Voltage reflection coefficient.

— "ratio of reflected voltage to incident voltage"

Wave propagating in +ve z dirⁿ : $V_0^+ e^{-\gamma z}$ (INCIDENT WAVE)

in opp. dirⁿ : $V_0^- e^{\gamma z}$ (REFLECTED WAVE)

1) AT LOAD, that's γl

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{V_0^-}{V_0^+} e^{2\gamma l}$$

Using ③ & ④ for V_0^+ & V_0^-

$$\Gamma_L = \frac{\cancel{1/2}\,(V_L - I_L Z_0)}{\cancel{1/2}\,(V_L + I_L Z_0)} \quad \div \text{ by } I_L$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

① → When load is perfectly matched i.e $Z_L = Z_0$ load is matched to characteristic impedance.

$\boxed{\Gamma_L = 0}$.

In general, Γ_L is a complex quantity (bcos of Z_0).

② → when load is short ckted, $\Gamma_L = -1$ $(Z_L = 0)$

③ → When load is open ckted, $Z_L = \infty$, $\Gamma_L = 1$

$$\frac{Z_L\left(1 - \frac{Z_0}{Z_L}\right)^{0}}{Z_L\left(1 + \frac{Z_0}{Z_L}\right)^{0}} = 1$$

$\boxed{0 \leq |\Gamma_L| \leq 1}$

For perfect mismatch — short ckt or open ckt!

that's $y\ \gamma z$

2) AT ANY z,

$$\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}}$$

instead of using ③ & ④

$$= \frac{V_0^-}{V_0^+} \cdot e^{2\gamma z} = \gamma_L e^{-2\gamma l} e^{2\gamma z}$$

Replace $z = l - l'$ from load also.

$$\boxed{\Gamma(z) = \gamma_L e^{-2\gamma l'}}$$

Q 1) A 50Ω transmission line of length l is open circuited. If the input impedance is $-J62\ \Omega$, determine l in terms of λ.

Sol. $Z_{in} = \cancel{50\Omega}\ -J62\ \Omega$; $Z_L = \infty$; $\alpha = 0$

$$Z_{in} = \frac{Z_0 [Z_L + Z_0 \tanh(J\beta l)]}{Z_0 + Z_L \tanh(J\beta l)}$$

$\beta = \frac{2\pi}{\lambda}$

Take out Z_L;

$$= \frac{Z_0 \left(1 + \frac{Z_0}{Z_L} \tanh \gamma l\right)}{\frac{Z_0}{Z_L} + \tanh(J\beta l)}$$

$$Z_{in} = \frac{Z_0}{J\tanh(\gamma l)}$$

$$Z_{in} = -JZ_0 \coth(\gamma l)$$

$$J62 = -J50 \cot(\gamma l)$$

$$Z_{in} = -JZ_0 \cot(\beta l) \quad \because \alpha = 0$$

$$-J62 = -J50 \cot(\beta l)$$

$\tan \beta l = \frac{50}{62}$

use β in c

$\tan^{-1}() = 0.6786 = \beta l$

$0.6786 \Rightarrow \frac{2\pi}{\lambda} \cdot l$

$l = \frac{0.678\lambda}{2\pi}$

Current reflection coefficient

– "ratio of reflected current to incident current". 9.11.23

– negative of voltage reflection coefficient

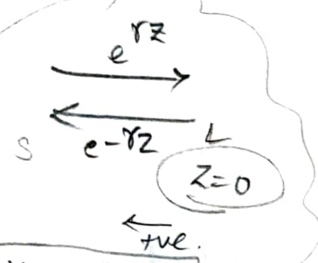
AT LOAD,

from z=0 $e^{-\gamma}$

$$= \frac{I_0^- e^{+\gamma l}}{I_0^+ e^{-\gamma l}} = \frac{\dfrac{-V_0^-}{\cancel{Z_0}} e^{\gamma l}}{\dfrac{V_0^+}{\cancel{Z_0}} e^{-\gamma l}}$$

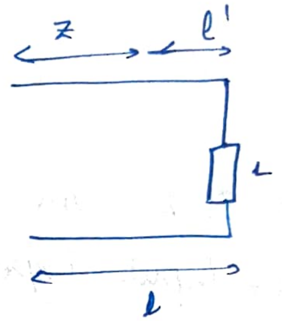
volt. r. c.

$$= \boxed{-\Gamma_L}$$



incident : S to L
reflected : L to S

☆☆



l' is the distance from the load.

$$V(l') = V_L^+ e^{\gamma l'} + V_L^- e^{-\gamma l'}$$

$$I(l') = I_L^+ e^{\gamma l'} + I_L^- e^{-\gamma l'}$$

V_L^+ & V_L^- are the incident voltages at the load.

$$\Gamma(l') = \left(\frac{V_L^-}{V_L^+}\right) \frac{e^{-\gamma l'}}{e^{\gamma l'}}$$

$$\boxed{\Gamma(l') = \Gamma_L e^{-2\gamma l'}}$$

⇒ Reflection coefficient at any z (from load end), (prev page)

$$\Gamma(z) = \Gamma_L e^{-2\gamma z}$$

$$= \Gamma_L e^{-2(\alpha + j\beta) z}$$

$$= \Gamma_L e^{-2\alpha z} \cdot e^{-j2\beta z}$$

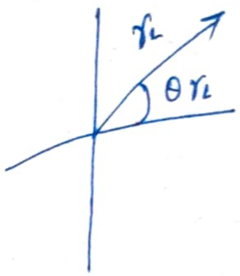
wk, $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$; can be complex; so mag. & dir^n.

$$\Gamma(z) = |\Gamma_L| e^{j\theta_{\Gamma_L}} \cdot e^{-2\alpha z} \cdot e^{-j2\beta z}$$

contributes to magnitude

$$\Gamma(z) = |\Gamma_L| e^{-2\alpha z} \cdot e^{j(\theta_L - 2\beta z)}$$

γ_L is a phasor, mag n dir^n



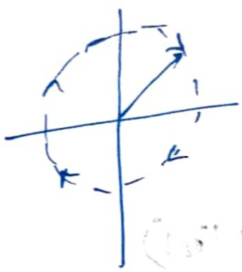
1) For ~~lossless~~ general line, ~~($\alpha \neq 0$)~~

As z ↑ses,

magnitudes ↓es

phase ↓ses clockwise, spiralling (rotates)

2) For lossless line, $(\alpha = 0)$



As z ↑ses,

magnitude of phasor remains same. (for all values of z)

angle rotates clockwise

$$\Rightarrow \gamma(z) = |\gamma_L| e^{j(\theta_{\gamma_L} - 2\beta z)}$$

For this phasor to complete one revolution, what will be the distance travelled? $2\beta z$ is the term deciding where it'll be there?

$$2\beta z_0 = 2\pi$$

$$\beta = \frac{2\pi}{\lambda}$$

$$2 \cdot \frac{2\pi}{\lambda} \cdot z_0 = 2\pi$$

$$\boxed{z_0 = \frac{\lambda}{2}}$$

1 complete revolution is equal to a distance $\frac{\lambda}{2}$.

