3

Introduction to S-Parameters

3.1 Introduction

This chapter describes circuit parameters that are based on power flow. The open-circuit Z-parameters and the short-circuit Y-parameters are used in many of the circuit computer-aided design (CAD) programs. We can calculate the network parameters of circuits made from well-known components such as resistors, capacitors, and inductors. However, the circuit parameters of transistors and other types of semiconductors must be measured. As frequencies rise, it becomes increasingly difficult to measure voltage and current on the ports of a circuit.

This chapter introduces transmission lines and the mathematical expression that is used to describe their voltage and current. A detailed discussion of the electromagnetic foundation of transmission line theory is beyond the scope of this book. Instead, we focus on the propagation of power and signals and show how the power flow relates to voltage and current at any point along a transmission line. Transmission lines are an integral part of the transmission of power and signals in high-frequency circuits. In microwave circuits, rather than using resistors, capacitors, or inductors, transmission lines are employed to connect components and produce a complex impedance.

Further, we will develop another set of circuit parameters based on power flow in a transmission line. Whereas voltage and current are difficult to measure at microwave frequencies, power flow can be measured easily with directional couplers. Scattering parameters, or S-parameters, are ratios of power flow amplitude and phase in a circuit. S-parameters are usually listed in the transistor's data sheets. Since S-parameter power vectors are related to voltage and current in a transmission line, we will show the relation between S-, Z-, Y-, and chain parameters.

In the last section of this chapter, some of the more common transmission line technologies are introduced. Coaxial cables and waveguides are very common interconnected transmission media found in RF and microwave systems. Planar transmission lines are used in microwave circuits because these lines can easily be manufactured on printed circuit boards. Microwave circuits are commonly made using a microstrip transmission line structure. Such a structure consists of a conducting strip of a specified width and thickness suspended above a uniform ground plane by an insulating substrate or circuit board material. Microstrip lines are an efficient transmission medium and easy to manufacture. Elements of microstrip lines are used to interconnect components and create the complex impedances needed to design a circuit.

3.2 Transmission Lines

A transmission line is a generic term describing a medium or system of propagating energy from one point to another. As frequencies increase, the wires and interconnects between circuit components become larger compared to the wavelength of the signal flowing along them. Consider an interconnect between several components shown in Figure 3-1. At low frequencies, this wire can be considered part of the node connecting these components. At high frequencies, the wire's physical length introduces a noticeable delay to the signal traveling from one end to the other. For example, a 10 GHz signal has a wavelength of 3 cm. A signal entering one end with a positive voltage will appear a half-wavelength later as a negative voltage. When the voltage or current is measured at each end of the wire, or along it, a different value will be found at different points along the line. Solid wire interconnects can no longer be considered internal to the node, and the current entering one end of the wire is not necessarily equal to the current leaving the other end. The wire must be considered as a distributed element and modeled as many nodes distributed along the wire. This is called a transmission line.

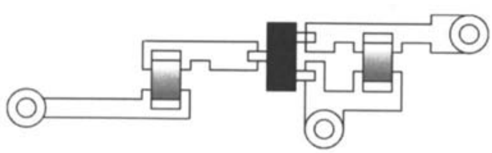

Figure 3-1 *A circuit with two components connected with a circuit trace.*

In this section, we will consider uniform transmission lines only. A uniform transmission line is a propagation medium in which wavelength, impedance and loss are constant along the entire length of the line. We will begin by modeling the transmission line as a series of circuit elements. Next, these circuit elements will be used to find the characteristic impedance, loss-per-unit length, and speed of propagation of the transmission line.

Before studying transmission lines, we will define the way we express voltage and current in a circuit. Consider a sinusoidal voltage source where the voltage is given by

$$V = V_o \cos(2\pi ft) \tag{3.1}$$

where f is the frequency of the source voltage and t is time. Equation 3.1 describes a voltage that oscillates in time from positive to negative in a sinusoidal fashion. Suppose there is a second part, an imaginary component, to this voltage that helps in the analysis of the circuit and networks. By superposition, we can add this imaginary signal to the real signal and then remove it (or ignore it) when it is no longer needed. This imaginary part is the sin component of the voltage. We can write

$$V = V_o\left(\cos(2\pi ft) + j\sin(2\pi ft)\right) = V_o e^{j2\pi ft} = V_o e^{j\omega t} \tag{3.2}$$

As will be shown, this exponential representation is a convenient way to express the voltage and current on a transmission line. Now that we have the basic equation of the voltage in the time domain, we can combine it with the effects of transmission line length. We add an imaginary component to the exponent to describe the way voltage changes at different points along the transmission line given by x.

$$V = V_o e^{j\omega t - j\beta x} \tag{3.3}$$

One other term is needed to characterize the loss (or gain) in a signal along the length of a transmission line. The general expression for voltage on a transmission line is

$$V = V_o e^{-\gamma + j\omega t} \tag{3.4}$$

where

$$\gamma = x(j\beta + \alpha) \tag{3.5}$$

In Equation 3.5, the frequency is expressed in radians/second, or $2\pi f$, and α is the loss per wavelength. This describes a wave propagating in one direction along the transmission line. When β is positive, the wave moves along the transmission line in the positive x direction. This is called the forward-propagating wave. We can define a wave that travels backward along the transmission line by this method.

$$V = V_r e^{j\omega t + x(j\beta + \alpha)} \tag{3.6}$$

Waves can propagate in both directions simultaneously. The total voltage on the transmission line is the superposition of the forward- and backward-traveling waves given by

$$V = \left(V_f e^{-\gamma} + V_r e^{\gamma}\right) e^{j\omega t} \tag{3.7}$$

Figure 3-2(a) shows the forward-traveling wave at time t along the transmission line, and Figure 3-2(b) shows the reflected wave at time t. Notice the sinusoidal variation of the voltage with distance along the line and the attenuation of the wave as it propagates. Current along the transmission line is expressed in the same way (V_s is replaced with I_s) and is separated into forward-traveling current waves and reverse current waves.

$$I = \left(I_f e^{-\gamma} + I_r e^{\gamma}\right) e^{j\omega t} \tag{3.8}$$

The wavelength denotes the distance along the line where the voltage goes through one complete cycle, i.e., for example, from one voltage peak to the next. In terms of the voltage vector V, the wavelength is the distance Δx, which the phase of the wave vector goes 360 degrees, or 2π radians.

$$\lambda = \frac{2\pi}{\beta} \tag{3.9}$$

The propagation velocity is the distance the wave travels (Δx) along the transmission line in a certain time (Δt). If we look at the voltage at time $t + \Delta t$ along the transmission line, we must find a location $x + \Delta x$ where the voltage has passed through 2π of phase change.

$$\omega \Delta t = 2\pi = \beta \Delta x \tag{3.10}$$

The change in distance divided by the equivalent change in time is velocity.

$$v = \frac{\omega}{\beta} \tag{3.11}$$

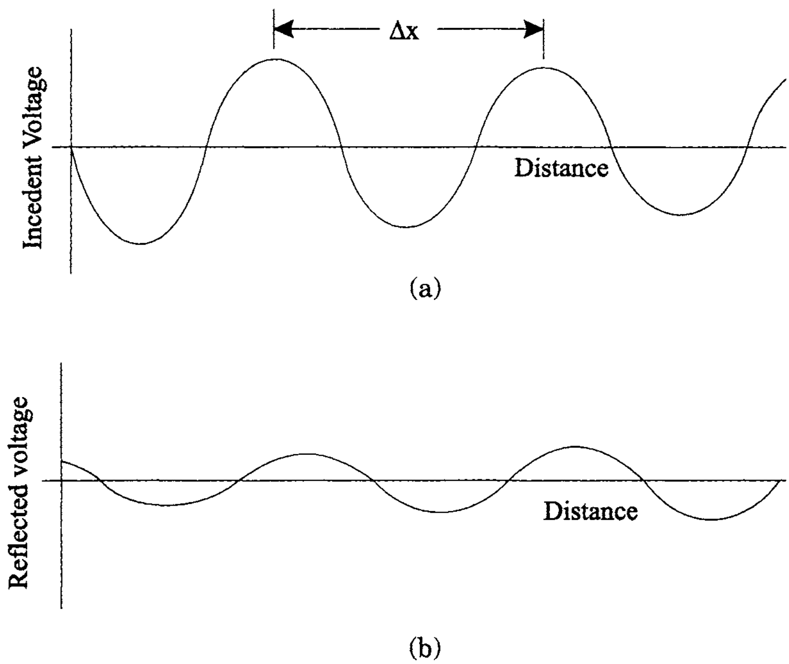

Figure 3-2 *The forward (a) and reflected (b) voltage waves along the transmission line at an instant in time.*

