



# **LINEAR INTEGRATED CIRCUITS**

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#### **Unit 2** Simultaneous equations with m < 1 (attenuation)



• Linear op amp transfer function is limited to equation of straight line y = +/-mx+/-b, where m is gain, b is offset, x is input and y is output

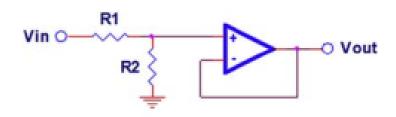
Table 5.1: The Gain and Offset Matrix

	b < 0	b = 0	ь>0
m > 1	Case 2 (Section 4.4.2)	Noninverting gain (Section 2.3)	Case 1 (Section 4.4.1)
m = 1	Section 5.4	Noninverting buffer	
m < 1		Section 5.2	Section 5.3
m = 0	Negative reference or regulator (Chapter 21)	Ground	Positive reference or regulator (Chapter 20)
m < -1	Section 5.7	Section 5.5	Section 5.6
m ≥ −1	Case 4 (Section 4.4.4)	Inverting gain (Section 2.4)	Case 3 (Section 4.4.3)
	m = 1 m < 1 m = 0 m < -1	m > 1	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Table represents different combination of **m** and **b** 

# Unit 2 Non inverting attenuator with zero offset, positive offset and negative offset



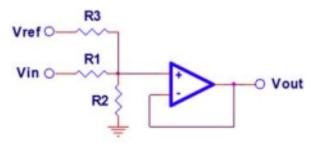


Circuit diagram

Vout = 
$$m \times Vin$$

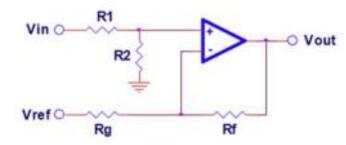
$$m = \frac{R2}{R1 + R2}$$
 Gain

Non inverting attenuator with Zero offset



Vout = 
$$m \times Vin + b$$
  
 $m = \frac{1/R1}{1/R1 + 1/R2 + 1/R3}$   
 $b = Vref \times \frac{1/R3}{1/R1 + 1/R3 + 1/R3}$ 

Non inverting attenuator with positive offset



Circuit diagram

Vout = 
$$m \times Vin - b$$
  

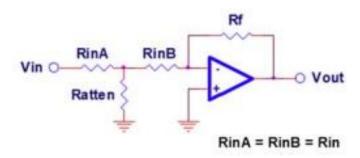
$$m = \left(\frac{R2}{R1 + R2}\right) \times \left(1 + \frac{Rf}{Rg}\right)$$

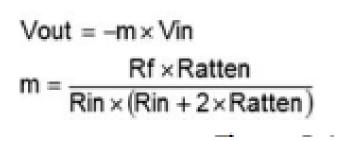
$$b = Vref \times \frac{Rf}{Rg}$$

Non inverting attenuator with negative offset

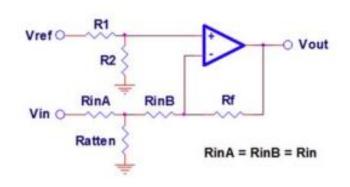
# Unit 2 Inverting attenuator with zero offset, positive offset and negative offset







Inverting attenuator with Zero offset



$$Vout = -m \times Vin + b$$

$$m = \frac{Rf \times Ratten}{Rin \times (Rin + 2 \times Ratten)}$$

$$b = Vref \times \left(\frac{R2}{R1 + R2}\right) \times \left(1 + \frac{Rf}{Rin + Rin ||Ratten|}\right)$$

**Inverting attenuator with Positive offset** 

$$\begin{aligned} &Vout = -m \times Vin - b \\ &m = \frac{Rf \times Ratten}{Rin \times (Rin + 2 \times Ratten)} \\ &b = Vref \times \frac{Rf}{Rg} \end{aligned}$$

Inverting attenuator with Negative offset

#### **Unit 2** Development of non ideal op amp equations

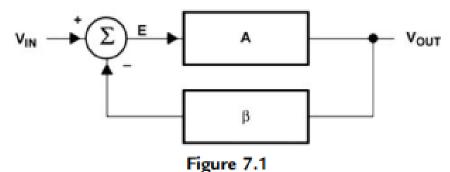


#### General summary points while considering non ideal scenario

- Concept of DC errors and AC errors
  - DC errors are offset voltage and input bias current, they are constant over entire frequency range
  - AC errors are CMRR, PSRR, differential gain, they can not be ignored at high frequencies
- Inaccuracies related to op amp can be minimized using negative feedback
- Stability is usually an criteria when operating frequency is high
- Internally compensated and externally compensated op amp circuits are used for better stability.

### **Unit 2** Development of non ideal op amp equations





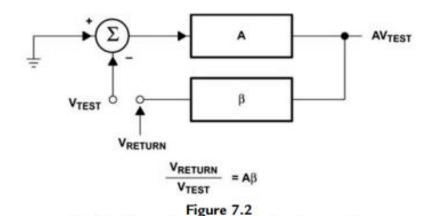
Feedback system block diagram.

Transfer function,

$$\frac{V_{OUT}}{V_{IN}} = \frac{A}{1 + A\beta}$$

When loop gain is large, transfer function is,

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{\beta}$$



Feedback loop broken to calculate loop gain.

