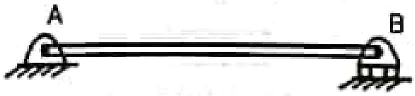
# Solutions: Chapter 3

# Coplanar Forces : Equilibrium

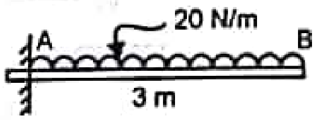
## Exercise 3.1 Equilibrium of Single Body

**P1.** Draw neat FBDs for the following supported bodies in equilibrium. Take all plane surfaces as smooth.

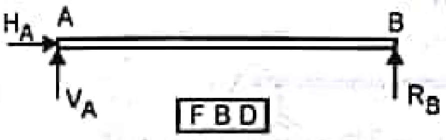



a) Beam AB

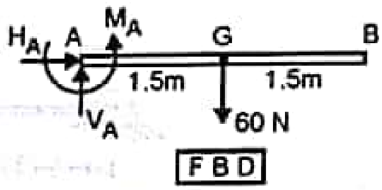

b) Beam AB

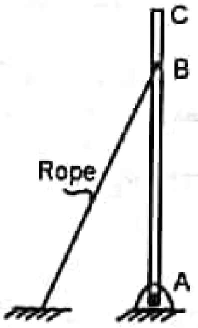


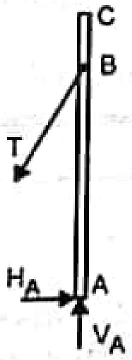

c) Fixed Beam AB



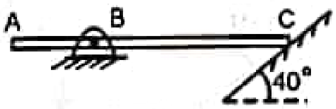




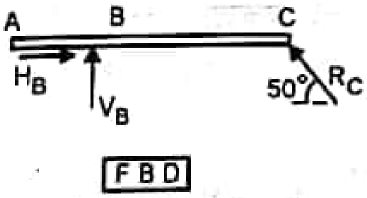



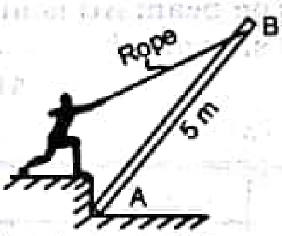




d) Pole AC

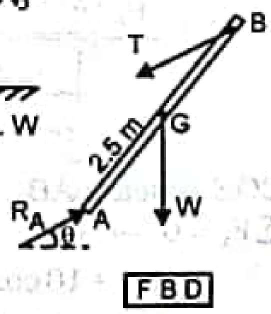


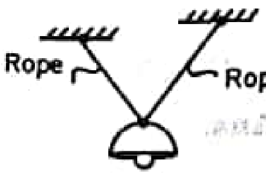

e) Beam AC



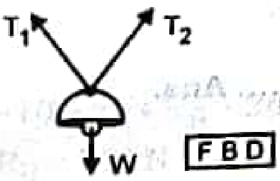



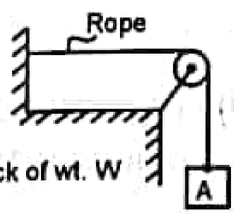

f) Rod AB of Wt. W



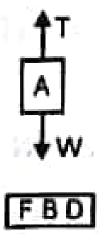



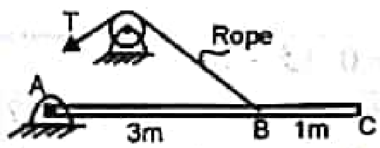

g) Lamp of wt. W



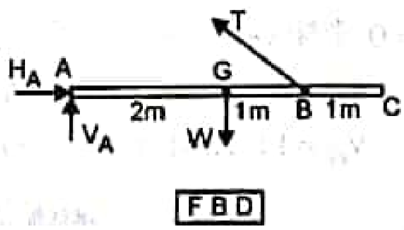



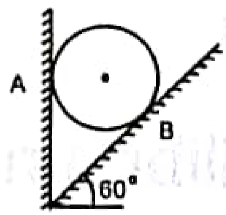

h) Block of wt. W



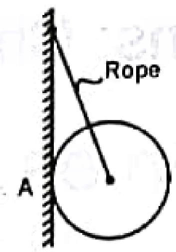




i) Beam AC of wt. W

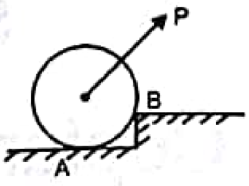




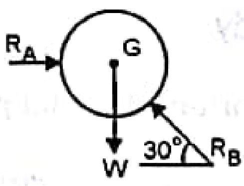

j) Sphere of wt. W



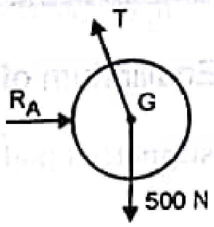

k) Sphere of wt. 500 N



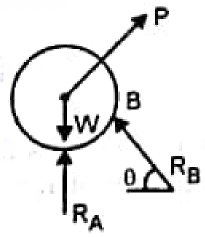

l) Wheel of wt. W




FBD

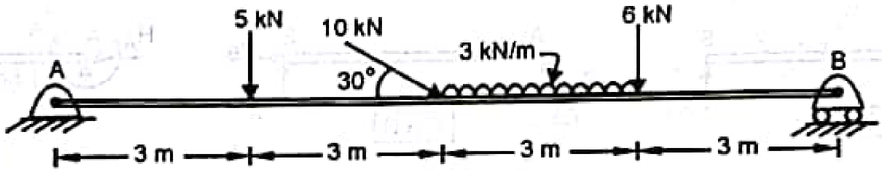



FBD

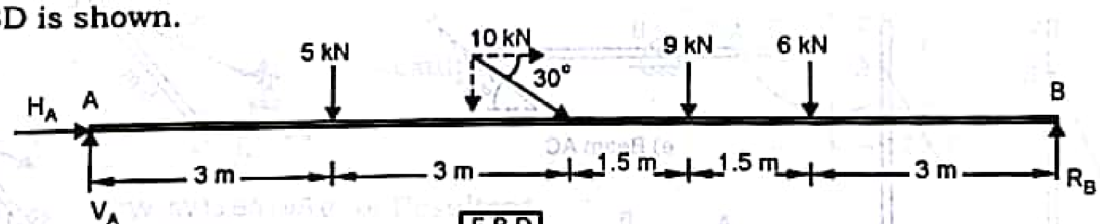



FBD

**P2.** A beam AB is loaded as shown. Find support reactions.




**Solution:** The beam AB is in equilibrium. It is supported by a hinge and roller. The FBD is shown.




FBD

COE – Beam AB

$\Sigma F_x = 0 \rightarrow +ve$

$$H_A + 10\cos 30 = 0$$

$\therefore \quad H_A = -8.66\,kN \quad$ Or $\quad H_A = 8.66\,kN \leftarrow$ ......... **Ans.**

$\Sigma M_A = 0 \circlearrowleft +ve$

$$-(5\times3)-(10\sin30\times6)-(9\times7.5)-(6\times9)+(R_B\times12)=0$$

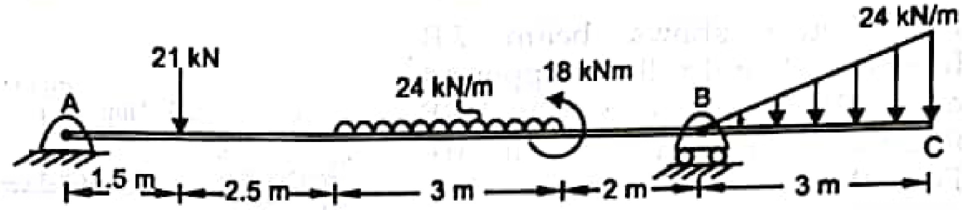
$\therefore \quad R_B = 13.875\,kN \quad$ Or $\quad R_B = 13.875\,kN \uparrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow +ve$

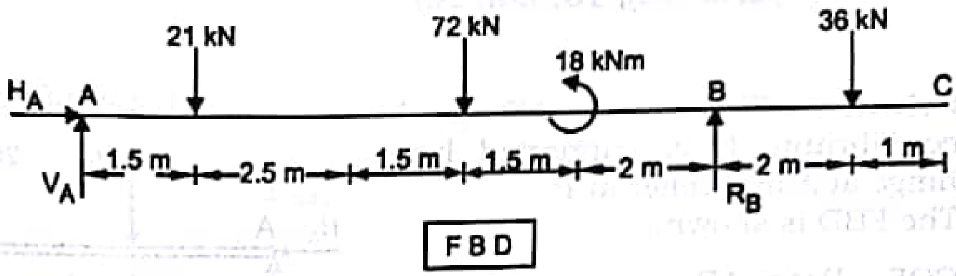
$$V_A - 5 - 10\sin30 - 9 - 6 + 13.875 = 0$$

$\therefore \quad V_A = 11.125\,kN \quad$ Or $\quad V_A = 11.125\,kN \uparrow$ ......... **Ans.**

**P3.** Determine the support reactions for the beam shown in figure.
***(KJS May 15 M.U Dec 15)***




**Solution:** The beam AC is in equilibrium, supported by a hinge at A and a roller at B. The FBD is shown.




COE – Beam AC

$$\Sigma M_A = 0 \ \circlearrowleft +ve$$

$$-(21\times1.5)-(72\times5.5)+18+(R_B\times9)-(36\times11)=0$$

$\therefore \quad R_B = 89.5$ kN $\quad$ or $\quad R_B = 89.5$ kN $\uparrow$ ......... **Ans.**

$$\Sigma F_y = 0 \ \uparrow +ve$$

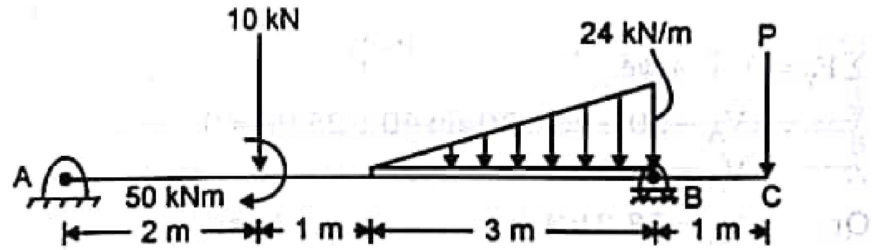
$$V_A - 21 - 72 + R_B - 36 = 0$$

$\therefore \quad V_A - 21 - 72 + 89.5 - 36 = 0$

$\therefore \quad V_A = 39.5$ kN $\quad$ or $\quad V_A = 39.5$ kN $\uparrow$ ......... **Ans.**

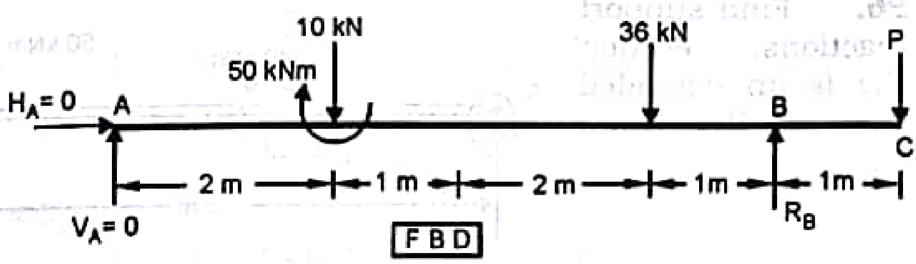
Since there is no horizontal force on the beam, $H_A = 0$ ......... **Ans.**

**P4.** Find analytically the support reaction at B and load P for the beam shown in figure if reaction at support A is zero.
***(M.U. Dec 08, 11)***




**Solution:** The beam AC is in equilibrium. It is supported by a hinge at A and roller at B. The FBD is shown. It is also given that reaction at A is zero.

i.e. $\quad H_A = V_A = 0$




COE – Beam AC

$\Sigma M_B = 0 \ \circlearrowleft +ve$

$+(10\times4)+(36\times1)-50-(P\times1)=0 \quad \therefore \quad P = 26$ kN ......... **Ans.**

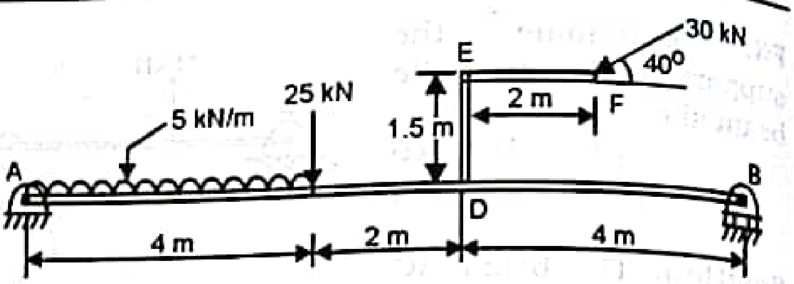
$\Sigma F_y = 0 \ \uparrow +ve$

$$-10 - 36 + R_B - 26 = 0$$

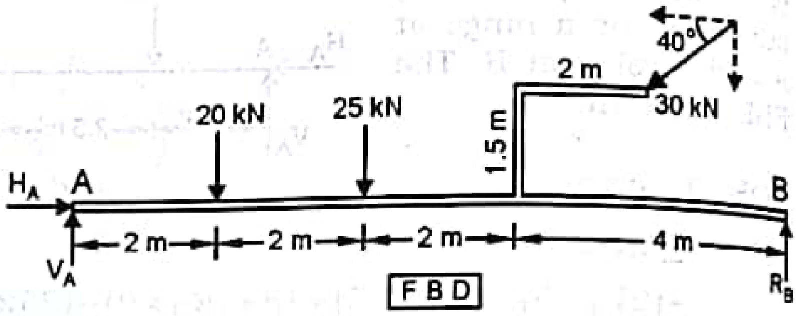
$\therefore \quad R_B = 72$ kN $\quad$ Or $\quad R_B = 72$ kN $\uparrow$ ......... **Ans.**

**P5.** Figure shows beam AB hinged at A and roller supported at B. The L shaped portion DEF is an extended part of beam AB. For the loading shown, find support reactions.

