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Homework 2

1. Prove, using induction, that for radix-r, the largest number that can be represented with N digits is $r^N - 1$

Claim: $\sum_{i=0}^{n-1} R^i = R^n - 1$

Base Step: $\sum_{i=0}^{0} R^i = R^0 = 1 = 2^1 - 1$, where radix-r is 2

Inductive Step:

Assume $\sum_{i=0}^{n-1} R^i = R^n - 1$

Then,

$\sum_{i=0}^{n} R^i = \left(\sum_{i=0}^{n-1} R^i\right) + R^n = (R^n - 1) + R^n$.

$(R^n - 1) + R^n = R(R^n) - 1 = R^{n+1} - 1$

4.2: For Radix-r addition, the carry bits are always 0 or 1

Carry bits occur when either when performing radix add/sub.

For a fixed radix, r, the carry bits are always either 0 or 1, since if numbers b_1, b_2 base b add without exceeding the base, the carry will be 0 or nothing overflowed.
Since b_1, b_2 can never be greater than the base b,
$b_1 + b_2$ can never be equal to 2b. Therefore, if $b_1 + b_2$ overflows, the max carry bit will be a 1.
Therefore, the carry bits will always be either 0 or 1.

4.3: given the formal definition, derive the minimum and maximum two's complement numbers that can be represented in n bits.

The maximum 2's complement numbers that can be represented in n bits is: $2^{(n-1)} - 1$.

The minimum is $-(2^{(n-1)})$.

4.4. For a number B with magnitude less than 2^{N-2}, show that if B is represented by a 2's compliment number with N bits $b_{N-1} \ldots b_0$, then

$$-(b_{N-1} \ldots b_0)_2 = (\overline{b_{N-1} \ldots b_0})_2 + 1$$

4.5 Prove that Sign extension is value Preserving.

Claim: for value X, with bit width w, extending the width by 1 to X' with width $w+1$, $X = X'$.

throw that,

$-2^{w-1} = -2^{w} + 2^{w-1}$

then by looking at the weights of the upper bits,

$X = -2^{w-1} X_{w-1}$

$X' = -2^{w} X_{w-1} + 2^{w-1} X_{w-1} = -2^{w-1} X_{w-1} = -2^{w-1}$

Therefore sign extension is Preserving.

6.)

mod 2

a)

Q	213	106	53	26	13	6	3	1
R	1	0	1	0	1	0	1	1

11010101 = 213

b.)

Q	1627	813	406	203	101	50	25	12	6	3	1
R	1	1	0	1	1	0	1	0	0	1	1

1627 = 11001011011

c.)

Q	31773	15886	7943	3971	1985	992	496	248	124
R	1	0	1	1	1	0	0	0	0

62	31	15	7	3	1
0	1	1	1	1	1

31773 = 111110000011101

d.)

Q	103	51	25	12	6	3	1
R	1	1	1	0	0	1	1

103 = 1100111

7.)

a.) 222 - 47:

$$\begin{array}{r} 222 \\ +\ 952 \\ \hline 175 \end{array}$$

+1 = 175

b.) 101 - 143 =

$$\begin{array}{r} 1011 \\ +\ 9857 \\ \hline 0868 \end{array}$$

= 868

c.) 171 - 88 =

$$\begin{array}{r} 171 \\ +\ 912 \\ \hline 083 \end{array}$$

= 83

d.) 2720 - 127 =

$$\begin{array}{r} 12720 \\ +\ 9873 \\ \hline 2593 \end{array}$$

= 2593

8.

a.) -101

Q	101	50	25	12	6	3	1
R	1	0	1	0	0	1	1

= 01100101

Flip: 10011010 + 1 = $\boxed{10011011}$

b.) -57

Q	57	28	14	7	3	1
R	1	0	0	1	1	1

= 00111001 =

Flip 11000110 +1 = $\boxed{11000111}$

c.) -107

Q	107	53	26	13	6	3	1
R	1	1	0	1	0	1	1

= 01101011 =

Flip 10010100 +1 = $\boxed{10010101}$

d.) 63

Q	63	31	15	7	3	1
R	1	1	1	1	1	1

00111111

e.) 122

Q	122	61	30	15	7	3	1
R	0	1	0	1	1	1	1

= 01111010