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Assignment

Vector & Matrices.

Minimum Background test

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \quad Y = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad Z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

1. Inner product of y and z $= y^T z = \begin{bmatrix} 1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$= 2\cdot 1 + 3\cdot 3$

$= 11$

2. Product Xy $= \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 4\cdot 3 \\ 1\cdot 1 + 3\cdot 3 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$.

3. Invertible

(i) determinant $= 2\cdot 3 - 4\cdot 1 = 6 - 4 = 2$.

(ii) $XX' = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(iii) $X^{-1} = \dfrac{\text{Adj } A}{\det |A|} = \dfrac{\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}}{2} = \begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}$

(i), (ii) & (iii) prove that X is invertible.

4. Rank of $X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$.

The rank of $X = 2$.

Row 1 & Row 2 are linearly independent of each other.

Calculus

1. $y = x^3 + x - 5$

$\dfrac{dy}{dx} = x^2 + 1$

2. $f(x_1, x_2) = x_1 \sin(x_2) e^{-x_1}$

$\nabla f(x) = \begin{pmatrix} \partial_{x_1} f \\ \partial_{x_2} f \end{pmatrix}$ ∴ $\partial_{x_1} f = (1 - x_1) e^{-x_1} \sin(x_2)$

$\partial_{x_2} f = x_1 e^{-x_1} \cos(x_2)$

$= \begin{pmatrix} (1 - x_1) e^{-x_1} \sin(x_2) \\ x_1 e^{-x_1} \cos(x_2) \end{pmatrix}$

Probability & statistics.

$S = \{1, 1, 0, 1, 0\}$. 0 → heads, 1 → tails

1. Sample mean $= \dfrac{\sum_{i=1}^{5} V_i}{\#\text{ of sample}} = \dfrac{3}{5}$

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(2) Sample variance $= \dfrac{\Sigma(Y_i - \bar{Y})^2}{\text{no of sample} - 1} = \dfrac{\left(\frac{2}{5}\right)^2+\left(\frac{2}{5}\right)^2+\left(-\frac{3}{5}\right)^2+\left(\frac{2}{5}\right)^2+\left(-\frac{3}{5}\right)^2}{4}$

$= \dfrac{1.2}{4} = 0.3$

3. Probability of observing $= \left(\frac{1}{2}\right)^5 = \frac{1}{32} = 0.0312$

(4). $P(x = 1)$.

Now, make $P(S) = \prod_{i=1}^{5} p^{x_i}(1-p)^{1-x_i}$

$= p^{\sum_{i=1}^{5} x_i}(1-p)^{n - \sum_{i=1}^{5} x_i}$

Now maximize $P(S)$, we will get.

$\max(P(S)) = \left(\sum_{i=1}^{5} x_i\right)\log(P) + \left(n - \sum_{i=1}^{5} x_i\right)\log(1-P)$

$\dfrac{d(\max(P(S)))}{dP} = 0$

$\dfrac{1}{P}\sum_{i=1}^{5} x_i - \dfrac{1}{1-P}\left(n - \sum_{i=1}^{5} x_i\right) = 0$

$\dfrac{\sum_{i=1}^{5} x_i - Pn}{P(1-P)} = 0$

$Pn = \sum_{i=1}^{5} x_i$

$P = \dfrac{\sum_{i=1}^{5} x_i}{n}$

for $(x=1)$ $\quad P = \frac{1}{5}(3) = 3/5$.

5. $P(z = T \text{ and } y = b) = 0.1$.

$P(z = T \mid y = b) = \dfrac{P(z = T \,\&\, y = b)}{P(y = b)} = \dfrac{0.1}{0.1 + 0.15} = 0.4$

Big-O Notation

1. $f(n) = \ln(n) = \lg(n)$ both are equivalent

2. $f(n) = 3^n$, $g(n) = n^{100}$, $g(n) = O(f(n))$, $f(n)$ rapidly increase than $f(n)$ when $n \gg 1$.

(3) $f(n) = 3^n$, $g(n) = 2^n$ $\quad g(n) = O(f(n))$, here also $f(n)$ rapidly ↑se when $n \gg 1$.

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(4). $f(n) = 1000n^2 + 2000n + 4000$, $g(n) = 3n^2 + 1$.
$g(n)$ increase rapidly when $n >> 1$.

Medium Background Test

Algorithms.

Probability & Random variables

Probability :- True or False.

(a) False (b) True (c) False (d) False (e) True.

Discrete & continuous distributions.

Multivariate Gaussian $\frac{1}{\sqrt{(2\pi)^d \Sigma}} \exp\left(-\frac{1}{2} \cdot (x-\mu)^T \Sigma^{-1} (x-\mu)\right)$

Bernoulli $p^x (1-p)^{1-x}$

Uniform $\frac{1}{b-a}$ when $a \le x \le b$; 0 otherwise

Binomial $\binom{n}{x} p^x (1-p)^{n-x}$

Mean, Variance and entropy.

(a) $Var(X) = E[(X - E X)^2]$

$= E[X^2 - 2XE[X] + (E[X])^2]$.

$= E[X^2] - E[2XE[X]] + E[(E[X])^2]$.

$= E[X]^2 - 2E[X^2] + (E[X])^2$

$= E[X]^2 - E[X^2]$.

(b) If mean is P.

Variance $= P(1-P)$

Entropy $= -(1-P)\log(1-P) - P\log(P)$.

Law of large number and central limit theorem.

(a)

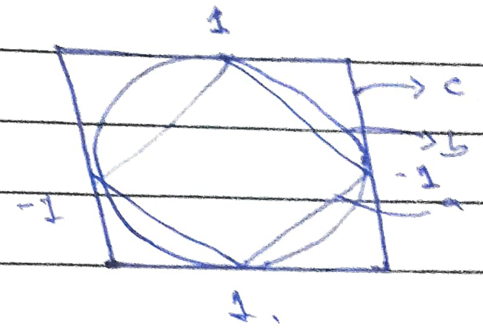
(b) $\hat{x}$ is avg. no. of heads.

coins is tossed n times $= \sqrt{n}(\hat{x} - \frac{1}{2})]^{n \to \infty} \to N($

This due to central limit theorm.

Linear Algebra.

Vector norm.



Geometry.

(a) $w^T x + b = 0$.

$w^T x_1 + b = 0 = w^T x_2 + b$.

$\Rightarrow w^T x_1 = w^T x_2$.

$\Rightarrow w^T(x_1 - x_2) = 0$

$\Rightarrow w$ is orthogonal to line.

(b) distance $= \dfrac{|w^T x|}{\|w\|_2}$

$= \dfrac{|-b|}{\|w\|_2}$

$= \dfrac{|b|}{\|w\|_2}$.