Ensemble Methods: Bagging and Boosting

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Machine Learning (CS771A)

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Some Simple Ensembles

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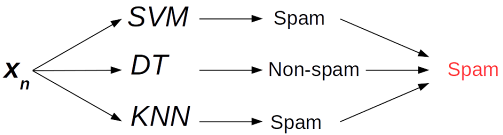

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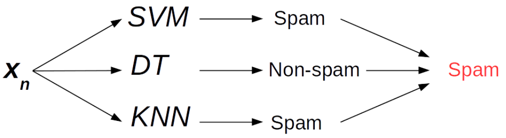
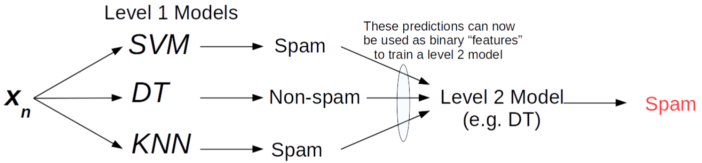


- "Stacking": Use predictions of multiple models as "features" to train a new model and use the new model to make predictions on test data



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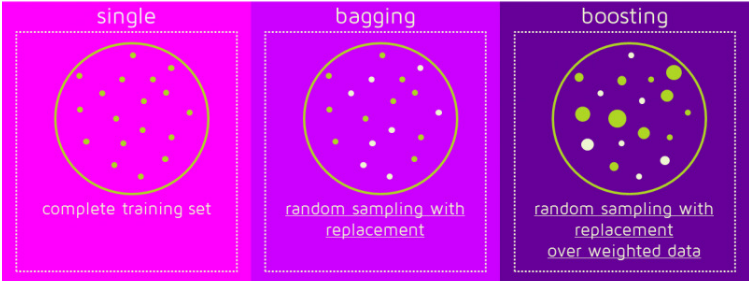

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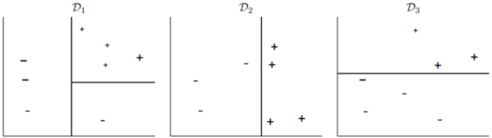
- Use an averaged model $h = \frac{1}{M}\sum_{m=1}^M h_m$ as the final model
- Useful for models with high variance and noisy data

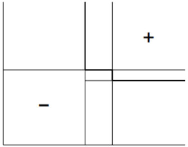
Bagging: illustration

Top: Original data, Middle: 3 models (from some model class) learned using three data sets chosen via bootstrapping, Bottom: averaged model

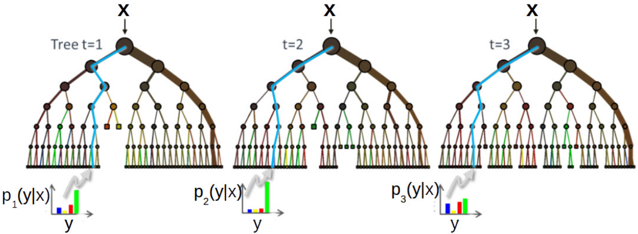


Middle Section: 3 Models (from some model class) learned using three data sets chosen via bootstrapping
This section depicts three models (shown generically) that are trained on subsets of the original data. Bootstrapping is a technique used to create these subsets. It involves randomly sampling the original data with replacement, meaning a data point can be chosen multiple times for a single subset and other data points might be left out altogether. Each of the three models you see is trained on a different bootstrapped subset of the original data (D).

This represents the final ensemble model created from bagging. The ensemble model makes predictions by averaging the predictions from the three individual models trained earlier (middle section).



Random Forests



- An ensemble of decision tree (DT) classifiers

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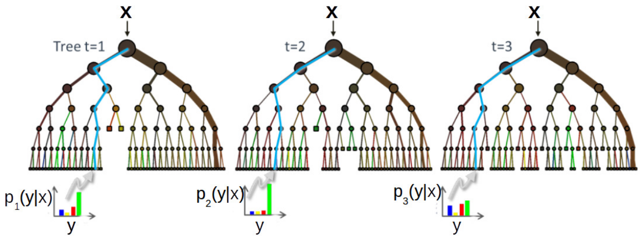

