JEE (ADVANCED) 2018 PAPER 1
PART-III MATHEMATICS

SECTION 1 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : $+4$ If only (all) the correct option(s) is (are) chosen.
  Partial Marks : $+3$ If all the four options are correct but ONLY three options are chosen.
  Partial Marks : $+2$ If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : $+1$ If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : $\mathbf{0}$ If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : -2 In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option) ,without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in -2 marks.

Q.1 For a non-zero complex number z, let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement(s) is (are) **FALSE?**

(A) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f: \mathbb{R} \rightarrow (-\pi, \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2,

$$\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$$

is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition

$$\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi,$$

lies on a straight line

Q.2 In a triangle PQR, let $\angle PQR = 30^\circ$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?

(A) $\angle QPR = 45^\circ$

(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^\circ$

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

(D) The area of the circumcircle of the triangle PQR is $100\,\pi$

Q.3 Let P_1: $2x + y - z = 3$ and P_2: $x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE?

(A) The line of intersection of P_1 and P_2 has direction ratios $1, 2, -1$

(B) The line

$$\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$$

is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

Q.4 For every twice differentiable function $f: \mathbb{R} \to [-2, 2]$ with $(f(0))^2 + \left(f'(0)\right)^2 = 85$, which of the following statement(s) is (are) TRUE?

(A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$

(C) $\lim\limits_{x \to \infty} f(x) = 1$

(D) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

Q.5 Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be two non-constant differentiable functions. If

$$f'(x) = \left(e^{(f(x)-g(x))}\right)g'(x) \quad \text{for all } x \in \mathbb{R},$$

and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Q.6 Let $f:[0,\ \infty) \rightarrow \mathbb{R}$ be a continuous function such that

$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)dt$$

for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE?

(A) The curve $y = f(x)$ passes through the point $(1, 2)$

(B) The curve $y = f(x)$ passes through the point $(2, -1)$

(C) The area of the region $\left\{(x, y) \in [0, 1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1 - x^2}\right\}$ is $\frac{\pi-2}{4}$

(D) The area of the region $\left\{(x, y) \in [0,1] \times \mathbb{R} : f(x) \leq y \leq \sqrt{1 - x^2}\right\}$ is $\frac{\pi-1}{4}$

SECTION 2 (Maximum Marks: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered as answer.
  Zero Marks : 0 In all other cases.

Q.7 The value of

$$\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$$

is ______ .

Q.8 The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is _____ .

Q.9 Let X be the set consisting of the first 2018 terms of the arithmetic progression $1, 6, 11, \ldots$, and Y be the set consisting of the first 2018 terms of the arithmetic progression $9, 16, 23, \ldots$. Then, the number of elements in the set $X \cup Y$ is _____.

Q.10 The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is _____ .

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

Q.11 For each positive integer n, let

$$y_n = \frac{1}{n}\left((n+1)(n+2)\cdots(n+n)\right)^{\frac{1}{n}}.$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is _____ .

Q.12 Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} \cdot \vec{b} = 0$. For some $x, y \in \mathbb{R}$, let $\vec{c} = x\,\vec{a} + y\,\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is _____ .

Q.13 Let a, b, c be three non-zero real numbers such that the equation

$$\sqrt{3}\, a \cos x + 2\, b \sin x = c,\ \ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right],$$

has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____ .

Q.14 A farmer F_1 has a land in the shape of a triangle with vertices at $P(0,0)$, $Q(1,1)$ and $R(2,0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ $(n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is _____ .

SECTION 3 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : $+3$ If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : -1 In all other cases.

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(*There are two questions based on PARAGRAPH "X", the question given below is one of them*)

Q.15 Let E_1E_2 and F_1F_2 be the chords of S passing through the point P_0 $(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve

(A) $x + y = 4$

(B) $(x-4)^2 + (y-4)^2 = 16$

(C) $(x-4)(y-4) = 4$

(D) $xy = 4$

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(*There are two questions based on PARAGRAPH "X", the question given below is one of them*)

Q.16 Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x+y)^2 = 3xy$

(B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$

(D) $x^2 + y^2 = x^2 y^2$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

Q.17 The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her is

(A) $\frac{3}{40}$ (B) $\frac{1}{8}$ (C) $\frac{7}{40}$ (D) $\frac{1}{5}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(There are two questions based on PARAGRAPH "A", the question given below is one of them)

Q.18 For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{7}{60}$ (D) $\frac{1}{5}$

END OF THE QUESTION PAPER