Error indicator E, proportional to signal and inversely proportional to loop gain

$$E = \frac{V_{IN}}{1 + A\beta}$$

#### **Unit 2** Development of non ideal op amp equations - Non Inverting amp



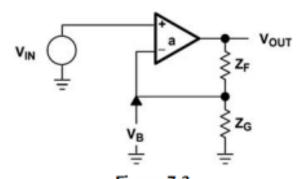


Figure 7.3 Noninverting op amp.

amplifier transfer equation.

$$V_{OUT} = a(V_{IN} \pm V_B)$$

----- 1

V<sub>B</sub> calculated based on resistor divider from V<sub>OUT</sub>

$$V_B = \frac{V_{OUT}Z_G}{Z_E + Z_G}$$
 for  $I_B = 0$  -----2

From 1 and 2,

$$V_{OUT} = aV_{IN} - \frac{aZ_GV_{OUT}}{Z_G + Z_F}$$

After simplification,

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}} \qquad ----3$$

In the form of closed loop function,

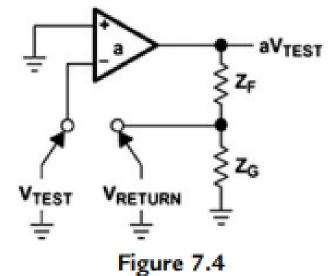
$$\frac{V_{OUT}}{V_{IN}} = \frac{A}{1 + A\beta} \qquad -----4$$

By comparing 3 and 4, loop gain is given by,

$$A\beta = \frac{aZ_G}{Z_G + Z_F}$$

#### Unit 2 Development of non ideal op amp equations - Non Inverting amp





Open-loop noninverting op amp.

$$\begin{split} V_{RETURN} &= \frac{aV_{TEST}Z_G}{Z_F + Z_G} \\ \frac{V_{RETURN}}{V_{TEST}} &= A\beta = \frac{aZ_G}{Z_F + Z_G} \end{split}$$

For measurement loop gain, break the loop, apply test signal at one end and measure voltage at the other end

#### Unit 2 Development of non ideal op amp equations Inverting amp



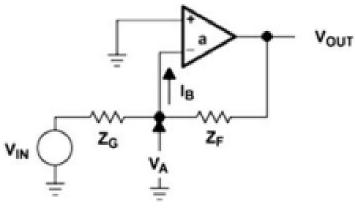


Figure 7.5 Inverting op amp.

$$V_{OUT} = -aV_A$$

Using superposition theorem, calculate voltage V<sub>A</sub>

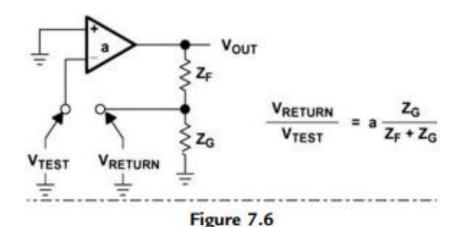
$$V_A = \frac{V_{IN}Z_F}{Z_G + Z_F} + \frac{V_{OUT}Z_G}{Z_G + Z_F}$$
 for  $I_B = 0$  .....5

Simplifying equation 5  $\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}}$ 

- Open loop gain is different compared to non inverting amp
- Loop gain is same compared to non inverting amp

#### Unit 2 Development of non ideal op amp equations - Inverting amp





Inverting op amp: feedback loop broken for loop gain calculation.

$$\frac{V_{RETURN}}{V_{TEST}} = \frac{aZ_G}{Z_G + Z_F} = A\beta \quad \text{Loop gain}$$

For measurement of loop gain, break the loop, apply test signal at one end and measure voltage at the other end

#### Unit 2 Development of non ideal op amp equations - Differential amp



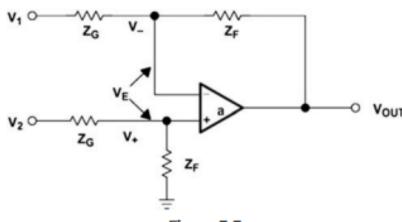


Figure 7.7
Differential amplifier circuit.

transfer equation.

$$V_{OUT} = aV_E = V_+ \pm V_-$$

$$a(V_+ + V_-)$$

Voltage at non inverting terminal is calculated as,

$$V_{+} = V_{2} \frac{Z_{F}}{Z_{F} + Z_{G}}$$

Voltage at inverting terminal is calculated as,

$$V_{-} = V_{1} \frac{Z_{F}}{Z_{F} + Z_{G}} - V_{OUT} \frac{Z_{G}}{Z_{F} + Z_{G}}$$

Voltage at output, using super position theorem,

$$V_{OUT} = a \left[ \frac{V_2 Z_F}{Z_F + Z_G} - \frac{V_1 Z_F}{Z_F + Z_G} - \frac{V_{OUT} Z_G}{Z_F + Z_G} \right] \quad .... 1 \label{eq:vout}$$

#### Unit 2 Development of non ideal op amp equations Differential amp



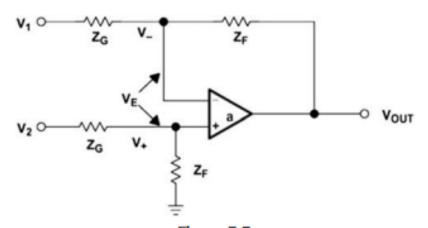
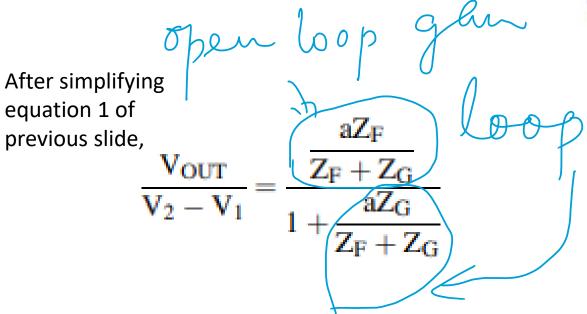
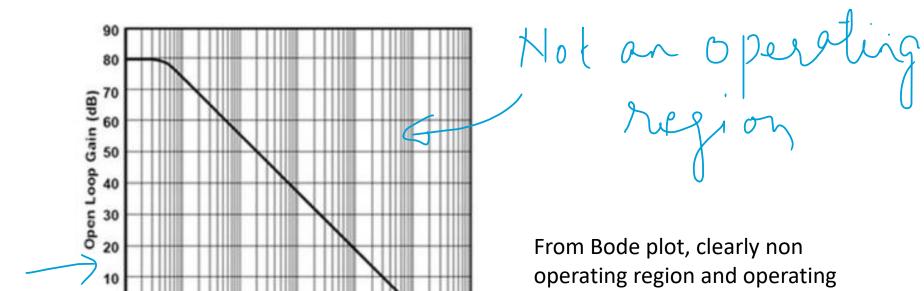


Figure 7.7
Differential amplifier circuit.



Loop gain is same compared to non inverting amp and inverting amp





region can be divided

Operating

Figure 7.8

Bode response of a typical op amp.

