



Applied Mathematics 2 - Dec 17

First Year Engineering (Semester 2)

Total marks: 80

Total time: 3 Hours

INSTRUCTIONS

- (1) Question 1 is compulsory.
- (2) Attempt any **three** from the remaining questions.
- (3) Draw neat diagrams wherever necessary.

- 1.a.** Evaluate $\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx$ (3 marks)
- 1.b.** Solve $(D^3+1)^2 y=0$ (3 marks)
- 1.c.** Solve the ODE $(y+\frac{1}{3}y^3+\frac{1}{2}x^2)dx+(x+xy^2)dy=0$ (3 marks)
- 1.d.** Use Taylor's series method to find a solution of $\frac{dy}{dx}=1+y^2, y(0)=0$ at $x=0.1$ taking $h=0.1$ correct to three decimal value. (3 marks)
- 1.e.** Given $\int_0^x \frac{dx}{x^2+y^2} = \frac{1}{a} \tan^{-1}(\frac{x}{a})$, using DUIS find the value of $\int_0^x \frac{dx}{(x^2+a^2)^2}$ (4 marks)
- 1.f.** Find the perimeter of the curve $r=a(1-\cos\theta)$ (4 marks)
- 2.a.** Solve $(D^3+D^2+D+1)y=\sin^2 x$ (6 marks)
- 2.b.** Change the order of integration $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y) dx dy$ (6 marks)
- 2.c.** Evaluate $\int \int_R \frac{2xy^2}{1+x^2y^2-y^4} dx dy$, where R is a triangle whose vertices are (0, 0), (1,1), (0,1). (8 marks)
- 3.a.** Find the volume enclosed by the cylinder $y^2=x$ and $y=x^2$. Cut off by the planes $z=0, x+y+z=2$ (6 marks)
- 3.b.** Using modified Euler's method, find an appropriate value of y at $x=0.2$ in two step taking $h=0.1$ and using iteration, given that $\frac{dy}{dx}=x+3y, y=1$ when $x=0$. $dydx=x+3y, y=1$ when $x=0$. (6 marks)
- 3.c.** Solve $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$ (8 marks)
- 4.a.** Show that $\int_0^{\infty} \sqrt{\frac{x^3}{a^3-x^3}}$ (6 marks)
- 4.b.** Solve $(D^2+2)y=e^x \cos x + x^2 e^{3x}$ (6 marks)



4.c. Use polar co-ordinates to evaluate $\int \int \frac{(x^2+y^2)^2}{x^2y^2}$ over the area Common to the circle $x^2+y^2=ax$ and $x^2+y^2=by, a>b>0$ (8 marks)

5.a. Solve $y dx + x(1-3x^2y^2)dy=0$ (6 marks)

5.b. Find the mass of a lamina in the form of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if the density at any point varies as the product of the distance from the axes of the ellipse. (6 marks)

5.c. Compute the value of $\int_0^{\pi/2} \sqrt{\sin x + \cos x}$ using

(i) Trapezoidal rule

(ii) Simpson's $(1/3)^{\text{rd}}$ rule

(iii) Simpson's $(3/8)^{\text{th}}$ rule by dividing into six subintervals (8 marks)

6.a. Evaluate $\int \int \int_V x^2 dx dy dz$ over the volume bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ (6 marks)

6.b. Change the order of integration and evaluate $\int_0^2 \int_{\sqrt{2}y}^2 \frac{x^2}{\sqrt{x^2-4y^2}}$ (6 marks)

6.c. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ (8 marks)