LINEAR INTEGRATED CIRCUITS

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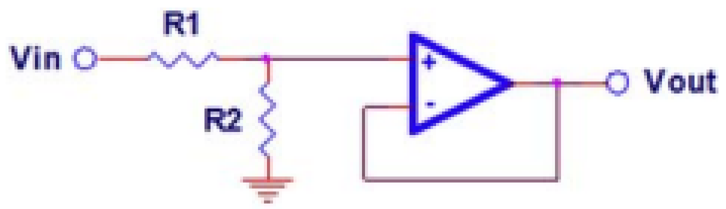
Unit 2 Simultaneous equations with m < 1

- Linear op amp transfer function is limited to equation of straight line $\underline{y = +/-mx+/-b}$

Table 5.1: The Gain and Offset Matrix

		b < 0	b = 0	b > 0
Noninverting	$m > 1$	Case 2 (Section 4.4.2)	Noninverting gain (Section 2.3)	Case 1 (Section 4.4.1)
	$m = 1$	Section 5.4	Noninverting buffer (Section 5.7)	
	$m < 1$		Section 5.2	Section 5.3
	$m = 0$	Negative reference or regulator (Chapter 21)	Ground	Positive reference or regulator (Chapter 20)
Inverting	$m < -1$	Section 5.7	Section 5.5	Section 5.6
	$m \geq -1$	Case 4 (Section 4.4.4)	Inverting gain (Section 2.4)	Case 3 (Section 4.4.3)

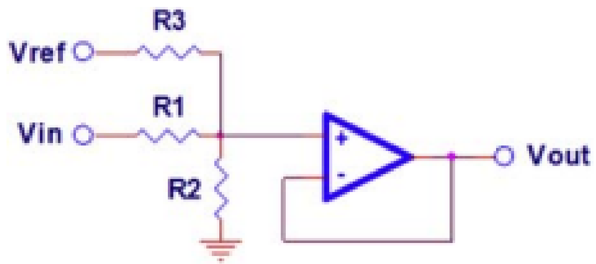
Unit 2 Non inverting attenuator with zero offset, positive offset and negative offset



$$Vout = m \times Vin$$

$$m = \frac{R2}{R1+R2}$$

Zero offset

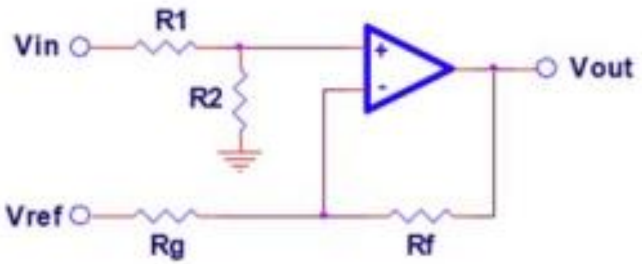


$$Vout = m \times Vin + b$$

$$m = \frac{1/R1}{1/R1 + 1/R2 + 1/R3}$$

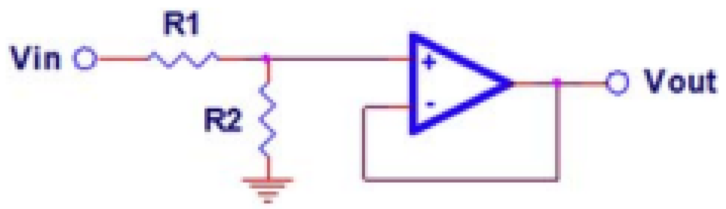
$$b = Vref \times \frac{1/R3}{1/R1 + 1/R2 + 1/R3}$$

Positive offset



Negative offset

Unit 2 Non inverting attenuator with zero offset, positive offset and negative offset



$$Vout = m \times Vin$$

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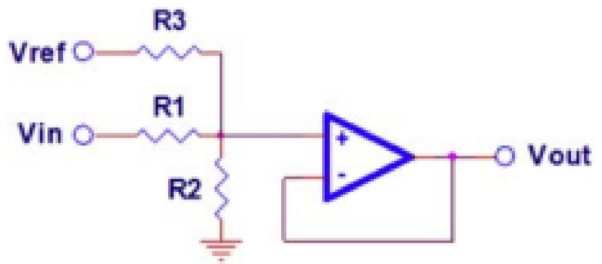
Zero offset

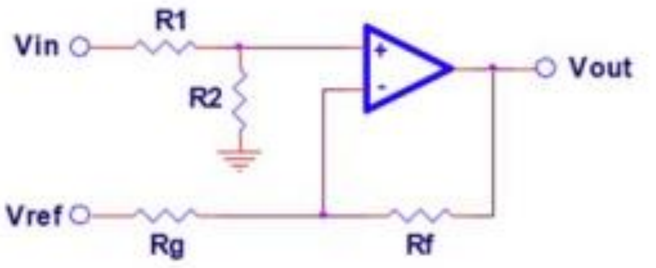


$$Vout = m \times Vin + b$$

$$m = \frac{1/R1}{1/R1 + 1/R2 + 1/R3}$$

$$b = Vref \times \frac{1/R3}{1/R1 + 1/R2 + 1/R3}$$

Positive offset



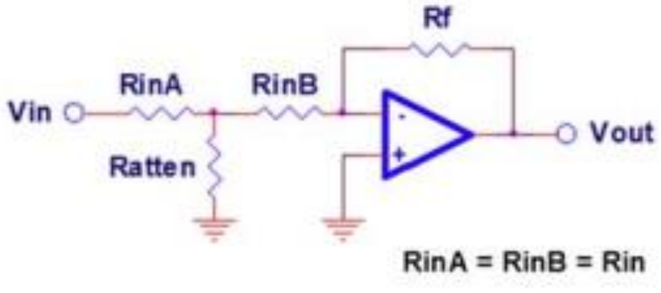
$$Vout = m \times Vin - b$$

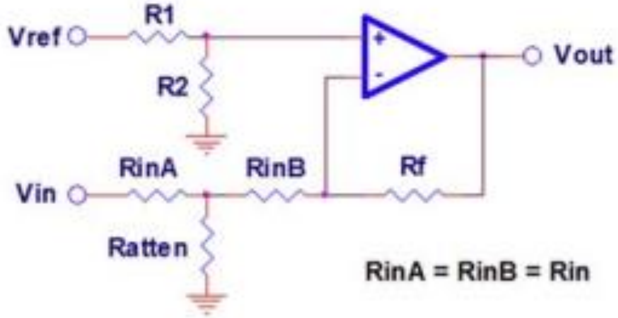
$$m = \left(\frac{R2}{R1+R2}\right) \times \left(1 + \frac{Rf}{Rg}\right)$$

$$b = Vref \times \frac{Rf}{Rg}$$

Negative offset

Unit 2 Inverting attenuator with zero offset, positive offset and negative offset

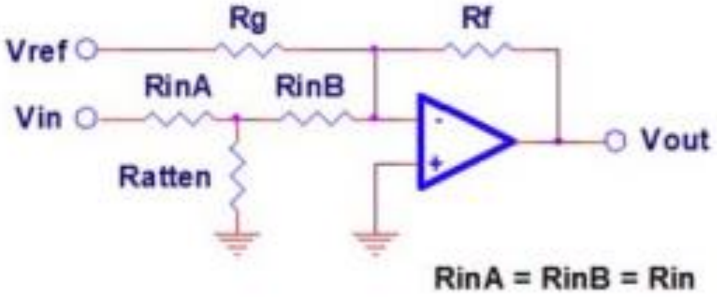




$$Vout = -m \times Vin + b$$

$$m = \frac{Rf \times Ratten}{Rin \times (Rin + 2 \times Ratten)}$$

$$b = Vref \times \left(\frac{R2}{R1 + R2}\right) \times \left(1 + \frac{Rf}{Rin + Rin \| Ratten}\right)$$

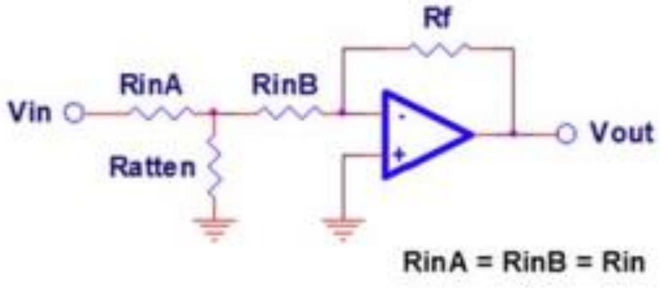


Zero offset

Positive offset

Negative offset

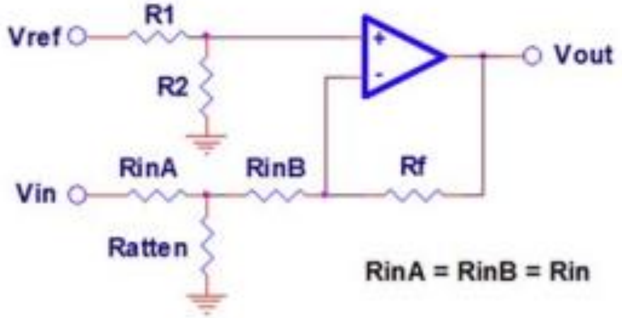
Unit 2 Inverting attenuator with zero offset, positive offset and negative offset



$$V_{out} = -m \times V_{in}$$

$$m = \frac{R_f \times R_{atten}}{R_{in} \times (R_{in} + 2 \times R_{atten})}$$

Zero offset

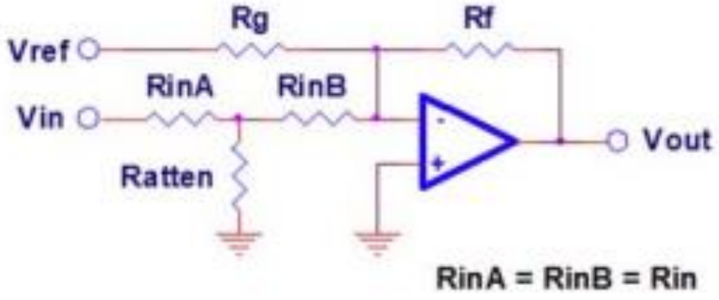


$$V_{out} = -m \times V_{in} + b$$

$$m = \frac{R_f \times R_{atten}}{R_{in} \times (R_{in} + 2 \times R_{atten})}$$

$$b = V_{ref} \times \left(\frac{R2}{R1 + R2}\right) \times \left(1 + \frac{R_f}{R_{in} + R_{in} \| R_{atten}}\right)$$