Frequency



#### When open loop gain is high

noninverting op amp:

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}} \implies \text{when } \alpha > > \implies \frac{V_{OUT}}{V_{IN}} = \frac{1}{2} + \frac{2}{2} + \frac{2}$$

inverting op amp stage:

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}} \label{eq:Vout}$$

When open loop gain is high

Results in ideal op amp relation

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## Different scenarios for changes in a, R<sub>G</sub> and R<sub>F</sub>

Table 7.1: Real Inverting Op Amp Stage Gains for a = 80 dB

Non ideal transfer function for inverting op amp

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

a	$R_G$	R <sub>F</sub>	Attempted	Actual	Error (%)
10,000 10,000 10,000	100,000 10,000 1000	100,000 100,000 100,000	-1 -10 -100	-0.9998 -9.9890 -99.0001	-0.0200 -0.1099 -0.9999
10,000 10,000 10,000 10,000	100 10 1	100,000 100,000 100,000 1.00E + 12	-1000 -10,000 -100,000 -1E + 12	-909.0083 -4999.7500 -9090.8264 -9999.9999	-9:0992 -50:0025 -90:9092 -100

Table 7.2: Real Noninverting Op Amp Stage Gains for a = 80 dB

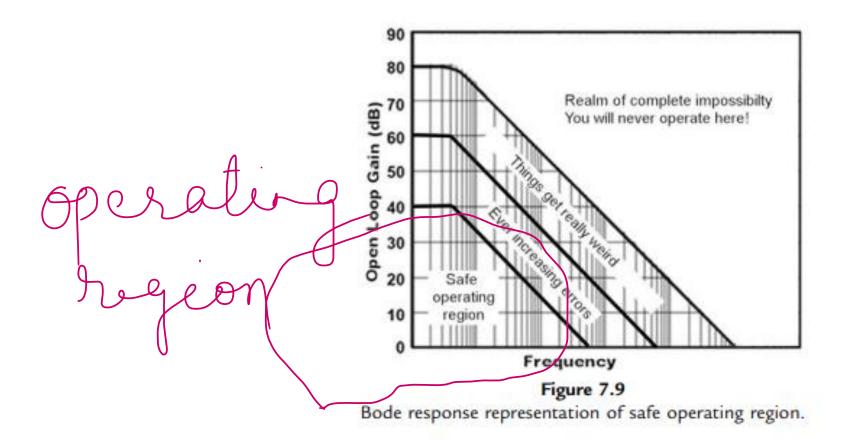
Non ideal transfer function for non inverting op amp

$$\frac{V_{\text{OUT}}}{V_{\text{IN}}} = \frac{a}{1 + \frac{aZ_{G}}{Z_{G} + Z_{I}}}$$

a	$R_G$	$R_{F}$	Attempted	Actual	Error (%)
10,000	100,000	100,000	2	1.9996	-0.0200
10,000	10,000	100,000	11	10.9879	-0.1099
10,000	1000	100,000	101	99.9901	-0.9999
10,000	100	100,000	1001	909.9173	9.0992
10,000	10	100,000	10,001	5000.2500	-50.0025
10,000	1	100,000	100,001	9090.9174	-90.9092
10,000	1	1.00E + 12	1E + 12	9999.9999	-100

higher

#### In actual scenario, operating region is much lesser





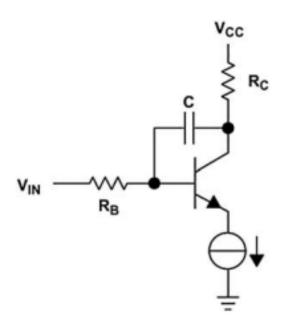
#### Unit 2 Voltage feedback op amp compensation



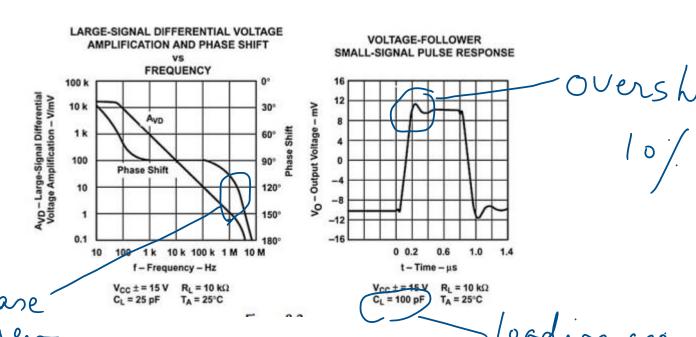
### **Background**

- Oscillations are considered as boundary between stability and non stability
- Poor stability circuit exhibits ringing and overshoot
- Phase margin is one measure for stability of the circuit
- Compensation provides patch between stability and performance
- Compensation network is by RC network





A capacitor C connected between input and output for compensation, called internal compensation capacitor, it is part of IC

