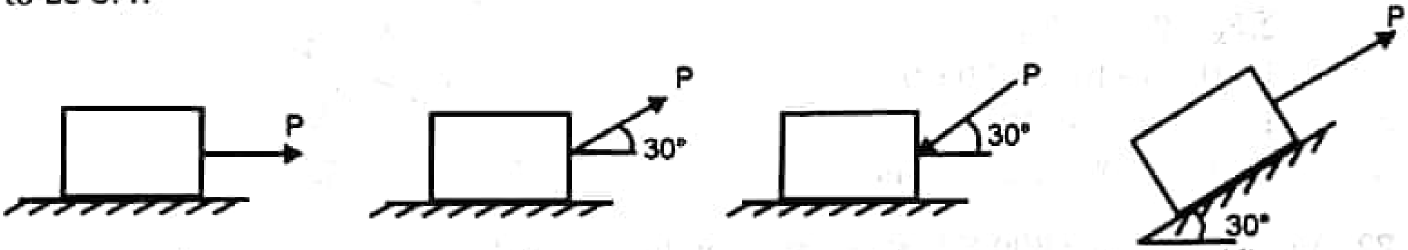
Solutions: Chapter 4

Friction

Exercise 4.1

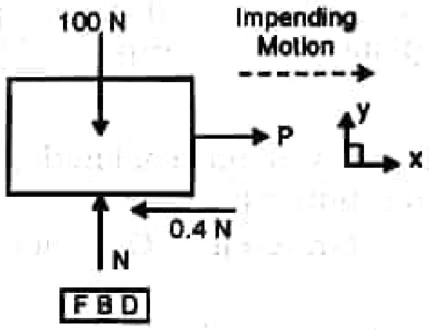
Blocks

P1. For the following cases find force P needed to just impend the motion of the block. Take weight of block to be 100 N and coefficient of static friction at the contact surface to be 0.4.



(a) (b) (c) (d)

Solution: Case (a) COE – Block



$$\Sigma F_y = 0$$

$$N - 100 = 0$$

$\therefore$ $N = 100\ \text{N}$

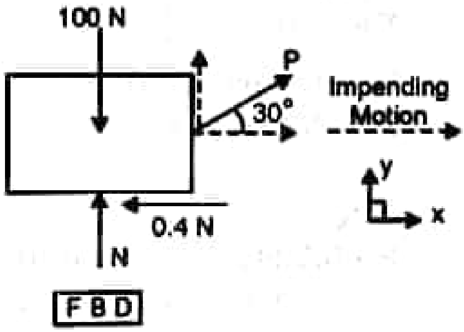
$$\Sigma F_x = 0$$

$$P - 0.4N = 0$$

$\therefore$ $P - 0.4 \times 100 = 0$

Or $P = 40\ \text{N}$ .... **Ans.**

Case (b) COE – Block



$$\Sigma F_x = 0$$

$$P \cos 30 - 0.4N = 0 \quad \text{......... (1)}$$

$$\Sigma F_y = 0$$

$$P \sin 30 + N - 100 = 0 \quad \text{......... (2)}$$

Solving equations (1) and (2) we get

$N = 81.24\ \text{N}$

and $P = 37.52\ \text{N}$ .... **Ans.**

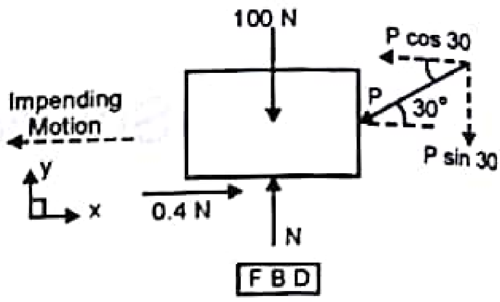
Case (c) COE – Block

$$\Sigma F_x = 0$$

$$-P\cos 30 + 0.4N = 0 \quad \text{.........} (1)$$

$$\Sigma F_y = 0$$

$$-P\sin 30 + N - 100 = 0 \quad \text{.........} (2)$$



Solving equations (1) and (2), we get

N = 130 N

and P = 60 N .... **Ans.**

Case (d) COE – Block

$$\Sigma F_y = 0$$

$$N - 100\cos 30 = 0$$

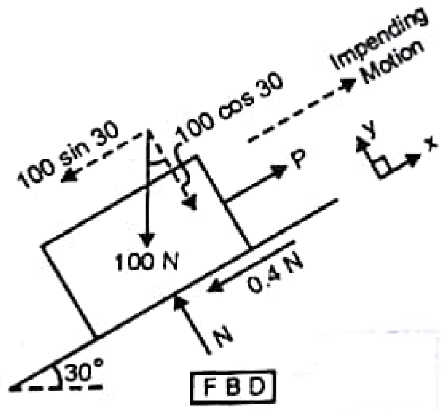
$\therefore$ N = 86.6 N

$$\Sigma F_x = 0$$

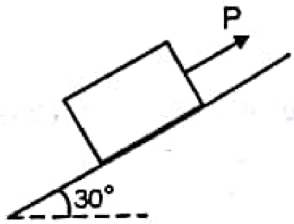
$$P - 0.4N - 100\sin 30 = 0$$

$\therefore$ $P - 0.4 \times 86.6 - 50 = 0$

$\therefore$ P = 84.64 N .... **Ans.**



P2. A block weighing 800 N has to rest on an incline of 30°. If the angle of limiting friction is 18°, Find the least and greatest force that need to be applied on the block, parallel to the plane so as to keep the block in equilibrium.



Solution: Given angle of limiting friction ϕ = 18°

Knowing $\tan\phi = \mu$

$\therefore$ $\tan 18 = \mu$ Or $\mu = 0.3249$

Case (1) P_{least}

When least force P is applied to keep the block in equilibrium, the block impends to move down.

COE – Block

$$\Sigma F_y = 0$$

$$N - 800\cos 30 = 0$$

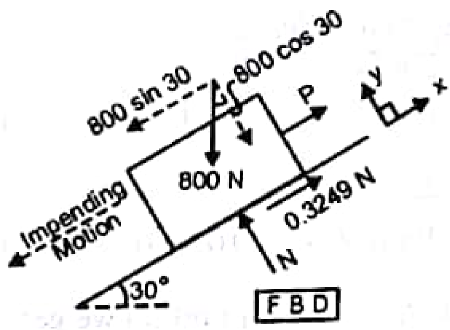
$\therefore$ N = 692.8 N

$$\Sigma F_x = 0$$

$$P + 0.3249N - 800\sin 30 = 0$$

$$P + 0.3249 \times 692.8 - 400 = 0$$

$\therefore$ $P_{Least} = 174.9$ N .... **Ans.**



Case (2) $P_{greatest}$

When greatest force P is applied for keeping the block in equilibrium, the block impends to move up the incline.

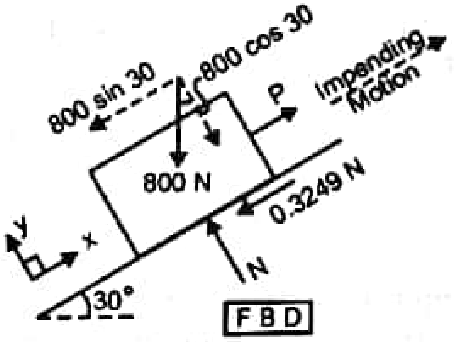
COE – Block

$$\Sigma F_x = 0$$

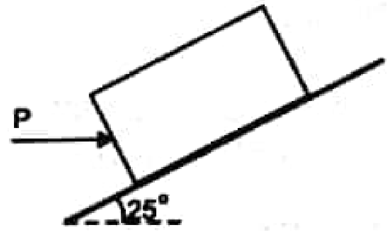
$$P - 800\sin 30 - 0.3249N = 0$$

$$P - 800\sin 30 - 0.3249 \times 692.8 = 0$$

$\therefore$ $P_{greatest} = 625.1$ N .... **Ans.**



P3. A block of weight 800 N is acted upon by a horizontal force P as shown in figure. If the coefficient of friction between the block and incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the value of P for impending motion up the plane. ***(MU Dec 15)***



Solution: To impend the motion up the plane, the frictional force i.e $\mu_s N$ will act down the inclined plane.

COE – Block

$$\Sigma F_y = 0$$

$$N\cos 25 - 0.35N\sin 25 - 800 = 0$$

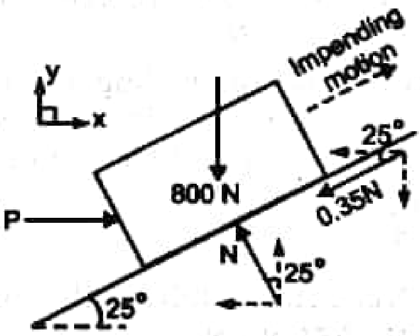
$\therefore$ $N = 1054.9$ N

$$\Sigma F_x = 0$$

$$P - N\sin 25 - 0.35N\cos 25 = 0$$

$\therefore$ $P - 1054.9\sin 25 - 0.35 \times 1054.9 \times \cos 25 = 0$

$\therefore$ $P = 780.4$ N .... **Ans.**

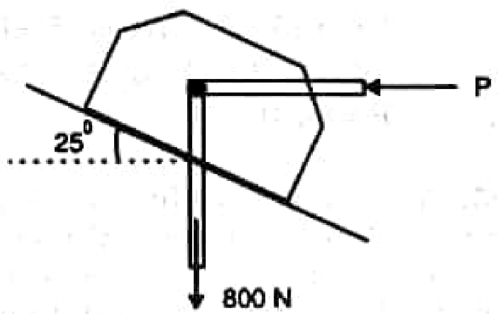


P4. A support block is acted upon by two forces as shown. Knowing $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the force P required

a. to start the block moving up the incline.

b. to keep it moving up.

c. to prevent it from sliding down.



Solution: a) To start the block moving up the incline.

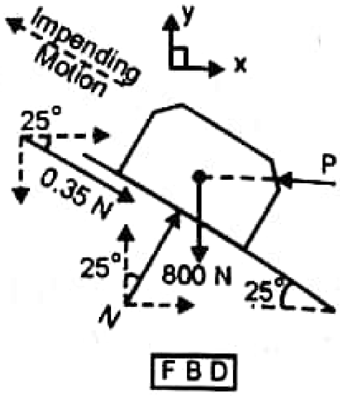
In this situation, the max. static frictional force i.e. $\mu_s N$ will act down the incline.

COE – Block

$$\Sigma F_y = 0$$

$$N\cos 25 - 0.35N\sin 25 - 800 = 0$$

$\therefore$ $N = 1054.9$ N



$$\Sigma F_x = 0$$

$$N\sin 25 + 0.35N\cos 25 - P = 0$$

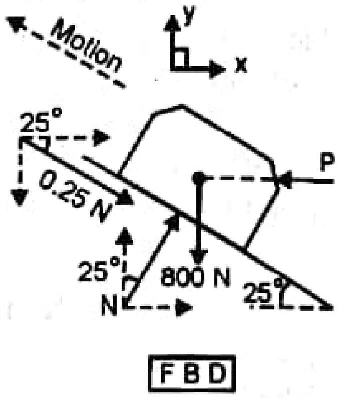
$$1054.9\sin 35 + 0.35\times 1054.9\times\cos 25 - P = 0$$

$\therefore$ $P = 780.4$ N .... **Ans.**

b) To keep it moving up the incline.

In this situation, the kinetic frictional force i.e. $\mu_k N$ will act down the incline.

COE – Block



$$\Sigma F_y = 0$$

$$N\cos 25 - 0.25N\sin 25 - 800 = 0$$

$\therefore$ $N = 999.18$ N

$$\Sigma F_x = 0$$

$$-P + 0.25N\cos 25 + N\sin 25 = 0$$

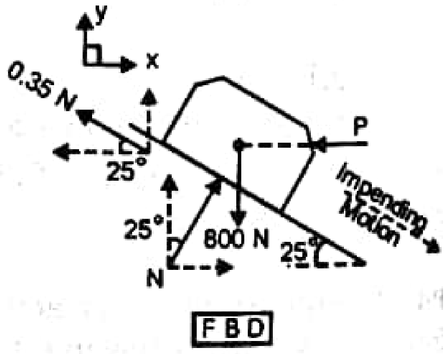
$$-P + 0.25\times 999.18\times\cos 25 + 999.18\sin 25 = 0$$

$\therefore$ $P = 648.6$ N .... **Ans.**

c) To prevent it from sliding down.

In this situation, the block impends to move down the incline and hence max. stati frictional force $\mu_s N$ will act up the incline.

COE – Block



$$\Sigma F_y = 0$$

$$N\cos 25 + 0.35N\sin 25 - 800 = 0$$

$\therefore$ $N = 758.85$ N

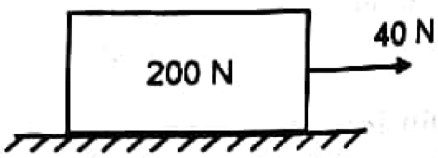
$$\Sigma F_x = 0$$

$$N\sin 25 - 0.35N\cos 25 - P = 0$$

$$758.85\sin 25 - 0.35\times 758.85\cos 25 - P = 0$$

$\therefore$ $P = 80$ N .... **Ans.**

P5. A block of weight 200 N rests on a horizontal surface. The co-efficient of friction between the block and the horizontal surface is 0.4. Find the frictional force acting on the block if a horizontal force of 40 N is applied to the block. ***(M. U. Dec 09)***

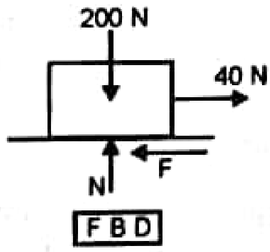


Solution: Let F be the frictional force acting at the horizontal rough surface, required to keep the block in equilibrium.

