Seat No.	

B.Sc. (Part - I) (Semester -I) Examination, November - 2018
MATHEMATICS
Calculus (Paper - II)
Sub. Code : 59674

Day and Date : Thursday, 22- 11 - 2018 **Total Marks : 50**

Time : 3.00 p.m. to 5.00 p.m.

Instructions :
1) **All questions are compulsory.**
2) **Figures to the right indicate full marks.**

Q1) Select the correct alternative for each of the following : **[10]**

a) If $y = \sin(3x - 5)$ then $y_3 =$ ________.

i) $3^3 \sin(3x - 5)$ ii) $3^3 \cos\left(3x - 5 + \frac{3\pi}{2}\right)$

iii) $3^3 \sin\left(3x - 5 + \frac{3\pi}{2}\right)$ iv) $3^3 \sin\left(3x - 5 + \frac{\pi}{2}\right)$

b) The product of curvature and radius of curvature at any point on a curve is equal to ___________.

i) -1 ii) 1

iii) 0 iv) $\frac{1}{2}$

c) The value of $\lim_{x \to 2}\left(\frac{x^2 - x - 2}{x - 2}\right) =$ ___________.

i) 0 ii) 1

iii) 2 iv) 3

d) If $z = xy\, f\left(\frac{y}{x}\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$ __________.

i) $2z$ ii) z

iii) $3z$ iv) 0

e) n^{th} derivative of $\sin x \cos x$ is __________.

i) $2^n \sin x$ ii) $2^n.\sin(2x)$

iii) $2^n \sin\left(2x + \frac{n\pi}{2}\right)$ iv) $2^{n-1}\sin\left(2x + \frac{n\pi}{2}\right)$

f) If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$ ________.

i) 0 ii) 1

iii) 2 iv) 3

g) If $y = (\sin^{-1}x)^2$ then $\left(1 - x^2\right)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} =$ _________.

i) 2 ii) 4

iii) 0 iv) -2

h) If $u = \log\left(\frac{x^5 + y^5}{x + y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ________.

i) 2 ii) 4

iii) 6 iv) 2

i) The Infinite series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots\infty$ is the expansion of ______.

i) $\sin x$ ii) e^x

iii) e^{-x} iv) $\cos x$

j) The radius of curvature of the curve S = $a\log \operatorname{cosec}\psi$ is ____________.

i) $c \operatorname{cosec} \psi$ ii) $c^2 \operatorname{cosec}^2 \psi$

iii) $-c \cot \psi$ iv) $c \tan \psi$

Q2) Attempt any two of the following: **[20]**

a) If $x = f(t)$ and $y = g(t)$ are differentiable functions of t, then prove that

$$\rho = \frac{\left(\dot{x}^2 + \dot{y}^2\right)^{3/2}}{\dot{x}\,\ddot{y} - \ddot{x}\dot{y}}.$$

b) State and prove Leibnitz's theorem.

c) Define homogeneous function of two variables. If $z = f(x, y)$ is a homogeneous function of two variables x and y of degree 'n' then prove that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = nz$.

Q3) Attempt any four of the following: **[20]**

a) If $y = e^{m\tan^{-1}x}$, then prove that $(1 + x^2)y_{n+1} + (2nx - m)y_n + n(n-1)y_{n-1} = 0$.

b) If $u = \sin^{-1}\left(\dfrac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right)^{1/2}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{12}\tan u$.

c) Find radius of curvature for the curve $\sqrt{x} + \sqrt{y} = 1$ at $\left(\dfrac{1}{4}, \dfrac{1}{4}\right)$.

d) Expand $\cos x$ by Maclaurin's series.

e) If $z = f(x, y)$, $x = e^u \cos v$ and $y = e^u \sin v$, then prove that $x\dfrac{\partial z}{\partial v} + y\dfrac{\partial z}{\partial u} = e^{2u}\dfrac{\partial z}{\partial y}$.

f) Find $\lim\limits_{x\to 0}\dfrac{3^x - 2^x}{x}$.

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