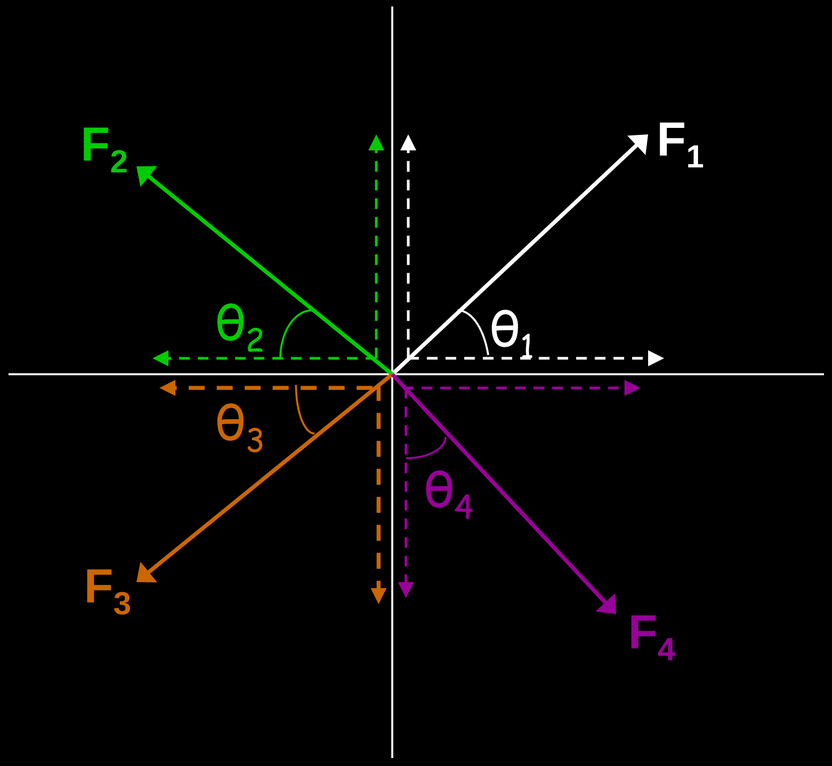
FORCE

Force is a physical quantity which when applied on a system changes or tends to change the state of the system.

REPRESENTATION OF FORCE VECTOR

Force, in general, is written in vector form as :-

$$F = F_X.i + F_Y.j \text{ (N)}$$



$$F_1 = F_1\cos\theta_1.i + F_1\sin\theta_1.j$$

$$F_2 = -F_2\cos\theta_2.i + F_2\sin\theta_2.j$$

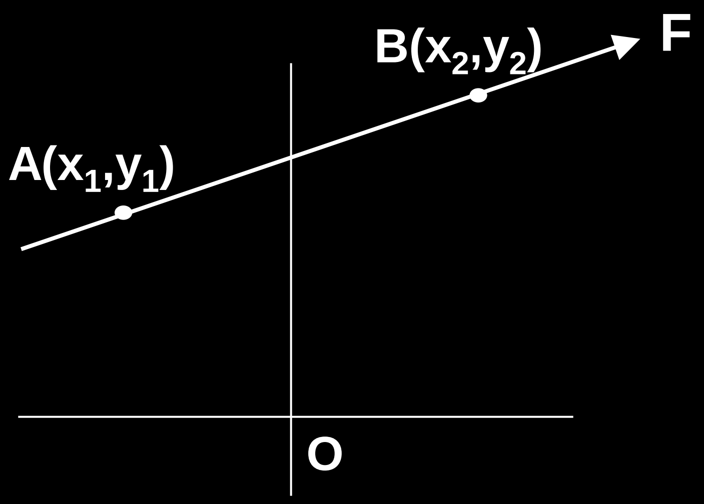
$$F_3 = -F_3\cos\theta_3.i - F_3\sin\theta_3.j$$

$$F_4 = F_4\sin\theta_4.i - F_4\cos\theta_4.j$$

REPRESENTATION OF FORCE VECTOR

$$\vec{F} = F.\,\vec{ab}$$

Unit vector



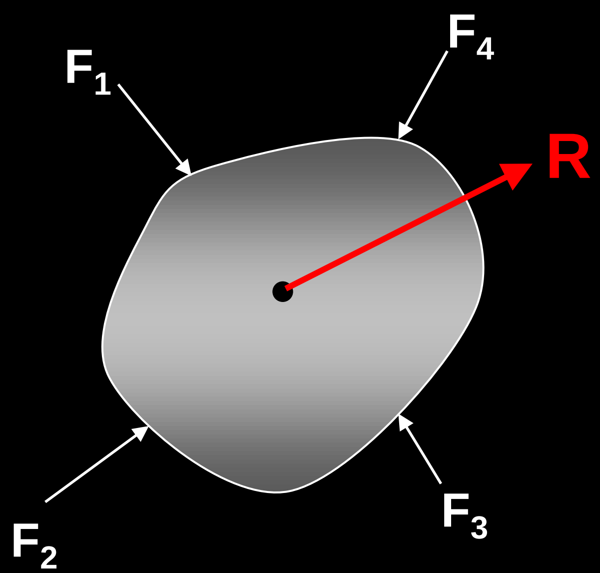
$$\vec{F} = F \times \left[\frac{(x_2 - x_1).i + (y_2 - y_1).j}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \right]$$

RESULTANT FORCE

IT IS A SINGLE FORCE THAT REPLACES AN ENTIRE GIVEN SYSTEM OF FORCES (MOST IMPORTANTLY) HAVING THE SAME EFFECT AS THAT OF THE GIVEN FORCE SYSTEM.

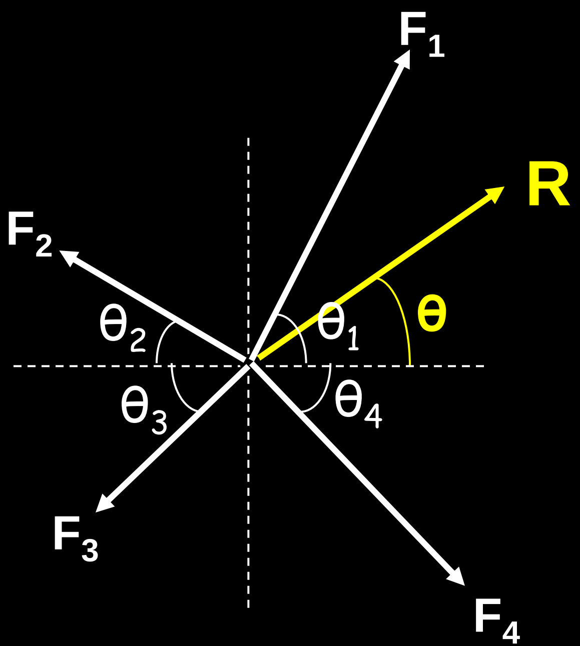
RESULTANT FORCE

- **MAGNITUDE**
- **DIRECTION**
- **POINT OF APPLICATION**



RESULTANT OF CONCURRENT FORCES

CONSIDER A CONCURRENT FORCE SYSTEM AS SHOWN IN FIGURE.



X – COMPONENT OF RESULTANT FORCE

$R_X = \sum F_X$

$= (F_1\cos\theta_1)-(F_2\cos\theta_2)-(F_3\cos\theta_3)+(F_4\cos\theta_4)$

Y – COMPONENT OF RESULTANT FORCE

$R_y = \sum F_y$

$= (F_1\sin\theta_1)+(F_2\sin\theta_2)-(F_3\sin\theta_3)-(F_4\sin\theta_4)$

$R = \sqrt{(R_X^2 + R_Y^2)}$ & $\theta = \tan^{-1}\{ R_Y / R_X \}$

THE RESULTANT OF A CONCURRENT FORCE SYSTEM ALWAYS PASSES THROUGH THE POINT OF CONCURRENCE !

MOMENT OF A FORCE

MOMENT IS DEFINED AS THE ROTATIONAL EFFECT OF A FORCE ABOUT ANY POINT.

MATHEMATICALLY, THE MAGNITUDE OF MOMENT IS GIVEN BY THE PRODUCT OF MAGNITUDE OF THE FORCE AND IT'S PERPENDICULAR DISTANCE FROM A GIVEN REFERENCE POINT.

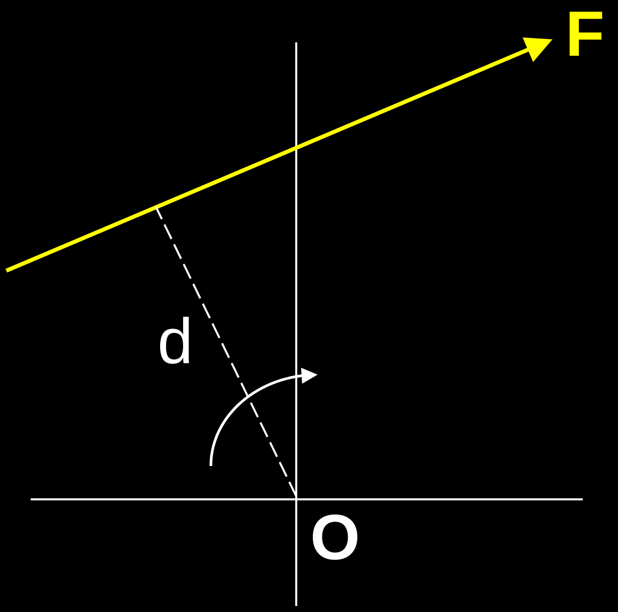
MOMENT ABOUT 'O'

$$M_o = \pm \{ F \times d \}$$

DEPENDING ON DIRECTION OF ROTATION.