COE – Block

$$\Sigma F_y = 0$$

$$N - 200 = 0 \quad \text{or} \quad N = 200 \text{ N}$$



$$\Sigma F_x = 0$$

$$40 - F = 0 \quad \text{or} \quad F = 40 \text{ N}$$

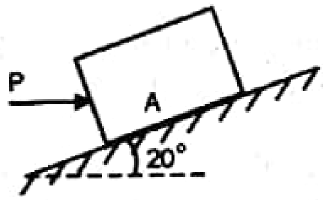
∴ F = 40 N is required frictional force to keep the block in equilibrium. .... **Ans.**

Now, the max. frictional force the surface can produce = $F_{max} = \mu_s N$

∴ $F_{max} = 0.4 \times 200$ or $F_{max} = 80$ N

Since $F_{required} < F_{max}$, it implies the block is in equilibrium (not impending) and the frictional force produced at the horizontal surface = 40 N .... **Ans.**

P6. A horizontal force P is applied to the block A of mass 50 kg kept on an inclined plane as shown. If P = 200 N, find whether the block is in equilibrium. Also find the magnitude and direction of frictional force.
Take $\mu_s = 0.25$. ***(SPCE Nov 12)***

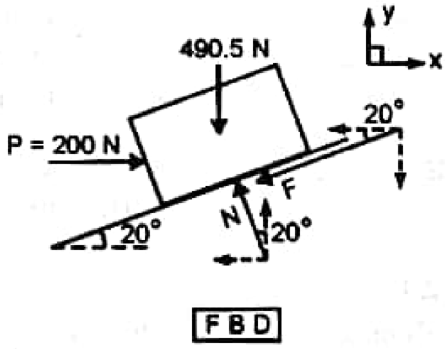


Solution: Let F be the frictional force required to keep the block in equilibrium.

COE – Block

$$\Sigma F_x = 0 \rightarrow + ve$$

$$-F\cos 20 - N\sin 20 + 200 = 0 \quad \text{......... (1)}$$



$$\Sigma F_y = 0 \uparrow + ve$$

$$-F\sin 20 + N\cos 20 - 490.5 = 0 \quad \text{......... (2)}$$

Solving equations (1) and (2) we get

∴ $F = 20.18$ N and $N = 529.3$ N

∴ $F = 20.18$ N is required to keep the block in equilibrium.

The maximum frictional force 'F_{max}' the surface can provide = μ_s N

∴ $F_{max} = 0.25 \times 529.3 = 132.3$ N

Since $F_{required} < F_{max}$ the block is in equilibrium with the help of a frictional force F = 20.18 N acting down the rough plane. .... **Ans.**

P7. A car of 1000 kg mass is to be parked on the same 10° incline year around. The static coefficient of friction between the tires and the road varies between the extremes of 0.05 and 0.9. Is it always possible to park the car at this place? Assume that the car can be modeled as a particle. ***(M.U. May 08)***

Solution: The minimum angle of inclination of a plane with the horizontal for which a body kept on it will just slide down on its own without the application of any external force is known as angle of repose (α).'

Angle of Repose α is related to coefficient of friction μ as $\boxed{\tan\alpha = \mu}$

A body kept on a slope of angle θ slides down on its own if the angle θ is greater than Angle of Repose α.

If $\mu = 0.05$ $\therefore$ $\tan\alpha = \mu = 0.05$ or $\alpha = 2.862°$

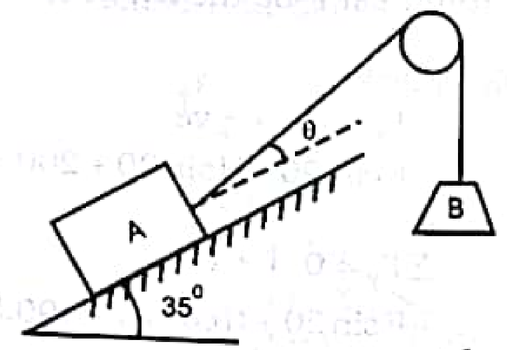
Since slope of road θ = 10° is greater than angle of repose α = 2.86°, the car would slide down if $\mu = 0.05$.

If $\mu = 0.9$ $\therefore$ $\tan\alpha = \mu = 0.9$ or $\alpha = 41.98°$

Since slope of road θ = 10° is smaller than angle of repose α = 41.98°, the car would not slide down if $\mu = 0.9$.

From above we conclude that the car cannot be parked all the year round without sufficient braking system. .... **Ans.**

P8. Block A of weight 2000 N is kept on a plane inclined at 35°. It is connected to weight B by an in-extensible string passing over a smooth pulley. Determine weight of B so that B just moves down. Take θ = 20° and μ = 0.2.



Solution: This is a system of two blocks connected to each other by a rope.

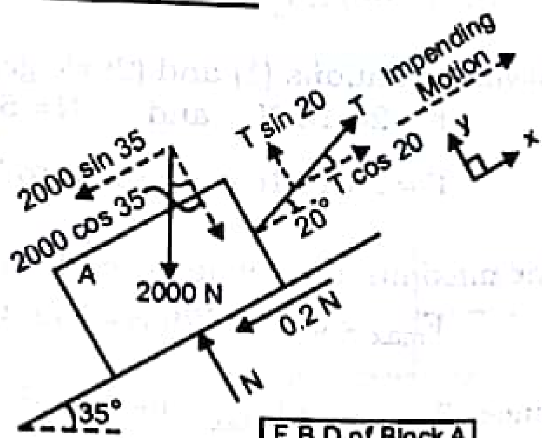
COE – Block A

$\Sigma F_x = 0$

$T\cos 20 - 0.2N - 2000\sin 35 = 0$ .... (1)

$\Sigma F_y = 0$

$T\sin 20 + N - 2000\cos 35 = 0$ .... (2)



F B D of Block A

Solving equations (1) and (2), we get

T = 1462.96 N

Isolating block B

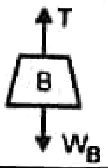
COE – Block B

$$\Sigma F_y = 0$$

$\therefore \quad T - W_B = 0$

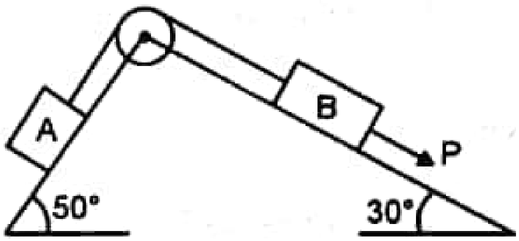
Or $\quad W_B = T$

$\therefore \quad W_B = 1462.96\,\text{N}$ .... **Ans.**



F B D of Block B

P9. Two blocks A and B of weight 500 N and 750 N respectively are connected by a cord that passes over a frictionless pulley as shown in figure. μ between the block A and inclined plane is 0.4 and the between the block B and the inclined plane is 0.3. Determine the force P to be applied to block B to produce the impending motion of block B down the plane. ***(M. U. Dec 09)***



Solution: This is a system of two blocks connected to each other by a rope. As block B impends to slide down the slope, it pulls block A up the slope. Isolating the blocks by cutting the rope.

COE – Block A

$$\Sigma F_y = 0$$

$$N_1 - 500\cos 50 = 0$$

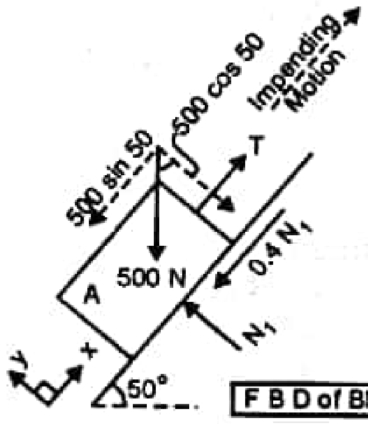
$\therefore \quad N_1 = 321.39\ \text{N}$

$$\Sigma F_x = 0$$

$$T - 500\sin 50 - 0.4N_1 = 0$$

$$T - 500\sin 50 - 0.4 \times 321.39 = 0$$

$\therefore \quad T = 511.6\ \text{N}$



F B D of Block A

COE – Block B

$$\Sigma F_y = 0$$

$$N_2 - 750\cos 30 = 0$$

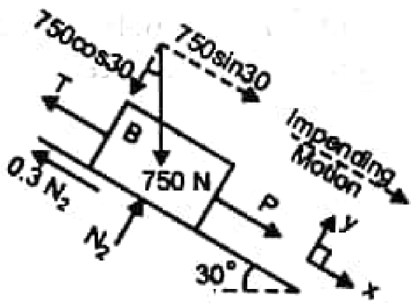
$\therefore \quad N_2 = 649.52\ \text{N}$

$$\Sigma F_x = 0$$

$$P - T - 0.3N_2 + 750\sin 30 = 0$$

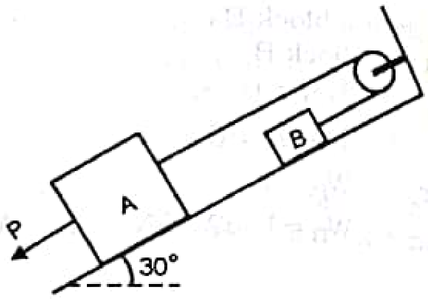
$$P - 511.6 - 0.3 \times 649.52 + 750\sin 30 = 0$$

$\therefore \quad P = 331.45\ \text{N}$ .... **Ans.**



F B D of Block B

P10. Determine the force P to cause motion to impend. Take masses A and B as 9 kg and 4 kg respectively and coefficient of static friction as 0.25. The force P and rope are parallel to the inclined plane. Assume smooth pulley.



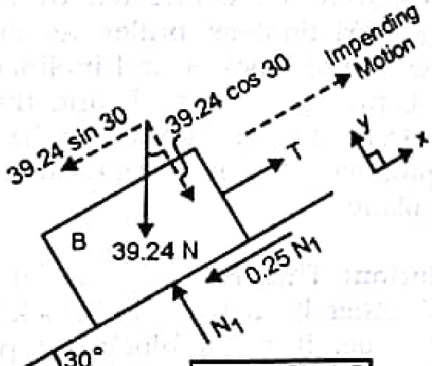
Solution: This is a system of two blocks connected to each other by a rope. As the force P applied to block A, impends to pull it down the plane, the block B impends to travel up the plane.

Isolating Block B

COE – Block B

$$\Sigma F_y = 0$$

$$N_1 - 39.24\cos 30 = 0 \quad \therefore \quad N_1 = 33.98\,N$$



F B D - Block B

$$\Sigma F_x = 0$$

$$T - 39.24\sin 30 - 0.25N_1 = 0$$

$$T - 39.24\sin 30 - 0.25\times 33.98 = 0$$

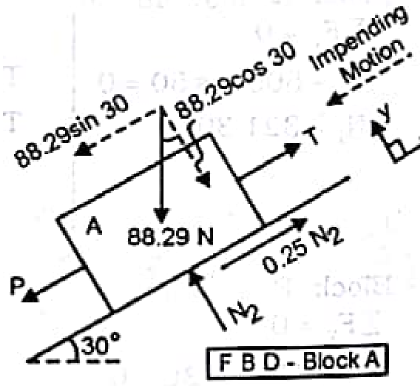
$$\therefore \quad T = 28.11\,N$$

Isolating Block A

COE – Block A

$$\Sigma F_y = 0$$

$$N_2 - 88.29\cos 30 = 0 \quad \therefore \quad N_2 = 76.46\,N$$



F B D - Block A

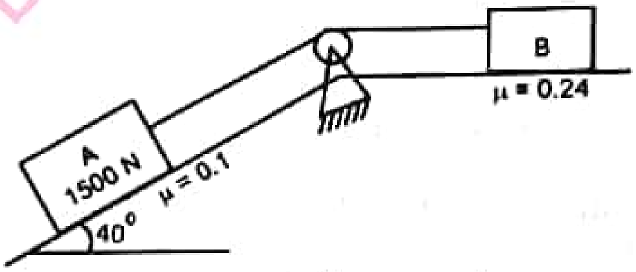
$$\Sigma F_x = 0$$

$$-P + T + 0.25N_2 - 88.29\sin 30 = 0$$

$$-P + 28.11 + 0.25\times 76.46 - 88.29\sin 30 = 0$$

$$\therefore \quad P = 3.08\,N \quad \text{......... Ans.}$$

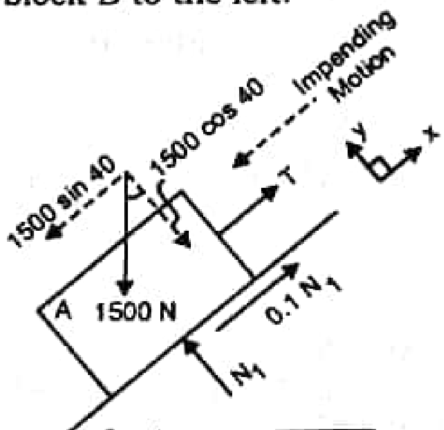
P11. What is the minimum value of mass of block B required to maintain the equilibrium? The rope connecting A and B passes over a frictionless pulley.



Solution: This is a system of two blocks connected to each other by a rope.
Here the block A impends to slide down the plane, pulling block B to the left.
Isolating The blocks by cutting the rope.
Isolating Block A
COE – Block A

$$\Sigma F_y = 0$$
$$N_1 - 1500\cos 40 = 0$$
$$\therefore \quad N_1 = 1149\,N$$

$$\Sigma F_x = 0$$
$$T + 0.1N_1 - 1500\sin 40 = 0$$
$$\therefore \quad T + 0.1 \times 1149 - 1500\sin 40 = 0$$
$$\therefore \quad T = 849.3\ N$$



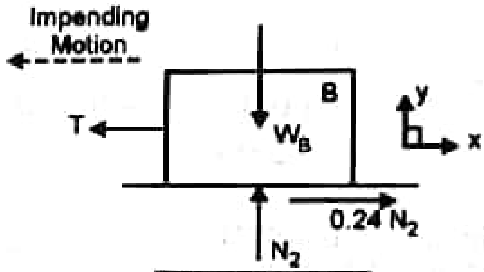
F B D of Block A

Isolating Block B
COE – Block B

$$\Sigma F_x = 0$$
$$-T + 0.24N_2 = 0$$
$$-849.3 + 0.24N_2 = 0 \quad \therefore \quad N_2 = 3538.7\,N$$

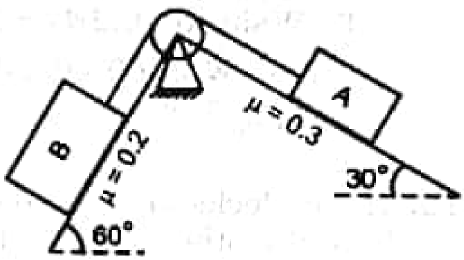
$$\Sigma F_y = 0$$
$$N_2 - W_B = 0 \quad \therefore \quad W_B = 3538.7\ N$$

Or mass of block B = 360.7 kg .... **Ans.**



F B D of Block B

P12. Determine the least and greatest value of weight W of block B for the equilibrium of the whole system. Take weight of block A = 100 N.