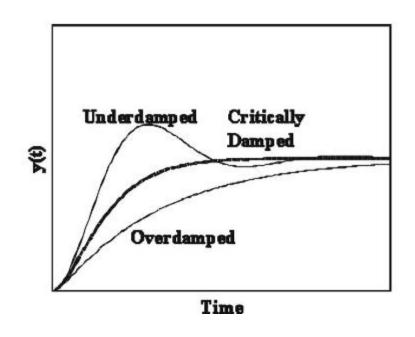


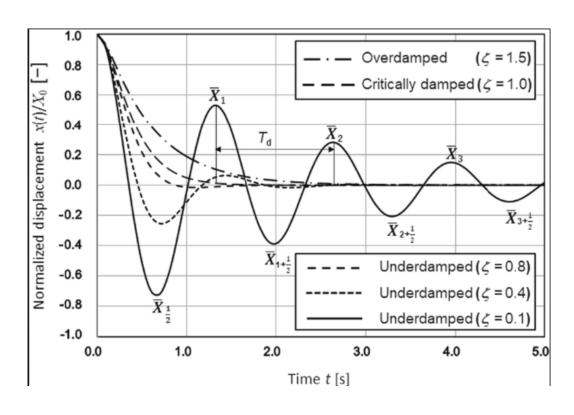
Plot of internally compensated op amp

changes phase margen



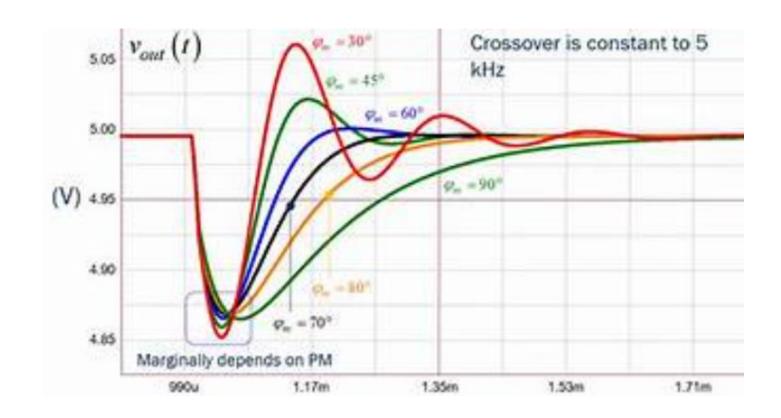
### Underdamped vs Overdamped





Critically damped circuits are preferred over other types. They settle faster with minimum overshoot





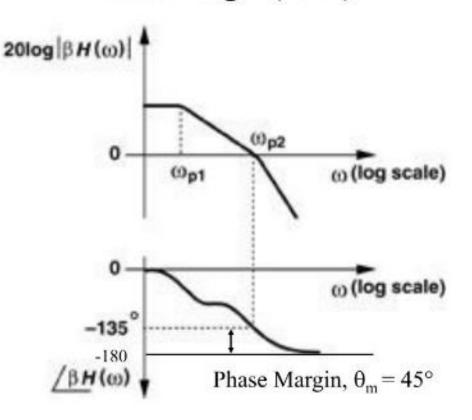
For critically damped circuits, phase margin has to be around 60 degrees



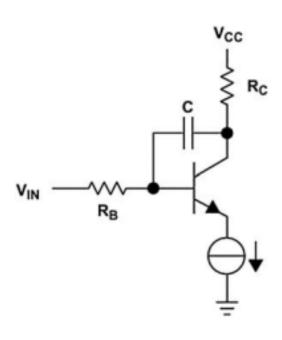
Phase Margin (cont.)

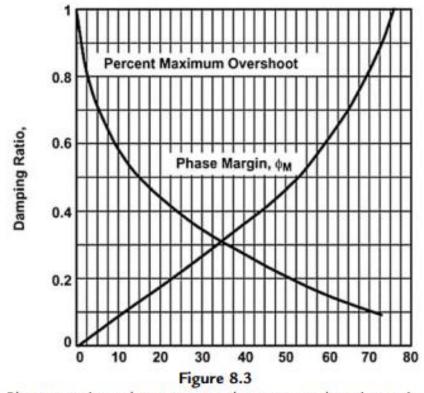


- Decrease in gain for pole frequency
- Phase shift of -90 degrees at pole frequency
- Increase in gain for zero frequency
- Phase shift of +90 degrees at pole frequency









Phase margin and percent overshoot versus damping ratio.

Plot of internally compensated op amp, measure of phase margin with damping ratio and overshoot (Data sheet for TL03X)

#### **Unit 2** Importance of external compensation

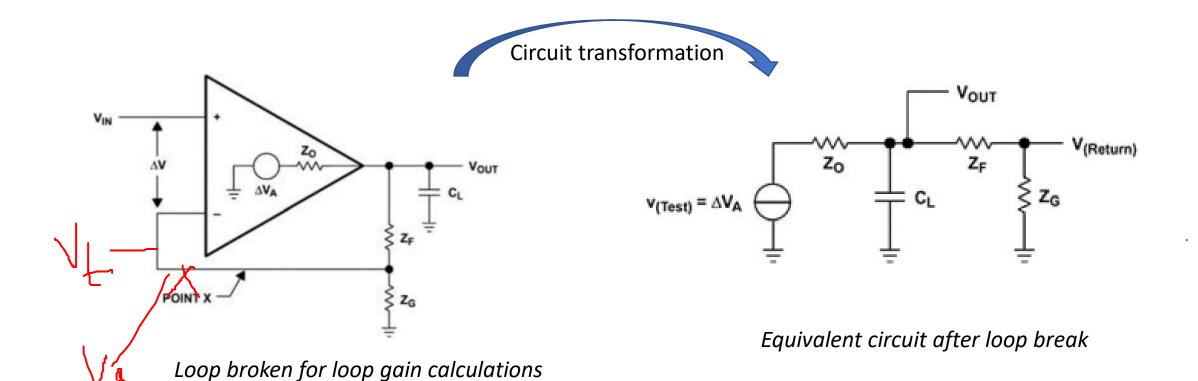


#### **Summary points**

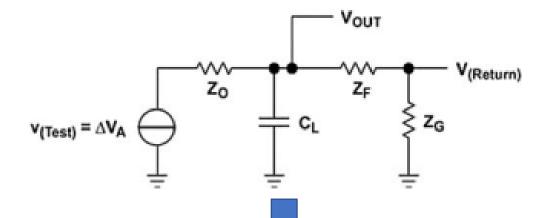
- High frequency noise reduction by closed loop configuration
- Improve phase margin in turn improving stability
- Reduce overshoot by having better phase margin
- Compensation can be tailored to the circuit requirement



- In this type of compensation, an output capacitor is added
- Combination of output capacitor and output impedance forms dominant pole (a low frequency pole)







Apply Thevenin's theorem in Figure 1 to separate Z<sub>o</sub> and C<sub>L</sub>

$$V_{TH} = \frac{\Delta Va}{Z_0 C_{LS} + 1}$$

$$Z_{TH} = \frac{Z_O}{Z_O C_{LS} + 1}$$

Calculate V<sub>return</sub> voltage from Figure 2 using resistive divider theorem