Positive offset



$$V_{out} = -m \times V_{in} - b$$

$$m = \frac{R_f \times R_{atten}}{R_{in} \times (R_{in} + 2 \times R_{atten})}$$

$$b = V_{ref} \times \frac{R_f}{R_g}$$

Negative offset

Unit 2 Development of non ideal op amp equations

- Concept of DC errors and AC errors
- Inaccuracies can be minimized using negative feedback
- Stability is usually an criteria when operating frequency is high
- Internally compensated and externally compensated to improve stability

Unit 2 Development of non ideal op amp equations

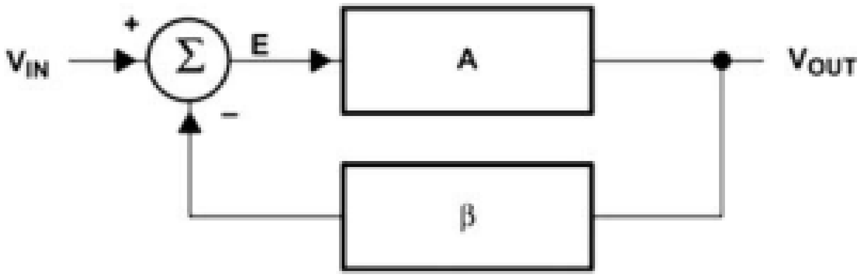


Figure 7.1
Feedback system block diagram.

$$\frac{V_{OUT}}{V_{IN}} = \frac{A}{1 + A\beta}$$

When loop gain is large,

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{\beta}$$

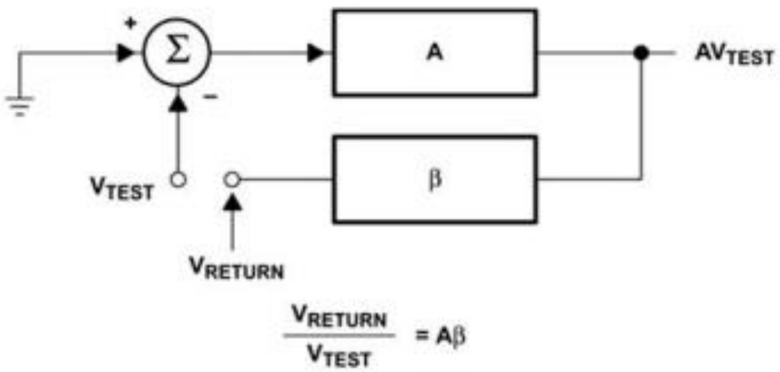


Figure 7.2
Feedback loop broken to calculate loop gain.

Error indicator, proportional to signal and inversely proportional to loop gain

$$E = \frac{V_{IN}}{1 + A\beta}$$

Unit 2 Development of non ideal op amp equations Non Inverting amp

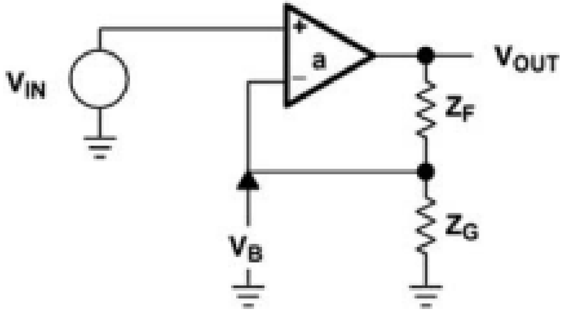


Figure 7.3
Noninverting op amp.

amplifier transfer equation.

$$V_{OUT} = a(V_{IN} \pm V_B)$$

--------- 1

V_B calculated based on resistor divider from V_{OUT}

$$V_B = \frac{V_{OUT} Z_G}{Z_F + Z_G} \quad \text{for } I_B = 0$$ --------- 2

From 1 and 2,

$$V_{OUT} = aV_{IN} - \frac{aZ_G V_{OUT}}{Z_G + Z_F}$$

After simplification,

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

In the form of closed loop function,

$$\frac{V_{OUT}}{V_{IN}} = \frac{A}{1 + A\beta}$$

loop gain $\rightarrow$ $$A\beta = \frac{aZ_G}{Z_G + Z_F}$$

Unit 2 Development of non ideal op amp equations Non Inverting amp

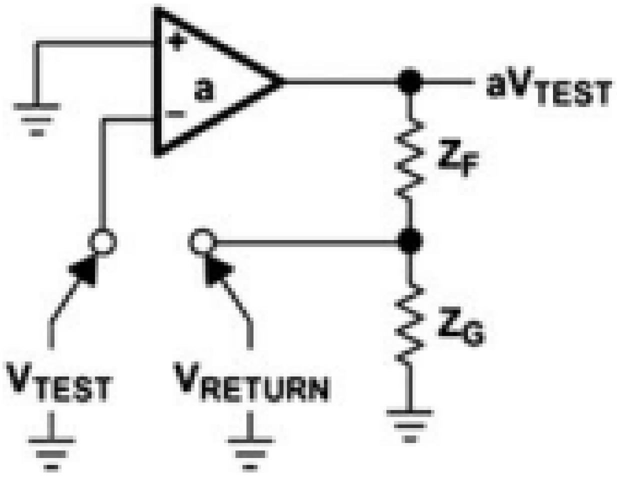


Figure 7.4
Open-loop noninverting op amp.

$$V_{RETURN} = \frac{aV_{TEST}Z_G}{Z_F + Z_G}$$

$$\frac{V_{RETURN}}{V_{TEST}} = A\beta = \frac{aZ_G}{Z_F + Z_G}$$

For measurement, break the loop, apply test signal at one end and measure voltage at the other end

Unit 2 Development of non ideal op amp equations Inverting amp

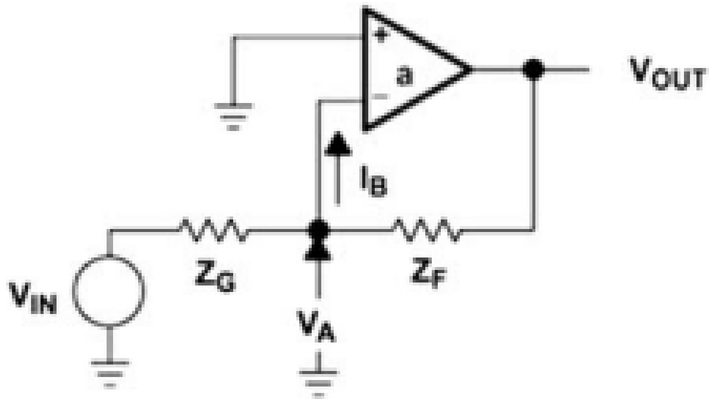


Figure 7.5
Inverting op amp.

$$V_A = \frac{V_{IN}Z_F}{Z_G + Z_F} + \frac{V_{OUT}Z_G}{Z_G + Z_F} \quad \text{for } I_B = 0$$

$$V_{OUT} = -aV_A$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

open loop gain

loop gain

- Open loop gain is different compared to non inverting amp
- Loop gain is same compared to non inverting amp

Unit 2 Development of non ideal op amp equations Inverting amp

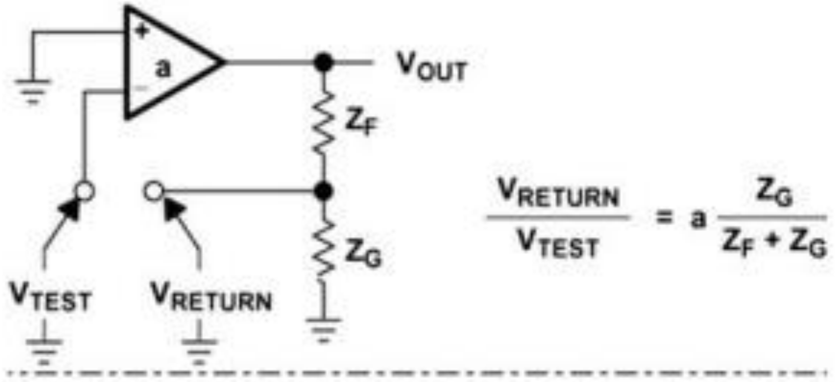


Figure 7.6
Inverting op amp: feedback loop broken for loop gain calculation.

$$\frac{V_{RETURN}}{V_{TEST}} = \frac{aZ_G}{Z_G + Z_F} = A\beta$$

For measurement, break the loop, apply test signal at one end and measure voltage at the other end

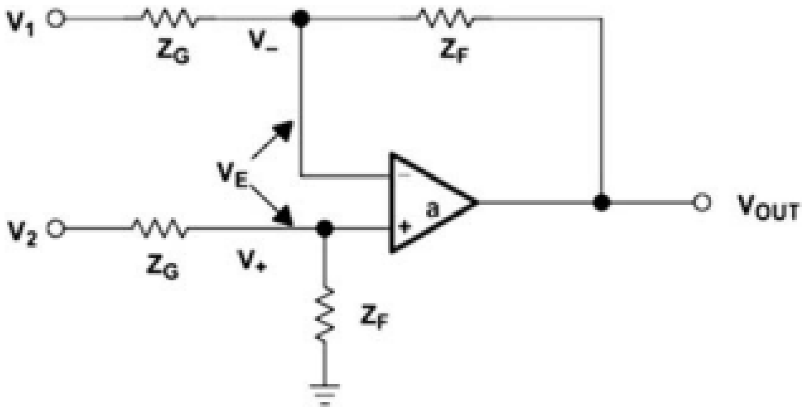


Figure 7.7
Differential amplifier circuit.

$$V_+ = V_2 \frac{Z_F}{Z_F + Z_G}$$

$$V_- = V_1 \frac{Z_F}{Z_F + Z_G} - V_{OUT} \frac{Z_G}{Z_F + Z_G}$$

: transfer equation.

$$V_{OUT} = aV_E = V_+ \pm V_-$$

$$V_{OUT} = a\left[\frac{V_2 Z_F}{Z_F + Z_G} - \frac{V_1 Z_F}{Z_F + Z_G} - \frac{V_{OUT} Z_G}{Z_F + Z_G}\right]$$

Unit 2 Development of non ideal op amp equations Differential amp

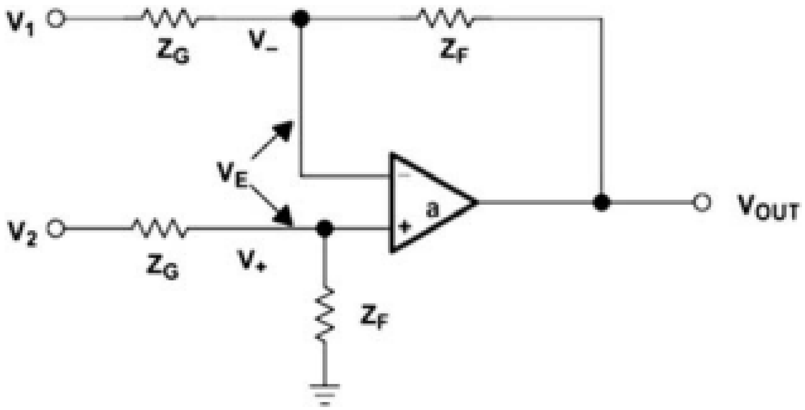


Figure 7.7
Differential amplifier circuit.

