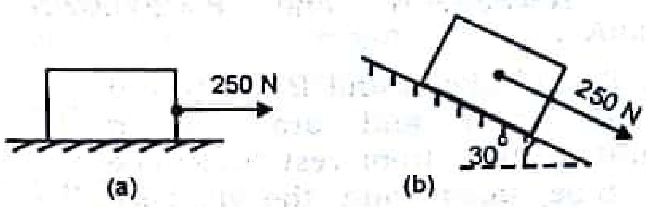
Solutions: Chapter 10

Kinetics of Particles: Newton's Second Law

Exercise 10.1

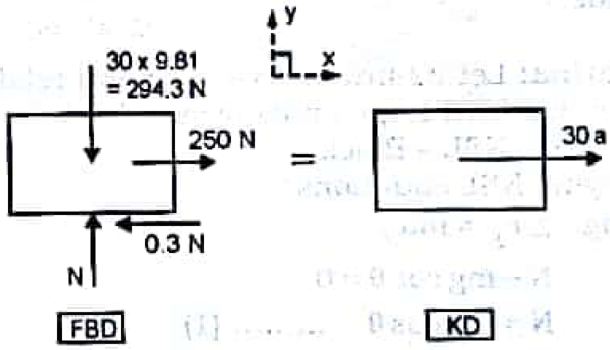
N.S.L. – Rectilinear Motion

P1. A block of mass 30 kg is placed on a plane. μ_k between the block and plane is 0.3. If a force of 250 N is acting on the block, find its acceleration for the two cases shown.



Solution: a) Kinetics – NSL – Block

Applying NSL equations



Using $\Sigma F_y = ma_y$

$$N - 294.3 = 0$$

$\therefore$ $N = 294.3\ N$

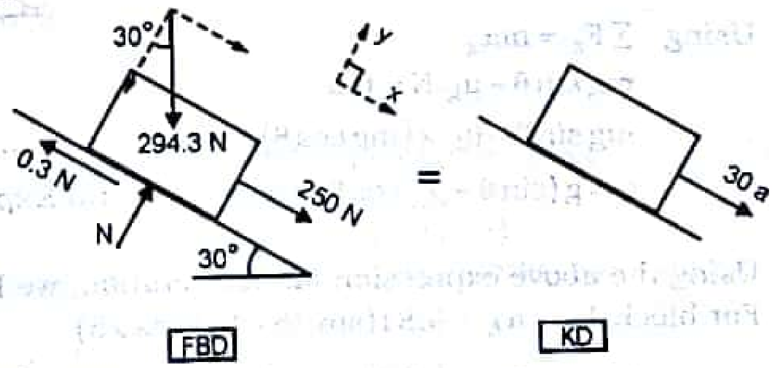
Using $\Sigma F_x = ma_x$

$$250 - 0.3 \times (294.3) = 30a$$

$\therefore$ $a = 5.39\ m/s^2$ .... **Ans.**

b) Kinetics – NSL – Block

Applying NSL equations



Using $\Sigma F_y = ma_y$

$$N - 294.3 \cos 30 = 0$$

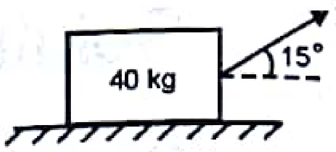
$\therefore$ $N = 254.87\ N$

Using $\Sigma F_x = ma_x$

$$250 + 294.3 \sin 30 - 0.3 \times (254.87) = 30a$$

$\therefore$ $a = 10.689\ m/s^2$ .... **Ans.**

P2. Find force P required to accelerate the block shown in figure at $2.5\ m/s^2$. Take $\mu = 0.3$ ***(M. U. Dec 11)***

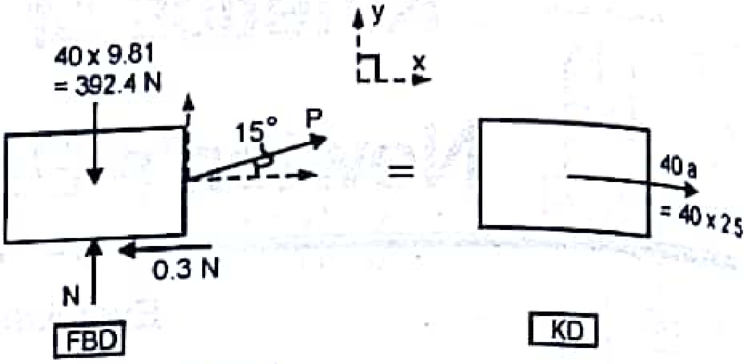


Solution: Kinetics – NSL – Block

Applying NSL equations

Using $\Sigma F_y = ma_y$

$N - 392.4 + P\sin 15 = 0$ .... (1)



Using $\Sigma F_x = ma_x$

$-0.3N + P\cos 15 = 40a$

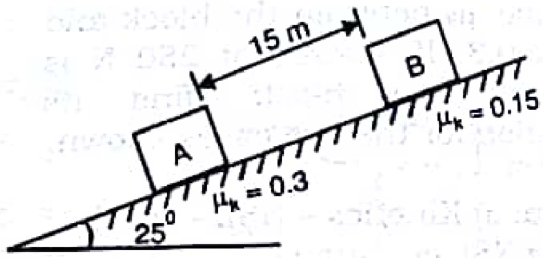
$-0.3N + P\cos 15 = 40 \times 2.5$ .... (2)

.... given $a = 2.5\ m/s^2$

Solving equations (1) and (2) we get

$\therefore$ $N = 338.4\ N$ and $P = 208.62\ N$ .... **Ans.**

P3. Two blocks A and B are placed 15 m apart and are released simultaneously from rest. Calculate the time taken and the distance traveled by each block before they collide.



Solution: Let us first derive a general relation for acceleration of a block sliding down a rough inclined slope on its own.

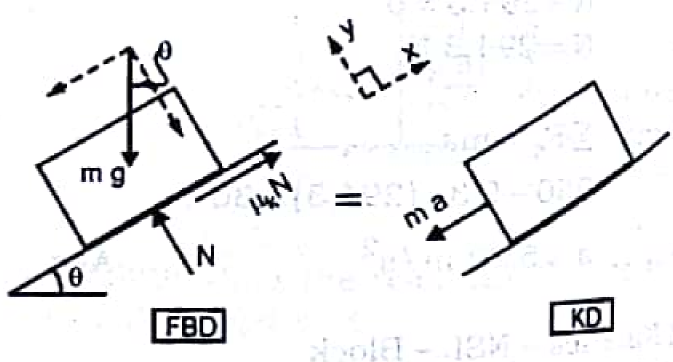
Kinetics – NSL – Block

Applying NSL equations

Using $\Sigma F_y = ma_y$

$N - mg\cos\theta = 0$

$N = mg\cos\theta$ .... (1)



Using $\Sigma F_x = ma_x$

$mg\sin\theta - \mu_k.N = ma$

$\therefore$ $mg\sin\theta - \mu_k \times (mg\cos\theta) = ma$ .... (2)

$\therefore$ $a = g(\sin\theta - \mu_k\cos\theta)$ .... General Expression

Using the above expression for acceleration, we have

For block A; $a_A = 9.81(\sin 25 - 0.3\cos 25)$

$\therefore$ $a_A = 1.479\ m/s^2$

For block B; $a_B = 9.81(\sin 25 - 0.15\cos 25)$

$\therefore$ $a_B = 2.812\ m/s^2$

Kinematics – Let the block A travel x metres and block B travel (15 + x) metres before the blocks collide.

Block – A

Rectilinear Motion – Uniform Acceleration

$u = 0$

$v = -$

$s = x$

$a = 1.479\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

$x = 0 + \frac{1}{2} \times 1.479 \times t^2$ .... (1)

Block – B

Rectilinear Motion – Uniform Acceleration

$u = 0$

$v = -$

$s = 15 + x$

$a = 2.812\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

$15 + x = 0 + \frac{1}{2} \times 2.812 \times t^2$ .... (2)

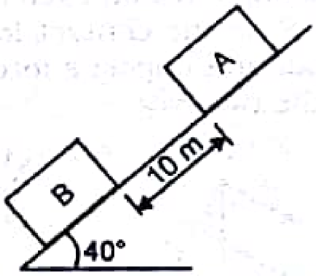
Solving equations (1) and (2) we get, $t = 4.744$ sec and $x = 16.643$ m

$\therefore$ Block A travels 16.643 m,

Block B travels 31.643 m and they take 4.744 sec to collide .... **Ans.**

P4. Two blocks A of weight 500 N and B of weight 300 N are 10 m apart on an inclined plane as shown in figure. $\mu = 0.2$ for block A and 0.3 for block B. If the blocks begin to slide down simultaneously calculate the time and the distance travelled by each block when block A touches block B.

(M. U. Dec 09)



Solution: Using general expression for acceleration of a block sliding down a rough inclined slope, we have

$a = g(\sin\theta - \mu_k \cos\theta)$ .... General Expression

For block A; $a_A = 9.81(\sin 40 - 0.2\cos 40)$

$\therefore$ $a_A = 4.803\ m/s^2$

For block B; $a_B = 9.81(\sin 40 - 0.3\cos 40)$

$\therefore$ $a_B = 4.051\ m/s^2$

Kinematics – Let the block B travel x metres and block A travel (10 + x) metres before they collide.

Block – A

Rectilinear Motion – Uniform Acceleration

$u = 0$

$v = -$

$s = 10 + x$

$a = 4.803\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

Block – B

Rectilinear Motion – Uniform Acceleration

$u = 0$

$v = -$

$s = x$

$a = 4.051\ m/s^2$

$t = t$

Using $s = ut + \frac{1}{2}at^2$

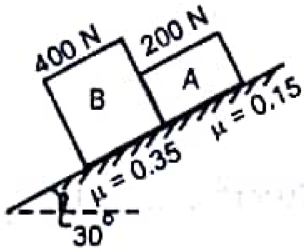
$$10 + x = 0 + \frac{1}{2} \times 4.803 \times t^2 \quad \text{......... (1)}$$

$$x = 0 + \frac{1}{2} \times 4.051 \times t^2 \quad \text{......... (2)}$$

Solving equations (1) and (2) we get, $t = 5.158$ sec and $x = 53.89$ m

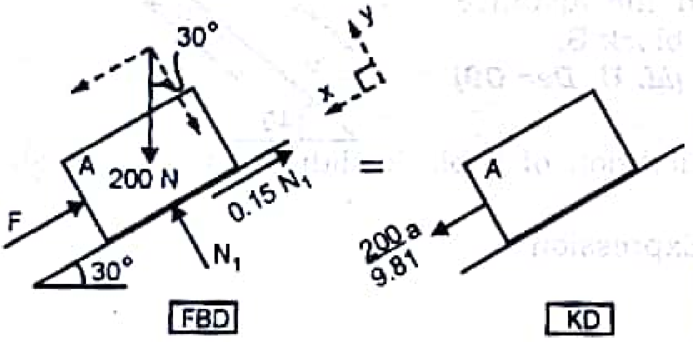
$\therefore$ Block B travels 53.89 m, Block A travels 63.89 m and they take 5.16 sec before they collide .... **Ans.**

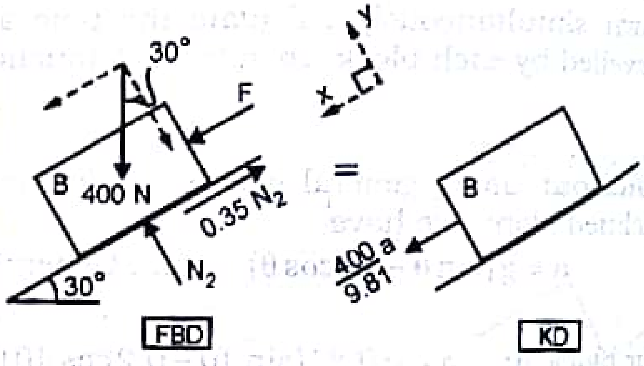
P5. Two blocks A and B rest on an inclined plane as shown. Find their acceleration on being released from rest. Also find the contact force between the blocks.



Solution: Since μ_k of block B is higher than μ_k of block A, indicates that block B is a slow mover than block A. Therefore block A pushes block B and both the blocks exerting force on each other travel down the slope with a common acceleration.

Let F be the contact force between blocks (bock A exerts a push force on block B. An equal and opposite force is given by block B to block A) which is seen only on isolation of the two.