Example 3.1: Given a voltage of 2cos(4t + 3x) on a transmission line running in the x direction, find the wavelength and propagation velocity of the signal. If the current is I = .5cos(4t + 3x + π), what is the impedance of the transmission line at this frequency?

$$V = 2\cos(4t + 3x)$$

$$V = 2e^{j4t+j3x}$$

$$\omega = 4 \text{ rad/s}$$

$$\beta = 3$$

$$\lambda = \frac{2\pi}{3}$$

$$v = \frac{4}{3}$$

$$Z = \frac{2e^{j4t+j3x}}{0.5e^{j4t+j3x+j\pi}}$$

$$Z = 4e^{j\pi} = -4$$

From this point in our discussion, we will assume that there is a frequency term ($e^{j\omega t}$) associated with any voltage or current on a transmission line. Since we know there is a frequency term associated with all of these quantities, we will not write $e^{j\omega t}$ in every equation. It will be easier to work with voltage and current if we do not have to put $e^{j\omega t}$ every time. However, we must keep in mind that a frequency term accompanies all of the voltage and current terms. Some authors give these variables a special font or put a tilde over the letter. In this book, we will simply ignore the frequency term until we determine the value of a component such as a capacitor or an inductor.

A transmission line can be represented by an equivalent circuit that accounts for its distributed nature. We start with a small segment of transmission line that has a length of Δx. The equivalent circuit of a transmission line with a series resistance $R\Delta x$, an inductance $L\Delta x$, a shunt conductance of $G\Delta x$ and a capacitance of $C\Delta x$ is shown in Figure 3-3. If the length of the transmission line element Δx is small, Kirchhoff's laws will apply.

$$V(x-\Delta x) = V(x) - I(x)(sL+R)\Delta x \qquad 3.12$$

$$I(x-\Delta x) = I(x) - V(x)(sC+G)\Delta x \qquad 3.13$$

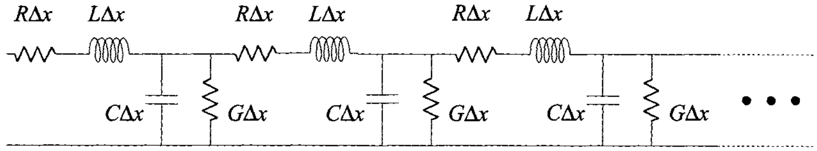

Figure 3-3 *Lumped component equivalent of a transmission line.*

If we divide both sides of the equation by Δx, rearrange the terms and take the limit as Δx approaches zero, we receive two differential equations in the form

$$-(sC+G)V(x) - (sL+R)I(x) = \frac{\partial I(x)}{\partial x} \qquad 3.14$$

$$-(sL+R)I(x)=\frac{\partial V(x)}{\partial x} \tag{3.15}$$

It is no coincidence that the equations for the total voltage and current on a transmission line, Equations 3.7 and 3.8, are solutions to these differential equations. When these formulas are used, Equations 3.14 and 3.15 become

$$I=I_f e^{-\gamma}+I_r e^{\gamma} \tag{3.16}$$

$$V=V_f e^{-\gamma}+V_r e^{\gamma} \tag{3.17}$$

$$-(sL+R)\left(I_f e^{-\gamma}+I_r e^{\gamma}\right)=-\gamma V_f e^{-\gamma}+\gamma V_r e^{\gamma} \tag{3.18}$$

$$-(sL+R)\left(V_f e^{-\gamma}+V_r e^{\gamma}\right)=-\gamma I_f e^{-\gamma}+\gamma I_r e^{\gamma} \tag{3.19}$$

These equations must hold everywhere along the transmission line for all x.

$$(sL+R)I_f=\gamma V_f \tag{3.20}$$

$$(sC+G)V_f=\gamma I_f \tag{3.21}$$

Solving Equations 3.20 and 3.21 for γ yields the propagation constant in terms of the circuit elements.

$$\gamma=\sqrt{(sL+R)(sC+G)} \tag{3.22}$$

The characteristic impedance of a transmission line is the ratio of the forward-traveling voltage wave and forward-traveling current wave.

$$Z_o=\frac{V_f}{I_f} \tag{3.23}$$

$$=\frac{sL+R}{\gamma} \tag{3.24}$$

$$=\sqrt{\frac{sL+R}{sC+G}} \tag{3.25}$$

Characteristic impedance can also be found from the reverse traveling voltage and current waves.

$$\frac{V_r}{I_r} = \frac{-(sL+R)}{\gamma} = -Z_o \qquad 3.26$$

Example 3.2: A transmission line has a resistance of 2 ohms/m, an inductance of 1.5 microhenry/meter, 0.1 mhos/meter and 0.10 micofarid/meter. Find the characteristic impedance, loss, and propagation velocity at 1 MHz. The characteristic impedance is found using Equation 3.25.

$$Z_o = \sqrt{\frac{j2\pi(1E6)(1.5E-6)+2}{j2\pi(1E6)(1E-7)+0.1}}$$

$$Z_o = 3.924 - j0.7282\Omega$$

The propagation constant is found from Equation 3.22.

$$\gamma = \sqrt{(j2\pi(1E6)(1.5E-6)+2)(j2\pi(1E6)(1E-7)+0.1)}$$

$$\gamma = 0.4517 + j2.434$$

The loss is the real part of the propagation constant,

$$\alpha = 0.4517$$

and the propagation velocity is found using Equation 3.11.

$$v = \frac{2\pi(1E6)}{2.434} = 2.581 \text{ m/s}$$

Impedance is defined as the ratio of voltage and current traveling in the same direction on a transmission line. Another important quantity is the *reflection coefficient,* which is the ratio of forward- and reverse-traveling waves.

$$\Gamma = \frac{V_f}{V_r} \qquad 3.27$$

The reflected wave normally will be less than the incident, or forward, wave. The reflection coefficient will be a complex number with a magnitude of 1.0 or less. Because the forward and reflected voltage may differ in phase, the reflection coefficient will have an argument, or phase angle component.

In this section, we have postulated the existence of a forward- and reverse-traveling wave regardless of where these waves originate. There are two ways that these waves are created — the simplest being two voltage or current sources on either end of the transmission line. A more interesting and more common source of these two waves is the use of one source, or generator, and a circuit element that causes a mismatch on the transmission line. Consider the case where a transmission line is leading to a circuit element Z_L, as shown in Figure 3-4. The voltage and current at this termination, or load, is related by Ohm's law.

$$Z_L = \frac{V}{I} = \frac{V_f\, e^{-\gamma} + V_r e^{\gamma}}{I_f e^{-\gamma} - I_r e^{\gamma}} \qquad 3.28$$

$$= \frac{V_f\, e^{-\gamma} + V_r e^{\gamma}}{\frac{V_f}{Z_o} e^{-\gamma} - \frac{V_r}{Z_o} e^{\gamma}} \qquad 3.29$$

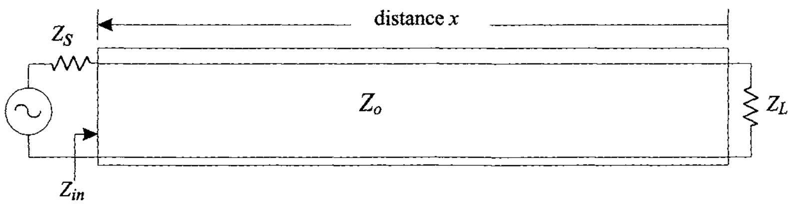

Figure 3-4 *Transmission line with a characteristic impedance of Z_o terminated with a load impedance of Z_L.*

