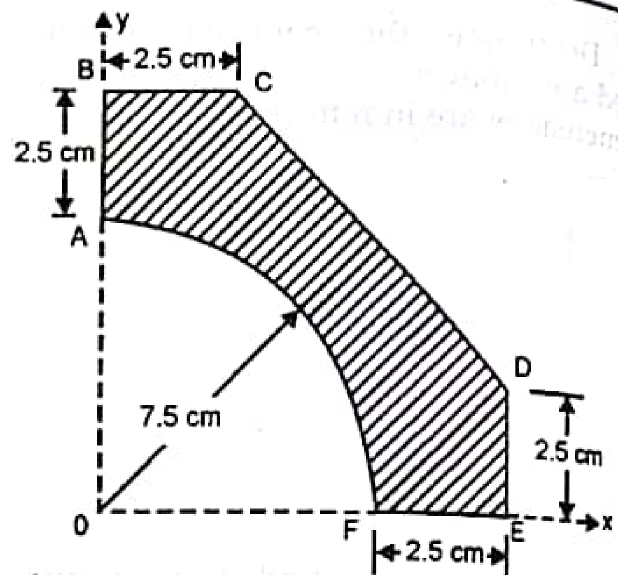


Part	Area A cm^2	Co-ordinates		A x cm^3	A y cm^3
		x cm	y cm		
1. Rectangle	$5\times9=45$	2.5	4.5	112.5	202.5
2. Rt. Triangle	$\frac{1}{2}\times4.5\times3=6.75$	−1.5	8	−10.12	54
3. Semicircle	$\frac{\pi\times2.5^2}{2}=9.817$	2.5	−1.061	24.54	−10.42
4. Quarter circle	$-\frac{\pi\times3^2}{4}=-7.068$	3.726	7.727	−26.34	−54.62
	$\Sigma A=54.5$			$\Sigma Ax=100.58$	$\Sigma Ay=191.46$

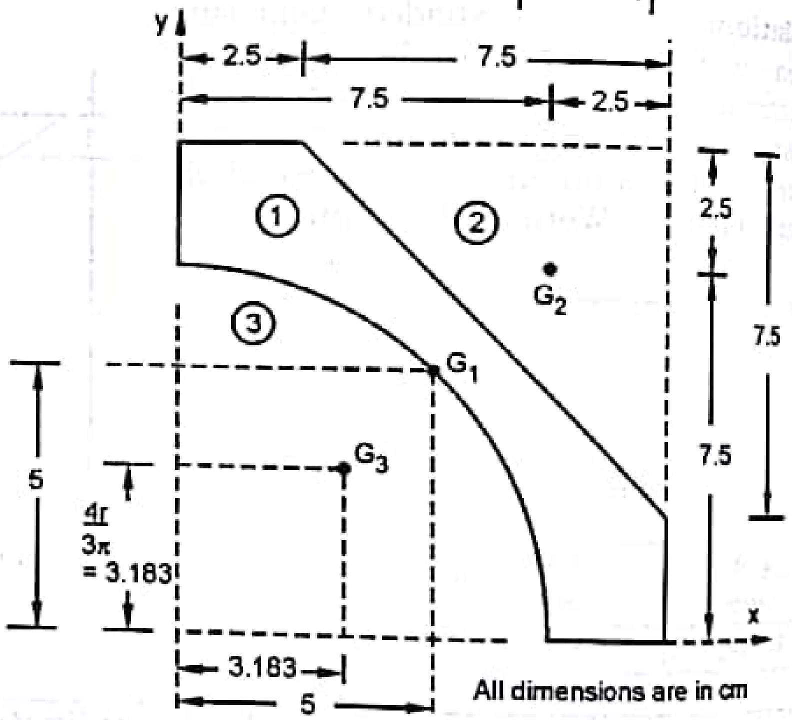
Using $\overline{X}=\frac{\Sigma Ax}{\Sigma A}=\frac{100.58}{54.5}=1.845$ cm and $\overline{Y}=\frac{\Sigma Ay}{\Sigma A}=\frac{191.46}{54.5}=3.513$ cm

$\therefore\ \overline{X},\overline{Y}=(1.845,3.513)$ cm .... **Ans.**

P12. Find the centroid of the shaded area shown in figure. Note that OAF is a quarter part of a circle of radius 7.5 cm.