open loop gain

$$\frac{V_{OUT}}{V_2 - V_1} = \frac{\frac{aZ_F}{Z_F + Z_G}}{1 + \frac{aZ_G}{Z_F + Z_G}}$$

loop gain

- Loop gain is same compared to non inverting amp and inverting amp

Operating Region

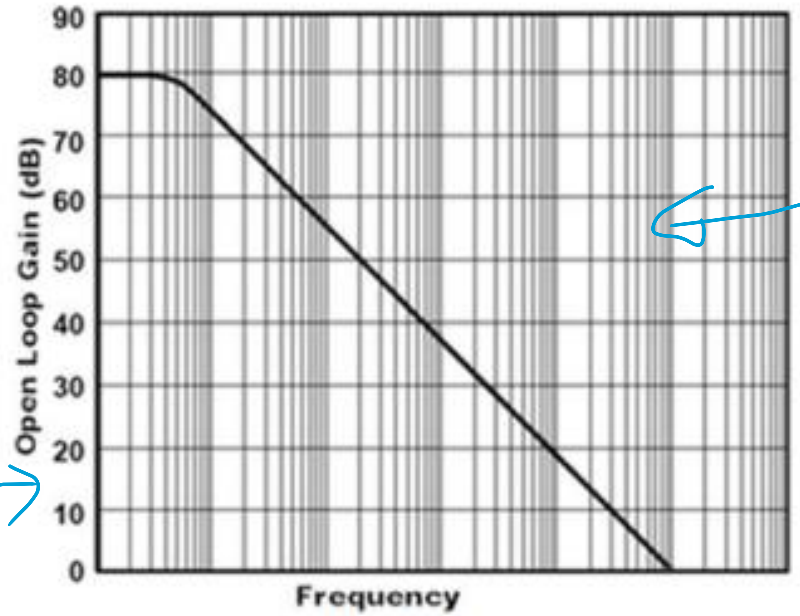


Figure 7.8
Bode response of a typical op amp.

Not an operating region

When open loop gain is high

noninverting op amp:

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

when $a \gg$. $\frac{V_{OUT}}{V_{IN}} = 1 + \frac{Z_F}{Z_G}$

When open loop gain is high

inverting op amp stage:

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

When $a \gg$. $\frac{V_{OUT}}{V_{IN}} = -\frac{Z_F}{Z_G}$

Table 7.1: Real Inverting Op Amp Stage Gains for a = 80 dB

a	R_G	R_F	Attempted	Actual	Error (%)
10,000	100,000	100,000	−1	−0.9998	−0.0200
10,000	10,000	100,000	−10	−9.9890	−0.1099
10,000	1000	100,000	−100	−99.0001	−0.9999
10,000	100	100,000	−1000	−909.0083	−9.0992
10,000	10	100,000	−10,000	−4999.7500	−50.0025
10,000	1	100,000	−100,000	−9090.8264	−90.9092
10,000	1	1.00E + 12	−1E + 12	−9999.9999	−100

Error is more for higher gain

Table 7.2: Real Noninverting Op Amp Stage Gains for a = 80 dB

a	R_G	R_F	Attempted	Actual	Error (%)
10,000	100,000	100,000	2	1.9996	−0.0200
10,000	10,000	100,000	11	10.9879	−0.1099
10,000	1000	100,000	101	99.9901	−0.9999
10,000	100	100,000	1001	909.9173	−9.0992
10,000	10	100,000	10,001	5000.2500	−50.0025
10,000	1	100,000	100,001	9090.9174	−90.9092
10,000	1	1.00E + 12	1E + 12	9999.9999	−100

operating region

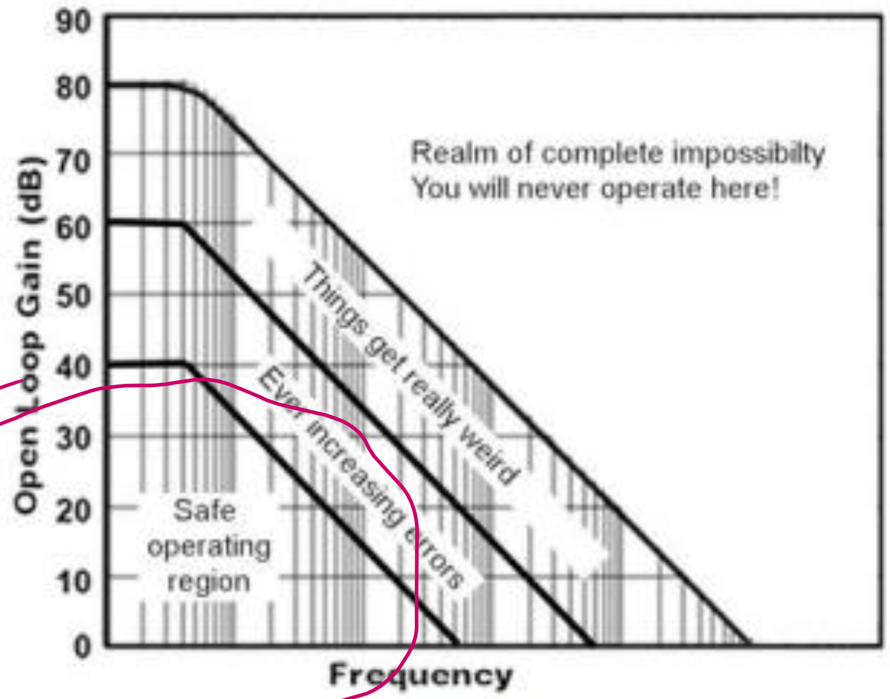


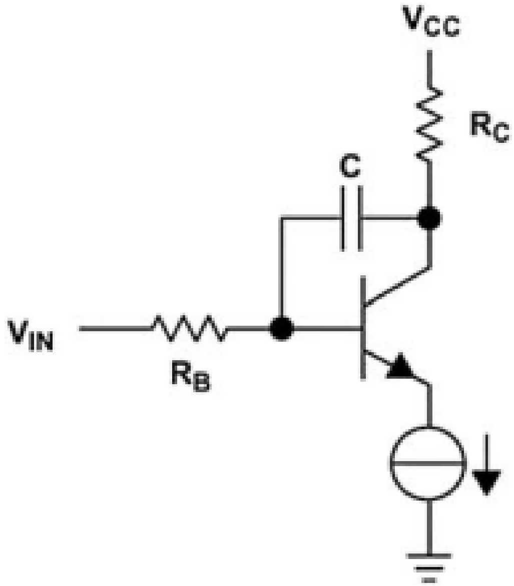
Figure 7.9

Bode response representation of safe operating region.

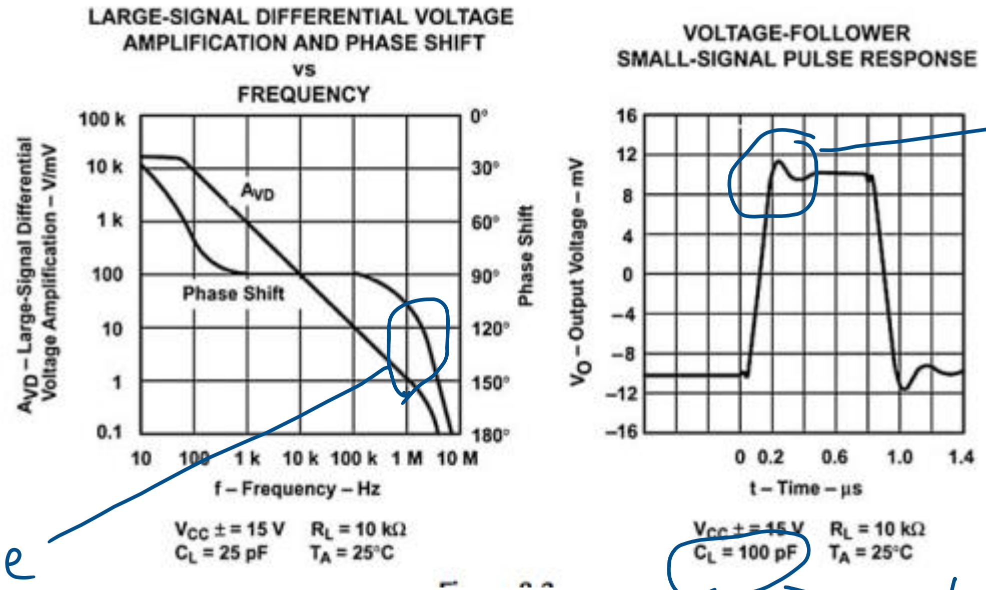
Background

- Oscillations are considered as boundary between stability and non stability
- Poor stability circuit exhibits ringing and overshoot
- Phase margin is one measure for stability
- Compensation provides patch between stability and performance
- Compensation network is by RC network

Unit 2 Voltage feedback op amp compensation Internal compensation



A capacitor C connected between input and output for compensation, called internal compensation capacitor