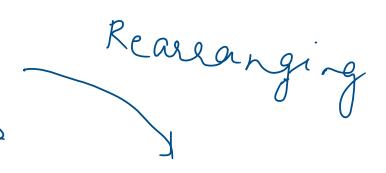
$$V_{RETURN} = \frac{V_{TH}Z_G}{Z_G + Z_F + Z_{TH}} = \frac{\Delta Va}{Z_OC_Ls + 1} \left( \frac{Z_G}{Z_F + Z_G + \frac{Z_O}{Z_OC_Ls + 1}} \right) \quad . \label{eq:VRETURN}$$

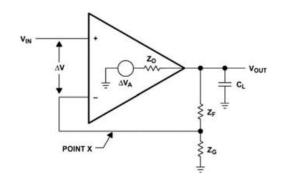
Figure 2

Figure 1



$$V_{RETURN} = \frac{V_{TH}Z_G}{Z_G + Z_F + Z_{TH}} = \frac{\Delta Va}{Z_OC_Ls + 1} \left( \frac{Z_G}{Z_F + Z_G + \frac{Z_O}{Z_OC_Ls + 1}} \right)$$
Here  $\Delta V_{\alpha} = \Delta V_{\dagger}$ 



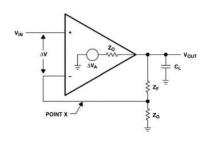


$$\frac{V_{RETURN}}{V_{TEST}} = A\beta = \frac{\frac{aZ_G}{Z_F + Z_G + Z_O}}{\frac{(Z_F + Z_G)Z_OC_Ls}{Z_F + Z_G + Z_O} + 1}$$

When  $(Z_F + Z_O) >> Z_O$ 

$$A\beta = \frac{aZ_G}{Z_F + Z_G} \left( \frac{1}{Z_O C_L s + 1} \right)$$





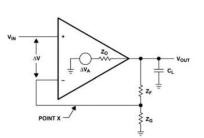
$$A\beta = \frac{aZ_G}{Z_F + Z_G} \left( \frac{1}{Z_O C_L s + 1} \right) \quad ----1$$

In case op-amp a Second order

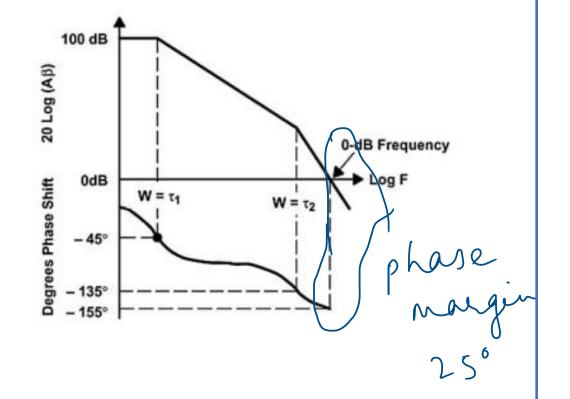
$$a = \frac{K}{(s + \tau_1)(s + \tau_2)}$$
 .....2

From 1 and 2, Loop gain is equal to

$$A\beta = \frac{K}{(s+\tau_1)(s+\tau_2)} \frac{Z_G}{Z_F + Z_G} \frac{1}{Z_O C_L s + 1} \label{eq:beta}$$

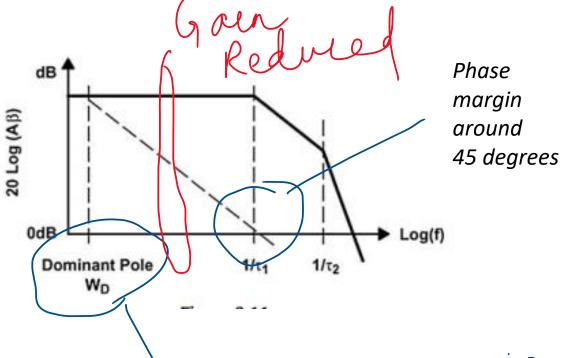


#### **Bode plot without compensation**

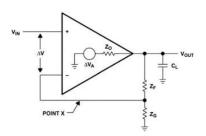


#### **Bode plot with compensation**





Additional pole By compensation terhuisne



Loop gain 
$$A\beta = \frac{aZ_G}{Z_F + Z_G} \left( \frac{1}{Z_O C_L s + 1} \right)$$

$$\frac{AB - aZG}{ZG + ZF}$$



Closed loop transfer function is given by (From slide 7)

$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{Z_F + Z_G}{Z_G}$$

Which represents gain of non inverting amp in ideal conditions

#### **Unit 2 Gain compensation**

Loop gain parameter and closed loop parameters are related

Loop gain 
$$A\beta = \frac{aZ_G}{Z_G + Z_F}$$

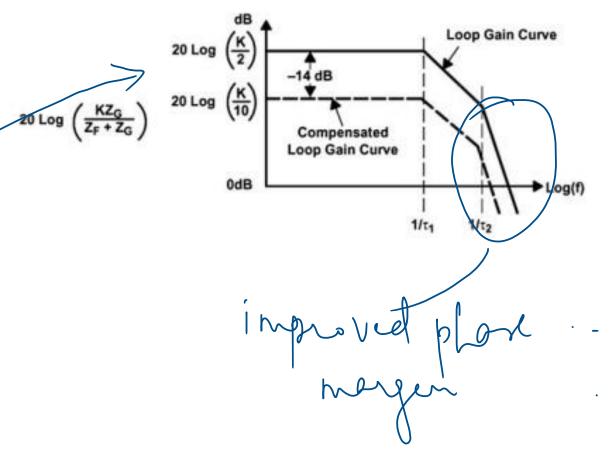
Closed Loop gain (Non ideal) 
$$\frac{V_{OUT}}{V_{IN}} = \frac{a}{1 + \frac{aZ_G}{Z_G + Z_F}}$$

Closed Loop gain (Ideal) 
$$\frac{V_{OUT}}{V_{IN}} = \frac{Z_F + Z_G}{Z_G}$$

- Example, Change non inverting amp closed loop gain from 2 to 10
- Loop gain will reduce by -14db