(Important for Smith chart!)

→ Standing wave : got by ~~sketching~~ sum of voltages of Inc & Ref waves

~~It is~~ "Standing wave pattern is obtained by plotting amplitude of resultant of voltages corresponding to incident and reflected waves on a lossless line". $\alpha = 0$

b/cos $\frac{V_0^-}{V_0^+} = \Gamma_L$

$$V(z) = V_0 \sin(\omega t + \beta z) + V_0 \Gamma_L \sin(\omega t - \beta z)$$

$$= \underbrace{V_0 \sin(\omega t + \beta z)}_{\text{Incident}} + \underbrace{V_0 |\Gamma_L| \sin(\omega t - \beta z + \theta_{\gamma_L})}_{\text{Reflected}}$$

→ for waveform:

bcos reflected has extra θ_{γ_L}

arrows: direction of rotation of

Incident & Reflected
↓ Anticlockwise — sin(z↑) angle ↑ses
↓ Clockwise — sin(−βz↓) angle ↓ses

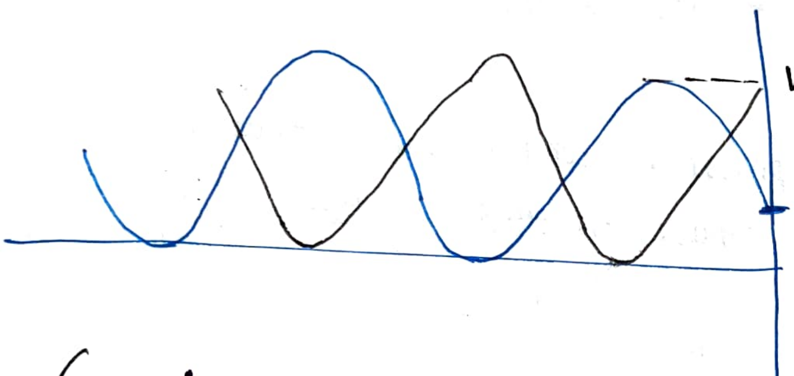
Sketching sum of resultant (standing wave defⁿ):

→ $V(z) = V_0 \sin(\omega t + \beta z) + V_0 |\gamma_L| \sin(\omega t - \beta z + \theta_{\gamma_L})$

$$V_{max} = V_0 [1 + |\gamma_L|]$$

$$V_{min} = V_0 [1 - |\gamma_L|]$$

Corresponding to voltage.



(- for current: b cos expⁿ for I(z) there is −ve)

$[I(z) = I_0^+ \sin(\omega t + \beta z) + I_0^- \sin(\omega t - \beta z + \theta)]$

→ V_{max} : when both in phase → Addⁿ of incid + refl.

V_{min} : when both out of phase

VSWR — Voltage Standing Wave Ratio. - ratio of maxᵐ voltage to minᵐ voltage.

denoted as S (our TB) / ρ (other TB)

$$S = \frac{V_{max}}{V_{min}} = \frac{\cancel{V_0}[1 + |\gamma_L|]}{\cancel{V_0}[1 - |\gamma_L|]}$$

$$\boxed{S = \frac{1 + |\gamma_L|}{1 - |\gamma_L|}}$$

; in terms of reflection coefficient

If $|\gamma_L| = 0$, $S = 1$

$|\gamma_L| = 1$, $S = \infty$

} All lossless

i) **Short circuited load** $Z_L = 0$

$$\boxed{\gamma_L} = \frac{Z_L - Z_0}{Z_L + Z_0} = \boxed{-1}$$

inc. & refl. are out of phase :

$-V_0$ V_0

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

$$Z_{sc} = \boxed{Z_{in} = jZ_0 \tan(\beta l)}$$

If length of transmission line is varying from 0 to $\lambda/4$

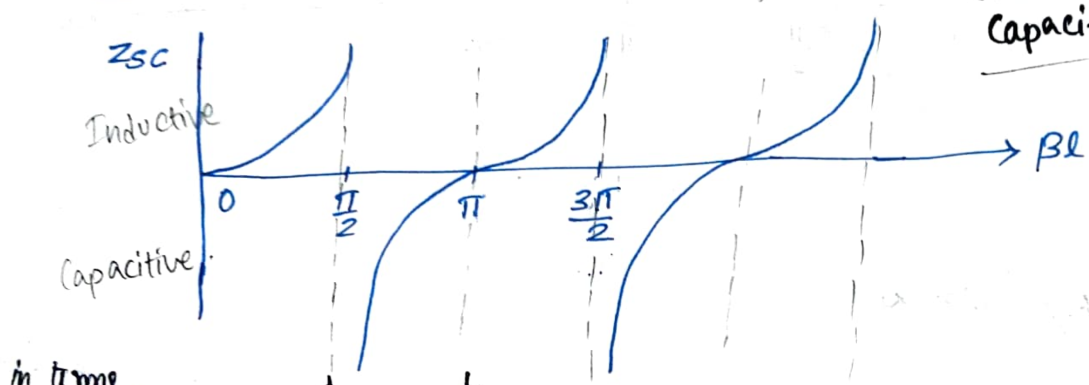
$l \to 0$ to $\frac{\lambda}{4}$; $Z = j0$ to $j\infty$

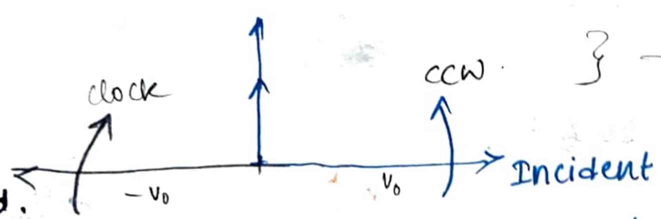
$l \to \frac{\lambda}{4}$ to $\frac{\lambda}{2}$; $Z = -j\infty$ to $j0$.