Isolating block - A

Applying NSL equations

Using $\Sigma F_y = ma_y$

$$N_1 - 200\cos 30 = 0$$

$\therefore$ $N_1 = 173.2$ N

Using $\Sigma F_x = ma_x$

$$-F - 0.15N_1 + 200\sin 30 = \frac{200}{9.81}a$$

$$-F - 0.15(173.2) + 100 = 20.387a$$

$$F + 20.387a = 74.02 \quad \text{......... (1)}$$

Isolating block - B

Applying NSL equations

Using $\Sigma F_y = ma_y$

$$N_2 - 400\cos 30 = 0$$

$\therefore$ $N_2 = 346.4$ N

Using $\Sigma F_x = ma_x$

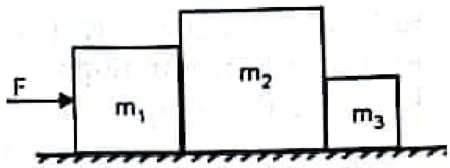
$$F - 0.35(346.4) + 400\sin 30 = \frac{400}{9.81}a$$

$$F - 40.77a = -78.76 \quad \text{......... (2)}$$

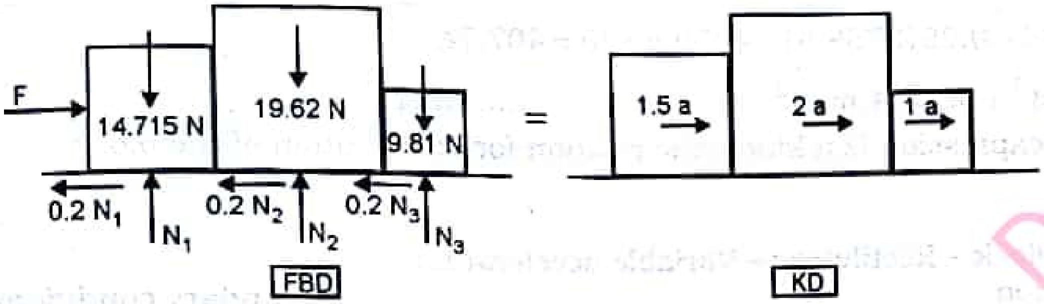
Solving equations (1) and (2)

$F = 23.09$ N .... **Ans.** and $a = 2.498$ m/s² .... **Ans.**

P6. Three blocks m_1 and m_2 and m_3 of masses 1.5 kg, 2 kg and 1 kg respectively are placed on a rough surface with $\mu = 0.20$ as shown. If a force F is applied to accelerate the blocks at 3 m/s^2 what will be the force that 1.5 kg block exerts on 2 kg block. ***(M.U. Dec 12)***



Solution: This is a system of three blocks resting against each other. All the blocks move with a common acceleration 'a'. Applying NSL to the system



Applying NSL equations to the System

Using $\Sigma F_x = ma_x$

$$F - 0.2N_1 - 0.2N_2 - 0.2N_3 = 1.5a + 2a + 1a$$

$\therefore$ $F - 0.2 \times 14.715 - 0.2 \times 19.62 - 0.2 \times 9.81 = 4.5 \times 3$

or $F = 22.329$ N

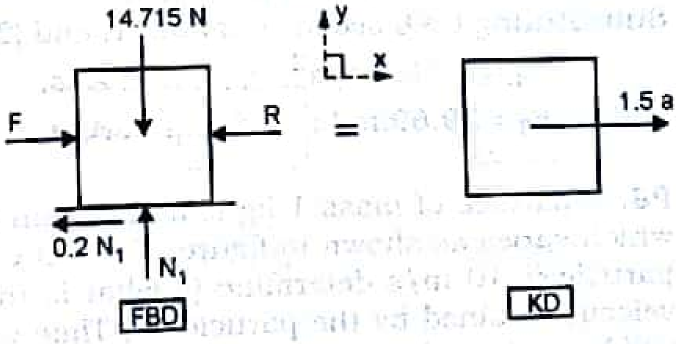
$a = 3\ m/s^2$......... given

also $N_1 = m_1 g = 14.715$ N

$N_2 = m_2 g = 19.62$ N

$N_3 = m_3 g = 9.81$ N

Let R be the force that the 1.5 kg block exerts on the 2 kg block, which is the same as the 2 kg block exerts on the 1.5 kg block.



Isolating the 1.5 kg block

NSL – Block 1

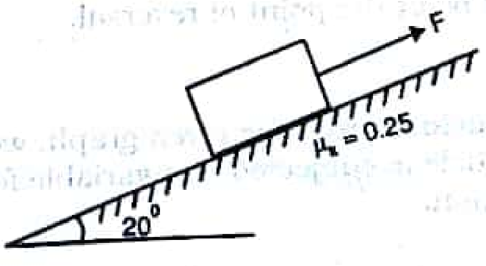
$$\Sigma F_x = ma_x$$

$$F - R - 0.2\ N_1 = 1.5a$$

$\therefore$ $22.329 - R - 0.2 \times 14.715 = 1.5 \times 3$

$\therefore$ $R = 14.886$ N .... **Ans.**

P7. A package of total weight 4000 N is being pulled up a slope by a steel cable exerting a force $F = (50\ t^2 + 2500)$ N. Knowing that the velocity of the package was 5 m/s at t = 0, find the acceleration and velocity of the package at t = 8 sec.



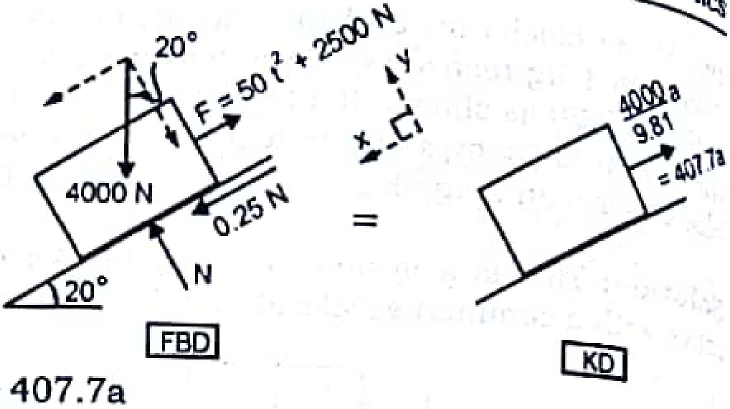
Solution: Kinetics – NSL – Block

Applying NSL equations

Using $\Sigma F_y = ma_y$

$$N - 4000\cos 20 = 0$$

$\therefore \quad N = 3758.8 \text{ N}$



Using $\Sigma F_x = ma_x$

$$50t^2 + 2500 - 0.25(3758.8) - 4000\sin 20 = 407.7a$$

$$a = 0.1226t^2 + 0.4714 \text{ m/s}^2 \qquad \text{......... (1)}$$

The above expression is a kinematic relation for acceleration of the block.

Kinematics

Motion of block – Rectilinear – Variable acceleration

Given relation

$$a = 0.1226t^2 + 0.4714 \text{ m/s}^2$$

Boundary conditions

At t = 0, v = 5 m/s

Integrating using $a = \dfrac{dv}{dt}$

$\therefore \quad dv = 0.1226t^2 + 0.4714 \, dt$

$\therefore \quad \int_5^v dv = \int_0^t 0.1226t^2 + 0.4714 \, dt$

or $\quad v = 0.04086\, t^3 + 0.4714t + 5 \text{ m/s} \qquad \text{......... (2)}$

Velocity and acceleration at t = 8 sec.

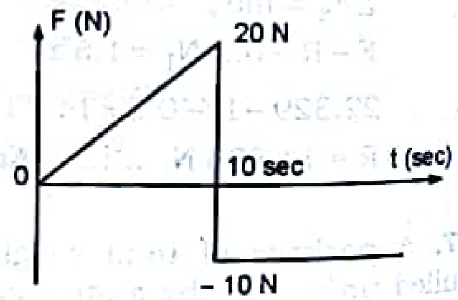
Substituting t = 8 sec in equation (1) and (2), we get

$a_8 = 8.318 \text{ m/s}^2$ .... **Ans.**

and $\quad v_8 = 29.69 \text{ m/s}$ .... **Ans.**

P8. A particle of mass 1 kg is acted upon by a force F which varies as shown in figure. If initial velocity of the particle is 10 m/s determine (i) what is the maximum velocity attained by the particle. (ii) Time when particle will be at the point of reversal.

(M.U. Dec 13)



Solution: From the given graph, we note that for the first ten seconds (stage 1), the particle is subjected to a variable force F = 2 t N , which results in variable acceleration motion.

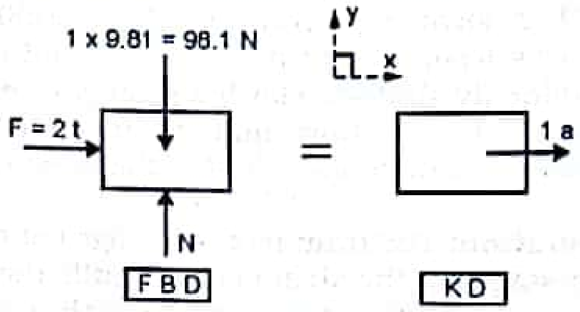
Kinetics – stage (1) – NSL

Applying NSL equations

Using $\Sigma F_x = ma_x$

$$2t = 1a$$

$\therefore \quad a = 2t \text{ m/s}^2 \quad \text{.........} (1)$



Kinematics – stage (1) – Variable acceleration

Given relation

$$a = 2t \text{ m/s}^2$$

Integrating using $a = \dfrac{dv}{dt}$

$\therefore \quad dv = 2t\ dt$

$\therefore \quad \int_{10}^{v} dv = \int_{0}^{t} 2t\ dt$

$\therefore \quad v - 10 = t^2$

or $\quad v = t^2 + 10 \text{ m/s} \quad \text{.........} (2)$

substituting t = 10 sec in equation (a), we get

$$v = 10^2 + 10 = 110 \text{ m/s}$$

$\therefore$ The particles maximum velocity is 110 m/s at t = 10 sec. .... **Ans.**

From graph we note that after 10 seconds of motion (stage 2), the particle is subjected to a constant negative force of 10 N, which results in uniform deceleration of the particle.

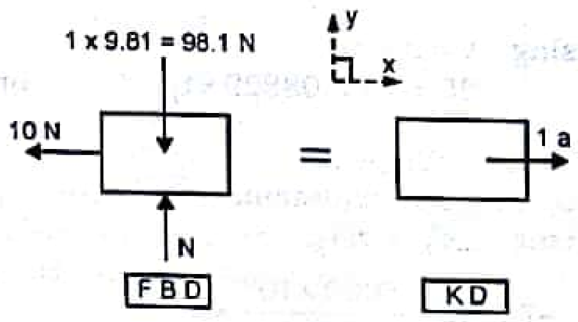
Kinetics – stage (2) – NSL

Applying NSL equations

Using $\Sigma F_x = ma_x$

$$-10 = 1a$$

or $\quad a = -10 \text{ m/s}^2$



Kinematics – stage (2) – uniform acceleration

The particle would be at the point of reversal when its velocity momentarily becomes zero.

$u = 110 \text{ m/s}, \ v = 0, \ s = -\ , \ a = -10 \text{ m/s}^2, \ t = t \text{ sec}$

Using $\quad v = u + at$

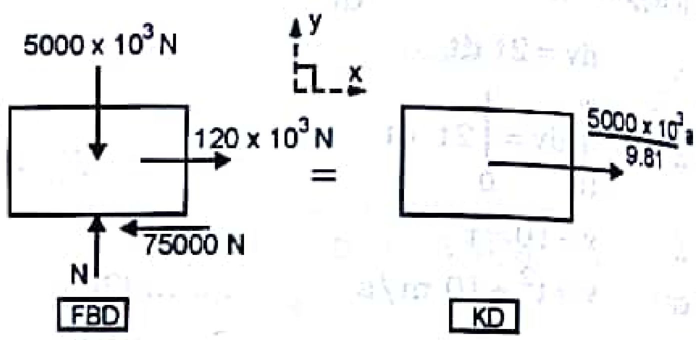
$$0 = 110 + (-10) \times t \qquad \text{Or} \qquad t = 11 \text{ sec}$$

$\therefore$ Particle reverses at t = 10 + 11 = 21 sec .... **Ans.**

P9. A locomotive train weighing 5000 kN exerts a constant pull of 120 kN. The train starts from rest on a level track and attains a speed of 90 kmph, after which steam is suddenly shut off, slowly bringing it to a halt. If the tractive resistance is 15 N/kN of the weight of the train, find the total distance covered by the train and the total time to cover this distance. What is the maximum power developed by the engine?

Solution: The train has two stages of rectilinear motion.
In stage (1), the steam engine pulls the train with a 120 kN force.
In stage (2), the steam engine is shut causing the train to come to a halt.
The tracks after a resistance force of 15 N/kN = 15 × 5000 = 75000 N throughout the motion.