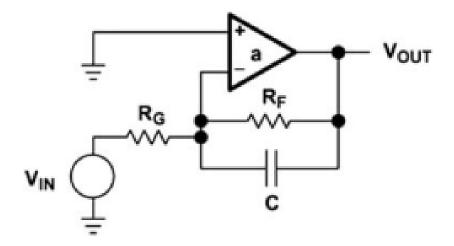


#### Gain compensation – Bode plot

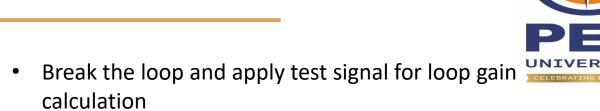


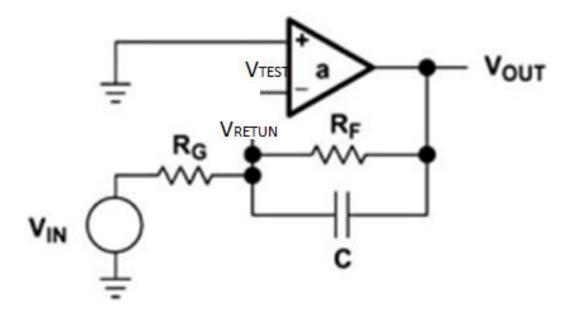
Improvement observation from the Bode plot

- In this compensation, C is added across feedback resistor
- C is because of parasitic capacitance



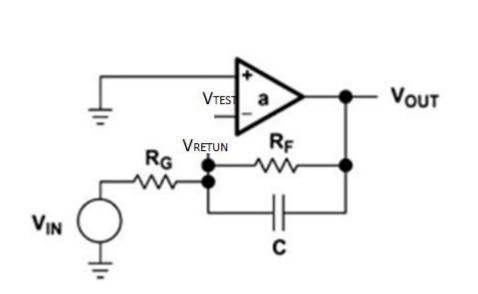
Circuit diagram







By using voltage divider theorem for the circuit shown, we can write loop gain as



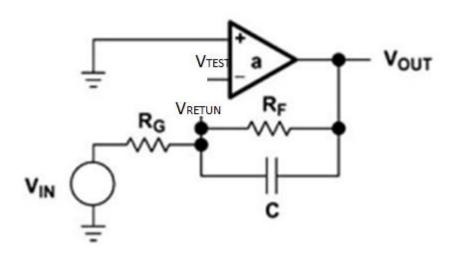
Victur = 
$$aR_G(SCR_f+1)$$
  
 $R_G(SCR_f+1)+R_F$ 

**OR** 

$$A\beta = \left(\frac{R_G}{R_G + R_F}\right) \left(\frac{R_F C_S + 1}{R_G \|R_F C_S + 1}\right)$$







$$a = \frac{K}{(s + \tau_1)(s + \tau_2)}$$

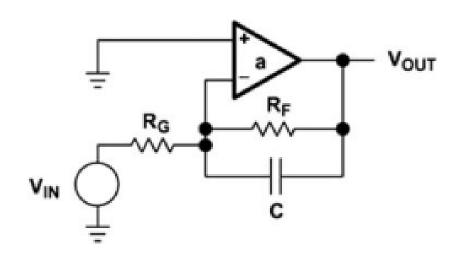
We can rewrite loop gain equation as,

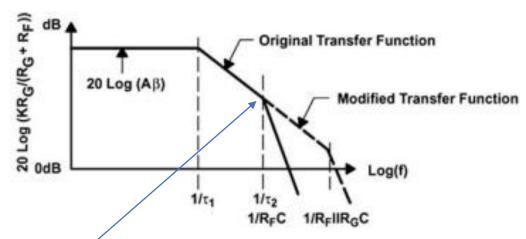
$$A\beta = \left(\frac{R_G}{R_G + R_F}\right) \left(\frac{R_F C s + 1}{R_G \|R_F C s + 1}\right) \left(\frac{K}{(s + \tau_1)(s + \tau_2)}\right)$$



$$A\beta = \left(\frac{R_G}{R_G + R_F}\right) \left(\frac{R_F C s + 1}{R_G \|R_F C s + 1}\right) \left(\frac{K}{(s + \tau_1)(s + \tau_2)}\right)$$
   
 
$$2 \text{ The left of the last o$$

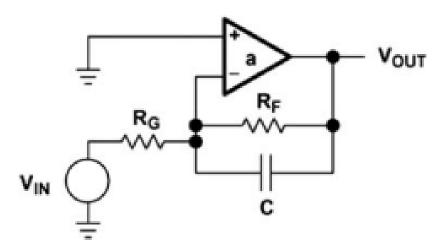
#### **Bode plot of lead compensation**





- Zero placed near second pole
- R<sub>F</sub> has to larger compared to R<sub>G</sub> in parallel with R<sub>F</sub>
- Improves phase margin





Transfer function of inverting amp is given by

Closed Loop gain (Non ideal)

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}} - \cdots - 10$$

\_\_\_

When a is infinity, transfer function shown in 10 can be seen as,

$$\frac{V_{OUT}}{V_{IN}} = -\frac{Z_F}{Z_{IN}} \ge C_F$$

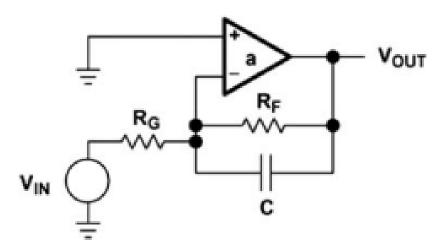
Substituting  $R_F \| C$  for  $Z_F$  and  $R_G$  for  $Z_G$ 

Transfer function is given by

$$\frac{\text{Closed Loop}}{\text{gain (Ideal)}} \quad \frac{V_{OUT}}{V_{IN}} = -\frac{R_F}{R_G} \left( \frac{1}{R_F Cs + 1} \right)$$

----1





Transfer function of inverting amp is given by

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{-aZ_F}{Z_G + Z_F}}{1 + \frac{aZ_G}{Z_G + Z_F}} - \dots - 10$$

