① Convert PDA to CFG —

$M = \{(P, q), (0, 1), (x, z), \delta, q, z\}$

Transition function δ is defined by —

$\delta(q_0, 1, z) \Rightarrow \{q, xz\}$

$\delta(q, 1, x) \Rightarrow \{q, xx\}$

$\delta(q, \epsilon, x) \Rightarrow \{q, \epsilon\}$

$\delta(q, 0, x) \Rightarrow \{P, x\}$

$\delta(P, 1, x) \Rightarrow \{P, \epsilon\}$

$\delta(P, 0, z) \Rightarrow \{q, z\}$

$q_0 = \{q\}$

$Q = \{P, q\}$

$\Sigma = T = \{0, 1\}$

$V \cup T = \{x, z\} = \Gamma$

$S = \{z\}$

$\Rightarrow$

① Add the productions for start symbol 'S'

Rule ① $S \rightarrow [q_0 z q_i]$ for each $q_i \in Q$.

$Q = \{P, q\} \Rightarrow q_i$ Two symbols i.e. P & q so two productions

$S \rightarrow [q z q]$ / A

$S \rightarrow [q z P]$ // B

$S \rightarrow A | B$

② Add the production for $\delta(q, 1, Z) \Rightarrow (q, XZ)$ (q_i a B ; q_j G C_2)

Rule ③ $\delta(q_i, a, B) \Rightarrow (q_j, G C_2)$ then

$$[q_i \; B \; P_1] \Rightarrow a\,[q_j \; G \; P_2]\,[P_2 \; C_2 \; P_1]$$

for P_1 & P_2 $\in Q$

$P_1 \to q \Rightarrow P_2 < q, p$ & $P_1 \to p \Rightarrow P_2 < p, q$

q_i P_1 ; q_j G P_2 ; P_2 C_2 P_1

(i) $[q \; Z \; q]_{//A} \Rightarrow 1\,[q \; X \; q]_{//E}\,[q \; Z \; q]_{A}$

$[q \; Z \; q]_{A} \Rightarrow 1\,[q \; X \; p]_{//F}\,[p \; Z \; q]_{C}$

$A \to 1EA \mid 1FC$

$B \to 1EB \mid 1FD$

q_i P_1 ; q_j G P_2 ; P_2 C_2 P_1

(ii) $[q \; Z \; p]_{//B} \Rightarrow 1\,[q \; X \; q]_{//E}\,[q \; Z \; p]_{//B}$

$[q \; Z \; p]_{//B} \Rightarrow 1\,[q \; X \; p]_{//F}\,[p \; Z \; p]_{//D}$

③ Add the production for $\delta(q, 1, X) \Rightarrow (q, XX)$ (q_i a B ; q_j G C_2)

Rule No. ③

q_i B P_1 ; q_j G P_2 ; P_2 C_2 P_1

(v) $[q \; X \; q]_{//E} \Rightarrow 1\,[q \; X \; q]_{//E}\,[q \; X \; q]_{//E}$

$[q \; X \; q]_{//E} \Rightarrow 1\,[q \; X \; p]_{//F}\,[p \; X \; q]_{//G}$

(vi) $[q \; X \; p]_{//F} \Rightarrow 1\,[q \; X \; q]_{//E}\,[q \; X \; p]_{F}$

$[q \; X \; p]_{//F} \Rightarrow 1\,[q \; X \; p]_{F}\,[p \; X \; p]_{H}$

$E \to 1EE \mid 1FG$

$E \to 1EF \mid 1FH$

④ Add the production for $\delta(q, \epsilon, x) \Rightarrow (q, \epsilon)$
$q_i \qquad\qquad q_j$

Rule ②

$\delta(q_i, \epsilon, B) \Rightarrow (q_j, \epsilon)$ Popping.

$[q_i \overset{B}{\underline{\quad}} q] \Rightarrow \underline{\epsilon}\,[q_j \overset{\epsilon}{=} q] \Rightarrow \underline{\epsilon}$

$\therefore [q \times q]_{,\epsilon} \Rightarrow \underline{\epsilon}$ $\qquad E \to \epsilon$

⑤ Add the production for $\delta(\overset{q_i}{q}, \underline{\overset{a}{o}}, \overset{B}{x}) \Rightarrow (\overset{q_j}{P}, x)$

Rule ⑤ $\qquad P \Rightarrow q \overset{q}{<_P}$

$[\overset{q_i}{q} \times \underline{\overset{P}{q}}]_{,E} \Rightarrow o\,[\underline{P \times q}]_{,G}$ (vi) $\qquad E \to oG$