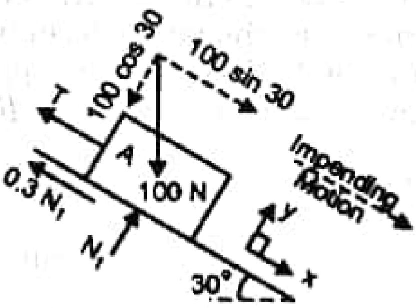


Solution: For least value of W, block A tends to slide down the slope, pulling block B up the plane.

For greatest value of W, block B tends to slide down the slope, pulling block A up the plane.
Any value of W between least and greatest will keep the system of blocks in equilibrium.

a) Let us first find the least value of W
Isolating block A
COE – Block A

$$\Sigma F_y = 0$$
$$N_1 - 100\cos 30 = 0 \quad \therefore \quad N_1 = 86.6\,N$$

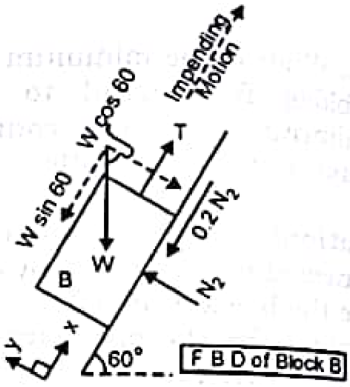


F B D of Block A

$\Sigma F_x = 0$

$-T - 0.3N_1 + 100\sin 30 = 0$

$\therefore \quad -T - 0.3 \times 86.6 + 50 = 0 \quad$ Or $\quad T = 24.02$ N



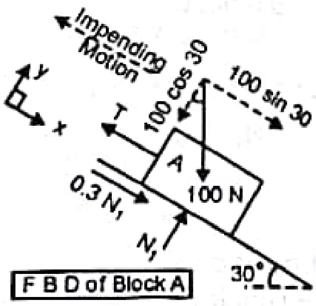
F B D of Block B

Isolating Block B

COE – Block B

$\Sigma F_y = 0$

$N_2 - W\cos 60 = 0 \quad \therefore \; N_2 = 0.5W$



F B D of Block A

$\Sigma F_x = 0$

$-W\sin 60 - 0.2N_2 + T = 0$

$-W\sin 60 - 0.2 \times 0.5W + 24.02 = 0$

$\therefore \quad W_{Least} = 24.86$ N .... **Ans.**

B) Now let us find the greatest value of W. Block B tends to slide down, pulling block A up the plane.

Isolating block A

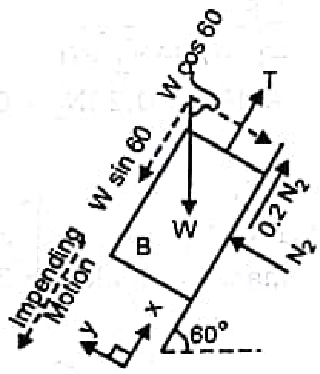
COE – Block A

$\Sigma F_x = 0$

$-T + 0.3N_1 + 100\sin 30 = 0$

$-T + 0.3 \times 86.6 + 50 = 0$

$\therefore \quad T = 75.98$ N



F B D of Block B

Isolating block B

COE – Block B

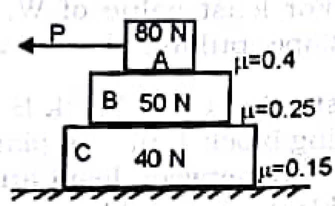
$\Sigma F_x = 0$

$T - W\sin 60 + 0.2N_2 = 0$

$75.98 - W\sin 60 + 0.2 \times 0.5W = 0$

$\therefore \quad W_{greatest} = 99.19$ N .... **Ans.**

P13. Three blocks are placed on the surface one above the other as shown in figure. The static coefficient of friction between the surfaces is shown. Determine the maximum value of P that can be applied before any slipping takes place. ***(M.U. May 08)***

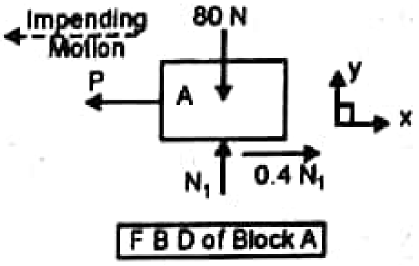


Solution: When force P is applied, any one of the three possibilities may take place.

Possibility (1) – Block A slides over B and B and C remains stationary.

Let us analyse block A alone.

COE – Block A



$$\Sigma F_y = 0$$

$$N_1 - 80 = 0 \qquad \text{Or} \qquad N_1 = 80\,N$$

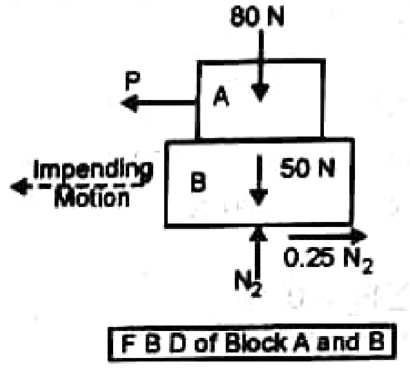
$$\Sigma F_x = 0$$

$$-P + 0.4N_1 = 0$$

$$-P + 0.4 \times 80 = 0 \qquad \text{Or} \qquad P = 32\ N$$ .... for A to slide or B

Possibility (2) – Block A and B remains together and acting as one unit slide over block C which remains stationary.

COE – System of Block A and B



$$\Sigma F_y = 0$$

$$N_2 - 80 - 50 = 0$$

Or $N_2 = 130\,N$

$$\Sigma F_x = 0$$

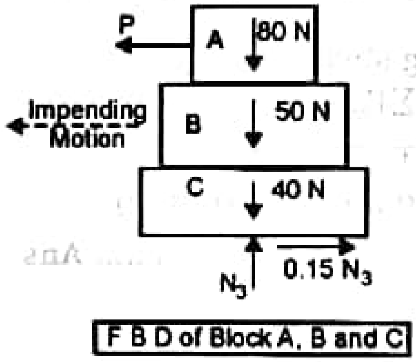
$$-P + 0.25N_2 = 0$$

$$-P + 0.25 \times 130 = 0$$

Or $P = 32.5\ N$ .... for A and B together sliding over C

Possibility (3) – Block A, B and C remains together and acting as a single unit slide over the ground.

COE – System of Block A, B and C



$$\Sigma F_y = 0$$

$$N_3 - 80 - 50 - 40 = 0$$

Or $N_3 = 170\,N$

$$\Sigma F_x = 0$$

$$-P + 0.15N_3 = 0$$

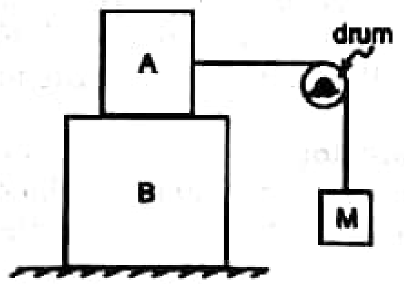
$$-P + 0.15 \times 170 = 0$$

Or $P = 25.5\ N$ .... for A, B and C together sliding over ground

∴ The max. value of P that can be applied is 25.5 N, before any slipping takes place.

.... **Ans.**

P14. The mass of A is 23 kg and mass of B is 36 kg. The coefficient of friction is 0.4 between A and B and 0.2 between ground and block B. Assume smooth drum. Determine the minimum value of mass M at impending motion. ***(M.U. May 14)***

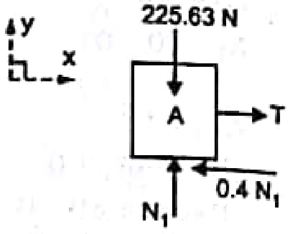


Solution: There are two possibilities. One is that block A moves over block B, while the other possibility is that both blocks A and B move together over the ground.

1st possibility: Let block A move over block B. Let T be the tension in the rope.
Applying COE

$\Sigma F_y = 0$

$N_1 - 225.63 = 0 \quad \therefore \quad N_1 = 225.63\ N$



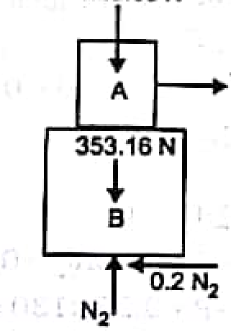
F B D - Block A

$\Sigma F_x = 0$

$T - 0.4 N_1 = 0$

$\therefore \quad T - 0.4 \times 225.63 = 0 \quad$ or $\quad T = 90.25\ N$

2nd possibility: Both blocks A and B move together over the ground.
Applying COE



F B D - Block A & B

$\Sigma F_y = 0$

$N_2 - 225.63 - 353.16 = 0$

$\therefore \quad N_2 = 578.79\ N$

$\Sigma F_x = 0$

$T - 0.2 N_2 = 0$

$\therefore \quad T - 0.2 \times 578.79 = 0 \quad$ or $\quad T = 115.76\ N$

Since tension T required to move A over B is less that T required to move A and B together over the ground, the block A is set in motion at T = 90.25 N

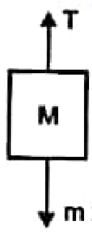
Isolating Mass M

$\Sigma F_y = 0 \quad \uparrow +ve$

$T - M \times g = 0$

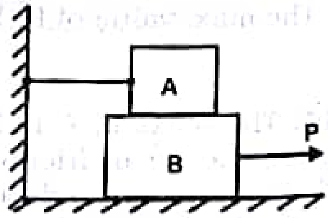
$\therefore \quad 90.25 - M \times 9.81 = 0$

or $\quad M = 9.2\ kg$ .... **Ans.**



F B D - Mass M

P15. Block A weighing 1000 N rests over a block B weighing 2000 N as shown. Block A is tied to a wall with a horizontal string. If $\mu = 0.25$ between blocks A and B and $\mu = 1/3$ between block B and floor, determine the force P needed to move the block if (a) P is horizontal,
(b) P acts at 30° upwards to horizontal. ***(VJTI Dec 11)***



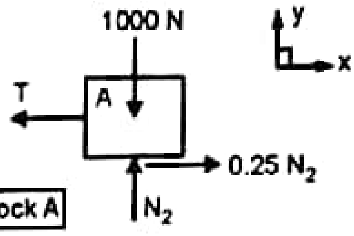
Solution: a) Force P is horizontal.
This is a system of two blocks resting on each other. Isolating the blocks as shown. The contact surface between the two blocks is rough.

COE – Block A

$\Sigma F_y = 0 \uparrow +ve$

$N_2 - 1000 = 0$

$\therefore \quad N_2 = 1000$ N



F B D of block A

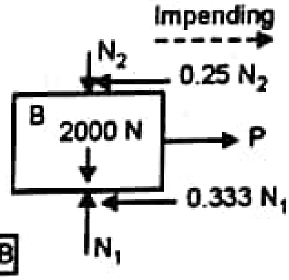
COE – Block B

$\Sigma F_y = 0 \uparrow +ve$

$N_1 - N_2 - 2000 = 0$

$\therefore \quad N_1 - 1000 - 2000 = 0$

or $\quad N_1 = 3000$ N



F B D of block B

$\Sigma F_x = 0 \rightarrow +ve$

$P - 0.25N_2 - 0.333N_1 = 0$

$\therefore \quad P - 0.25 \times 1000 - 0.333 \times 3000 = 0$

or $\quad P = 1250$ N .... **Ans.**

b) Force P acts at 30° to horizontal on block B

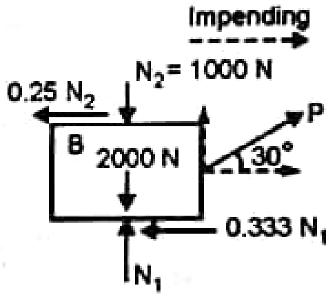
COE – Block B

$\Sigma F_y = 0 \uparrow +ve$

$N_1 - N_2 - 2000 + P\sin 30 = 0$

$N_1 - 1000 - 2000 + P\sin 30 = 0$

$\therefore \quad N_1 + P\sin 30 = 3000$ .... (1)



F B D of block B

$\Sigma F_x = 0 \rightarrow +ve$

$-0.333N_1 + P\cos 30 - 0.25N_2 = 0$

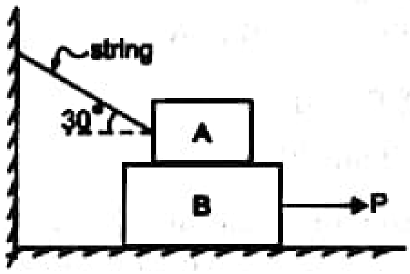
$-0.333N_1 + P\cos 30 - 0.25 \times 1000 = 0$

$\therefore \quad -0.333N_1 + P\cos 30 = 250$ .... (2)

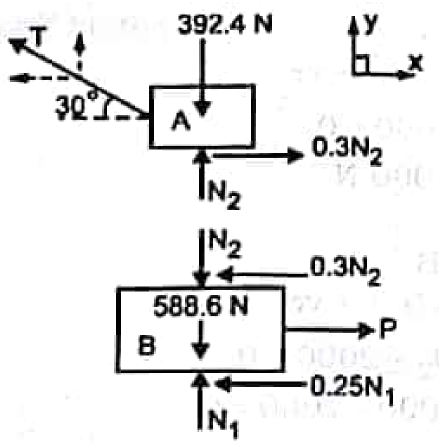
Solving equations (1) and (2) we get

$\therefore \quad N_1 = 2395$ N and $P = 1210.4$ N .... **Ans.**

P16. Find force required to pull block B as shown. Coefficient of friction between A and B is 0.3 and between B and floor is 0.25. Mass of A = 40 kg and B = 60 kg. ***(MU Dec 15)***



Solution: This is a system of two connected blocks, resting over each other. Isolating the blocks as shown.