Kinetics – Stage (1) – NSL
Applying NSL equations
Using $\Sigma F_x = ma_x$



$$120\times10^3 - 75000 = \frac{5000\times10^3}{9.81}\times a$$

$$\therefore \quad a = 0.08829\ \text{m/s}^2$$

Kinematics – Stage (1) – Uniform acceleration

$u = 0$, $v = 25$ m/s, $s = x_1$, $a = 0.08829$ m/s², $t = t_1$

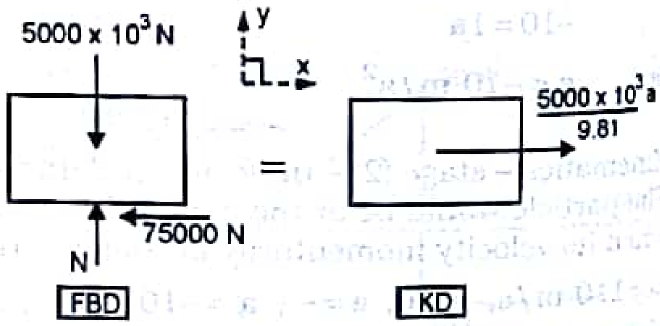
using $v^2 = u^2 + 2as$

$$25^2 = 0 + 2\times0.08829\times x_1 \quad \text{or} \quad x_1 = 3539.5\ \text{m}$$

using $v = u + at$

$$25 = 0 + 0.08829\times t_1 \quad \text{or} \quad t_1 = 283.16\ \text{sec}$$

Kinetics – Stage 2
Applying NSL equations
Using $\Sigma F_x = ma_x$



$$-75000 = \frac{5000\times10^3}{9.81}\times a$$

$$\therefore \quad a = -0.14715\ \text{m/s}^2$$

Kinematics – Stage 2 – uniform acceleration
$u = 25$ m/s, $v = 0$, $s = x_2$,
$a = -0.14715$ m/s², $t = t_2$

using $v^2 = u^2 + 2as$

$$0 = 25^2 + 2\times(-0.14715)\times x_2$$

$$\therefore \quad x_2 = 2123.7\ \text{m}$$

using $v = u + at$

$0 = 25 - 0.14715 \times t_2$

$\therefore \quad t_2 = 169.89$ sec

Total distance covered by train $= x_1 + x_2 = 3539.5 + 2123.7 = 5663.2$ m

Total time $= t_1 + t_2 = 283.16 + 169.89 = 435.05$ sec .... **Ans.**

Power $= \dfrac{\text{Work}}{\text{time}} = \dfrac{F \times s}{t} = F \times v$

$\therefore \quad$ Max. Power $= F \times v_{max}$

$= 120 \times 10^3 \times 25 = 3000 \times 10^3$ W $= 3000$ kW .... **Ans.**

P10. A vertical lift of total mass 750 kg acquires an upward velocity of 3 m/s over a distance of 4 m moving with constant acceleration starting from rest. Calculate the tension in the cable. ***(M.U. Dec 12)***

Solution: Let us first work with kinematics of the lift.
Motion of lift – Rectilinear – Uniform Acceleration
u = 0, v = 3 m/s, s = 4 m, a = a m/s², t = -

Using $v^2 = u^2 + 2as$

$3^2 = 0 + 2 \times a \times 4$

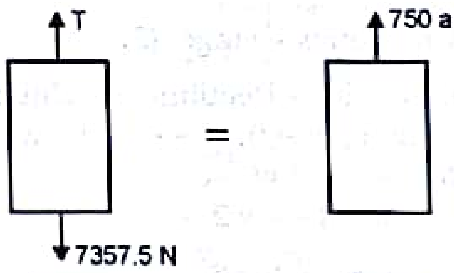
$\therefore \quad a = 1.125 \text{ m/s}^2$

Now applying kinetics to the lift
Applying NSL equations
Using $\Sigma F_y = ma_y$

$T - 7357.5 = 750a$

$\therefore \quad T - 7357.5 = 750 \times 1.125$

or $\quad T = 8201.25$ N .... **Ans.**



FBD KD

P11. a) A 400 kg lift carrying a 60 kg person travels vertically up starting from rest and acquires a velocity of 4 m/s in a distance of 3 m of motion. Find the tension in the cable supporting the lift and the force transmitted by the person on the lift floor. Does the person feel normal or heavy or light.

b) The lift is now brought to a halt from the speed of 4 m/s, in 2 sec time. What is the force exerted by the man on the floor of the lift and how does he feel now.

Solution: a) Kinematics – Stage (1)

Motion of lift – Rectilinear – Uniform Acceleration

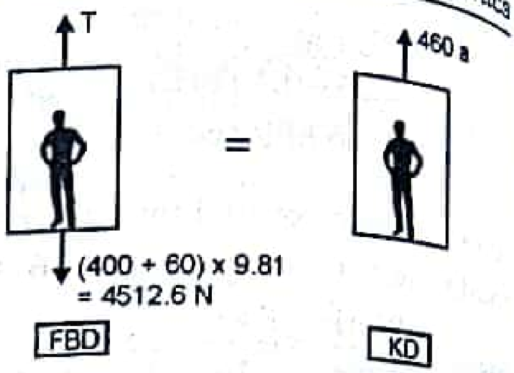
u = 0, v = 4 m/s, s = 3 m, a = a, t = -

Using $v^2 = u^2 + 2as$

$4^2 = 0 + 2 \times a \times 3$

$\therefore \quad a = 2.667 \text{ m/s}^2$

Kinetics – Stage (1) – NSL – System of lift and person



Applying NSL equations

Using $\Sigma F_y = ma_y$

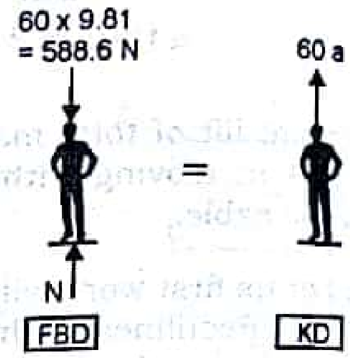
$T - 4512.6 = 460a$

$T - 4512.6 = 460 \times 2.667$

$\therefore$ $T = 5739.4$ N .... **Ans.**

Isolating the person.
Let N be the normal reaction from the floor.

Kinetics – NSL – Person



Applying NSL equations

Using $\Sigma F_y = ma_y$ $\uparrow +ve$

$N - 588.6 = 60a$

$N - 588.6 = 60 \times 2.667$

$\therefore$ $N = 748.62$ N

$\therefore$ The force transmitted by the person on the floor = N = 748.62 N .... **Ans.**
Also the person feels heavy since N > W .... **Ans.**

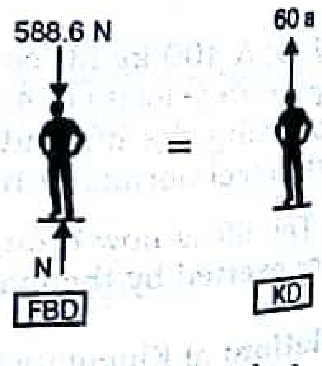
b) Kinematics – stage (2)

Motion of lift – Rectilinear – Uniform Acceleration
u = 4 m/s, v = 0, s = –, a = a, t = 2 sec
Using $v = u + at$

$0 = 4 + a \times 2$

$\therefore$ $a = -2\ m/s^2$

Kinetics – stage (2) – NSL – Person



Applying NSL equations
Using $\Sigma F_y = ma_y$ $\uparrow +ve$

$N - 588.6 = 60a$

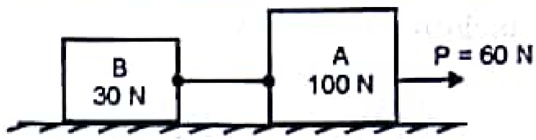
$N - 588.6 = 60 \times (-2)$

$\therefore$ $N = 468.6$N

$\therefore$ The force transmitted by the person on the floor = N = 468.6 N .... **Ans.**

$\therefore$ The person feels light since N < W .... **Ans.**

P12 Two blocks A and B are connected by a rope and move on a rough horizontal plane under a pull of 60 N as shown. If μ is 0.2 between the blocks and the surface, find the acceleration of the blocks and the tension in the rope. ***(VJTI Dec 11)***



Solution: This is dependent system of string connected blocks. Both the blocks will move with a common acceleration i.e. $a_A = a_B$ .... (1)

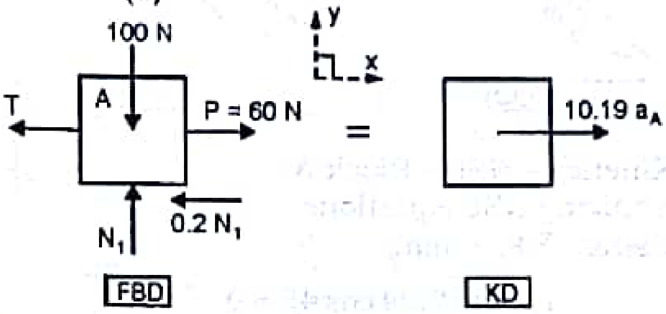
Isolating block A

NSL – block A

Using $\Sigma F_y = ma_y$

$N_1 - 100 = 0$

$\therefore \quad N_1 = 100$ N



Using $\Sigma F_x = ma_x$

$-T + 60 - 0.2N_1 = 10.19\ a_A$

$-T + 60 - 0.2 \times 100 = 10.19\ a_A$

$\therefore \quad 10.19\ a_A + T = 40$ .... (2)

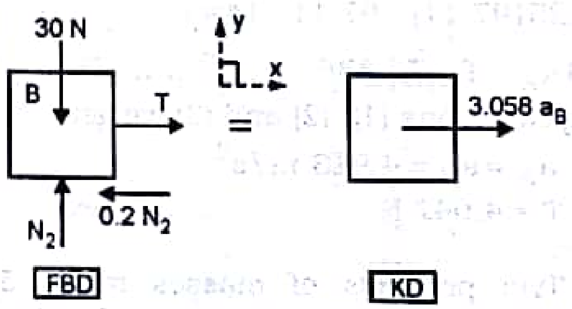
Isolating block B

NSL – block B

Using $\Sigma F_y = ma_y$

$N_2 - 30 = 0$

$\therefore \quad N_2 = 30$ N



Using $\Sigma F_x = ma_x$

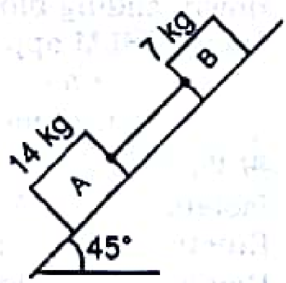
$T - 0.2N_2 = 3.058 a_B$

$T - 0.2 \times 30 = 3.058 a_B$

$\therefore \quad 3.058\ a_B - T = -6$ .... (3)

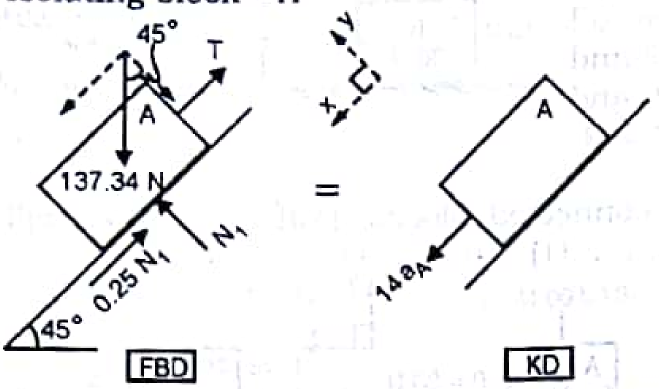
Solving we get, $a_A = a_B = 2.566\ m/s^2$ and $T = 13.846$ N .... **Ans.**

P13. Two masses 14kg and 7kg connected by a flexible inextensible cord rest on a plane inclined at 45° with the horizontal as shown in figure. When the masses are released what will be the tension T in the cord? Assume the coefficient of friction between the plane and the 14 kg mass as 0.25 and between the plane and the 7kg mass as 0.375 ***(VJTI May 08)***



Solution: This is a system of two blocks connected by a common string. Using CSLM, we get $a_A = a_B$ .... (1)

Isolating block – A



Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_y = ma_y$

$$N_1 - 137.34\cos 45 = 0$$

$\therefore \quad N_1 = 97.11\ \text{N}$

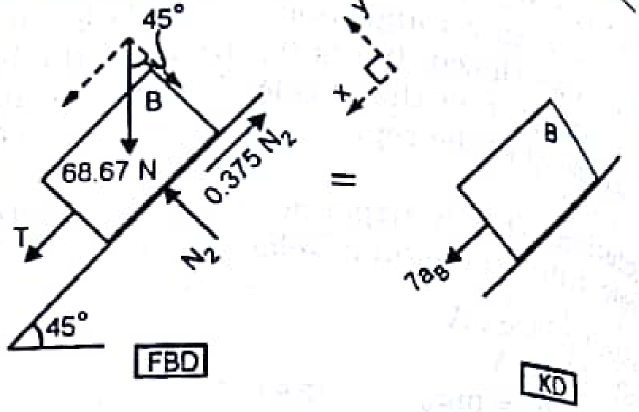
Using $\Sigma F_x = ma_x$

$$-T - 0.25N_1 + 137.34\sin 45 = 14a_A$$

$$-T - 0.25(97.11) + 97.11 = 14a_A$$

$\therefore \quad 14a_A + T = 72.836$ .... (2)