+ve → ANTICLOCKWISE

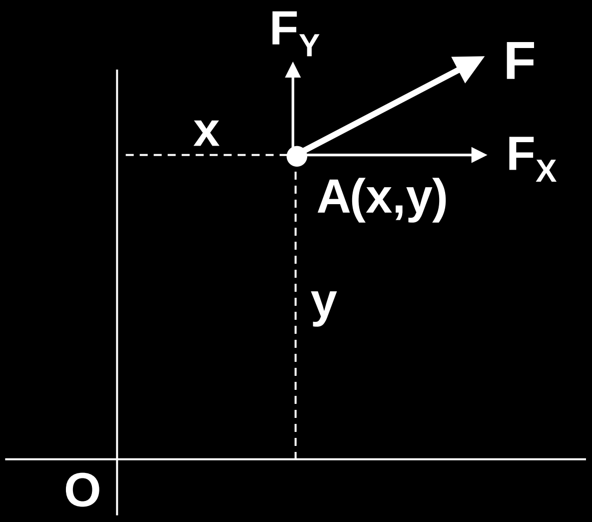
- ve → CLOCKWISE.



VARIGNON'S THEOREM

MOMENT OF A FORCE ABOUT ANY POINT IS ALSO EQUAL TO MOMENT DUE TO IT'S COMPONENTS ABOUT THE SAME POINT.

$$M_O = -(F_X \times y) + (F_y \times x)$$



NOW, LET US HAVE A SMALL EXERCISE !

FIND THE MOMENT OF THE FORCES SHOWN BELOW ABOUT THE ORIGIN.

$M_1 = +(5\cos 50 \times 3) + (5\sin50 \times 5)$

$= 28.8$ N-m

$M_2 = +(8\cos 30 \times 3) + (8\sin30 \times 3)$

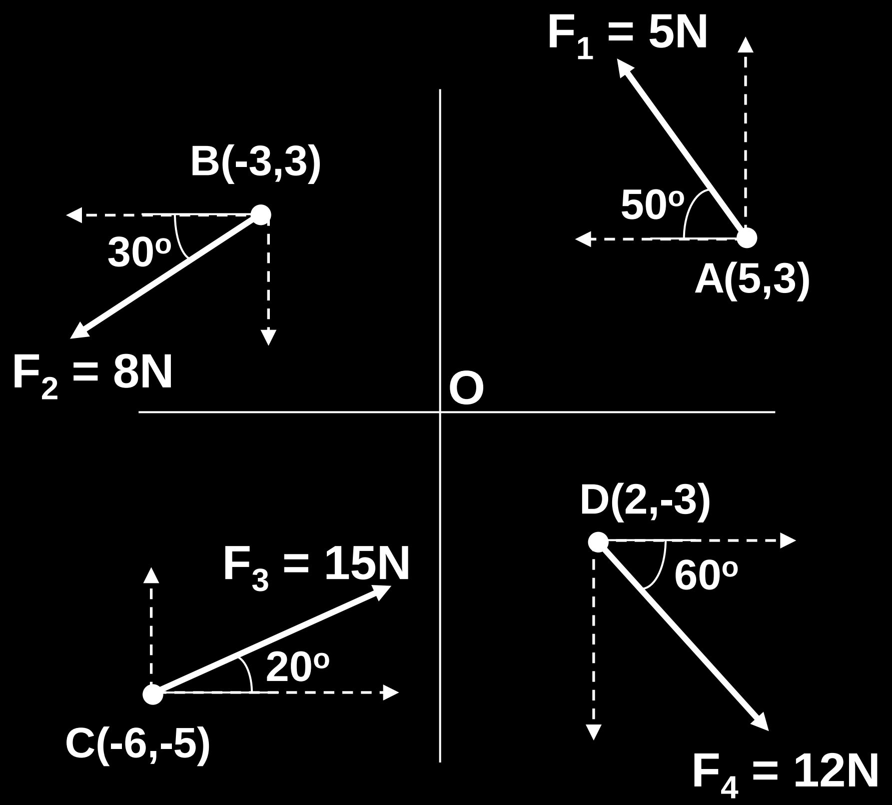
$= 32.78$ N-m

$M_3 = +(15\cos 20 \times 5) - (15\sin20 \times 6)$

$= 39.7$ N-m

$M_4 = +(12\cos 60 \times 3) - (12\sin60 \times 2)$

$= -2.79$ N-m



MOMENT VECTOR

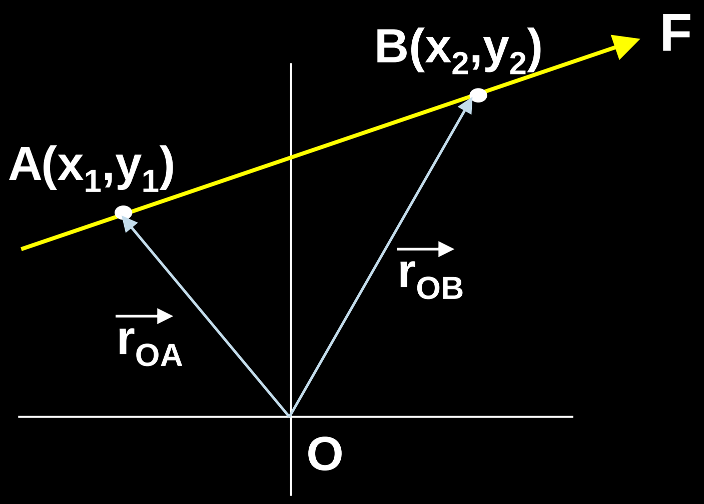
$$\vec{M}_O = \vec{r} \times \vec{F}$$

POSITION VECTOR OF ANY POINT ON THE LINE OF ACTION OF THE FORCE

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}$$

'OR'

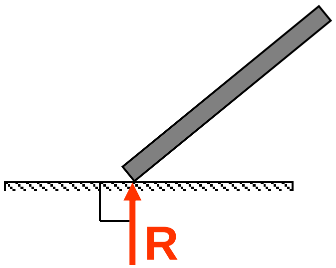
$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$



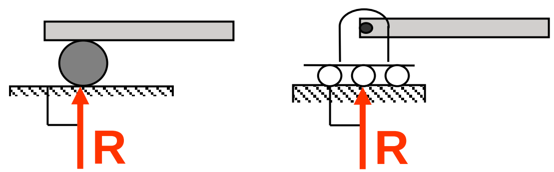
TYPES OF COPLANAR FORCES

- COPLANAR FORCES
  - CONCURRENT
  - PARALLEL
  - GENERAL COPLANAR

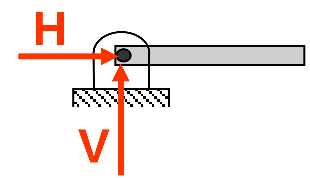
DIFFERENT TYPES OF SUPPORTS



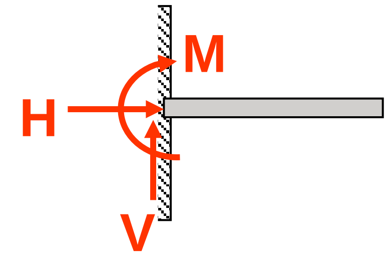
Plane support



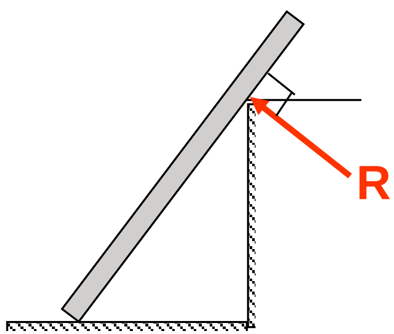
Roller support



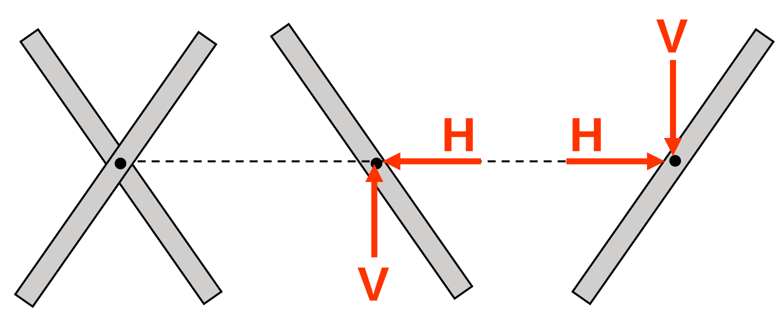
Hinge support



Fixed support



Knife edge



Pin - joint

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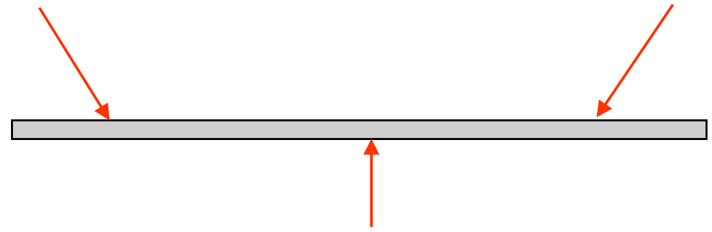
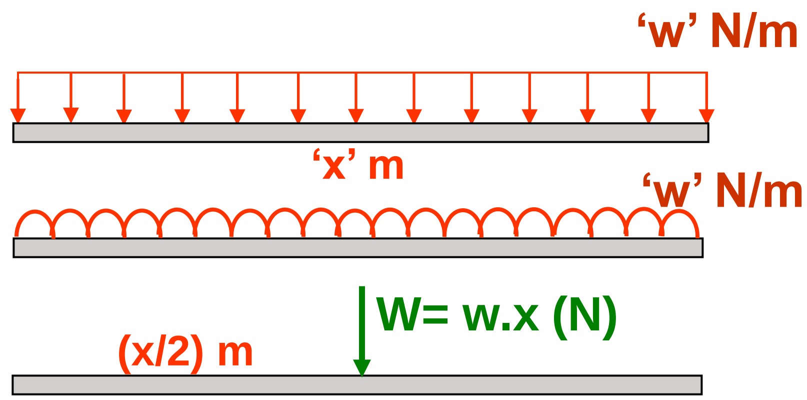
DIFFERENT TYPES OF LOADS