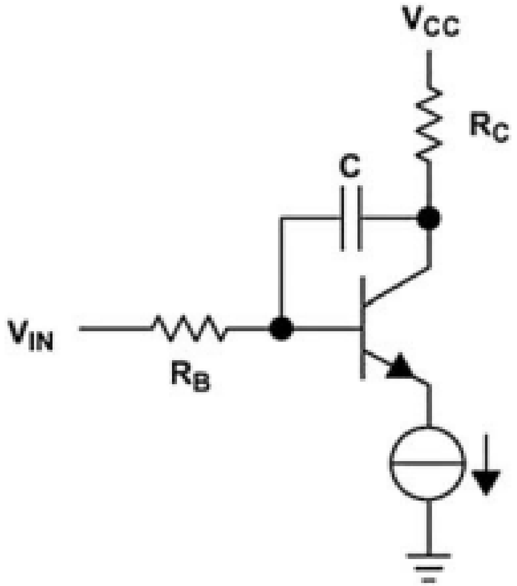


Phase margin 72°

overshoot 10%

Plot of internally compensated op amp

loading capacitor changes phase margin



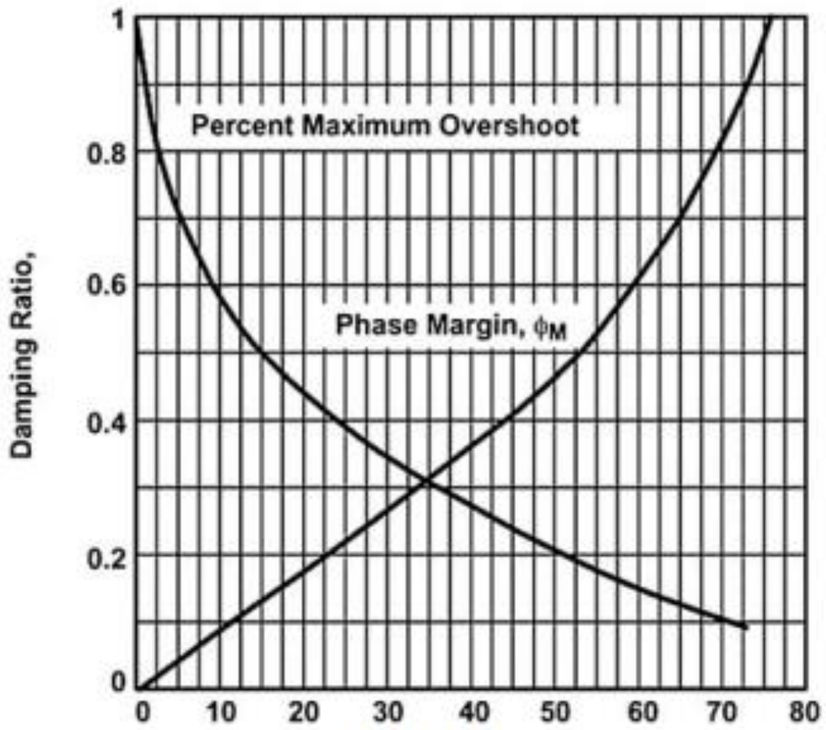


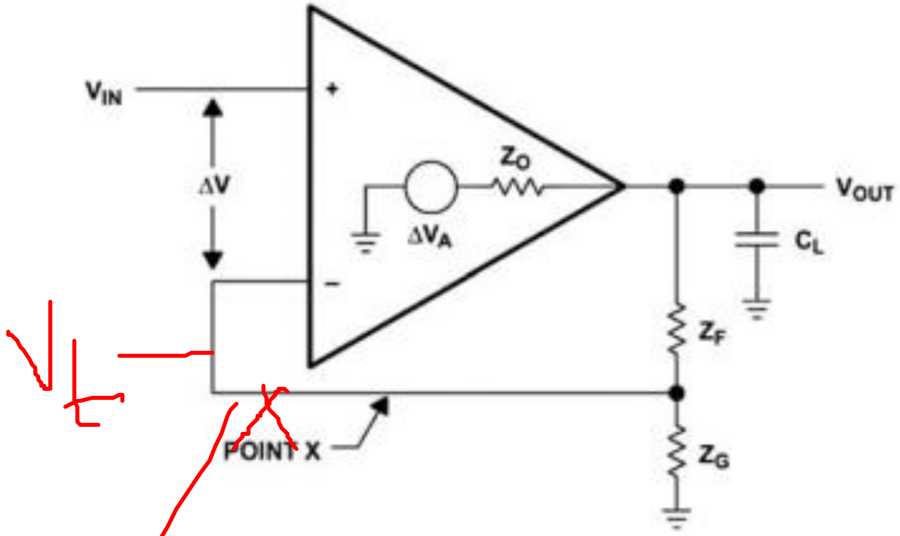
Figure 8.3
Phase margin and percent overshoot versus damping ratio.

Plot of internally compensated op amp, measure of phase margin with damping ratio and overshoot (Data sheet for TL03X)

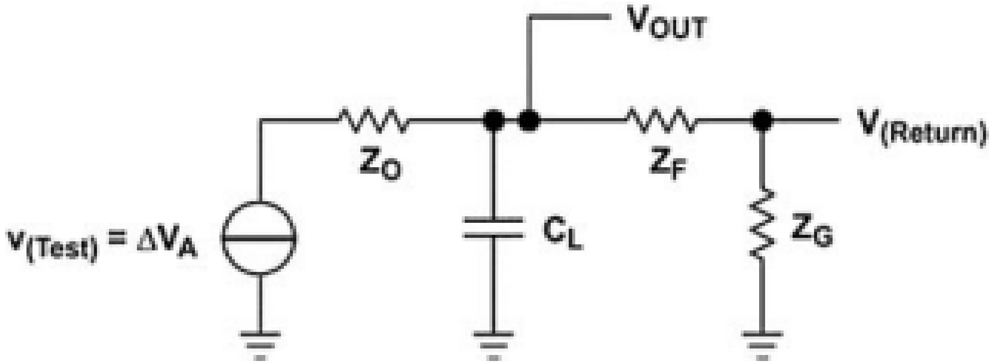
 Importance of external compensation

- High frequency noise reduction
- Improve phase margin
- Reduce overshoot
- Compensation can be tailored to the circuit requirement

- Output capacitor added
- Combination of output capacitor and output impedance forms dominant pole

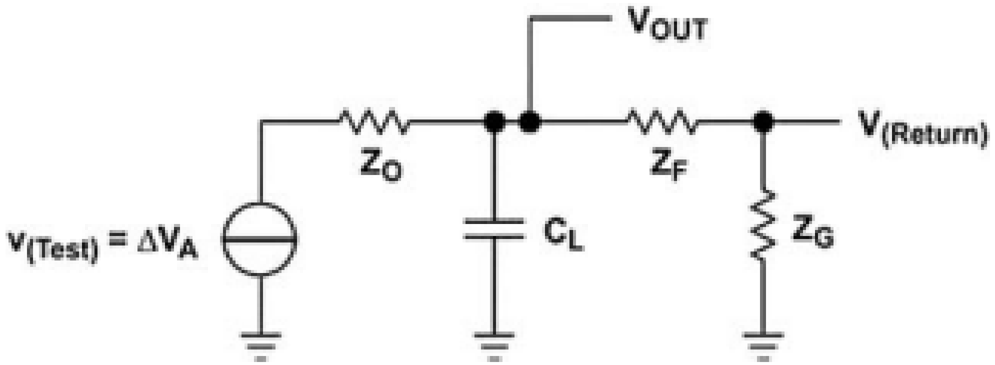


Loop broken for loop gain calculations



Equivalent circuit after loop break

Unit 2 Dominant pole compensation



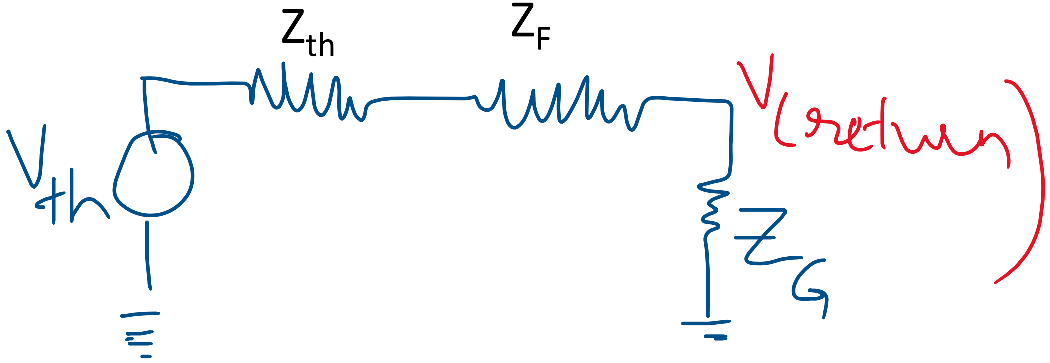
Apply Thevenin's theorem to separate Z_o and C_L

$$V_{TH} = \frac{\Delta V_a}{Z_O C_L s + 1}$$

$$Z_{TH} = \frac{Z_O}{Z_O C_L s + 1}$$

Calculate V_{return} voltage

$$V_{RETURN} = \frac{V_{TH} Z_G}{Z_G + Z_F + Z_{TH}} = \frac{\Delta V_a}{Z_O C_L s + 1}\left(\frac{Z_G}{Z_F + Z_G + \frac{Z_O}{Z_O C_L s + 1}}\right)$$

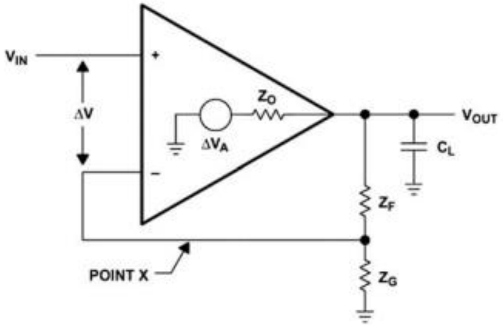


Unit 2 Dominant pole compensation

$$V_{RETURN} = \frac{V_{TH} Z_G}{Z_G + Z_F + Z_{TH}} = \frac{\Delta V a}{Z_O C_L s + 1}\left(\frac{Z_G}{Z_F + Z_G + \frac{Z_O}{Z_O C_L s + 1}}\right)$$

Rearranging

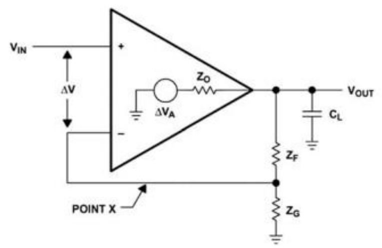
Here $\Delta V_a = a V_{test}$



$$\frac{V_{RETURN}}{V_{TEST}} = A\beta = \frac{\frac{aZ_G}{Z_F + Z_G + Z_O}}{\frac{(Z_F + Z_G)Z_O C_L s}{Z_F + Z_G + Z_O} + 1}$$

When $(Z_F+Z_O) >> Z_O$

$$A\beta = \frac{aZ_G}{Z_F + Z_G}\left(\frac{1}{Z_O C_L s + 1}\right)$$



$$\frac{V_{RETURN}}{V_{TEST}} = A\beta = \frac{\dfrac{aZ_G}{Z_F + Z_G + Z_O}}{\dfrac{(Z_F + Z_G)Z_O C_L s}{Z_F + Z_G + Z_O} + 1}$$

In case op-amp a second order system

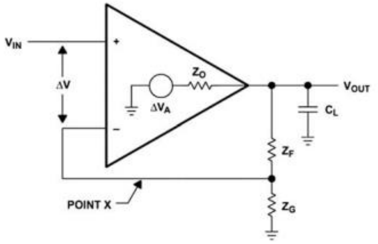
$$a = \frac{K}{(s + \tau_1)(s + \tau_2)}$$