COE – Block A

$\Sigma F_x = 0 \rightarrow +ve$

$-T\cos 30 + 0.3N_2 = 0$ .... (1)

$\Sigma F_y = 0 \uparrow +ve$

$T\sin 30 + N_2 - 392.4 = 0$ .... (2)

Solving equations (1) and (2)

$T = 115.86$ N and $N_2 = 334.47$ N

COE – Block B

$\Sigma F_y = 0 \uparrow +ve$

$N_1 - N_2 - 588.6 = 0$

$\therefore \quad N_1 - 334.47 - 588.6 = 0$

or $\quad N_1 = 923.07$ N

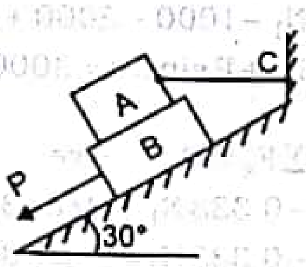
$\Sigma F_x = 0 \rightarrow +ve$

$P - 0.25N_1 - 0.3N_2 = 0$

$\therefore \quad P - 0.25 \times 923.07 - 0.3 \times 334.47 = 0$

or $\quad P = 331.1$ N .... **Ans.**

P17. Block A of mass 27 kg rests on block B of mass 36kg. Block A is restrained from moving by a horizontal rope to the wall at C. What force P, parallel to the plane inclined at 30° with the horizontal is necessary to start B down the plane? Assume μ for all surfaces = 0.33. ***(VJTI May 08)***



Solution: This is system of two blocks resting on each other. Isolating the blocks as shown. The contact surface between the two blocks is rough.

COE – Block A

$\Sigma F_x = 0$

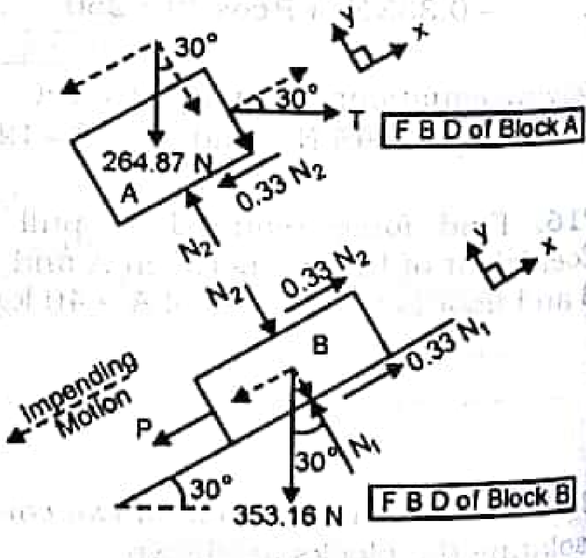
$T\cos 30 - 0.33N_2 - 264.87\sin 30 = 0$ .... (1)

$\Sigma F_y = 0$

$-T\sin 30 + N_2 - 264.87\cos 30 = 0$ .... (2)

Solving equations (1) and (2) we get

$T = 296.9$N and $N_2 = 377.8$ N



COE – Block B

$\Sigma F_y = 0$

$N_1 - N_2 - 353.16\cos 30 = 0$

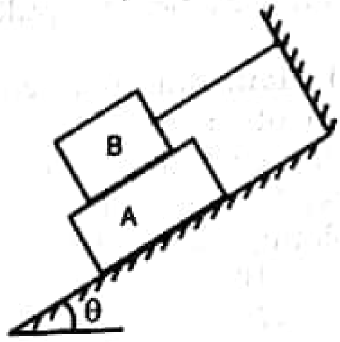
$\therefore \quad N_1 - 377.8 - 353.16\cos 30 = 0 \qquad$ Or $\qquad N_1 = 683.6$ N

$$\Sigma F_x = 0$$

$$-P + 0.33N_2 + 0.33N_1 - 353.16\sin 30 = 0$$

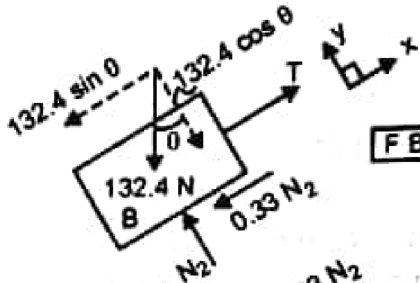
$\therefore \quad -P + 0.33 \times 377.8 + 0.33 \times 683.6 - 353.16\sin 30 = 0$

Or $\quad P = 173.68$ N .... **Ans.**

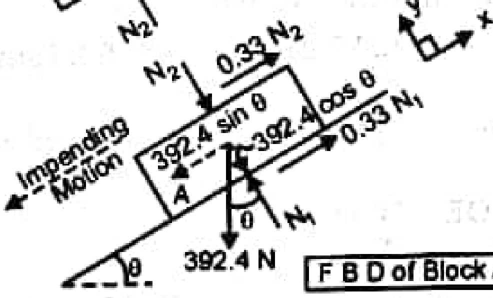


P18. What should be the value of 'θ' so that the motion of block A impends down the plane?
Take m_A = 40 kg and m_B = 13.5 kg.
μ = 1/3 for all surfaces. ***(M. U. Dec 11)***

Solution: This is a system of two blocks resting on each other. Isolating the blocks as shown.



F B D of Block B



F B D of Block A

COE – Block B

$$\Sigma F_y = 0$$

$$N_2 - 132.4\cos\theta = 0$$

$\therefore \quad N_2 = 132.4\cos\theta$ .... (1)

$$\Sigma F_x = 0$$

$$T - 132.4\sin\theta - 0.33N_2 = 0$$

$$T - 132.4\sin\theta - 0.33 \times 132.4\cos\theta = 0$$

$\therefore \quad T = 132.4\sin\theta + 43.69\cos\theta = 0$ .... (2)

COE – Block A

$$\Sigma F_y = 0$$

$$N_1 - N_2 - 392.4\cos\theta = 0$$

$$N_1 - 132.4\cos\theta - 392.4\cos\theta = 0$$

or $\quad N_1 = 524.8\cos\theta$ .... (3)

$$\Sigma F_x = 0$$

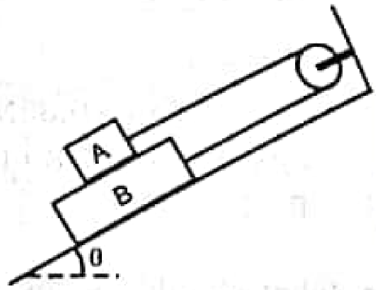
$$-392.4\sin\theta + 0.33N_1 + 0.33N_2 = 0$$

$$-392.4\sin\theta + 0.33 \times 524.8\cos\theta + 0.33 \times 132.4\cos\theta = 0$$

$\therefore \quad 392.4\sin\theta = 219.06\cos\theta$

$\therefore \quad \tan\theta = 0.5582 \qquad$ or $\qquad \theta = 29.17°$ .... **Ans.**

P19. Block A has a mass of 25 kg and block B has a mass of 15 kg. Knowing $\mu_s = 0.2$ for all surfaces, determine value of θ for which motion impends. Assume frictionless pulley.



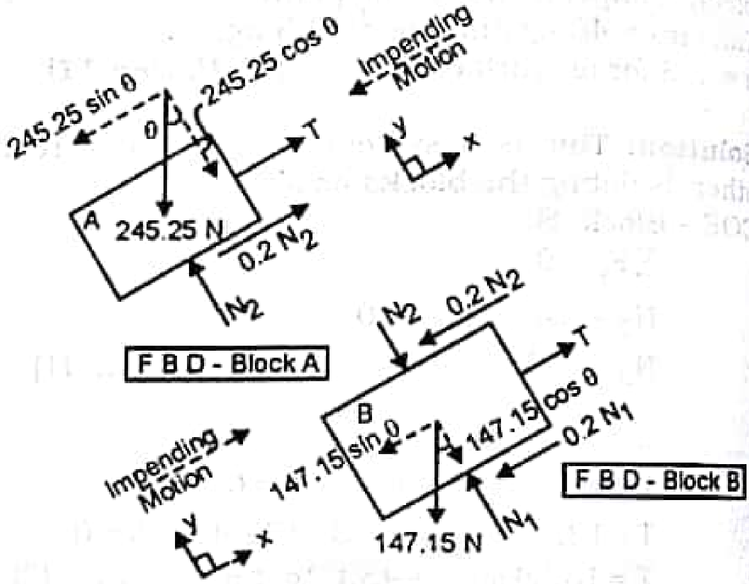
Solution: This is a system of two blocks connected to each other by a rope. Let us assume at a particular value of θ, block A impends to slide down, this initiates block B to slide up.

Isolating the blocks

COE – Block A

$\Sigma F_x = 0$

$T + 0.2N_2 - 245.25\sin\theta = 0$ .... (1)



$\Sigma F_y = 0$

$N_2 - 245.25\cos\theta = 0$

Or $N_2 = 245.25\cos\theta$ .... (2)

Eliminating N_2 we get

$T + 0.2(245.25\cos\theta) - 245.25\sin\theta = 0$

Or $T = 245.25\sin\theta - 49.05\cos\theta$ .... (A)

COE – Block B

$\Sigma F_x = 0$

$T - 0.2N_1 - 0.2N_2 - 147.15\sin\theta = 0$ ... (3)

$\Sigma F_y = 0$

$N_1 - N_2 - 147.15\cos\theta = 0$

Substituting for N_2

$N_1 - 245.25\cos\theta - 147.15\cos\theta = 0$

Or $N_1 = 392.4\cos\theta$ .... (4)

Eliminating N_1 and N_2 from equation (3), we get

$T - 0.2(392.4\cos\theta) - 0.2(245.25\cos\theta) - 147.15\sin\theta = 0$

Or $T = 127.53\cos\theta + 147.15\sin\theta$ .... (B)

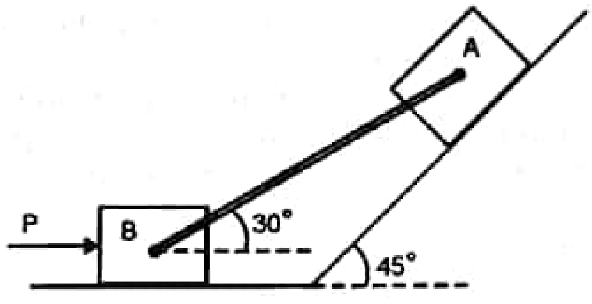
Equating equations (A) and (B), we get

$$245.25\sin\theta - 49.05\cos\theta = 127.53\cos\theta + 147.15\sin\theta$$

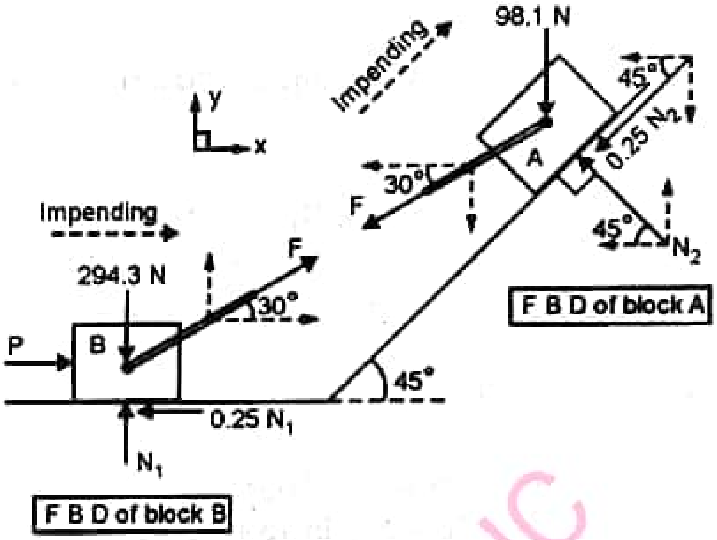
$$98.1\sin\theta = 176.58\cos\theta$$

$\therefore$ $\theta = 60.94°$ .... **Ans.**

P20. A block A of mass 10 kg rests on a rough inclined plane as shown. This block is connected to another block B of mass 30 kg resting on a rough horizontal plane by a rigid bar inclined at an angle of 30°. Find the horizontal P required to be applied to block B to just move block A in the upward direction. Take $\mu = 0.25$ for all contact surfaces. ***(SPCE Nov 12)***



Solution: This is a system of two blocks connected by a rod. As the block B is pushed to the right, the block A slides up the slope.
Isolating the blocks by cutting the rod. On doing so, the force F in the rod is exposed as shown.
Let us assume the force F in the rod is tensile.



COE – Block A

$$\Sigma F_x = 0 \ \rightarrow + \text{ve}$$

$$-F\cos 30 - N_2\cos 45 - 0.25N_2\cos 45 = 0$$

$$\therefore\ -F\cos 30 - 0.884\,N_2 = 0 \quad \text{......... (1)}$$

$$\Sigma F_y = 0 \ \uparrow + \text{ve}$$

$$-F\sin 30 + N_2\sin 45 - 0.25N_2\sin 45 - 98.1 = 0$$

$$\therefore\ -F\sin 30 + 0.53\,N_2 = 98.1 \quad \text{......... (2)}$$

Solving equations (1) and (2), we get

$\therefore$ $N_2 = 94.26$ N and $F = -96.22$ N or $F = 96.22$ N (Compressive)

COE – Block B

$$\Sigma F_y = 0 \ \uparrow + \text{ve}$$

$$N_1 + F\sin 30 - 294.3 = 0$$

$$\therefore\ N_1 + (-96.22)\sin 30 - 294.3 = 0$$

or $N_1 = 342.4$ N

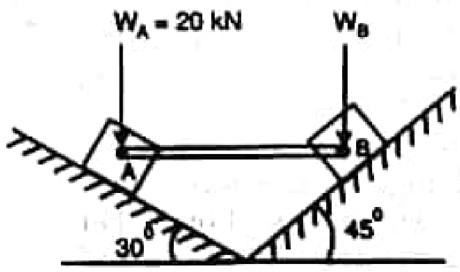
$$\Sigma F_x = 0 \ \rightarrow + \text{ve}$$

$$P - 0.25N_1 + F\cos 30 = 0$$

$$\therefore\ P - 0.25 \times 342.4 + (-94.26)\cos 30 = 0$$

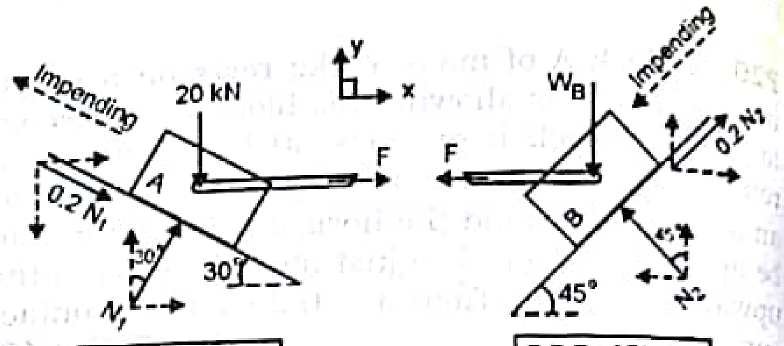
or $P = 167.2$ N .... **Ans.**

P21. Find the maximum value of W_B for the rod AB to remain horizontal. Also find the corresponding axial force in the rod.
Take $\mu = 0.2$ for all contact surfaces.