Inductive reactance

Capacitive reactance

→ Z_{sc} vs βl :





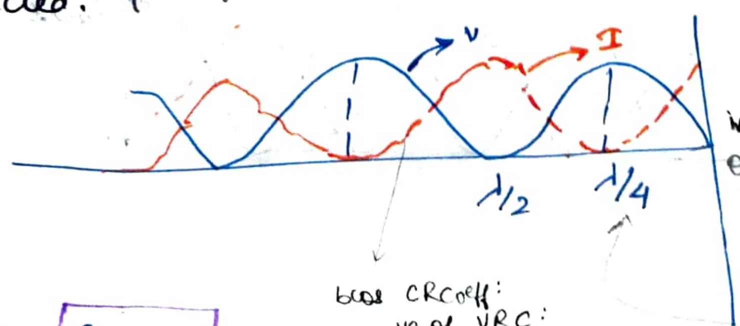
} Total phase $= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

when both travelled $\pi/2$ for maxm.

First Maxima $= \frac{\lambda}{4}$

First Minima $= 0$

Voltage standing wave :



in terms of wavelength

$2\beta Z_0 = \pi$

$2 \cdot \frac{2\pi}{\lambda} Z_0 = \cancel{\pi}$

$$\boxed{Z_0 = \frac{\lambda}{4}}$$

bcoz CR coeff : $-$ve of VRC : $\gamma_L = 1$

• VSWR ; $\boxed{S = \infty}$

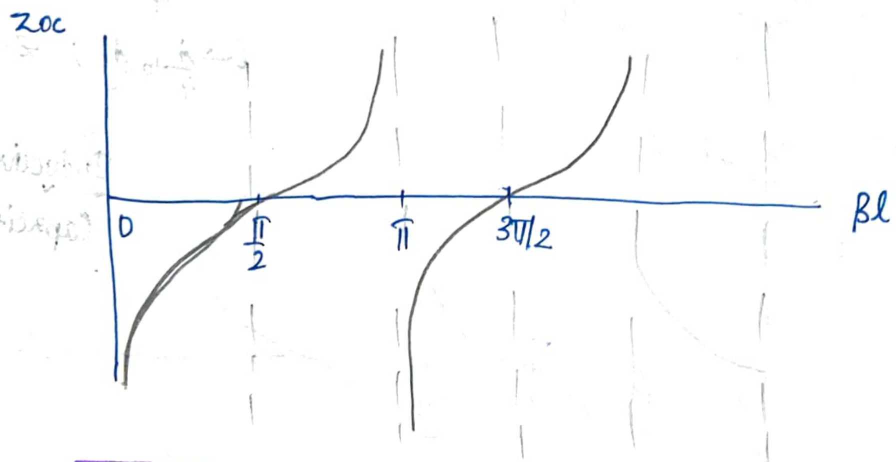
2) Open circuited load: $Z_L = \infty$

$$\boxed{\Gamma_L = 1}$$

$$Z_{oc} = Z_{in} = Z_0 \cdot \frac{Z_L}{Z_L}\left[\frac{1 + j\cancel{\frac{Z_0}{Z_L}}^{\,0}\tan(\beta l)}{\cancel{\frac{Z_0}{Z_L}}^{\,0} + j\tan(\beta l)}\right]$$

$$\boxed{Z_{oc} = Z_{in} = \frac{Z_0}{j\tan\beta l}} = \underline{-jZ_0\cot\beta l}.$$

Z_{oc} vs βl:



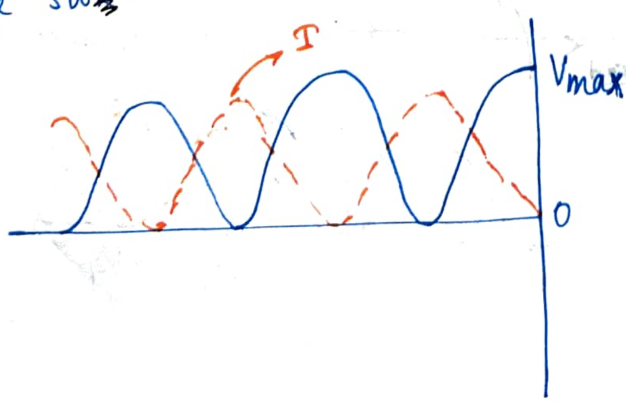
• VSWR; $\boxed{S = \infty}$

16.11.'23

• $\Gamma_L = 1$, $S = \infty$

↳ means if inc. wave $\longrightarrow$ dirⁿ, then refl. wave also same dirⁿ $\longrightarrow$ ccw, cw, so addⁿ. (starts at maxima)

Voltage SW:



$\Rightarrow V_0(1+|\Gamma_L|)$
$= V_0(1+1)$
$= \underline{\underline{2V_0}}$

3) Matched load :

$Z_L = Z_0$

$\boxed{\Gamma_L = 0}$ → Reflection coefficient;

So that means THERE's NO REFLECTION.

∴ No standing wave Pattern.

Ideally preferred.

cos during reflection; "there's loss of signal".

$\boxed{Z_{in} = Z_0}$ ← Substitute in eqn.

• VSWR; $\boxed{S = 1}$

Average Power

$$\boxed{P_{av} = V_{rms} I_{rms} \cos\theta}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$= \frac{1}{2}(V_m I_m \cos\theta)$$

$$P_{av} = \frac{1}{2} Re\{V_s(l) . I_s^*(l)\}$$

+ cos from load end.

for a lossless line : ($\alpha = 0$)

$$= \frac{1}{2} Re\{(V_0^+ e^{J\beta l} + \Gamma_L V_0^+ e^{-J\beta l})(I_0^+ e^{J\beta l} + \Gamma_L I_0^+ e^{-J\beta l})^*\}$$

$$= \frac{1}{2} Re\{(V_0^+ e^{J\beta l} + \Gamma_L V_0^+ e^{-J\beta l})\left(\frac{V_0^+}{Z_0} e^{J\beta l} - \Gamma_L \frac{V_0^+}{Z_0} e^{-J\beta l}\right)^*\}$$