Isolating block – B



Kinetics – NSL – Block B
Applying NSL equations
Using $\Sigma F_y = ma_y$

$$N_2 - 68.67\cos 45 = 0$$

$\therefore \quad N_2 = 48.56\ \text{N}$

Using $\Sigma F_x = ma_x$

$$T - 0.375N_2 + 68.67\sin 45 = 7a_B$$

$$T - 0.375(48.56) + 48.56 = 7a_B$$

$\therefore \quad 7a_B - T = 30.347$ .... (3)

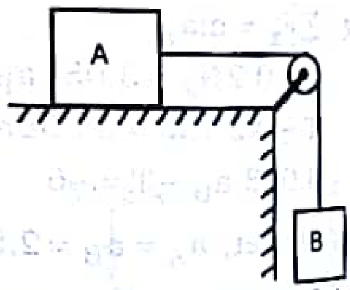
Solving equations (1), (2) and (3) we get

$a_A = a_B = 4.913\ \text{m/s}^2$

$T = 4.047$ N .... **Ans.**

P14. Two particles of masses m_A = 5 kg and m_B = 10 kg are supported as shown. find the acceleration of the blocks

a) if the horizontal surface is smooth
b) if the horizontal surface is rough having coefficient of kinetic friction $\mu_k = 0.4$



Solution: This is system of two blocks connected by a common string. Block B travels down, sliding block A to the right.
Using CSLM approach, we get,

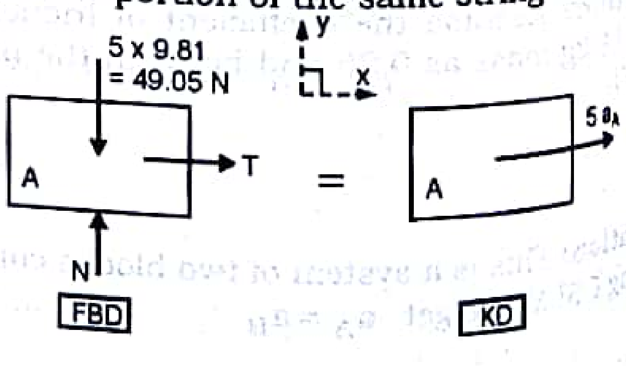
$a_A = a_B$ .... (1)

[since one portion of string holds A and one portion of the same string holds B]

a) $\mu_k = 0$

Isolating block A
Kinetics – NSL – Block A
Using $\Sigma F_x = ma_x$

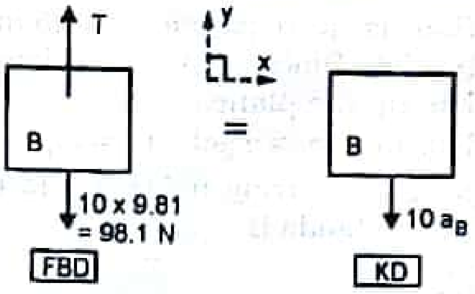
$T = 5a_A$ .... (2)



Isolating block B
Kinetics – NSL – Block B

Using $\Sigma F_y = ma_y$

$$T - 98.1 = -10\,a_B \qquad .... (3)$$



Solving equations (1), (2) and (3) we get

$$a_A = a_B = 6.54\ \text{m/s}^2$$

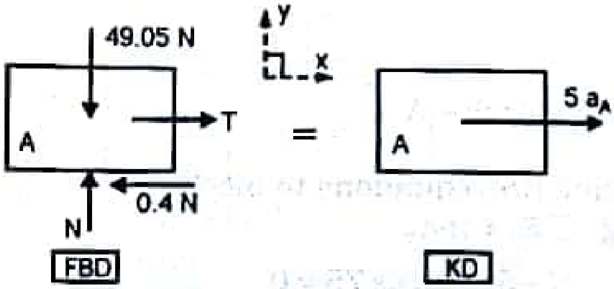
and $\quad T = 32.7\ \text{N} \qquad$ .... **Ans.**

b) $\mu_k = 0.4$

Isolating block A

Kinetics – NSL – Block A

Applying NSL equations



Using $\Sigma F_y = ma_y$

$$N - 49.05 = 0$$

$$\therefore \quad N = 49.05\ \text{N}$$

Using $\Sigma F_x = ma_x$

$$T - 0.4N = 5\,a_A$$

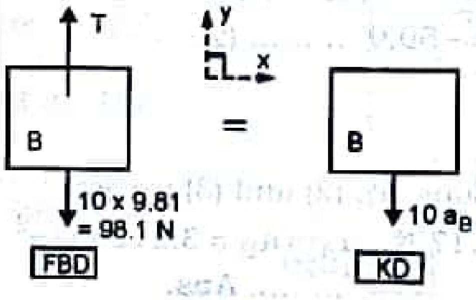
$$T - 0.4 \times 49.05 = 5a_A$$

$$\therefore \quad T - 5a_A = 19.62 \qquad .... (4)$$

Isolating block B

Kinetics – NSL – Block B

Applying NSL equations



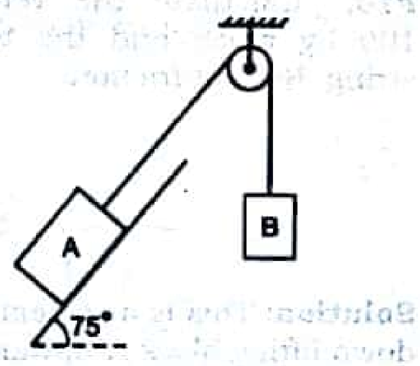
Using $\Sigma F_y = ma_y$

$$T - 98.1 = -10\,a_B \qquad .... (5)$$

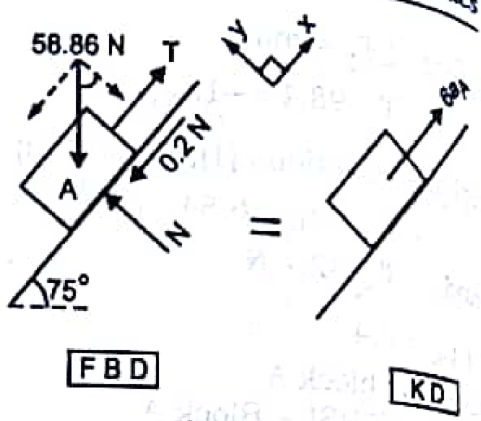
Solving equations (1), (4) and (5) we get

$$a_A = a_B = 5.232\ \text{m/s}^2 \quad \text{and} \quad T = 45.78\ \text{N}$$ .... **Ans.**

P15. Block A and B of mass 6 kg and 12 kg respectively are connected by a string passing over a smooth pulley. Neglect mass of pull. If coefficient of kinetic friction between the block A and the inclined surface is 0.2, determine acceleration of block A and block B. ***(MU Dec 15)***



Solution: This is a dependent system of string connected blocks. Block B travels down, causing block A to slide up the plane.
Using CSLM approach, we get $a_A = a_B$ .... (1)
[Since one portion of string holds A and one portion of the same string holds B.



Isolating block – A

Applying NSL equations to block A
Using $\Sigma F_y = ma_y$

$$N - 58.86\cos 75 = 0$$

$\therefore$ $N = 15.23$ N

Using $\Sigma F_x = ma_x$

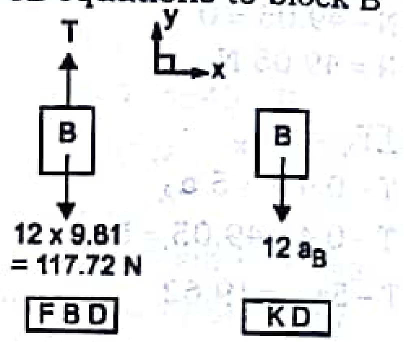
$$T - 58.86\sin 75 - 0.2N = 6a_A$$

$\therefore$ $T - 58.86\sin 75 - 0.2\times 15.23 = 6a_A$

or $6a_A - T = -59.9$ .... (2)

Isolating block – B

Applying NSL equations to block B



Using $\Sigma F_y = ma_y$

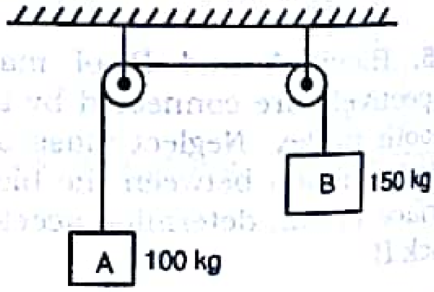
$$T - 117.72 - 12a_B$$

or $12a_B + T = 117.72$ .... (3)

Solving equations (1), (2) and (3) we get

$T = 79.17$ N, $a_A = a_B = 3.212\ m/s^2$ .... **Ans.**

P16. Calculate the vertical acceleration of the 100 kg block and the tension in the connecting string. Neglect friction.



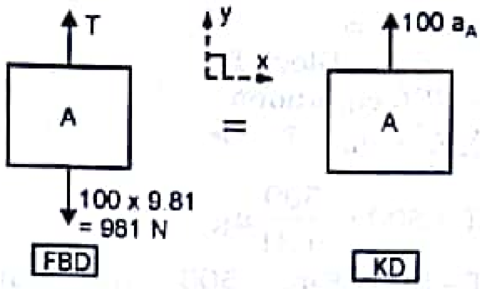
Solution: This is a system of two blocks connected by a common string. Block B travels down lifting block A upwards. Using CSLM approach, we get,

$a_A = a_B$ .... (1)

[since one portion of string holds A and one portion of the same string holds B]

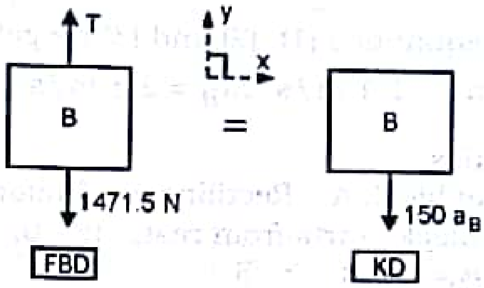
Isolating block A
Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_y = ma_y \uparrow +ve$

$$T - 981 = 100\,a_A \quad \text{......... (2)}$$



Isolating block B
Kinetics – NSL – Block B
Applying NSL equations
Using $\Sigma F_y = ma_y \uparrow +ve$

$$T - 1471.5 = -150\,a_B \quad \text{......... (3)}$$

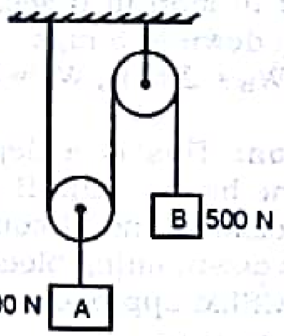


Solving equations (1), (2) and (3) we get

$$a_A = a_B = 1.962\ m/s^2 \quad \text{and} \quad T = 1177.2\ N$$ .... **Ans.**

P17. Determine the tension in the string and the velocity of 1500N block shown in the figure 5 sec after starting from rest.

(VJTI May 10)



Solution: This is a dependent system of two blocks connected by a common string. 1500 N block A travels down, lifting 500 N block B upwards.