***(M.U. May 13, Dec 16)***




**Solution:** The beam AB is in equilibrium. It is supported by a hinge at A and roller at B.
The FBD is shown.




COE – Beam AB

$\Sigma M_A = 0$ ↺ + ve

$$-(20\times2)-(25\times4)-(30\sin40\times8)+(30\cos40\times1.5)+(R_B\times10)=0$$

$\therefore \quad R_B = 25.98\,\text{kN}$

Or $\quad R_B = 25.98\,\text{kN}\uparrow$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow +\text{ve}$

$H_A - 30\cos40 = 0$

$\therefore \quad H_A = 22.98\,\text{kN}$

Or $\quad H_A = 22.98\,\text{kN}\rightarrow$ ......... **Ans.**

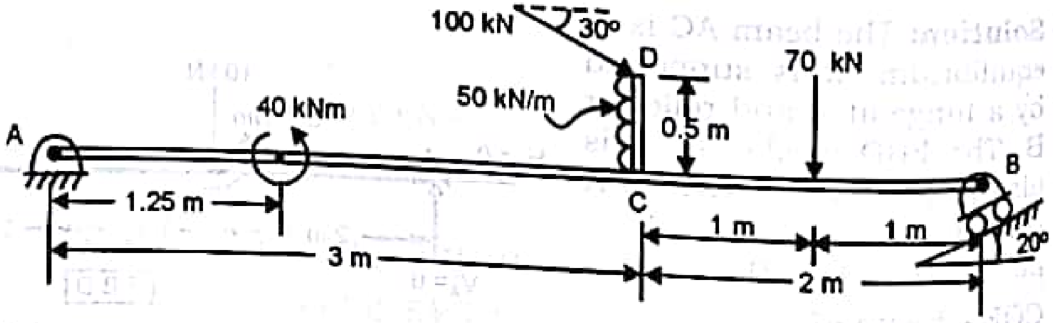
$\Sigma F_y = 0 \uparrow +\text{ve}$

$V_A - 20 - 25 - 30\sin40 + 25.98 = 0$

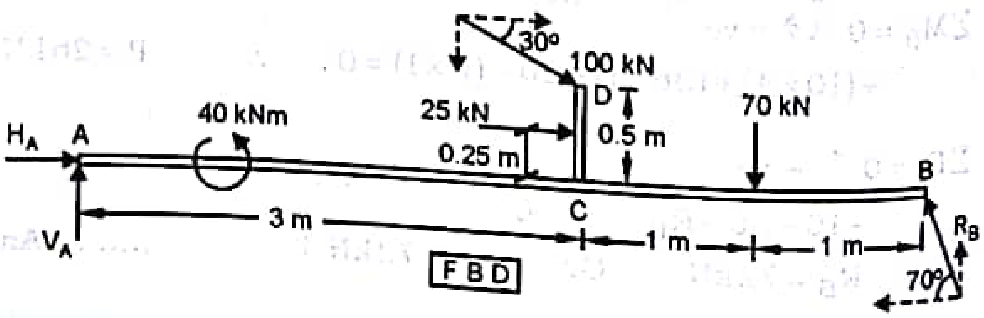
$\therefore \quad V_A = 38.3\,\text{kN}$

Or $\quad V_A = 38.3\,\text{kN}\uparrow$ ......... **Ans.**

**P6.** Find support reactions. Portion CD is an extended part of the beam AB.




**Solution:** The beam AB is in equilibrium. It is supported by a hinge at A and roller at B. The FBD is shown.



COE – Beam AB

$\Sigma M_A = 0$ ↺ + ve

$40-(100\sin 30\times 3)-(100\cos 30\times 0.5)-(25\times 0.25)-(70\times 4)+(R_B\sin 70\times 5)=0$

$\therefore \quad R_B = 93.55\,kN$

Or $\quad R_B = 93.55\,kN,\ \theta = 70^\circ$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow +ve$

$H_A + 25 + 100\cos 30 - 93.55\cos 70 = 0$

$\therefore \quad H_A = -79.6\,kN$

Or $\quad H_A = 79.6\,kN \leftarrow$ ......... **Ans.**

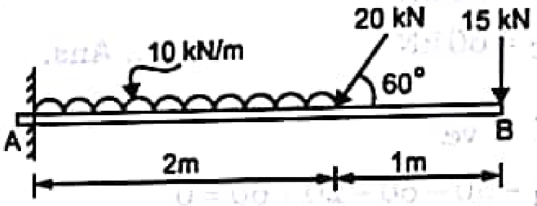
$\Sigma F_y = 0 \uparrow +ve$

$V_A - 100\sin 30 - 70 + 93.55\sin 70 = 0$

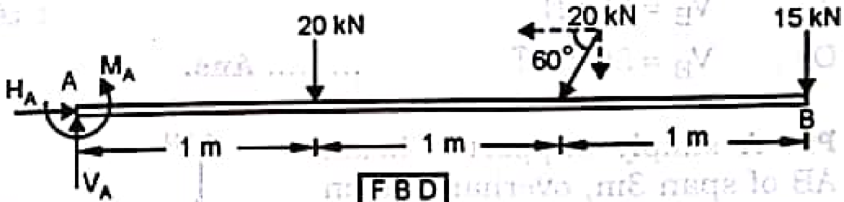
$\therefore \quad V_A = 32.09\,kN$

Or $\quad V_A = 32.09\,kN \uparrow$ ......... **Ans.**

**P7.** Determine the reactions developed in the cantilever beam as shown in figure.
*(VJTI Nov 10)*




**Solution:** The beam AB is in equilibrium. It is supported by a fixed support at A.
The FBD is shown.




COE – Beam AB

$\Sigma F_x = 0 \rightarrow +ve$

$H_A - 20\cos 60 = 0$

$\therefore \quad H_A = 10\,kN$ Or $H_A = 10\,kN \rightarrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow +ve$

$V_A - 20 - 20\sin 60 + 15 = 0$

$\therefore \quad V_A = 52.32\,kN$ Or $V_A = 52.32\,kN \uparrow$ ......... **Ans.**

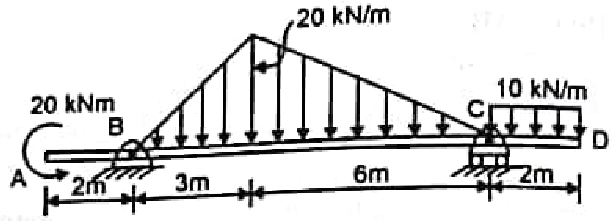
$\Sigma M_A = 0$ ↺ + ve

$+M_A - (20\times 1) - (20\sin 60\times 2) - (15\times 3) = 0$

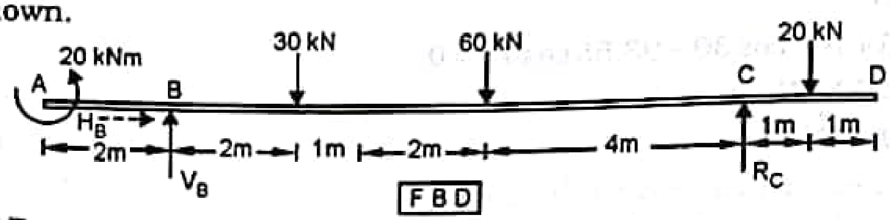
$\therefore \quad M_A = 99.64\,kNm$ Or $M_A = 99.64\,kNm$ ↺ ......... **Ans.**

**P8.** Find the reactions at the supports of the beam loaded as shown in figure.

***(M. U. Dec 09)***




**Solution:** The beam AD is in equilibrium. It is supported by hinge at B and roller at D. The FBD is shown.




COE – Beam AD

$\Sigma F_x = 0 \rightarrow +ve$

$H_B = 0$ ......... **Ans.**

$\Sigma M_B = 0 \circlearrowleft +ve$

$+20-(30\times2)-(60\times5)-(20\times10)+(R_C\times9)=0$

$\therefore \quad R_C = 60\,kN$

Or $\quad R_C = 60\,kN \uparrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow +ve$

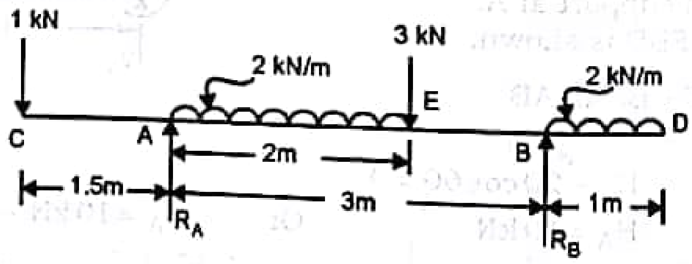
$V_B - 30 - 60 - 20 + 60 = 0$

$\therefore \quad V_B = 50\,kN$

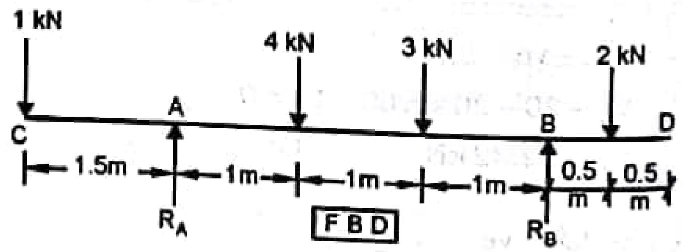
Or $\quad V_B = 50\,kN \uparrow$ ......... **Ans.**

**P9.** A simply supported beam AB of span 3m, overhanging on both sides is loaded as shown in figure. Find the support reactions.

***(VJTI May 10)***




**Solution:** The beam CD is in equilibrium. It is simply supported at A and B. The FBD is shown.




COE – Beam CD

$\Sigma M_A = 0 \circlearrowleft +ve$

$+(1\times1.5)-(4\times1)-(3\times2)-(2\times3.5)+(R_B\times3)=0$

$\therefore \quad R_B = 5.167\,kN \quad$ Or $\quad R_B = 5.167\,kN \uparrow$ ......... **Ans.**

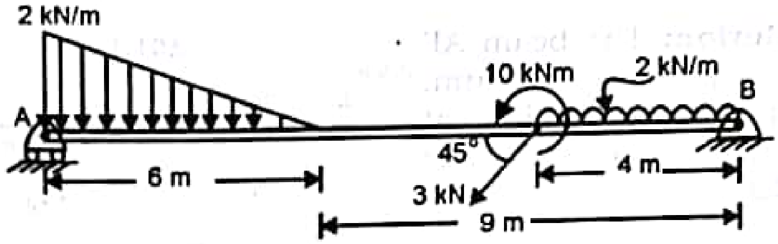
$\Sigma F_y = 0 \uparrow + ve$

$-1 + R_A - 4 - 3 + 5.167 - 2 = 0$

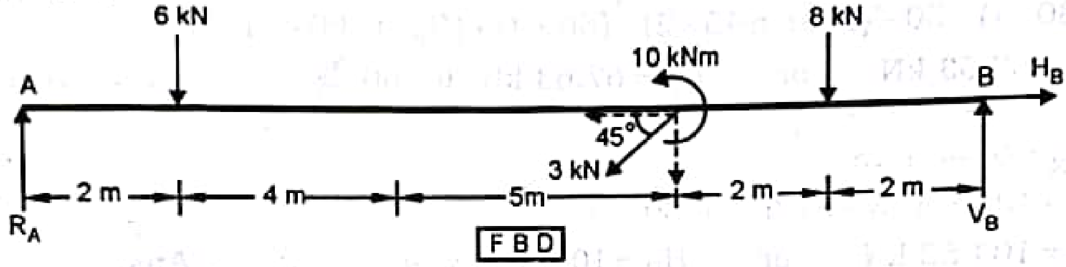
$\therefore \quad R_A = 4.833\,kN$

Or $\quad R_A = 4.833\,kN \uparrow$ ......... **Ans.**

**P10.** Find the reactions at the supports of the beam AB loaded as shown. ***(MU May 11)***




**Solution:** The beam AB is in equilibrium. It is supported by a roller at A and a hinge at B. The FBD is shown.




FBD

COE – Beam AB

$\Sigma M_A = 0 \circlearrowleft + ve$

$-(6 \times 2) - (3 \sin 45 \times 11) - (8 \times 13) + 10 + (V_B \times 15) = 0$

$\therefore \quad V_B = 8.622\,kN$

Or $\quad V_B = 8.622\,kN \uparrow$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow + ve$

$-(3 \cos 45) + H_B = 0$

$\therefore \quad H_B = 2.12\,kN$

Or $\quad H_B = 2.12\,kN \rightarrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow + ve$

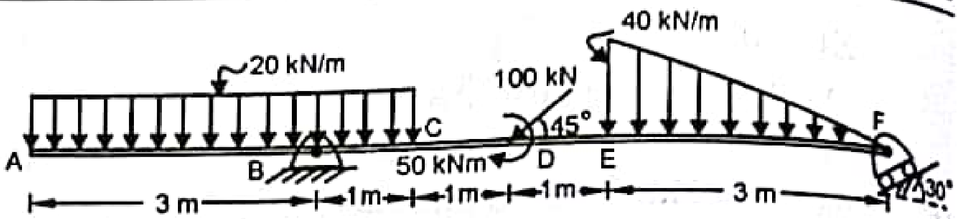
$R_A - 6 - 3 \sin 45 - 8 + 8.622 = 0$

$\therefore \quad R_A = 7.5\,kN$

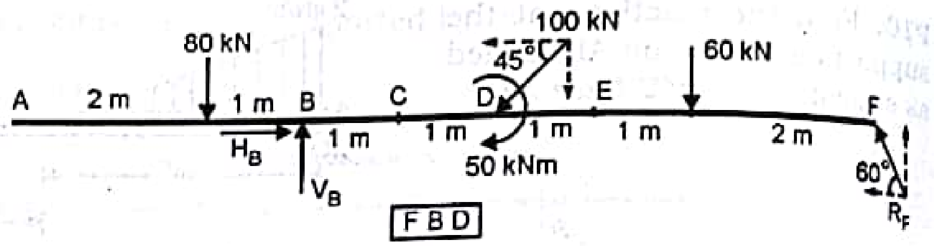
Or $\quad R_A = 7.5\,kN \uparrow$ ......... **Ans.**

**P11.** Find the support reactions for the beam loaded and supported as shown in figure.

***(M.U. Dec 12)***




**Solution:** The beam AF is in equilibrium, supported by a hinge at B and a roller at F. The FBD is shown.




COE – Beam AF

$\Sigma M_B = 0$ ↺ + ve

$$+(80\times1)-50-(100\sin45\times2)-(60\times4)+(R_F\sin60\times6)=0$$

∴ $R_F = 67.63$ kN or $R_F = 67.63$ kN $\theta = 60°$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow$ + ve

$$H_B - 100\cos45 - 67.63\cos60 = 0$$

∴ $H_B = 104.52$ kN or $H_B = 104.52$ kN → ......... **Ans.**

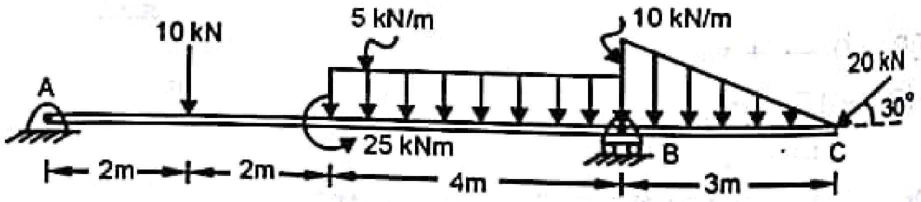
$\Sigma F_y = 0 \uparrow$ + ve

$$V_B - 80 - 100\sin45 - 60 + 67.63\sin60 = 0$$

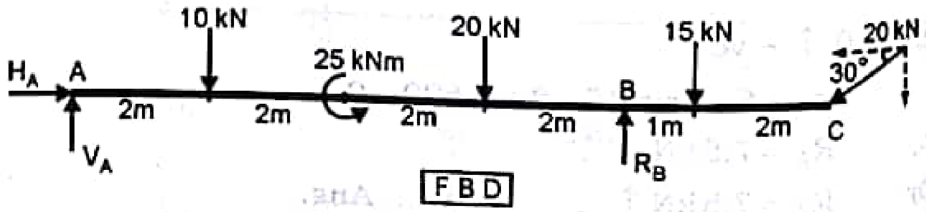
∴ $V_B = 152.14$ kN or $V_B = 152.14$ kN ↑ ......... **Ans.**

**P12.** Determine support reactions for the beam shown.

***(VJTI Nov 12)***




**Solution:** The beam AC is in equilibrium. It is supported by a hinge at A and a roller at B. The FBD is shown.




COE – Beam AC

$\Sigma M_A = 0$ ↺ + ve

$$-(10\times2)+25-(20\times6)+(R_B\times8)-(15\times9)-(20\sin30\times11)=0$$

∴ $R_B = 45$ kN or $R_B = 45$ kN ↑ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow +ve$

$H_A - 20\cos 30 = 0$

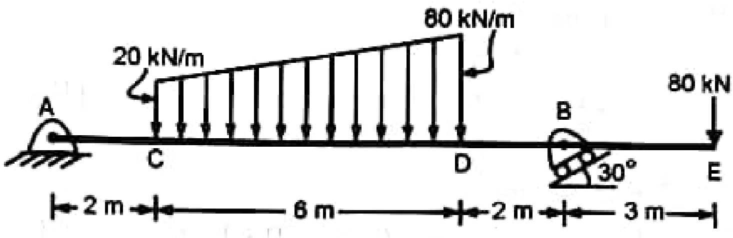
$\therefore \quad H_A = 17.32\ kN \quad$ or $\quad H_A = 17.32\ kN \rightarrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow +ve$

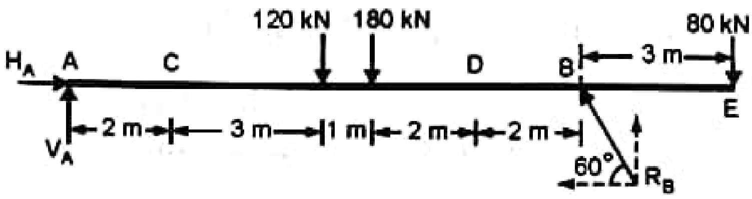
$V_A - 10 - 20 + 45 - 15 - 20\sin 30 = 0$

$\therefore \quad V_A = 10\ kN \quad$ or $\quad V_A = 10\ kN \uparrow$ ......... **Ans.**

**P13.** Find support reactions at A and B for the beam shown in figure. ***(M.U. Dec 13)***




**Solution:** The beam AB is in equilibrium. It is supported by a hinge and roller.
The beam supports a point load of 80 kN and a trapezoidal load which varies from 20 kN/m to 80 kN/m.
The trapezoidal load can be converted into two point loads $L_1$ and $L_2$.




