4/17/19 Assignment-8 1001669806

Task 1

Sahara- Sensor S in Sahara
Maine - Sensor S in Maine

P(Maine) = 0.05 P(¬Maine) = 1-0.05
= 0.95
∴ P(Sahara) = 0.95

Sensor is in →	Maine	Sahara
Temp ≥ 80	0.20	0.90
Temp < 80	0.80	0.10

a) $P(\text{Maine} / \text{Temp} < 80) = \dfrac{P(\text{Maine} \wedge \text{Temp} < 80)}{P(\text{Temp} < 80)}$

$= \dfrac{\cancel{0.80}}{0.\cancel{80 + 0.10}}$

$= \dfrac{P(\text{Temp} < 80 \mid \text{Maine}) \cdot P(\text{Maine})}{P(\text{Temp} < 80 \mid \text{Maine}) \cdot P(\text{Maine}) + P(\text{Temp} < 80 \mid \text{Sahara}) \cdot P(\text{Sahara})}$

$= \dfrac{0.80 \times 0.05}{0.80 \times 0.05 + 0.10 \times 0.95}$

$= \dfrac{0.04}{0.135} = 0.2962$

b) temp1 < 80 is the first email of tempolature reading below 80.

temp 2 < 80 is the second email of tempolature reading below 80.

We have to find $P(temp2 < 80 \mid temp1 < 80)$

$$P(temp2 < 80 \mid temp1 < 80) = \frac{P(temp1 < 80 \wedge temp2 < 80)}{P(temp1 < 80)} \rightarrow ①$$

$$P(temp1 < 80 \wedge temp2 < 80) = P(temp2 < 80 \wedge temp1 < 80 \mid Maine) \cdot P(Maine) + P(temp2 < 80 \wedge temp1 < 80 \mid Sahala) \cdot P(Sahala)$$

$$= P(temp2 < 80 / Maine) \cdot P(temp1 < 80 \mid Maine) \cdot P(Maine) + P(temp2 < 80 \mid Sahala) \cdot P(temp2 < 80 / Sahala) \cdot P(Sahala)$$

$$= 0.80 \times 0.80 \times 0.05 + 0.10 \times 0.10 \times 0.95$$

$$= 0.032 + 0.0095$$

$$= \underline{0.0415}$$

$$P(temp1 < 80) = P(temp1 < 80 \mid Maine) \cdot P(Maine) + P(temp1 < 80 \mid Sahala) \cdot P(Sahala)$$

$$= 0.8 \times 0.05 + 0.1 \times 0.95$$

$$= 0.135$$

$$P(temp2 < 80 \mid temp1 < 80) = \frac{0.0415}{0.135} = 0.3074$$

c) temp3<80 is the third leading of temp below 80.

$P(temp1<80 \wedge temp2<80 \wedge temp3<80)$

$= P(temp1<80 \mid Moine) \cdot P(temp2<80 \mid Moine) \cdot P(temp3<80 \mid Moine) \cdot P(Moine)$

$+ P(temp1<80 \mid Sahara) \cdot P(temp2<80 \mid Sahara) \cdot P(temp3<80 \mid Sahara) \cdot P(Sahara)$

$= 0.8 \times 0.8 \times 0.8 \times 0.05 + 0.1 \times 0.1 \times 0.1 \times 0.95$

~~= 0.02561~~

$= 0.02655$

Task 2

a) A can have 5 values.
B can have 7 values.

No. of A values $= 5^1$

No of B values $= 7^{10}$

Total number to store in joint distribution Table $= 5 \times 7^{10} - 1$

b) ~~P(A) = ...~~

$P(A) = 5 - 1 = 4$

$P(B/A) = 5 \times (7-1) = 30$

For all 10 B, $30 \times 10 = 300$

The most space-efficient way will need $300 + 4 = 304$ values.

Task-3

a) Consider B is True

$P(A|B) = \alpha\, P(A, B)$

$= \alpha[P(A, B, C=True) + P(A, B, C=False)]$

$= \alpha[(0.048\ \ 0.012) + (0.196\ \ 0.294)]$

$= \alpha[0.244\ \ 0.306]$

$= (0.4395\ \ 0.5604) = (0.4395\ \ 0.5604)$

Consider B is False,

$P(A|B) = \alpha[P(A, B=False, C=True) + P(A, B=False, C=False)]$

$= \alpha[(0.192\ \ 0.048) + (0.084\ \ 0.126)]$

$= \alpha[0.276\ \ 0.174]$

$= (0.613\ \ 0.387)$

	B = T	B = F
A = T	0.4395	0.613
A = F	0.5604	0.387

b) Consider B = True, C = True

$P(A|B,C) = \alpha[P(A, B=T, C=T)]$

$= \alpha[(0.048\ \ 0.012)]$

$= (0.8\ \ 0.2)$

Consider $B = \text{True}, C = \text{False}$

$$P(A|B,C) = \alpha[\ P(A, B=T, C=F)]$$

$$= \alpha[\ (0.196 \quad 0.294)]$$

$$= (0.4 \quad 0.6)$$

Consider $B = \text{False} \quad C = \text{True}$

$$P(A/B,C) = \alpha[\ P(A, B=F, C=T)]$$

$$= \alpha[(0.192 \quad 0.048)]$$

$$= (0.8 \quad 0.2)$$

Consider $B = \text{False} \quad C = \text{False}$

$$P(A|B,C) = \alpha[\ P(A, B=F, C=F)]$$

$$= \alpha[(0.084 \quad 0.126)]$$

$$= (0.4 \quad 0.6)$$

	C = T		C = F	
	B=T	B=F	B=T	B=F
A=T	0.8	0.8	0.4	0.4
A=F	0.2	0.2	0.6	0.6

c) When B = True,

~~P(A, B | C)~~

$P(A, C | B) = \alpha [P(A=T\ C=T \quad A=T\ C=F \quad A=F\ C=T \quad A=F\ C=F)]$

$= \alpha [\langle 0.048 \quad 0.196 \quad 0.012 \quad 0.294 \rangle]$

$= \langle 0.0872 \quad 0.3564 \quad 0.0218 \quad 0.5345 \rangle$

when B = False

$P(A, C | B) = \alpha [P(A=T\ C=T \quad A=T\ C=F \quad A=F\ C=T \quad A=F\ C=F)]$

$= \alpha [\langle 0.192 \quad 0.084 \quad 0.048 \quad 0.126 \rangle]$

$= \langle 0.4267 \quad 0.1867 \quad 0.1067 \quad 0.28 \rangle$

	C = T		C = F	
	B = T	B = F	B = T	B = F
A = T	0.0872	0.4267 ~~0.192~~	0.3564	0.1867
A = F	0.0218	0.1067	0.5345	0.28

d) Yes, Given B, A is conditionaly independent of C

Because

$P(A | B) = \alpha [P(A, B, C=T) + P(A, B, C=F)]$

We get 4 values and we consider both C = True & C = False while calculating. Hence, it is independent of C.