Using CSLM approach, we get,

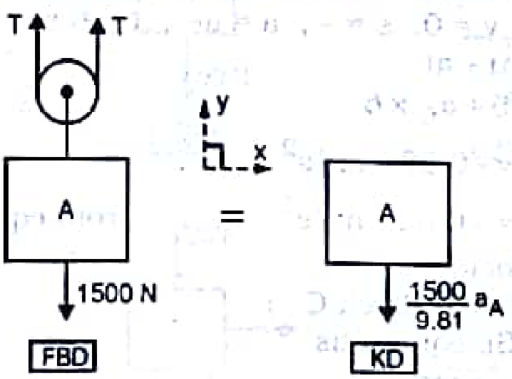
$$2a_A = a_B \quad \text{......... (1)}$$

[since two portions of string holds block A, while only one portion of string holds B]

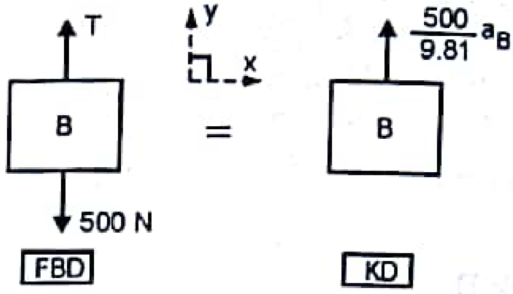
Isolating block A
Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_y = ma_y \uparrow +ve$

$$2T - 1500 = \frac{-1500}{9.81}a_A$$

$$\therefore \quad 2T + 152.9a_A = 1500 \quad \text{......... (2)}$$



Isolating block B
Kinetics – NSL – Block B
Applying NSL equations
Using $\Sigma F_y = ma_y \uparrow +ve$



$$T - 500 = \frac{500}{9.81} a_B$$

$\therefore \quad T - 50.968\, a_B = 500 \quad \text{.........} (3)$

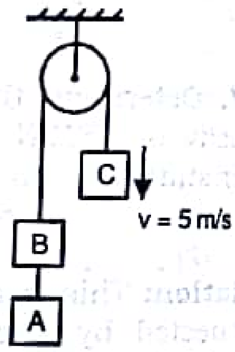
Solving equations (1), (2) and (3) we get

$a_A = 1.4\ m/s^2, a_B = 2.8\ m/s^2$ and $T = 642.86$ N .... **Ans.**

Kinematics
Motion of block A – Rectilinear – Uniform Acceleration
1500 N block starts from rest, $u = 0$, $v = v$, $s = -$, $a = 1.4\ m/s^2$, $t = 5$ sec
using $v = u + at$

$v = 0 + 1.4 \times 5 \quad \therefore \quad v = 7\ m/s$ .... **Ans.**

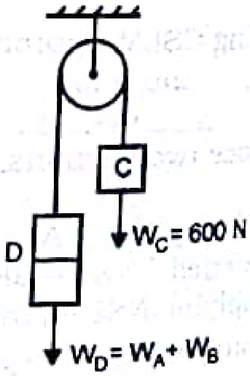
P18. a) Determine the weight W_A of block A required to bring the system to stop in 6 sec, if at the instant shown, the block C is moving down at 5 m/s.
Given W_B = 200 N, W_C = 600 N. Take the pulley to be smooth.



Solution: This is a dependent system of three blocks. Let us combine block A and B as a single block say D. So we now have two blocks C and D connected by a common string. As block C travels down, lifting block D upwards.
Using CSLM approach, we get,

$a_C = a_D$ .... (1)

[since one portion of string holds block C and one portion of the same string holds D]



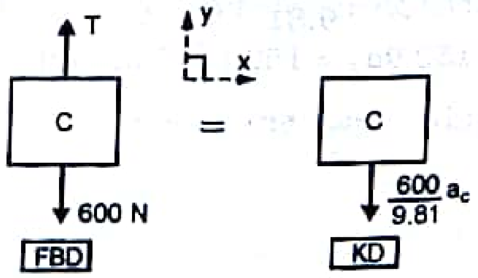
Kinematics – Block C
Motion of block C – Rectilinear – Uniform Acceleration
$u = 5\ m/s$, $v = 0$, $s = -$, $a = a_C$, $t = 6$ sec
Using $v = u + at$

$0 = 5 + a_C \times 6$

$\therefore \quad a_C = -0.833\ m/s^2$

also $\quad a_D = -0.833\ m/s^2$ .... from eqn. (1)

Isolating block C
Kinetics – NSL – Block C
Applying NSL equations
Using $\Sigma F_y = ma_y$

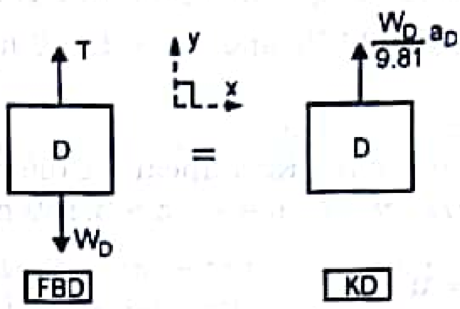


$$T - 600 = \frac{-600}{9.81} \times a_C$$

$\therefore \quad T - 600 = -61.16 \times (-0.833)$

$\therefore \quad T = 650.95$ N

Isolating block D
Applying NSL equations
Using $\Sigma F_y = ma_y$ $\uparrow +ve$



$$T - W_D = \frac{W_D}{9.81} \times a_D$$

$$\therefore \quad 650.95 - W_D = \frac{W_D}{9.81} \times (-0.833)$$

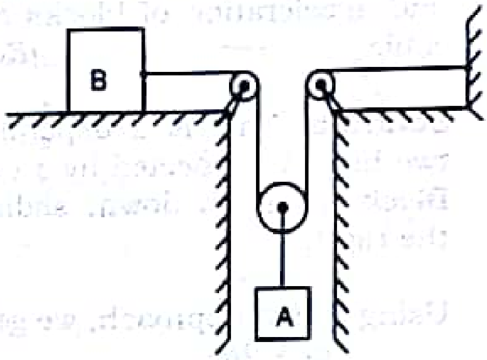
Solving we get $W_D = 711.35$ N

Now $W_D = W_A + W_B$

$\therefore \quad 711.35 = W_A + 200$

or $W_A = 511.35$ N .... **Ans.**

P19. At a given instant 50 N block A is moving downwards with a speed of 1.8 m/sec. determine its speed 2 sec. later. Block 'B' has a weight 20 N, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of pulleys and chord. ***(M.U May 09)***

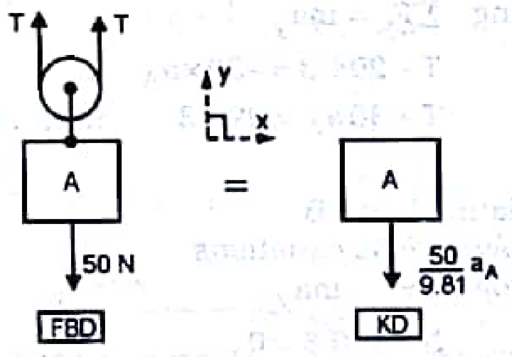


Solution: This is a dependent system of two blocks connected by a common string.
Block A travels down, sliding block B to the right.
Using CSLM approach, we get,

$2a_A = a_B$ .... (1)

[since two portions of the string holds A while one portion of the same string holds B]

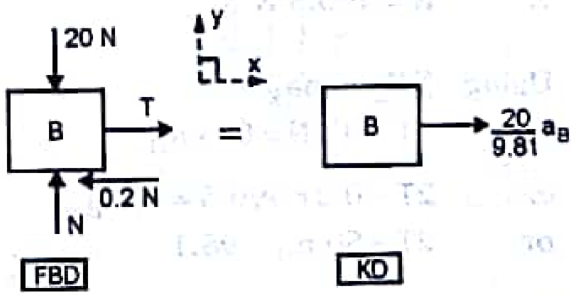
Isolating block A
Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_y = ma_y$ $\uparrow +ve$



$$2T - 50 = \frac{-50}{9.81} \times a_A$$

$$\therefore \quad 2T + 5.097\, a_A = 50 \qquad \text{......... (2)}$$

Isolating block B
Applying NSL equations
Using $\Sigma F_y = ma_y$



$$N - 20 = 0$$

$$\therefore \quad N = 20 \text{ N}$$

Using $\Sigma F_x = ma_x$

$$T - 0.2N = \frac{20}{9.81} \times a_B$$

$\therefore \quad T - 0.2 \times 20 = 2.039\, a_B$ or $T - 2.039\, a_B = 4$ .... (3)

Solving equations (1), (2) and (3) we get

$$T = 16.92 \text{ N}, \text{ and } a_A = 3.169 \text{ m/s}^2$$

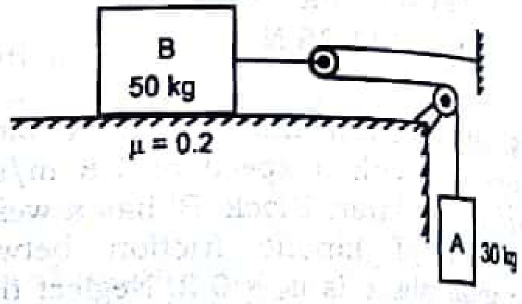
Kinematics – Block A
Motion of block A – Rectilinear – Uniform Acceleration
$u = 1.8$ m/s, $v = v$, $s = -$, $a = 3.169$ m/s², $t = 2$ sec

Using $v = u + at$

$$v = 1.8 + 3.169 \times 2$$

$\therefore$ $v = 8.138$ m/s .... **Ans.**

P20. If the system is released from rest find acceleration of blocks and tension in cable. ***(KJS Nov 15)***

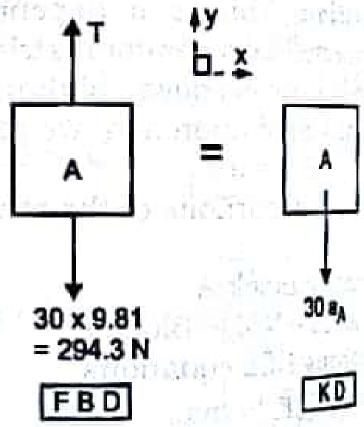


Solution: This is a dependent system of two blocks connected by a common string. Block A travels down, sliding block B to the right.

Using CSLM approach, we get,

$$a_A = 2a_B \quad \text{......... (1)}$$

[since one portion of the string holds A while two portions of the same string holds B]



Isolating block A
Applying NSL equations
Using $\Sigma F_y = ma_y$ $\uparrow$ + ve

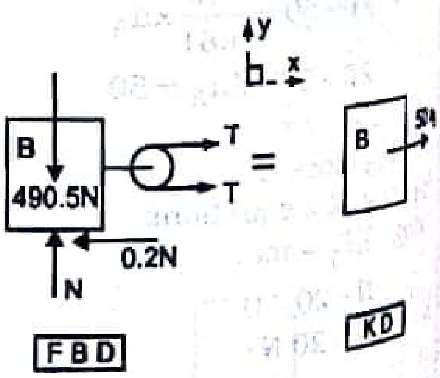
$$T - 294.3 = -30 \times a_A$$

$\therefore$ $T + 30a_A = 294.3$ .... (2)

Isolating block B
Applying NSL equations
Using $\Sigma F_y = ma_y$

$$N - 490.5 = 0$$

$\therefore$ $N = 490.5$ N



Using $\Sigma F_x = ma_x$

$$2T - 0.2N = 50 \times a_B$$

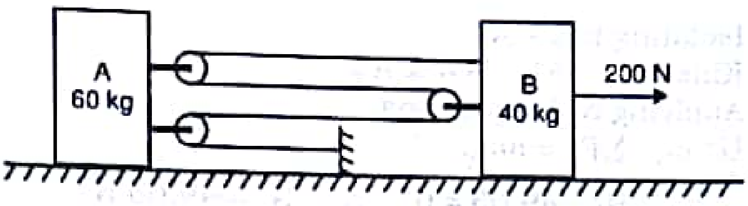
$\therefore$ $2T - 0.2 \times 490.5 = 50a_B$

or $2T - 50\ a_B = 98.1$ .... (3)

Solving equations (1), (2) and (3), we get

$T = 121.2$ N, $a_A = 5.77$ m/s² and $a_B = 2.885$ m/s² ...... **Ans.**

P21. Two blocks A and B are connected by an inextensible string as shown. Neglecting the effect of friction, determine the acceleration of each block and tension in the cable.

