talent sprint | Indian Institute of Science भारतीय विज्ञान संस्थान

1. Let the random variable X denote the time a person waits for an elevator to arrive. Suppose the longest one would need to wait for the elevator is 2 minutes, so that the possible values of X (in minutes) are given by the interval $[0, 2]$. A possible PDF for X is given by

$$f(x) = \begin{cases} x, & for\ 0 \leq x \leq 1 \\ 2 - x, & for\ 1 < x \leq 2 \\ 0, & otherwise \end{cases}$$

(a). Compute the expected value of X

(b). Compute the variance of X

(c). Compute the standard deviation of X

2.a. Explain in detail what is meant by continuous and discrete random variables. What do you understand by Probability Mass Function (PMF)?

2.b. Let S be the set of integers and $f(x)$ be defined as:

$$f(x) = \begin{cases} k(7x + 3) & if\ x = 1, 2\ or\ 3 \\ 0 & otherwise \end{cases}$$

For what value of k is f a PMF?

3.a. Find the constant c such that the function

$$f(x) = \begin{cases} cx^2, & for\ 0 < x < 3 \\ 0, & otherwise \end{cases}$$

is a density function

and

3.b. Compute $P(1 < X < 2)$ for the density function as described in $8.a$.

4.a. The distribution function for a random variable X is:

$$F(x) = \begin{cases} 1 - e^{-2x} & for\ x \geq 0 \\ 0 & for\ x < 0 \end{cases}$$

Find the density function.

4.b. Having found the density function in $4.a.$, determine the probability that $X > 2$.

4.c. Having found the density function in 9.*a.*, determine the probability that $-3 < X \leq 4$.

5. The number of life insurance policies sold per day by a life insurance salesman is Poisson distributed. The salesman sells on the average 3 life insurance policies per week. We assume five working days per week. Calculate the probabilities that the salesman will sell

(a) One or more policies in a week.

(b) 2 or more policies but less than 5 policies in a week.

(c) One policy in a day.

6. The probability of a student passing an exam is 0.2. Ten students took the exam.

(a) What is the probability that at least two students passed the exam?

(b) What is the expected number of students who passed the exam?

(c) How many students must take the exam to make the probability at least 0.99 that a student will pass the exam?

7. The number of vehicles passing per minute through a junction on a busy road is Poisson distributed, with the average rate being 300 per hour.

(a) Find the probability that none passes in a given minute.

(b) What is the expected number of vehicles passing in two minutes?

8. Fashion in Vogue is an e-commerce company that sells apparel for men. It is observed that about 10% of their customers return the items purchased by them for many reasons (such as size, color, and material mismatch). On a particular day, 20 customers purchased items from Fashion in Vogue. Calculate:

(a) Probability that exactly 5 customers will return the items.

(b) Probability that a maximum of 5 customers will return the items.

(c) Probability that more than 5 customers will return the items purchased by them.

(d) Average number of customers who are likely to return the items.

(e) The variance and the standard deviation of the number of returns.