Q. ~~ASSIGNMENT 4.~~ for a rectangular waveguide ~~submission~~ with dimensions of $a = 1.07\,cm$ $b = 0.43\,cm$. filled with a medium $\varepsilon_r = 2.08$ find the cutoff frequencies for the first 5 modes.

$$f_{cutoff} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

01
10
20
11
02

$$f_c = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_{C01} = \frac{c}{2\pi\sqrt{2.08}}\sqrt{\left(\frac{\pi}{1.07\times10^{-2}}\right)^2} = 24.19\ GHz$$

$$f_{C10} = \frac{c}{2\pi\sqrt{2.08}}\sqrt{\left(\frac{\pi}{0.43\times10^{-2}}\right)^2} = \cancel{24.18\ GHz}\ 9.72\ GHz.$$

$$f_{C20} = \frac{c}{2\pi\sqrt{2.08}}\sqrt{\left(\frac{2\pi}{1.07\times10^{-2}}\right)^2} = 19.44\ GHz.$$

$$f_{C11} = \frac{c}{2\pi\sqrt{2.08}}\sqrt{\left(\frac{\pi}{1.07\times10^{-2}}\right)^2 + \left(\frac{\pi}{0.43\times10^{-2}}\right)^2} = 26.07\ GHz$$

$$f_{C02} = \frac{c}{2\pi\sqrt{2.08}}\sqrt{\left(\frac{2\pi}{0.43\times10^{-2}}\right)^2} = 51.03\ GHz$$

Q. for an airfield rectangular waveguide, ratio of dimensions $a:b = 2:1$. a cut off frequency of TM_{11} mode is 15 GHz find dimensions a & b.

$$f_{cmn} = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$15\,GHz = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{2b}\right)^2 + \left(\frac{\pi}{b}\right)^2}.$$

$$\frac{15\times10^9}{3\times10^8} = \sqrt{\frac{\pi^2}{4b^2} + \frac{\pi^2}{b^2}}.$$

$$50 = \sqrt{\frac{\pi^2+4\pi^2}{4b^2}} \Rightarrow 2500 = \frac{\pi^2+4\pi^2}{4b^2}$$

$$4b^2 = \frac{\pi^2+4\pi^2}{2500} \Rightarrow b^2 = \frac{\pi^2+4\pi^2}{2500\times4} \Rightarrow b = \cancel{1.1\,cm}\ 1.118\,cm.$$

$a = \cancel{2b = 2.2\,cm} = 2b = 2.236\,cm.$

Q. a rectangular waveguide with dimensions $a = 1.07\,cm$, $b = 0.43\,cm$ is filled with a certain dielectric material which exhibits a cut off frequency of 31.03 GHz for TM_{21} mode find the dielectric constant of the medium.

$$f_c = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.$$

$$31.03\times10^9 = \frac{3\times10^8}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{2\pi}{1.07\times10^{-2}}\right)^2 + \left(\frac{\pi}{0.43\times10^{-2}}\right)^2}$$

$$31.03\times10^9 = \frac{47.74\times10^6}{\sqrt{\varepsilon_r}} \times 937.33.$$

$$(\sqrt{\varepsilon_r})^2 = \left(\frac{47.74\times10^6 \times 937.33}{31.03\times10^9}\right)^2 = 2.08$$

Q. for a rectangular waveguide filled with a dielectric material $\epsilon_r = 2.08$ cut-off frequency for TE10 mode is 9.72 GHz. when the waveguide is now filled with air determine the cutoff frequency for TE10 mode

$$(f_c)_d = \frac{c}{2\sqrt{\epsilon_r}} \sqrt{\left(\frac{1}{a}\right)^2}$$

$$(f_c)_{air} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2}$$

$$(f_{c10})_d = \frac{(f_{c10})_{air}}{\sqrt{\epsilon_r}} \Rightarrow (f_{c10})_{air} = \sqrt{\epsilon_r}\,(f_{c10})_d.$$

$$= 14.08 \text{ GHz}.$$

Q. a cavity resonator with its height : width : length in the ratio 1 : 2 : 3 is required to ~~with~~ operate at 10 GHz of the dominant mode find the dimensions of the cavity

cavity resonator,
dominant mode : TE_{101}.
height : width : length = 1 : 2 : 3.