Solution: The given shaded composite area can be obtained by taking a square of 10 cm × 10 cm (Part 1), subtracting a rt. angle triangle of base and height of 7.5 cm (Part 2) and subtracting a quarter circle of radius 7.5 cm (Part 3) from it.

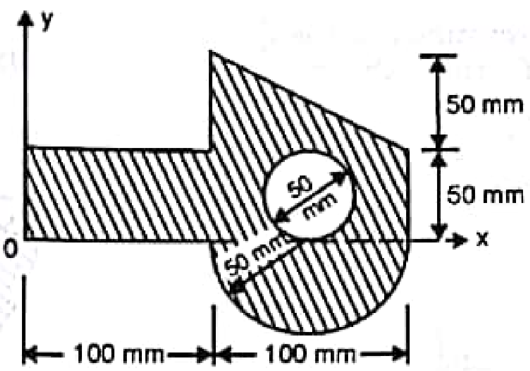


Part	Area A cm^2	Co-ordinates		Ax cm^3	Ay cm^3
		x cm	y cm		
1. Square	10 × 10 = 100	5	5	500	500
2. Rt. Triangle	$-(\frac{1}{2} \times 7.5 \times 7.5)$ = − 28.125	7.5	7.5	− 210.94	− 210.94
3. Quarter Circle	$-(\pi \times 7.5^2)/4$ = − 44.179	3.183	3.183	− 140.63	− 140.63
	ΣA = 27.696			ΣAx = 148.43	ΣAy = 148.43

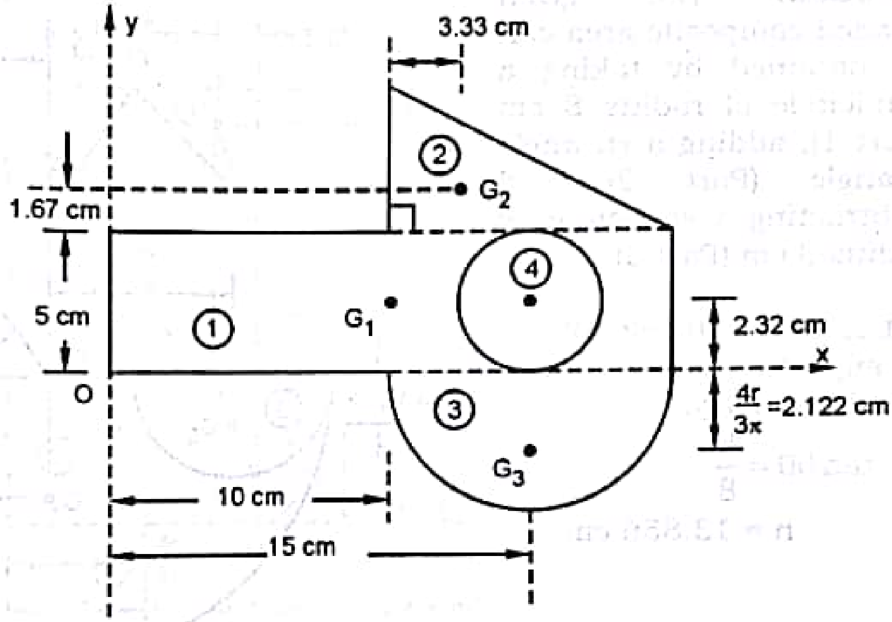
Using $\overline{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{148.43}{27.696} = 5.359$ cm and $\overline{Y} = \frac{\Sigma Ay}{\Sigma A} = \frac{148.43}{27.696} = 5.359$ cm

$\therefore$ $\overline{X}, \overline{Y}$ = (5.359, 5.359) cm .... **Ans.**

P13. Determine the co-ordinates of the centroid of the lamina shown with respect to origin. Note that the circle of diameter 50 mm is cut out from the plane lamina, with centre at (150, 25) mm.