When the location of the load impedance is at $x = 0$, the plane of reference,

$$Z_L = Z_o \frac{V_f + V_r}{V_f - V_r} \qquad 3.30$$

Solving Equation 3.30 for V_f and V_r, the reflection coefficient in terms of the characteristic impedance of the transmission line (Z_o) and the load impedance (Z_L) can be found to be

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad 3.31$$

At other points along the transmission line the voltage and current of the waves are different. The reflection coefficient changes as we move along the transmission line. This is demonstrated in Equation 3.29, which can be rewritten in terms of Z_o and Z_L

$$Z_{in} = Z_o \frac{Z_L \cosh\gamma\, x + Z_o \sinh\gamma\, x}{Z_o \cosh\gamma\, x + Z_L \sinh\gamma\, x} \qquad 3.32$$

where Z_{in} is the input impedance and x is the length of the transmission line. If the transmission line is lossless, Equation 3.32 becomes

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta\; x}{Z_o + jZ_L \tan\beta\; x} \qquad 3.33$$

The reflection coefficient along the transmission line changes as we move along the line. Using the forward- and reverse-traveling waves from Equation 3.17 in Equation 3.27, the reflection coefficient at location x is

$$\Gamma(x) = \Gamma_o e^{-2\gamma x} \qquad 3.34$$

where Γ_o is the reflection coefficient at $x = 0$. When the transmission line is lossless,

$$\Gamma(x) = \Gamma_o (\cos 2\beta x - j\sin 2\beta x) \qquad 3.35$$

When moving along a lossless transmission line, the magnitude of the reflection coefficient stays constant. The argument, or angle, of $\Gamma(x)$ varies as $\exp(-2\beta x)$ as the reference plane moves in the x direction.

The reflection coefficient is frequently used in the analysis and design of microwave circuits to cancel unwanted reflections or cause a specific circuit response. These circuits are designed using components or circuit elements that cause reflections on a transmission line. The reflection coefficient is such a valuable tool that a method of graphing impedances and admittances in relation to the reflection coefficients they create has been developed.

Example 3.3: Figure 3-5 shows a transmission line with a characteristic impedance of 75 ohms that is terminated by a 100 ohm load. Find the reflection coefficient at any point along the transmission line. The reflection coefficient at the end of the transmission line where $x = 0$ is

$$\Gamma_o = \frac{100 - 75}{100 + 75} = \frac{25}{175} = 0.143$$

The reflection coefficient at any value of x is

$$\Gamma(x) = 0.143(\cos 2\beta x - j\sin 2\beta x)$$

In terms of wavelengths we can write

$$\Gamma(x) = 0.143\left(\cos\frac{4\pi}{\lambda}x - j\sin\frac{4\pi}{\lambda}x\right)$$

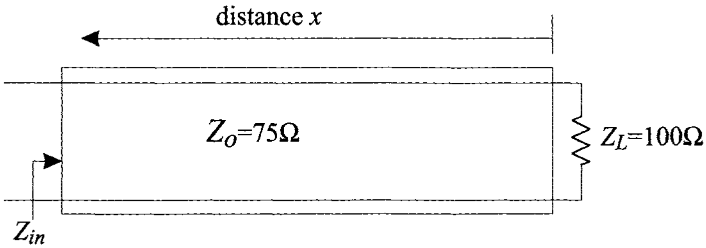

Figure 3-5 *The transmission line for Example 3.3.*

3.3 Introduction to the Smith Chart

The Smith chart is a graphical tool used to visualize, analyze, and design high-frequency circuits. It was invented by Phillip Smith [5] and has become an indispensable tool for microwave engineers. Figure 3-6 shows a Smith chart that has constant impedance lines. Two sets of circles can be seen on the Smith chart. In one set, the circles are complete and nested, one inside the other, all touching at the far right-hand side of the chart. These are circles of constant resistance, which will be discussed later in this section. The other set consists of partial circles, all touching the horizontal centerline of the chart. These are constant reactance circles, or plots, of the constant imaginary part of impedance.

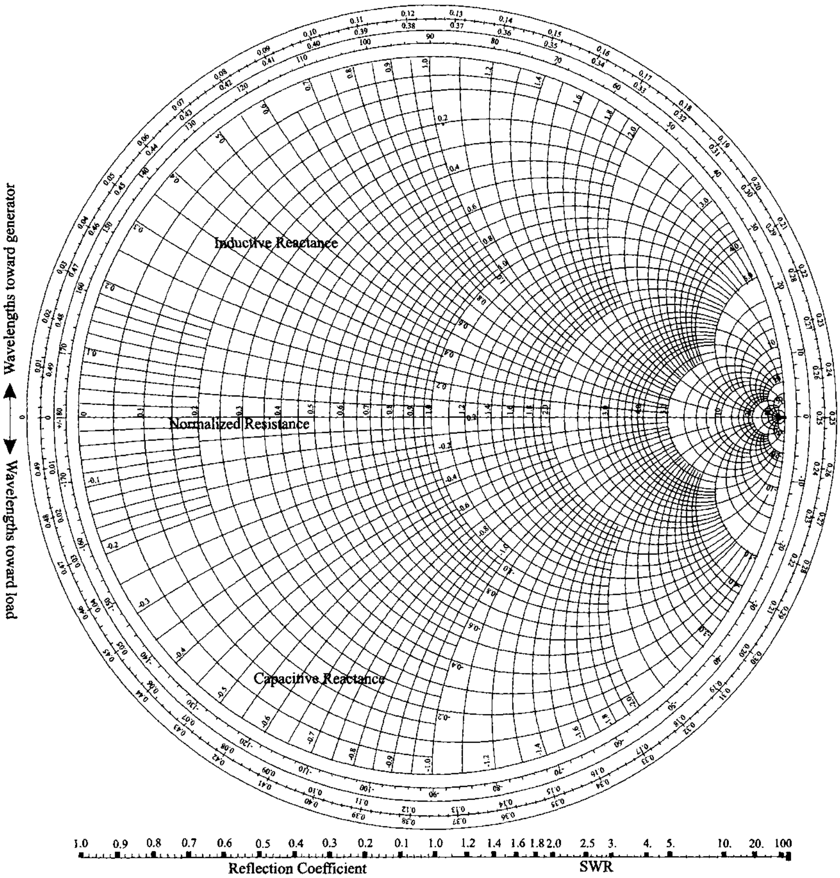

Figure 3-6 *The Smith chart plotting constant impedance circles.*

Figure 3-7 shows a Smith chart with constant admittance lines. On the admittance Smith chart, the full circles are the constant conductance lines. The partial circles are the constant susceptance lines. One common characteristic of the admittance and the impedance Smith chart is the location of the open and short on both charts. In fact, a reflection coefficient will be plotted in the same place on both charts.

If a rectangular grid is superimposed onto the Smith chart, the real and imaginary part of the reflection coefficient can be plotted in Cartesian coordinates. The horizontal axis is the real part of the reflection coefficient, and the vertical axis is the imaginary part. The reflection coefficient equals zero at the center of the Smith chart. The edge of the Smith chart corresponds to all points where the reflection coefficient is equal to 1. When a reflection coefficient is expressed as a magnitude A and angle α, it is plotted on the Smith chart at a radius of A from the center of the chart, at an angle α from the positive x-axis.