Solution: This is a system of two blocks connected by a rod.

For the condition that weight of block B i.e. W_B is maximum keeping the rod AB horizontal, the block B impends to slide down, causing impending motion of block A up the slope. Isolating the blocks by cutting the rod. On doing so, the force F (assumed tensile) in the rod is exposed as shown.



F B D of Block A   F B D of Block B

COE – Block A

$\Sigma F_y = 0$

$N_1 \cos 30 - 0.2N_1 \sin 30 - 20 = 0$ $\quad \therefore \quad N_1 = 26.11$ kN

$\Sigma F_x = 0$

$F + N_1 \sin 30 + 0.2 N_1 \cos 30 = 0$

$F + 26.11 \sin 30 + 0.2 \times 26.11 \cos 30 = 0$

$\therefore \quad F = -17.576$ kN $\quad$ Or $\quad F = 17.576$ kN (Comp.)

COE – Block B

$\Sigma F_x = 0$

$-F - N_2 \sin 45 + 0.2 N_2 \cos 45 = 0$

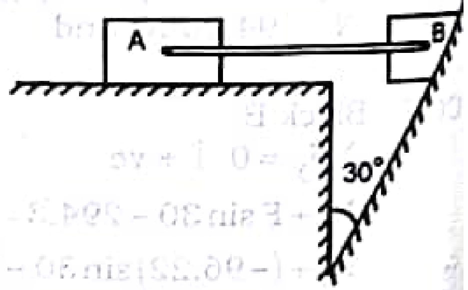
$-(-17.576) - N_2 \sin 45 + 0.2 \times N_2 \cos 45 = 0$ $\quad \therefore \quad N_2 = 31.07$ kN

$\Sigma F_y = 0$

$-W_B + 0.2 N_2 \sin 45 + N_2 \cos 45 = 0$

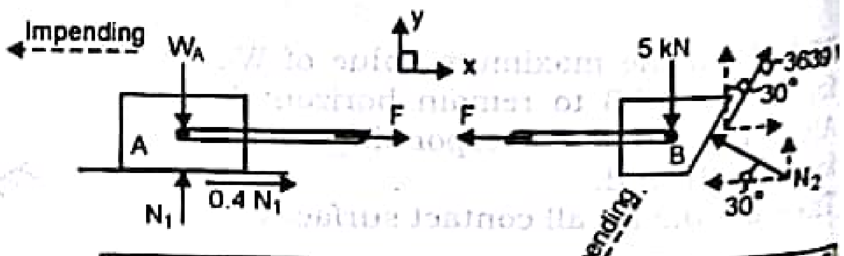
$-W_B + 0.2 \times 31.07 \sin 45 + 31.07 \cos 45 = 0$ $\quad \therefore \quad W_B = 26.36$ kN .... **Ans.**

P22. Two blocks connected by a horizontal link AB are supported on two rough planes, μ between block A and horizontal surface is 0.4. The limiting angle of friction between block B and inclined plane is 20°. What is the smallest weight W of the block A for which equilibrium of the system can exist, if the weight of block B is 5 kN?



(VJTI Mar 11)

Solution: This is a system of two blocks connected by a rod. For the condition that weight of block A W_A is least, the block B impends to slide down, causing impending motion of block A to the left. Isolating the blocks by cutting the rod. On doing so, the force F in the rod is exposed as shown. Let us assume the force F in the rod is tensile in nature.



F B D of Block A   F B D of Block B

Knowing $\tan\phi = \mu$ $\therefore$ $\tan 20 = \mu$

Or $\mu = 0.3639$ ... for Block B

COE – Block B

$\Sigma F_y = 0$

$N_2 \sin 30 + 0.3639 N_2 \cos 30 - 5 = 0$ $\therefore$ $N_2 = 6.134$ kN

$\Sigma F_x = 0$

$-F - N_2 \cos 30 + 0.3639 N_2 \sin 30 = 0$

$-F - 6.134 \cos 30 + 0.3639 \times 6.134 \sin 30 = 0$

$\therefore$ $F = -4.196$ kN Or $F = 4.196$ kN (compressive) .... **Ans.**

COE – Block A

$\Sigma F_x = 0$

$F + 0.4 N_1 = 0$

$-4.196 + 0.4 N_1 = 0$

Or $N_1 = 10.49$ kN

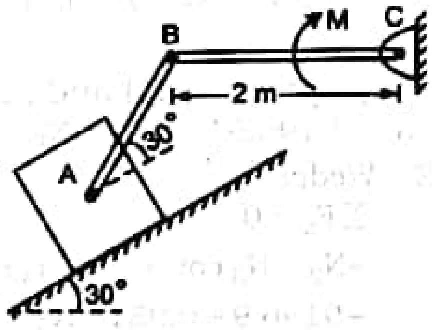
$\Sigma F_y = 0$

$N_1 - W_A = 0$

$10.49 - W_A = 0$

$\therefore$ $W_A = 10.49$ kN .... **Ans.**

P23. For the figure shown mass of block A is 200 kg. Linkages are smooth. Rod BC is horizontal. The coefficient of friction between block A and plane is 0.2. Calculate momentum to just start the motion of block A up the plane. ***(KJS May 15)***



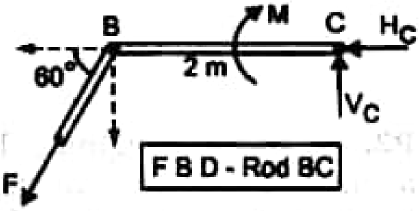
Solution: This is a system of connected bodies consisting of block A and Rod AB and BC.

Isolating Block A by cutting rod AB. Let F be the force in rod AB.

COE – Block

$\Sigma F_x = 0$

$F \cos 30 - 0.2 N - 1962 \sin 30 = 0$ ... (1)



F B D - Rod BC

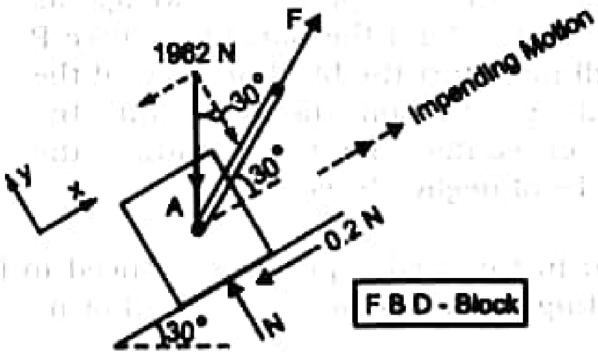
$\Sigma F_y = 0$

$F \sin 30 + N - 1962 \cos 30 = 0$ ... (2)

Solving equations (1) and (2), we get

$F = 1367.3$ N

and $N = 1015.5$ N



F B D - Block

COE – Rod BC

$\Sigma M_C = 0$ ↺ + ve

$+(F \sin 60 \times 2) - M = 0$

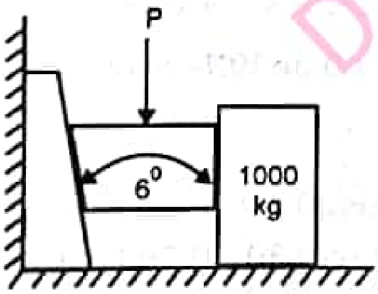
$+(1367.3 \sin 60 \times 2) - M = 0$

$\therefore$ $M = 2368.2$ Nm .... **Ans.**

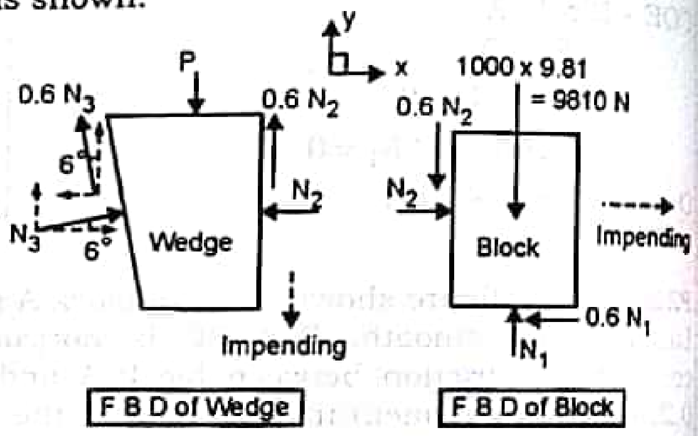
Exercise 4.2

Wedges

P1. The horizontal position of the 1000 kg block is adjusted by 6° wedge. If coefficient of friction for all surfaces is 0.6, determine the least value of force P required to move the block. ***(NMIMS June 07)***



Solution: In this wedge problem we need to find force P required to just shift the block to the right. Isolating the wedge and block as shown.



COE – Block

$$\Sigma F_x = 0$$

$$N_2 - 0.6N_1 = 0 \quad \text{......... (1)}$$

$$\Sigma F_y = 0$$

$$-0.6N_2 + N_1 - 9810 = 0 \quad \text{......... (2)}$$

Solving equations (1) and (2) we get

$$N_2 = 9196.9\,N \quad \text{and} \quad N_1 = 15328\,N$$

COE – Wedge

$$\Sigma F_x = 0$$

$$-N_2 + N_3\cos 6 - 0.6N_3 \sin 6 = 0$$

$$-9196.9 + 0.9318\ N_3 = 0$$

$$\therefore \quad N_3 = 9870\,N$$

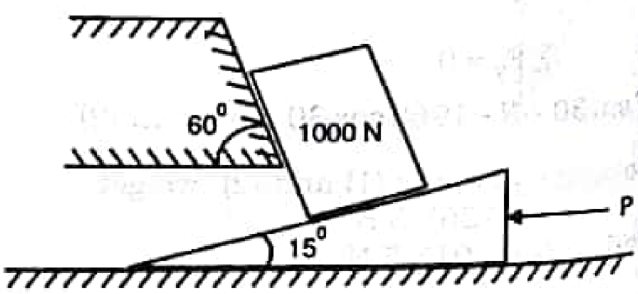
$$\Sigma F_y = 0$$

$$-P + 0.6N_2 + N_3 \sin 6 + 0.6N_3 \cos 6 = 0$$

$$-P + 0.6\times 9196.9 + 9870 \sin 6 + 0.6\times 9870 \cos 6 = 0$$

$$\therefore \quad P = 12439\,N \quad \text{.........} \ \textbf{Ans.}$$

P2. A block weighing 1000 N is raised against a surface inclined at 60° to the horizontal by means of a 15° wedge as shown in figure. Find the horizontal force P which will just start the block to move if the coefficient of friction between all the surfaces of contact be 0.2. Assume the wedge to be of negligible weight.



Solution: In this wedge problem we need to find force P required to just lift the block up. Isolating the wedge and block as shown.

COE – Block

$\Sigma F_x = 0$

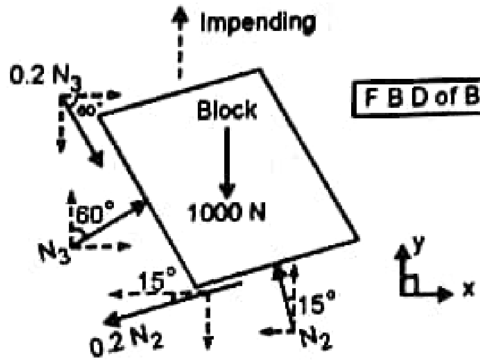
$-N_2 \sin 15 - 0.2 N_2 \cos 15 + N_3 \sin 60 + 0.2 N_3 \cos 60 = 0$

$-0.452 N_2 + 0.966 N_3 = 0$ .... (1)

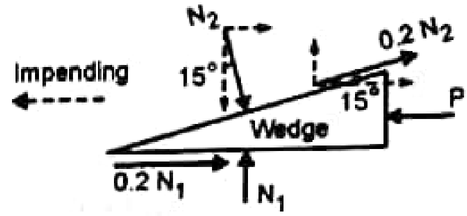
$\Sigma F_y = 0$

$N_2 \cos 15 - 0.2 N_2 \sin 15 + N_3 \cos 60 - 0.2 N_3 \sin 60 - 1000 = 0$

$0.9142 N_2 + 0.3268 N_3 = 1000$ .... (2)



F B D of Block



F B D of Wedge

Solving equations (1) and (2) we get

$N_2 = 937.1 N$ and $N_3 = 438.5 N$

COE – Wedge

$\Sigma F_y = 0$

$N_1 - N_2 \cos 15 + 0.2 N_2 \sin 15 = 0$

$N_1 - 937.1 \cos 15 + 0.2 \times 937.1 \sin 15 = 0$

$\therefore \quad N_1 = 856.7 \text{ N}$

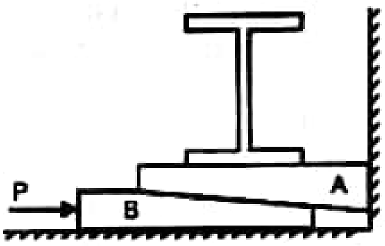
$\Sigma F_x = 0$

$-P + 0.2 N_1 + N_2 \sin 15 + 0.2 N_2 \cos 15 = 0$

$-P + 0.2 \times 856.7 + 937.1 \sin 15 + 0.2 \times 937.1 \cos 15 = 0$

$\therefore \quad P = 594.9 \text{ N}$ .... **Ans.**

P3. The vertical position of the 200 kg mass I section is being adjusted by two 15° wedges as shown. Find force P to just raise the mass. Take $\mu = 0.2$ for all surfaces.