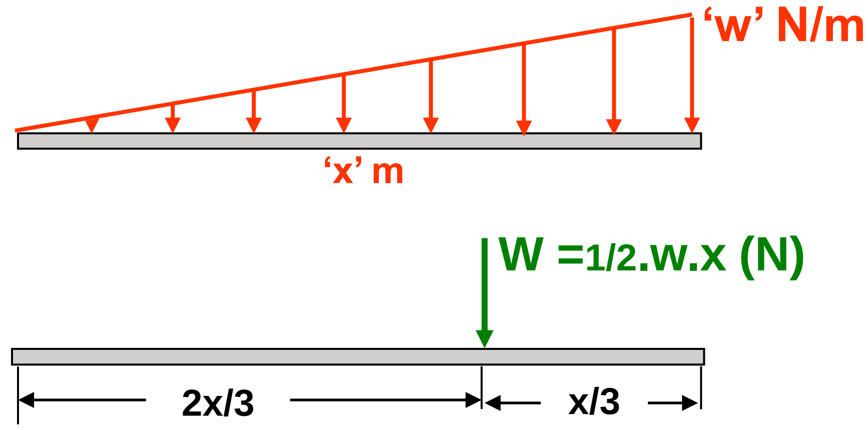
1. POINT LOAD



2. UNIFORMLY DISTRIBUTED LOAD (UDL)



3. UNIFORMLY VARIABLE LOAD (UVL)



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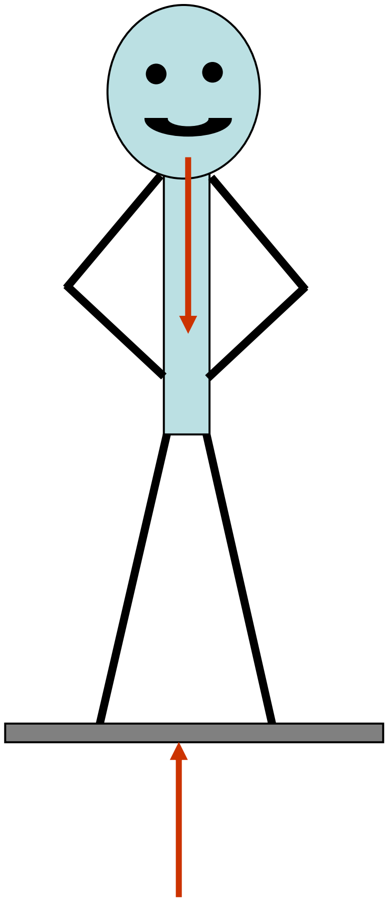
FREE BODY DIAGRAM (F.B.D.)



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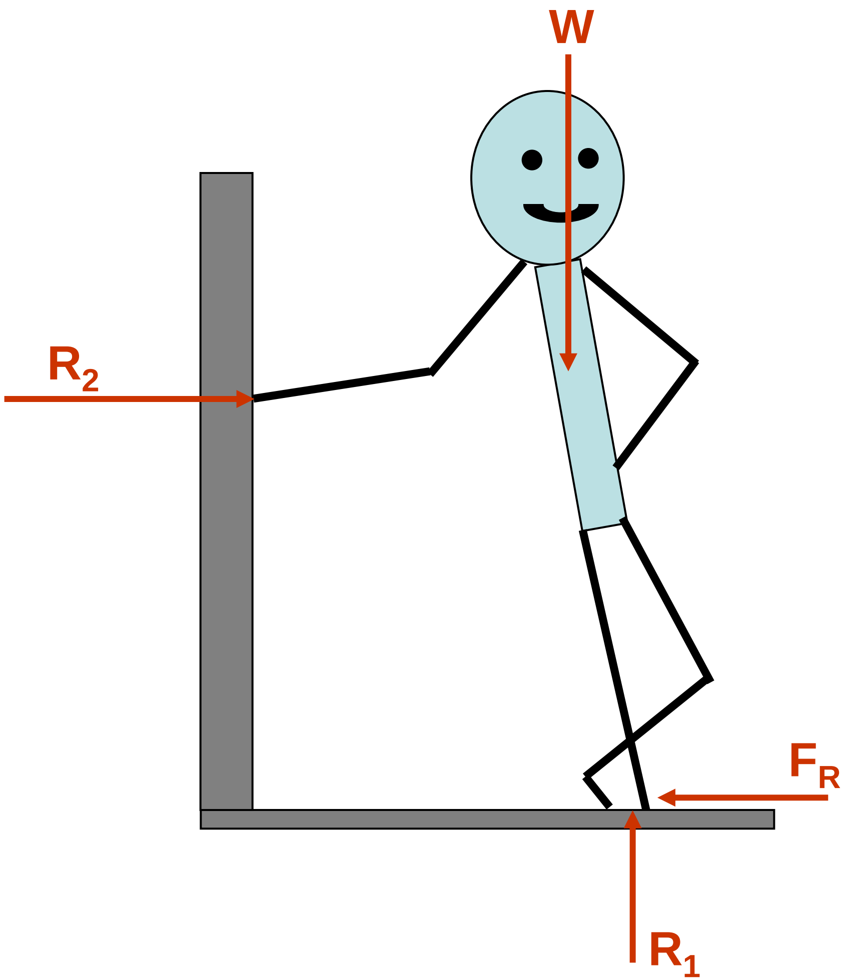
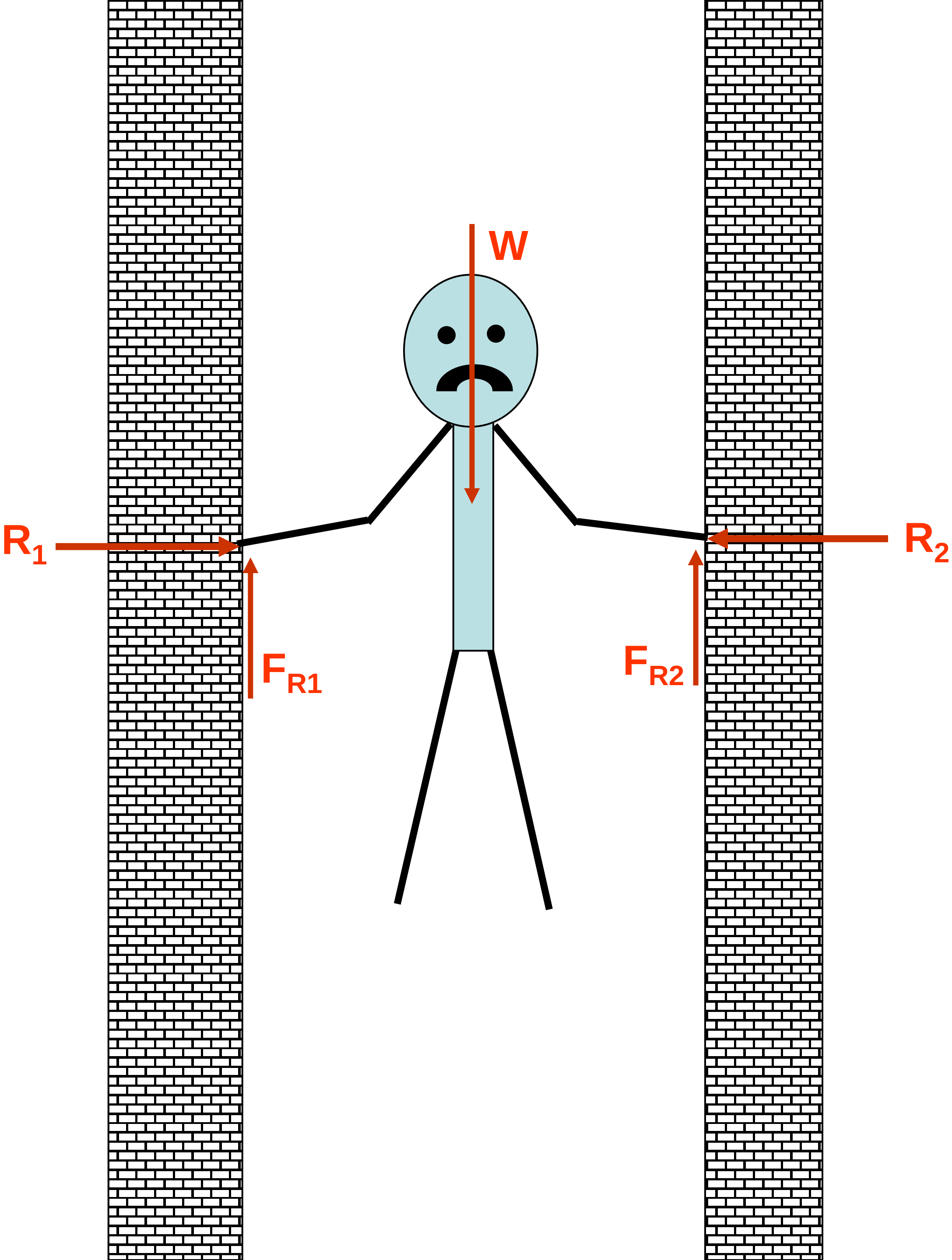
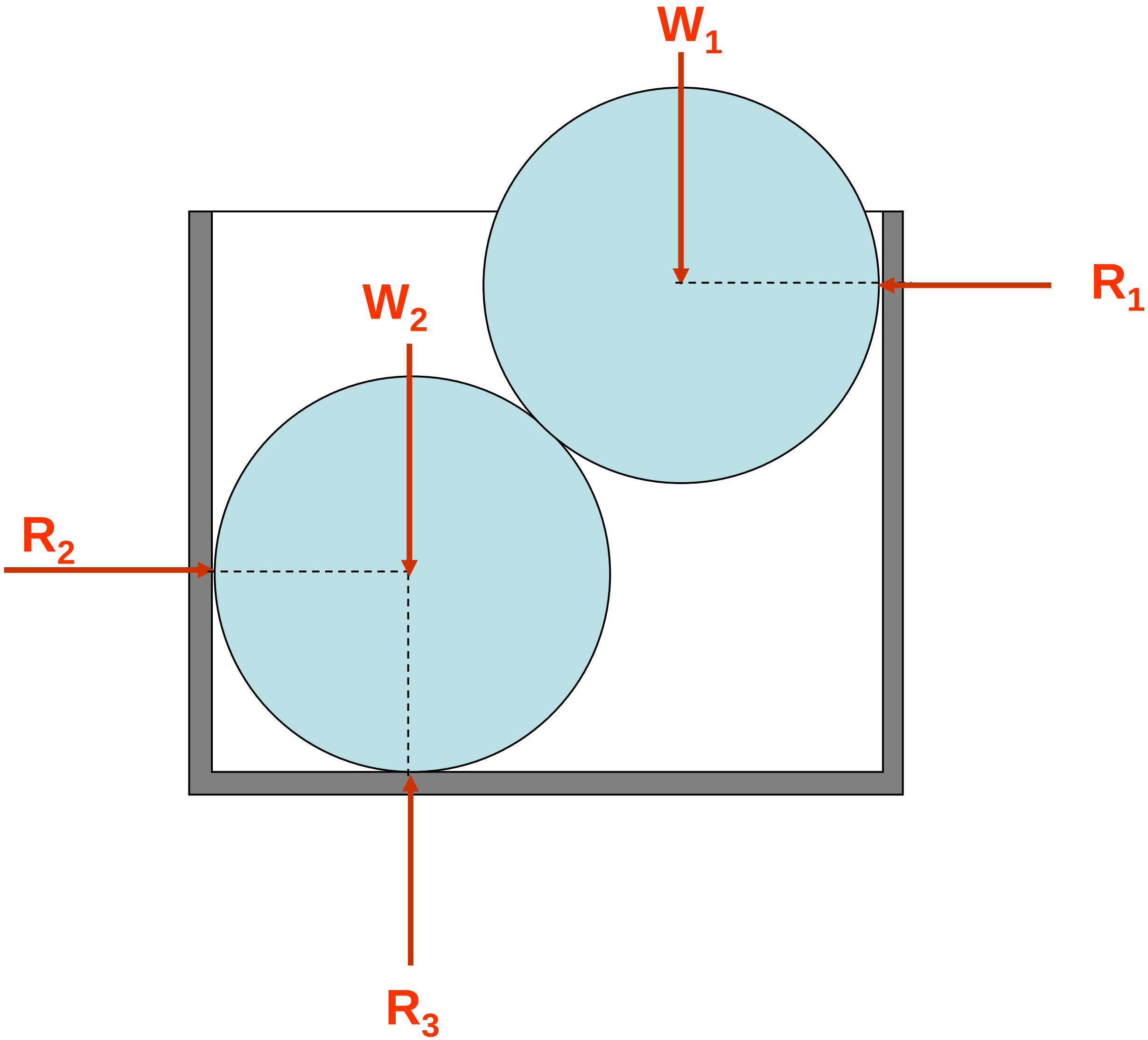