$L_1 = 20 \times 6 = 120\ kN \downarrow$ ... acting at midpoint of CD.

$L_2 = \frac{1}{2} \times (80 - 20) \times 6 = 180\ kN \downarrow$ ... acting at 2 m to the left from D.

COE – Beam AB

$\Sigma M_A = 0 \circlearrowleft +ve$

$-(120 \times 5) - (180 \times 6) + (R_B \sin 60 \times 10) - (80 \times 13) = 0$

$\therefore \quad R_B = 314.08\ kN \quad$ Or $\quad R_B = 314.08\ kN, \theta = 60°$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow +ve$

$H_A - R_B \cos 60 = 0$

$\therefore \quad H_A - 314.1\cos 60 = 0$

$\therefore \quad H_A = 157.04 kN \quad$ Or $\quad H_A = 157.04 kN \rightarrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow +ve$

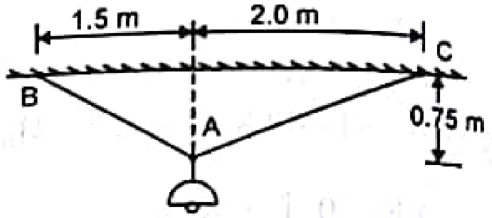
$V_A - 120 - 180 + R_B \sin 60 - 80 = 0$

$\therefore \quad V_A - 120 - 180 + 314.08 \sin 60 - 80 = 0$

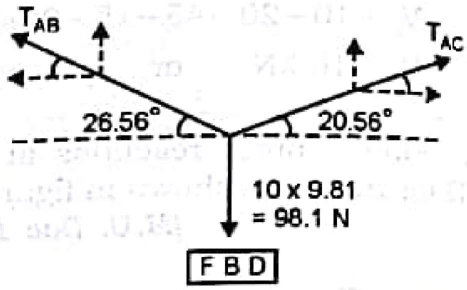
$\therefore \quad V_A = 108\ kN \quad$ Or $\quad V_A = 108\ kN \uparrow$ ......... **Ans.**

**P14.** The figure shows a 10 kg lamp supported by two cables AB and AC. Find the tension in each cable

***(VJTI May 08)***




**Solution:** The lamp is in equilibrium. It is supported by two ropes AB and AC. The FBD is shown.




COE – Lamp

$\Sigma F_x = 0 \rightarrow +ve$

$-T_{AB}\cos 26.56 + T_{AC}\cos 20.56 = 0$ ......... (1)

$\Sigma F_y = 0 \uparrow +ve$

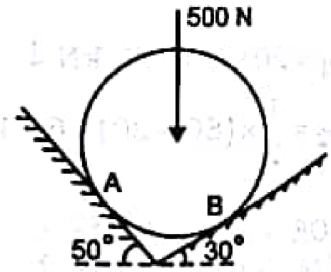
$T_{AB}\sin 26.56 + T_{AC}\sin 20.56 - 98.1 = 0$ ......... (2)

Solving equations (1) and (2), we get

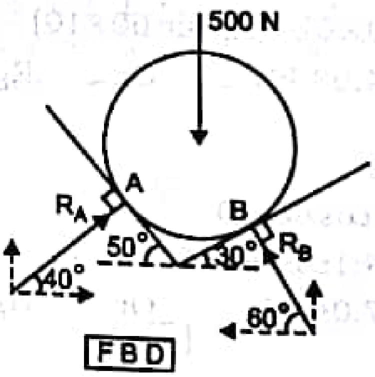
$T_{AB} = 125.3$ N ......... **Ans.**

$T_{AC} = 119.7$ N ......... **Ans.**

**P15.** A cylinder of 500 N weight is kept between two smooth inclined planes as shown. Find reactions at contact points A and B. ***(M.U Dec 14)***




**Solution:** The cylinder is in equilibrium. It is supported by two smooth surfaces. The FBD is shown.




COE - Cylinder

$\Sigma F_x = 0 \rightarrow +ve$

$R_A\cos 40 - R_B\cos 60 = 0$ ......... (1)

$\Sigma F_y = 0 \uparrow +ve$

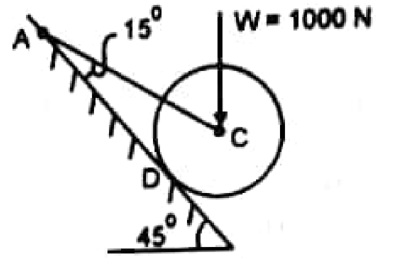
$R_A\sin 40 + R_B\sin 60 - 500 = 0$ ......... (2)

Solving equations (1) and (2), we get

$R_A = 253.86$ N, $\theta = 40°$ ......... **Ans.**

$R_B = 388.93$ N, $\theta = 60°$ ......... **Ans.**

**P16.** A roller of weight W = 1000 N rest on a smooth incline plane. It is kept from rolling down the plane by a string AC. Find the tension in the string and reaction at the point of contact D. ***(M.U. Dec 08, VJTI May 10)***

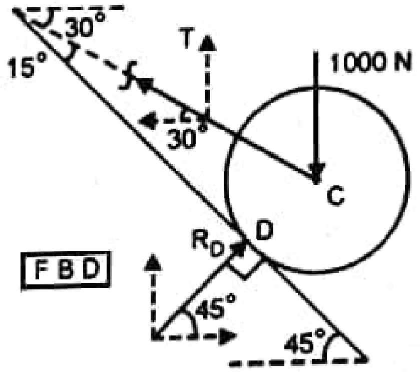



**Solution:** The sphere is in equilibrium. It is supported by a smooth surfaces at D and by a rope. The FBD is shown.

COE – Sphere

$\Sigma F_x = 0 \rightarrow +ve$

$R_D \cos 45 - T\cos 30 = 0$ ......... (1)




$\Sigma F_y = 0 \uparrow +ve$

$R_D \sin 45 + T\sin 30 - 1000 = 0$ ......... (2)

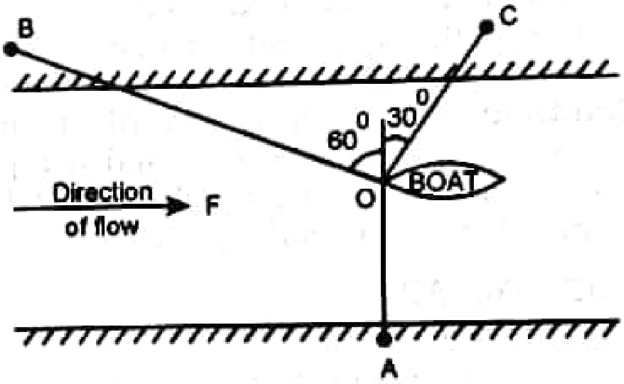
Solving equations (1) and (2), we get

$R_D = 896.57$ N, $\theta = 45°$ and $T = 732.05$ N ......... **Ans.**

**P17.** A small boat is held in static by means of three inextensible taut ropes OA, OB and OC as shown. The water in the river exerts a force on the boat in the direction of flow.

a) If the tension in OA and OB are 1 kN and 0.6 kN respectively, determine the force F, exerted by the flow on the boat and the tension in rope OC.

b) If rope OC breaks, will the boat remain in equilibrium? What would then be the new values of tension in ropes OA and OB after OC breaks?




**Solution:** a) The boat is in equilibrium. It is supported by three ropes OA, OB and OC. Let F be the force on the boat exerted by the flowing river. Also given $T_{OA}$ = 1 kN and $T_{OB}$ = 0.6 kN. The FBD is shown.

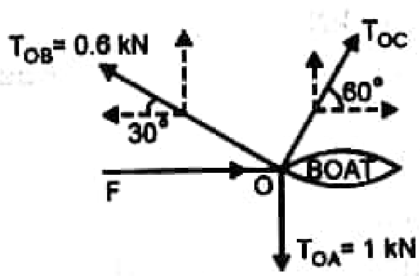
COE – Boat

$\Sigma F_y = 0 \uparrow +ve$

$T_{OC} \sin 60 + T_{OB} \sin 30 - T_{OA} = 0$

$\therefore$ $T_{OC} \sin 60 + 0.6 \times \sin 30 - 1 = 0$

$\therefore$ $T_{OC} = 0.8083$ kN ......... **Ans.**




F B D - Case (a)

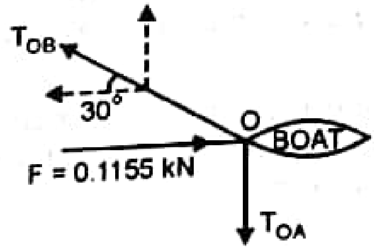
$\Sigma F_x = 0 \rightarrow +ve$

$T_{OC} \cos 60 - T_{OB} \cos 30 + F = 0$

$\therefore$ $0.8083 \cos 60 - 0.6 \times \cos 30 + F = 0$

$\therefore$ $F = 0.1155$ kN ......... **Ans.**

b) If rope OC breaks it implies $T_{OC} = 0$. The boat will still remain in equilibrium because three concurrent forces $T_{OA}$, $T_{OB}$ and F can keep the boat in equilibrium as per the concept of three force body. The new FBD is shown.




F B D - Case (b)

COE – Boat

$$\Sigma F_x = 0 \rightarrow +ve$$

$$0.1155 - T_{OB}\cos 30 = 0$$

$$\therefore \quad T_{OB} = 0.1333\,\text{kN} \quad \text{......... Ans.}$$

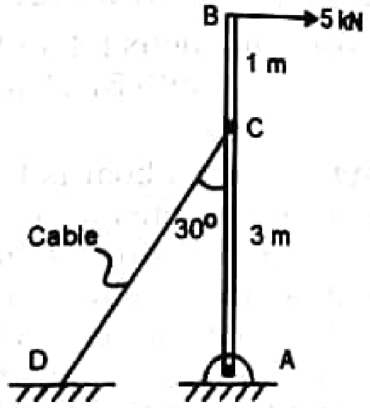
$$\Sigma F_y = 0 \uparrow +ve$$

$$T_{OB}\sin 30 - T_{OA} = 0$$

$$\therefore \quad 0.1333\sin 30 - T_{OA} = 0$$

$$\therefore \quad T_{OA} = 0.0667\ \text{kN} \quad \text{......... Ans.}$$

**P18.** A straight vertical mast 4 m long and self weight 1200 N is pinned to the ground and stayed by means of a cable at a distance of 3 m from the bottom as shown. If a horizontal force of 5 kN acts at the top, determine the tension in the cable and reaction at the hinge.



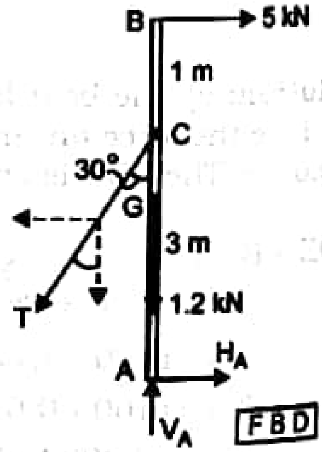

**Solution:** The vertical pole AB is in equilibrium. It is supported by a hinge at A and a rope at C. The FBD is shown. Note that the weight of 1.2 kN acts through the centre of gravity G of the pole.

COE – Pole AB

$$\Sigma M_A = 0 \circlearrowleft +ve$$

$$-(5\times 4) + (T\sin 30\times 3) = 0$$

$$\therefore \quad T = 13.33\ \text{kN} \quad \text{......... Ans.}$$




FBD

$$\Sigma F_x = 0 \rightarrow +ve$$

$$H_A + 5 - T\sin 30 = 0$$

$$\therefore \quad H_A + 5 - 13.33\sin 30 = 0$$

$$\therefore \quad H_A = 1.66\ \text{kN}$$

$$\text{or} \quad H_A = 1.66\ \text{kN} \rightarrow \quad \text{......... Ans.}$$

$$\Sigma F_y = 0 \uparrow +ve$$

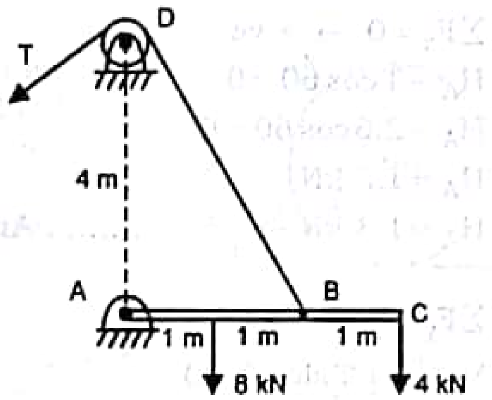
$$V_A - T\cos 30 - 1.2 = 0$$

$$\therefore \quad V_A - 13.33\cos 30 - 1.2 = 0$$

$$\therefore \quad V_A = 12.74\ \text{kN}$$

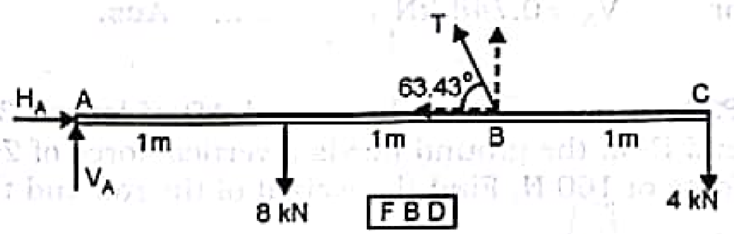
$$\text{or} \quad V_A = 12.74\ \text{kN} \uparrow \quad \text{......... Ans.}$$

**P19.** Determine the tension T in the cable and the reaction at pin support A for the beam loaded as shown. Neglect weight of the beam and the size of the pulley.




**Solution:** The beam AC is in equilibrium. It is supported by a hinge at A and a rope at B. Since the rope passes over a smooth pulley at D, the tension T remains the same throughout the rope. The FBD is shown.