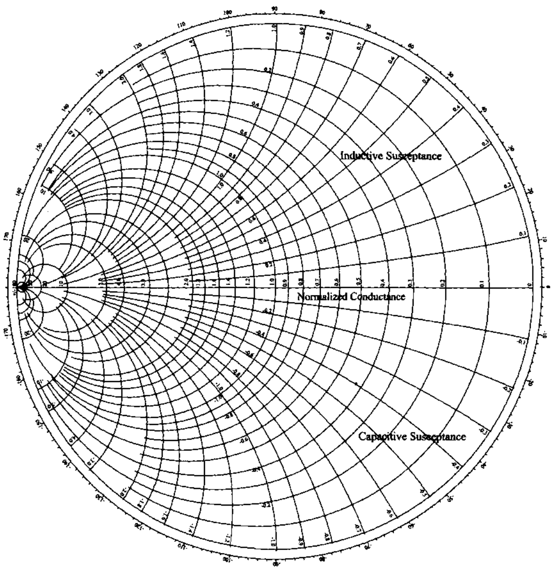

Figure 3-7 *The Smith chart plotting constant admittance circles.*

The simplest and most common way to use a Smith chart is to convert a load impedance to a reflection coefficient and vice versa. The reflection coefficient is a function of characteristic impedance Z_o and the load impedance. When we normalize the load impedance by the characteristic impedance, the reflection coefficient becomes

$$\Gamma = \frac{Z-1}{Z+1} \quad 3.36$$

where $Z = Z_L/Z_o$. Constant resistance circles appear on the Smith chart as circles that are always centered on the real axis and touch the same point at the far right side of the chart. With $\Gamma = u + jv$ and $Z = x + jy$, Equation 3.36 becomes

$$u + jv = \frac{x + jy - 1}{x + jy + 1} \quad 3.37$$

which can be written as

$$jy + x - jv + vy - jvx - u - juy - ux = 1 \tag{3.38}$$

Equation 3.34 is separated into the real and imaginary parts:

$$jy - jv - jvx - juy = 0 \tag{3.39}$$

and

$$x + vy - u - ux = 1 \tag{3.40}$$

We first solve Equations 3.39 and 3.40 for y.

$$y = \frac{v(1+x)}{1-u} \tag{3.41}$$

We then solve for x by substituting Equation 3.41 back into Equations 3.39 and 3.40 so that it too can be eliminated.

$$\begin{aligned} v^2 &= \left(u - \frac{x}{1-x}\right)^2 = \frac{x^2 = x - 1}{(x = 1)^2} \\ (u-1)^2 &= \left(v - \frac{1}{2y}\right)^2 = \frac{1}{4}y^2 \end{aligned} \tag{3.42}$$

This exercise yields the equation of a circle.

$$(x-a)^2 + (y-b)^2 = r^2$$

where the center of the circle is at $x = a, y = b$, and r is the radius. By comparing the equation of a circle to Equation 3.42, circles of constant resistance are centered at $b = 0$ in all cases, and the radius is always equal to a. In other words, the circles are always centered on the x-axis. Since the radius and the x location of the center are the same, all the resistance circles will meet on the x-axis when $\Gamma = 1$. Another equation of a circle appears that is centered at $a = 1$ in all cases and that r is equal to b.

Example 3.4: Using the Smith chart in Figure 3-6 with a characteristic impedance of 50 ohms, plot the following load impedances:

- A. 120 $-j75$ ohms,
- B. 30 $+j120$,
- C. a 50 ohm load with any possible imaginary part,
- D. a $-j30$ ohm load with any possible real part.

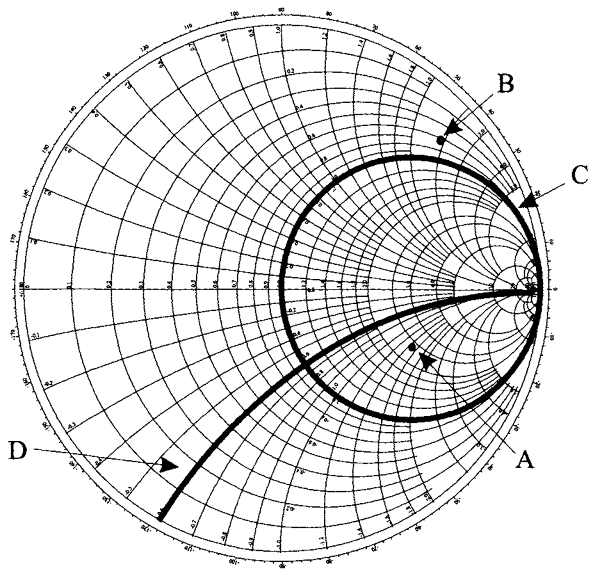

Figure 3-8 *The impedances in Example 3.4 are plotted on the 50 ohm Smith chart.*

The Smith chart is valuable for plotting impedance along a transmission line. Consider the transmission line shown in Figure 3-5. The reflection coefficient at the load (when $x = 0$) is not matched to the transmission line impedance. We can easily plot this point on the Smith chart. Let us begin to add length to the transmission line so that the distance increases. If the transmission line is lossless, Equation 3.35 shows that the magnitude of the reflection coefficient remains constant and only the angle changes. When $\Gamma(x)$ is plotted on the Smith chart, the point moves in a circle, centered in the middle of the Smith chart, and rotates in the clock-wise direction. The arrow in Figure 3-6 labeled "toward the generator" shows the direction of rotation as transmission line length is added. If the transmission line is shortened, the reflection coefficient, as plotted on the Smith chart, rotates counter-clockwise. This direction is shown in Figure 3-6 by the arrow labeled "toward the load."

Example 3.5: Figure 3-9 shows two transmission lines with one of the lines terminated. Plot Z_{in} on the Smith chart. A load with an impedance of 45 -j20 ohms is connected to a 50 ohm transmission line that is one-eighth of a wavelength long. Then, the 50 ohm transmission line is connected to a 75 ohm transmission line that is three-eights of a wavelength long. Find the input impedance of the circuit using the Smith chart.

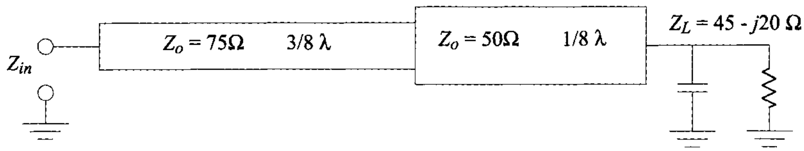

Figure 3-9 *Circuit for Example 3.5.*

Since the transmission line on the right is a 50 ohm line, we begin with a 50 ohm Smith chart. The termination impedance of 45 $-j20$ is plotted at a point labeled Z_L on the Smith chart in Figure 3-10(a). The impedance is transformed along path B by the 45 degrees long transmission line ending at point C.

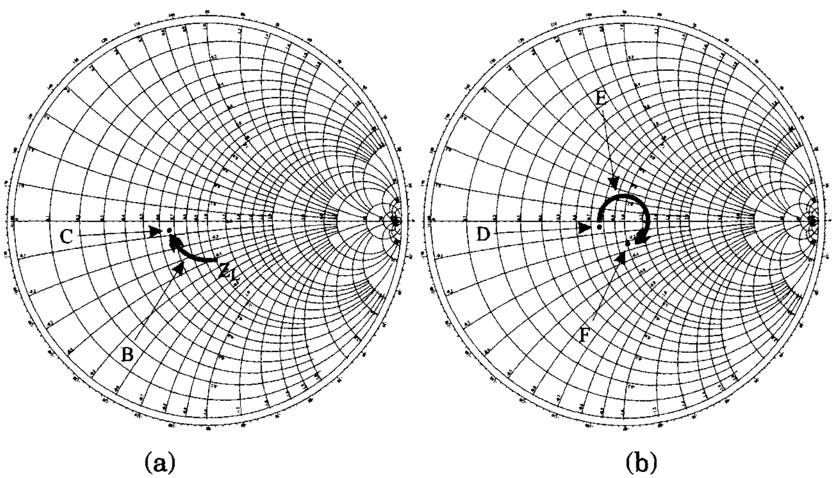

Figure 3-10 *The 50 ohm Smith chart (a) and the 75 ohm Smith chart (b) for Example 3.5.*

The next transmission line is a 75 ohm characteristic impedance; therefore, point C is transferred to a 75 ohm Smith chart in Figure 3-10(b). The impedance at point C is found by multiplying the reading on the Smith chart in Figure 3-10(a) by 50 ohms. The impedance is then divided by 75 ohms and plotted on the 75 ohm Smith chart in Figure 3-10(b) at point D. Finally, the reflection coefficient at point D is transformed around the Smith chart by the last transmission line, shown as path E, which is 3/8 of a wavelength long. The input impedance and reflection coefficient of the circuit is plotted at point F.