For lossless line, Z_0 = Real

$$= \frac{1}{2} \frac{|V_0^+|^2}{Z_0} . Re\{(e^{J\beta l} + \Gamma_L e^{-J\beta l})(e^{J\beta l} - \Gamma_L e^{-J\beta l})^*\}$$

$$= \frac{1}{2} \frac{|V_0^+|^2}{Z_0} . Re\{(e^{J\beta l} + \Gamma_L e^{-J\beta l})(e^{-J\beta l} - \Gamma_L^* e^{J\beta l})\}$$

$$= \frac{1}{2} \frac{|V_0^+|^2}{Z_0} . Re\{e^{J\beta l}(e^{-J\beta l} - \Gamma_L^* e^{J\beta l}) + \Gamma_L e^{-J\beta l}(e^{-J\beta l} - \Gamma_L^* e^{J\beta l})\}$$

$$= \frac{|V_0^+|^2}{2Z_0} Re\{1 + \underbrace{\Gamma_L e^{-J2\beta l} - \Gamma_L^* e^{J2\beta l}}_{} - |\Gamma_L|^2\}$$

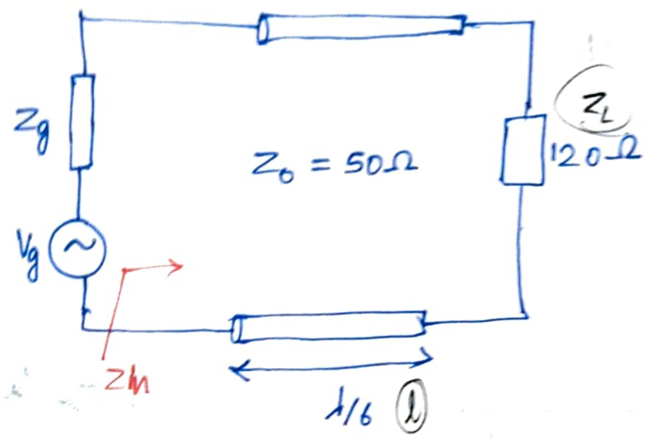
they are complex conjugates to each other.

$Re\{a + Jb - (a - Jb)\} = \underline{0}$

$$\boxed{P_{av} = \frac{|V_0^+|^2}{2Z_0}[1 - |\Gamma_L|^2]}$$

$P_t = P_i - P_r$ → Power dissipated by load/ Avg. power/ Total power.

Q.1) A Lossless transmission line is as shown in the figure.



Find γ, VSWR and Z_{in} at the generator end.

Sol. i) $\gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{120-50}{120+50} = \dfrac{70}{170} = 0.411$

ii) VSWR, $S = \dfrac{1+|\gamma_L|}{1-|\gamma_L|} = 2.39$

iii) Z_{in},

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

$\Rightarrow \beta l = ?$

$$\beta l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{6} = \frac{\pi}{3}$$

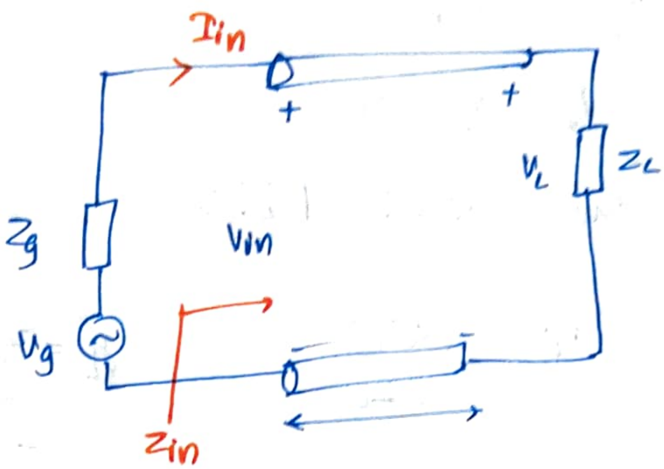
$$Z_{in} = 50 \left[\frac{120 + j50 \tan(\pi/3)}{50 + j120 \tan(\pi/3)} \right] = 50 \left[\frac{120 + j\frac{50}{\sqrt{3}}}{50 + j\frac{120}{\sqrt{3}}} \right]$$

$$= \frac{6000 + j\frac{2500}{\sqrt{3}}}{50 + j\frac{120}{\sqrt{3}}} = \frac{7399.3\angle 35.81}{213.77\angle 76.47} = 34.61\angle -40.6\,\Omega$$

Casio Degrees!

2) A transmission line shown in the figure is 40m long and has $V_g = 15\angle 0°$ V_{rms}, $Z_0 = 30 + J60\ \Omega$, $V_L = 5\angle -48°$ V_{rms}, $Z_g = 0$.

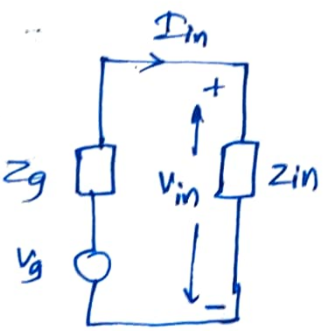
If the line is matched to the load and $Z_g = 0$, calculate i) input impedance; Z_{in} ii) sending end current and voltage (I_{in} & V_{in}).



Sol. i) $Z_{in} = Z_0$ For matched load.

$= 30 + J60\ \Omega$

ii)



→ In general, (if i.e Z_g was not zero)

$$\boxed{V_{in} = V_g \cdot \frac{Z_{in}}{Z_{in} + Z_g}}$$

here $Z_g = 0$

$V_{in} = V_g$

→ $$\boxed{I_{in} = \frac{V_g}{Z_g + Z_{in}}}$$

$$I_{in} = \frac{15\angle 0°}{67.08\angle 63.43} = 0.223\angle -63.43\ A.$$

ebook

3) A 50Ω lossless transmission line is terminated by a load impedance 50−J75Ω. If the incident power is 100mW, find the power dissipated by the load.

Sol.

