Graph Colouring:

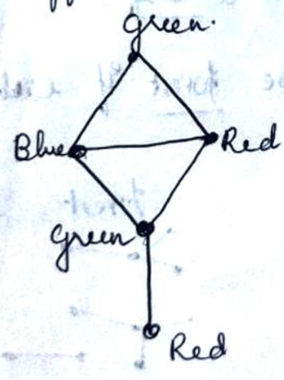
A graph which can be represented by atleast one plane drawing in which the edges meet only at the vertices is called a planar graph.

A graph which cannot be represented by a plane drawing in which the edges meet only at the vertices is called a non-planar graph

Given a planar or non-planar graph G, if we assign colours to its vertices in such a way that no two adjacent vertices have the same colour, then we say that the graph G is properly coloured.

In other words, proper colouring of a graph means assigning colours to its vertices such that adjacent vertices have different colours.

Eg:



Chromatic Number :-

A graph G is said to be k-colourable if we can properly colour it with k (number of) colours.

A graph G which is k-colourable but not (k-1) colourable is called a k-chromatic graph.

A k-chromatic graph is a graph that can be properly coloured with k colours but not with less than k colours.

If a graph G is k-chromatic, then k is called the chromatic number of G

Thus, the chromatic number of a graph is the minimum number of colours with which the graph can be properly coloured.

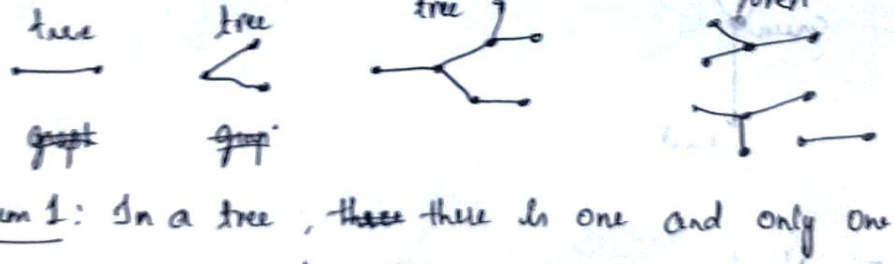
The chromatic number of a graph G is usually denoted by $\chi(G)$

Trees and their Basic properties

A graph G is said to be a tree if it is connected and has no cycle.

A pendant vertex of a tree is called a leaf.

A disconnected graph is said to be forest if each component of it is a tree.



Theorem 1: In a tree, ~~there~~ there is one and only one path between every pair of vertices.

Theorem 2: If a graph G there is one and only one path between every pair of vertices, then G is a tree.

Theorem 3: A tree with n vertices has n-1 edges

Theorem 4: Any connected graph with n vertices and $(n-1)$ edges is a tree. 13

Theorem 5: A connected graph G is a tree if and only if adding an edge between any two vertices in G create exactly one cycle in it.

Minimally connected graph:

A connected graph is said to be minimally connected if the removal of any edges from it disconnects the graph.

Rooted Trees:

A directed tree is a directed graph whose underlying graph is a tree.

A directed tree T is called a rooted tree if (i) T contains a unique vertex, called the root, whose in-degree is equal to 0, and (ii) the in-degree of all other vertices of T are equal to 1.

* Root of a tree denoted by r.

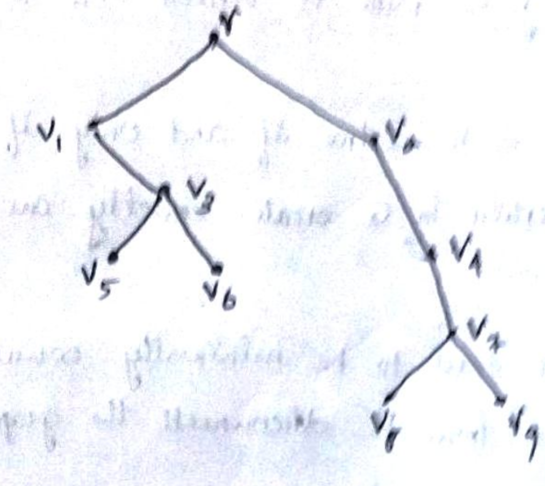
* A vertex v (other than the root r) of a rooted tree is said to be at kth level or has level k. If the path from r to v is of length k.

* If v_1 and v_2 are two vertices such that v_1 has a lower level number, then we say that v_1 is an ancestor of v_2, or that v_2 is a descendant of v_1.