3.4 Wave Vectors and S-Parameters

At high frequencies it is difficult to measure the voltage and current at a terminal of a device or network. On the other hand, directional couplers can easily measure the power flow into or out of a circuit. Hence, it is suitable to describe the electrical properties of a circuit by some means of power flow or ratios thereof.

Concerning the incident voltage and current in a transmission line, it would be more convenient to normalize these by the characteristic impedance (Z_o). We can define a normalized incident vector a in terms of the incident voltage and current.

$$a = \frac{V_f}{\sqrt{Z_o}} = I_f\sqrt{Z_o} = \frac{V_f + Z_o I_f}{2\sqrt{Z_o}} \tag{3.43}$$

We can also define a normalized reflected vector b, which is a function of the reflected voltage and current.

$$b = \frac{V_r}{\sqrt{Z_o}} = -I_r\sqrt{Z_o} = \frac{V_r - Z_o I_r}{2\sqrt{Z_o}} \tag{3.44}$$

In terms of a and b, the incident power is

$$P^+ = |a|^2 \tag{3.45}$$

The reflected power is

$$P^- = |b|^2 \tag{3.46}$$

The reflection coefficient is

$$\Gamma = \frac{b}{a} \tag{3.47}$$

The actual power delivered to the load will be

$$P = |a|^2 - |b|^2 \tag{3.48}$$

and the VSWR of the termination is

$$\rho = \frac{|a| + |b|}{|a| - |b|} \tag{3.49}$$

With the new set of variables (a and b) it is simpler to compare the calculated circuit response to the power flow measured with directional couplers and power meters.

Usually, there is very little interest in the actual values of a and b. Rather, the interest lies in their ratio, which defines reflection and transmission of power in a network. The reflection coefficient Γ in Equation 3.47 is the S-parameter representation of a one port. Just as with Z- and Y-parameters, S-parameters can be applied to n-port networks by defining incoming and outgoing waves at each port:

$$a_i = \frac{1}{2}\left(\frac{V_i + Z_o I_i}{\sqrt{Z_o}}\right) \tag{3.50}$$

$$b_j = \frac{1}{2}\left(\frac{V_j - Z_o I_j}{\sqrt{Z_o}}\right) \tag{3.51}$$

where i and j are the port number. For the two-port network shown in Figure 3-7, the S-parameters are defined as

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \tag{3.52}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \tag{3.53}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \tag{3.54}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \tag{3.55}$$

The power wave leaving Port 1 is the sum of the power reflected from the input port ($S_{11}a_1$) plus the power passing through the two-port from Port 2 to Port 1 ($S_{21}a_2$).

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{3.56}$$

Correspondingly, the power wave leaving Port 2 is

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{3.57}$$

or, in matrix representation,

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{3.58}$$

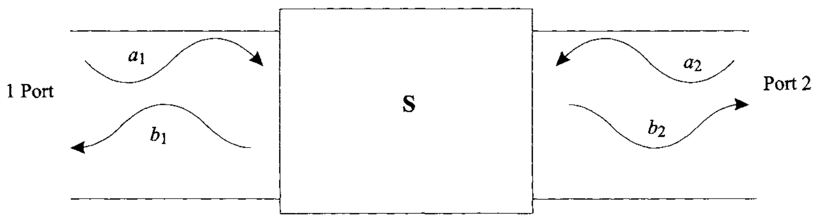

Figure 3-11 *A two-port network and the incident vectors a_1 and a_2 and outgoing waves b_1 and b_2 at the two ports.*

Since the waves a_i and b_i are functions of voltage and current vectors, the S-parameters can be related to the Z-, Y-, and ABCD-parameters of the network. Let us look at the matrix of S-parameters and column vectors **a**' and **b**' in Equation 3.58

$$\mathbf{b}' = \mathbf{S}\mathbf{a}' \tag{3.59}$$

where

$$\mathbf{a}' = \frac{1}{\sqrt{Z_o}}(\mathbf{V} + Z_o\mathbf{I}) \tag{3.60}$$

$$\mathbf{b}' = \frac{1}{\sqrt{Z_o}}(\mathbf{V} - Z_o\mathbf{I}) \tag{3.61}$$

and **V** and **I** are the column vectors consisting of the voltages on the two-port terminals and the current flowing into the terminals. The impedance matrix of the two port is defined as

$$\mathbf{V} = \mathbf{ZI} \tag{3.62}$$

Inserting Equation 3.62 into Equations 3.60 and 3.61 yields

$$\mathbf{a}' = \frac{1}{\sqrt{Z_o}}(\mathbf{ZI} + Z_o\mathbf{I}) \qquad 3.63$$

$$\mathbf{b}' = \frac{1}{\sqrt{Z_o}}(\mathbf{ZI} - Z_o\mathbf{I}) \qquad 3.64$$

We find that the S-parameter matrix can be expressed as a function of the Z-parameter matrix.

$$\mathbf{S} = \frac{\mathbf{a}'}{\mathbf{b}'} = \frac{\mathbf{ZI} - Z_o\mathbf{I}}{\mathbf{ZI} + Z_o\mathbf{I}} = (\mathbf{Z} - Z_o)(\mathbf{Z} + Z_o)^{-1} \qquad 3.65$$

Note that the constant Z_o in Equation 3.60 was converted to a diagonal matrix by multiplying Z_o by the identity matrix. S-parameters can be related to other network parameters by performing similar derivations to the one above.

Example 3.6: Consider a 75 ohm transmission line that is terminated with a $35 + j120$ ohm load. We want to find the reflection coefficient at the load and the VSWR using the wave vectors.

Normalizing the voltage source to 1, the incident voltage and current are (as if the transmission line were perfectly matched)

$$V^+ = \frac{1}{2} \qquad\qquad I^+ = \frac{1}{150}$$

The reflected voltage and current are found to be

$$V^- = \frac{75 - (35 + j120)}{2(75 + (35 + j120))} = \frac{40 - j120}{220 + j240} = \frac{4}{-5 + j9}$$

$$I^- = \frac{75 - (35 + j120)}{150(75 + (35 + j120))} = \frac{4}{-375 + j675}$$

The vectors a and b are

$$a = \frac{1}{2\sqrt{75}} \qquad\qquad b = \frac{4}{\sqrt{75}(-5 + j9)}$$

and the reflection coefficient and the VSWR are

$$\Gamma = S_L = \frac{8}{-5+j9} \qquad \rho = \frac{\frac{1}{2}+\frac{4}{\sqrt{106}}}{\frac{1}{2}-\frac{4}{\sqrt{106}}} = 7.97:1$$

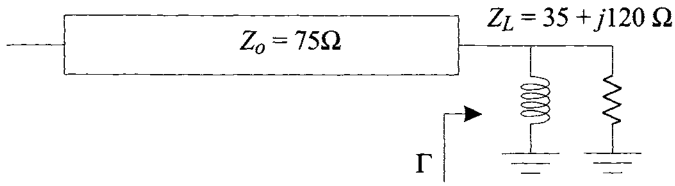

Figure 3-12 *The circuit for Example 3.6 with Z_o = 75 ohms and Z_L = 35 + j120 ohms.*

Example 3.7: Consider the circuit shown in Figure 3-13. The termination in Example 3.6 is connected to the same 75 ohm source through a transmission line 3/8 λ long. At the load, the parameters calculated above are the same. At the generator, the length of the transmission line has shifted the phases. In terms of incident voltage and current at the generator,

$$V_g^+ = \frac{1}{2}\exp\left(j2\pi\frac{3}{8}\right) = -\frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

$$I_g^+ = \frac{1}{150}\exp\left(j2\pi\frac{3}{8}\right) = \frac{1}{150}\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