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}\left[1-|\Gamma_L|^2\right]$$

$$P_t = P_i - P_r$$

$$P_{av} = P_i\left(1-|\Gamma_L|^2\right)$$

Incompl

$$P_i = \frac{|V_0^+|^2}{2Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{\cancel{50} - J75 - \cancel{50}}{50 - J75 + 50}$$

$$= \frac{-J75}{250 - J75}$$

$$= \frac{75\angle -90^\circ}{261.007\angle -16.69}$$

$$= 0.28\angle -73.31^\circ$$

load

20.11.23

inc. + refl.

$$\Rightarrow V(x) = V_0 e^{+\gamma x} + V_0 \Gamma_L e^{-\gamma x}$$

$$= V_0\left(e^{\gamma x} + \Gamma_L e^{-\gamma x}\right)$$

$$I(x) = I_0 e^{\gamma x} - I_0 \Gamma_L e^{-\gamma x}$$

3llly the + refl. but −ve of VRC.

$$= \frac{V_0}{Z_0}\left(e^{\gamma x} - \Gamma_L e^{-\gamma x}\right)$$

$$\rightarrow Z(x) = \frac{V(x)}{I(x)} = Z_0 \frac{\left(e^{\gamma x} + \Gamma_L e^{-\gamma x}\right)}{\left(e^{\gamma x} - \Gamma_L e^{-\gamma x}\right)}$$

Take $e^{\gamma x}$ out;

$$= \frac{Z_0 \left(1 + \gamma_L e^{-2\gamma x}\right)}{\left(1 - \gamma_L e^{-2\gamma x}\right)} = Z_0 \left(\frac{1 + \Gamma(x)}{1 - \Gamma(x)}\right)$$

capital gamma

$$\boxed{\Gamma(x) = \Gamma_L e^{-2\gamma x}} = \boxed{|\Gamma_L| e^{-2\alpha x} \cdot \angle \theta_{\Gamma_L} - 2\beta x}$$

contributes to ampl.

$\rightarrow Z_{max} = \frac{V_{max}}{I_{min}}$; Wherever there is voltage maximum, there is current minimum

$$= \frac{V_0 (1 + |\Gamma_L|)}{I_0 (1 - |\Gamma_L|)}$$

$$\boxed{Z_{max} = Z_0 S}$$

Illy

$\rightarrow Z_{min} = \frac{V_{min}}{I_{max}}$

$$= \frac{V_0 (1 - |\Gamma_L|)}{I_0 (1 + |\Gamma_L|)}$$

$$\boxed{Z_{min} = \frac{Z_0}{S}}$$

Q1) A 600Ω transmission line is 150 m long. It operates at 400 KHz with $\alpha = 2.4$ mNp/m and $\beta = 0.0212$ rad/m load impedance of $424.3 \angle 45°$. Find

i) length of the line in terms of λ.

★ Refl. coeff @ load. ii) Γ_L

★. Refl. coeff from Z_L @ length l (so src) iii) $\Gamma(x)$ at $x = l$

iv) Z at $x = l$

Sol. Given,

$Z_0 = 600\,\Omega$

$l = 150\,m$

$Z_L = 424.3 \angle 45° = 300 + J300$

$\alpha = 2.4 \times 10^{-3}$

$\beta = 0.0212$

$f = 400$ KHz.

i) WK, $\beta = \frac{2\pi}{\lambda}$

$0.0212 = \frac{2\pi}{\lambda}$ rad. calci

$\lambda = 296.37°$ m.

~~$x = 150$ m~~

$150\,\lambda = 296.37\,x\,l$

$\boxed{l = 0.506\,\lambda}$

ii) $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{300 + J300 - 600}{300 + J300 + 600}$ ↳ Degree.

$= \frac{-300 + J300}{900 + J300} = \frac{424.26 \angle 135°}{948.68 \angle 18.43°}$

$\boxed{\Gamma_L = 0.447 \angle 116.57}$

θ_{Γ_L}

CALC:
Whenever β, rad

iii) $\Gamma(x) = |\Gamma_L| e^{-2\gamma x}$

$116.57 - 2\beta x$

$= |\Gamma_L| e^{-2\alpha x} \angle \theta_{\Gamma} - 2\beta x$

$$\boxed{\Gamma(x) = 0.2175 \angle -247.}$$

iv) $$z(x) = \frac{Z_0 (e^{\gamma x} + \Gamma_L e^{-\gamma x})}{(e^{\gamma x} - \Gamma_L e^{-\gamma x})}$$

$$= Z_0 \left(\frac{1 + \Gamma(x)}{1 - \Gamma(x)} \right)$$

$$= 600 \left(\frac{1 + 0.2175 \angle -247}{1 - 0.2175 \angle -247} \right)$$

$$= 600 \left(\frac{1 + (-0.081 + J0.2)}{1 - (-0.081 + J0.2)} \right)$$

$$= 600 \left(\frac{1 - 0.081 + J0.2}{1 + 0.081 - J0.2} \right) = 600 \left(\frac{0.919 + J0.2}{1.081 - J0.2} \right)$$

$$= 513.19 \angle 22.75 .$$

SMITH CHART

→ Normalised Impedance chart
Normalised admittance chart

Since many values of Z_L and Z_0 map to same Γ,

complex → real + imaginary

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_r + j\Gamma_i \quad \text{--- (1)}$$

Normalised impedance,

$$z_L = \frac{Z_L}{Z_0} = r + jx$$

From (1),

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$\Gamma(z_L + 1) = z_L - 1$$

$$\Gamma z_L + \Gamma - z_L + 1 = 0$$

$$z_L(1-\Gamma) = 1+\Gamma$$

$$z_L = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} \times \frac{(1-\Gamma_r + j\Gamma_i)}{(1-\Gamma_r + j\Gamma_i)}$$

$\boxed{(a-jb)(a+jb) = a^2+b^2}$

Easy calc^n:

$\underbrace{1+j\Gamma_i}-\underbrace{\Gamma_r}$

$\underbrace{1+j\Gamma_i}+\underbrace{\Gamma_r}$

$\boxed{(a+b)(a-b) = a^2-b^2}$

$$(z_L)\quad \underbrace{r}_{\text{real}} + \underbrace{jx}_{\text{Im.}} = \frac{1-\Gamma_r^2 - \Gamma_i^2 + 2j\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

i) Equating real parts,

$$r = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Eqn. of circle w/ centre (a,b):

$$(x-a)^2 + (y-b)^2 = r^2$$

$$x = \Gamma_r$$

$$y = \Gamma_i$$

$$r(1-\Gamma_r)^2 + r\Gamma_i^2 = 1 + \Gamma_r^2 - 2\Gamma_r \;\Gamma_i^2$$

$$r(1-2\Gamma_r + \Gamma_r^2)$$

$$r + r\Gamma_r^2 - r2\Gamma_r + r\Gamma_i^2 = 1 - \Gamma_r^2 - \Gamma_i^2 - 2\Gamma_r$$

$$r - r\Gamma_r^2 + \Gamma_r^2 - r2\Gamma_r = 1 - r\Gamma_i^2 - \Gamma_i^2 - 2\Gamma_r$$

$$\rightarrow \Gamma_r^2(r+1) - 2r\Gamma_r + \Gamma_i^2(r+1) = 1-r$$

$$\Gamma_r^2 - \frac{2r}{r+1}\Gamma_r + \Gamma_i^2 = \frac{1-r}{1+r}$$

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \frac{r^2}{(r+1)^2}$$

$$\boxed{\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2}$$

$$\boxed{\text{center : } \left(\frac{r}{r+1}, 0\right)}$$

$$\boxed{\text{Radius : } \left(\frac{1}{1+r}\right)}$$

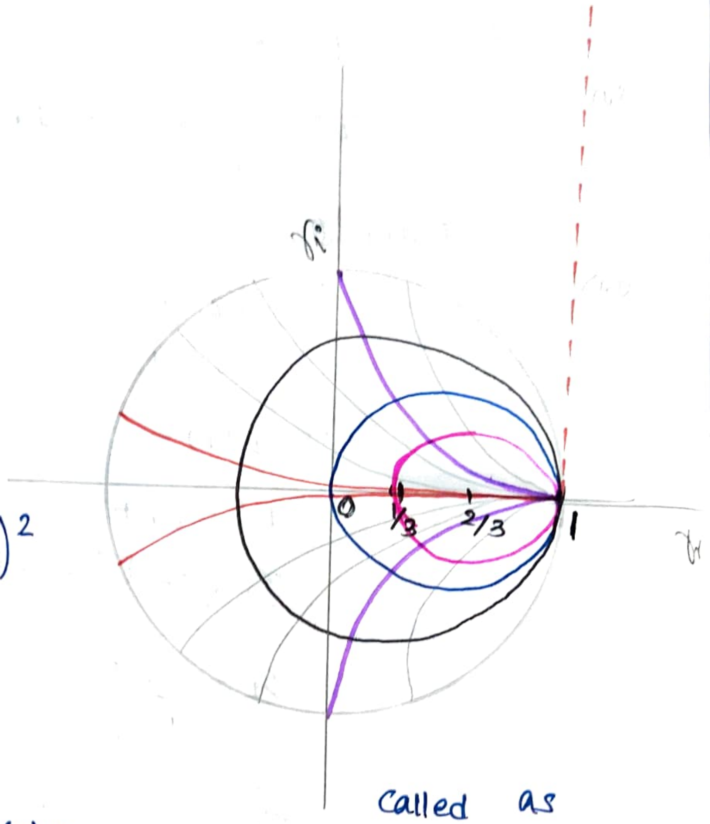
1) when $r = 0$

$$\Gamma_r^2 + \Gamma_i^2 = 1$$

2) when $r = \frac{1}{2}$

$$\left(\Gamma_r - \frac{1}{3}\right)^2 + \Gamma_i^2 = \left(\frac{2}{3}\right)^2$$

w/ centre $1/3$, sketch a circle of radius $2/3$



3) when $r = 1$

$$\left(\Gamma_r - \frac{1}{2}\right)^2 + \Gamma_i^2 = \left(\frac{1}{2}\right)^2$$

centre. $1/2$, and radius $1/2$

Called as "constant r-circle" or resistance circles.

→ when $r = \infty$, smallest circle

4) when $r = 2$

$$\left(\Gamma_r - \frac{2}{3}\right)^2 + \Gamma_i^2 = \left(\frac{1}{3}\right)^2$$

centre $2/3$, radius $1/3$

That's why x is -ve (or) +ve

x - can be $\overbrace{\text{cap}}^{-ve}$ / $\overbrace{\text{Ind.}}^{+ve}$ reactance

but r - resistance cannot be -ve.

ii) equating imaginary parts,

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x = \frac{2\Gamma_i}{1 - 2\Gamma_r + \Gamma_r^2 + \Gamma_i^2}$$

$$\boxed{(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2}$$

1) when $x = 0$

$$(\Gamma_r - 1)^2 + (\Gamma_i - \infty)^2 = (\infty)^2$$

2) when $x = \frac{1}{2}$

$$(\Gamma_r - 1)^2 + (\Gamma_i - 2)^2 = 2^2$$

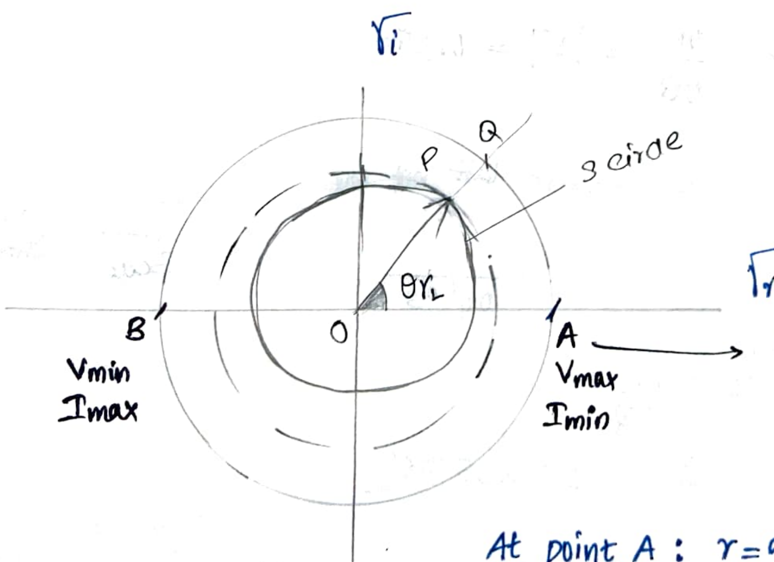
center: (1, 2)
Radius: 2.

3) when $x = 1$

$$(\Gamma_r - 1)^2 + (\Gamma_i - 1)^2 = 1^2$$

—

⇒



@ pt. A voltage is maximum; Vmax so Imin.

Z_0, Z_L

$z_L = \frac{Z_L}{Z_0} = r + jx$

normalised impedance.