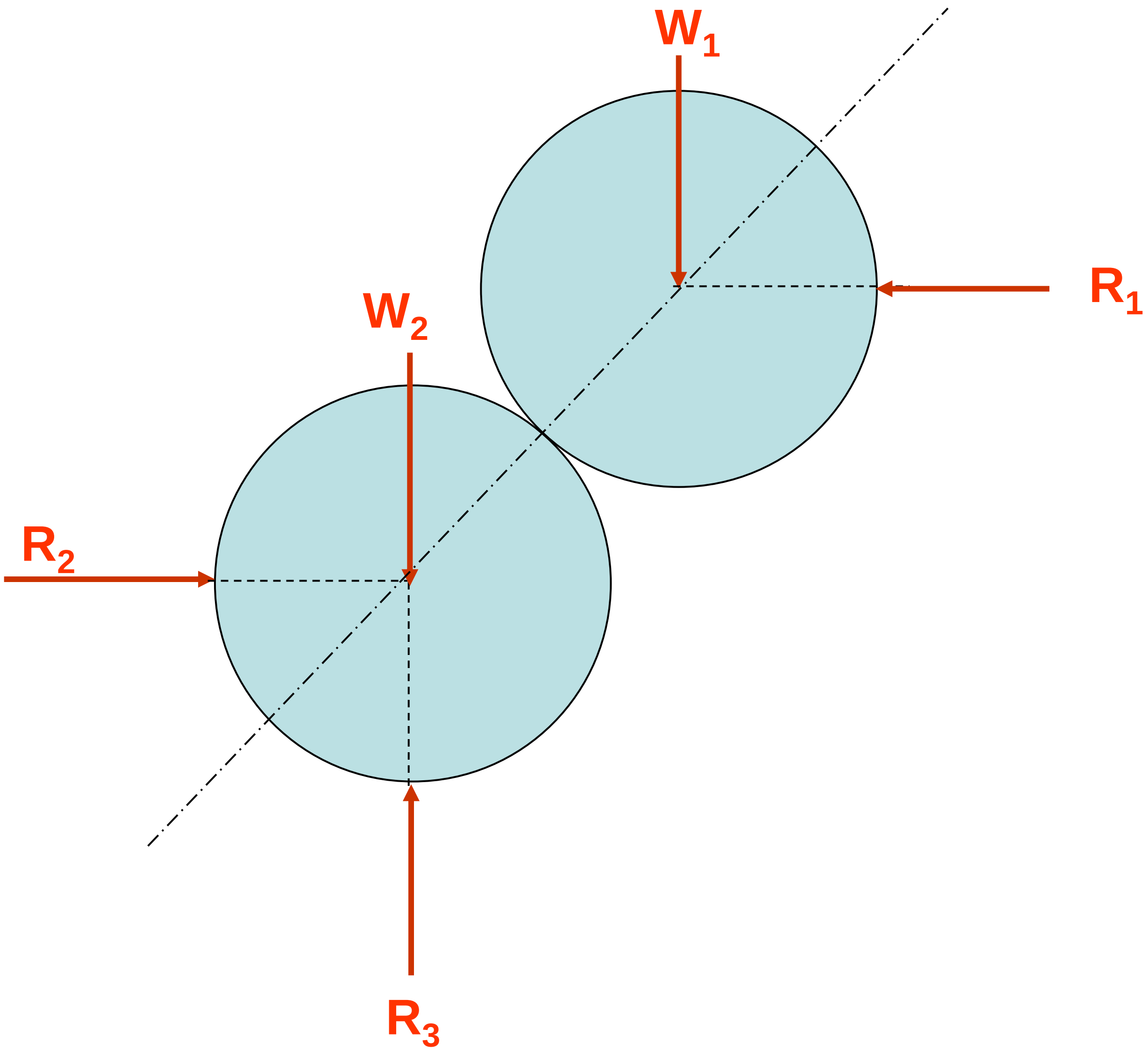
PREPARED BY M.B.RAO



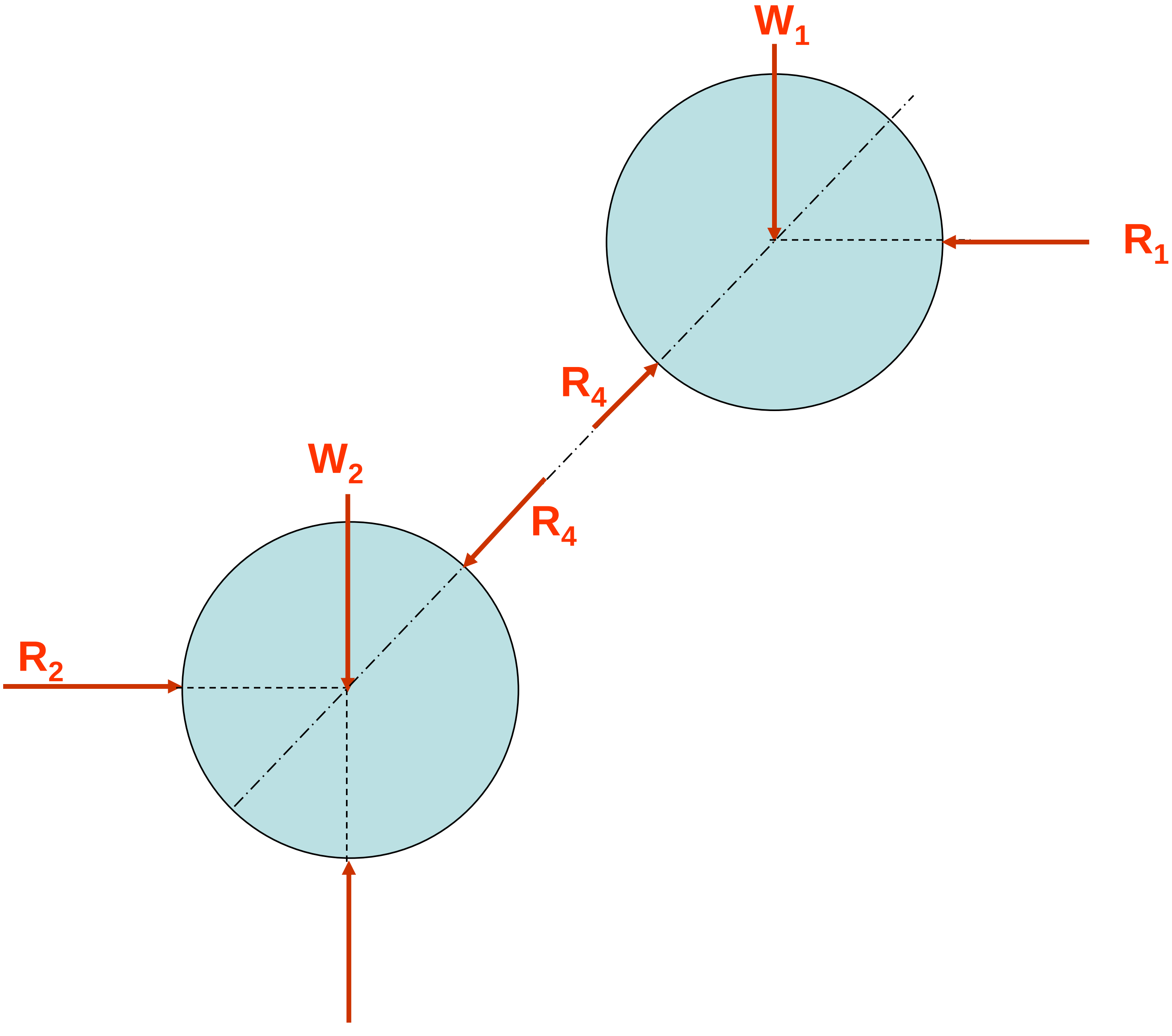
PREPARED BY
M.B.RAO



PREPARED BY M.B.RAO



W1
W2
R1
R2
R3



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HENCE, F.B.D. IS THE DIAGRAM OF A SYSTEM THAT IS ISOLATED FROM THE SURROUNDING (ie. FROM ALL CONTACT POINTS) AND SHOWN WITH ALL THE EXTERNAL FORCES AND MOMENTS ACTING ON IT.