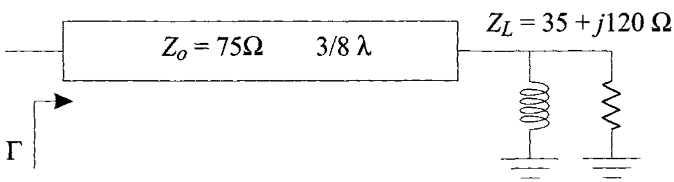

Figure 3-13 *The circuit of Example 3.7 with a transmission line 3/8 of a wavelength long.*

The reflected voltage and current are

$$V_g^- = \frac{4}{-5+j9}\exp\left(-j2\pi\frac{3}{8}\right) = \frac{2\sqrt{2}+j2\sqrt{2}}{5-j9}$$

$$I_g^- = \frac{4}{-375 + j675} \exp\left(-j2\pi \frac{3}{8}\right) = \frac{2\sqrt{2} + j2\sqrt{2}}{375 - j675}$$

The vectors a and b are

$$a' = \frac{1}{2\sqrt{75}} \left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$b' = \frac{2\sqrt{2} + j2\sqrt{2}}{\sqrt{75}(+5 - j9)}$$

The reflection coefficient and the VSWR are

$$S_{in} = \Gamma_{in} = \frac{b'}{a'} = \frac{\dfrac{4}{\sqrt{75}(-5 + j9)} \exp\left(-j2\pi \dfrac{3}{8}\right)}{\dfrac{1}{2\sqrt{75}} \exp(j2\pi \dfrac{3}{8})} = \frac{8}{-5 + j9} \exp\left(-j4\pi \frac{3}{8}\right)$$

$$= \frac{8 \exp(-j\pi \frac{3}{2})}{-5 + j9} = \frac{j8}{5 - j9}$$

$$\rho = 7.97:1 \qquad \left(\left|-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right| = 1\right)$$

Note that the phase of the reflection coefficient has been shifted by two times the electrical length because the waves must reach the load and then return through the length of transmission line.

Example 3.8: Consider a two-port network with Port 2 terminated, as shown in Figure 3-14. Suppose we want to know the input reflection coefficient Γ_i. The ratio of the reflected wave from the load (a_2) to the incident wave on the load (b_2) is

$$\Gamma_L = \frac{a_2}{b_2}$$

The incident wave on the output of the two-port is

$$a_2 = \Gamma_L b_2$$

Substituting this in Equation 3.56 and 3.57, the response of the two-port is given by

$$b_1 = S_{11}a_1 + S_{12}\Gamma_L b_2$$

$$b_2 = S_{21}a_1 + S_{22}\Gamma_L b_2$$

By solving the second equation for b_2

$$b_2 = \frac{S_{21}a_1}{1 - S_{22}\Gamma_L}$$

we can solve the first equation and find the input reflection coefficient.

$$\Gamma_i = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

The S-parameters can be manipulated algebraically just as any of the other linear parameters.

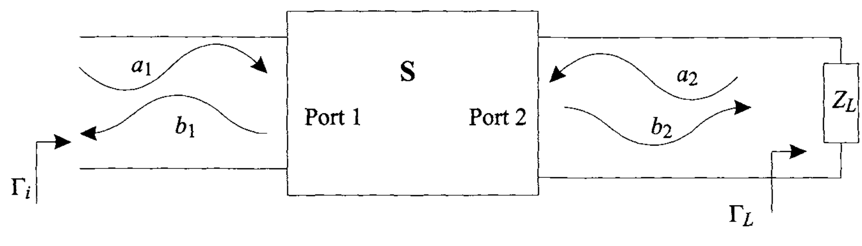

Figure 3-14 *The terminated two-port in Example 3.8.*

This has been a discussion of the basics of S-parameters and their relation to the other linear network parameters. The voltage across a load or network port is split into two components. One component is the voltage that would exist if the load or port were perfectly matched to the transmission line (otherwise known as the incident voltage), and the second component is the reflected voltage, or difference between the actual and incident voltages. The concept is the same for current. Voltage and current are the defining quantities of the incident and reflected waves a and b.

This discussion has simplified a few concepts. Most applications have a characteristic impedance of 50 ohms. In general, however, the characteristic impedance on the i^{th} port of a network can be any complex number Z_{si} other than 0 or infinity. In this case, we must define the column vectors $\boldsymbol{a}'$ and $\boldsymbol{b}'$ for an n-port as

$$\mathbf{a}' = \mathbf{R_s}(\mathbf{V} + \mathbf{Z_s I}) \tag{3.66}$$

$$\mathbf{b}' = \mathbf{R_s}\left(\mathbf{V} + \mathbf{Z_s}^* \mathbf{I}\right) \tag{3.67}$$

where R_s and Z_s are diagonal matrices whose i^{th} components are given by

$$\frac{1}{2}\frac{1}{\sqrt{\mathrm{Re}(Z_{si})}} \tag{3.68}$$

and Z_{si} respectively. We now derive new formulas for V, I and the conversion formulas for Y- and Z-parameters. Using the basics given in this chapter, we can now discuss power gain, insertion loss, load matching, and stability, categories which are needed for circuit design.

The following list of equations allows us to convert between two-port S-parameters and Z-, Y-, and chain parameters:

$$S_{11} = \frac{(z_{11}-1)(z_{22}+1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \tag{3.69}$$

$$= \frac{(1-y_{11})(1+y_{22}) + y_{12}y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \tag{3.70}$$

$$= \frac{Z_oA + B - C(Z_o)^2 - Z_oD}{Z_oA + B + C(Z_o)^2 + Z_oD} \tag{3.71}$$

$$S_{12} = \frac{2z_{12}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \tag{3.72}$$

$$= \frac{-2y_{12}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \tag{3.73}$$

$$= \frac{2Z_o(AD - BC)}{Z_oA + B + C(Z_o)^2 + Z_oD} \tag{3.74}$$

$$S_{21} = \frac{2z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \tag{3.75}$$

$$= \frac{-2y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \quad 3.76$$

$$= \frac{2Z_o}{Z_oA + B + C(Z_o)^2 + Z_oD} \quad 3.77$$

$$S_{22} = \frac{(z_{11}+1)(z_{22}-1) - z_{12}z_{21}}{(z_{11}+1)(z_{22}+1) - z_{12}z_{21}} \quad 3.78$$

$$= \frac{(1+y_{11})(1-y_{22}) + y_{12}y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \quad 3.79$$

$$= \frac{-Z_oA + B - C(Z_o)^2 + Z_oD}{Z_oA + B + C(Z_o)^2 + Z_oD} \quad 3.80$$

In Equations 3.69 to 3.79, $z_{ij} = Z_{ij}/Z_o$ and $y_{ij} = Z_o\, Y_{ij}$.

$$z_{11} = \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \quad 3.81$$

$$z_{12} = \frac{2S_{12}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \quad 3.82$$

$$z_{21} = \frac{2S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \quad 3.83$$

$$z_{22} = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \quad 3.84$$

$$y_{11} = \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}} \quad 3.85$$

$$y_{12} = \frac{-2S_{12}}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}} \quad 3.86$$

$$y_{21} = \frac{-2S_{21}}{(1+S_{11})(1+S_{22}) - S_{12}S_{21}} \quad 3.87$$

$$y_{22} = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}} \quad 3.88$$

$$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}} \quad 3.89$$

$$B = Z_o \frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}} \quad 3.90$$

$$C = \frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{Z_o 2S_{21}} \quad 3.91$$

$$D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}} \quad 3.92$$

S-parameters and transmission lines are the two main tools of the microwave engineer. Manufacturers measure transistors and publish the S-parameters in data sheets. Transistors are usually built into a circuit with transmission lines and other components. All these parts are designed to work in unison to amplify a signal by a predictable amount. Many other types of circuits are designed to filter, radiate, modulate, demodulate, or control microwave signals. There are a number of different types of transmission lines that are useful in certain applications.