* If v_1 and v_2 are two vertices such that v_1 has a lower level number and there is an edge from v_1 to v_2, then v_1 is called the parent of v_2 or v_2 is called the child of v_1.

* Two vertices with a common parent are referred to as siblings.

* In a rooted tree a vertex whose out-degree is 0 is called a leaf and a vertex which is not a leaf is called an internal vertex



(i) v_1 & v_2 are at first level

v_3, v_4 are at second level.

v_5, v_6, v_7 are at third level.

v_8, v_9 are at fourth level.

(ii) v_1 is ancestor of v_3, v_5, v_6 or v_3, v_5, v_6 are the descendants of v_1.

v_2 is the ancestor of v_4, v_7, v_8, v_9

(iii) v_1 is ~~parent~~ parent of v_3 (or v_3 is child of v_1)

(iv) v_5 and v_6 are siblings.

(v) v_5, v_6, v_8, v_9 are ~~leaves~~ leaves,

<u>m-ary Tree</u> :-

A rooted tree T is called an <u>m-ary</u> tree if every internal vertex of T is of out-degree $\leq m$; that is if every internal vertex of T has at most m children.

A rooted tree T is called a <u>complete</u> <u>m-ary</u> tree if every internal vertex of T is of out-degree m; that is every internal vertex of T has exactly m children.

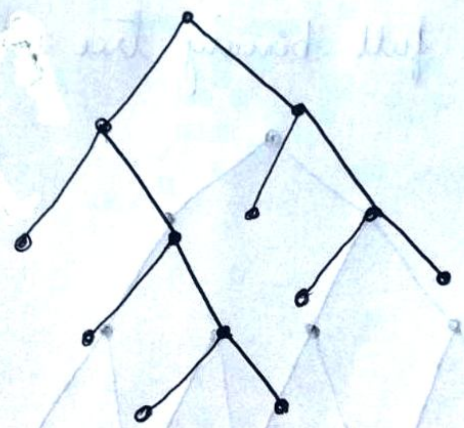
Binary Tree:-

An m-ary tree for which m=2 is called a binary tree. A rooted tree T is called a binary tree if every vertex of T is of out-degree ≤ 2; that is if every vertex has at most two children.

A complete m-ary tree for which m=2 is called a complete binary tree.

In other words, a rooted tree T is called a complete binary tree if every internal vertex of T is of out-degree 2; that is, if every internal vertex of T is of out-degree 2; that is if every internal vertex has exactly two children.

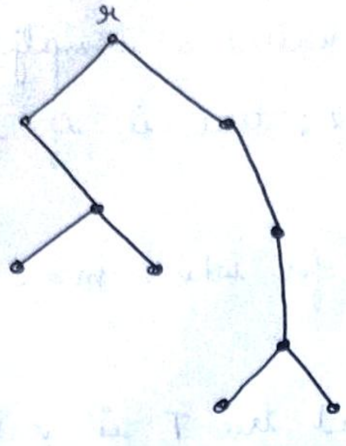
Example for a complete binary tree is:



Balanced Tree:

If T is a rooted tree and h is the largest level number achieved by a leaf of T, then T is said to have height h. A rooted tree of height h is said to be balanced if the level number of every leaf is h or h-1.

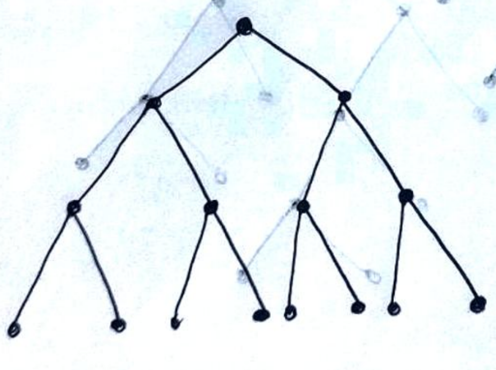
Example for balanced tree :



Full binary Tree :

Let T be a complete binary tree of height h. Then T is called a full binary tree if all the leaves in T are at level h.

Example for full binary tree :



NOTE: Let T be a complete m-ary tree of order n with p leaves and q internal vertices.

Then (a) $n = mq + 1 = \frac{mp-1}{m-1}$

(b) $p = (m-1)q + 1 = \frac{(m-1)n+1}{m}$

(c) $q = \frac{n-1}{m} = \frac{p-1}{m-1}$