1. Explain what is Maximum Likelihood Estimation (MLE).

2. Suppose that X is a discrete random variable with the following probability mass function: where $0 \leq \theta \leq 1$ is a parameter. The following 10 independent observations

X	0	1	2	3
$P(X)$	$2\theta/3$	$\theta/3$	$2(1-\theta)/3$	$(1-\theta)/3$

were taken from such a distribution: (3, 0, 2, 1, 3, 2, 1, 0, 2, 1). What is the maximum likelihood estimate of θ.

3. A coin is flipped 100 times. Given that there were 55 heads, find the maximum likelihood estimate for the probability p of heads on a single toss.

Maximum likelihood for continuous distributions

4. Suppose that the lifetime of Badger brand light bulbs is modeled by an exponential distribution with (unknown) parameter λ. We test 5 bulbs and find they have lifetimes of 2, 3, 1, 3, and 4 years, respectively. What is the Maximum Likelihood Estimate (MLE) for λ?

5. Let X and Y be two discrete random variables with joint probability mass functions $p_{XY}(1,2) = p_{XY}(3,4) = \frac{1}{2}$. What is the co-variance matrix of the following matrix of X and Y?