COE – Beam AC




$$\Sigma M_A = 0 \quad \circlearrowleft + ve$$

$$-(8\times1)-(4\times3)+(T\sin 63.43\times2)=0$$

$\therefore \quad T = 11.18\ kN$ ......... **Ans.**

$$\Sigma F_x = 0 \quad \rightarrow + ve$$

$$H_A - T\cos 63.43 = 0$$

$\therefore \quad H_A - 11.18\cos 63.43 = 0$

$\therefore \quad H_A = 5\ kN$

or $\quad H_A = 5\ kN \rightarrow$ ......... **Ans.**

$$\Sigma F_y = 0 \quad \uparrow + ve$$

$$V_A - 8 - 4 + T\sin 63.43 = 0$$

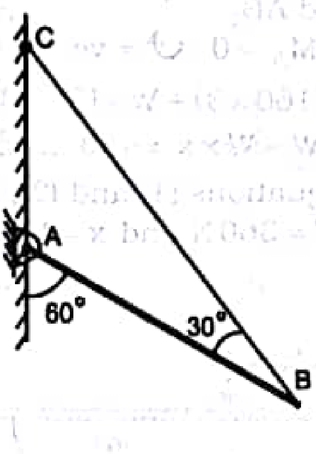
$\therefore \quad V_A - 12 + 11.18\sin 63.43 = 0$

$\therefore \quad V_A = 2\ kN$

or $\quad V_A = 2\ kN \uparrow$ ......... **Ans.**

**P20.** A prismatic bar AB of length 6 m and weight 3 kN is hinged to a wall and supported by a cable BC. Find hinge reaction and tension in cable BC.

*(MU Dec 15)*




**Solution:** COE – System

$$\Sigma M_A = 0 \quad \circlearrowleft + ve$$

$$-(3\times2.6)-(T\cos 60\times3)+(T\sin 60\times5.2)=0$$

$\therefore \quad T = 2.6\ kN$ ......... **Ans.**

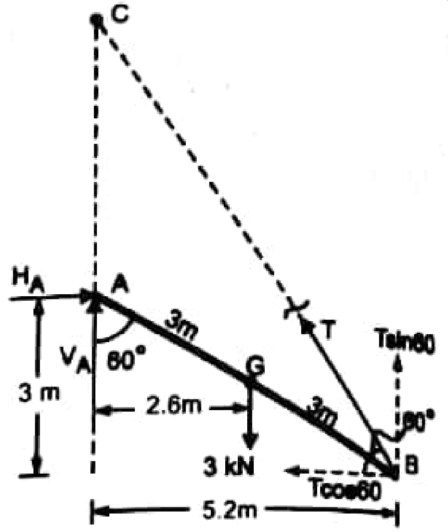
$$\Sigma F_x = 0 \rightarrow +\text{ ve}$$

$$H_A - T\cos 60 = 0$$

$\therefore$ $$H_A - 2.6\cos 60 = 0$$

$\therefore$ $$H_A = 1.3\text{ kN}$$

or $$H_A = 1.3\text{ kN} \rightarrow$$ ......... **Ans.**




$$\Sigma F_y = 0 \uparrow +\text{ ve}$$

$$V_A - 3 + T\sin 60 = 0$$

$$V_A - 3 + 2.6\sin 60 = 0$$

$\therefore$ $$V_A = 0.748\text{ kN}$$

or $$V_A = 0.748\text{ kN} \uparrow$$ ......... **Ans.**

**P21.** A non uniform heavy rod AB of length 3 m lies on horizontal ground. To lift the end B off the ground needs a vertical force of 200 N. To lift A end off the ground needs a force of 160 N. Find the weight of the rod and the position of centre of mass.

***(M.U. May 08)***

**Solution:** The non-uniform rod AB is in equilibrium. Let W be the weight of the rod acting through its centre of gravity G. located at some distance x from A.

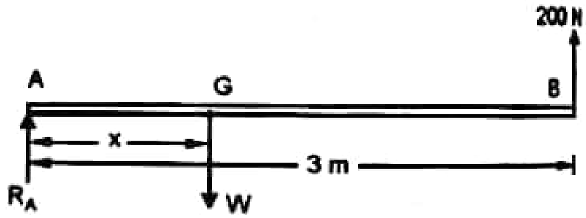
Case (a): Rod is just lifted by 200 N force applied at B. The rod is now supported only at end A by reaction force $R_A$.

COE – Rod AB

$$\Sigma M_A = 0 \circlearrowleft +\text{ ve}$$

$$+(200 \times 3) - W \times x = 0$$

$\therefore$ $$W \times x = 600$$ ......... (1)




Case (a) End B is lifted

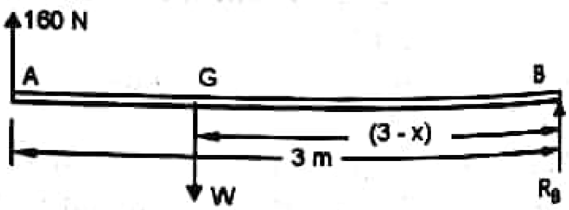
Case (b) Rod is just lifted by 160 N applied at A. The rod is now supported only at end B by reaction force $R_B$.

COE – Rod AB

$$\Sigma M_A = 0 \circlearrowleft +\text{ ve}$$

$$-(160 \times 3) + W \times (3 - x) = 0$$

$\therefore$ $$3W - W \times x = 480$$ ......... (2)

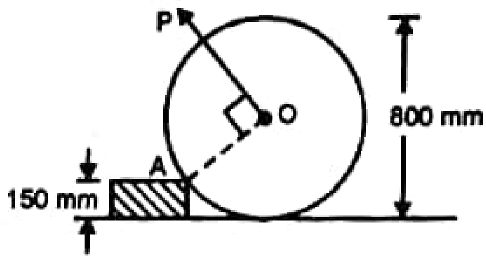



Case (b) End A is lifted

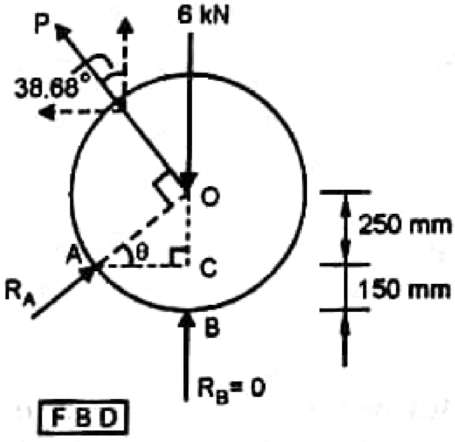
Solving equations (1) and (2), we get

$$W = 360\text{ N and } x = 1.667\text{ m}$$ ......... **Ans.**

**P22.** (a) A uniform wheel of 800 mm diameter, weighing 6 kN rests against a rigid rectangular block of 150 mm height as shown in figure. Find the least pull, through the center of the wheel, required just to turn the wheel over the edge A of the block. Also find the reaction on the block. Take all the surface to be smooth. ***(VJTI Nov 09)***




**Solution:** The wheel is on the verge of turning over the edge A, but is still in equilibrium. It is supported by an edge at A and a smooth surface at ground B. Note the edge at A offers a reaction force $R_A$ at some angle θ as shown. The smooth surface at B offers reaction force $R_B$. But $R_B = 0$ since the wheel would lose contact with the surface at B as it turns over the edge A. The FBD is shown.




COE – Wheel

$$\Sigma M_A = 0 \quad \circlearrowleft + ve$$

$$-(6 \times 312.25) + (P \times 400) = 0$$

$\therefore$ $P = 4.684$ kN ......... **Ans.**

$$\Sigma F_x = 0 \rightarrow + ve$$

$$R_A \cos\theta - P\sin 38.68 = 0$$

$\therefore$ $R_A \cos\theta - 4.684 \sin 38.68 = 0$

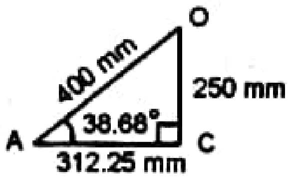
$\therefore$ $R_A \cos\theta = 2.927$ ......... (1)

$$\Sigma F_y = 0 \uparrow + ve$$

$$R_A \sin\theta + P\cos 38.68 - 6 = 0$$

$\therefore$ $R_A \sin\theta + 4.684 \cos 38.68 - 6 = 0$

$\therefore$ $R_A \sin\theta = 2.343$ ......... (2)

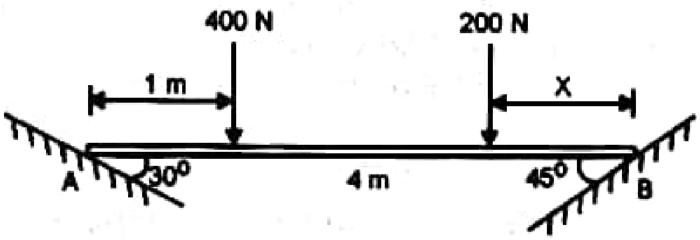



Triangle AOC

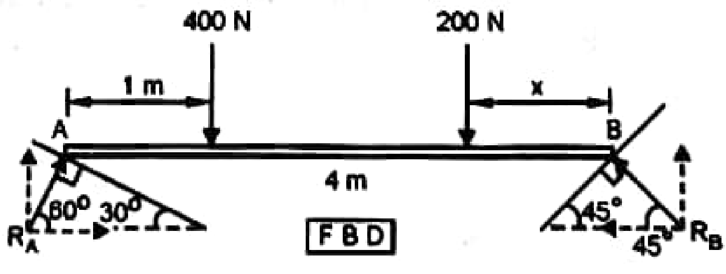
Solving equations (1) and (2), we get

$R_A = 3.749$ kN, $\theta = 38.68°$ ......... **Ans.**

**P23.** A weightless bar is placed in a horizontal position on the smooth inclines as shown. Find x at which the 200 N force should be placed from point B to keep the bar horizontal.




**Solution:** The rod AB is in equilibrium. It is supported by two smooth surfaces at A and B. The FBD is shown.




COE – Rod AB

$$\Sigma F_x = 0 \rightarrow + ve$$

$$R_A \cos 60 - R_B \cos 45 = 0 \quad ......... (1)$$

$$\Sigma F_y = 0 \uparrow + ve$$

$$R_A \sin 60 + R_B \sin 45 - 400 - 200 = 0 \quad ......... (2)$$

Solving equations (1) and (2), we get $R_A = 439.2$ N and $R_B = 310.58$ N.

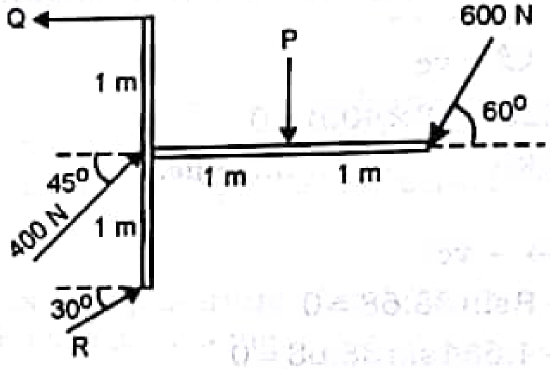
$$\Sigma M_B = 0 \circlearrowleft + ve$$

$$-(R_A \sin 60 \times 4) + (400 \times 3) + (200 \times x) = 0$$

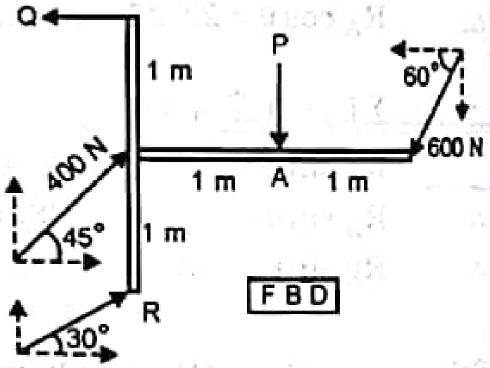
$\therefore$ $$-439.2 \sin 60 \times 4 + 1200 + 200x = 0$$

$\therefore$ $x = 1.607$ m ......... **Ans.**

**P24.** The force system shown has neither a resultant force nor a resultant couple.
Find magnitude of forces P, Q and R.
Hint: The system is in equilibrium.




**Solution:** Since the resultant of the force system is zero, implies it is in equilibrium.




COE – System

$$\Sigma F_x = 0 \rightarrow + ve$$

$$-Q + R \cos 30 + 400 \cos 45 - 600 \cos 60 = 0$$

$\therefore$ $$-Q + R \cos 30 = 17.157 \quad ......... (1)$$

$$\Sigma M_A = 0 \circlearrowleft + ve$$

$$+(Q \times 1) + (R \cos 30 \times 1) - (R \sin 30 \times 1) - (400 \sin 45 \times 1) - (600 \sin 60 \times 1) = 0$$

$\therefore$ $$Q + 0.366R = 802.46 \quad ......... (2)$$

On solving (1) and (2) we get
$Q = 558.97$ N and $R = 665.26$ N ......... **Ans.**

$$\Sigma F_y = 0 \uparrow + ve$$

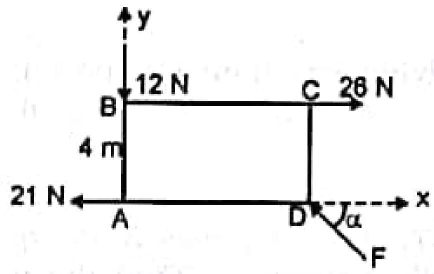
$$-P + R \sin 30 + 400 \sin 45 - 600 \sin 60 = 0$$

$\therefore$ $$-P + 665.26 \sin 30 + 400 \sin 45 - 600 \sin 60 = 0$$

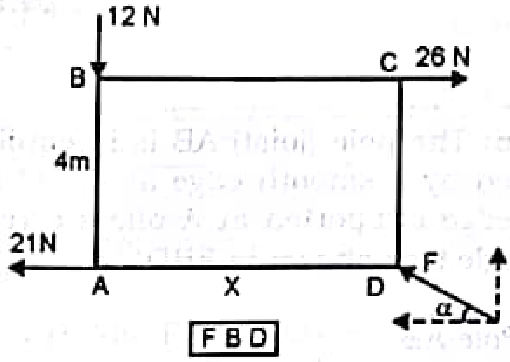
$\therefore$ $P = 95.86$ N ......... **Ans.**

**P25.** Forces act on the plate ABCD as shown in figure. The distance AB is 4 m. Given that the plate is in equilibrium find.
(i) force F (ii) angle $\alpha$ and (iii) the distance AD

***(M.U. May 08)***




**Solution:** The plate is in equilibrium.
Let distance AD = x
COE – Plate




$$\Sigma F_x = 0 \rightarrow +ve$$

$$-F\cos\alpha - 21 + 26 = 0$$

$\therefore$ $F\cos\alpha = 5$ ......... (1)

$$\Sigma F_y = 0 \uparrow +ve$$

$$F\sin\alpha - 12 = 0$$

$\therefore$ $F\sin\alpha = 12$ ......... (2)

Solving (1) and (2) we get

$F = 13$ N and $\alpha = 67.38°$ ......... **Ans.**

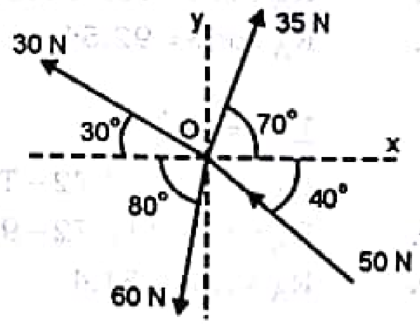
$$\Sigma M_A = 0 \circlearrowleft +ve$$

$$+(F\sin\alpha \times x) - 26\times 4 = 0$$

$\therefore$ $13\sin 67.38 \times x - 104 = 0$

or $x = 8.66$ m ......... **Ans.**

**P26.** Determine the equilibrant of the force system shown in figure. ***(SPCE Nov 12)***




**Solution:** Let E be the equilibrant force acting at some angle θ in the first quadrant acting as shown.
Since the concurrent system is in equilibrium,