At point A : $r = \infty, x = \infty$

$z_L = \infty + j\infty$

$Z_L = \infty$

open circuit.

iii) At point B : $r = 0, x = 0$

$z_L = 0 + J0$

$Z_L = 0$

Short circuit.

1) A lossless transmission line with $Z_0 = 50\Omega$ is 30m long and operates at 2MHz. The line is terminated with a load $Z_L = (60 + J40)\Omega$. If $u = 0.6c$ on the line, find

i) Γ — using Smith chart.

ii) S

iii) Z_{in}

Sol. $z_L = \frac{Z_L}{Z_0} = \frac{60 + J40}{50} = 1.2 + J0.8$

i) OP = 2.7
OQ = 7.7. } from chart.

ii) $\frac{OP}{OQ} = |\Gamma| = 0.35$

iii) Bottom-most last but one.

Centre

← Scale.

0.4 0.3

(1) $\Gamma_L = \frac{OP}{OQ}$

$\Gamma_L = 0.35 \angle 56°$

iv) Draw circle w/ OP radius. = comes as 2.1 = S (2)

(3) $\lambda = \frac{u}{f} = \frac{0.6c}{2M} = 90\text{ m}.$

For lossless,

one revolution $= \frac{\lambda}{2} = 360°$; $\lambda = 720°$ $\boxed{0.5\lambda = 720}$

$\frac{\lambda}{3} = 240°$

$90 \rightarrow \lambda$

$30 \rightarrow \frac{\lambda}{3}$

Starting from P, clockwise (cos expected), $56 + 240 = 184°$

extend 1.0 to 184.

→ Check where intersection.

we get $r = 0.48$

$x = 0.04$

But this is normalised.

→ $z_{in} = 0.48 + j0.04$

$Z_{in} = Z_0 z_{in}$

$\underline{Z_{in} = 24 + j2}$

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—.

2) A 70Ω lossless line has $S = 1.6$ and $\theta_r = 300°$. If the line is 0.6λ long. Obtain

1) Γ, Z_L, Z_{in}

2) The distance of first voltage minimum from the load.

(r/Γ_r axis)

Sol. → From 1.0 as centre, draw a circle (S) passing through 1.6 because $S = 1.6$

→ Draw a line joining the centre (origin) at 300° in Anticlockwise dim or 60° clockwise

→ Pt. of intersection of this line w/ S circle $= Z_L$

$\longrightarrow (1.15, 0.5)$

$\therefore z_L = 1.15 - j0.5$

but we need Z_L,

$\rightarrow \quad Z_L = Z_0 \, z_L$

$= 70\,(1.15 - j0.5)$

$\rightarrow \quad |\Gamma| = \dfrac{OP}{OQ} = \dfrac{1.7}{7.6} = 0.22$ → ① either directly from Refl. coeff below. after getting OP

$\gamma = 0.22 \angle 300^\circ$ → ② Get OP, OQ from diag & divide.

$\rightarrow \quad z_{in} = ?$

WKT, one revolution; $360^\circ = \dfrac{\lambda}{2}$ in ta

Method 1:

Given, $0.6\lambda = l$

$0.6 \times 720 = 432^\circ$ → from 0.334 point

$432 - 360 = 12^\circ$

$z_{in} = 0.68 - j0.25$ → below

$Z_{in} = Z_0 \, z_{in}.$

Method 2:

From 0.334, mark 0.6λ

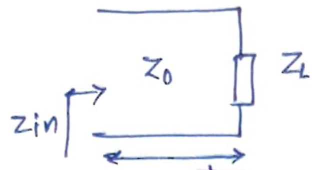
1 rev till 0.334 = 0.5

then add $+ 0.1\lambda$ to it

So. 0.334
+0.1
0.434 ✓

2) $0.5 - 0.334 = 0.166.$

QUARTER WAVE TRANSFORMER



We want inp. impedance for this; $\lambda/4$ = length of transmission line

$$Z_{in} = Z_0 \left(\frac{Z_L + JZ_0 \tan \beta l}{Z_0 + JZ_L \tan \beta l} \right)$$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi/2 .$$

$$= Z_0 \left(\frac{Z_L + JZ_0 \tan \pi/2}{Z_0 + JZ_L \tan \pi/2} \right)$$

$\div$ Nr & Dr by $\tan \pi/2$

$$= Z_0 \left(\frac{\frac{Z_L}{\tan \pi/2} + JZ_0}{\frac{Z_0}{\tan \pi/2} + JZ_L} \right)$$

$$\boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$$

Input Impedance for a quarter wave transformer.

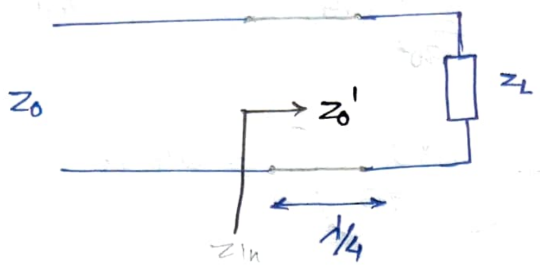
23/11/23

→ Quarter wave matching.

$Z_0 = Z_L$

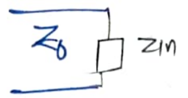
We have transformer whose char. imp $= Z_0$

load imp $= Z_L$



Z_0' = char. imp. of quarter wave transformer.

$$Z_{in} = \frac{(Z_0')^2}{Z_L}$$



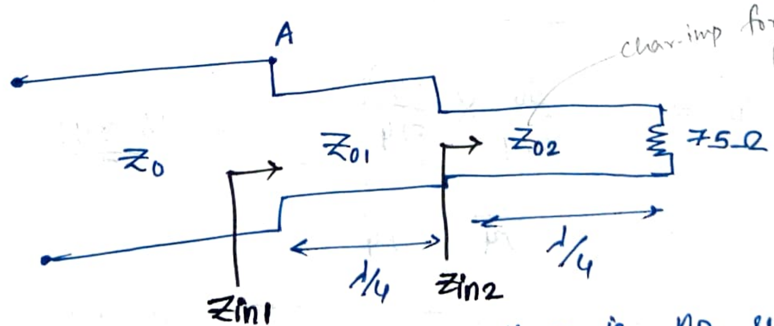
$$Z_{in} = \boxed{\frac{(Z_0')^2}{Z_L} = Z_0}$$

This condition should be satisfied. for matching

→ At what frequency matching happens?

