Lecture 1

Sequence of real numbers

Large collection of numbers $x_1, x_2, \cdots, x_n, \cdots \quad \forall \quad x_i \in \mathbb{R}$ in some particular order is called a sequence.

mathematically, sequence is $X : \mathbb{N} \longrightarrow \mathbb{R}, X(i) = x_i$.

Ex: {1, 1/2, 1/3, $\cdots$ } can be expressed as a function as

$$X : \mathbb{N} \longrightarrow \mathbb{R}, X(i) = \frac{1}{i}$$

$$or$$

$$\left\{X(n) = \frac{1}{n}\right\}_{n=1}^{\infty}$$

$$or$$

$$\left\langle X(n) \right\rangle_{n=1}^{\infty}$$

Ex: {c, c, $\cdots$ } = $\left\{X(n) = c\right\}_{n=1}^{\infty}$ is a constant sequence.

Ex: {-1, 1, -1, 1, $\cdots$ } = $\left\{X(n) = (-1)^n\right\}_{n=1}^{\infty}$

- The sequence $\left\{X(n) = \frac{1}{n}\right\}$ approaches 0 as $x \longrightarrow \infty$.

Convergence of a sequence $\left\{X(n)\right\}_{n=1}^{\infty}$

Sequence converges to L if for every $\epsilon > 0$, there exists a stage $n_1 \in \mathbb{N}$, such that $|x_n - L| < \epsilon \quad \forall \quad n \geq n_1$.

In other words we can say that for the sequence $\{x_1, x_2, \cdots, x_{n_1}, \cdots\}$ we can find a stage n_1 such that all x_n after x_{n_1} lies in the epsilon neighbourhood of L.

Thus it follows that,

$$\lim_{n \to \infty} x_n = L$$

- ϵ neighbourhood of L can be represented as $N_\epsilon(L)$ and it is also refered to as epsilon ball of L & denoted by $B(L, \epsilon)$.

Ex: $\{1, \frac{1}{2}, \frac{1}{3}, \cdots\}$ = $\left\{X(n) = \frac{1}{n}\right\}_{n=1}^{\infty}$

We can take L = 0 as $\lim_{n \to \infty} x_n = 0$ & $n_1 = 3, \epsilon = 1/2$.
So all values for $n \geq 3$ lies within the ϵ neighbourhood of zero.

Ex: $\left\{X(n) = (-1)^n\right\}$ = {1, -1, 1, -1, $\cdots$}
We are unable to find any stage n_1 such that for all $n \geq n_1$ values lie within the ϵ neighbourhood of L.

⇒ Divergent sequence & limit does not exists.

> **Theorem.** Limit of a sequence is unique.

#semester-1 #mathematics #real-analysis