Solution: In this wedge problem we need to find force P required to just lift the 200 kg I section up. Isolating the wedges A and B as shown.

COE – Wedge A

$\Sigma F_x = 0$

$N_3 \sin 15 + 0.2 N_3 \cos 15 - N_2 = 0$

$0.452 N_3 - N_2 = 0$ .... (1)

$\Sigma F_y = 0$

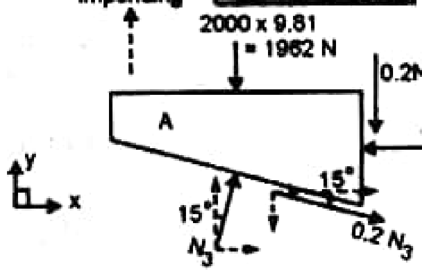
$N_3 \cos 15 - 0.2 N_3 \sin 15 - 0.2 N_2 - 1962 = 0$

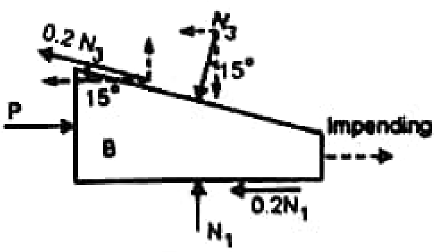
$0.9142 N_3 - 0.2 N_2 = 1962$ .... (2)

Solving equations (1) and (2) we get

$N_3 = 2381.6 N$ and $N_2 = 1076.5 N$

F B D of Wedge A





F B D of Wedge B

COE – Wedge B

$$\Sigma F_y = 0$$

$$N_1 - N_3 \cos 15 + 0.2 N_3 \sin 15 = 0$$

$$N_1 - 2381.6 \cos 15 + 0.2 \times 2381.6 \sin 15 = 0$$

$$\therefore \quad N_1 = 2177.2 \text{ N}$$

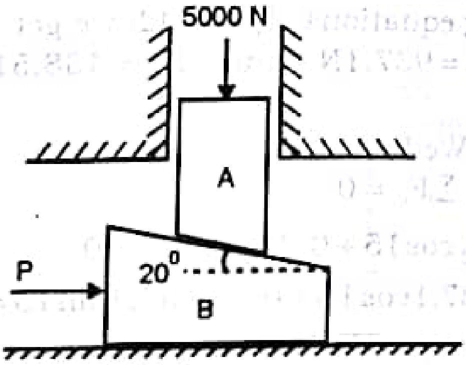
$$\Sigma F_x = 0$$

$$P - 0.2 N_1 - 0.2 N_3 \cos 15 - N_3 \sin 15 = 0$$

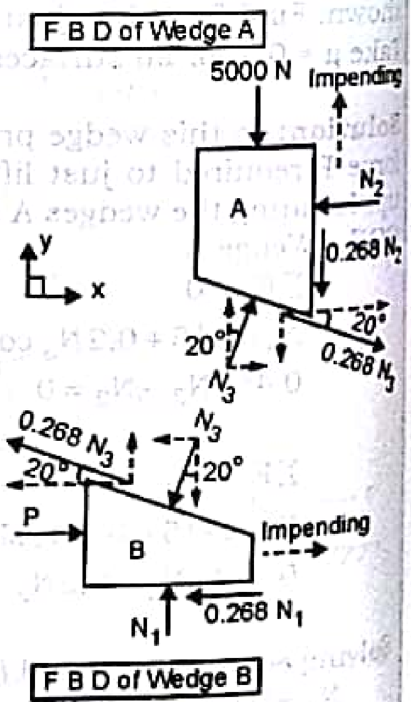
$$P - 0.2 \times 2177.2 - 0.2 \times 2381.6 \cos 15 - 2381.6 \sin 15 = 0$$

$$\therefore \quad P = 1512 \text{ N}$$ .... **Ans.**

P4. Wedge A supports a load of W = 5000 N which is to be raised by forcing the wedge B under it. The angle of friction for all surfaces is 15°. Determine the necessary force P to initiate upward motion of the load. Neglect the weight of the wedges.
Hint: Wedge A comes in contact with the right wall, as the load gets lifted.



Solution: In this wedge problem we need to find force P required to just lift the 5000 N load up. Isolating the wedges A and B as shown.
Knowing $\tan \phi = \mu \ \therefore \ \tan 15 = \mu$ or $\mu = 0.268$



COE – Wedge A

$$\Sigma F_x = 0$$

$$N_3 \sin 20 + 0.268 N_3 \cos 20 - N_2 = 0$$

$$0.5938 N_3 - N_2 = 0 \quad \text{.........(1)}$$

$$\Sigma F_y = 0$$

$$N_3 \cos 20 - 0.268 N_3 \sin 20 - 0.268 N_2 - 5000 = 0$$

$$0.848 N_3 - 0.268 N_2 = 5000 \quad \text{.........(2)}$$

Solving equations (1) and (2) we get

$$N_3 = 7258.2 \text{N} \quad \text{and} \quad N_2 = 4310 \text{N}$$

COE – Wedge B

$$\Sigma F_y = 0$$

$$N_1 - N_3 \cos 20 + 0.268 N_3 \sin 20 = 0$$

$$N_1 - 7258.2 \cos 20 + 0.268 \times 7258.2 \sin 20 = 0$$

$$\therefore \quad N_1 = 6155.2 \text{ N}$$

$\Sigma F_x = 0$

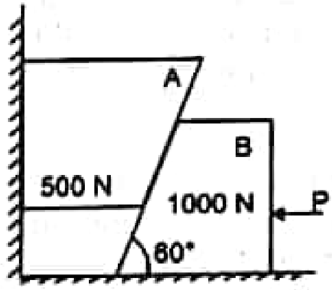
$P - 0.268N_1 - 0.268N_3 \cos 20 - N_3 \sin 20 = 0$

$P - 0.268 \times 6155.2 - 0.268 \times 7258.2 \cos 20 - 7258.2 \sin 20 = 0$

$\therefore \quad P = 5959.9$ N .... **Ans.**

P5. Two blocks A and B are resting against the wall and floor as shown in figure. Find minimum value of P that will hold the system in equilibrium. $\mu = 0.25$ at the floor, $\mu = 0.3$ at the wall and $\mu = 0.2$ between the blocks.

(VJTI Nov 09, M.U. Dec 12)



Solution: In this wedge problem wedge A under is own weight impends to move down pushing wedge B to the right. Force P prevents this and hold the system in equilibrium.

COE – Wedge A

$\Sigma F_x = 0$

$N_2 + 0.2N_3 \cos 60 - N_3 \sin 60 = 0$

$\therefore N_2 - 0.766N_3 = 0$ .... (1)

$\Sigma F_y = 0$

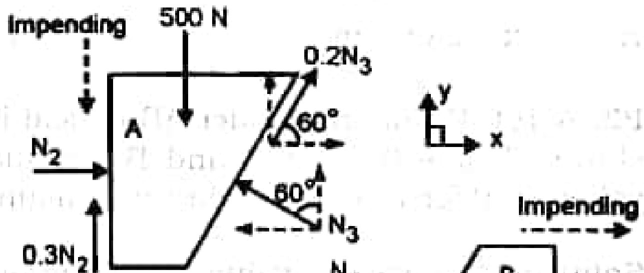
$0.3N_2 + 0.2N_3 \sin 60 + N_3 \cos 60 - 500 = 0$

$0.3N_2 + 0.6732N_3 = 500$ .... (2)

Solving equations (1) and (2) we get

$N_2 = 424$ N and $N_3 = 553.7$ N

F B D of Wedge A



COE – Wedge B

$\Sigma F_y = 0$

$-N_3 \cos 60 - 0.2N_3 \sin 60 + N_1 - 1000 = 0$

$-553.7 \cos 60 - 0.2 \times 553.7 \sin 60 + N_1 - 1000 = 0$

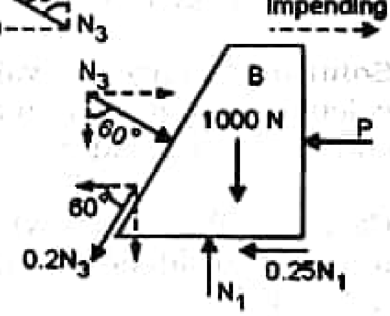
$\therefore \quad N_1 = 1372.7$ N

$\Sigma F_x = 0$

$-P - 0.25N_1 + N_3 \sin 60 - 0.2N_3 \cos 60 = 0$

$-P - 0.25 \times 1372.7 + 553.7 \sin 60 - 0.2 \times 553.7 \cos 60 = 0$

$\therefore \quad P = 80.97$ N .... **Ans.**

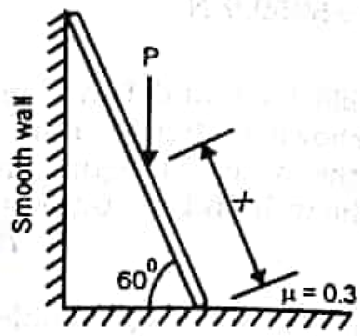


F B D of Wedge B

Exercise 4.3

Ladders

P1. A person of weight P = 600 N ascends the 5 m ladder of weight 400 N as shown. How far up the ladder may the person climb before sliding motion of ladder takes place.



Solution: For maximum distance 'x' the person climes, the ladder impends to slip down and move away from the wall.

COE – ladder

$$\Sigma F_y = 0 \uparrow +ve$$

$$N - 400 - 600 = 0$$

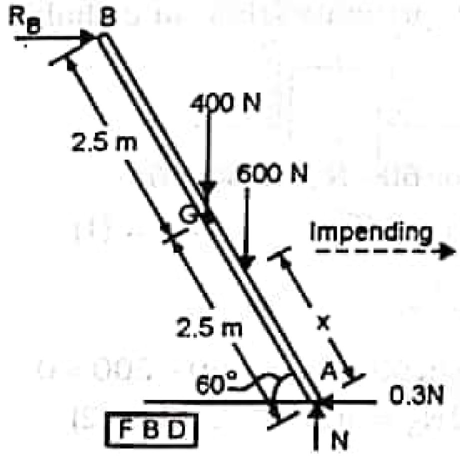
$$\therefore \quad N = 1000 \text{ N}$$

$$\Sigma F_x = 0 \rightarrow +ve$$

$$R_B - 0.3N = 0$$

$$R_B - 0.3 \times 1000 = 0$$

$$\therefore \quad R_B = 300 \text{ N}$$



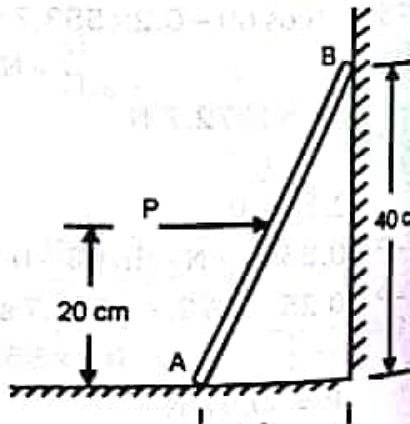
$$\Sigma M_A = 0 \circlearrowleft +ve$$

$$-(R_B \times 5 \sin 60) + (400 \times 2.5 \cos 60) + (600 \times x \cos 60) = 0$$

$$-(300 \times 5 \sin 60) + 500 + 300x = 0$$

$$\therefore \quad x = 2.663 \text{ m} \quad \text{......... Ans.}$$

P2. A 100 N uniform ladder AB is held in equilibrium as shown. If μ = 0.15 at A and B, calculate the range of values of P for which equilibrium is maintained.



Solution: Range of values of P implies we need to calculate minimum and maximum value of P required for equilibrium of ladder.

Case (1) $P_{minimum}$: When P is minimum, the ladder impends to slip down and away from the wall.

COE – ladder

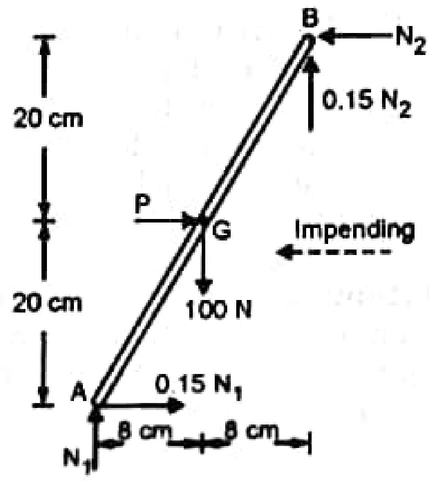
$$\Sigma F_y = 0 \uparrow +ve$$

$$N_1 + 0.15N_2 - 100 = 0 \quad \text{......... (1)}$$

$\Sigma M_G = 0$ ↺ + ve

$0.15N_1 \times 20 - N_1 \times 8 + N_2 \times 20 + 0.15N_2 \times 8 = 0$

$-5N_1 + 21.2N_2 = 0$ .... (2)



F B D for $P_{minimum}$

Solving equations (1) and (2) we get,

$N_1 = 96.58\,N$ and $N_2 = 22.78\,N$

$\Sigma F_x = 0$ → + ve

$0.15N_1 - N_2 + P = 0$

$0.15 \times 96.58 - 22.78 + P = 0$

∴ $P_{minimum} = 8.293$ N .... **Ans.**

Case (2) $P_{maximum}$: When P is maximum, the ladder impends to slip up and towards the wall.