THE TWO GENERAL CONDITIONS OF EQUILIBRIUM ARE :-

$\sum$ FORCES = 0 & $\sum$ MOMENTS = 0

(i) CONCURRENT FORCES :- $\sum F_X = 0$ & $\sum F_Y = 0$

(ii) PARALLEL FORCES :- $\sum F_X = 0$ 'OR' $\sum F_Y = 0$
&
$\sum M_Z = 0$

(iii) GENERAL COPLANAR :- $\sum F_X = 0$, $\sum F_Y = 0$
&
$\sum M_Z = 0$

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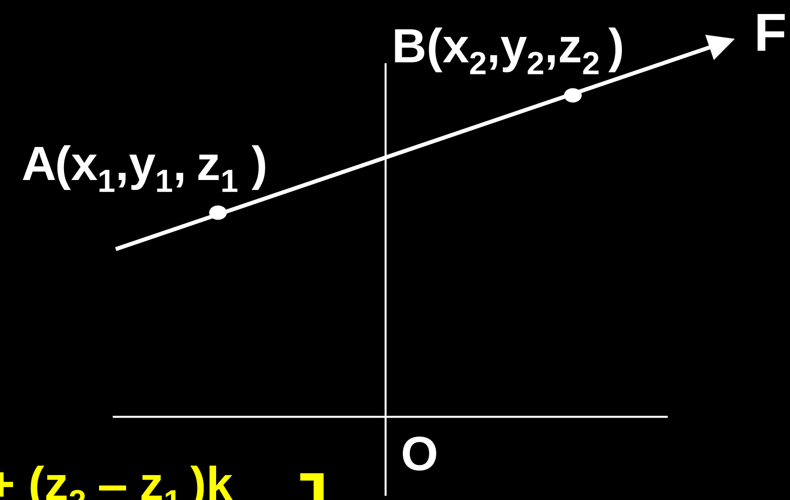
REPRESENTATION OF FORCE VECTOR

$$\vec{F} = F.\,\vec{ab}$$

$\vec{ab}$: Unit vector

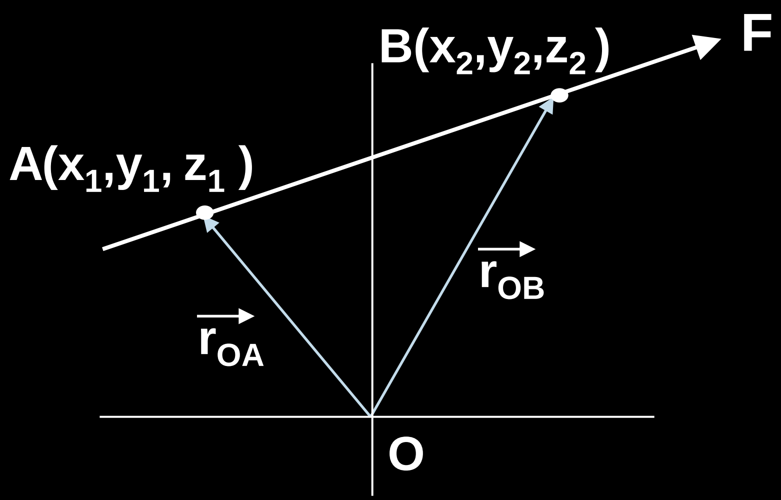


$$\vec{F} = F \times \left[\frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}} \right]$$

MOMENT VECTOR

$$\vec{M}_O = \vec{r} \times \vec{F}$$

POSITION VECTOR OF ANY POINT ON THE LINE OF ACTION OF THE FORCE



$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}$$

'OR'

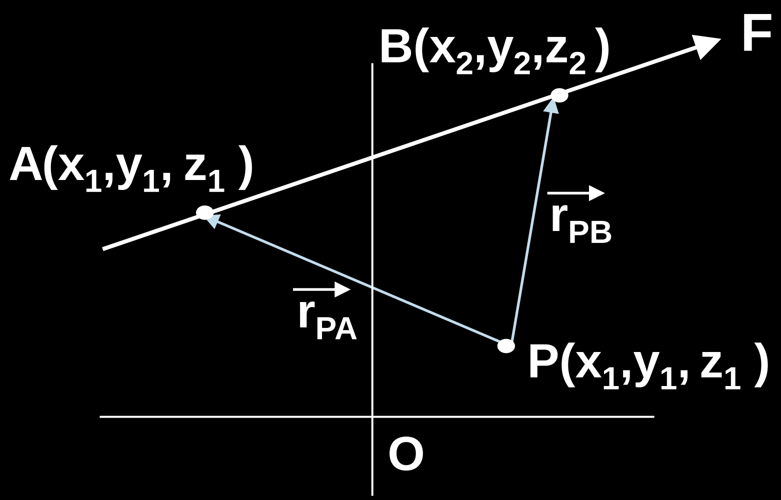
$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$

MOMENT VECTOR

$$\vec{M}_P = \vec{r}_{PA} \times \vec{F}$$

'OR'

$$\vec{M}_P = \vec{r}_{PB} \times \vec{F}$$



1] Find the resultant of following force system :

F_1 = 3i + 4j (N) acting at (3,0,2)m ; F_2 = -7k (N) acting at (5,-2,2)m ;

F_3 = 2i - 4j + k (N) acting at (0,0,0)m ; F_4 = -5i + 6k (N) acting at (-3,-4,1)m ;

M_1 = 10i - 30j + 10k (N-m) and M_2 = 8i -24j -2k (N-m).

$$\vec{R} = \Sigma \vec{F} = 0$$

$$\vec{M}_O = \Sigma (\vec{r} \times \vec{F}) + \vec{M}_1 + \vec{M}_2 = 0$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 3 & 4 & 0 \end{vmatrix} = -8i + 6j + 12k$$

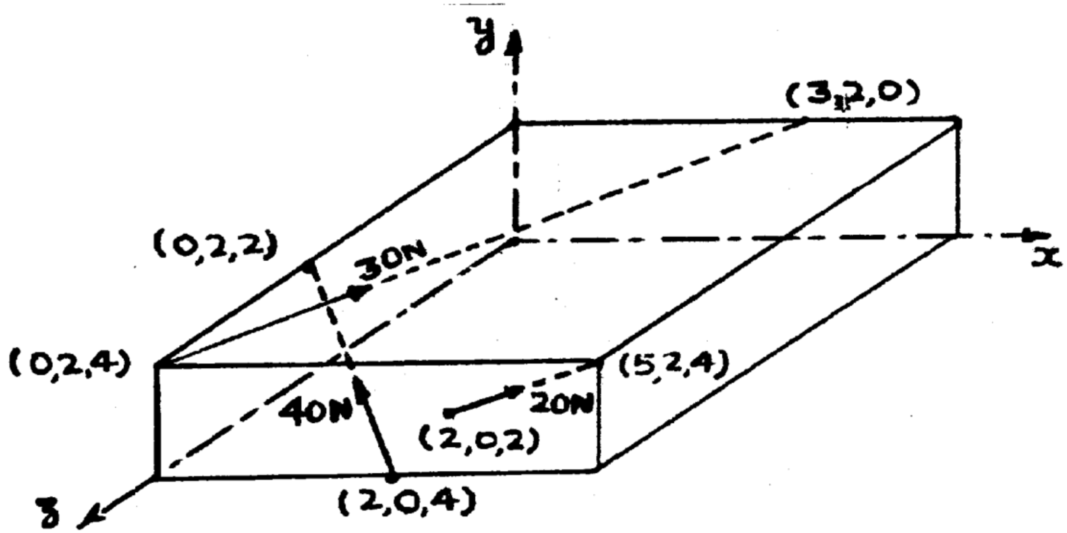
$$\vec{r}_2 \times \vec{F}_2 = 14i + 35j$$

$$\vec{r}_3 \times \vec{F}_3 = 0$$

$$\vec{r}_4 \times \vec{F}_4 = -24i + 13j - 20k$$

Presentation by M.B.Rao

2]



$$\vec{F_1} = 20 \times \left[\frac{3i + 2j + 2k}{\sqrt{3^2 + 2^2 + 2^2}} \right] = 14.55i + 9.7j + 9.7k \text{ (N)}$$

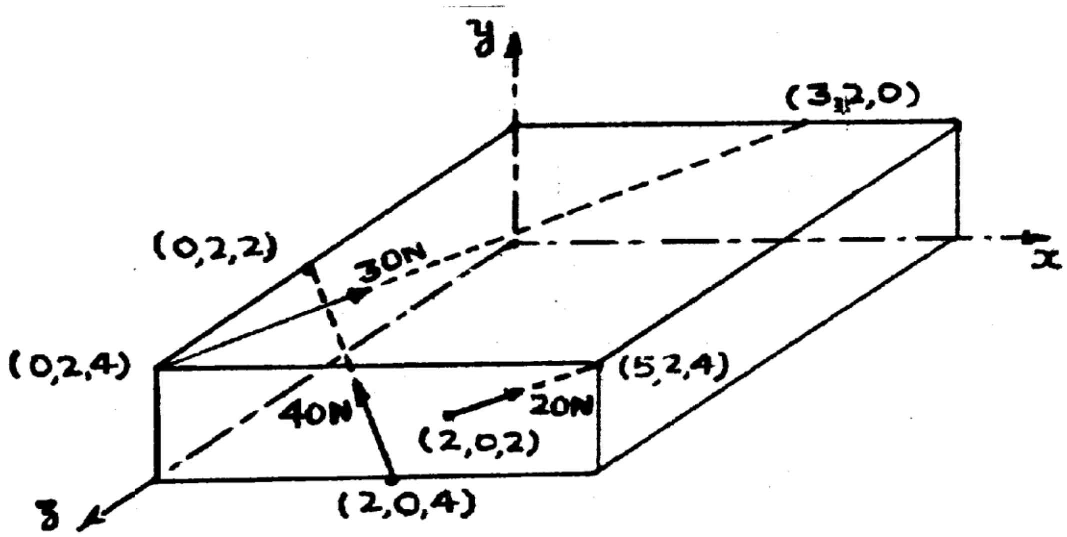
$$\vec{F_2} = 30 \times \left[\frac{3i + 0j - 4k}{\sqrt{3^2 + 4^2}} \right] = 18i - 24k \text{ (N)}$$

$$\vec{F_3} = 40 \times \left[\frac{-2i + 2j - 2k}{\sqrt{3\,(2^2)}} \right] = -23.09i + 23.09j - 23.09k \text{ (N)}$$

$$\underline{\vec{R} = 9.46i + 32.79j - 37.39k \text{ (N)}}$$

Presentation by M.B.Rao

2]