$b : a : d = 1 : 2 : 3$

$$f_r = \frac{\cancel{\kappa} c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$10 \times 10^9 = \frac{3\times10^8}{2\pi} \sqrt{\cancel{\left(\frac{\pi}{2b}\right)^2} + \cancel{\left(\frac{\pi}{3b}\right)^2}} = \left(\frac{1}{2b}\right)^2 + \left(\frac{1}{3b}\right)^2$$

$$\frac{10^{10} \times 2\pi}{3 \times 10^8} = \sqrt{\cancel{\frac{4\pi^2}{b^2}} + \cancel{\frac{9\pi^2}{b^2}}}.$$

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$$\frac{10\times10^{9}\times2\pi}{3\times10^{8}} = \sqrt{\left(\frac{\pi}{2b}\right)^2+\left(\frac{\pi}{3b}\right)^2}$$

$$209.43 = \sqrt{\frac{\pi^2}{4b^2}+\frac{\pi^2}{9b^2}} = \sqrt{\frac{13\pi^2}{36b^2}}$$

$$\cancel{209.43 = \sqrt{\frac{13b^2\pi^2}{36b^2}}}$$

$b = 9\times10^{-3}\,m \approx 9mm.$

$a = 2b = 18mm.$

$d = 3b = 27mm.$

Q. a resonator with dimensions $a = 18mm$ $b = 9mm$. $d = 27mm$. is operating in TE_{111} mode find the resonating frequency of resonator for $\epsilon_r = 4$.

$$f_r = \frac{c}{2\pi\sqrt{\epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2+\left(\frac{p\pi}{d}\right)^2}$$

$$= \frac{3\times10^{8}}{4\pi}\sqrt{\left(\frac{\pi}{18\times10^{-3}}\right)^2+\left(\frac{\pi}{9\times10^{-3}}\right)^2+\left(\frac{\pi}{27\times10^{-3}}\right)^2}$$

$f_r = 9.7GHz.$

Q. an airfield rectangular ~~f~~ waveguide with dimens of $a = 6cm$, $b = 4cm$ the signal frequency is 3GHz compute the following TE_{10} and TE_{11} modes

i. cut off frequency
ii. wavelength in waveguide
iii. phase constant & phase velocity
iv. group velocity in a waveguide

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$TE_{10} \rightarrow$

i. $f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 2.5\,GHz.$

ii. $\lambda = \frac{c}{f\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 180\times10^{-3}$

iii. $\beta = \frac{\omega}{c}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 34.73.$

$v_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 542.72\times10^6\ m/s.$ $\qquad v_g = \frac{c^2}{v_p} =$

$TM_{11} \rightarrow$

i. $f_c = 4.5\,GHz$; operating frequency is 3GHz so TM mode will not ~~operate~~ propagate.

→ the radiation problem.

$div\ \vec{E} = \frac{\rho_v}{\varepsilon_0}(\vec{r},t)$ — ①

$div\ \vec{B} = 0.$ — ②

$curl\ \vec{E} = -\frac{\partial \vec{B}}{\partial t}(\vec{r},t)$ — ③

$curl\ \vec{B} = \mu_0\vec{j}(\vec{r},t) + \mu_0\varepsilon_0\frac{\partial \vec{E}}{\partial t}(\vec{r},t)$ — ④.

from ②

$div\ \vec{B} = 0.$

$div.\ curl\ \vec{A} \neq 0 = 0$

$\vec{B} = \nabla\times\vec{A} \longrightarrow$ ⑤.

from ③.

$\nabla\times\vec{E} + \frac{\partial}{\partial t}(\nabla\times\vec{A}) = 0.$

$\left[\nabla\times \vec{E} + \frac{\partial \vec{A}}{\partial t}\right] = 0.$