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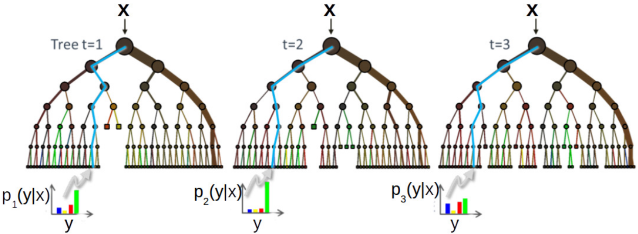
https://www.youtube.com/watch?v=J4Wdy0Wc_xQ&ab_channel=StatQuestwithJoshStarmer



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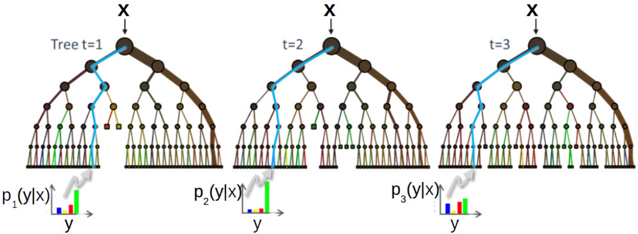

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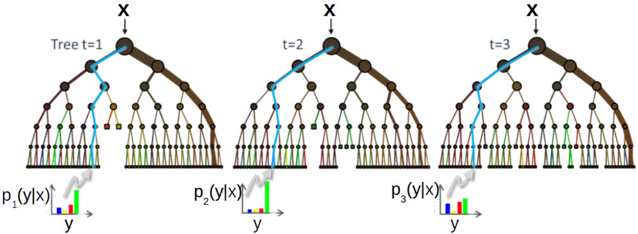

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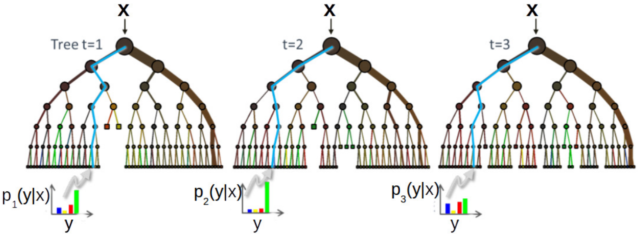

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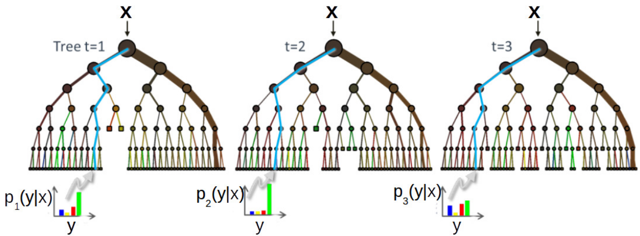

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- All DTs usually have the same depth
- Each DT will split the training data differently at the leaves
- Prediction for a test example votes on/averages predictions from all the DTs

Boosting

- The basic idea

The Power of Weak Learners:

Boosting is a powerful ensemble technique that focuses on improving the performance of weak learners. A weak learner is a basic model that only performs slightly better than random guessing on the training data.
Making Weak Learners Strong:

The core idea of boosting is to iteratively train and combine weak learners in a way that focuses on the hardest examples for the current model. This progressively improves the overall accuracy of the ensemble.
Boosting Algorithm Steps:

Train a Weak Model: You start by training a weak model on the entire training data.
Calculate Errors: The errors for each training example are computed based on the model's predictions.
Increase Difficulty Focus: The training data is reweighted, giving higher weights to examples that the model classified incorrectly in the previous step. This essentially makes the next model focus more on the challenging cases.
Re-train with Weights: A new weak model is trained using the reweighted training data. This new model should perform better on the previously difficult examples.
Iteration: Steps 2-4 are repeated for multiple iterations. With each iteration, the models focus on the remaining errors from previous models, progressively improving the overall accuracy.
Combining the Weak Learners:

After multiple iterations, you have a collection of weak models. Boosting algorithms use different strategies to combine the predictions from these models to get a final prediction. Some common approaches include weighted voting or using a meta-model that learns how to weight the individual weak learner predictions.
By focusing on the difficult examples and iteratively improving weak models, boosting algorithms can achieve significantly better performance compared to a single weak learner. This makes boosting a valuable technique for building robust and accurate machine learning models.

Boosting

- The basic idea
  - Take a weak learning algorithm

Let's imagine you have a dataset for classifying handwritten digits (0-9). Here's how boosting might work on this example:

Start Simple: Begin with a weak learner, like a simple decision tree with only a few rules. This model might perform slightly better than random guessing at identifying the digits.