3.5 Microstrip Transmission Lines

There are many physical transmission line structures used in microwave circuits; common transmission lines include waveguide, fiber optic cable, and coaxial cable. This section covers in some detail the most common type of transmission lines used in microwave amplifiers: the microstrip line.

Figure 3-15 shows some of the transmission structures used in microwave circuits. Waveguides are common in high-power and low-loss applications. Coaxial cables are frequently used to transmit signals from an antenna to an amplifier. Striplines are conducting strips sandwiched between two conducting ground planes. These strips are insulated from the ground planes by a dielectric. Voltage and current waves propagate along the strips guided by two ground planes. Characteristic impedance is mainly a function of the strip width, the dielectric constant of the substrate, and the thickness of the dielectric. Slot lines have a nonconducting slot where the wave propagates guided by two grounds on either side of the gap, as shown in Figure 3-15(d). Coplanar waveguides are similar to slotlines, but contain a conduction strip in the middle of the slot. A dielectric image line, shown in Figure 3-15(f), is common at frequencies above 60 GHz. An image line is a slab of dielectric material bonded to a conducting ground plane. The wave is guided by the surface of the slab and the ground plane.

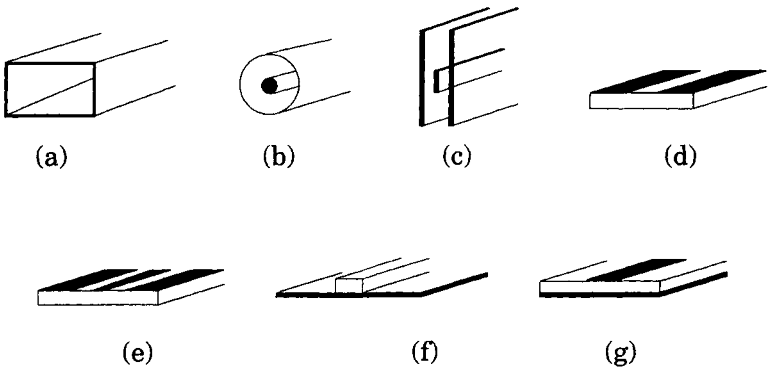

Figure 3-15 *Diagram of some microwave transmission structure: waveguide (a), coaxial cable (b), stripline (c), slot line (d), coplanar waveguide (e), image line (f) and microstrip line (g).*

Microstrip lines are the most common form of microwave transmission lines used in amplifiers. A microstrip line is a narrow, conducting strip attached to the top of a dielectric sheet. The bottom of the sheet is covered with a conductor, serving as a ground plane. Signals propagate along the strip. The impedance of a microstrip line primarily depends on the width and thickness of the microstrip line and the dielectric constant of the material. The velocity of propagation is also related to these parameters. The width of a microstrip line is sometimes expressed as a ratio of strip width to substrate thickness, or w/h ratio. The characteristic impedance of a coaxial line can be calculated by solving the electromagnetic boundary value equations. This calculation does not work for microstrip lines because there is no closed-form solution. However, some very good approximations have been developed [1, 3]. One of these approximations is given below for calculating the characteristic impedance given the relative dielectric constant of the substrate and the w/h ratio of the microstrip line. When the microstrip is narrow ($w/h < 3.3$), the characteristic impedance is

$$Z_o = \frac{119.9}{\sqrt{2(\varepsilon_r + 1)}}\left\{ \ln\left[4\frac{h}{w} + \sqrt{16\left(\frac{h}{w}\right)^2 + 2}\right] - \frac{1}{2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right)\right\} \qquad 3.93$$

when the microstrip is wide ($w/h > 3.3$), the characteristic impedance is

$$Z_o = \frac{119.9\pi}{2\sqrt{\varepsilon_r}}\left\{\frac{w}{2h} + \frac{\ln 4}{\pi} + \frac{\ln(e\pi^2/16)}{2\pi}\left(\frac{\varepsilon_r - 1}{\varepsilon_r^2}\right) + \frac{\varepsilon_r + 1}{2\pi\varepsilon_r}\left[\ln\frac{e\pi}{2} + \ln\left(\frac{w}{2h} + 0.94\right)\right]\right\}^{-1} \qquad 3.94$$

Figure 3-16 shows the characteristic impedance of a microstrip line as a function of width-to-height ratio. Each line represents the relation between impedance and w/h for different substrate dielectric constants.

It is more useful to obtain an expression for finding the w/h ratio given the desired characteristic impedance of the microstrip line and the dielectric constant. For narrow strips, $Z_o > 44 - 2\,\varepsilon_r$

$$\frac{w}{h} = \left(\frac{\exp H'}{8} - \frac{1}{4\exp H'}\right)^{-1} \qquad 3.95$$

where

$$H' = \frac{Z_o\sqrt{2(\varepsilon_r + 1)}}{119.9} - \frac{1}{2}\left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(\ln\frac{2}{\pi} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right) \qquad 3.96$$

When the strip is wide, $Z_o < 44 - 2\,\varepsilon_r$,

$$\frac{w}{h} = \frac{2}{\pi}\{(d-1) - \ln(2d-1)\} + \frac{\varepsilon_r - 1}{\pi\varepsilon_r}\left\{\ln(d-1) + 0.293 - \frac{0.517}{\varepsilon_r}\right\} \qquad 3.97$$

where

$$d = \frac{59.95\,\pi^2}{Z_o\sqrt{\varepsilon_r}} \qquad 3.98$$

The relative dielectric constant of the microstrip line is approximated by

$$\varepsilon_{eff} = \frac{\varepsilon_r}{0.96 + \varepsilon_r(0.109 - 0.004\varepsilon_r)\{\log(10 + Z_o) - 1\}} \qquad 3.99$$

The propagation velocity of the signal in a microstrip line is given by

$$v_g = \frac{c}{\sqrt{\varepsilon_o\varepsilon_{eff}}} \qquad 3.100$$

Figure 3-17 shows the ratio of wavelength on the microstrip line to free space wavelength as a function of w/h and substrate dielectric.

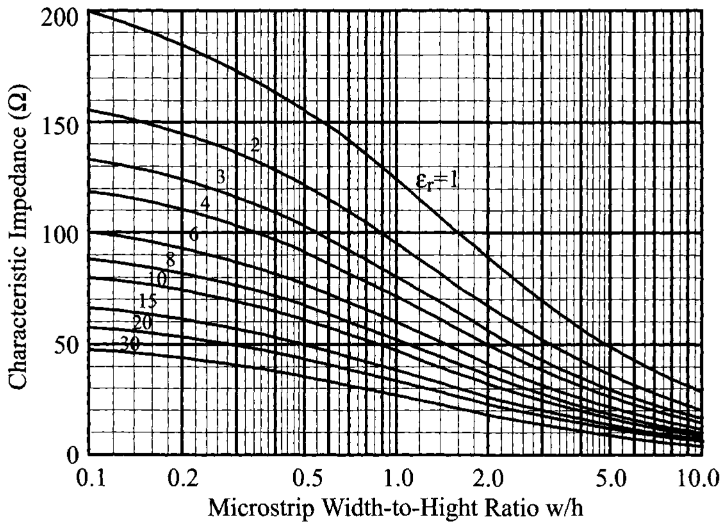

Figure 3-16 *Plot of characteristic impedance of microstrip lines.*

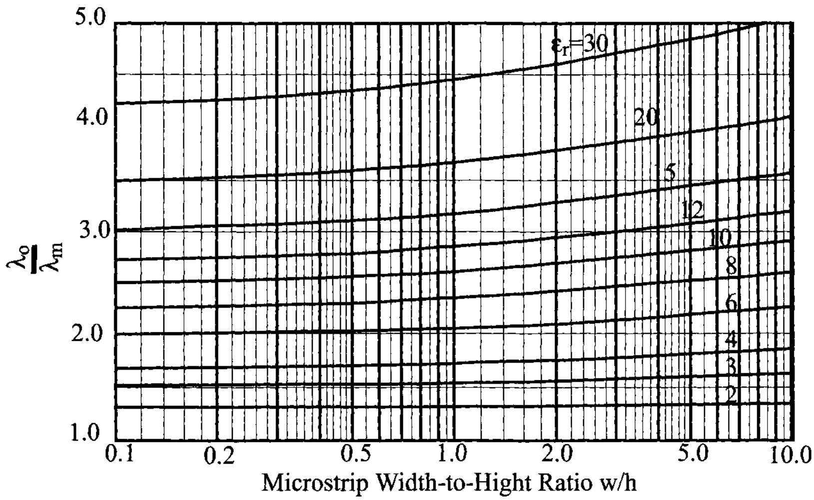

Figure 3-17 *Plot of wavelength on microstrip lines.*

One disadvantage of microstrip lines lies in the fact that they exhibit frequency-dispersive effects, i.e., the propagation characteristics of the microstrip line changes with frequency. However, this effect is negligible unless the frequency is high enough for the microstrip line or the substrate height to become an appreciable fraction of a wavelength. The relative permittivity experiences the most change as frequency increases. A few approximations have been published [2, 3, 4]; one approximation is by Edwards where h is the substrate height in millimeters.