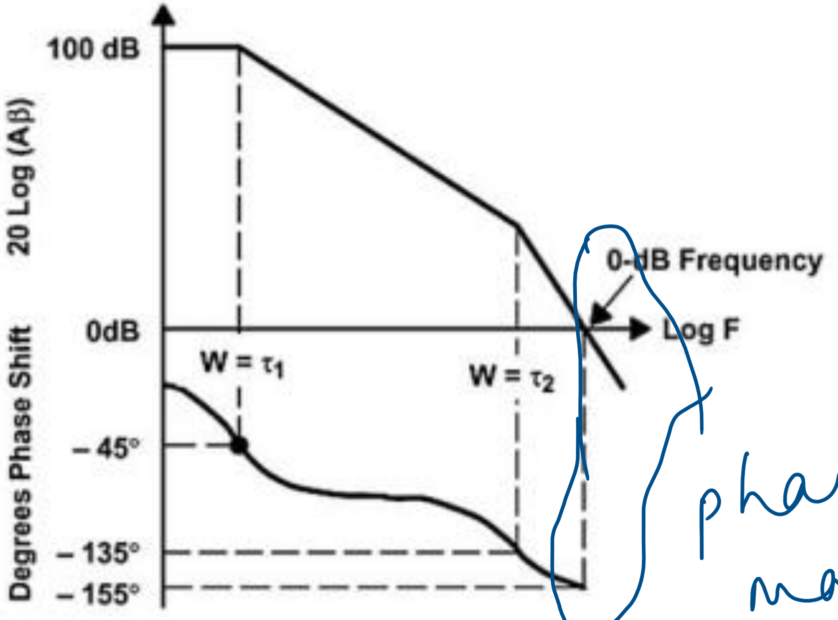
Loop gain is equal to

$$A\beta = \frac{K}{(s + \tau_1)(s + \tau_2)} \frac{Z_G}{Z_F + Z_G} \frac{1}{Z_O C_L s + 1}$$

It is a three pole System

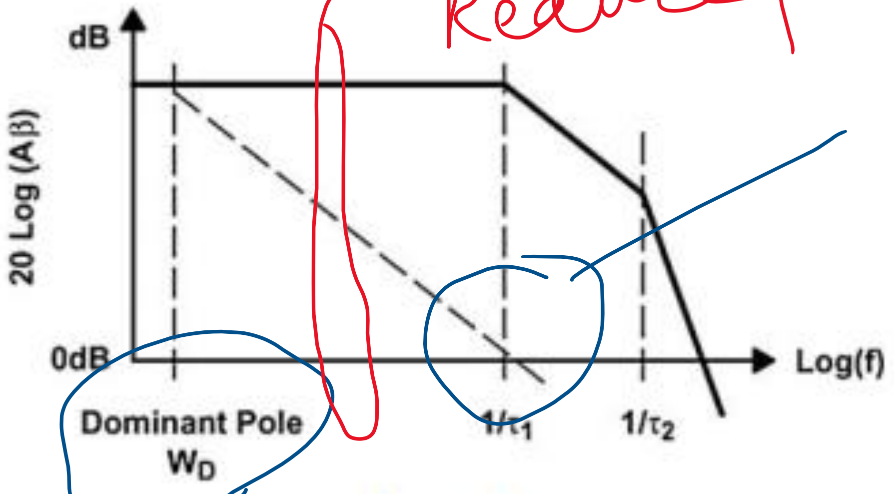
Unit 2 Dominant pole compensation





phase margin 25°

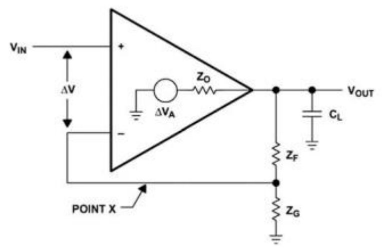
Gain Reduced



Phase margin around 45 degrees

Additional pole By compensation technique

Unit 2 Dominant pole compensation



Loop gain $$A\beta = \frac{aZ_G}{Z_F + Z_G}\left(\frac{1}{Z_O C_L s + 1}\right)$$

when $Z_0 << Z_F$

$$A\beta = \frac{aZ_G}{Z_G + Z_F}$$

Closed loop transfer function is given by

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

when $a = \infty$

$$\frac{V_{OUT}}{V_{IN}} = \frac{Z_F + Z_G}{Z_G}$$

Which represents gain of non inverting amp

Unit 2 Gain compensation

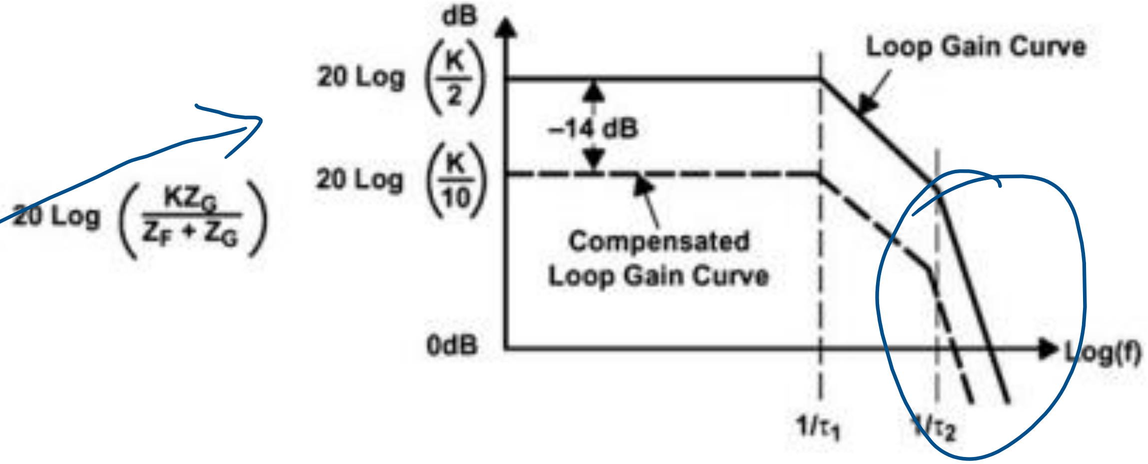
- Loop gain parameter and closed loop parameters are related

$$A\beta = \frac{aZ_G}{Z_G + Z_F}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

- Example, Change non inverting amp gain from 2 to 10

- Loop gain will reduce by -14db

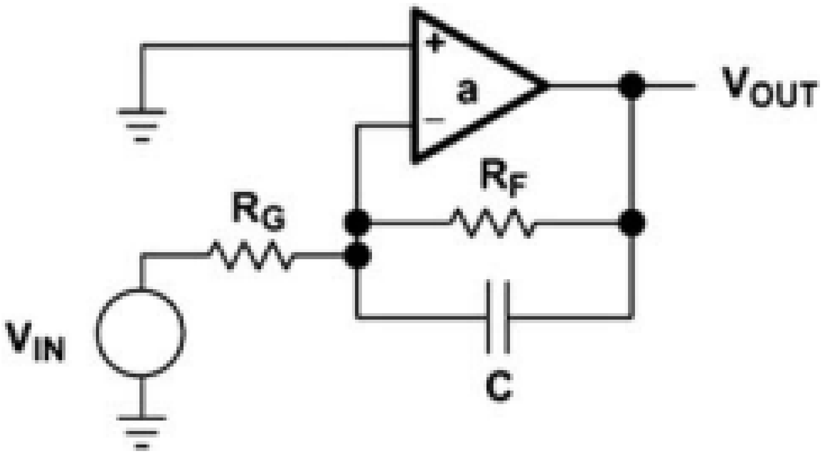


improved phase margin

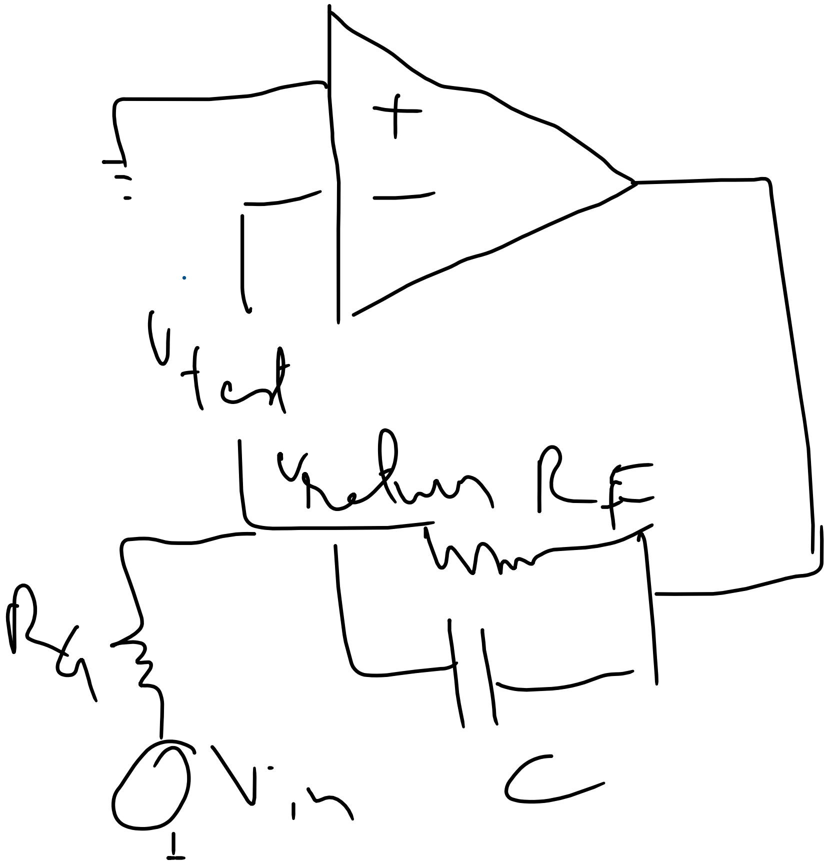
Improvement observation from the Bode plot