Identify the Difficult Examples: Analyze the mistakes made by the first model. Assign higher weights to the data points where the model made errors (e.g., mistaking a 3 for an 8).

Train the Second Learner: Train a new, weak decision tree, but this time focus on the weighted data. This means the model will pay more attention to the previously misclassified digits.

Iteration: Repeat steps 2 and 3. The second decision tree should perform better on the initially challenging examples the first model struggled with.

Combine for Improved Results: Now you have two weak models. Boosting algorithms determine how to combine their predictions for a final classification. This could involve weighted voting where the more accurate model's vote has higher weight.

Essentially, boosting uses each weak learner to find and address the errors of the previous one. With each iteration, the models focus on the remaining difficult examples, progressively improving the overall accuracy of classifying handwritten digits.

Here are some additional points to consider:

Boosting algorithms can be more sensitive to noisy data compared to other techniques like bagging.
There are different boosting algorithms with variations in how they weight examples and combine models (e.g., AdaBoost, Gradient Boosting). They each have their strengths and weaknesses depending on the data and task.
By leveraging the concept of boosting, you can turn a collection of weak learners into a much stronger ensemble model for various machine learning tasks.

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  5. Go back to step 2

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  - Normalize D_{t+1} so that it sums to 1: $D_{t+1}(n) = \frac{D_{t+1}(n)}{\sum_{m=1}^{N} D_{t+1}(m)}$

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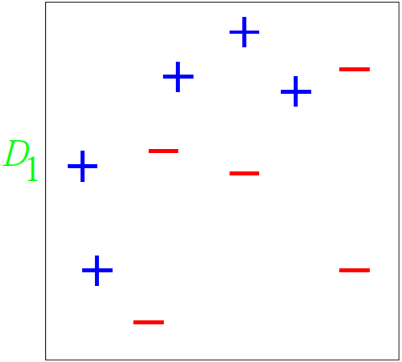
- Output the "boosted" final hypothesis $H(\boldsymbol{x}) = \text{sign}(\sum_{t=1}^{T} \alpha_t h_t(\boldsymbol{x}))$

AdaBoost: Example

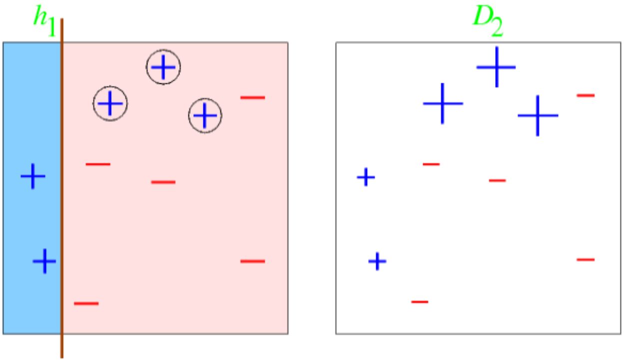
Consider binary classification with 10 training examples

Initial weight distribution D_1 is uniform (each point has equal weight $= 1/10$)



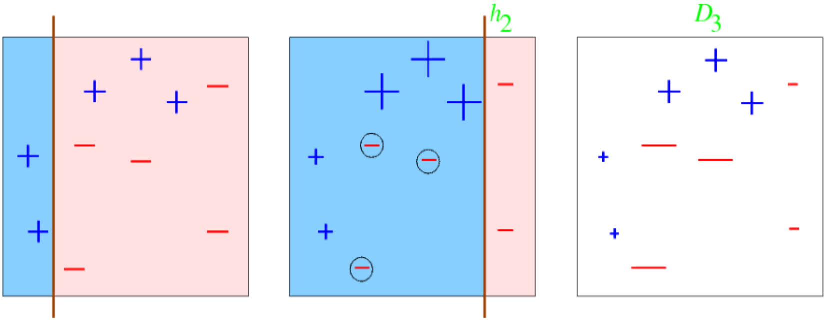
Each of our weak classifers will be an **axis-parallel linear classifier**