Freq. corresponding to $\frac{\lambda}{4}$.

Q1) Two $\frac{\lambda}{4}$ transformers in tandem (connected together) are to connect a 50Ω line to a 75Ω load as shown in the figure.



Determine Z_{01} if $Z_{02} = 30\Omega$, and there is no reflected wave to the left of A.

Sol. $Z_{in2} = \frac{(Z_{02})^2}{75} = \frac{(30)^2}{75}\Omega$

$Z_{in1} = \frac{(Z_{01})^2}{Z_{in2}} = \cancel{Z_0}$

Since no reflection to left of A means has to be matched.

$\frac{Z_{01}^2 (75)}{30^2} = 50$

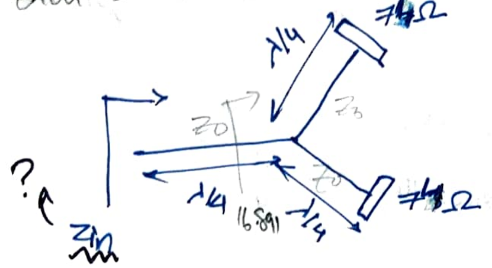
$\cancel{\frac{Z_{01}}{Z_{in2}}} \cancel{/1}$

$Z_{01} = \left(\frac{50 \times 30^2}{75}\right)^{\frac{1}{2}} = 24.49.$

Q2) Two identical antennas with load impedance 74Ω are fed with 3 identical 50Ω quarter wave lossless transmission lines as shown in the figure.

Acting as load even if not mentioned in question.

each is Z_0.



Calculate input impedance at the source.

Sol.

They are parallel.

$Z_{in}' = \frac{\frac{50^2}{74} \times \frac{50^2}{74}}{\frac{50^2}{74} + \frac{50^2}{74}} = \underline{16.891}$

$\therefore Z_{in} = \frac{50^2}{Z_{in}'(Z_L)} = \frac{50^2}{16.891} = \underline{148.007\ \Omega}$

Maam's method :

$$Z_{in_1} = Z_{in_2} = \frac{(50)^2}{74}\,\Omega$$

Admittance $$y_{in_1} = y_{in_2} = \frac{74}{(50)^2}$$

Becos when in llel, easier if admittance!

$$y = y_{in_1} + y_{in_2} = \frac{148}{(50)^2}$$

$$Z = \frac{1}{y}$$

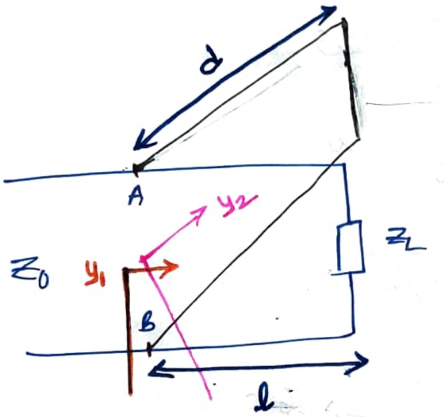
$$Z_{in} = \frac{50^2}{\frac{1}{y}} = 148\,\Omega$$

SINGLE STUB MATCHING

(using Smith chart.)

By using i) short ckt line
ii) open ckt line
(In llel to main transmission line)

In syllabus.



If matched,

WK, $\boxed{\frac{Z_L}{Z_0} = 1}$ → Normalised Impedance = 1, so we can say admittance also = 1

Why we took? cos llel so easier.

$$y_1 = 1 + JX$$

For matching, $y = 1$ (only real part)

Suppose y_1 has JX
So y_2 should have $-JX$
So that $y = y_1 + y_2 = 1$

$$\therefore\ y_2 = -JX$$

At AB admittance,

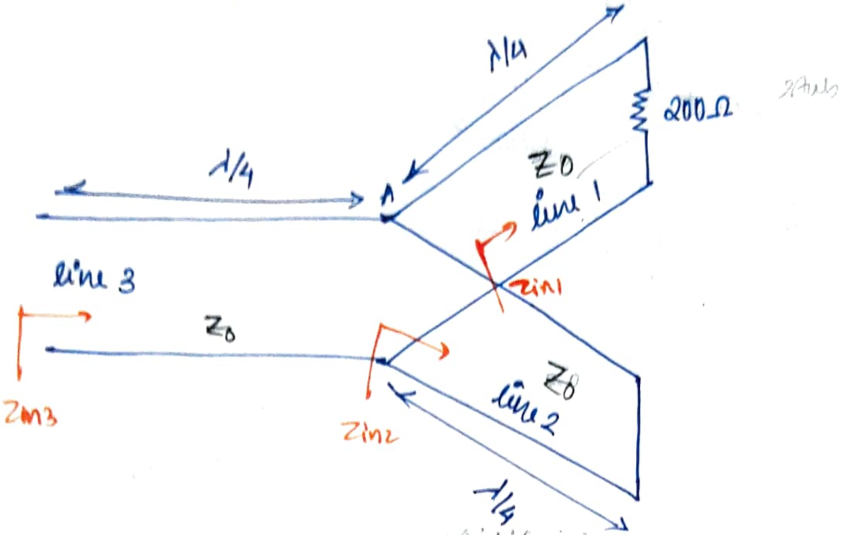
$$y = y_1 + y_2 = 1 + JX - JX$$

$$\underline{y = 1}.$$

Q) Consider 3 lossless lines as shown in the figure. If $Z_0 = 50\,\Omega$, Calculate i) Z_{in} looking into line 1

ii) Z_{in} looking into line 2

iii) Z_{in} looking into line 3.



Sol. i) $Z_{in1} = \dfrac{50^2}{200} = 12.5\,\Omega$

ii) $Z_{in2} = \dfrac{50^2}{0} = \infty$

iii) $Z_{in3} = \dfrac{Z_0^2}{Z_L}$;

At A,

$Y = \dfrac{200}{(50)^2} + 0\Sigma$

$Z_{in1} \parallel Z_{in2} = \dfrac{12.5\,\infty}{12.5 + \infty} = \dfrac{\infty(12.5)}{\infty\left(\frac{12.5}{\infty} + 1\right)}$

$= 12.5$

$Z_{in3} = \dfrac{(Z_0)^2}{12.5} = 200\,\Omega.$