COE – ladder

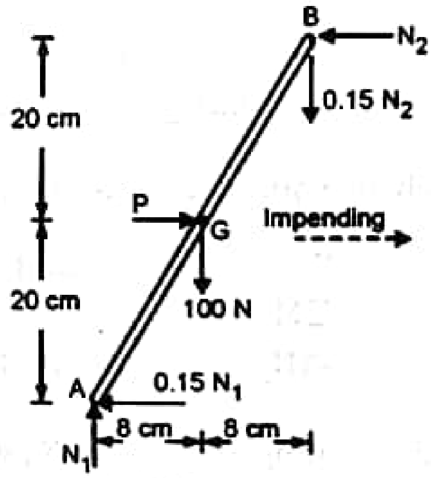
$\Sigma F_y = 0$ ↑ + ve

$N_1 - 0.15N_2 - 100 = 0$ .... (3)

$\Sigma M_G = 0$ ↺ + ve

$-N_1 \times 8 - 0.15N_1 \times 20 + N_2 \times 20 - 0.15N_2 \times 8 = 0$

$-11N_1 + 18.8N_2 = 0$ .... (4)



F B D for $P_{maximum}$

Solving equations (3) and (4) we get,

$N_1 = 109.6\,N$ and $N_2 = 64.14\,N$

$\Sigma F_x = 0$ → + ve

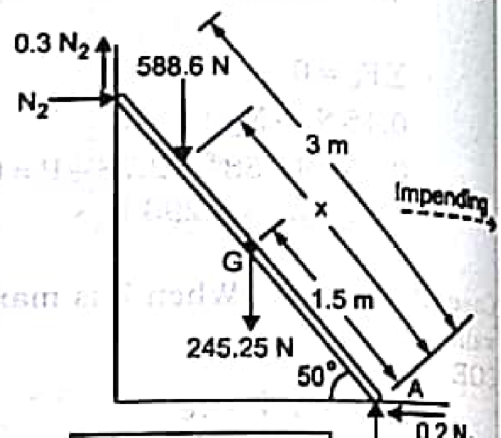
$P - 0.15N_1 - N_2 = 0$

$P - 0.15 \times 109.6 - 64.14 = 0$

∴ $P_{maximum} = 80.58$ N .... **Ans.**

P3. A ladder AB of length 3 m and mass 25 kg is resting against a vertical wall and a horizontal floor. The ladder makes an angle 50° with the floor. A man of mass 60 kg tries to climb the ladder. (a) How much distance along the ladder he will be able to climb if μ between ladder and floor is 0.2 and between ladder and wall is 0.3.
(b) also find the angle the ladder should make with the horizontal such that the man climb till the top of the ladder. **(M.U Dec 14)**

Solution: (a) Let x be the maximum distance the person of mass 60 kg = 588.6 N weight climbs, before causing the ladder to slip down and away from the wall.



F B D - Person at x metre from bottom

COE – ladder

$$\Sigma F_x = 0 \rightarrow +ve$$

$$-0.2N_1 + N_2 = 0 \quad \text{.........} (1)$$

$$\Sigma F_y = 0 \uparrow +ve$$

$$N_1 + 0.3N_2 - 588.6 - 245.25 = 0 \quad \text{.........} (2)$$

Solving equations (1) and (2), we get

$$N_1 = 786.6 \text{ N} \quad \text{and} \quad N_2 = 157.3 \text{ N}$$

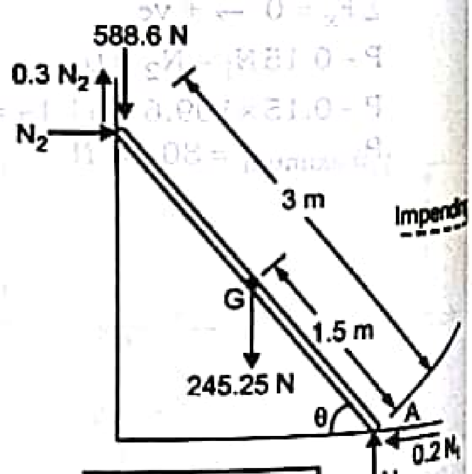
$$\Sigma M_A = 0 \circlearrowleft +ve$$

$$-(N_2 \times 3\sin 50) - (0.3N_2 \times 3\cos 50) + (588.6 \times x\cos 50) + (245.25 \times 1.5\cos 50) = 0$$

Substituting $N_2 = 157.3$ N, we get

$\therefore \quad \boxed{x = 0.571 \text{ m}}$ .... **Ans.**

b) Let the person reach the top of ladder kept at minimum angle θ. The impending motion of ladder is away from the wall.



F B D - Person at top of ladder

COE – ladder

$$\Sigma M_A = 0 \circlearrowleft +ve$$

$$-(N_2 \times 3\sin\theta) - (0.3N_2 \times 3\cos\theta)$$

$$+ (588.6 \times 3\cos\theta) + (245.25 \times 1.5\cos\theta) = 0$$

Substituting $N_2 = 157.3$ N, we get

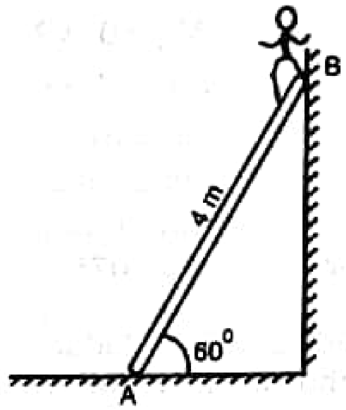
$$-(157.3 \times 3\sin\theta) - (0.3 \times 157.3 \times 3\cos\theta)$$

$$+ (588.6 \times 3\cos\theta) + (245.25 \times 1.5\cos\theta) = 0$$

$\therefore \quad -471.9\sin\theta + 1992\cos\theta = 0$

$\therefore \quad \tan\theta = 4.22 \quad \text{or} \quad \theta = 76.67°$ .... **Ans.**

P4. A 150 N uniform ladder 4 m long supports a 500 N weight person at its top. Assuming the wall to be smooth, find the minimum coefficient of friction which is required at the bottom rough surface to prevent the ladder from slipping.



Solution: For minimum value of μ, the ladder impends to slip down and away from the wall. The frictional force at A is μN.

COE – ladder

$\Sigma F_y = 0 \uparrow +ve$

$N - 150 - 500 = 0$

$\therefore \quad N = 650 \text{ N}$

$\Sigma M_A = 0 \circlearrowleft +ve$

$-(150 \times 2\cos 60) - (500 \times 4\cos 60) + (R_B \times 4\sin 60) = 0$

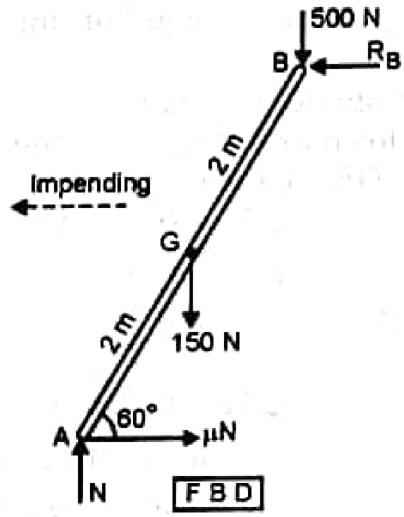
$\therefore \quad R_B = 332 \text{ N}$

$\Sigma F_x = 0 \rightarrow +ve$

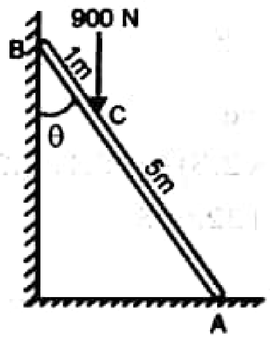
$\mu N - R_B = 0$

$\mu \times 650 - 332 = 0$

$\therefore \quad \mu = 0.5108$ .... **Ans.**



P5. The ladder shown is 6m long and is supported by a horizontal floor and a vertical wall. μ between floor and ladder is 0.4 and between wall and ladder is 0.25. The weight of ladder is 200 N. The ladder also supports a vertical load of 900 N at C. Determine the greatest value of θ for which the ladder may be placed without slipping. ***(VJTI Nov 10)***



Solution: For greatest value of θ, the ladder impends to slip down and away from the wall.

COE – ladder

$\Sigma F_x = 0 \rightarrow +ve$

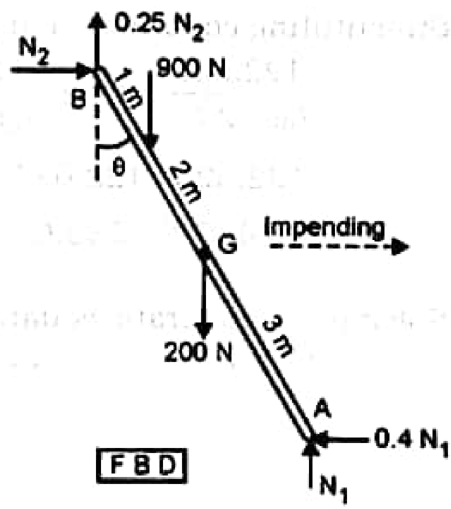
$-0.4 N_1 + N_2 = 0$ .... (1)

$\Sigma F_y = 0 \uparrow +ve$

$N_1 + 0.25 N_2 - 900 - 200 = 0$ .... (2)

Solving equation (1) and (2), we get

$N_1 = 1000 \text{N}$ and $N_2 = 400 \text{N}$



$$\Sigma M_B = 0 \quad \circlearrowleft + ve$$

$$-(900 \times 1 \sin\theta) - (200 \times 3 \sin\theta) - (0.4 N_1 \times 6 \cos\theta) + (N_1 \times 6 \sin\theta) = 0$$

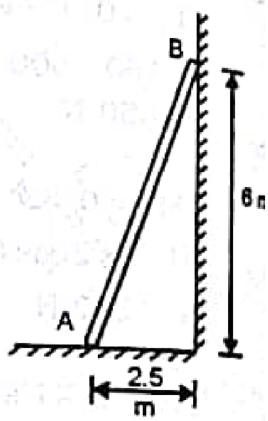
$$-900 \sin\theta - 600 \sin\theta - 2.4 \times 1000 \times \cos\theta + 1000 \times 6 \sin\theta = 0$$

$$4500 \sin\theta = 2400 \cos\theta$$

$\therefore \quad \tan\theta = 0.5333$

or $\quad \theta = 28.07°$ .... **Ans.**

P6. A 6.5 m ladder AB of mass 10kg leans against a wall as shown. Assuming that the coefficient of static friction μ is the same at both surface of contact, determine the smallest value of μ for which equilibrium can be maintained. ***(SPCE Dec 10)***



Solution: For smallest value of μ, the ladder impends to slide down and away from the wall.

COE – ladder

$$\Sigma F_x = 0 \rightarrow + ve$$

$$\mu N_1 - N_2 = 0$$

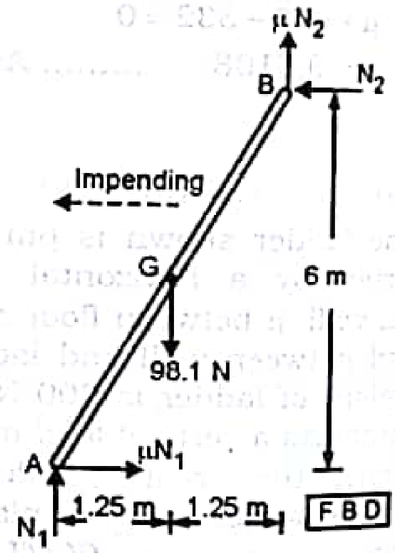
or $\quad N_2 = \mu N_1$ .... (1)

$$\Sigma F_y = 0 \uparrow + ve$$

$$N_1 + \mu N_2 - 98.1 = 0$$

$$N_1 + \mu(\mu N_1) - 98.1 = 0$$

or $\quad N_1 + \mu^2 N_1 - 98.1 = 0$ .... (2)



$$\Sigma M_B = 0 \quad \circlearrowleft + ve$$

$$+(\mu N_1 \times 6) - (N_1 \times 2.5) + (98.1 \times 1.25)$$

$\therefore \quad N_1(6\mu - 2.5) = -122.625$

Or $\quad N_1 = \dfrac{-122.625}{6\mu - 2.5}$ .... (3)

Substituting equation (3) in (2), we get

$$\frac{-122.625}{6\mu - 2.5} + \mu^2 \times \left(\frac{-122.625}{6\mu - 2.5}\right) - 98.1 = 0$$

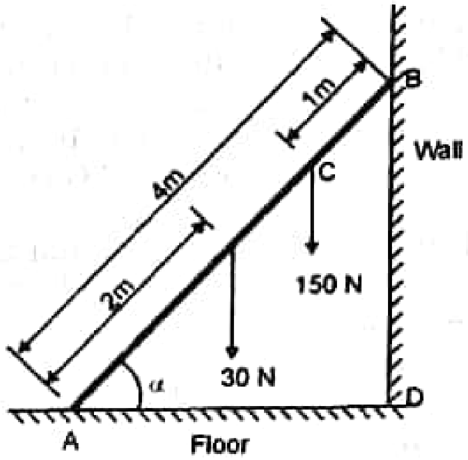
$$-122.625 - 122.625\mu^2 = 588.6\mu - 245.25$$

$\therefore \quad 122.625\mu^2 + 588.6\mu - 122.625 = 0$

Solving the quadratic equation, we get

$\mu = 0.2$ .... **Ans.**

P7. A ladder shown is 4 m long and is supported by a horizontal floor and vertical wall. The coefficient of friction at the wall is 0.25 and that at the floor is 0.5. The weight of ladder is 30 N. It also supports a vertical load of 150 N at 'C'. Determine the least value of 'α' at which the ladder may be placed without slipping to the left. ***(VJTI Nov 09)***



Solution: For the least value of α, the ladder impends to slip down and away from the wall.