\_\_\_

$$a = \frac{K}{(s + \tau_1)(s + \tau_2)}$$
 ----11

When a is infinity, transfer function shown in 10 can be seen as,

$$\frac{V_{OUT}}{V_{IN}} = -\frac{Z_F}{Z_{IN}} \ge C_{C}$$

Substituting  $R_F \| C$  for  $Z_F$  and  $R_G$  for  $Z_G$ 

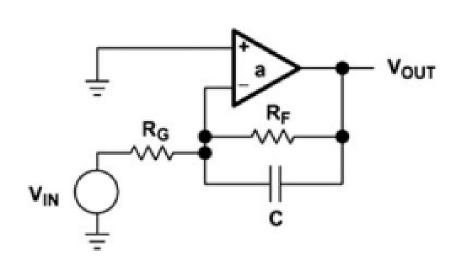
Transfer function is given by

Closed Loop gain (Ideal) 
$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_F}{R_G} \left( \frac{1}{R_F C s + 1} \right)$$

---12



#### Behavior on bode plot for 10, 11,12



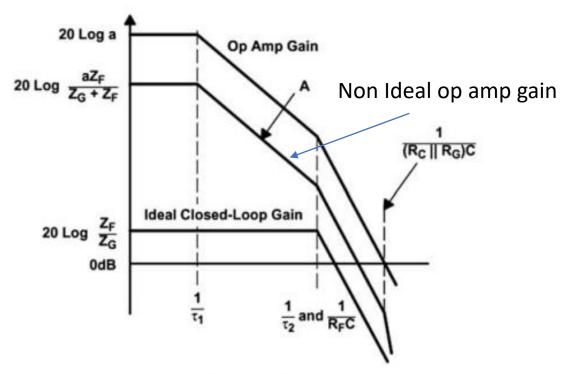
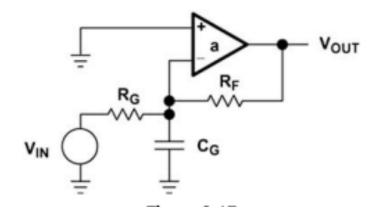


Figure 8.15
Inverting op amp with lead compensation.



Stray capacitance is added due to PCB trace This circuit is unstable because of three poles



Circuit diagram

#### **Loop gain**

$$A\beta = \left(\frac{R_G}{R_G + R_F}\right) \left(\frac{1}{R_G \|R_F C s + 1}\right) \left(\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}\right)$$



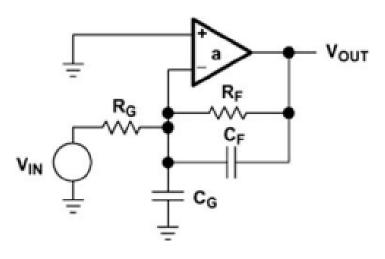
# Compensation capacitor is added parallel to feedback resistor

#### Loop gain for the circuit shown is

$$A\beta = \begin{bmatrix} \frac{R_G}{R_GC_Gs+1} \\ \frac{R_G}{R_GC_Gs+1} + \frac{R_F}{R_FC_Fs+1} \end{bmatrix} \begin{pmatrix} K \\ (\tau_1s+1)(\tau_2s+1) \end{pmatrix}$$

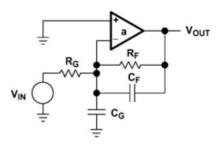
If 
$$R_G C_G = R_F C_F$$

$$\frac{\text{Loop gain is}}{R_G + R_F} \left[ \frac{R_G}{(\tau_1 s + 1)(\tau_2 s + 1)} \right]$$



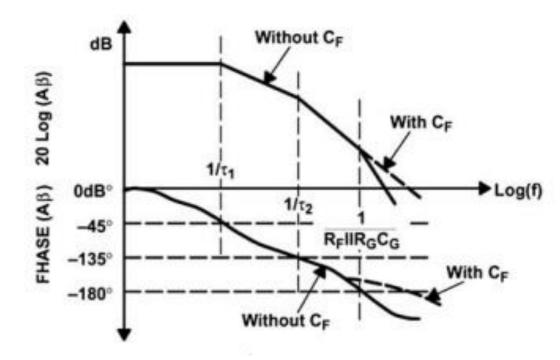
Circuit diagram



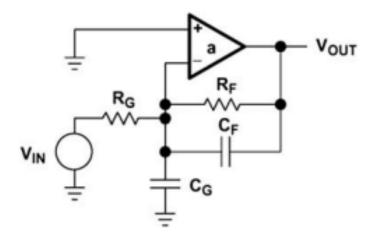


$$A\beta = \left[ \frac{\frac{R_{G}}{R_{G}C_{G}s+1}}{\frac{R_{G}}{R_{G}C_{G}s+1} + \frac{R_{F}}{R_{F}C_{F}s+1}} \right] \left( \frac{K}{(\tau_{1}s+1)(\tau_{2}s+1)} \right)$$

- With addition of compensation capacitor, it cancels pole and zero.
- It acts like open loop gain, a two pole system







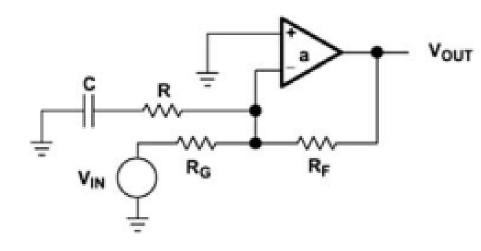
In compensated attenuation circuit, closed loop gain of inverted amp does not change. Capacitor has not effect on gain

#### **Closed loop gain**

$$\frac{V_{OUT}}{V_{IN}} = \frac{\frac{R_F}{R_F C_F s + 1}}{\frac{R_G}{R_G C_G s + 1}}$$
 When  $R_F C_F = R_G C_G$  
$$\frac{V_{OUT}}{V_{IN}} = -\left(\frac{R_F}{R_G}\right)$$



