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Assignment-1
CS202A

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~~K-Sudoku Solver~~

I) K-Sudoku puzzle pair Solver

first of all given any K-Sudoku we have

no of rows = K^2

no of Columns = K^2

no of blocks (Sub-grids) = K^2

and we assign number $m \in \{1, 2, 3, \dots K^2\}$

for example for 3-Sudoku $K=3$

we have no of rows = 9

no of columns = 9

no of blocks = 9

ad each cell is filled with $m \in \{1, 2, 3, \dots 9\}$

And there are 2 such sudoku in this Question

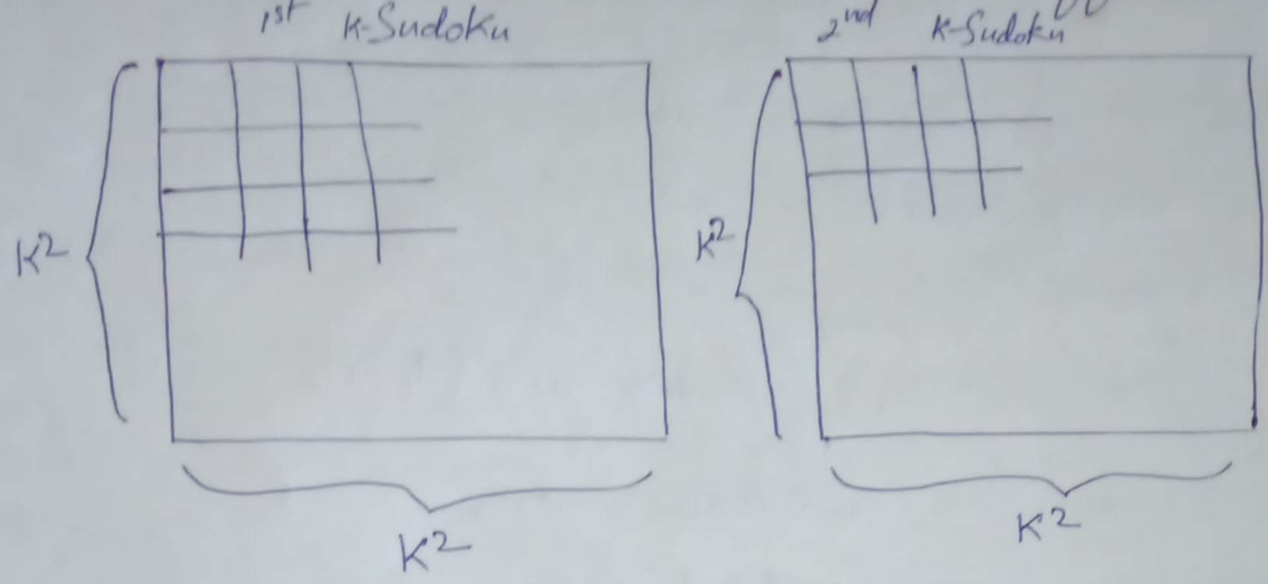
Now we have to express the contraints.

The contraints are—

1) There is atleast one number in each cell.
2) There is atmost one number in each cell.
3) A number appears at most once in each row
4) A number appears at most once in each Column.

5) A number appears at most once in each of K×K blocks (sub-grids)

6) The number in corresponding cell of pair of K-Sudoku must be different.



S_{rcin} ::= states the r^{th} row and c^{th} column of i^{th} sudoku contains a number "n"

where $r \in \{1, 2, 3, \dots K^2\}$
$c \in \{1, 2, 3, \dots K^2\}$
$i \in \{1, 2\}$
$n \in \{1, 2, 3, \dots K^2\}$

Let $D = \{1, 2, 3, \dots, K^2\}$

Converting Constraint to formula (Propositional Logic)

1) If we want to express that there is atleast

One number in one cell we encode:

$$\phi_{rci} = (S_{rci1} \vee S_{rci2} \vee S_{rci3} \vee \ldots \ldots \vee S_{rci(K)^2})$$

To express that there is atleast one number in each cell we can conjunct all possible ϕ_{rci}

$$\therefore \boxed{\phi = \bigwedge_{r,c,i \in D} \phi_{rci}}$$

2) In order to say that there is at most one digit in each cell we have to exclude that for every possible digit pair S_{rcid} and S_{rcik} (where $d \neq K$) are both true for the same cell.

$$\psi_{rci} = \bigwedge_{\substack{d,K \in D \\ d \neq K}} \neg(S_{rcid} \wedge S_{rcik})$$

To express that there is at most one number in each cell we conjunct all ψ_{rci}

$$\psi = \bigwedge_{\substack{r,c \in D \\ i \in \{1,2\}}} \psi_{rci}$$

3) For the 3rd constraint we similarly do

$$\omega_{rid} = \bigwedge_{c \in D} \bigwedge_{c'=c+1}^{K^2} \neg(S_{rcid} \wedge S_{rc'id})$$

$$\omega = \bigwedge_{\substack{r,d \in D \\ i \in \{1,2\}}} \omega_{rid}$$

4) $$\Pi_{cid} = \bigwedge_{r \in D} \bigwedge_{r' = r+1}^{k^2} \neg (S_{rcid} \wedge S_{r'cid}$$

$$\Pi = \bigwedge_{\substack{r, d \in D \\ i \in \{1,2\}}} \Pi_{cid}$$

5) for the 5^{th} Constraint, a number appears at most once in each $K \times K$ blocks (sub-grid), we use the already used at most pattern

Let $G = \{ \{1, 2, 3, \cdots K\}, \{K+1, K+2, \cdots 2K\}, \ldots \{K^2-K+1, K^2-K+2, \cdots, K^2\} \}$

$$\therefore \lambda_{id} = \bigwedge_{\substack{(r,c), (r',c') \in I \times J \\ I, J \in G}} \neg (S_{rcid} \wedge S_{r'c'id})$$

$$\therefore \lambda = \bigwedge_{\substack{i \in \{1,2\} \\ d \in D}} \lambda_{id}$$

6) for the 6^{th} constraint we can write

$$\sigma_{rcd} = \neg(S_{rc1d} \wedge S_{rc2d})$$

$$\sigma = \bigwedge_{r,c,d \in D} \neg(S_{rc1d} \wedge S_{rc2d})$$

we encode these in Solver together with the initial setup of the sudoku. Say some θ

$\therefore$ A solution for a pair K-Sudoku can be read off from the satisfying assignment returned by the SAT solver of $\boxed{\theta \wedge \phi \wedge \psi \wedge \omega \wedge \pi \wedge \lambda \wedge \sigma}$

II K-Sudoku pair generator.

for generating a pair of sudoku of dimension k we use following strategy.

a) find an assignment for $\phi \wedge \psi \wedge \omega \wedge \pi \wedge \lambda \wedge \sigma$ with out any initial setup (or blank board).

b) Then use the assignment and randomly delete any model fact to create blank (holes) in sudoku pair

c) Solve sudoku again. If uniquely satisfiable, we create more blanks untill sudoku is not "SATISFIABLE uniquely".

d) we print the last satisfiable (uniquely) sudoku-pair.