



UE22EC351B

**WAVEGUIDES, CAVITIES AND RADIATION FROM POINT
SOURCE**

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Problem 1:

A waveguide that is rectangular has its dimensions in the ratio $a:b=2:1$ is supporting TE₁₁ mode, and it has cut-off frequency of 9 GHz. (a) Determine the dimensions of the waveguide.

Solution:

TE_{mn} mode $m=1, n=1$

Unless otherwise specified, free space is present inside the waveguide.

Width is greater than the height ($a > b$)

$$f_{cmn} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_{cmn} = \frac{c}{2\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Problems based on Unit 2

In free space $\mu_r = 1, \epsilon_r = 1$

$$9 \times 10^9 = \frac{3 \times 10^8}{2\sqrt{1 \times 1}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$\frac{a}{b} = \frac{2}{1}$$

$$a = 2b$$

$$60^2 = \left(\frac{1}{2b}\right)^2 + \left(\frac{1}{b}\right)^2$$

$$3600 = \left(\frac{1}{2b}\right)^2 + \left(\frac{1}{b}\right)^2$$

$$b = 0.018\text{m}$$

and

$$a = 2b = 2(0.018)$$

$$a = 0.037\text{m}$$

Problem 2:

If the operating frequency of the wave is 18GHz and cut-off frequency of 9GH, determine

- (i) wave number or phase constant β .
- (ii) Phase velocity V_g
- (iii) wave impedance of the wave Z_g .

Solution: (i) wave number or phase constant β .

$$\beta = \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$
$$\beta = \omega / c \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\omega = 2\pi f = 36\pi \times 10^9$$

$$\sqrt{\mu \epsilon} = \sqrt{\mu_0 \epsilon_0} = \frac{1}{c} = 1/3 \times 10^8$$

$$\beta = 36 \pi 10^9 / 3 \times 10^8 \sqrt{1 - \left(\frac{9}{18}\right)^2}$$

$$\beta = 326.483 \text{ rad/m}$$

(ii) Phase velocity V_g

$$V_g = \frac{c}{\sqrt{\mu_r \epsilon_r} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \omega / \beta$$

$$V_g = 3 \times 10^8 / \sqrt{1 \times 1} \sqrt{1 - \left(\frac{9}{18}\right)^2}$$

$$V_g = 3.464 \times 10^8 \text{ m/s}$$

(iii) wave impedance of the wave Z_g .

$$Z_g = \eta_o / \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \omega\mu / \beta$$

$$Z_g = 120\pi / \sqrt{1 - \left(\frac{9}{18}\right)^2}$$

$$Z_g = 435.31 \, \Omega$$

Problem 3:

An air-filled rectangular waveguide of inside dimensions 7x3.5cm operates in dominant mode(TE₁₀).

- (i) Find the cut-off frequency.
- (ii) Determine the phase velocity of the wave in the guide at a frequency of 3.5 GHz.
- (iii) Determine the guided wavelength at the same frequency.

Solution: (i) the cut-off frequency $m = 1, n = 0$

$$f_{cmn} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2}$$
$$f_{c10} = \frac{1}{2a\sqrt{\mu\epsilon}}$$

where in air filled medium $\sqrt{\mu\epsilon} = \sqrt{\mu_0\epsilon_0} = \frac{1}{c} = 1/3 \times 10^8$

$$f_{c10} = \frac{c}{2\sqrt{\mu_0\epsilon_0}},$$

$$f_{c10} = \frac{3 \times 10^8}{2 \times 7 \times 10^{-2} \sqrt{1 \times 1}}$$

$$f_{c10} = 2.14 \text{ GHz}$$

(ii) the phase velocity of the wave in the guide at a frequency of 3.5 GHz

$$V_g = \frac{c}{\sqrt{\mu_r \epsilon_r} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$V_g = \frac{3 \times 10^8}{\sqrt{1 \times 1} \sqrt{1 - \left(\frac{2.14}{3.5}\right)^2}}$$

$$V_g = 3.78 \times 10^8 \text{ m/s}$$

(iii) the guided wavelength at the same frequency

$$\lambda_g = \frac{\lambda}{\sqrt{\mu_r \epsilon_r} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad \text{where } \lambda = c/f = 3 \times 10^8 / 3.54 = 0.085$$
$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{2.14}{3.5}\right)^2}}$$

$$\lambda_g = \frac{0.085}{0.79}$$

$$\lambda_g = 0.108$$

Problems based on Unit 2

Problem 4:

A rectangular waveguide resonator is operating at 9 GHz in dominant mode. If the resonator dimensions are in the ratio of $a:b:d=2:1:4$, determine the dimensions of the resonator, if it is filled with dielectric having $\epsilon_r = 4$.

Solution: Dominant mode TE_{101} , $m=1$, $n=0$, $p=1$

$$a=2b \text{ and } d=4b, \quad f_r = 9\text{GHz}$$

$$\mu_r \epsilon_r = 1 \times 4 = 4$$

$$f_r = \frac{c}{2\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$
$$9 \times 10^9 = \frac{3 \times 10^8}{2\sqrt{4}} \sqrt{\left(\frac{1}{2b}\right)^2 + \left(\frac{0}{b}\right)^2 + \left(\frac{1}{4b}\right)^2}$$

Problems based on Unit 2

$$3x10x4 = \sqrt{\left(\frac{1}{2b}\right)^2 + \left(\frac{1}{4b}\right)^2}$$

$$b = 4.658\text{mm}$$

$$a = 9.317\text{mm}$$

$$d = 18.63\text{mm}$$



THANK YOU

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