COE – ladder

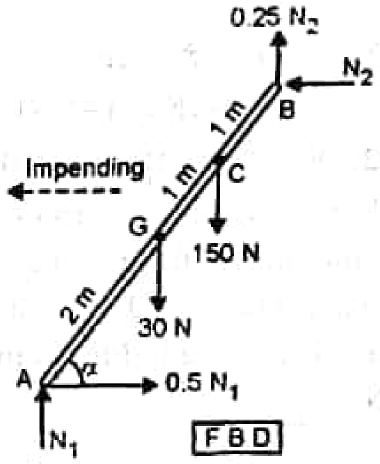
$\Sigma F_x = 0 \rightarrow +\text{ve}$

$0.5N_1 - N_2 = 0$ .... (1)

$\Sigma F_y = 0 \uparrow +\text{ve}$

$N_1 + 0.25N_2 - 30 - 150 = 0$

$N_1 + 0.25N_2 = 180$ .... (2)



Solving equation (1) and (2), we get

$N_1 = 160\,N$ and $N_2 = 80\,N$

$\Sigma M_A = 0$ ↺ +ve

$$-(30\times 2\cos\alpha) - (150\times 3\cos\alpha) + (0.25N_2\times 4\cos\alpha) + (N_2\times 4\sin\alpha) = 0$$

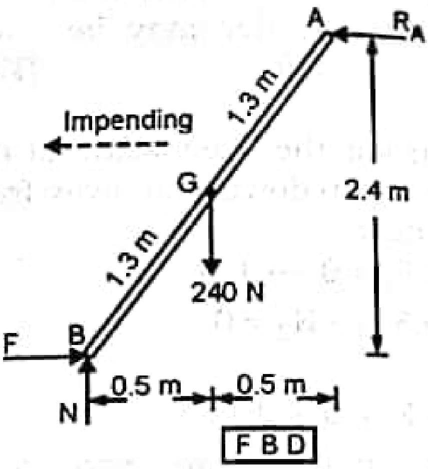
$$-60\cos\alpha - 450\cos\alpha + 0.25\times 80\times 4\cos\alpha + 80\times 4\sin\alpha = 0$$

$$-430\cos\alpha + 320\sin\alpha = 0$$

$\therefore$ $\tan\alpha = 1.3437$ or $\alpha = 53.34°$ .... **Ans.**

P8. A uniform ladder of length 2.6m and weight 240 N is placed against a smooth vertical wall at A, with its lower end 1 m from the wall on the floor at B. The coefficient of friction between the ladder and the floor is 0.3. Find the frictional force acting on the ladder at the point of contact between the ladder and the floor. Will the ladder remain in equilibrium in this position? Give reason. *(VJTI May 06)*

Solution: Let F be the frictional force acting at the floor, required to keep the ladder in equilibrium.



COE – ladder

$\sum F_y = 0$ ↑ + ve

$N - 240 = 0$

$\therefore \quad N = 240$ N

$\sum M_A = 0$ ↺ + ve

$+(F \times 2.4) - (N \times 1) + (240 \times 0.5) = 0$

$2.4F - (240 \times 1) + 120 = 0$

$\therefore \quad F = 50$ N .... Frictional force required for equilibrium

Now the maximum frictional force the rough floor can produce is $F_{max} = \mu_s \times N$

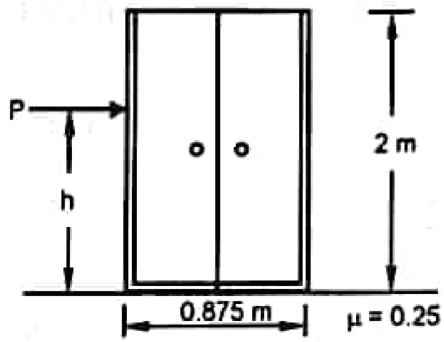
$\therefore \quad F_{max} = 0.3 \times 240$ or $F_{max} = 72$ N

Since $F < F_{max}$ the ladder is in equilibrium and the frictional force at the floor is F = 50 N .... **Ans.**

Exercise 4.4

Tipping

P1. A 1500 N cupboard is to be shifted to the right by a horizontal force P as shown. Find the force P required to just cause the motion and the maximum height upto which it can be applied.



Solution: To find force P to just move the cupboard.

COE – Cupboard

$\Sigma F_y = 0 \uparrow +ve$

$N - 1500 = 0$

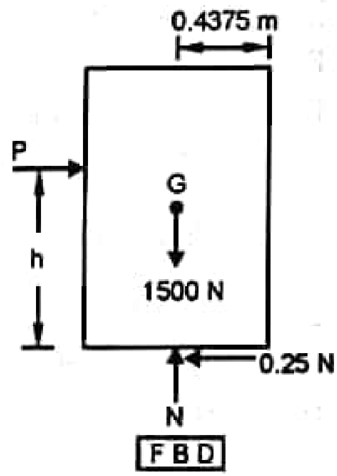
$\therefore \quad N = 1500 \text{ N}$

$\Sigma F_x = 0 \rightarrow +ve$

$P - 0.25N = 0$

$\therefore \quad P - 0.25 \times 1500 = 0$

or $\quad P = 375 \text{ N}$ .... **Ans.**



To find maximum height upto which force P can be applied.

At maximum height condition, the cupboard is on the verge of tipping about A. The normal reaction N and friction force F shifts to A.

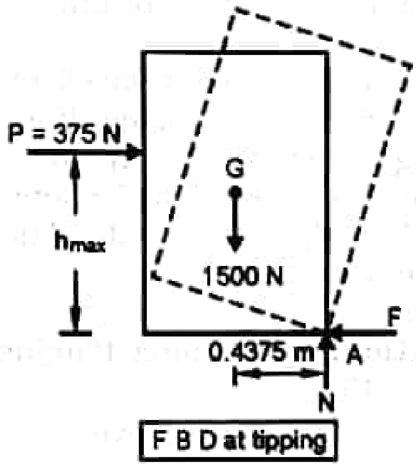
COE – Cupboard

$\Sigma M_A = 0 \circlearrowright +ve$

$-(P \times h_{max}) + (1500 \times 0.4375) = 0$

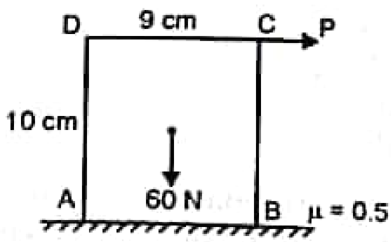
$-(375 \times h_{max}) + 656.25 = 0$

or $\quad h_{max} = 1.75 \text{ m}$ .... **Ans.**



P2. For the block shown in figure, find the minimum value P, which will just disturb the equilibrium of the system.

(M.U Dec 12)



Solution: The equilibrium of the block can be disturbed either by sliding of the block to the right or by tipping about the edge B.

Possibility (1) – The block slides to the right.

COE – Block

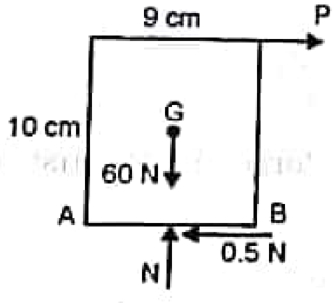
$$\Sigma F_y = 0 \uparrow + ve$$

$$N - 60 = 0 \quad \therefore \quad N = 60\ N$$

$$\Sigma F_x = 0 \rightarrow + ve$$

$$P - 0.5N = 0$$

$$\therefore \quad P - 0.5 \times 60 = 0 \quad \text{or} \quad P = 30\ N$$



FBD

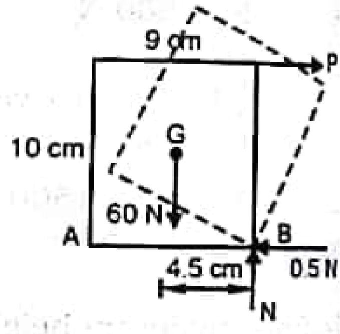
Possibility (2) – The block tips about edge B.

COE – Block

$$\Sigma M_B = 0 \circlearrowleft + ve$$

$$-(P \times 10) + (60 \times 4.5) = 0$$

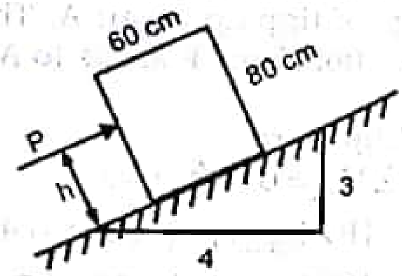
$$\text{or} \quad P = 27\ N$$



F B D at tipping

∴ From above values of P obtained, we can say that the block would loose its equilibrium by tipping at P = 27 N. .... **Ans.**

P3. A homogeneous block of weight W rests upon the inclined plane. If μ = 0.3, determine the greatest height h at which a force P parallel to the inclined plane may be applied so that the block will slide up the plane without tipping over.



Solution: To find force P to just move the block.

COE – Block

$$\Sigma F_y = 0 \quad + ve$$

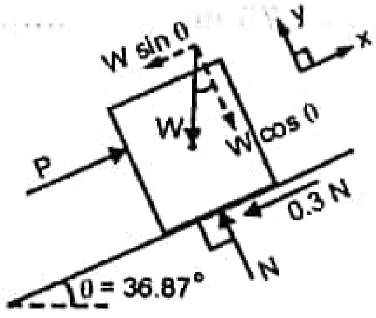
$$N - W\cos 36.87 = 0 \quad \therefore \quad N = 0.8\,W$$

$$\Sigma F_x = 0 \quad + ve$$

$$P - 0.3N - W\sin 36.87 = 0$$

$$\therefore \quad P - 0.3 \times 0.8\,W - 0.6\,W = 0$$

$$\text{or} \quad P = 0.84\,W$$



FBD

To find greatest height h at which the block slides without tipping.

At maximum height condition, the block is on the verge of tipping about edge A. The normal reaction N and friction force F shifts to A.

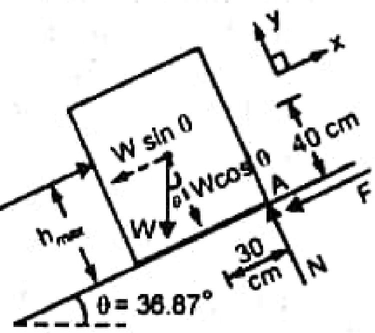
COE – Block

$$\Sigma M_A = 0 \quad \circlearrowleft + ve$$

$$-(P \times h_{max}) + (W\cos 36.87 \times 30) + (W \sin 36.87 \times 40) = 0$$

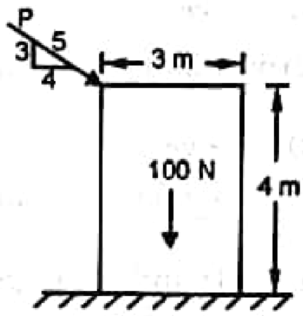
$$-(0.84\,W \times h_{max}) + 48\,W = 0$$

or $\quad h_{max} = 57.14$ cm .... **Ans.**



F B D at tipping

P4. A block weighing 100N, rests on the floor for which the coefficient of static friction is $\mu_s = 0.5$. Determine, if the block slips or tips and the smallest magnitude of force P that will cause impending motion of the block. ***(SPCE Dec 10)***



Solution: At some smallest force P, the block can slide to the right or tip about edge A.

Possibility (1) – The block slips to the right.

COE – Block

$$\Sigma F_x = 0 \quad \rightarrow + ve$$

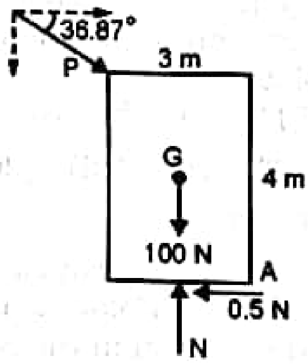
$$P\cos 36.87 - 0.5\,N = 0 \qquad \text{......... (1)}$$

$$\Sigma F_y = 0 \quad \uparrow + ve$$

$$-P\sin 36.87 + N - 100 = 0 \qquad \text{......... (2)}$$

Solving equations (1) and (2) we get

$P = 100$ N and $N = 160$ N



F B D

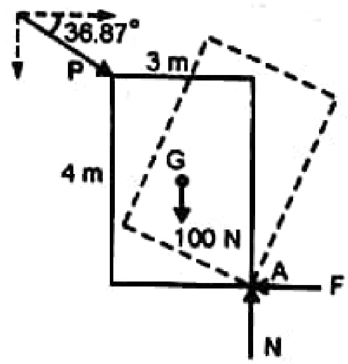
Possibility (2) – The block tips about edge A. In this situation the normal reaction N and friction force F shift to A

COE – Block

$$\Sigma M_A = 0 \quad \circlearrowleft + ve$$

$$-(P\cos 36.87 \times 4) + (P \sin 36.87 \times 3) + (100 \times 1.5) = 0$$

$\therefore \quad P = 107.1$ N

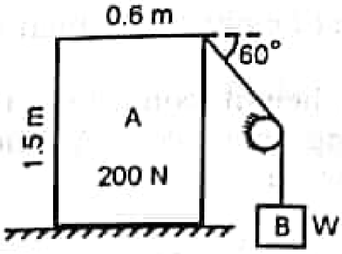


F B D at tipping

$\therefore$ From above values of P obtained, we can say that the block starts moving by slipping at P = 100 N.

.... **Ans.**

P5. Block A weighing 200 N is connected to another block of weight W by a cord passing over a smooth pulley. The weight W is slowly increased. Find its value for which motion just impends.
Take μ at floor = 0.2



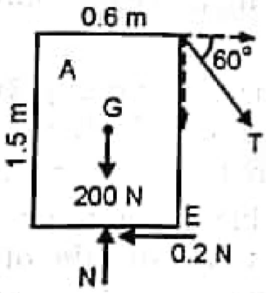
Solution: Block B of weight W can move down either by sliding of the block A to the right or by tipping of block A about edge E.
Possibility (1) – The block A slides to the right.
Isolating block A by cutting the cord.
COE – Block A

$$\Sigma F_y = 0 \uparrow + ve$$

$$N - T\sin 60 - 200 = 0 \quad \text{.........} \ (1)$$

$$\Sigma F_x = 0 \rightarrow + ve$$

$$-0.2N + T\cos 60 = 0 \quad \text{.........} \ (1)$$



F B D of block A

Solving equations (1) and (2) we get

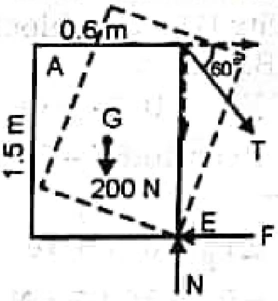
$$N = 306 \text{ N} \qquad \text{and} \qquad T = 122.4 \text{ N}$$

Possibility (2) – The block A tips about edge E. The normal reaction N and friction force F shift to edge E.
COE – Block A

$$\Sigma M_E = 0 \ \circlearrowleft + ve$$

$$-(T\cos 60 \times 1.5) + (200 \times 0.3) = 0$$

$\therefore \quad T = 80 \text{ N}$



F B D at tipping

$\therefore$ From above values of T obtained, we can say that the motion of block A is initiated by tipping about E when the tension in the cord reaches T = 80 N.

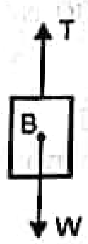
COE – Block B

$$\Sigma F_y = 0 \uparrow + ve$$

$$T - W = 0$$

$\therefore \quad 80 - W = 0$

or $\quad W = 80 \text{ N}$ .... **Ans.**



F B D of block B

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