Solution: The shaded composite figure can be obtained by taking a rectangle (Part 1), adding a rt. angle triangle (Part 2), also adding a semicircle (Part 3) and subtracting a circle (Part 4). Working in cm units.

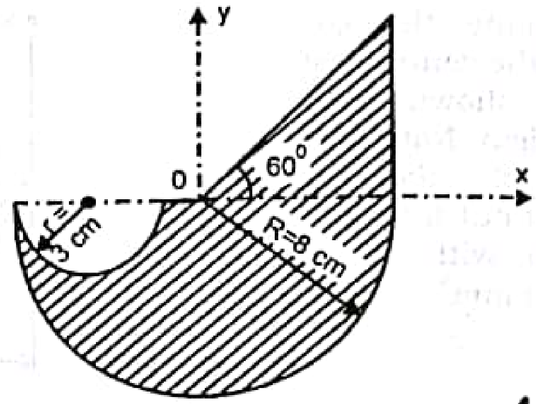


Part	Area A cm²	Co-ordinates		A x cm³	A y cm³
		x cm	y cm		
1. Rectangle	20 × 5 = 100	10	2.5	1000	250
2. Rt. Triangle	$(\frac{1}{2} \times 10 \times 5)$ = 25	13.33	6.67	333.3	166.7
3. Semicircle	$(\pi \times 5^2)/2$ = 39.27	15	– 2.122	589	– 83.33
4. Circle	– $(\pi \times 2.5^2)$ = – 19.63	15	2.5	– 294.5	– 49.09
	ΣA = 144.64			$\Sigma A x$ = 1627.8	$\Sigma A y$ = 284.28

Using $\bar{X} = \frac{\Sigma A x}{\Sigma A} = \frac{1627.8}{144.64} = 11.254$ cm and $\bar{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{284.28}{144.64} = 1.965$ cm

$\therefore$ $\bar{X}, \bar{Y} = (11.254, 1.965)$ cm .... **Ans.**

P14. Determine the centroid of the shaded portion shown.