- R and C used for compensation
- Compensation circuit adds Pole and Zero

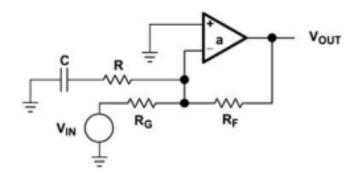


Circuit diagram

#### loop gain of the circuit diagram shown

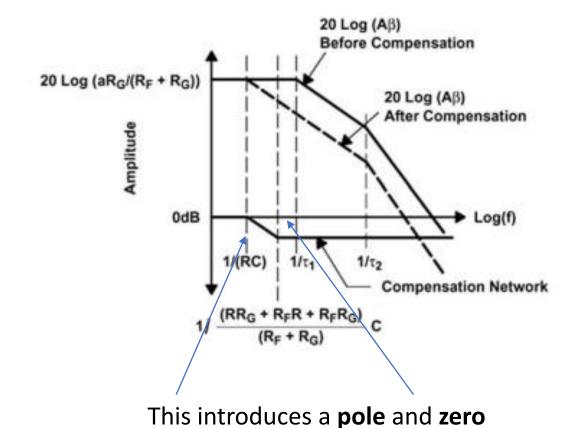
$$A\beta = \frac{K}{(\tau_{1}s+1)(\tau_{2}s+1)} \frac{R_{G}}{R_{G}+R_{F}} \frac{RCs+1}{(RR_{G}+RR_{F}+R_{G}R_{F})} \frac{RCs+1}{(R_{G}+R_{F})} \frac{RCs+1}{$$





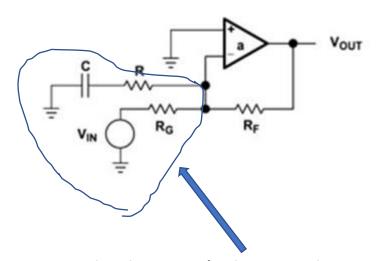
$$A\beta = \frac{K}{(\tau_{1}s+1)(\tau_{2}s+1)} \frac{R_{G}}{R_{G}+R_{F}} \frac{RCs+1}{\frac{(RR_{G}+RR_{F}+R_{G}R_{F})}{(R_{G}+R_{F})}Cs+1}$$

#### **Gain plot**

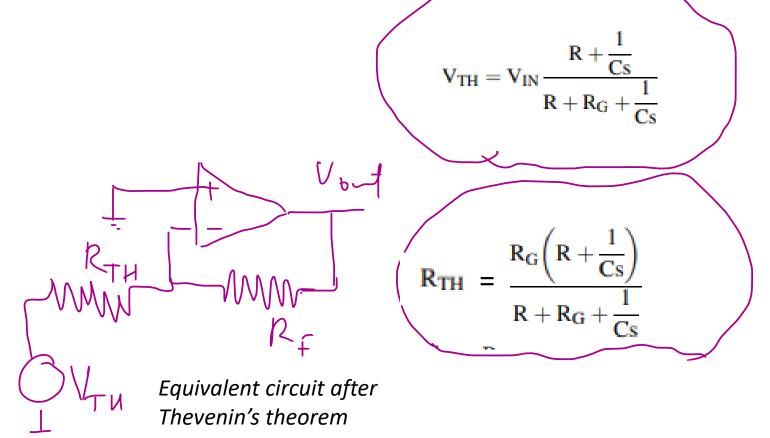




Ideal closed loop gain can be calculated, use Thevenin's theorem first



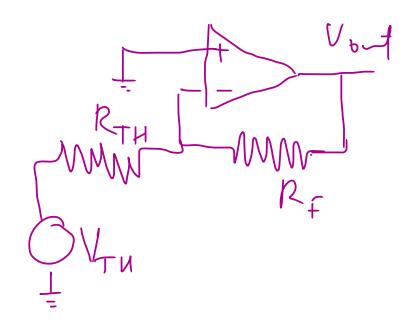
Apply Thevenin's theorem here





Ideal closed loop gain calculated, based on inverting amp configuration

$$V_{OUT} = -V_{TH} \frac{R_F}{R_{TH}}$$



Substitute  $V_{TH}$  and  $R_{TH}$  (refer previous slide)

$$-\frac{V_{OUT}}{V_{IN}} = \frac{R + \frac{1}{Cs}}{R + R_G + \frac{1}{Cs}} \frac{R_F}{R_G \left(R + \frac{1}{Cs}\right)} = \frac{R_F}{R_G}$$

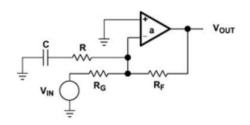
$$\frac{R_F}{R_G + \frac{1}{Cs}}$$

$$\frac{R_F}{R_G + \frac{1}{Cs}}$$

$$\frac{R_F}{R_G + \frac{1}{Cs}}$$

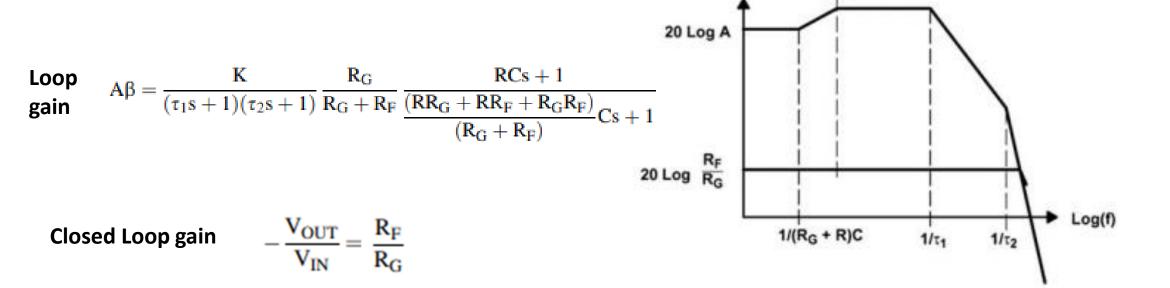
$$\frac{R_F}{R_G + \frac{1}{Cs}}$$





#### Bode plot of lead lag compensation techniques for closed loop gain

1/RC



# **Unit 3 Comparison of compensation schemes**



Scheme	Advantages	Disadvantages
Internal compensation	No need for extra component	Under certain load capacitance, it is unstable
Dominant pole compensation	Suitable for high load capacitance	Load capacitance make op amp to ring
Gain compensation	Good in terms of stability	Gain reduces
Lead compensation	Increases bandwidth	Reduces closed loop gain
Compensated attenuator	Useful scheme when stray capacitance seen at inverting input	Needs matching two RC time constants
Lead lag compensation	Increased bandwidth	More external components



# **Textbook:**

Op Amp for Everyone : Bruce Carter and Ron Mancini Fifth Edition 2017





# **THANK YOU**

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