After Round 1



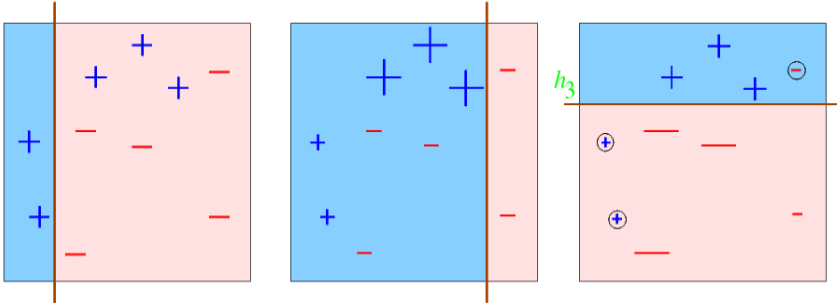
- Error rate of h_1: $\epsilon_1 = 0.3$; weight of h_1: $\alpha_1 = \frac{1}{2}\ln((1-\epsilon_1)/\epsilon_1) = 0.42$
- Each misclassified point upweighted (weight multiplied by $\exp(\alpha_2)$)
- Each correctly classified point downweighted (weight multiplied by $\exp(-\alpha_2)$)

After Round 2



- Error rate of h_2: $\epsilon_2 = 0.21$; weight of h_2: $\alpha_2 = \frac{1}{2}\ln((1-\epsilon_2)/\epsilon_2) = 0.65$
- Each misclassified point upweighted (weight multiplied by $\exp(\alpha_2)$)
- Each correctly classified point downweighted (weight multiplied by $\exp(-\alpha_2)$)

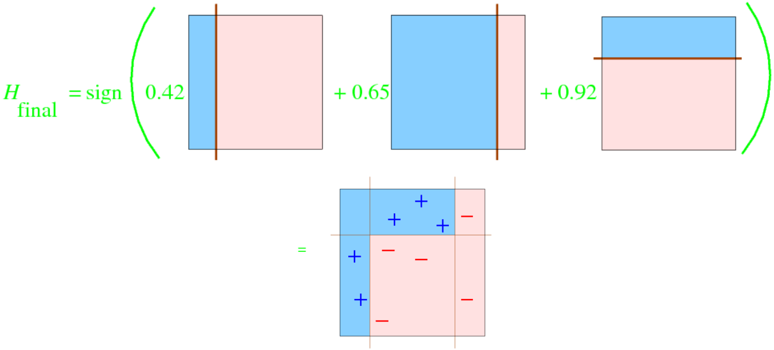
After Round 3



- Error rate of h_3: $\epsilon_3 = 0.14$; weight of h_3: $\alpha_3 = \frac{1}{2}\ln((1-\epsilon_3)/\epsilon_3) = 0.92$
- Suppose we decide to stop after round 3
- Our ensemble now consists of 3 classifiers: h_1, h_2, h_3

Final Classifier

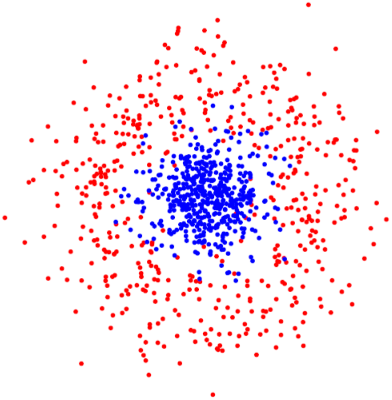
- Final classifier is a weighted linear combination of all the classifiers
- Classifier h_i gets a weight α_i



- Multiple weak, linear classifiers **combined** to give a strong, nonlinear classifier

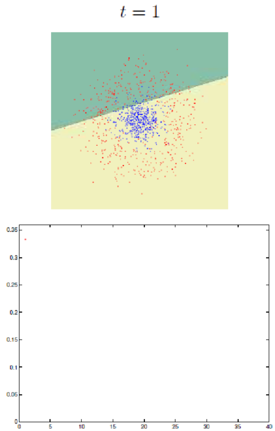
Another Example

- Given: A nonlinearly separable dataset
- We want to use Perceptron (linear classifier) on this data



AdaBoost: Round 1

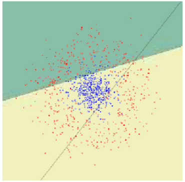
- After round 1, our ensemble has 1 linear classifier (Perceptron)
- Bottom figure: X axis is number of rounds, Y axis is training error

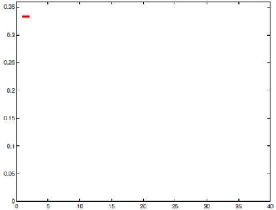


AdaBoost: Round 2

- After round 2, our ensemble has 2 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 2$