Unit 2 Lead compensation

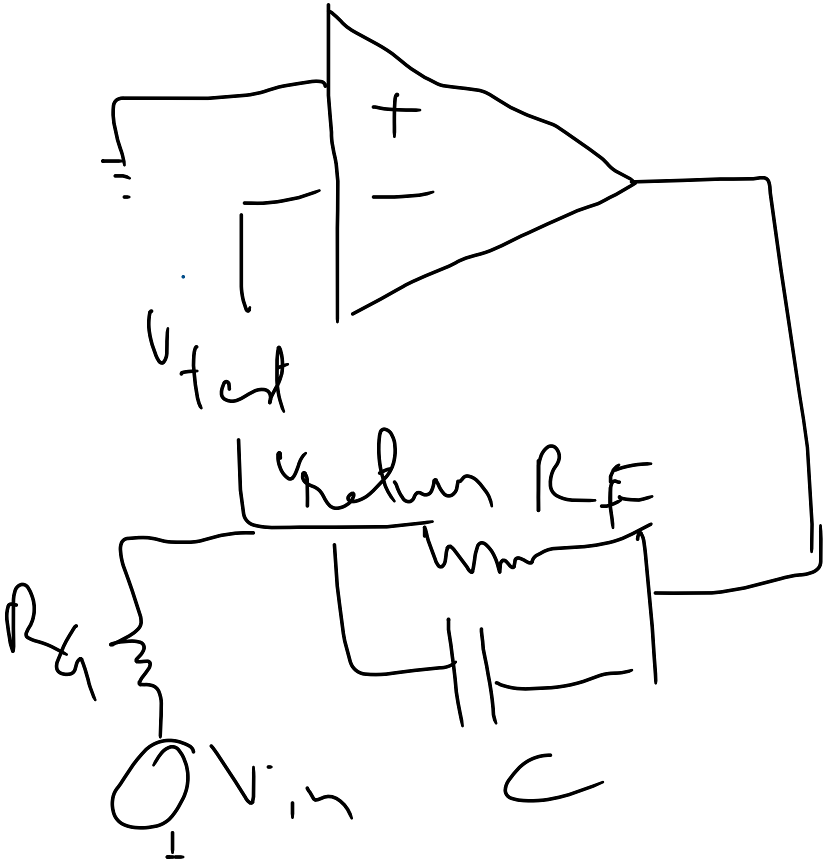
- C is because of parasitic capacitance



- Break the loop for loop gain calculation



Unit 2 Lead compensation



Contribution of open loop op amp

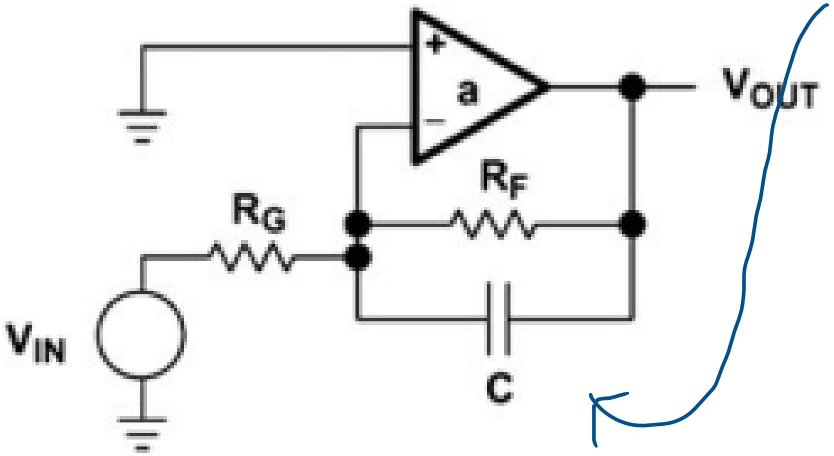
$$A\beta = \left(\frac{R_G}{R_G + R_F}\right)\left(\frac{R_F C s + 1}{R_G \| R_F C s + 1}\right)\left(\frac{K}{(s+\tau_1)(s+\tau_2)}\right)$$

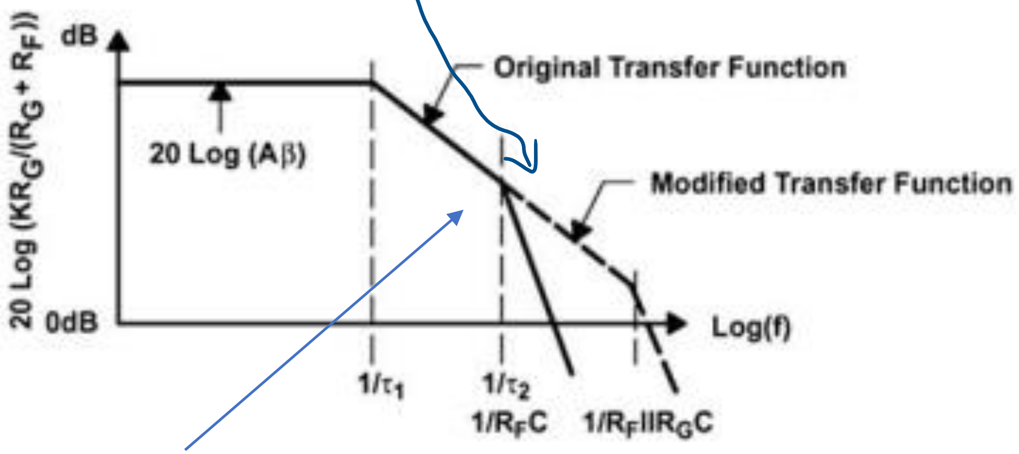
$$\frac{V_{return}}{V_{test}} = \frac{a R_G (s C R_F + 1)}{R_G (s C R_F + 1) + R_F}$$

Unit 2 Lead compensation

$$A\beta = \left(\frac{R_G}{R_G + R_F}\right)\left(\frac{R_F C s + 1}{R_G \| R_F C s + 1}\right)\left(\frac{K}{(s+\tau_1)(s+\tau_2)}\right)$$

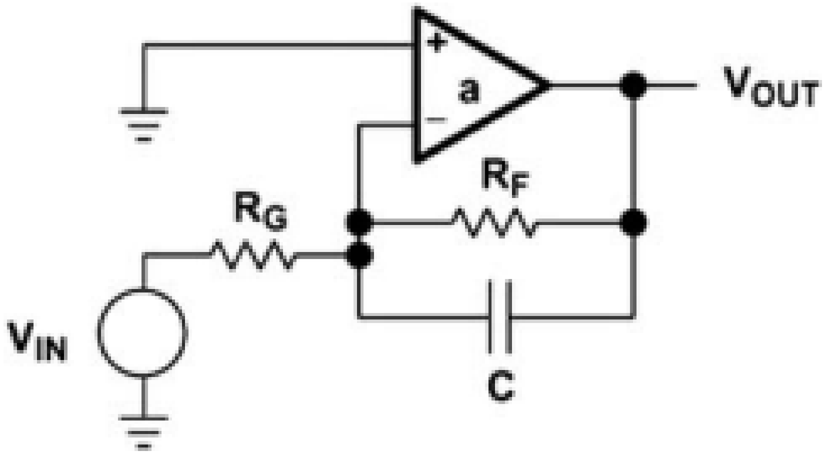
Zero introduced





- Zero placed near pole 2
- R_F has to larger compared to R_G in parallel with R_F

Unit 3 Lead compensation



Transfer function of inverting amp is given by

$$\frac{V_{OUT}}{V_{IN}} = \frac{\dfrac{-aZ_F}{Z_G + Z_F}}{1 + \dfrac{aZ_G}{Z_G + Z_F}} \qquad \text{------10}$$

$$a = \frac{K}{(s+\tau_1)(s+\tau_2)} \qquad \text{----11}$$

When **a** is infinity, transfer function can be seen as,

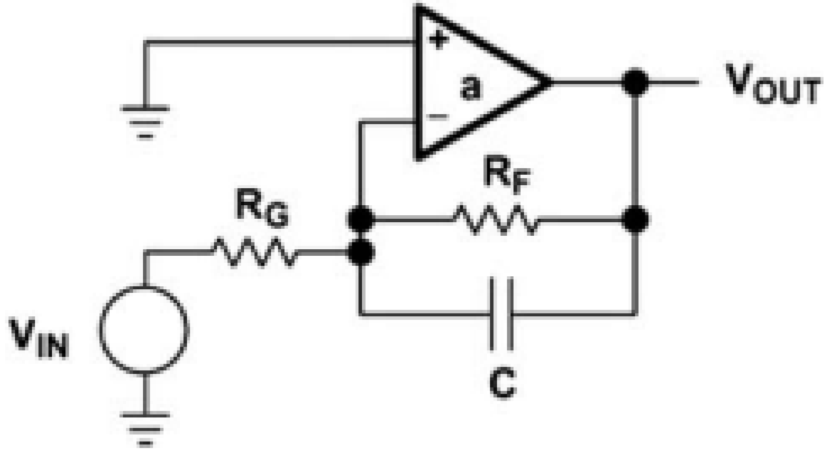
$$\frac{V_{OUT}}{V_{IN}} = -\frac{Z_F}{\cancel{Z_{IN}}} \quad Z_G$$

Substituting $R_F||C$ for Z_F and R_G for Z_G

Transfer function is given by

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_F}{R_G}\left(\frac{1}{R_F C s + 1}\right) \qquad \text{----12}$$

Unit 2 Lead compensation



Behavior on bode plot for 10, 11,12

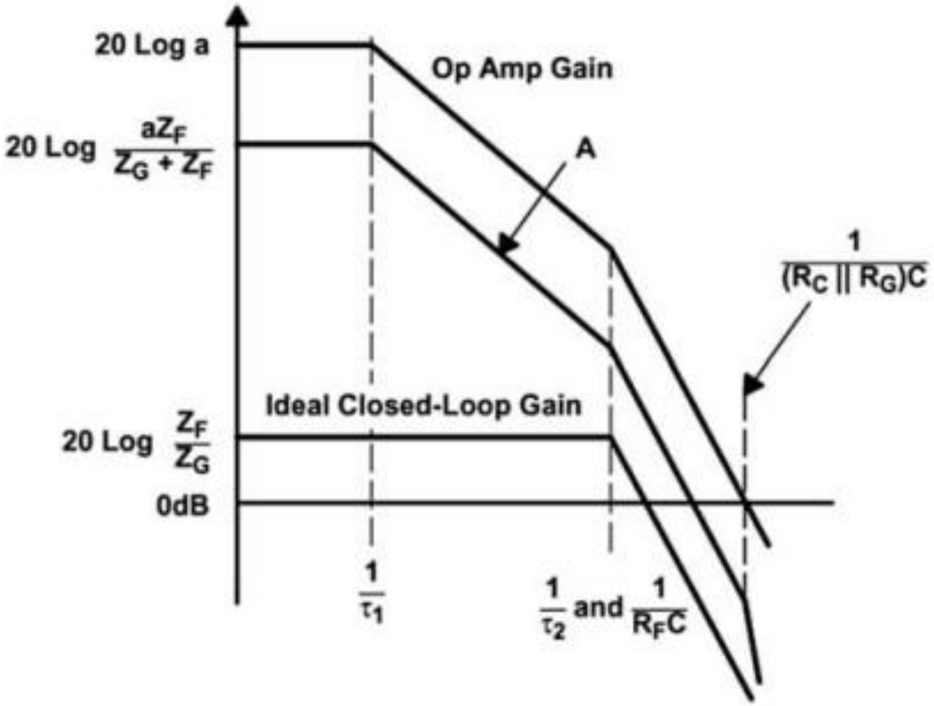
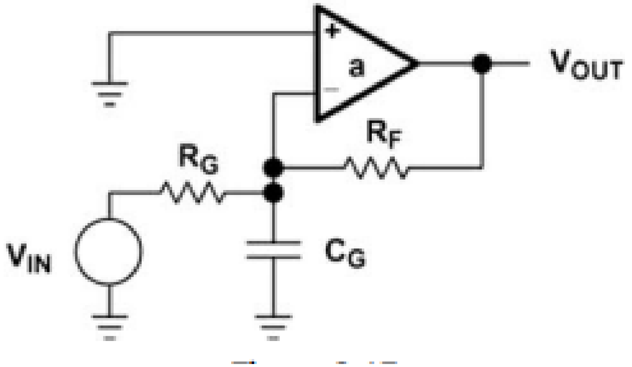


Figure 8.15
Inverting op amp with lead compensation.