Taking Moments about the Origin-

$$\vec{r_1} \times \vec{F_1} = \begin{vmatrix} i & j & k \\ 2 & 0 & 2 \\ 14.55 & 9.7 & 9.7 \end{vmatrix}$$

$$= -19.4i + 9.7j + 19.4k \text{ (N-m)}$$

$$\vec{r_2} \times \vec{F_2} = \begin{vmatrix} i & j & k \\ 0 & 2 & 4 \\ 18 & 0 & -24 \end{vmatrix}$$

$$= -48i + 72j - 36k \text{ (N-m)}$$

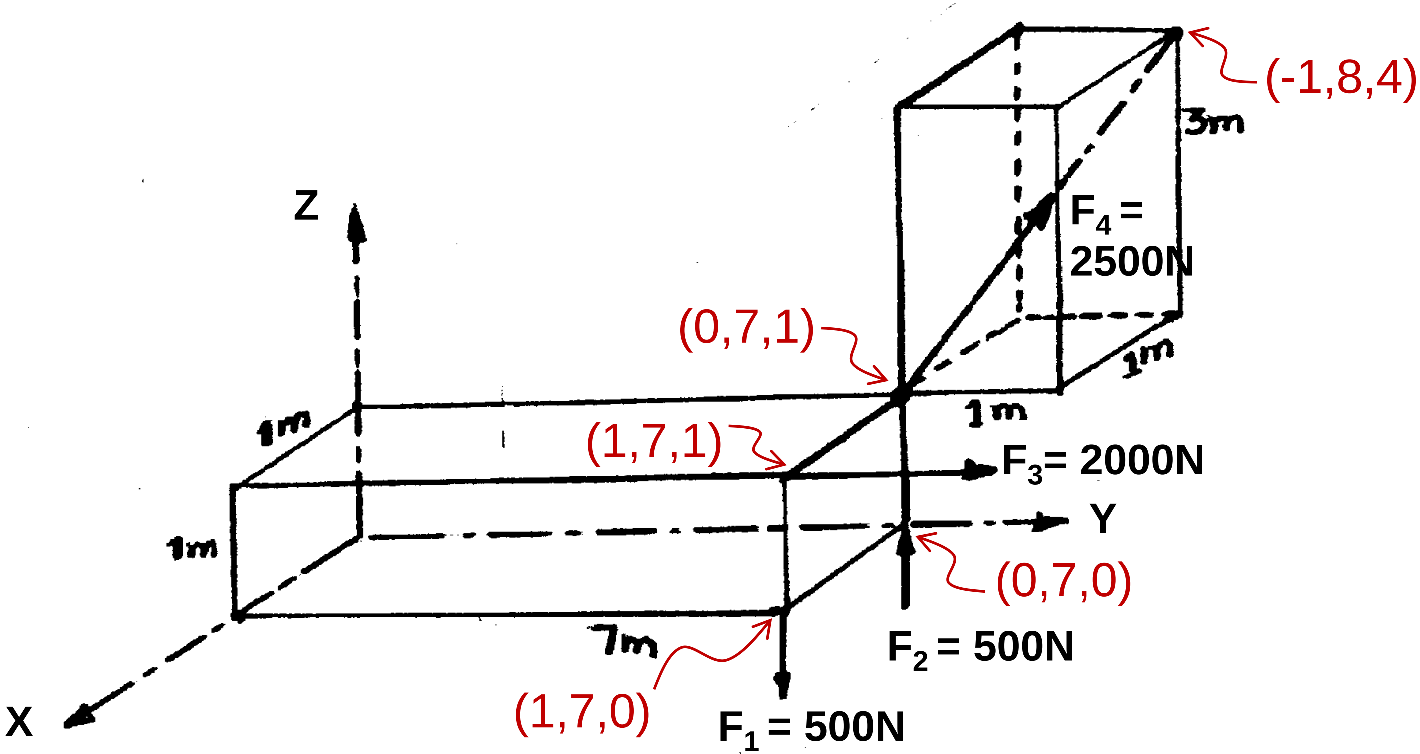
$$\vec{r_3} \times \vec{F_3} = \begin{vmatrix} i & j & k \\ 0 & 2 & 2 \\ -23.09 & 23.09 & -23.09 \end{vmatrix}$$

$$= -92.36i - 46.18j + 46.18k \text{ (N-m)}$$

$$\vec{M_o} = -159.76i + 35.52j + 29.58k \text{ (N-m)}$$

Presentation by M.B.Rao

3]



$\vec{F_1}$ = -500k (N)

$\vec{F_2}$= +500k (N)

$\vec{F_3}$= +2000j (N)

$$\vec{F_4} = 2500 \times \left[\frac{-i + j + 3k}{\sqrt{11}}\right]$$

= -753.78i + 753.78j +2261.3k (N)

$\vec{R}$ = -753.78i + 2753.78j + 2261.3k (N)

$\vec{M_O} = \Sigma(\vec{r} \times \vec{F})$

= -159.8i + 35.6j + 29.6k (N-m)

Presentation by M.B.Rao

4] Forces F_1 = 1KN, F_2 = 3KN, F_3 = 2KN, F_4 = 5KN and F_5 = 2KN act along the line joining the corners of a block as shown in the figure. Find the resultant force and resultant moment about the origin.

$\vec{F}_1 = -0.8i + 0.6k$ (KN)

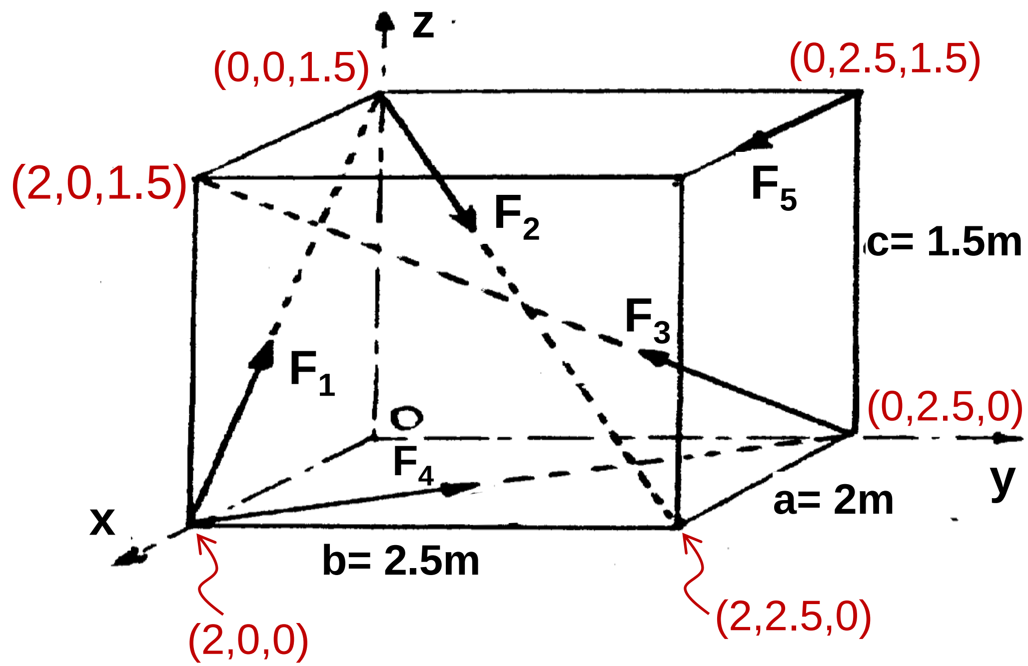
$\vec{F}_2 = 1.69i + 2.12j - 1.27k$ (KN)

$\vec{F}_3 = 1.13i - 1.41j + 0.84k$ (KN)

$\vec{F}_4 = -3.12i + 3.9j$ (KN)

$\vec{F}_5 = 2i$ (KN)

$\vec{R} = 0.91i + 4.61j + 0.175k$ (KN)



$\vec{r}_1 \times \vec{F}_1 =$

$\vec{r}_2 \times \vec{F}_2 =$

$\vec{r}_3 \times \vec{F}_3 =$

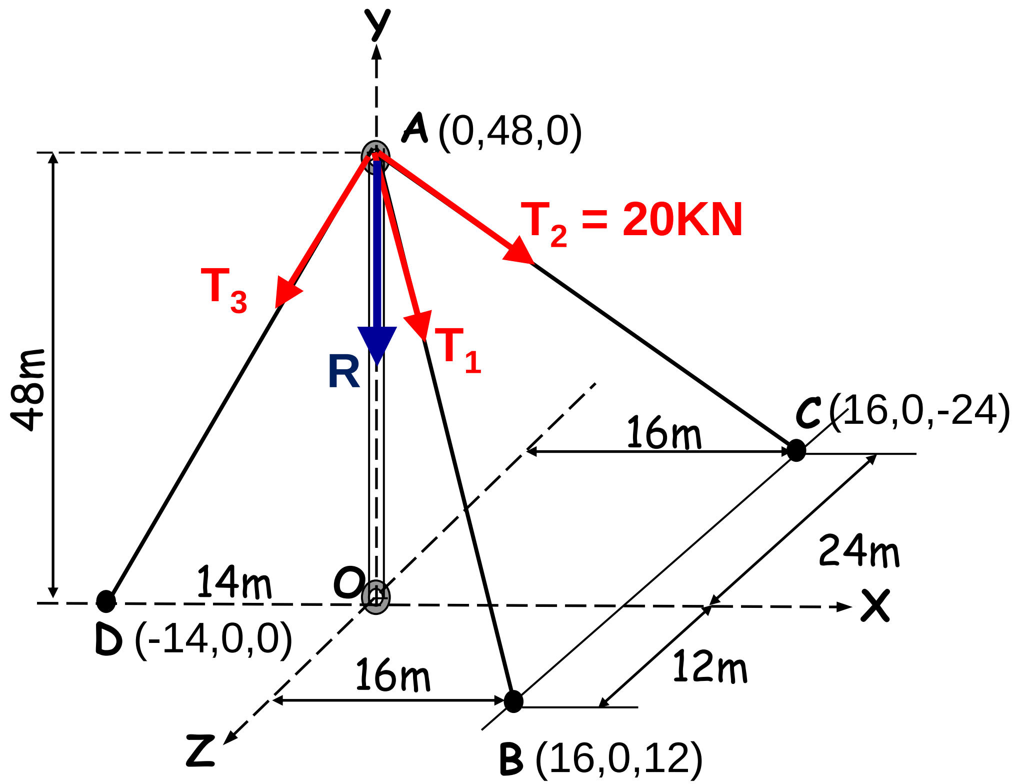
$\vec{r}_4 \times \vec{F}_4 =$

$\vec{r}_5 \times \vec{F}_5 =$

$\vec{M}_o = -1.06i + 4.35j - 0.03k$ (KN-m)