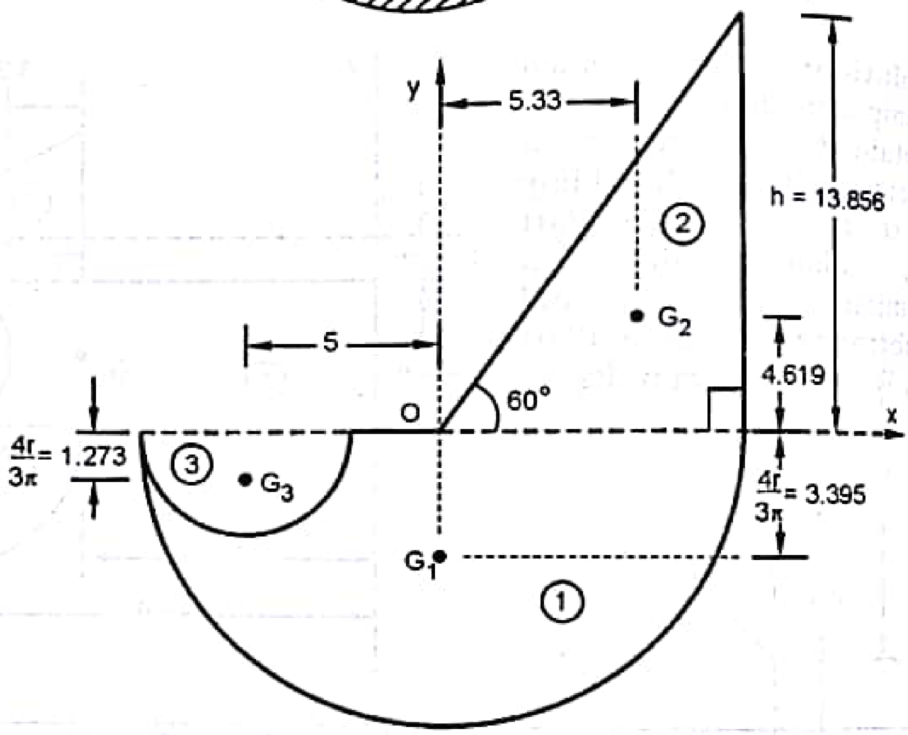


Solution: The given shaded composite area can be obtained by taking a semicircle of radius 8 cm (Part 1), adding a rt. angle triangle (Part 2) and subtracting a semicircle of radius 3 cm (Part 3).

Let h be the height of the rt. angle triangle,
From geometry,

$$\tan 60 = \frac{h}{8}$$

or $h = 13.856$ cm



All dimensions are in cm

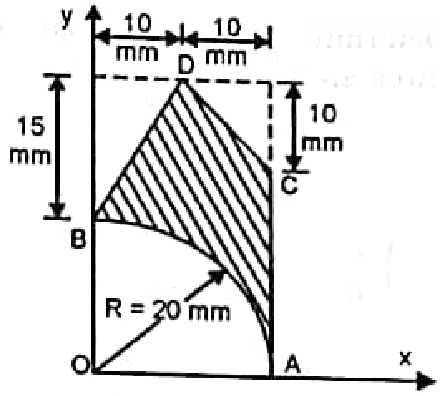
Part	Area A cm²	Co-ordinates		A x cm³	A y cm³
		x cm	y cm		
1. Semicircle	$(\pi \times 8^2)/2 = 100.53$	0	− 3.395	0	− 341.3
2. Rt. Triangle	$(\frac{1}{2} \times 8 \times 13.856) = 55.42$	5.33	4.619	295.4	256
3. Semicircle	$-(\pi \times 3^2)/2 = -14.137$	− 5	− 1.273	70.68	18
	$\Sigma A = 141.81$			$\Sigma A x = 366$	$\Sigma A y = -67.33$

Using $\overline{X} = \frac{\Sigma A x}{\Sigma A} = \frac{366}{141.81} = 2.581$ cm and $\overline{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{-67.33}{141.81} = -0.474$ cm

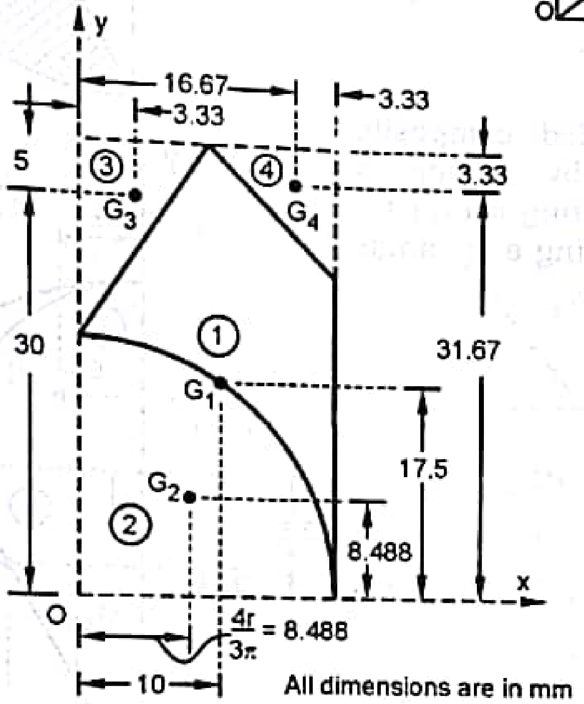
$\therefore \quad \overline{X}, \overline{Y} = (2.581, -0.474)$ cm .... **Ans.**

P15. Find centroid of shaded plane area.

