PART III : MATHEMATICS

SECTION 1 (Maximum Marks: 21)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +3	If only the bubble corresponding to the correct option is darkened
Zero Marks	: 0	If none of the bubbles is darkened
Negative Marks:	-1	In all other cases

Q.37 The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

[A] $14x + 2y - 15z = 1$ [B] $14x - 2y + 15z = 27$

[C] $14x + 2y + 15z = 31$ [D] $-14x + 2y + 15z = 3$

Q.38 Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP} \cdot \overrightarrow{OQ} + \overrightarrow{OR} \cdot \overrightarrow{OS} = \overrightarrow{OR} \cdot \overrightarrow{OP} + \overrightarrow{OQ} \cdot \overrightarrow{OS} = \overrightarrow{OQ} \cdot \overrightarrow{OR} + \overrightarrow{OP} \cdot \overrightarrow{OS}$$

Then the triangle PQR has S as its

[A] centroid [B] circumcentre

[C] incentre [D] orthocenter

Space for rough work

Answers for the above questions

Ans for Q.37: (C)	Ans for Q.38: (D)

Q.39 If $y = y(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, \qquad x > 0$$

and $y(0) = \sqrt{7}$, then $y(256) =$

[A] 3 [B] 9 [C] 16 [D] 80

Q.40 If $f: \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, $f(1) = 1$, then

[A] $f'(1) \leq 0$ [B] $0 < f'(1) \leq \frac{1}{2}$

[C] $\frac{1}{2} < f'(1) \leq 1$ [D] $f'(1) > 1$

Space for rough work

Answers for the above questions

Ans for Q.39: (A)	Ans for Q.40: (D)

Q.41 How many 3×3 matrices M with entries from $\{0,1,2\}$ are there, for which the sum of the diagonal entries of M^TM is 5?

[A] 126 [B] 198 [C] 162 [D] 135

Q.42 Let $S=\{1,2,3,\ldots,9\}$. For $k=1,2,\ldots,5,$ let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1+N_2+N_3+N_4+N_5=$

[A] 210 [B] 252 [C] 125 [D] 126

Q.43 Three randomly chosen nonnegative integers x,y and z are found to satisfy the equation $x+y+z=10$. Then the probability that z is even, is

[A] $\frac{36}{55}$ [B] $\frac{6}{11}$ [C] $\frac{1}{2}$ [D] $\frac{5}{11}$

Space for rough work

Answers for the above questions

Ans for Q.41: (B)	Ans for Q.42: (D)	Ans for Q.43: (B)

SECTION 2 (Maximum Marks: 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options [A], [B], [C] and [D]. **ONE OR MORE THAN ONE** of these four options is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened
Zero Marks	: 0	If none of the bubbles is darkened
Negative Marks	: -2	In all other cases

- For example, if [A], [C] and [D] are all the correct options for a question, darkening all these three will get +4 marks; darkening only [A] and [D] will get +2 marks; and darkening [A] and [B] will get -2 marks, as a wrong option is also darkened

Q.44 If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\, dt$, then

[A] $g'\left(\frac{\pi}{2}\right) = -2\pi$

[B] $g'\left(-\frac{\pi}{2}\right) = 2\pi$

[C] $g'\left(\frac{\pi}{2}\right) = 2\pi$

[D] $g'\left(-\frac{\pi}{2}\right) = -2\pi$

Q.45 Let α and β be nonzero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

[A] $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

[B] $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

[C] $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

[D] $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

Answers for the above questions

Q.44: Due to internal review, all candidates are awarded +4 marks.	**Q.45: Due to internal review, all candidates are awarded +4 marks.**

Q.46 If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$, then

[A] $f(x)$ is increasing in $(0,\infty)$

[B] $f(x)$ is decreasing in $(0,\infty)$

[C] $f(x) > e^{2x}$ in $(0,\infty)$

[D] $f'(x) < e^{2x}$ in $(0,\infty)$

Q.47 Let $f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then

[A] $\lim_{x\to1^-} f(x) = 0$

[B] $\lim_{x\to1^-} f(x)$ does not exist

[C] $\lim_{x\to1^+} f(x) = 0$

[D] $\lim_{x\to1^+} f(x)$ does not exist

Q.48 If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

[A] $f'(x) = 0$ at exactly three points in $(-\pi,\pi)$

[B] $f'(x) = 0$ at more than three points in $(-\pi,\pi)$

[C] $f(x)$ attains its maximum at $x = 0$

[D] $f(x)$ attains its minimum at $x = 0$

Space for rough work

Answers for the above questions

Ans for Q.46: (A) and (C)

Ans for Q.47: (A) and (D)

Ans for Q.48: (B) and (C)

Q.49 If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \leq y \leq x, 0 \leq x \leq 1\}$ into two equal parts, then

[A] $0 < \alpha \leq \frac{1}{2}$

[B] $\frac{1}{2} < \alpha < 1$

[C] $2\alpha^4 - 4\alpha^2 + 1 = 0$

[D] $\alpha^4 + 4\alpha^2 - 1 = 0$

Q.50 If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)}\, dx$, then

[A] $I > \log_e 99$

[B] $I < \log_e 99$

[C] $I < \frac{49}{50}$

[D] $I > \frac{49}{50}$

Space for rough work

Answers for the above questions

Ans for Q.49: (B) and (C)	Ans for Q.50: (B) and (D)

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO** paragraphs
- Based on each paragraph, there are **TWO** questions
- Each question has **FOUR** options [A], [B], [C], and [D]. **ONLY ONE** of these four options is correct
- For each question, darken the bubble corresponding to the correct option in the ORS
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened
Zero Marks : 0 In all other cases

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

Q.51 $|\ \overrightarrow{OX}\times\overrightarrow{OY}\ |=$

[A] $\sin(P+Q)$ [B] $\sin 2R$ [C] $\sin(P+R)$ [D] $\sin(Q+R)$

Q.52 If the triangle PQR varies, then the minimum value of

$$\cos(P+Q)+\cos(Q+R)+\cos(R+P)$$

is

[A] $-\frac{5}{3}$ [B] $-\frac{3}{2}$ [C] $\frac{3}{2}$ [D] $\frac{5}{3}$

Space for rough work

Answers for the above questions

Ans for Q.51: (A)	Ans for Q.52: (B)

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

Q.53 $a_{12} =$

[A] $a_{11} - a_{10}$ [B] $a_{11} + a_{10}$ [C] $2a_{11} + a_{10}$ [D] $a_{11} + 2a_{10}$

Q.54 If $a_4 = 28$, then $p + 2q =$

[A] 21 [B] 14 [C] 7 [D] 12

END OF THE QUESTION PAPER

Space for rough work

Answers for the above questions

Ans for Q.53: (B)	Ans for Q.54: (D)

SPACE FOR ROUGH WORK