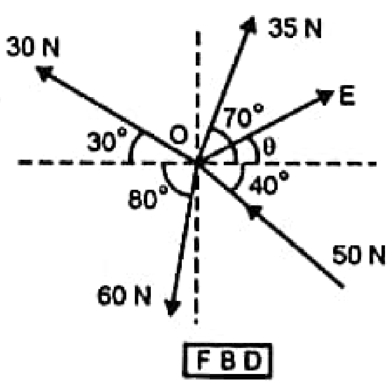
Applying COE

$$\Sigma F_x = 0 \rightarrow +ve$$

$$E\cos\theta + 35\cos 70 - 30\cos 30 - 60\cos 80 - 50\cos 40 = 0$$

$\therefore$ $E\cos\theta = 62.73$ ......... (1)

or $E\cos\theta = 62.73$ N$\rightarrow$




$$\Sigma F_y = 0 \uparrow +ve$$

$$E\sin\theta + 35\sin 70 + 30\sin 30 - 60\sin 80 + 50\sin 40 = 0$$

$\therefore$ $E\sin\theta = -20.94$ ......... (1)

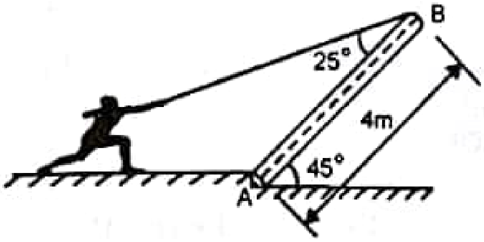
or $E\sin\theta = 20.94$ N $\downarrow$

Solving equations (1) and (2), we get

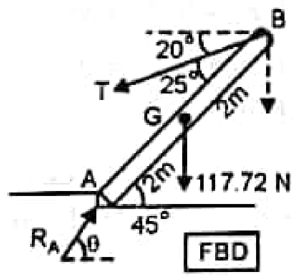
$E = 66.13$ N and $\theta = 18.46°$ ......... **Ans.**

**P27.** A man raises a 12 kg joist of length 4m by pulling the rope. Find the tension in the rope and the reaction at A.

***(M.U Dec 2010)***




**Solution:** The pole (joist) AB is in equilibrium. It is supported by a smooth edge at A and by a rope at B. The edge supported at A offers a reaction $R_A$ at some angle θ as shown in FBD.




COE – Pole AB

$\Sigma M_A = 0$ ↺ + ve

$-(117.72 \times 2\cos 45) - (T\sin 20 \times 4\cos 45) + (T\cos 20 \times 4\sin 45) = 0$

∴ $T = 98.48$ N ......... **Ans.**

$\Sigma F_x = 0 \rightarrow$ + ve

$R_A \cos\theta - T\cos 20 = 0$

∴ $R_A \cos\theta - 98.48\cos 20 = 0$

∴ $R_A \cos\theta = 92.54$ ......... (1)

$\Sigma F_y = 0 \uparrow$ + ve

$R_A \sin\theta - 117.72 - T\sin 20 = 0$

∴ $R_A \sin\theta - 117.72 - 98.48\sin 20 = 0$

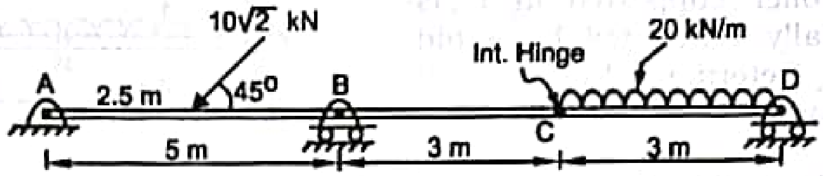
∴ $R_A \sin\theta = 151.4$ ......... (2)

Solving equations (1) and (2), we get

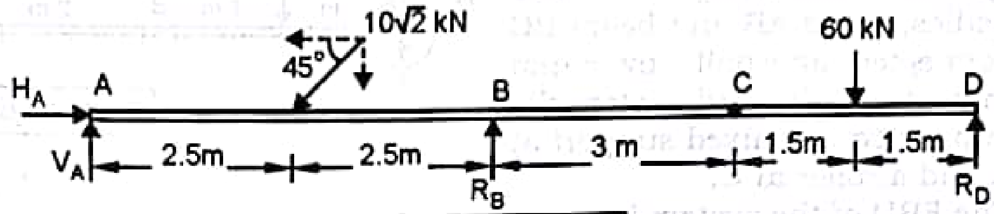
$R_A = 177.4$ N, $\theta = 58.56°$ ......... **Ans.**

## Exercise 3.2 Equilibrium of Connected Bodies

**P1.** A two span beam ABCD is loaded as shown. Calculate support reaction.

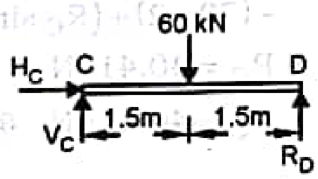



**Solution:** This is a connected system consisting of two bodies, beam AC and beam CD connected internally by a pin (hinge) at C and externally supported by a hinge at A and two rollers one each at B and D. The FBD of the system is shown.



F B D of System

Isolating beam CD from the system as shown.



F B D of beam CD

COE– Beam CD

$\Sigma M_C = 0$ ↺ + ve

$+(R_D \times 3) - (60 \times 1.5) = 0$

$\therefore \quad R_D = 30\,\text{kN} \quad \text{Or} \quad R_D = 30\,\text{kN}\uparrow$ ......... **Ans.**

COE– System

$\Sigma M_A = 0$ ↺ + ve

$-(10\sqrt{2}\sin 45 \times 2.5) + (R_B \times 5)$

$-(60 \times 9.5) + (30 \times 11) = 0$

$\therefore \quad R_B = 53\,\text{kN} \quad \text{Or} \quad R_B = 53\,\text{kN}\uparrow$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow + \text{ve}$

$H_A - 10\sqrt{2}\cos 45 = 0$

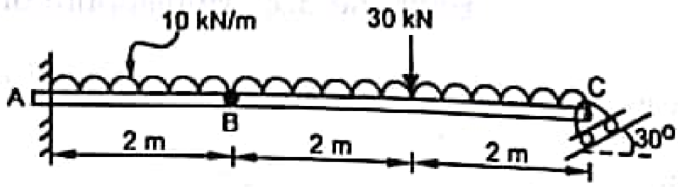
$\therefore \quad H_A = 10\text{ kN} \quad \text{Or} \quad H_A = 10\text{ kN} \rightarrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow + \text{ve}$

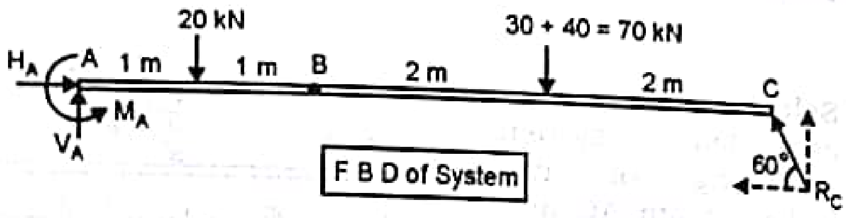
$V_A - 10\sqrt{2}\sin 45 + 53 - 60 + 30 = 0$

$\therefore \quad V_A = -13\text{ kN} \quad \text{Or} \quad V_A = 13\text{ kN}\downarrow$ ......... **Ans.**

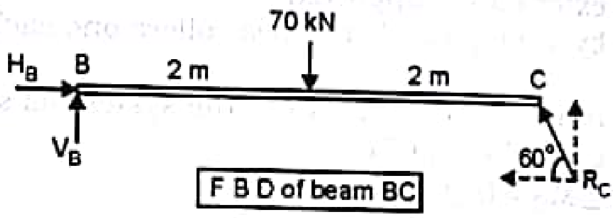
**P2.** A beam ABC, fixed at A and roller supported at C is internally connected by a pin at B. Determine the support reactions.




**Solution:** This is a connected system consisting of two bodies, beam AB and beam BC connected internally by a pin (hinge) at B and externally supported by a fixed support at A and a roller at C.
The FBD of the system is shown.




Isolating beam BC from the system as shown.
COE– Beam BC




$\Sigma M_B = 0 \quad \circlearrowleft + ve$

$-(70 \times 2) + (R_C \sin 60 \times 4) = 0$

$\therefore \quad R_C = 40.41 \text{kN}$

Or $\quad R_C = 40.41 \text{kN} \quad \theta = 60°$ ......... **Ans.**

COE – System

$\Sigma M_A = 0 \quad \circlearrowleft + ve$

$M_A - (20 \times 1) - (70 \times 4) + (40.41 \sin 60 \times 6) = 0$

$\therefore \quad M_A = 90 \text{kNm} \qquad$ Or $\qquad M_A = 90 \text{kNm} \circlearrowleft$ ......... **Ans.**

$\Sigma F_x = 0 \rightarrow + ve$

$H_A - 40.41 \cos 60 = 0$

$\therefore \quad H_A = 20.2 \text{ kN} \qquad$ Or $\qquad H_A = 20.2 \text{ kN} \rightarrow$ ......... **Ans.**

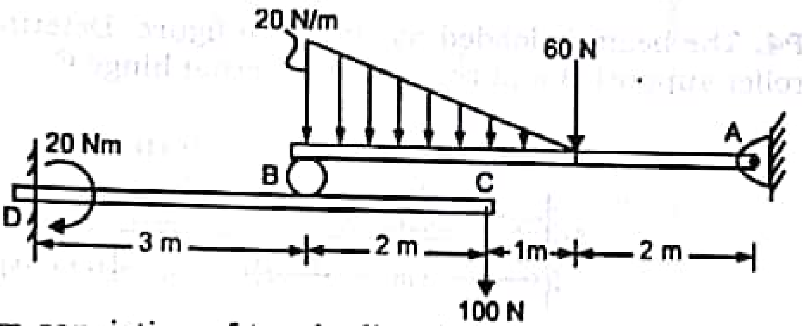
$\Sigma F_y = 0 \uparrow + ve$

$V_A - 20 - 70 + 40.41 \sin 60 = 0$

$\therefore \quad V_A = 55 \text{ kN} \qquad$ Or $\qquad V_A = 55 \text{ kN} \uparrow$ ......... **Ans.**

**P3.** Find the reactions at hinge A, fixed support D. Also find reaction at roller on CD.

***(KJS Nov 15)***




**Solution:** This is a connected system consisting of two bodies, beam AB and beam CD connected internally by a roller at B and externally supported by a fixed support at D and hinge at A. The FBD of the system is shown.

Isolating beam AB from the system as shown.
COE– Beam AB

$\Sigma M_B = 0$ ↺ + ve

$-(30\times 1)-(60\times 3)+(V_A\times 5)=0$

∴ $V_A = 42\text{N}$ Or $V_A = 42\text{N}\uparrow$

$\Sigma F_y = 0 \uparrow + \text{ve}$

$R_B - 30 - 60 + V_A = 0$

∴ $R_B - 30 - 60 + 42 = 0$

∴ $R_B = 48\text{N}$ Or $R_B = 48\text{N}\uparrow$

Since no horizontal force acts on the beam, $H_A = 0$

∴ Hinge reactions at A: $H_A = 0$, $V_A = 42\text{N}\uparrow$ ......... **Ans.**

Roller reaction at B on beam AB : $R_B = 48\text{N}\uparrow$
Roller reaction at B on beam CD : $R_B = 48\text{N}\downarrow$ ......... **Ans.**

COE– System

$\Sigma F_y = 0 \uparrow + \text{ve}$

$V_D - 100 - 30 - 60 + V_A = 0$

∴ $V_D - 100 - 30 - 60 + 42 = 0$

∴ $V_D = 148\text{N}$ Or $V_D = 148\text{N}\uparrow$

$\Sigma M_D = 0$ ↺ + ve

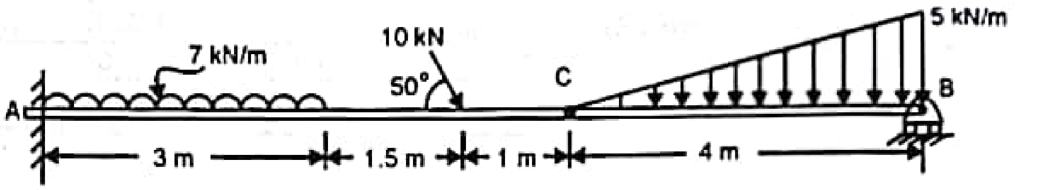
$+M_D - 20 - (100\times 5) - (30\times 4) - (60\times 6) + (42\times 8) = 0$

∴ $M_D = 664$ Nm Or $M_D = 664$ Nm ↺

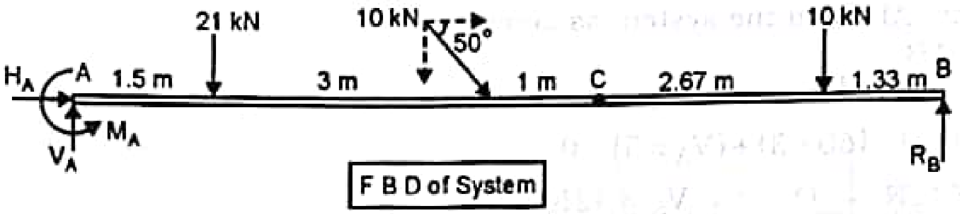
Since there is no horizontal force in the system, $H_D = 0$

∴ Fixed support reactions at D: $H_D = 0, V_D = 148\text{N}\uparrow$ and $M_D = 664$ Nm ↺ ......... **Ans.**

**P4.** The beam is loaded as shown in figure. Determine the reactions at fixed support A, roller support B and reaction at internal hinge C. ***(SPCE Dec 10)***

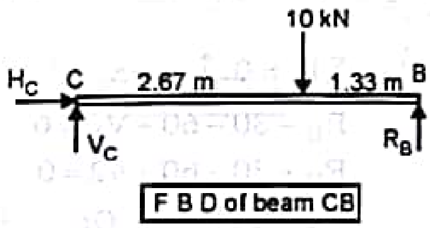



**Solution:** This is a connected system consisting of two bodies, beam AC and beam CB connected internally by a pin (hinge) at C and externally supported by a fixed support at A and roller at B. The FBD of the system is shown.




Isolating beam CB from the system as shown.