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{\text{eff}}}{1 + (h/Z_o)^{1.33}(0.43f^2 - 0.009f^3)} \qquad 3.101$$

When the frequency is too high, another substrate should be used for the circuit. The upper frequency limit is thought to occur when the widest line becomes approximately one-eighth of a wavelength wide. The characteristic impedance is fairly constant with frequencies up to this point.

Many microstrip circuit elements, including open-circuited transmission lines, short-circuited transmission lines, and transmission lines of differing impedance are used as tuning elements when designing microwave amplifiers. These elements are joined and bent causing discontinuities in the orderly flow of power along the transmission line. Sometimes the effects of the discontinuities must be considered. Most microwave computer aided design systems have junction elements such as a step in the width of a microstrip line, T-junctions, X-junctions, and bends, as shown in Figure 3-18.

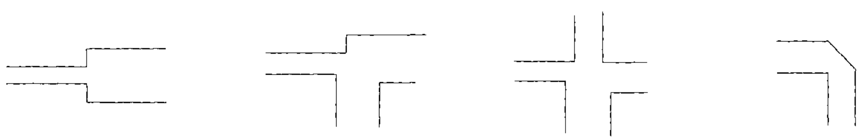

Figure 3-18 *Common discontinuities in microstrip lines: a step in width (a), a junction of three microstrip lines (b), a junction of four lines (c), and a 90 degree bend with a chamfered corner (d).*

Example 3.9: Find the width of a 50 ohm line on a sapphire substrate, ε_r=4.2, that is 0.65 mm thick. Since 50 is greater than 44-2 (4.2), Equations 3.95 and 3.96 are used to find the width of the 50 ohm line. First, we determine H'.

$$H' = \frac{50\sqrt{2(4.2+1)}}{119.9} - \frac{1}{2}\left(\frac{4.2-1}{4.2+1}\right)\left(\ln\frac{2}{\pi} + \frac{1}{4.2}\ln\frac{4}{\pi}\right) = 1.3905$$

We then calculate the width/height ratio.

$$\frac{w}{h} = \left(\frac{\exp 1.3905}{8} - \frac{1}{4 \exp 1.3905}\right)^{-1} = 2.2979$$

3.6 Summary

This chapter presented four valuable concepts for designing and building microwave amplifiers. First, we introduced transmission lines. Wires that interconnect components can no longer be treated as a part of the circuit node at high frequencies. Wires or circuit traces can be modeled as transmission lines with a characteristic impedance, propagation velocity, and length. In Section 3.2, we formulated the mathematical expressions for transmission lines. We found that, since the circuit interconnects must be treated as transmission lines, transmission lines can be used as the circuit elements. Also, instead of using capacitors and inductors, we can use small sections of transmission line. For example, at microwave frequencies, a short open-circuited transmission line connected in shunt can function much the same way a shunt capacitor does. In fact, if space permits, it is a preferred practice to use transmission lines wherever possible in circuit design.

Section 3.3 introduced the Smith chart, a graphical tool for analyzing and designing circuits using transmission line circuits. The Smith chart is a collection of constant impedance or admittance lines plotted on the reflection coefficient plane upon which transmission lines transform a reflection coefficient around in great circles. The Smith chart makes it easy to relate impedances to reflection coefficients and manipulate the reflection coefficients by adding transmission lines.

Further, since we will be designing microwave circuits with transmission lines, S-parameters are a convenient way to describe these circuits mathematically. Section 3.4 described the formulation of the S-parameter and how it relates to voltages and currents on a transmission line. Whereas Z-, Y-, and chain parameters are ratios of voltages and currents at the input and output of a circuit, S-parameters are ratios of traveling waves entering and leaving a circuit. These waves are fed into and extracted from a microwave circuit by transmission lines.

Finally, many different kinds of transmission lines can be used on the circuit board of a microwave amplifier. Microstrip lines are the most common type because of their ease of manufacture and low loss. Some examples of amplifiers using microstrip lines will be shown in the following chapters.

3.7 Problems

3.1 Find the wavelength, propagation velocity, and loss of a transmission line where $V = .34\ e^{j3e7t - j(35+.3)}$.

3.2 Find the voltage and current along a 300 ohm transmission line with a propagation velocity of 1E8 m/s and a loss of 0.2 dB/meter.

3.3 Find the wavelength and loss of a transmission line in terms of L, C, R, and G in Equation 3.22.

3.4 Find the equivalent L, C, R and G for a 75 ohm transmission line that has a propagation velocity of 1.2E8 m/s and a loss of 0.5 dB/m.

3.5 Find the VSWR and reflection coefficient of a 75 ohm load on a 50 ohm transmission line.

3.6 What is the reflection coefficient of a 50 ohm resistor in parallel with a 100 pF capacitor terminating a 50 ohm transmission line as a function of frequency?

3.7 Solve the input impedance and reflection coefficient of a lossless 75 ohm transmission line terminated by a 300 ohm resistor as a function of transmission line length.

3.8 Plot the following impedances on a 40 ohm Smith chart:
a) $10 - j70$
b) $80 + j10$
c) $40 + j40$
d) $20 - j80$
e) $30 + j0$.

3.9. One 75 ohm transmission line is split into two 100 ohm transmission lines, one a quarter of a wavelength long and the other three-eighths of a wavelength long. Both are terminated by 100 ohm loads, as shown in Figure 3-19. Find the reflection coefficient at the junction of the three transmission lines.

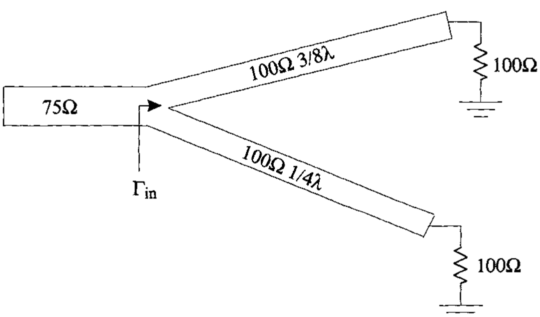

Figure 3-19 *The circuit in Problem 3.9.*

3.10 Plot the constant 30 ohm resistance line on the 50 ohm Smith chart.

3.11 Find the two-port S-parameters of a series impedance.

3.12 Find the two-port S-parameters of a shunt impedance.

3.13 Find the input reflection coefficient in a 50 ohm characteristic impedance network of a 40 ohm transmission line that is 1/4 wavelength long terminated with a 60 ohm resistor.

3.14 Using the low frequency approximation, find the width-to-height ratio of a 30 ohm, 50 ohm, 75 ohm and 100 ohm microstrip line on an aluminum substrate that has a relative dielectric constant of 9.9.

3.15 What is the impedance of a mircrostrip line that is 1.4 mm wide on a substrate that is 0.6 mm thick and has a relative dielectric constant of 2.2?

3.8 References

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