Unit 2 Compensated attenuation

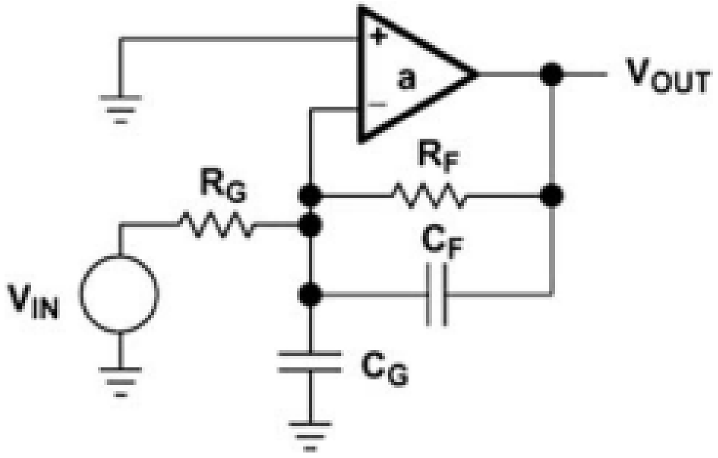
Stray capacitance is added due to PCB trace
This circuit is unstable because of three poles



Loop gain

$$A\beta = \left(\frac{R_G}{R_G + R_F}\right)\left(\frac{1}{R_G \| R_F C s + 1}\right)\left(\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}\right)$$

Compensation capacitor is added parallel to feedback resistor



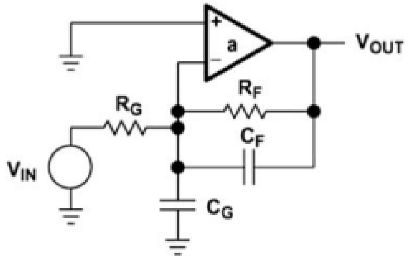
Loop gain

$$A\beta = \left[\frac{\dfrac{R_G}{R_GC_Gs+1}}{\dfrac{R_G}{R_GC_Gs+1}+\dfrac{R_F}{R_FC_Fs+1}}\right]\left(\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}\right)$$

If $R_GC_G = R_FC_F$

$$A\beta = \left[\frac{R_G}{R_G+R_F}\right]\left(\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}\right)$$

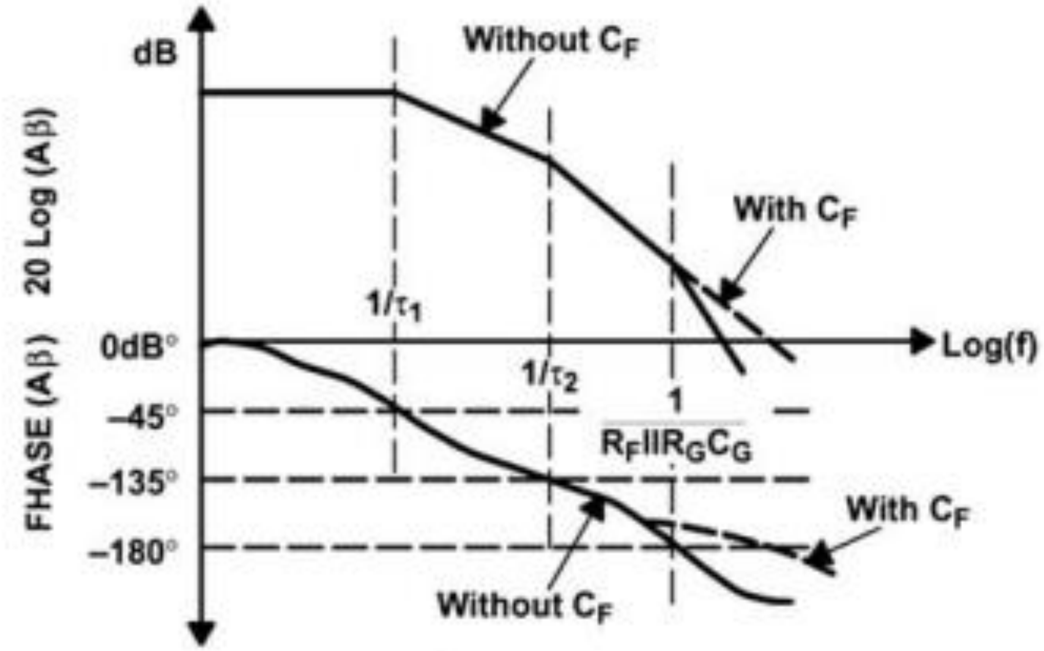
Unit 2 Compensated attenuation



Compensation capacitor is added, acts like open loop gain two pole system

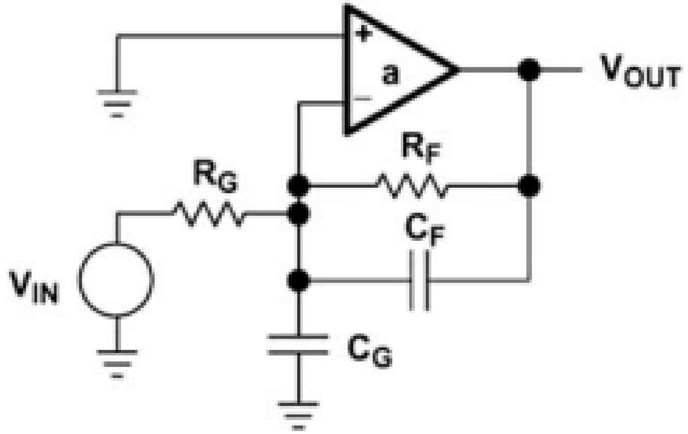
Gain plot

$$A\beta = \left[\frac{\frac{R_G}{R_G C_G s + 1}}{\frac{R_G}{R_G C_G s + 1} + \frac{R_F}{R_F C_F s + 1}}\right]\left(\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}\right)$$



Unit 2 Compensated attenuation

Closed loop gain of inverted amp does not change. Capacitor has not effect on gain



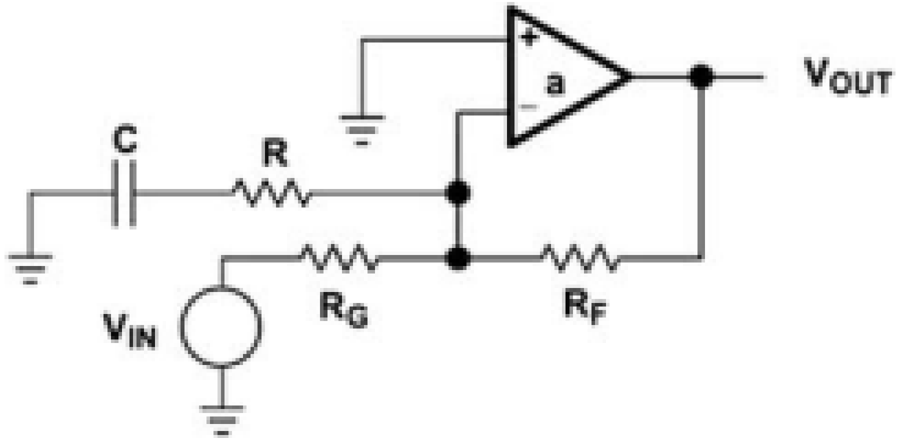
Closed loop gain

$$\frac{V_{OUT}}{V_{IN}} = \frac{\dfrac{R_F}{R_F C_F s + 1}}{\dfrac{R_G}{R_G C_G s + 1}}$$

When $R_F C_F = R_G C_G$ $\quad \dfrac{V_{OUT}}{V_{IN}} = -\left(\dfrac{R_F}{R_G}\right)$

Unit 2 Lead lag compensation

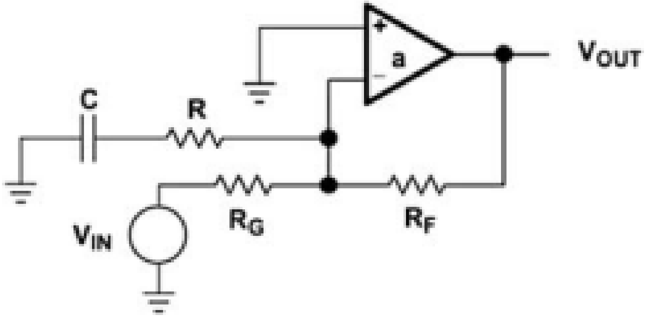
- R and C used for compensation
- Compensation circuit adds Pole and Zero



loop gain

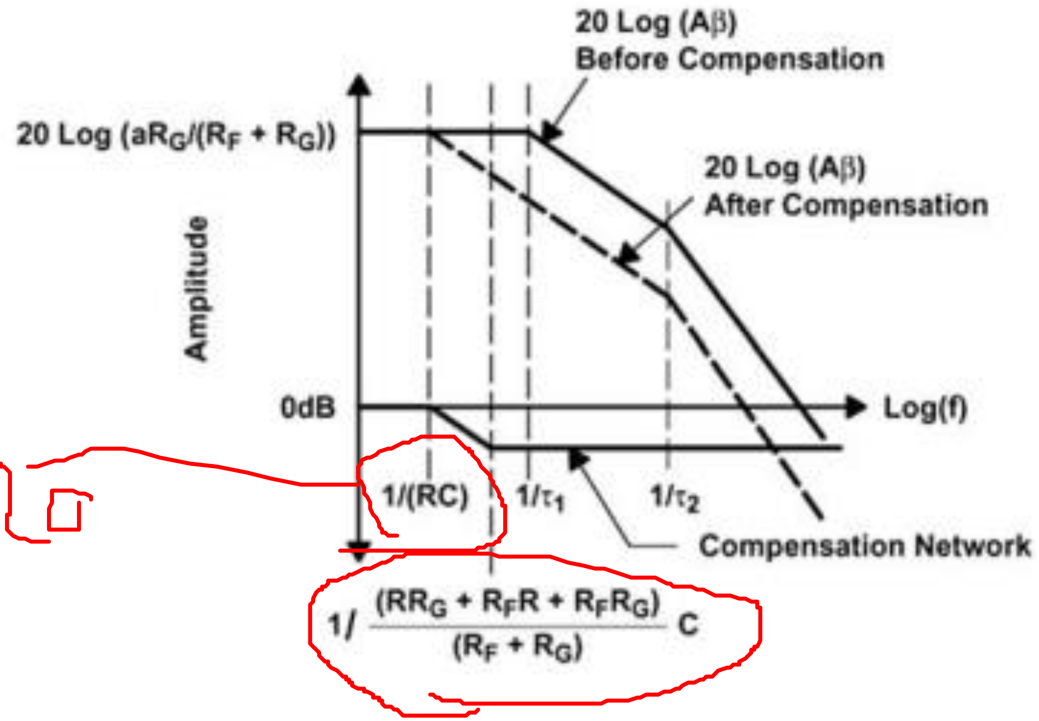
$$A\beta = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \frac{R_G}{R_G + R_F} \frac{RCs + 1}{\frac{(RR_G + RR_F + R_G R_F)}{(R_G + R_F)} Cs + 1}$$

