Assignment: 1

Date Of Submission: 10-03-2021

Write Python program to implement the following:
Submit the programs along with the results

Several of the computer exercises will rely on the following data.

	ω_1			ω_2			ω_3		
sample	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
1	−5.01	−8.12	−3.68	−0.91	−0.18	−0.05	5.35	2.26	8.13
2	−5.43	−3.48	−3.54	1.30	−2.06	−3.53	5.12	3.22	−2.66
3	1.08	−5.52	1.66	−7.75	−4.54	−0.95	−1.34	−5.31	−9.87
4	0.86	−3.78	−4.11	−5.47	0.50	3.92	4.48	3.42	5.19
5	−2.67	0.63	7.39	6.14	5.72	−4.85	7.11	2.39	9.21
6	4.94	3.29	2.08	3.60	1.26	4.36	7.17	4.33	−0.98
7	−2.51	2.09	−2.59	5.37	−4.63	−3.65	5.75	3.97	6.65
8	−2.25	−2.13	−6.94	7.18	1.46	−6.66	0.77	0.27	2.41
9	5.56	2.86	−2.26	−7.39	1.17	6.30	0.90	−0.43	−8.71
10	1.03	−3.33	4.33	−7.50	−6.32	−0.31	3.52	−0.36	6.43

1.(a) Write a procedure to generate random samples according to a normal distribution $N(\boldsymbol{\mu},\boldsymbol{\Sigma})$ in d dimensions.
(b) Write a procedure to calculate the discriminant function

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^t\boldsymbol{\Sigma}_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\boldsymbol{\Sigma}_i| + \ln P(\omega_i).$$

for a given normal distribution and prior probability $P(\omega_i)$.
(c) Write a procedure to calculate the Euclidean distance between two arbitrary points.
(d) Write a procedure to calculate the Mahalanobis distance between the mean $\boldsymbol{\mu}$ and an arbitrary point $\mathbf{x}$, given the covariance matrix $\boldsymbol{\Sigma}$.

2. Use your classifier from Problem 1(b) to classify the following 10 samples from the table above in the following way. Assume that the underlying distributions are normal.
(a) Assume that the prior probabilities for the first two categories are equal ($P(\omega_1) = P(\omega_2) = 1/2$ and $P(\omega_3) = 0$) and design a dichotomizer for those two categories using only the $x1$ feature value.
(b) Repeat all of the above, but now use *two* feature values, x_1, and x_2.
(c) Repeat, but use all *three* feature values.

3. Repeat Computer exercise 2 but for categories ω_1 and ω_3.

4. Repeat Computer exercise 2 but for categories ω_2 and ω_3.

5. Consider the three categories in Computer exercise 2, and assume $P(\omega_i) = 1/3$.

(a) What is the Mahalanobis distance between each of the following test points and each of the category means in Computer exercise 2: $(1, 2, 1)^t$, $(5, 3, 2)^t$, $(0, 0, 0)^t$, $(1, 0, 0)^t$.
(b) Classify those points.
(c) Assume instead that $P(\omega_1) = 0.8$, and $P(\omega_2) = P(\omega_3) = 0.1$ and classify the test points again.