FIR Filter Design, Analysis, and Quantization Using MATLAB

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1. Overview:

This report details the design of a low-pass FIR filter with stringent specifications (passband edge 0.2π, stopband edge 0.23π, and minimum stopband attenuation of 80 dB) using MATLAB. The design exploits the Parks–McClellan equiripple algorithm to obtain an optimal minimax solution. Fixed-point quantization in Q1.15 format is applied for hardware implementation, with explicit measures taken to prevent overflow.

2. FIR Filter Design and Parks–McClellan Algorithm:

The filter specifications require a narrow transition band and deep stopband attenuation. The `firpmord` function first estimates the minimum filter order, and we enforce a minimum of 150 taps to ensure robustness post quantization. The Parks–McClellan algorithm solves the Chebyshev approximation problem by minimizing the maximum error

$$E = \max_{\omega} |D(\omega) - H(\omega)| ,$$

where $D(\omega)$ is the desired frequency response and $H(\omega)$ is the actual filter response. By the alternation theorem, an optimal n^{th}-order filter must exhibit at least $n+2$ alternations in the error function. This yields an equiripple behavior in both passband and stopband, ensuring that the filter meets the 80 dB attenuation requirement with the minimum possible order. This optimality is critical in resource-constrained hardware implementations.

3. Frequency Response Analysis:

The MATLAB `freqz` function was used to compute the frequency response of the unquantized filter. The magnitude response (in dB) confirms that the designed filter achieves the required specifications, with a calculated stopband attenuation close to 80 dB and controlled passband ripple. Plots generated from MATLAB (not shown here) illustrate both the linear and logarithmic responses, validating the filter's performance.

4. Quantization and Overflow Handling:

To implement the filter in hardware, the floating-point coefficients were quantized to Q1.15 format (16-bit with 15 fractional bits). The conversion process involved:

$$b_{\text{scaled}} = \text{round}\left(b \times 2^{15}\right) ,$$

followed by clamping the values to the signed 16-bit range $[-2^{15}, 2^{15}-1]$. This scaling minimizes quantization error while ensuring that overflow does not occur during arithmetic operations. The quantized coefficients are then exported to a text file for use in Verilog, maintaining the filter's desired frequency response even after quantization.

5. Conclusion:

The Parks–McClellan equiripple algorithm was selected for its optimal minimax properties and uniform error distribution, which are essential for meeting tight specifications with minimal filter order. MATLAB was used to design, analyze, and quantize the FIR filter, yielding a design that achieves an 80 dB stopband attenuation while ensuring robustness against quantization and overflow. The resulting fixed-point coefficients facilitate seamless hardware implementation.