Unit 2 Lead lag compensation



$$A\beta = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \frac{R_G}{R_G + R_F} \frac{RCs + 1}{\frac{(RR_G + RR_F + R_G R_F)}{(R_G + R_F)} Cs + 1}$$

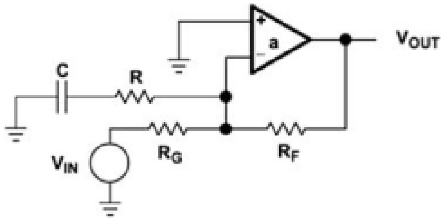
Gain plot



Zero

Pole

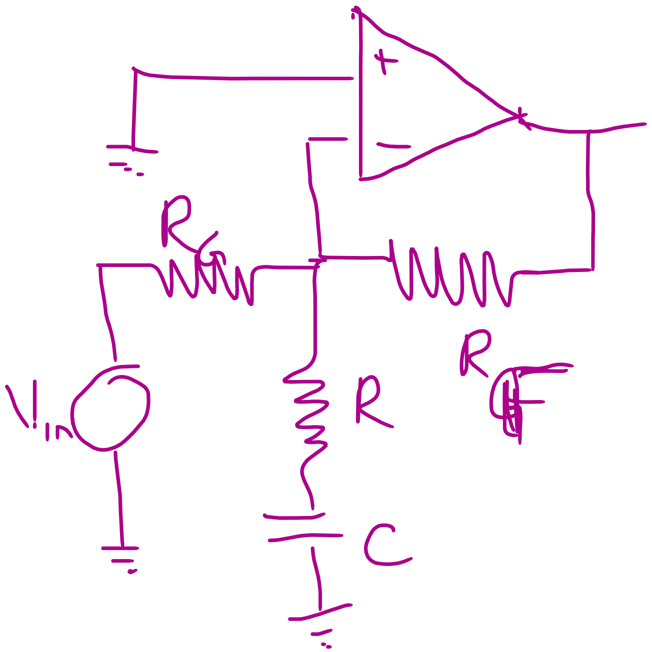
Unit 2 Lead lag compensation

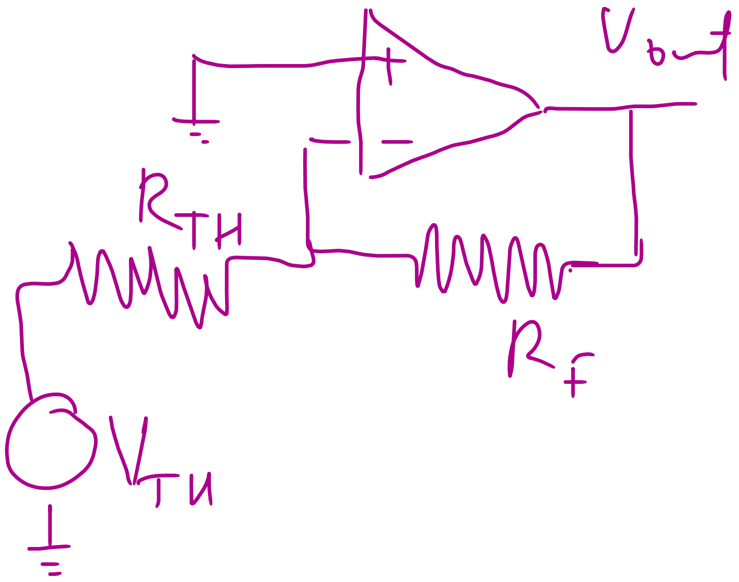


Ideal closed loop gain calculated

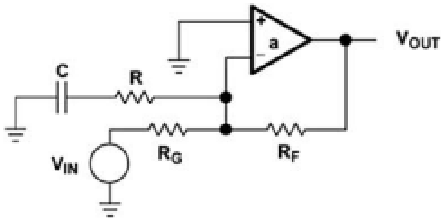
$$V_{TH} = V_{IN}\frac{R + \frac{1}{Cs}}{R + R_G + \frac{1}{Cs}}$$

$$R_{TH} = \frac{R_G\left(R + \frac{1}{Cs}\right)}{R + R_G + \frac{1}{Cs}}$$





Unit 2 Lead lag compensation

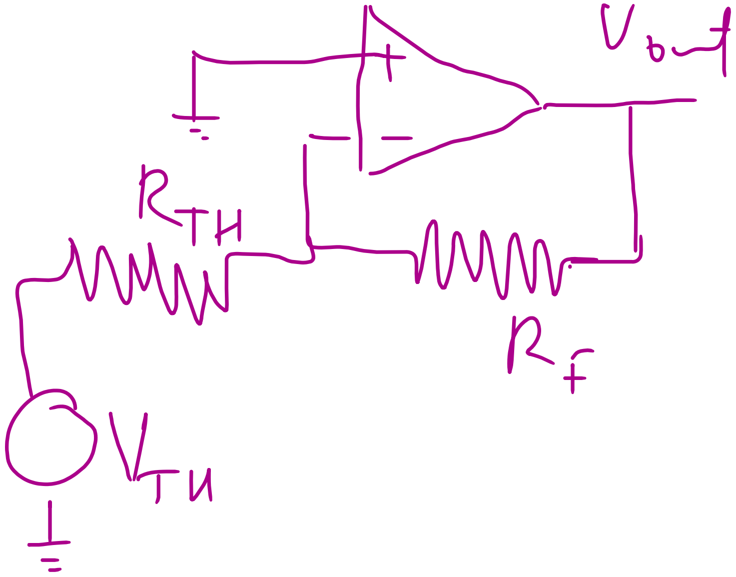


Ideal closed loop gain calculated

$$V_{OUT} = -V_{TH}\frac{R_F}{R_{TH}}$$

$$V_{TH} = V_{IN}\frac{R + \frac{1}{Cs}}{R + R_G + \frac{1}{Cs}}$$

Substitute Vth and Rth

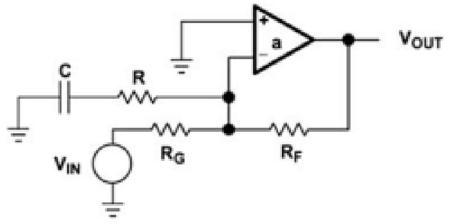


$$R_{TH}\frac{R_G\left(R + \frac{1}{Cs}\right)}{R + R_G + \frac{1}{Cs}}$$

$$-\frac{V_{OUT}}{V_{IN}} = \frac{R + \frac{1}{Cs}}{R + R_G + \frac{1}{Cs}}\frac{R_F}{\frac{R_G\left(R + \frac{1}{Cs}\right)}{R + R_G + \frac{1}{Cs}}} = \frac{R_F}{R_G}$$

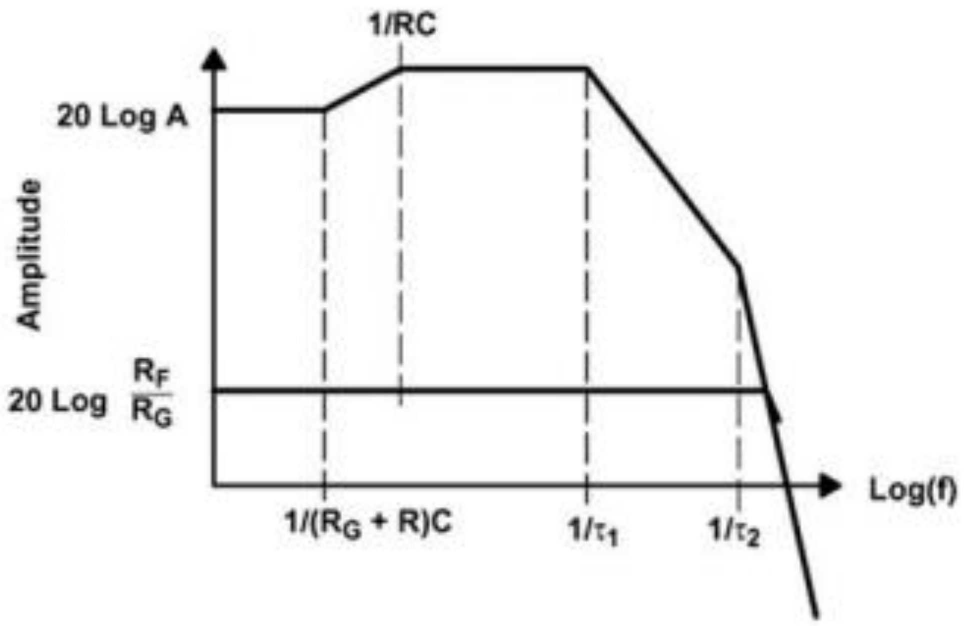
gain remains same

Unit 2 Lead lag compensation



$$A\beta = \frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \frac{R_G}{R_G+R_F} \frac{RCs+1}{\frac{(RR_G+RR_F+R_GR_F)}{(R_G+R_F)}Cs+1}$$

$$-\frac{V_{OUT}}{V_{IN}} = \frac{R_F}{R_G}$$



Unit 3 Comparison of compensation schemes

Scheme	Advantages	Disadvantages
Internal compensation	No need for extra component	Under certain load capacitance, it is unstable
Dominant pole compensation	Suitable for high load capacitance	Load capacitance make op amp to ring
Gain compensation	Good in terms of stability	Gain reduces
Lead compensation	Increases bandwidth	Reduces closed loop gain
Compensated attenuator	Useful scheme when stray capacitance seen at inverting input	Needs matching two RC time constants
Lead lag compensation	Increased bandwidth	More external components

Textbook :

Op Amp for Everyone : Bruce Carter and Ron Mancini Fifth Edition 2017

THANK YOU

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