COE– Beam CB




$\Sigma M_C = 0$ ↺ + ve

$-(10 \times 2.67) + (R_B \times 4) = 0$

∴ $R_B = 6.675$ kN Or $R_B = 6.675$ kN ↑... **Ans.**

$\Sigma F_y = 0$ ↑ + ve

$V_C - 10 + R_B = 0$

∴ $V_C - 10 + 6.675 = 0$ or $V_C = 3.325$ kN ......... **Ans.**

Since there is no horizontal force on the beam CB we have $H_C = 0$ ......... **Ans.**

COE– system

$\Sigma M_A = 0$ ↺ + ve

$+M_A - (21 \times 1.5) - (10 \sin 50 \times 4.5) - (10 \times 8.17) + (6.675 \times 9.5) = 0$

∴ $M_A = 84.26$ kNm Or $M_A = 84.26$ kNm ↺ ......... **Ans.**

$\Sigma F_x = 0$ → + ve

$H_A + 10 \cos 50 = 0$

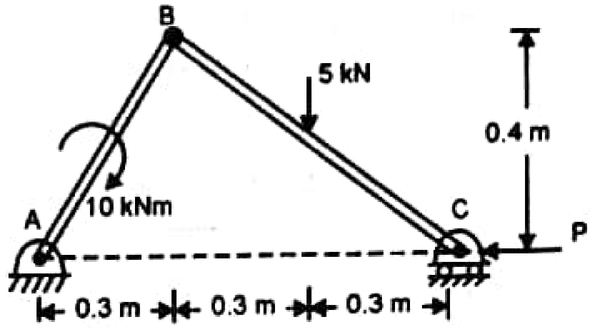
∴ $H_A = -6.428$ kN or $H_A = 6.428$ kN ← ......... **Ans.**

$\Sigma F_y = 0$ ↑ + ve

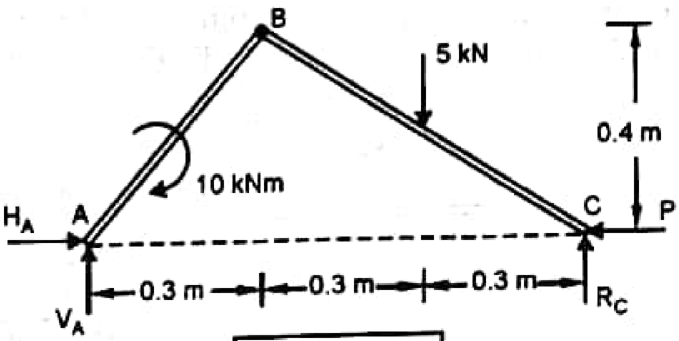
$V_A - 21 - 10 \sin 50 - 10 + 6.675 = 0$

∴ $V_A = 31.96$ kN or $V_A = 31.96$ kN ↑ ......... **Ans.**

**P5.** A two bar mechanism is internally pinned at B and externally supported as shown. It is subjected to external loads as shown. Calculate force P required at C to maintain the configuration.




**Solution:** This is a connected system consisting of two bodies, rod AB and rod BC. The rods are internally connected by a pin (hinge) at B and externally supported by a hinge at A and a roller at C. The FBD of system is shown.




F B D of system

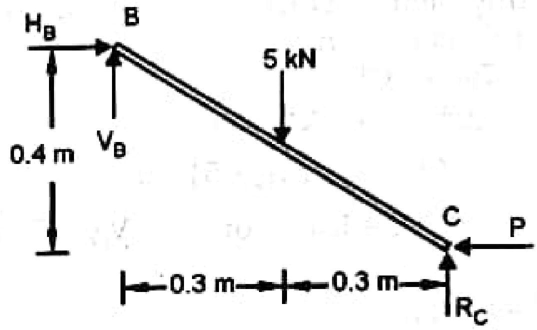
COE – System

$\Sigma M_A = 0$ ↺ + ve

$-10-(5\times0.6)+(R_C\times0.9)=0$

$\therefore$ $R_C = 14.44$ kN

or $R_C = 14.44$ kN ↑




F B D of rod BC

Isolating rod BC

COE – Rod BC

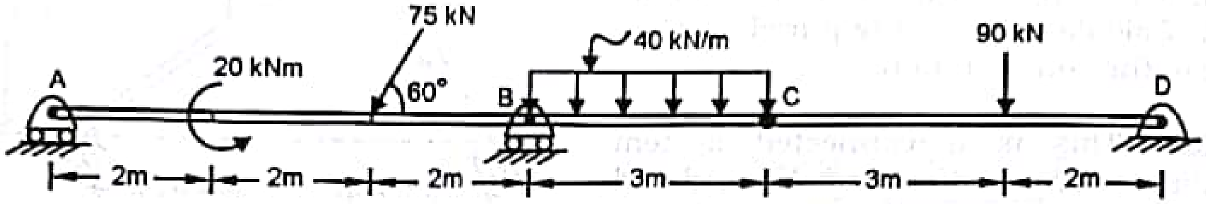
$\Sigma M_B = 0$ ↺ + ve

$-(5\times0.3)+(R_C\times0.6)-(P\times0.4)=0$

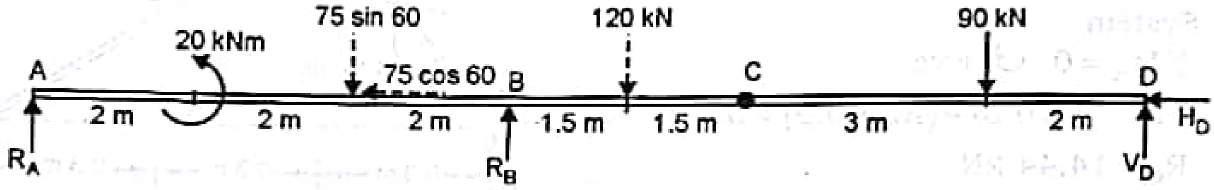
$-1.5+14.44\times0.6-0.4P=0$

or $P = 17.91$ kN ......... **Ans.**

**P6.** A beam is supported and loaded as shown. Find reactions at the supports using equations of equilibrium. C is an internal hinge. ***(SPCE Nov 12)***




**Solution:** This is a connected system consisting of two bodies, beam AC and beam CD, connected internally by a pin (hinge) at C and externally supported by two rollers at A and B and a hinge at D. The FBD of the system is shown.




F B D of System

Isolating beam CD from the system by opening the pin at C as shown.


F B D of Beam CD

COE – Beam CD

$\Sigma M_C = 0$ ↺ + ve

$-(90\times 3)+(V_D\times 5)=0$

$\therefore \quad V_D = 54$ kN or $V_D = 54$ kN ↑ ......... **Ans.**

COE – System

$\Sigma M_A = 0$ ↺ + ve

$+20-(75\sin 60\times 4)+(R_B\times 6)-(120\times 7.5)-(90\times 12)+(V_D\times 14)=0$

$\therefore \quad R_B = 243.97$ kN or $R_B = 243.97$ kN ↑ ......... **Ans.**

$\Sigma F_y = 0$ ↑ + ve

$R_A - 75\sin 60 - 120 - 90 + R_B + V_D = 0$

$\therefore \quad R_A - 75\sin 60 - 120 - 90 + 243.97 + 54 = 0$

$\therefore \quad R_A = -23.02$ kN or $R_A = 23.02$ kN ↓ ......... **Ans.**

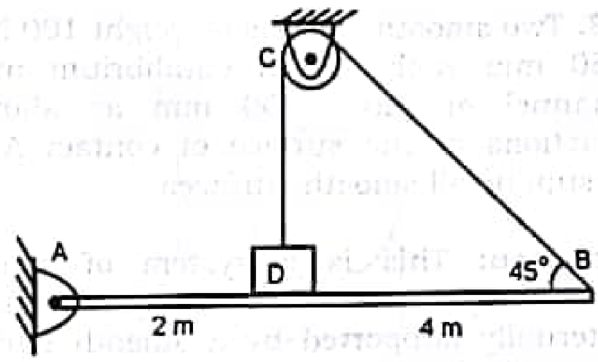
$\Sigma F_x = 0$ → + ve

$-75\cos 60 - H_D = 0$

$\therefore \quad H_D = -37.5$ kN or $H_D = 37.5$ kN → ......... **Ans.**

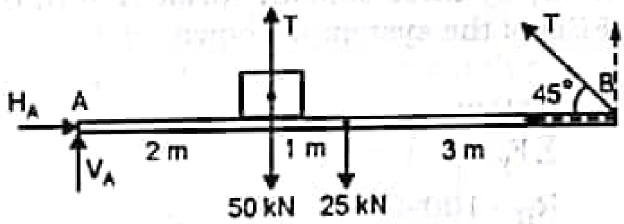
**P7.** A uniform beam AB hinged at A is kept horizontal by a 50 kN weight supported on it with the help of a string tied at B and passing over a smooth pulley at C as shown. The beam weight is 25 kN. Find the reaction at A and C.

***(VJTI Dec 13)***




**Solution:** Let T be the tension in the string connecting the rod to the block. Since the block is resting on the beam, the tension T in the string is not equal to the blocks weight.

FBD of the beam AB with the block resting on it is shown.




F B D - Beam AB

COE – Beam AB

$\Sigma M_A = 0$ ↺ + ve

$$+(T\times 2)-(50\times 2)-(25\times 3)+(T\sin 45\times 6)=0$$

$\therefore \quad T = 28.03$ kN

$\Sigma F_x = 0 \rightarrow$ + ve

$$H_A - T\cos 45 = 0$$

$\therefore \quad H_A - 28.03\cos 45 = 0$

$\therefore \quad H_A = 19.82$ N $\quad$ Or $\quad H_A = 19.82$ N $\rightarrow$ ......... **Ans.**

$\Sigma F_y = 0 \uparrow$ + ve

$$V_A + T + T\sin 45 - 50 - 25 = 0$$

$\therefore \quad V_A + 28.03 + 28.03\sin 45 - 50 - 25 = 0$

$\therefore \quad V_A = 27.15$ kN $\quad$ Or $\quad V_A = 27.15$ kN $\uparrow$ ......... **Ans.**

$\therefore$ Reaction at A ; $H_A = 19.82$ kN $\rightarrow$ and $V_A = 27.15$ kN $\uparrow$ ......... **Ans.**

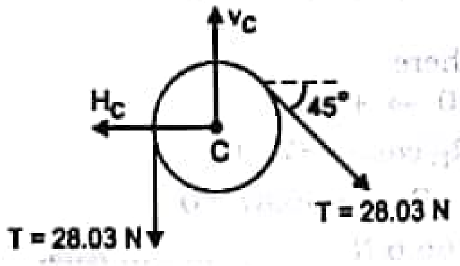
COE – Pulley

$\Sigma F_x = 0 \rightarrow$ + ve

$$-H_C + 28.03\cos 45 = 0$$

$\therefore \quad H_C = 19.82$ kN

or $\quad H_A = 19.82$ kN $\leftarrow$ ......... **Ans.**




FBD - Pulley C

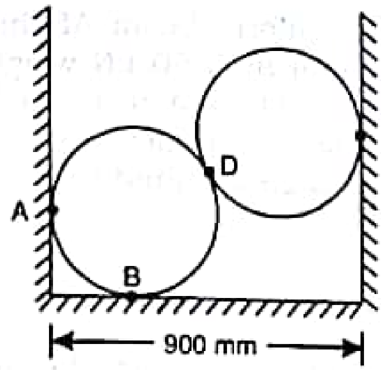
$\Sigma F_y = 0 \uparrow$ + ve

$$V_C - 28.03 - 28.03\sin 45 = 0$$

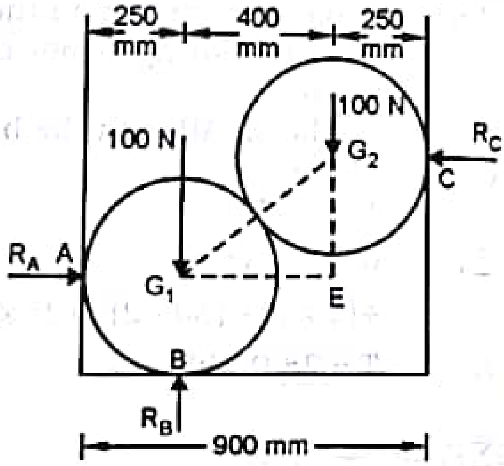
$\therefore \quad V_C = 47.85$ kN $\quad$ Or $\quad V_C = 47.85$ kN $\uparrow$ ......... **Ans.**

$\therefore$ Reaction at C ; $H_C = 19.82$ kN $\leftarrow$ and $V_C = 47.85$ kN $\uparrow$ ......... **Ans.**

**P8.** Two smooth spheres of weight 100 N and of radius 250 mm each are in equilibrium in a horizontal channel of width 900 mm as shown. Find the reactions at the surface of contact A, B, C and D assuming all smooth surfaces.




**Solution:** This is a system of connected bodies consisting of two bodies, left sphere and right sphere, internally supported by a smooth surface at D and externally by three smooth surfaces at A, B and C.
The FBD of the system is shown.



F B D of system

COE – System

$\Sigma F_y = 0 \uparrow +$ ve

$R_B - 100 - 100 = 0$

$\therefore \quad R_B = 200\text{ N}$

or $\quad R_B = 200\text{ N}\uparrow$ ... **Ans.**

$\Sigma M_{G1} = 0 \circlearrowleft +$ ve

$-(100 \times 400) + (R_C \times 300) = 0$

$\therefore \quad R_C = 133.3\text{ N}$

or $\quad R_C = 133.3\text{ N} \leftarrow$ ... **Ans.**

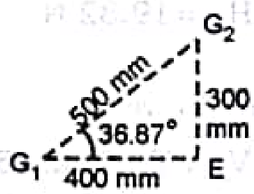
Note that forces $R_A$ and $R_B$ pass through $G_1$, while $R_C$ passes through $G_2$

$\Sigma F_x = 0 \rightarrow +$ ve

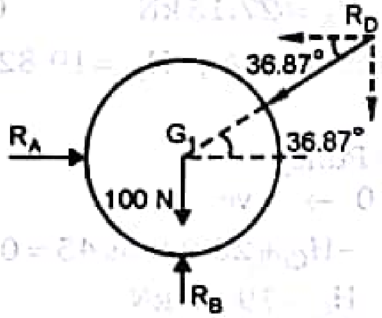
$R_A - 133.3 = 0$

$\therefore \quad R_A = 133.3\text{ N}$

or $\quad R_A = 133.3\text{ N} \rightarrow$ ... **Ans.**




Isolating left sphere.
The internal smooth surface at D offers reaction force $R_D$ normal to the tangent at D [i.e. along line $G_1G_2$] as shown.



F.B D of Left sphere

COE – Left sphere

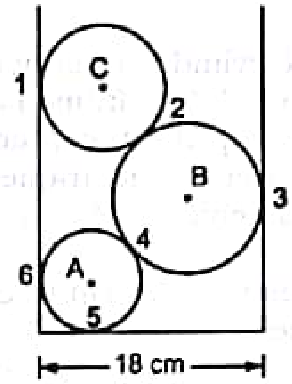
$\Sigma F_x = 0 \rightarrow +$ ve

$R_A - R_D \cos 36.87 = 0$

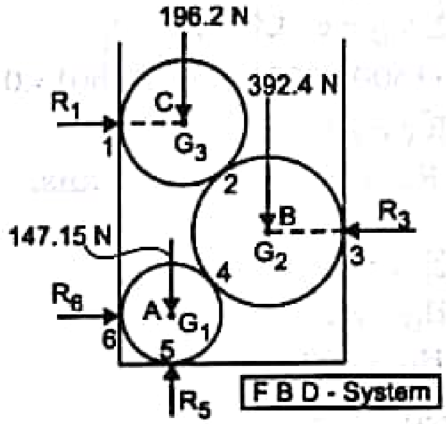
$\therefore \quad 133.3 - R_D \cos 36.87 = 0$

$\therefore \quad R_D = 166.6\text{ N}$ ......... **Ans.**

**P9.** Three right circular cylinders A, B and C are piled up in a rectangular channel as shown in figure. Determine the reactions at point 6 between cylinder A and vertical wall of the channel.
Cylinder A: radius = 4 cm, mass = 15 kg.
Cylinder B: radius = 6 cm, mass = 40 kg.
Cylinder C: radius = 5 cm, mass = 20 kg.
***(MU Dec 2015)***




**Solution:** This is a system of connected bodies, consisting of three spheres, internally supported by smooth surface at 2 and 4 and externally supported by smooth surface at 1, 3, 5 and 6. The F.B.D. of the system is shown.