$[q \times \underline{P}]_{,F} \Rightarrow o\,[P \times \underline{P}]_{,H}$ $\qquad F \to oH$

⑥ Add the production for $\delta(P, \boxed{1}, x) \Rightarrow (P, \epsilon)$

Rule No. ②

$\delta(q_i, \boxed{a}, B) \Rightarrow (q_j, \epsilon)$

$[q_i \overset{B}{\underline{\quad}} q_j] \Rightarrow \underline{a}$

(vii)

$\therefore [P^{\bullet} \times P]_{,H} \Rightarrow \mathbf{1}$ $\qquad H \to 1$

(7) Add the production for $\delta(p, a, Z) \Rightarrow (q, Z)$ (p: a B, q: C)

Rule No. (3)

(iii) $[pZq] \Rightarrow 0[pqZq]_A$ $\quad C \to 0A$

(iv) $[pZp]_D \Rightarrow 0[qZp]_B$ $\quad D \to 0B$

(8) Renaming of variables.

(i) $qZq \Rightarrow A$

(ii) $qZp \Rightarrow B$

(iii) $pZq \Rightarrow C$

(iv) $pZp \Rightarrow D$

(v) $qXq \Rightarrow E$

(vi) $qXp \Rightarrow F$

(vii) $pXq \Rightarrow G$

(viii) $pXp \Rightarrow H$

∴ The set of productions can be written as -

$S \to A \mid B$

$A \to 1EA \mid 1FC$

$B \to 1EB \mid 1FD$

$E \to 1EE \mid 1FG$ ← Remove

$F \to 1EF \mid 1FH$

$E \to \epsilon$

$E \to 0G$ ← Remove.

$F \to 0H$

$H \to 1$

$C \to 0A$

$D \to 0B$

⑨ Simplification of Grammar –

In the above grammar first identify the non-terminals that are not defined & eliminate the productions that refer to these productions.

=> Similarly use the procedure of eliminating the useless symbols & useless productions.

Here, Symbol G is not on the left side of production i.e. not defined.

So it can be eliminated the productions.

∴ Equivalent set of production in grammar –

$S \rightarrow A \mid B$

$A \rightarrow 1EA \mid 1Fc$

$B \rightarrow 1EB \mid 1FD$

$E \rightarrow 1EE \mid \epsilon$

$F \rightarrow 1EF \mid 1FH \mid 0H$

$H \rightarrow 1$

$C \rightarrow 0A$

$D \rightarrow 0B$

② Give the CFG generating the language accepted by the following PDA $M = \{(q_0, q_1), (0,1), (Z_0, x), \delta, q_0, Z_0, \phi\}$ when δ is given below.

Dec'13, May'15, May'16, Dec'16 for 9/12 Marks

$$\delta(q_0, 1, Z_0) = (q_0, xZ_0)$$
$$\delta(q_0, 1, x) = (q_0, xx)$$
$$\delta(q_0, 0, x) = (q_1, x)$$
$$\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$$
$$\delta(q_1, 1, x) = (q_1, \epsilon)$$
$$\delta(q_1, 0, Z_0) = (q_0, Z_0)$$

$\Rightarrow$ ⊛ $Q = \{q_0, q_1\}$, $T = \Sigma = \{0,1\}$, $VUT = \{Z_0, x\}$

$S = \{Z_0\}$, q_0.

① Add the productions for start symbol 'S'

Rule ①

$S \rightarrow [q_0 Z q_0]$ ∀A

$S \rightarrow [q_0 Z q_1]$ ∀B

$Q = \{q_0, q_1\} = q_i$

$S \rightarrow [q_0 Z q_i]$ for each $q_i \in Q$

$S \rightarrow A \mid B$

② Add the productions for $\delta(q_0, \underline{!}, Z_0) = (q_0, XZ_0)$

Rule No. ③

$$A\begin{cases}[q_0\ Z_0\ \underline{q_0}] \Rightarrow 1\,[q_0\ X\ \underline{q_0}]_{\#E}\,[q_0\ Z_0\ q_0]_{\#A} \\ [q_0\ Z_0\ \underline{q_0}] \Rightarrow 1\,[q_0\ X\ \underline{q_1}]_{\#F}\,[q_1\ Z_0\ q_0]_{F}\end{cases} \quad P_1 \Rightarrow q_0 \Rightarrow P_2 \begin{cases} q_0 \\ q_1 \end{cases}$$

$$B\begin{cases}[q_0\ Z_0\ \underline{q_1}] \Rightarrow 1\,[q_0\ X\ \underline{q_0}]_{\#E}\,[q_0\ Z_0\ q_1]_{IS} \\ [q_0\ Z_0\ \underline{q_1}] \Rightarrow 1\,[q_0\ X\ \underline{q_1}]_{\#F}\,[q_1\ Z_0\ q_1]\end{cases} \quad P_1 \Rightarrow q_1 \Rightarrow P_2 \begin{cases} q_0 \\ q_1 \end{cases}$$

