CSL-2020: Data Structures and Algorithms

HW-1

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Deadline: 09:55 AM, 2nd August 2022 Monsoon 2022, I Semester Max mark: 10

Submit the code on Mimir. Submit the proof either typeset or clearly handwriten in the class on 2nd August 2022.

1. (a) (Card trick 1:) You want to impress your friend (Alice) by demonstrating to her a card trick. Below are the steps for the same.

    1. Select 21 cards from a deck of playing cards and show them to Alice. She picks a card (mentally) but does not tell you which one is it. You distribute these 21 cards in 3 columns[1] and tell Alice to indicate in which column her chosen card is.
    2. Pick up one of the columns which doesn't contain the card (there are two such columns - pick anyone), then the column which contains the card, and finally the remaining column. This way, you have created a single pile of 21 cards again.
    3. Deal the cards in 3 columns again, starting from the left to the right, and repeating the process until there are no cards left in your hand (i.e. you have again created 3 columns of 7 cards each). Ask Alice to indicate the column which contains her card.
    4. Repeat step 2 then 3.
    5. Repeat step 2.

    Now by counting either from the top or from the bottom of the deck of 21 cards, the card chosen by Alice will be the 11th card !

    Write a C program to demonstrate this trick. Randomly pick a card during the process and verify that the trick works. The code should print the sequence of 21 cards whenever you create the pile of 21 cards (i.e. at the beginning, and then at the end of step 2 every time). Print the randomly picked card in Step 1, and verify that it matches at the end. Submit the code on Mimir before the class on next Tuesday. **(3)**

    (b) Explain by a mathematical argument why does this trick work. Submit the proof in the class on next Tuesday. **(2)**

[1] While dealing, distribute the cards horizontally in 3 columns. That is, put the cards in columns 1, 2, 3, 1, 2, 3, ... etc. Within a column, the card which appeared earlier lies below the ones which appeared afterwards.

2. (a) (Card trick 2): This time you are trying to impress Alice even more. You start with any 27 cards, and ask her to mentally choose one card without telling you which one she chose. You also ask her to choose a number between 1 to 27 and tell you this number. Let us say, she chooses the number n. Compute the base 3 representation of the number $(n-1)$ and let it be xyz. That is, $(n-1) = x \cdot 3^2 + y \cdot 3^1 + z \cdot 3^0$. You are going to use these numbers x, y and z in the trick.

   Now follow the steps below, which are similar to what we did previously but with some twist. The twist is that the specific pile which contains the chosen card will be mixed carefully with other cards depending on the value of x, y, and z. If the base 3 digit is 0 then the pile will be on top, if 1 then the pile will be the middle one, and if 2 then the pile will be the last one while mixing.

   Example: Say, Alice chooses 12 as her number. Then $n-1 = 11$ and $11 = 9+0+2 = 1 \cdot 3^2 + 0 \cdot 3^1 + 2 \cdot 3^0$. That is, $11_{(10)} = 102_{(3)}$, where the subscript denotes the base used in representing the number. In this example, $x = 1$, $y = 0$ and $z = 2$.

   1. Deal the 27 cards in 3 groups of 9 cards each (like in the previous question). Now ask Alice to inform you the pile to which her card belongs to.
   2. Depending on the value of z, put this pile in appropriate position while mixing the 3 piles to form a single set of 27 cards.[2]
   3. Deal the cards again to form 3 piles as earlier and again ask Alice to inform you the pile in which her card lies. Put this pile in its appropriate location depending on the value of y.
   4. Deal the cards the third time and again ask Alice for the pile containing her card. Put this pile in appropriate location depending on the value of x.
   5. Now, start dealing the cards from the single set of 27 cards. Interestingly, the n^{th} will be the one chosen by Alice !

   (3)

   (b) Explain by a mathematical argument why does this trick work. Submit the proof in the class on next Tuesday. **(2)**

[2]The rule was already mentioned. That is, if $z = 0$ then put this pile at the top and the other two below it. It does not matter where do you place the other two as long as they are below the pile in which we are interested. Similarly for $z = 1$ or 2.