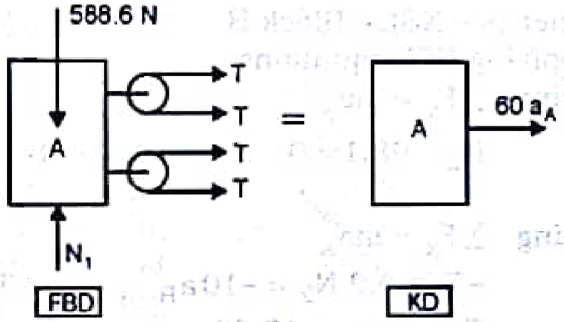


Solution: This is a dependent system of two blocks connected by a common string. Block B is pulled to the right causing block A to slide to the right.
Using CSLM approach, we get,

$4a_A = 3a_B$ .... (1)

[since 4 portions of string holds A, while 3 portions of the same string holds B]

Isolating block A

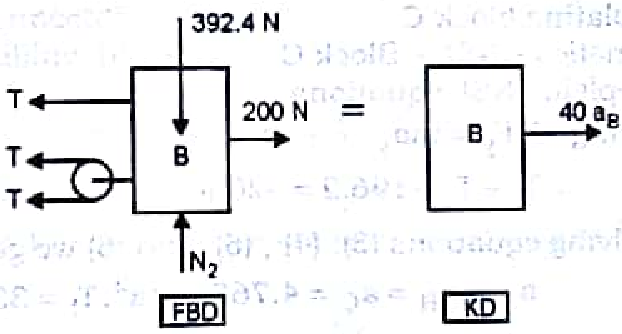


Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_x = ma_x$

$4T = 60a_A$

$\therefore \quad 4T - 60a_A = 0$ .... (2)

Isolating block B
Kinetics – NSL – Block B
Applying NSL equations
Using $\Sigma F_x = ma_x$



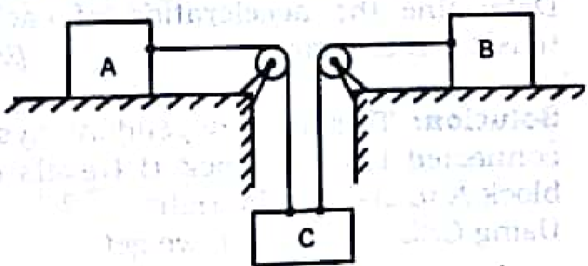
$200 - 3T = 40a_B$

$\therefore \quad 3T + 40a_B = 200$ .... (3)

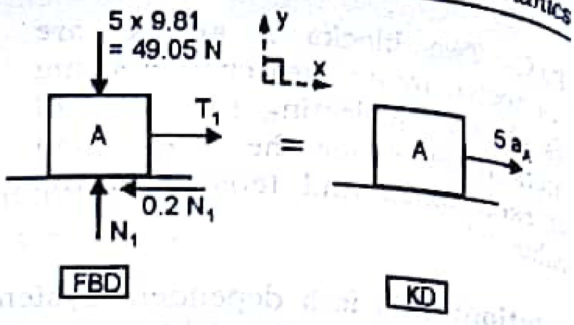
Solving equations (1), (2) and (3) we get,

$T = 30.51$ N, $a_A = 2.034$ m/s^2, $a_B = 2.712$ m/s^2 .... **Ans.**

P22. Masses A (5 kg), B (10 kg), C (20 kg) are connected as shown in the figure by inextensible cord passing over massless and frictionless pulleys. The coefficient of friction for masses A and B with ground is 0.2. If the system is released from rest, find the acceleration of the blocks and tension in the cords. ***(M.U Dec 10)***



Solution: This is a dependent system of three blocks connected by two different strings. Block C travels down causing block A to slide to the right and block B to slide to the left.
Using CSLM approach, for blocks A and C, we get, $a_A = a_C$ .... (1)
Using CSLM approach, for blocks B and C, we get, $a_B = a_C$ .... (2)
Combining equations (1) and (2) we get, we get, $a_A = a_B = a_C$ .... (3)



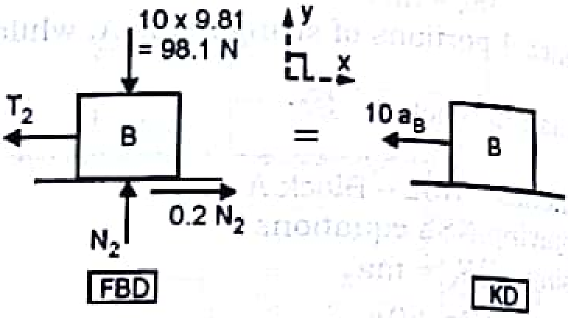
Isolating block A
Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_y = ma_y$

$N_1 - 49.05 = 0 \quad \therefore \quad N_1 = 49.05$ N

Using $\Sigma F_x = ma_x$

$T_1 - 0.2\ N_1 = 5a_A \quad \therefore \quad T_1 - 0.2 \times 49.05 = 5a_A$

$\therefore \quad T_1 - 5a_A = 9.81$ .... (4)



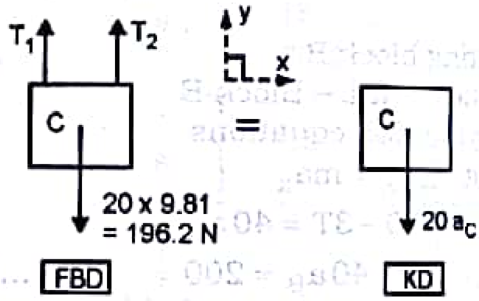
Isolating block B
Kinetics – NSL – Block B
Applying NSL equations
Using $\Sigma F_y = ma_y$

$N_2 - 98.1 = 0 \quad \therefore \quad N_2 = 98.1$ N

Using $\Sigma F_x = ma_x$

$-T_2 + 0.2\ N_2 = -10a_B \quad \therefore \quad -T_2 + 0.2 \times 98.1 = -10a_B$

or $\quad T_2 - 10a_B = 19.62$ .... (5)



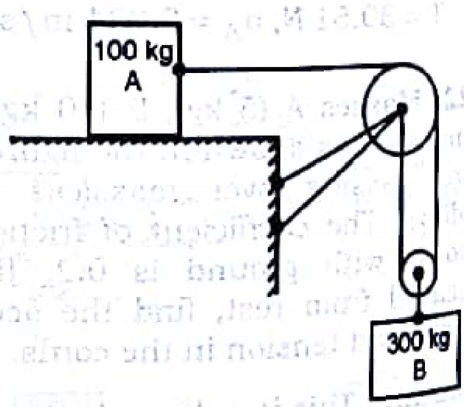
Isolating block C
Kinetics – NSL – Block C
Applying NSL equations
Using $\Sigma F_y = ma_y \quad \uparrow +$ ve

$T_1 + T_2 - 196.2 = -20\ a_C$ ....(6)

Solving equations (3), (4) , (5) and (6) we get,

$\therefore \quad a_A = a_B = a_C = 4.765\ \text{m/s}^2, T_1 = 33.63$ N and $T_2 = 67.27$ N .... **Ans.**

P23. The two blocks starts from rest. The horizontal plane and the pulley are frictionless. Determine the acceleration of each blocks and tension in the cord. ***(VJTI Nov 12)***



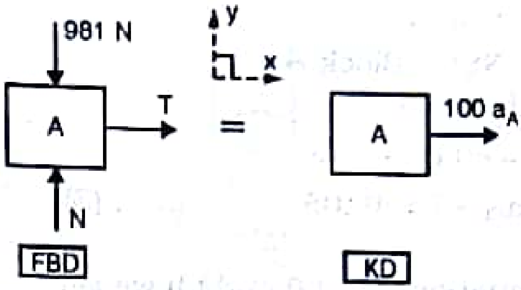
Solution: This is a dependent system of string connected blocks. Block B travels down causing block A to slide to the right.
Using CSLM approach, we get

$a_A = 2a_B$ .... (1)

[since one portion of string holds A, while two portions of the same string holds B.]

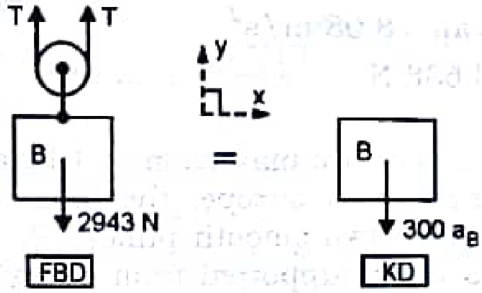
Isolating block A
Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_x = ma_x$

$$T = 100\ a_A \qquad \text{.........} (2)$$



Isolating block B
Kinetics – NSL – Block B
Applying NSL equations
Using $\Sigma F_y = ma_y$ $\uparrow$ + ve

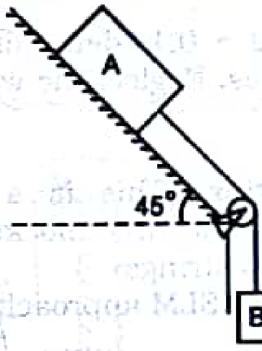
$$2T - 2943 = -300\ a_B \qquad \text{.........} (3)$$



Solving equations (1), (2 and (3) we get,

$\therefore$ $\quad T = 840.86$ N, $a_A = 8.408\ m/s^2$, $a_B = 4.204\ m/s^2$ .... **Ans.**

P24. Two blocks A and B are connected as shown. The string is inextensible. Mass A and B are 3 kg and 5 kg respectively. If coefficient of friction between A and inclined plane is 0.25, determine the tension in the string and accelerations of A and B. ***(M. U. Dec 14)***



Solution: This is system of two blocks connected by a common string. Block B travels down, pulling block A down the slope.

Using CSLM approach, we get,

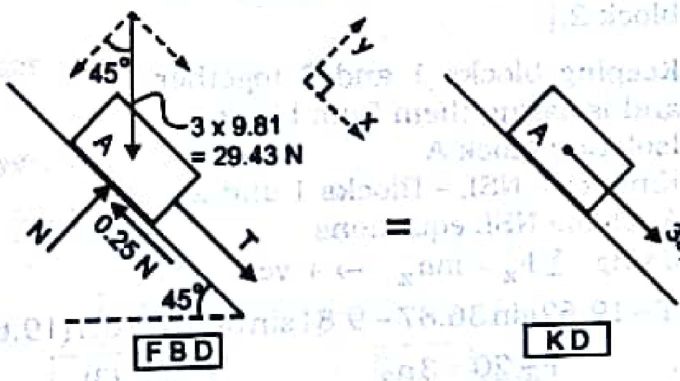
$$a_A = a_B \qquad \text{.........} (1)$$

[since one portion of string holds A and one portion of the same string holds B]

Isolating block A
Kinetics – NSL – Block A
Using $\Sigma F_y = ma_y$

$$N - 29.43 \cos 45 = 0$$

Or $\quad N = 20.81$ N



Using $\Sigma F_x = ma_x$

$$T - 0.25N + 29.43 \sin 45 = 3a_A$$

$\therefore$ $\quad T - 0.25 \times 20.81 + 20.81 = 3a_A$

Or $\quad 3a_A - T = 15.608 \qquad \text{.........} (2)$

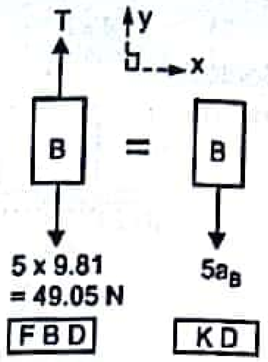
Isolating block B

Kinetics – NSL – Block B

Using $\Sigma F_y = ma_y$

$$T - 49.05 = -5a_B$$

$$5a_B + T = 49.05 \quad \text{......... (3)}$$



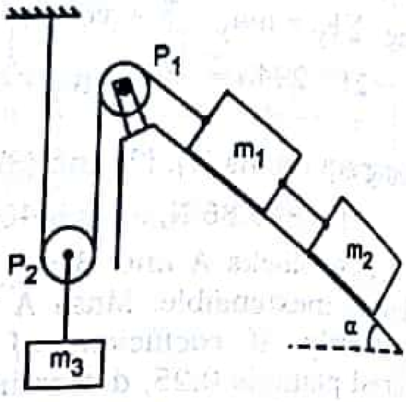
Solving equations (1), (2) and (3) we get

$$a_A = a_B = 8.08\ m/s^2$$

and $\quad T = 8.638\ N$ .... **Ans.**

P25. Figure shows two masses m_1 = 1 kg and m_2 = 2 kg connected by a rope, the other end of which pases over two smooth pulleys P_1 and P_2. If mass m_3 = 5 kg is supported from pulley P_2. If the inclination of the inclined plane is $\tan\alpha = \frac{3}{4}$, and μ = 0.1, determine the accelerations of the masses. Neglect the weight of pulleys.

(VJTI Dec 13)



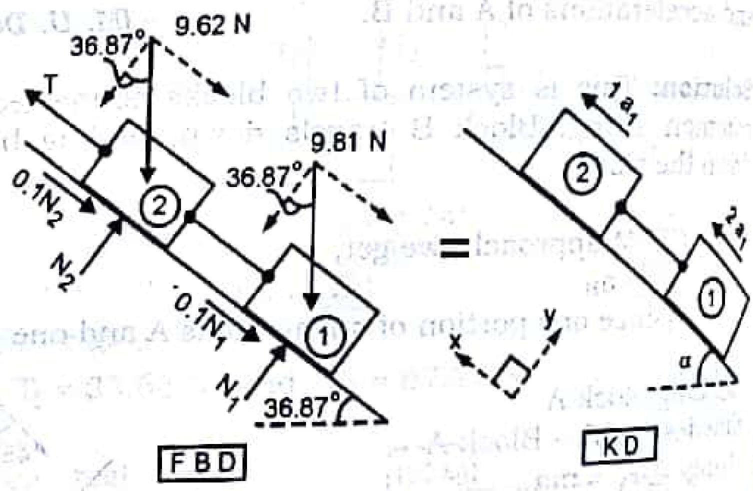
Solution: This is a dependent system of three blocks connected by two strings.