$A \to 1EA \mid 1FC$

$B \to 1EB \mid 1FD$

③ Add the productions for $\delta(q_0, \underline{!}, X) = (q_0, XX)$

Rule No. ③

$$E\begin{cases}[q_0\ X\ q_0] \Rightarrow 1\,[q_0\ X\ q_0]_{\#E}\,[q_0\ X\ q_0]_{\#E} \\ [q_0\ X\ q_0] \Rightarrow 1\,[q_0\ X\ q_1]_{\#F}\,[q_1\ X\ q_0]_{\#H}\end{cases} \quad E \to 1EE \mid 1FH$$

$$F\begin{cases}[q_0\ X\ q_1] \Rightarrow 1\,[q_0\ X\ q_0]_{\#E}\,[q_0\ X\ q_1]_{\#F} \\ [q_0\ X\ q_1] \Rightarrow 1\,[q_0\ X\ q_1]_{\#F}\,[q_1\ X\ q_1]_{\#G}\end{cases} \quad F \to 1EF \mid 1FG$$

④ Add the productions for $\delta(q_0, \underline{0}, X) = (q_1, X)$

Rule No. ②

$$P \begin{cases} q_0 \\ q_1 \end{cases}$$

$$[q_0\ X\ q_0]_{\#E} \Rightarrow 0\,[q_1\ X\ q_0]_{\#H} \quad E \to 0H$$

$$[q_0\ X\ q_1]_{\#F} \Rightarrow 0\,[q_1\ X\ q_1]_{\#G} \quad F \to 0G$$

(5) Add the production for $\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$ (q_i a B q_j C)

Rule No. (2) Popping.

$$[q_0\ Z_0\ q_0]_A \Rightarrow \epsilon \qquad A \to \epsilon$$

(6) Add the productions for $\delta(q_1, 1, X) = (q_1, \epsilon)$ (q_i a B q_j C)

Rule No (2)

$$[q_1\ X\ q_1]_G \Rightarrow 1 \qquad G \to 1$$

(7) Add the production for $\delta(q_1, 0, Z_0) = (q_0, Z_0)$ (q_i a B q_j C)

Rule No. 3

$$[q_1\ Z_0\ q_0]_C \Rightarrow 0[q_0\ Z_0\ q_0]_A \qquad C \to 0A$$
$$[q_1\ Z_0\ q_1]_D \Rightarrow 0[q_0\ Z_0\ q_1]_B \qquad D \to 0B$$

(8) Renaming of variables.

$[q_0\ Z_0\ q_0] \Rightarrow A$
$[q_0\ Z_0\ q_1] \Rightarrow B$
$[q_1\ Z_0\ q_0] \Rightarrow C$
$[q_1\ Z_0\ q_1] \Rightarrow D$
$[q_0\ X\ q_0] \Rightarrow E$
$[q_0\ X\ q_1] \Rightarrow F$
$[q_1\ X\ q_1] \Rightarrow G$
$[q_1\ X\ q_0] \Rightarrow H$

$S \to A|B$
$A \to 1EA|1FC$
$B \to 1EB|1FD$
$E \to 1EE|1FH$ ← Remove
$F \to 1EF|1FG$
$E \to 0H$ ←
$F \to 0G$
$A \to \epsilon$
$G \to 1$
$C \to 0A$
$D \to 0B$

(9) ∴ Simplification of grammar

Symbol H doesn't came on the left side of production so it has to be removed.

∴ Complete grammar is -

$S \rightarrow A|B$

$A \rightarrow 1EA \mid 1FC \mid \epsilon$

$B \rightarrow 1EB \mid 1FD$

$E \rightarrow 1EE$

$F \rightarrow 1EF \mid 1FG \mid 0G$

$G \rightarrow 1$

$C \rightarrow 0A$

$D \rightarrow 0B$