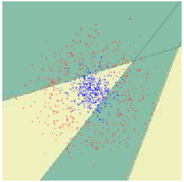


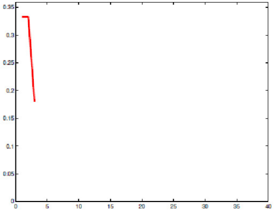


AdaBoost: Round 3

- After round 3, our ensemble has 3 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 3$

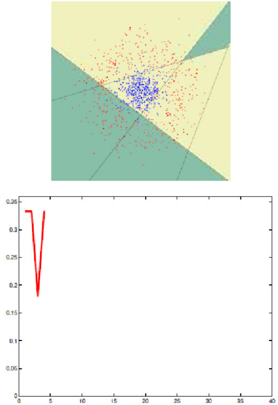




AdaBoost: Round 4

- After round 4, our ensemble has 4 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

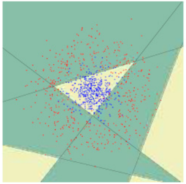
$t = 4$

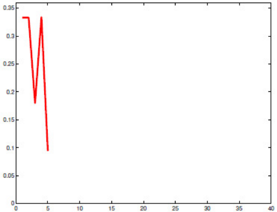


AdaBoost: Round 5

- After round 5, our ensemble has 5 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 5$

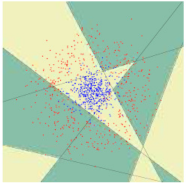


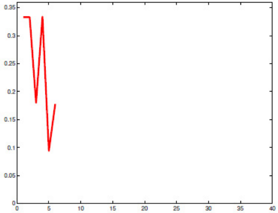


AdaBoost: Round 6

- After round 6, our ensemble has 6 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 6$

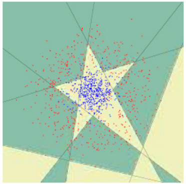


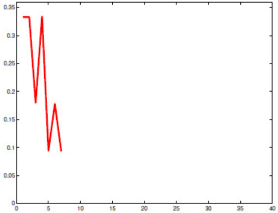


AdaBoost: Round 7

- After round 7, our ensemble has 7 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 7$

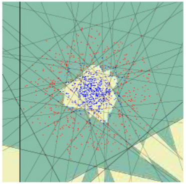


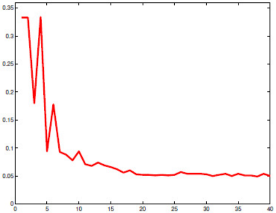


AdaBoost: Round 40

- After round 40, our ensemble has 40 linear classifiers (Perceptrons)
- Bottom figure: X axis is number of rounds, Y axis is training error

$t = 40$





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where $w_d = \sum_{t:i_t=d} 2\alpha_t s_t$ and $b = -\sum_t \alpha_t s_t$

Boosting: Some Comments

- For AdaBoost, given each model's error $\epsilon_t = 1/2 - \gamma_t$, the training error consistently gets better with rounds

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- Boosting in general can perform badly if some examples are outliers

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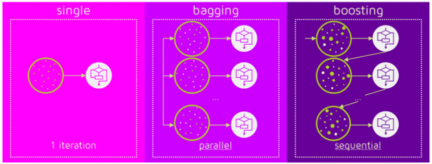

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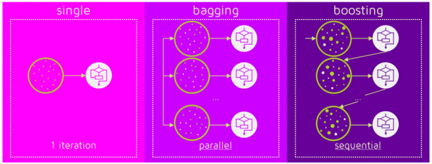

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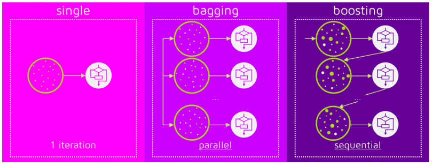

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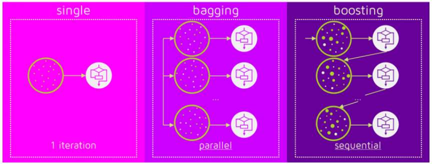

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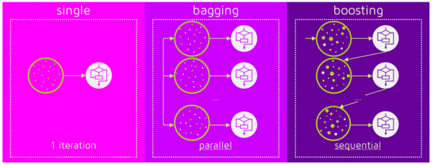

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- Bagging usually can't reduce the bias, boosting can (note that in boosting, the training error steadily decreases)
- Bagging usually performs better than boosting if we don't have a high bias and only want to reduce variance (i.e., if we are overfitting)