COE – System

$$\Sigma F_y = 0 \uparrow + ve$$

$$R_5 - 196.2 - 147.15 - 392.4 = 0$$

$\therefore$ $$R_5 = 735.75 \text{ N}$$

Isolating sphere A

COE – sphere A

$$\Sigma F_y = 0 \uparrow + ve$$

$$735.75 - 147.15 - R_4 \sin 36.87 = 0$$

$\therefore$ $$R_4 = 981 \text{ N}$$

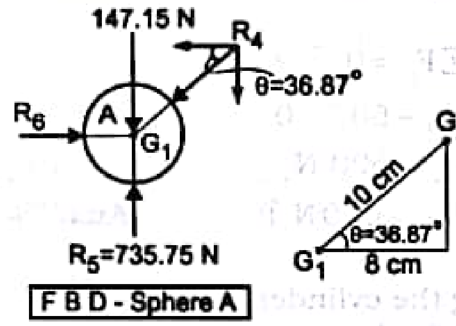
$$\Sigma F_x = 0 \rightarrow + ve$$

$$R_6 - R_4 \cos 36.87 = 0$$

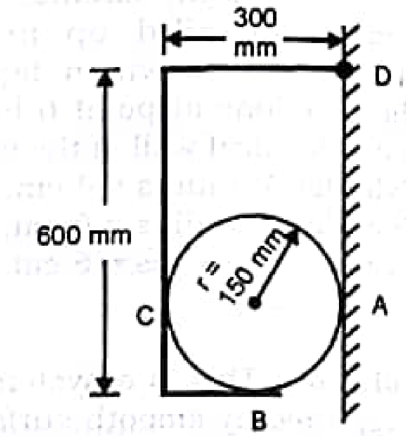
$\therefore$ $$R_6 - 981 \cos 36.87 = 0$$

$\therefore$ $$R_6 = 784.8 \text{ N}$$

or $$R_6 = 784.8 \text{ N} \rightarrow$$ ......... **Ans.**



**P10.** A 600N cylinder is supported by the frame BCD as shown in figure 4. The frame is hinged at D. Determine the reactions developed at contact points A, B, C and D. Neglect the weight of frame and assume all contact surfaces are smooth *(VJTI May 10)*




**Solution:** This is a system of connected bodies, consisting of two bodies, frame BCD and a cylinder internally supported by two surfaces at B and C and externally supported by a hinge at D and a smooth surface at A. The FBD of the system is shown.

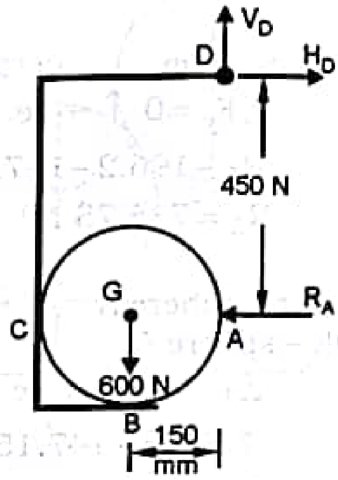
COE - System

$\Sigma M_D = 0$ ↺ + ve

$+(600 \times 150) - (R_A \times 450) = 0$

∴ $R_A = 200$ N

or $R_A = 200$ N ← ......... **Ans.**




F B D of system

$\Sigma F_x = 0$ → + ve

$H_D - R_A = 0$

∴ $H_D - 200 = 0$

∴ $H_D = 200$ N

or $H_D = 200$ N→ ......... **Ans.**

$\Sigma F_y = 0$ ↑ + ve

$V_D - 600 = 0$

∴ $V_D = 600$ N

or $V_D = 600$ N ↑ ......... **Ans.**

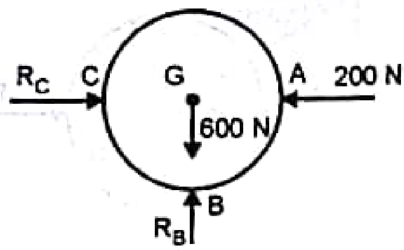
Isolating the cylinder

COE – Cylinder

$\Sigma F_x = 0$ → + ve

$R_C - 200 = 0$

∴ $R_C = 200$ N ......... **Ans.**



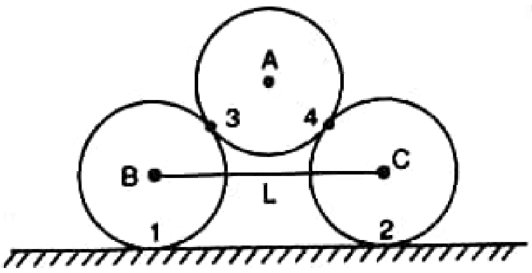

F B D of cylinder

$\Sigma F_y = 0$ ↑ + ve

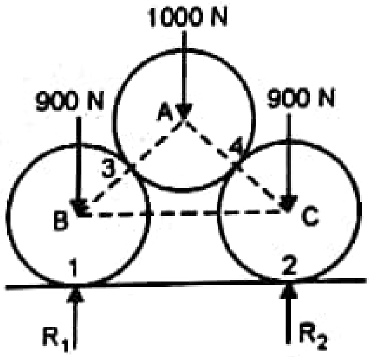
$R_B - 600 = 0$

∴ $R_B = 600$ N ......... **Ans.**

**P11.** Sphere A = 1000 N rests on two spheres B and C of weight 900 N each. The spheres B and C are connected by an inextensible string of length L = 600 mm. Assuming smooth contacts and radius of spheres to be 200 mm, determine the reactions at all contact points 1 to 4 and also the force in the string.




**Solution:** This is a system of connected bodies consisting of Three cylinders A, B and C, internally supported by two smooth surfaces one each at 3 and 4 and by a rope BC. The system is externally supported by two smooth surfaces one each at 1 and 2. The FBD of system is shown.




F B D of system

COE – System

$\Sigma M_B = 0$ ↺ + ve

$$-(1000\times300)-(900\times600)+(R_2\times600)=0$$

∴ $R_2 = 1400$ N

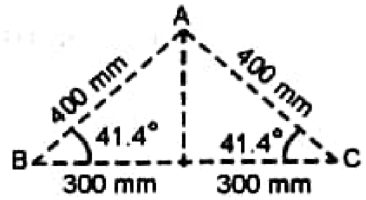
or $R_2 = 1400$ N ↑ ......... **Ans.**

$\Sigma F_y = 0$ ↑ + ve

$$R_1 - 900 - 1000 - 900 + 1400 = 0$$

∴ $R_1 = 1400$ N

or $R_1 = 1400$ N ↑ ......... **Ans.**




Triangle ABC

Isolating sphere B by **separating** sphere A from it at contact point 3 and cutting the rope. Let $R_3$ be the reaction at smooth surface 3 acting along line AB and T be the tension in the rope.

COE - Sphere B

$\Sigma F_y = 0$ ↑ + ve

$$1400 - 900 - R_3 \sin 41.4 = 0$$

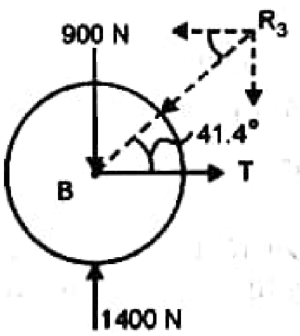
∴ $R_3 = 756$ N ......... **Ans.**

$\Sigma F_x = 0$ → + ve

$$T - R_3 \cos 41.4 = 0$$

∴ $T - 756 \cos 41.4 = 0$

or $T = 567$ N ......... **Ans.**



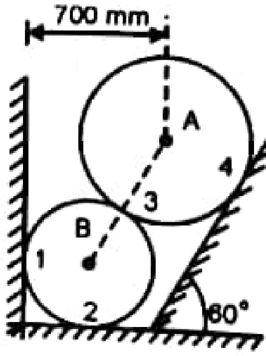

F B D of Sphere B

Also reaction at 4 i.e. $R_4 = R_3$ ...... due to symmetry of loading

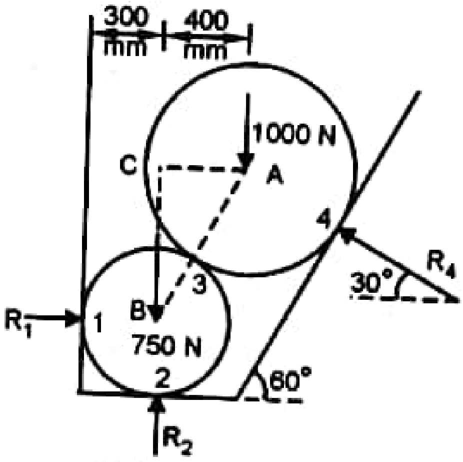
∴ $R_4 = 756$ N ......... **Ans.**

**P12.** Two spheres A and B of weight 1000 N and 750 N respectively are kept as shown in figure. Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A = 400 mm and radius of B = 300 mm.

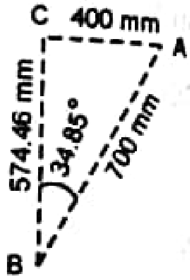
***(M.U May 11)***




**Solution:** The system consists of two spheres A and B internally supported by a smooth surface at 3 and externally supported by smooth surface at 1, 2 and 4. The FBD of the system is shown.




FBD of System




Triangle ABC

COE – System

$$\Sigma M_B = 0 \quad \circlearrowleft + ve$$

$$-(1000 \times 400) + (R_4 \cos 30 \times 574.46) + (R_4 \sin 30 \times 400) = 0$$

$\therefore \quad R_4 = 573.5 \text{ N}$

or $\quad R_4 = 573.5 \text{ N} \quad \theta = 30°$ **Ans.**

$\Sigma F_x = 0 \rightarrow + ve$

$R_1 - R_4 \cos 30 = 0$

$\therefore \quad R_1 - 573.5 \cos 30 = 0$

$\therefore \quad R_1 = 496.6 \text{ N} \quad$ or $\quad R_1 = 496.6 \text{ N} \rightarrow$ ......... **Ans.**

Note that forces $R_1$ and $R_2$ pass through B, while $R_4$ passes through A.

$\Sigma F_y = 0 \uparrow + ve$

$R_2 - 750 - 1000 + R_4 \sin 30 = 0$

$R_2 - 1750 + 573.5 \sin 30 = 0$

$\therefore \quad R_2 = 1463.3 \text{ N} \quad$ or $\quad R_2 = 1463.3 \text{ N} \uparrow$ ......... **Ans.**

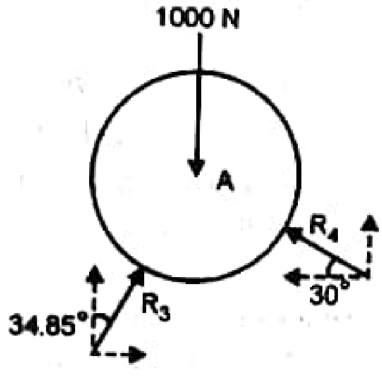
Isolating top sphere A. Let $R_3$ be the internal force seen on isolating, acting along line AB as shown.

$\Sigma F_x = 0 \rightarrow + ve$

$R_3 \sin 34.85 - R_4 \cos 30 = 0$

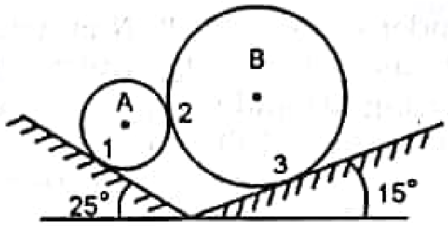
$\therefore \quad R_3 \sin 34.85 - 573.5 \cos 30 = 0$

$\therefore \quad R_3 = 869.1 \text{ N}$ ......... **Ans.**

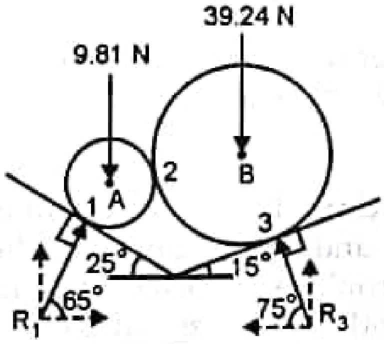



F B D of Sphere A

**P13.** Determine the reactions at points of contact 1, 2 and 3. Assume smooth surfaces.
Take $r_A = 1$ cm, $r_B = 4$ cm,
$m_A = 1$ kg, $m_B = 4$ kg.
***(NMIMS May 09, KJS Nov 15)***




**Solution:** The system consists of two connected bodies, sphere A and sphere B, internally supported by a smooth surface at 2 and externally supported by smooth surfaces at 1 and 3. The FBD of the system is shown.



F B D of System

COE – System

$\Sigma F_x = 0 \rightarrow +$ ve

$\therefore \quad R_1 \cos 65 - R_3 \cos 75 = 0 \quad \text{......... (1)}$

$\Sigma F_y = 0 \uparrow +$ ve

$R_1 \sin 65 + R_3 \sin 75 - 9.81 - 39.24 = 0$

$\therefore \quad R_1 \sin 65 - R_3 \sin 75 = 49.05 \quad \text{......... (2)}$

Solving equation (1) and (2) we get

$\therefore \quad R_1 = 19.75 \text{ N} \quad \theta = 65°$ ......... **Ans.**

$\therefore \quad R_3 = 32.25 \text{ N} \quad \theta = 75°$ ......... **Ans.**

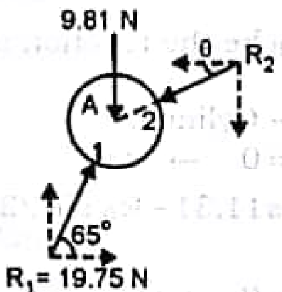
Isolating left sphere. Let $R_2$ be the internal force seen on isolation, acting along line AB at some angle θ as shown.

COE – Sphere A

$\Sigma F_x = 0 \rightarrow +$ ve

$\therefore \quad 19.75 \cos 65 - R_2 \cos \theta = 0$

$\therefore \quad R_2 \cos \theta = 8.347 \quad \text{......... (3)}$



F B D of sphere A

$\Sigma F_y = 0 \uparrow +$ ve

$19.75 \sin 65 - 9.81 - R_2 \sin \theta = 0$

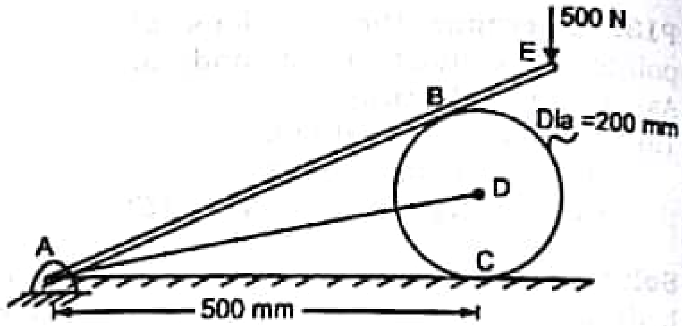
$\therefore \quad R_2 \sin \theta = 8.0895 \quad \text{......... (4)}$

Solving equation (3) and (4) we get

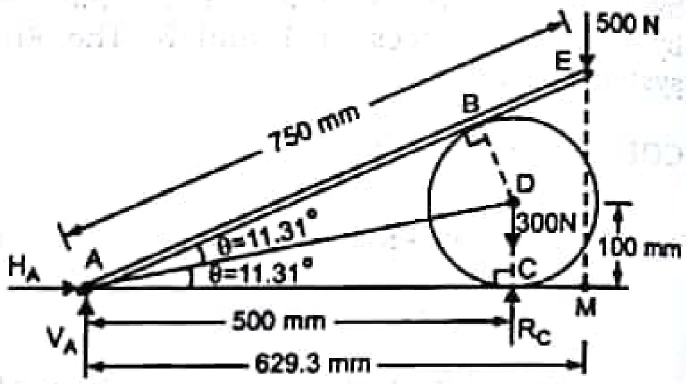
$\therefore \quad R_2 = 11.62 \text{ N} \quad \theta = 44.1°$ ......... **Ans.**

**P14.** A cylinder of weight 300 N is held in equilibrium as shown. Determine the tension in string AD and reaction at C and B. The length of AE = 750 mm.