(M.U Dec 10)



Solution:

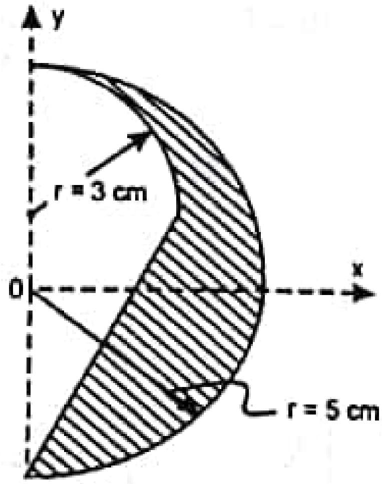


Part	Area A mm²	Co-ordinates		A x mm³	A y mm³
		x mm	y mm		
1. Rectangle	20 × 35 = 700	10	17.5	7000	12250
2. Qt. circle	$-(\pi \times 20^2)/4 = -314.16$	8.488	8.488	− 2666.6	− 2666.6
3. Rt. Triangle	$-(\frac{1}{2} \times 10 \times 15) = -75$	3.33	30	− 249.8	− 2250
4. Rt. Triangle	$-(\frac{1}{2} \times 10 \times 10) = -50$	16.67	31.67	− 833.5	− 1583.5
	ΣA = 260.84			ΣA x = 3250	ΣA y = 5750

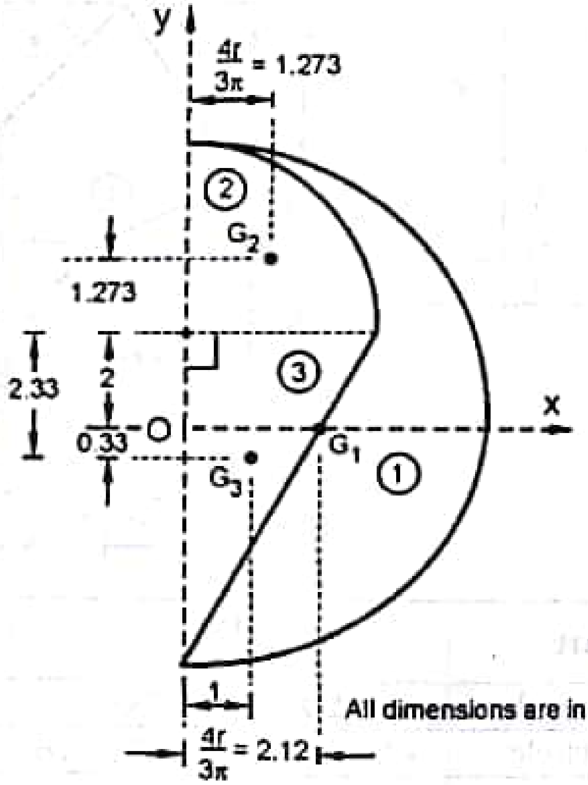
Using $\overline{X} = \frac{\Sigma A x}{\Sigma A} = \frac{3250}{260.84} = 12.46$ mm and $\overline{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{5750}{260.84} = 22.04$ mm

$\therefore \quad \overline{X}, \overline{Y} = (12.46\ ,\ 22.04)$ mm .... **Ans.**

P16. Determine the centroid of the shaded area shown.



Solution: The given shaded composite area can be obtained by taking a semicircle (Part 1), subtracting a quarter circle (Part 2) and subtracting a rt. angle triangle (Part 3).