Presentation by M.B.Rao

5] Knowing that the tension in AC is $T_{AC} = 20KN$, determine the required values of tension T_{AB} and T_{AD} so that the resultant of the three forces applied at A is vertical. Also calculate resultant force.



$$\vec{T}_1 = T_1 \times \left[\frac{16i - 48j + 12k}{\sqrt{16^2 + 48^2 + 12^2}} \right]$$

$$= T_1 \, [0.307i - 0.923j + 0.231k] \text{ (KN)}$$

$$\vec{T}_2 = 5.714i - 17.143j - 8.571k \text{ (KN)}$$

$$\vec{T}_3 = T_3 \, [-0.28i - 0.96j] \text{ (KN)} \quad \text{and} \quad \vec{R} = -R\, j \text{ (KN)}$$

$$R_x = \Sigma T_x \rightarrow \quad 0 = 0.307T_1 + 5.714 - 0.28T_3$$

$$R_y = \Sigma T_y \rightarrow \quad -R = -0.923T_1 - 17.143 - 0.96T_3$$

$$R_z = \Sigma T_z \rightarrow \quad 0 = 0.231T_1 - 8.571$$

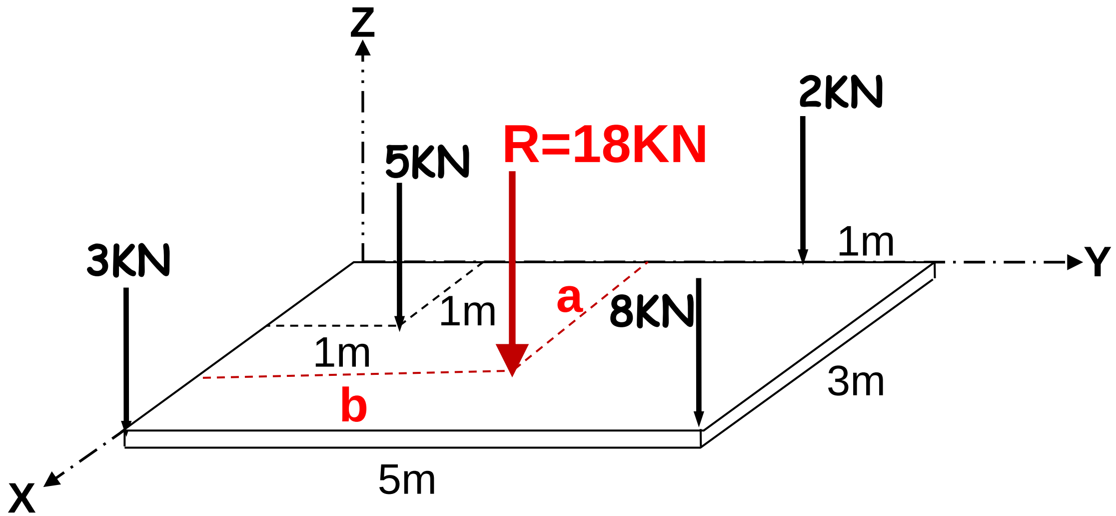
$T_{AB} = 37.1KN$

$T_{AD} = 61.1KN$

$R = 110.04KN$

Presentation by M.B.Rao

6] Four loads are applied on a rectangular plate of size 5m x 3m as shown in figure. Determine the location of point of intersection of the resultant force on the plane of the plate.



Moment of all forces = Moment due to Resultant force

$$\Sigma(\vec{r} \times \vec{F}) = \vec{r} \times \vec{R}$$

a = 2.11m, b = 2.944m

Presentation by M.B.Rao

EQUILIBRIUM OF COPLANAR FORCES

EQUILIBRIUM:

It is a state of rest or a state of uniform motion of a system. When a system is in equilibrium, then – **Σ Forces = 0**

Σ Moments (about any point) = 0

CONDITIONS OF EQUILIBRIUM FOR DIFFERENT TYPES OF COPLANAR FORCE SYSTEM:

(i) Concurrent Forces :- $\sum F_X = 0$ & $\sum F_Y = 0$

(ii) Parallel Forces :- $\sum F_X = 0$ 'or' $\sum F_Y = 0$ & $\sum M_Z = 0$

(iii) General Coplanar :- $\sum F_X = 0$, $\sum F_Y = 0$ & $\sum M_Z = 0$

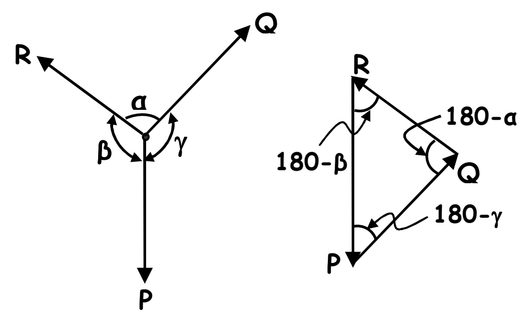
FREE BODY DIAGRAM (F.B.D.) :

It is the diagram of a system isolated from the surrounding (all the contact points) and shown with all the external forces, moments and reactions acting on it.

LAMI'S THEOREM :

" When three forces acting at a point are in equilibrium, then the ratio of magnitude of a force to the sine of the angle between the other two forces is always a constant ".

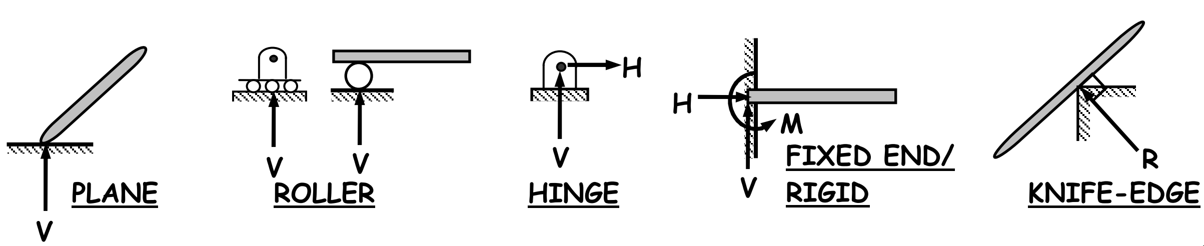
PROOF: When three force vectors acting at a point are in equilibrium, then their vector sum would be zero. Hence these force vectors when added (both in magnitude and direction) would form a closed triangle, as shown. By applying **sine-rule** to this triangle, we get –



$$\frac{P}{\sin(180-\alpha)} = \frac{Q}{\sin(180-\beta)} = \frac{R}{\sin(180-\gamma)}$$

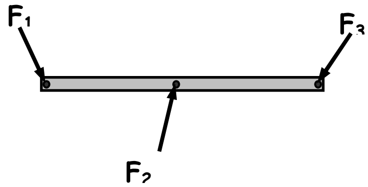
Hence,
$$\frac{P}{\sin(\alpha)} = \frac{Q}{\sin(\beta)} = \frac{R}{\sin(\gamma)}$$

DIFFERENT TYPES OF SUPPORTS AND REACTION EXERTED BY THEM :

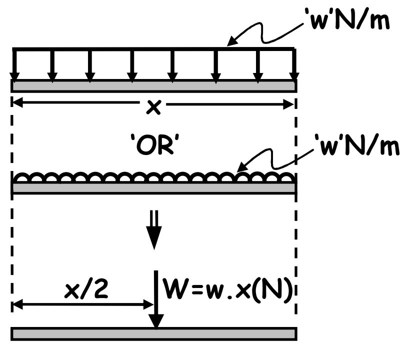


DIFFERENT TYPES OF LOADS :