***(M.U. Dec 14)***




**Solution:** This is a system consists of rod AE and a cylinder. The system is externally supported by a hinge at A and smooth surface at C. The internal supports are smooth surface at B and string AD.




COE – System

$$\Sigma M_A = 0 \circlearrowleft +ve$$

$$-(500 \times 692.3) - (300 \times 500) + (R_C \times 500) = 0$$

$\therefore \quad R_C = 992.3 \text{ N} \uparrow$ ...... **Ans.**

From geometry

$\angle DAB = \angle DAC = \theta$

$$\tan\theta = \frac{100}{500} \qquad \therefore \quad \theta = 11.31°$$

$$\text{Distance AM} = 750\cos 22.62° = 692.3 \text{ mm}$$

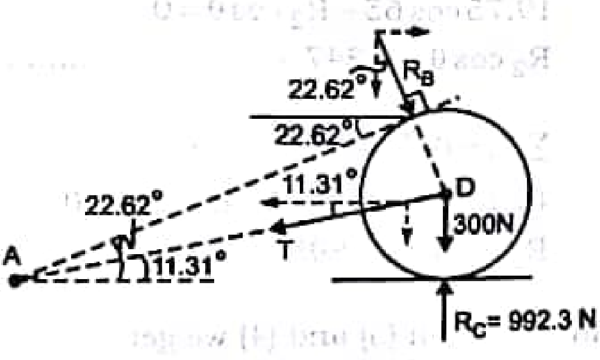
**Isolating the cylinder**

Let $R_B$ be the reaction at smooth surface B and T be the tension in string AD.

COE – Cylinder

$\Sigma F_x = 0 \quad \rightarrow +ve$

$$-T\cos 11.31 + R_B \sin 22.62 = 0 \text{ .......... (1)}$$




$\Sigma F_y = 0 \quad \uparrow +ve$

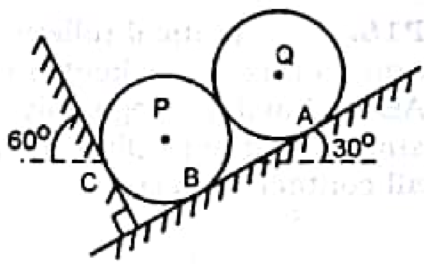
$$-T\sin 11.31 - R_B\cos 22.62 - 300 + 992.3 = 0 \text{ .......... (2)}$$

Solving equations (1) and (2), we get

$$R_B = 692.3 \text{ N}$$

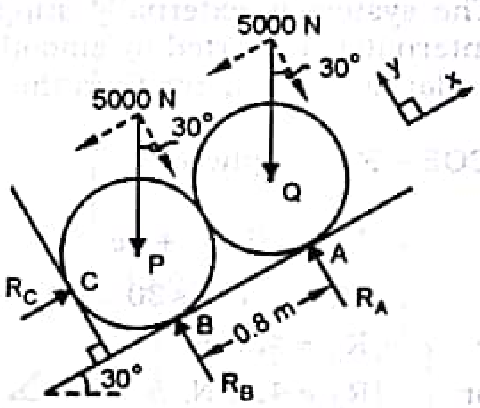
and $\quad T = 271.5 \text{ N} \uparrow$ ...... **Ans.**

**P15.** Two homogenous solid cylinders of identical weight of 5000 N and radius of 0.4 m are resting against inclined wall and sloping ground as shown. Assuming smooth surfaces find the reactions at A, B and C of the contact points.




**Solution:** This is a system of connected bodies consisting of two cylinders P and Q internally supported by a smooth surface & externally by three smooth surfaces at A, B & C.
The FBD of the system is shown.

Taking inclined axes x and y as shown. Note forces $R_A$ and $R_B$ act parallel to y-axis and force $R_C$ acts parallel to x axis. We need to resolve the weight force of cylinders P and Q as shown.




F B D of system

COE – System

$\Sigma F_x = 0$ □ + ve

$R_C - 5000\sin 30 - 5000\sin 30 = 0$

∴ $R_C = 5000\,N$

or $R_C = 5000\,N \quad \theta = 30°$ ......... **Ans.**

$\Sigma M_P = 0$ ↺ + ve

$+(R_A \times 0.8) - (5000\cos 30 \times 0.8) = 0$

∴ $R_A = 4330\ N$ or $R_A = 4330\ N \quad \theta = 60°$ ......... **Ans.**

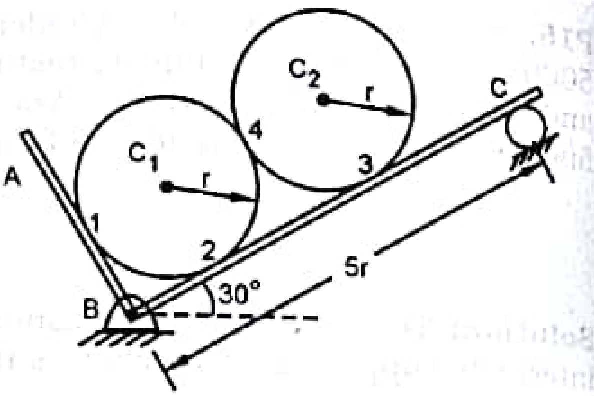
$\Sigma F_y = 0$ ↑ + ve

$R_A + R_B - 5000\cos 30 - 5000\cos 30 = 0$

$4330 + R_B - 8660 = 0$

∴ $R_B = 4330\ N$ or $R_B = 4330\ N \quad \theta = 60°$ ......... **Ans.**

**P16.** Two identical rollers each of weight 500 N and radius r are kept on a right angle frame ABC having negligible weight. Assuming smooth surfaces, find the reactions induced at all contact surfaces. ***(M.U. Dec 12)***




**Solution:** This is a system of three connected bodies viz a frame ABC and two spheres. The system is externally supported by a hinge at B and a roller at C. The system is internally supported by smooth surfaces at 1, 2, 3 and 4.
Isolating the spheres from the system as shown in figure.

COE – Right Sphere

$\Sigma F_y = 0$ □ + ve

$R_3 - 500\cos 30 = 0$

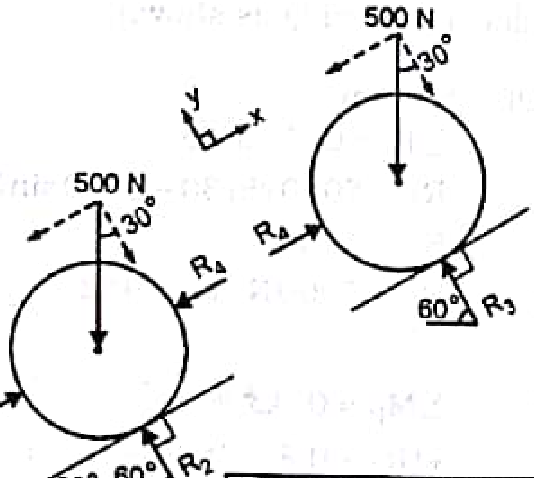
$\therefore$ $R_3 = 433$ N

or $R_3 = 433$ N, $\theta = 60°$ $\measuredangle$ ......... **Ans.**

$\Sigma F_x = 0$ □ + ve

$R_4 - 500\sin 30 = 0$

$\therefore$ $R_4 = 250$ N ......... **Ans.**




F B Ds of Isolated Spheres

COE – Left Sphere

$\Sigma F_y = 0 \uparrow + ve$

$R_2 - 500\cos 30 = 0$

$\therefore$ $R_2 = 433$ N or $R_2 = 433$ N, $\theta = 60°$ $\measuredangle$ ......... **Ans.**

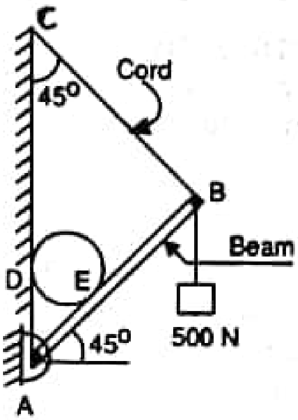
$\Sigma F_x = 0 \rightarrow + ve$

$R_1 - 500\sin 30 - R_4 = 0$

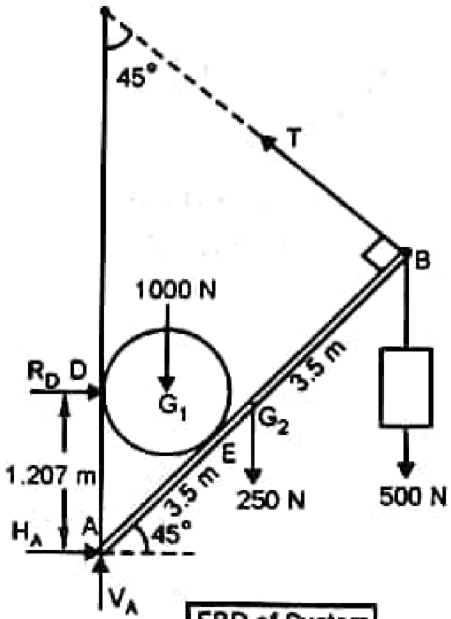
$R_1 - 500\sin 30 - 250 = 0$

$\therefore$ $R_1 = 500$ N or $R_1 = 500$ N, $\theta = 30°$ $\measuredangle$ ......... **Ans.**

**P17.** A cylinder of diameter 1 m and weighing 1000 N and another block weighing 500 N are supported by a beam of length 7 m and weighing 250 N with the help of a cord as shown.
If the surfaces of contact are frictionless, determine tension in cord and reaction at point of contacts.




**Solution:** This is a system of three connected bodies viz cylinder, beam AB and 500 N block, internally supported by a smooth surface at E (supporting cylinder on beam) and a rope (supporting block to beam). The system is externally supported by a hinge at A, smooth surface at D and a cord. The FBD of the system is shown.
Isolating the cylinder from the system



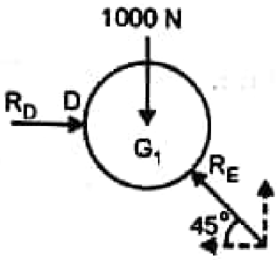
FBD of System

COE – cylinder

$\Sigma F_y = 0 \uparrow +ve$

$R_E \sin 45 - 1000 = 0$

$\therefore\ R_E = 1414.2\text{ N}$ ...... **Ans.**



FBD of Cylinder

$\Sigma F_x = 0 \rightarrow +ve$

$R_D - 1414.2\cos 45 = 0$

$\therefore\ R_D = 1000\text{ N}$ Or $R_D = 1000\text{ N} \rightarrow$ ......... **Ans.**

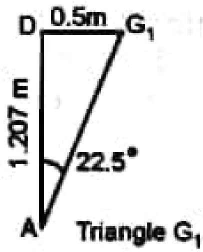
COE – System

$\Sigma M_A = 0$ ↺ + ve

$$+(T\times 7)-(500\times 7\cos 45)-(1000\times 0.5)-(250\times 3.5\cos 45)-(1000\times 1.207)=0$$

$\therefore\ T = 685.8\text{ N}$ ......... **Ans.**

Note that the surfaces at D and E are tangents to the cylinder. These tangents intersect at point A forming an angle of 45° between them. From geometry the line joining the centre $G_1$ to a point A, bisects the angle $\therefore\ \angle G_1AD = 22.5°$

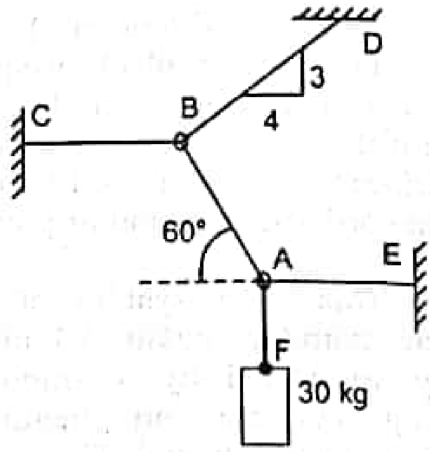


Triangle $G_1AD$

Now $\tan 22.5 = \dfrac{L(DG_1)}{L(DA)} = \dfrac{0.5}{L(DA)}$ $\therefore$ $L(DA) = 1.207$ m

**P18.** A 30 kg pipe is supported at 'A' by a system of five chords. Determine the force in each chord for equilibrium.

***(M. U. May 09)***




**Solution:** This is a system of a single body supported by a set of five cords. Of these cord BC, BD and AE are external supports while cord AF and AB are internal.



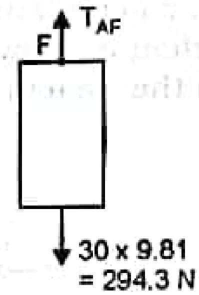
Isolating the 30 kg pipe

COE – Pipe

$$\sum F_y = 0 \uparrow +ve$$

$$T_{AF} - 294.3 = 0$$

$\therefore \quad T_{AF} = 294.3$ N ......... **Ans.**




F B D of Pipe

Isolating Joint A

COE – Joint A

$$\sum F_y = 0 \uparrow +ve$$

$$T_{AB} \sin 60 - 294.3 = 0$$

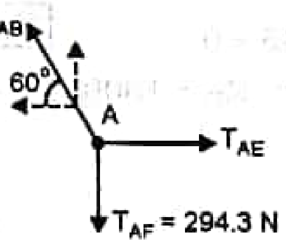
$\therefore \quad T_{AB} = 339.8$ N ......... **Ans.**

$$\sum F_x = 0 \rightarrow +ve$$

$$-T_{AB} \cos 60 + T_{AE} = 0$$

$$-339 \cos 60 + T_{AE} = 0$$

$\therefore \quad T_{AB} = 169.9$ N ......... **Ans.**




F B D of Joint A

Isolating Joint B

COE – Joint B

$$\sum F_y = 0 \uparrow +ve$$

$$T_{BD} \sin 36.87 - 339.8 \sin 60 = 0$$

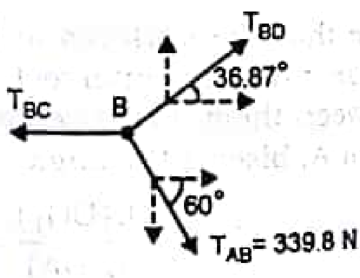
$\therefore \quad T_{BD} = 490.46$ N ......... **Ans.**

$$\sum F_x = 0 \rightarrow +ve$$

$$-T_{BC} + T_{BD} \cos 36.87 + 339.8 \cos 60 = 0$$

$$-T_{BC} + 490.46 \cos 36.87 + 339.8 \cos 60 = 0$$

$\therefore \quad T_{BC} = 562.3$ N ......... **Ans.**




F B D of Joint B

✱✚✱✚✱