Using CSLM approach, we get,

$a_1 = a_2$ .... (1)

$a_2 = 2a_3$ .... (2)

[since two portions of the string holds block 3, while one portion of the same string holds block 2.]



Keeping blocks 1 and 2 together and isolating them from block 3.

Isolating block A

Kinetics – NSL – Blocks 1 and 2

Applying NSL equations

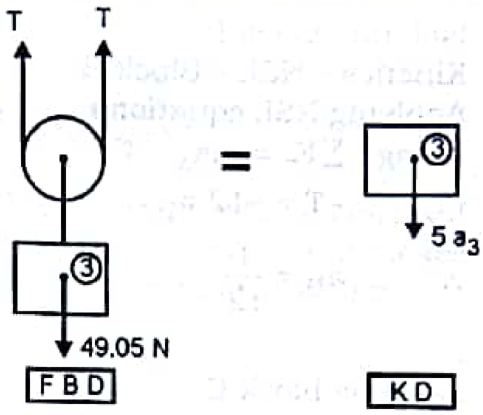
Using $\Sigma F_x = ma_x \quad \rightarrow +ve$

$$T - 19.62\sin 36.87 - 9.81\sin 36.87 - 0.1(19.62\cos 36.87) - 0.1(9.81\cos 36.87) = 1a_2 + 2a_1$$

$$\therefore \quad T - 20 = 3a_2 \quad \text{......... (3)}$$

Isolating block 3
Kinetics – N S L – Block 3
Using $\Sigma F_y = ma_y \quad \rightarrow +ve$

$$2T - 49.05 = -5a_3 \quad \text{......... (4)}$$



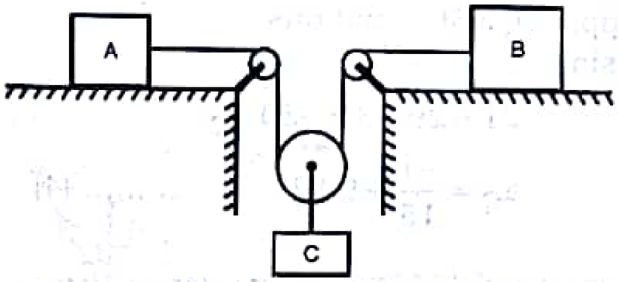
Solving equations (2), (3) and (4) we get

$$T = 23.19\ \text{N},\ a_2 = 1.064\ \text{m/s}^2$$

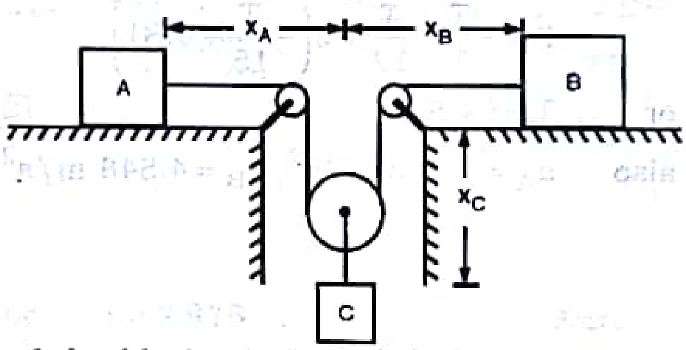
and $\quad a_3 = 0.532\ \text{m/s}^2$ .... **Ans.**

also $\quad a_1 = a_2 = 1.064\ \text{m/s}^2$ .... **Ans.**

P26. Block A (7 kg), B (12 kg), C (30 kg) are connected by an inextensible string as shown. If the system is released from rest find the accelerations of each block and tension in the cord. Assume smooth surfaces.



Solution: This is dependent system of three blocks connected by a common string. Block C travels down causing block A and B to slide to the right and to the left respectively.
Since three blocks are connected by a common string, we need to perform the complete CSLM approach as below



Let x_A, x_B and x_C be the variable portions of the blocks A, B and C. Total length of string, $L = x_A + x_B + 2\,x_C$. As C moves down, x_C increases, while x_A and x_B decreases with time.

Therefore correcting the above equation, we get

$$L = (-x_A) + (-x_B) + 2x_C \pm \text{constants}$$

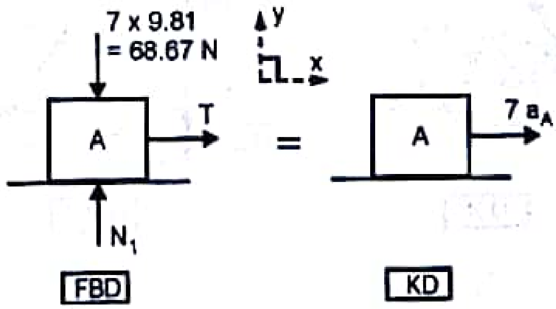
Differentiating the above equation twice, we get

$$0 = -a_A - a_B + 2a_C \quad \text{......... (1)}$$

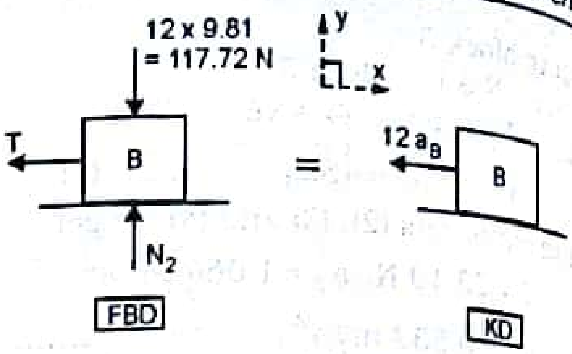
Isolating block A
Kinetics – NSL – Block A
Applying NSL equations
Using $\Sigma F_x = ma_x$

$$T = 7\ a_A$$

$$\therefore \quad a_A = \frac{T}{7} \quad \text{......... (2)}$$



Isolating block B
Kinetics – NSL – Block B
Applying NSL equations

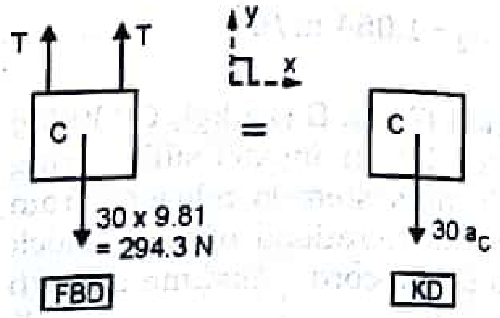


Using $\Sigma F_x = ma_x$

$$-T = -12\, a_B$$

$$\therefore \quad a_B = \frac{T}{12} \quad \text{......... (3)}$$

Isolating block C
Kinetics – NSL – Block C
Applying NSL equations



Using $\Sigma F_y = ma_y$

$$2T - 294.3 = -30\, a_C$$

$$\therefore \quad a_C = \frac{-T}{15} + 9.81 \quad \text{......... (4)}$$

Substituting equations (2), (3) and (4) in equation (1) we get,

$$0 = -\frac{T}{7} - \frac{T}{12} + 2\left(-\frac{T}{15} + 9.81\right)$$

or $\quad T = 54.57\text{ N},$ .... **Ans.**

also $\quad a_A = 7.796\text{ m/s}^2,\ a_B = 4.548\text{ m/s}^2,\ a_C = 6.172\text{ m/s}^2$ .... **Ans.**

Exercise 10.2

N.S.L. – Curvilinear Motion

P1. A steel bob of mass 5 kg tied to a string of 3 m length is whirled with a constant speed, such that the bob moves in a circle in the horizontal plane. If the string makes an angle of 30° with the vertical, find the speed of the bob and tension in the string.

Solution: The steel bob performs curvilinear motion. The bob describes a circle of radius = 3 sin 30 = 1.5 m in x – z plane as shown.

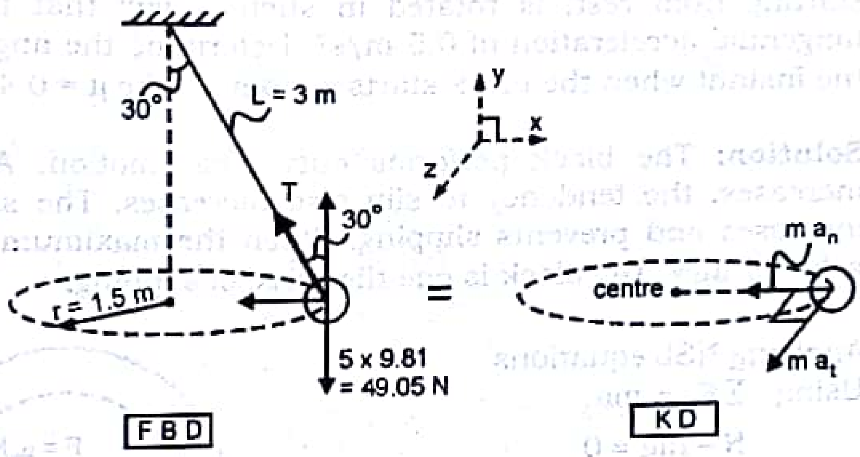
Kinetics – NSL – Steel bob

Applying NSL equations

Using $\Sigma F_y = ma_y$

$T\cos 30 - 49.05 = 0$

$\therefore \quad T = 56.64$ N .... **Ans.**



Using $\Sigma F_x = ma_x$

$-T\sin 30 = -ma_n$

$\therefore \quad 56.64\sin 30 = 5 \times a_n$

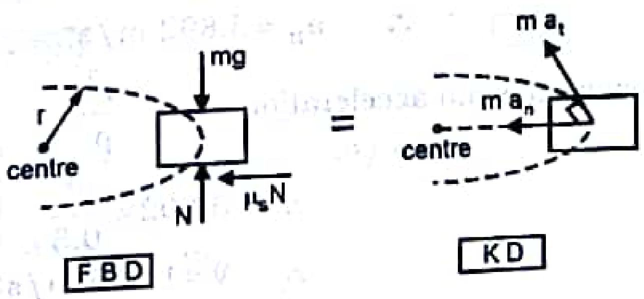
or $\quad a_n = 5.664\ m/s^2$

Knowing normal acceleration $\quad a_n = \dfrac{v^2}{\rho}$

$\therefore \quad 5.664 = \dfrac{v^2}{1.5} \quad$ or $\quad v = 2.915\ m/s$ .... **Ans.**

P2. A car weighing 12 kN goes round a flat curve of 90 m radius. Determine the uniform limiting speed of the car in order to avoid outward skidding. Take $\mu = 0.35$

Solution: The car performs curvilinear motion. It describes a circle of radius 90 m in the x-z plane as shown. As the car travels on the curve it tends to be thrown out of the curve. This motion is prevented by friction force developed at the ground. At the limiting speed (max. speed) the maximum frictional force = $\mu_S N$ is developed. Beyond this limiting speed, the car would skid.



Applying NSL equations

Using $\Sigma F_y = ma_y$

$N - mg = 0 \quad \therefore \quad N = mg$

Using $\Sigma F_x = ma_x$

$-\mu_S N = -ma_n \quad \therefore \quad \mu_S \times mg = ma_n$

or $\quad a_n = \mu_S \times g$

Knowing normal acceleration $a_n = \frac{v^2}{\rho}$

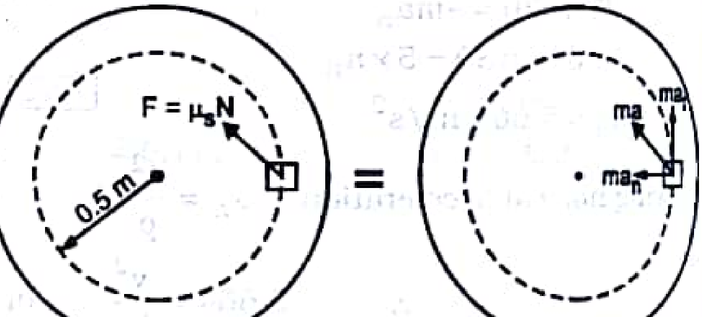
$\therefore \quad \mu_S \times g = \frac{v^2}{r}$ or $v = \sqrt{\mu_S \times g \times r}$ .... **Ans.**

Given $\mu_S = 0.35$ and $r = 90$ m

$v = \sqrt{0.35 \times 9.81 \times 90}$ or $v_{max} = 17.579$ m/s .... **Ans.**

P3. A small block rests on a turn table, 0.5 m away from its centre. The turn table, starting from rest, is rotated in such a way that the block undergoes a constant tangential acceleration of 0.5 m/s². Determine the angular velocity of the turn table at the instant when the block starts slipping. Take $\mu = 0.4$ ***(M.U Dec 15)***

Solution: The block performs curvilinear motion. As the speed of the turn table increases, the tendency to slip also increases. The static friction force automatically increases and prevents slipping. When the maximum static frictional force reaches a value of $\mu_S N$, the block is one the verge of slipping.