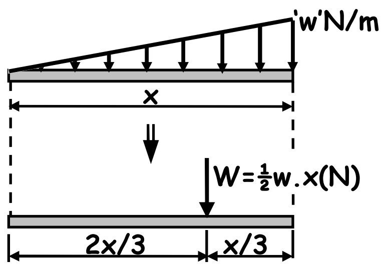
(i) Point Load

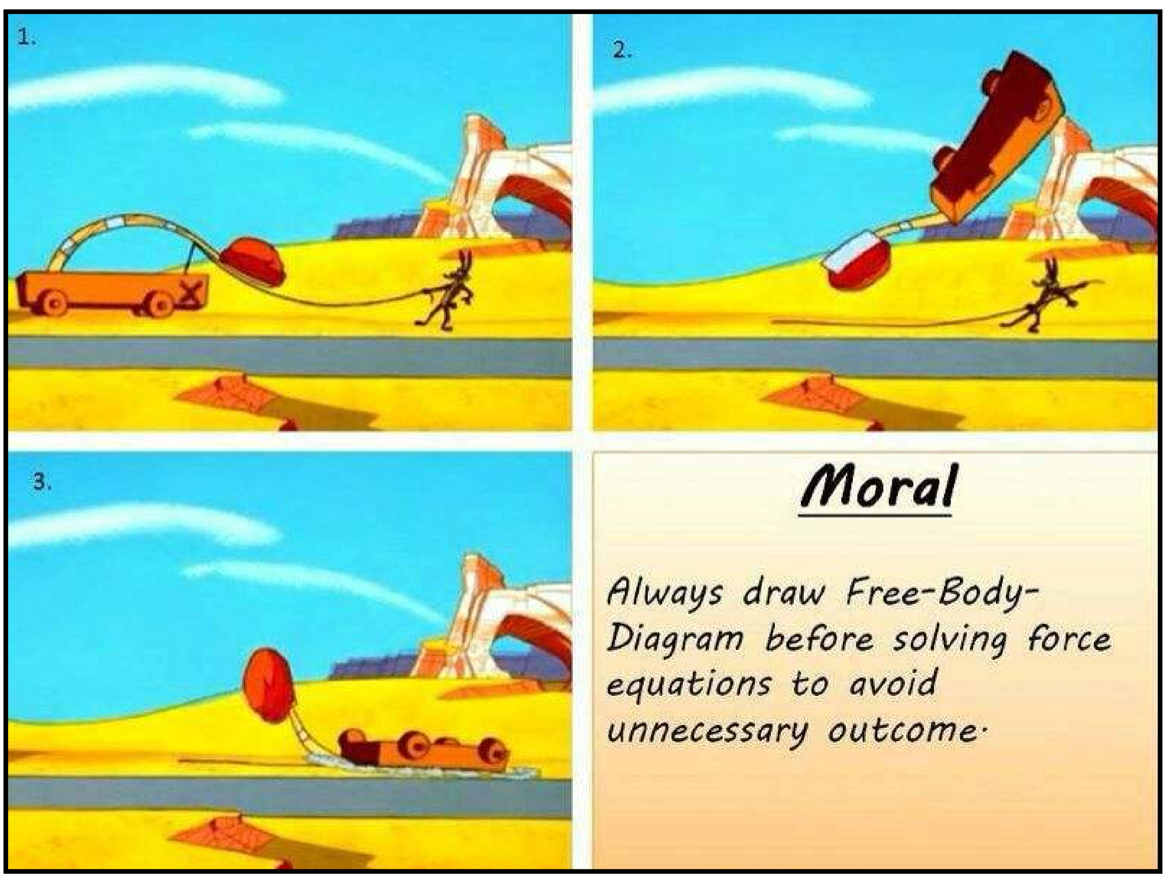


(ii) Uniformly Distributed Load (UDL)



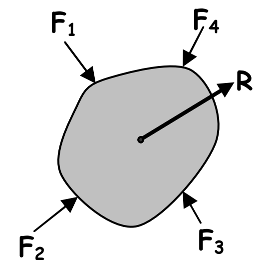
(iii) Uniformly Variable Load (UVL)





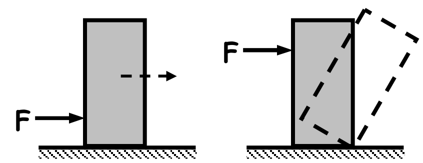
FORCE – AN INTRODUCTION

FORCE : It is a physical quantity which when applied on a system changes or tends to change the state of the system.



RESULTANT FORCE : It is a single force that replaces an entire system of forces having the same effect as that of the given force system.

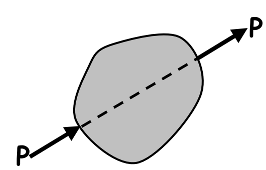
Consider a block placed on a plane and is being pushed by a force F. One needs to realize that the effect of this applied force depends on its point of application. Note that when the force is applied closer to the base, it makes the block slide. But when the same force is applied at the top makes the block topple about its corner.



Hence a force / resultant force has three properties :-

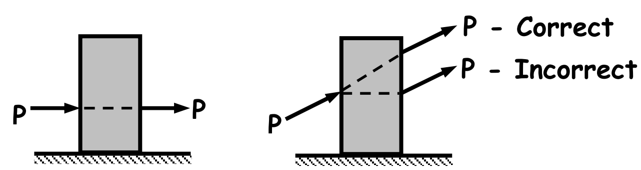
(i) Magnitude
(ii) Direction &
(iii) Point of Application.

LAW OF TRANSMISSIBILITY OF FORCE :



The effect of a force remains the same when it acts anywhere along its line of action.

Example:



REPRESENTATION OF FORCE VECTOR :-

Force, in general, is written in vector form as :- $\vec{F} = F_X.i + F_Y.j$ (N)

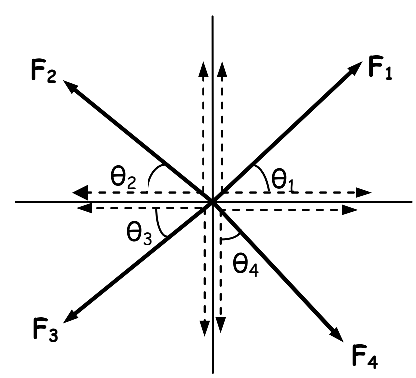
Consider 4 different forces acting as shown. They are written in vector form as follows:

$$\vec{F}_1 = F_1\cos\theta_1.i + F_1\sin\theta_1.j$$

$$\vec{F}_2 = -F_2\cos\theta_2.i + F_2\sin\theta_2.j$$

$$\vec{F}_3 = -F_3\cos\theta_3.i - F_3\sin\theta_3.j$$

$$\vec{F}_4 = F_4\sin\theta_4.i - F_4\cos\theta_4.j$$



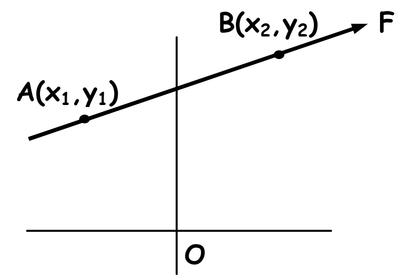
Then the resultant force is given by the vector sum of all the forces, and is written as :

$$\vec{R} = R_X.i + R_Y.j \text{ (N)}$$

where $R_X = \sum F_X$ → sum of all X – components of forces
$R_Y = \sum F_Y$ → sum of all Y – components of forces.

NOTE:
In certain cases, the direction of a force may also be given in terms of co-ordinates of any two points through which the force passes. In such case, the force may be written in vector as :- $\vec{F} = F.\,\vec{ab}$

$\Downarrow$
Unit vector



$$\vec{F} = F \times \left[\frac{(x_2 - x_1).i + (y_2 - y_1).j}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \right]$$

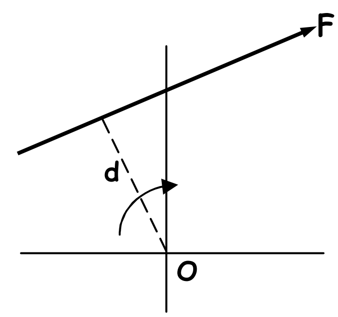
MOMENT OF A FORCE :-

It is understood now that as the point of application of a force is changed, it gives rise to a rotational effect. This rotational effect of the force at any point is called **"moment".**

Mathematically, the magnitude of the moment of a force about any point is given as the product of magnitude of the force and its perpendicular distance from the reference point.

ie., $M_O = \pm \{ F \times d \}$

+ → for anticlockwise rotation
– → for clockwise rotation.

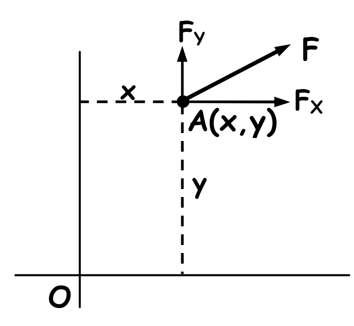


VARIGNON'S THEOREM :-

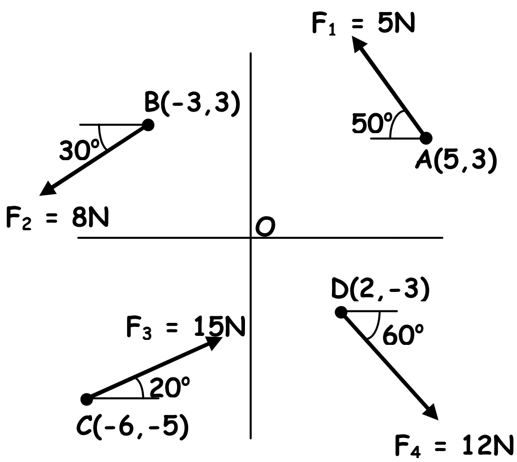
It states that – "the moment of a force about any point is also equal to the moment of components of the force about the same point ".

Consider a force of magnitude F acting at point A(x, y) as shown. By applying Varignon's theorem, the moment of this force is also given as -

$$M_O = -(F_X . y) + (F_y . x)$$



Now, let us have a small exercise! Find the moment of the individual forces given in the diagram shown below:



M_1 = +(5cos 50 x 3) + (5sin50 x 5)
= <u>28.8</u> N-m

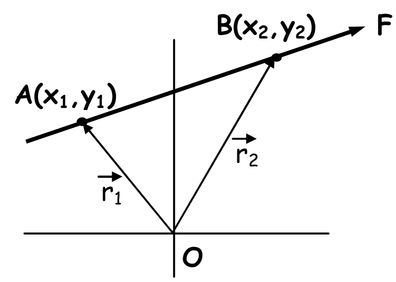
M_2 = +(8cos 30 x 3) + (8sin30 x 3)
= <u>32.78</u> N-m

M_3 = +(15cos 20 x 5) - (15sin20 x 6)
= <u>39.7</u> N-m

M_4 = +(12cos 60 x 3) - (12sin60 x 2)
= <u>-2.79</u> N-m

<u>MOMENT VECTOR</u> :-

Consider a force of magnitude 'F' passing through points 'A' and 'B' whose position vectors are as shown in the figure.



Then the moment of the force about the origin is given in the vector form as : $\vec{M}_O = \vec{r} \times \vec{F}$

Where $\vec{r}$ is the position vector of any point through which the force passes.

<u>TYPES OF COPLANAR FORCE SYSTEM</u> :-