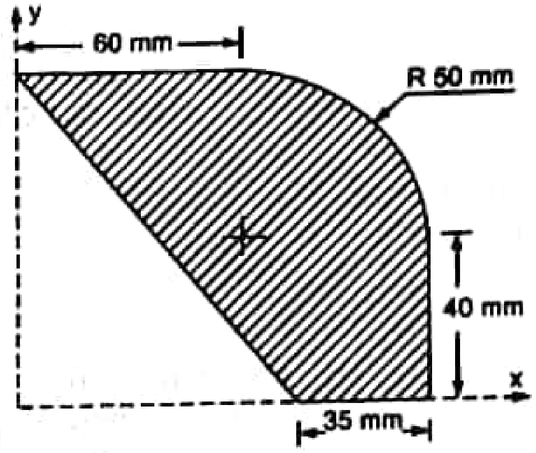
Part	Area A cm²	Co-ordinates		Ax cm³	Ay cm³
		x cm	y cm		
1. Semicircle	$(\pi \times 5^2)/2 = 39.27$	2.122	0	83.33	0
2. Qt. circle	$-(\pi \times 3^2)/4 = -7.068$	1.273	3.273	−9	−23.13
3. Rt. Triangle	$-(\frac{1}{2} \times 3 \times 7) = -10.5$	1	−0.33	−10.5	3.5
	$\Sigma A = 21.702$			$\Sigma Ax = 63.83$	$\Sigma Ay = -19.63$

Using $\overline{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{63.83}{21.702} = 2.941$ cm and $\overline{Y} = \frac{\Sigma Ay}{\Sigma A} = \frac{-19.63}{21.702} = -0.9045$ cm

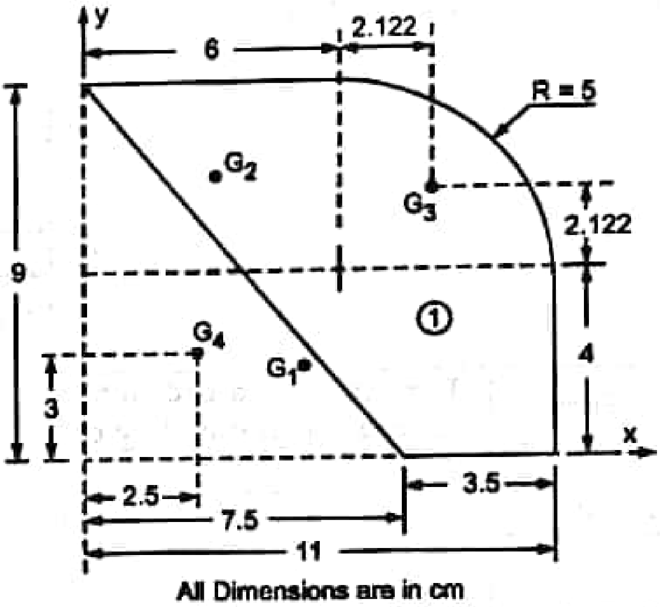
$\therefore$ $\overline{X}, \overline{Y} = (2.941, -0.9045)$ cm .... **Ans.**

P17. Determine the centre of gravity of the shaded area

(M.U May 14)



Solution: (Working in cm units). The given shaded area can be obtained by taking a rectangle of 11 cm × 4 cm (Part 1), adding another rectangle 6 cm × 5 cm (Part 2), adding a quarter circle of radius 5 cm (Part 3) and subtracting a rt. Angle triangle of base 7.5 cm ht, 9 cm (Part 4) from it.



Part	Area A cm²	Co-ordinates		A.x cm³	A.y cm³
		x cm	y cm		
1. Rectangle	$11\times4=44$	5.5	2	242	88
2. Rectangle	$6\times4=30$	3	6.5	90	195
3. Quarter Circle	$\frac{\pi\times5^2}{4}=19.63$	8.122	6.122	159.4	120.2
4. Rt. Angled Triangle	$-\frac{1}{2}\times7.5\times9$ $=-33.75$	2.5	3	– 84.37	– 101.2
	$\Sigma A=59.88$			ΣAx $=407.03$	ΣAy $=302$

Using $\overline{X}=\frac{\Sigma Ax}{\Sigma A}=\frac{407.03}{59.88}=6.797$ cm and $\overline{Y}=\frac{\Sigma Ay}{\Sigma A}=\frac{302}{59.88}=5.043$ cm

$\therefore\quad \overline{X},\overline{Y}=(6.797, 5.043)$ cm .... **Ans.**