Applying NSL equations

Using $\Sigma F_y = ma_y$

$N - mg = 0$

$\therefore \quad N = mg$

Using $\Sigma F_x = ma_x$

$\mu_S N = ma$

$\therefore \quad 0.4 \times (m \times 9.81) = m \times a$

$\therefore \quad a = 3.924 \ m/s^2$

Now, Total acceleration $a = \sqrt{a_n^2 + a_t^2}$

$3.924^2 = \sqrt{a_n^2 + 0.5^2}$

$\therefore \quad a_n = 3.892 \ m/s^2$

Knowing normal acceleration $a_n = \frac{v^2}{\rho}$

$\therefore \quad 3.892 = \frac{v^2}{0.5}$

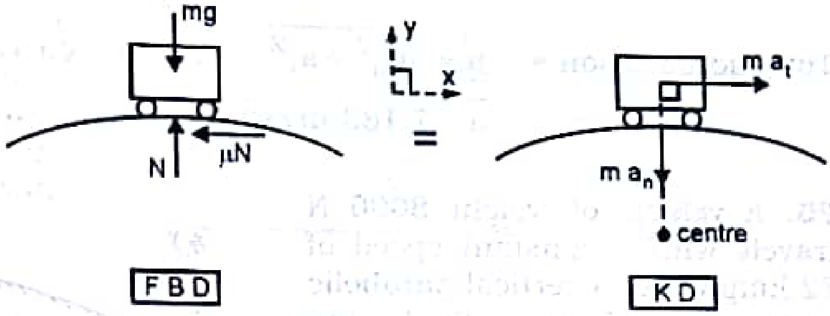
$\therefore \quad v = 1.395 \ m/s$

Using $v = r\omega$

$\therefore \quad 1.395 = 0.5 \times \omega$ or $\omega = 2.79$ rad/s .... **Ans.**

P4. Figure shows two vehicles moving at 90 kmph on a road. Knowing that $\mu = 0.5$ between road and the tyres. Determine total acceleration of each vehicle when the brakes are suddenly applied and the wheels skid.

Solution: Cars A and B performs curvilinear motion in a vertical plane. When brakes are applied, friction force μN acts at the ground surface.



Kinetics – NSL – Car A

Applying NSL equations

Using $\Sigma F_y = ma_y$

$$N - mg = -m\,a_n$$

$\therefore$ $N = m(9.81 - 3.125)$

or $N = 6.685\ m$

Using $\Sigma F_x = ma_x$

$$-\mu_s N = m\,a_t$$

$$-0.5 \times 6.685 = m\,a_t$$

$\therefore$ $a_t = -3.3425\ m/s^2$

$v = 90\ \text{kmph} = 25\ m/s$

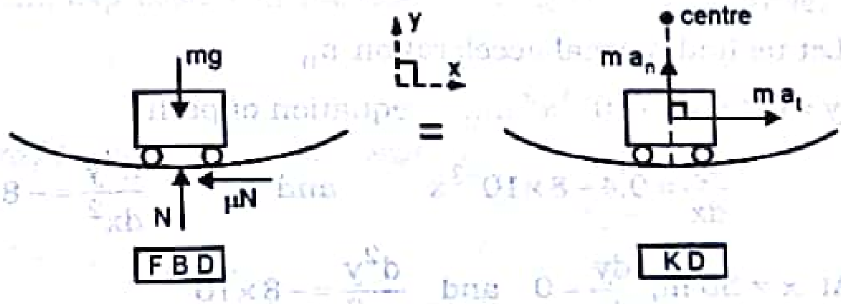
$$a_n = \frac{v^2}{\rho} = \frac{25^2}{200}$$

$$= 3.125\ m/s^2$$

Total acceleration $= a = \sqrt{a_n^{\,2} + a_t^{\,2}}$ $\therefore$ $a = \sqrt{3.125^2 + 3.3425^2}$

or $a = 4.576\ m/s^2$ .... **Ans.**

Kinetics-NSL-Car B



Applying NSL equations

Using $\Sigma F_y = ma_y$

$$N - mg = m\,a_n$$

$\therefore$ $N = m(g + a_n)$

$\therefore$ $N = m(9.81 + 3.125)$

or $N = 12.935\ m$

$v = 90\ \text{kmph} = 25\ m/s$

$$a_n = \frac{v^2}{\rho} = \frac{25^2}{200}$$

$$= 3.125\ m/s^2$$

Using $\Sigma F_x = ma_x$

$$-\mu_s N = m a_t$$

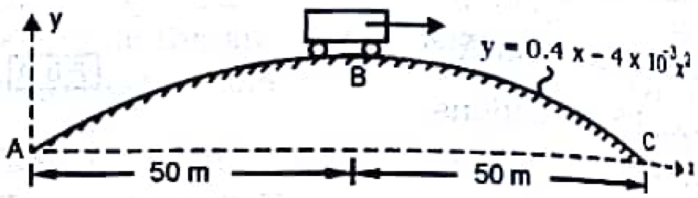
$$-0.5 \times 12.935\ m = m a_t$$

$$\therefore \quad a_t = -6.468\ m/s^2$$

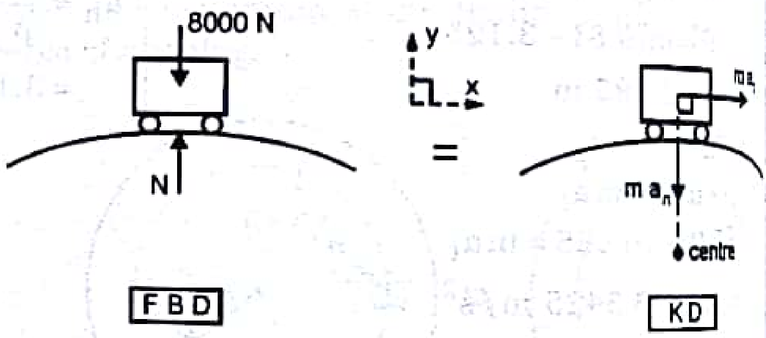
Total acceleration = $a = \sqrt{a_n^2 + a_t^2}$ $\quad \therefore \quad a = \sqrt{3.125^2 + 6.468^2}$

or $\quad a = 7.183\ m/s^2$ .... **Ans.**

P5. A vehicle of weight 8000 N travels with a constant speed of 72 kmph over a vertical parabolic curve as shown. Find the pressure exerted by the tyres on the road at the peak B.



Solution: The vehicle performs curvilinear motion in a vertical plane. Let N be the reaction to the pressure exerted by the tyres on the road.



NSL – Vehicle

Using $\Sigma F_y = ma_y$

$$N - 8000 = -m a_n$$

$$\therefore \quad N - 8000 = -815.49 a_n \quad \text{......... (1)}$$

$$m = \frac{8000}{9.81} = 815.49\ kg$$

$$v = 72\ kmph = 20\ m/s$$

Let us find normal acceleration a_n

$y = 0.4x - 4 \times 10^{-3} x^2$ .... equation of path

$$\frac{dy}{dx} = 0.4 - 8 \times 10^{-3} x \quad \text{and} \quad \frac{d^2y}{dx^2} = -8 \times 10^{-3}$$

At $x = 50$ m, $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = -8 \times 10^{-3}$

Knowing

$$\text{radius of curvature } \rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| \quad \therefore \quad \rho = \left| \frac{1+0}{-8 \times 10^{-3}} \right| \quad \text{or} \quad \rho = 125\ m$$

Using $a_n = \dfrac{v^2}{\rho} = \dfrac{20^2}{125} = 3.2 \text{ m/s}^2$

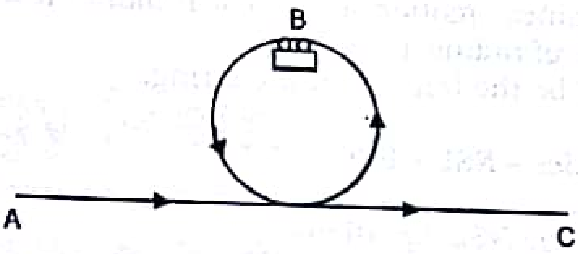
Substituting value of a_n in equation (1)

$$N - 8000 = -814.49 \times 3.2$$

Or $\quad N = 5390.4$ N

.... **Ans.**

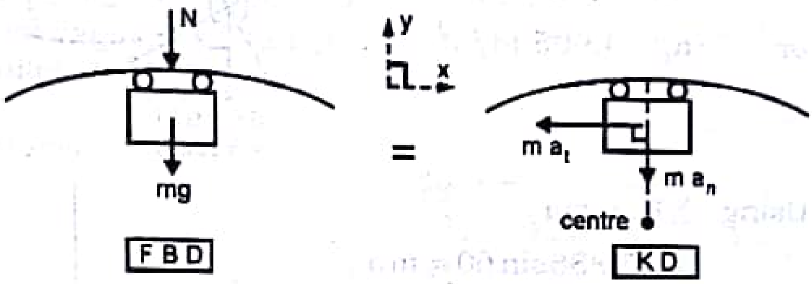
P6. In a roller coaster ride, the coaster traveling around the loop ABC needs to have a certain minimum velocity at the peak B. If the loop diameter is 10 m, find this velocity.



Solution: The roller coaster performs curvilinear motion in a vertical plane. It describes a circle of radius 5 m. Let us analyse it at the peak of curve.

Kinetics-NSL-Roller coaster

Applying NSL equations



Using $\Sigma F_y = ma_y \quad \uparrow +ve$

$$-N - mg = -m a_n$$

$\therefore \quad 0 - mg = -m a_n$

or $\quad a_n = g$

but $\quad a_n = \dfrac{v^2}{\rho}$

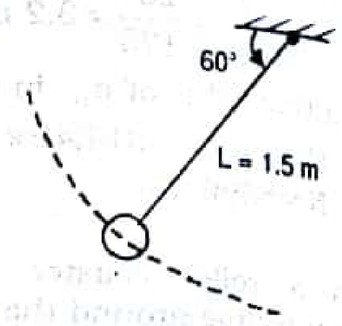
Subsisting $N = 0$ since for minimum speed condition at peak B, the roller coaster tends to loose contact with the surface at B, causing normal reaction N to become zero.

$\therefore \quad g = \dfrac{v^2}{\rho} \quad$ or $\quad v = \sqrt{g r}$ .... expression for minimum speed at peak of curve.

Substituting $r = 5$ m

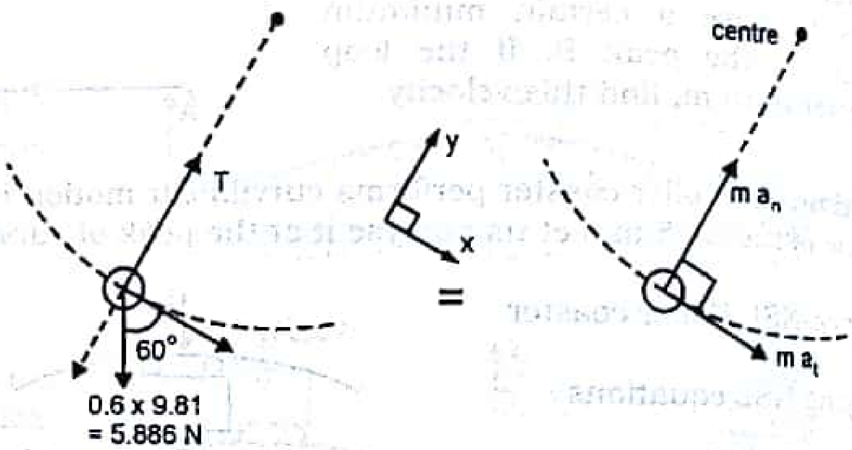
$v = \sqrt{9.81 \times 5} \quad$ or $\quad v = 7$ m/s .... **Ans.**

P7. A pendulum of mass 600 gms, has a speed of 3.6 m/s at the position shown. Find the tension in the string and the total acceleration at this instant.



Solution: The bob of pendulum performs curvilinear motion in a vertical plane. It describes a circle of radius 1.5 m.
Let T be the tension in the string.

Kinetics – NSL – Bob



Applying NSL equations

Using $\Sigma F_x = ma_x$

$$5.886 \cos 60 = m a_t$$

$$2.943 = 0.6\ a_t$$

or $a_t = 4.905\ m/s^2$

Using $\Sigma F_y = ma_y$

$$T - 5.886 \sin 60 = m a_n$$

$$T - 5.097 = 0.6 \times 8.64$$

$\therefore\ T = 10.281$ N .... **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{3.6^2}{1.5}$$

or $a_n = 8.64\ m/s^2$

Total acceleration $a = \sqrt{a_n^2 + a_t^2}$ $a = \sqrt{8.64^2 + 4.905^2}$ $\therefore\ a = 9.935\ m/s^2$ ...... **Ans.**

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