

⑥ $S \rightarrow 0SB$
 $B \rightarrow 0S \mid 1S \mid 0$ Test if 010^4 is in language

$$\Rightarrow M = \{ (q), \underbrace{(0, 1)}_T, \underbrace{(0, 1, 5, 13)}_{\text{VUT}}, \delta, q, s, \phi \}$$
$$\delta \Rightarrow \delta(q, \epsilon, s) \Rightarrow \{q, \text{OBB}\}$$
$$\delta(q, \epsilon, B) = \{(q, 0s), (q, 1s), (q, 0)\}$$
$$\delta(q, 0) \Rightarrow (q, \epsilon) \quad 1 \quad \pi$$
$$\delta(q, 1) \Rightarrow (q, \epsilon)$$

Acceptance of 0.10^4 by $N = 010000$

$$\delta(q, 010000, s) \xrightarrow{\delta(q, \epsilon, s)} (q, 0BB) \xrightarrow{\delta(q, 010000, 0BB)} (q, 010000, 0BB)$$
$$\delta(q, 0, 0) \Rightarrow (q, \epsilon) \rightarrow (q, \epsilon 10000, BB)$$
$$\delta(q, \epsilon, B) \Rightarrow (q, 15) \rightarrow (q, 10000, 15B)$$

$\delta(q, 1, 1) \Rightarrow (q, \epsilon)$ (q, 0000, 530)

$$\delta(q, \epsilon, \text{B}) \Rightarrow (q, \overset{0BB}{\text{B}}) (q, \overset{\text{B}}{\text{0000}}, \overset{\text{B}}{\text{0BBB}})$$
$$\delta(q, 0, 0) \Rightarrow (q, \epsilon) \rightarrow (q, \epsilon 000, BBB)$$
$$\delta(q, \epsilon, B) \Rightarrow (q, 0) \quad (q, \underline{000}, \underline{0}BB)$$
$$\underline{d(q, 0, 0) = (q, \epsilon), (q, 00, BB)}$$
$$\bar{a}(q, \epsilon, \delta) \Rightarrow (q, 0) \quad (q, \underline{00}, \underline{0B})$$
$$\delta(q, 0, 0) \rightarrow (q, \epsilon) \quad (q, 0, B)$$
$$\underline{\delta(q, \epsilon, B)} \Rightarrow (q, 0) \quad (q, \underline{\delta}, \underline{0})$$
$$\partial(q, 0, 0) \Rightarrow (q, \epsilon) \quad (q, \epsilon, \epsilon)$$
$$\therefore \underline{010^4 \in L}$$

$$\textcircled{7} \quad S \rightarrow aABB \mid aAA$$

$$A \rightarrow aBB \mid a$$

$$B \rightarrow bBB \mid A$$

$$\Rightarrow M = \{ (q), (a, b), (q, b, s, A, B), \delta, q, s, \phi \}$$

$$\delta: \left. \begin{aligned} \delta(q, \epsilon, S) &\Rightarrow \{ (q, aABB), (q, aAA) \} \\ \delta(q, \epsilon, A) &\Rightarrow \{ (q, aBB), (q, a) \} \\ \delta(q, \epsilon, B) &\Rightarrow \{ (q, bBB), (q, A) \} \end{aligned} \right\} \checkmark$$

$$\left. \begin{aligned} \delta(q, a, a) &\Rightarrow (q, \epsilon) \\ \delta(q, b, b) &\Rightarrow (q, \epsilon) \end{aligned} \right\} \top$$

$$\textcircled{8} \quad S \rightarrow SS \mid (S) \mid ()$$

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Parent's
Matching

$$\Rightarrow \delta(q, \epsilon, S) \Rightarrow \{ (q, SS), (q, (S)), (q, ()) \} \Rightarrow \checkmark$$

$$\left. \begin{aligned} \delta(q, (, ()) &\Rightarrow (q, \epsilon) \\ \delta(q, (, ()) &\Rightarrow (q, \epsilon) \end{aligned} \right\} \top$$

$$\begin{aligned} \textcircled{1} \quad \delta(q, ((), S) &\Rightarrow \underline{\delta(q, \epsilon, S) \Rightarrow (q, (S))} \quad (q, \underline{(()}, \underline{(S)}) \\ &\quad \underline{\delta(q, (, ()) \Rightarrow (q, \epsilon)} \quad (q, \underline{(()}, \underline{S)}) \\ &\quad \underline{\delta(q, \epsilon, S) \Rightarrow (q, (S))} \quad (q, \underline{()}, \underline{(S)}) \\ &\quad \underline{\delta(q, (, ()) \Rightarrow (q, \epsilon)} \quad (q, \underline{) }, \underline{)) } \\ &\quad \underline{\delta(q, (,)) \Rightarrow (q, \epsilon)} \quad (q, \underline{S }, \underline{) } \\ &\quad \underline{\delta(q, (,)) \Rightarrow (q, \epsilon)} \quad (q, \epsilon, \epsilon) \end{aligned}$$

* PDA to CFG Conversion:-

CFG to PDA Conversion is easy but PDA to CFG Conversion is more difficult.

The basic idea is to consider any two states p, q of PDA M . & think about what strings could be consumed in executing M from p to q .

⇒ Those strings will be represented by a variable

• $V = \{P, a, q\}$ in grammar G .

⇒ Thus S is start symbol, will stand for all strings consumed in going from q_p to an accept state & for PDA we have to consider the stack

⇒ we can construct a grammar G for any PDA M such that
 $L(G) = L(M)$

⇒ The variables of the CFG, constructed will be of the form,

$[P \overset{\text{State}}{\times} q]$, where $p, q \in Q$ & $\overset{\text{Stack symbol}}{\times} \in \Gamma$

Let PDA is given by, -

$$M = \{Q, \Sigma, \Gamma, \delta, q_0, \underline{z}, \phi\}$$

↪ Initial stack symbol.

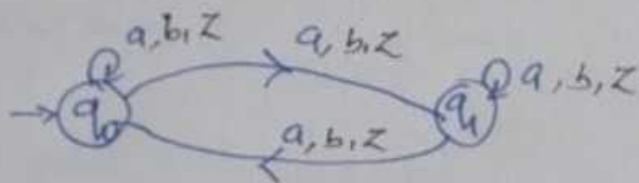
Then an equivalent CFG is given by -

$$G = (V, \underline{\Sigma}, P, S) \text{ where}$$

$$V = \{S, [P \times q] \mid P, q \in Q \text{ \& \& } x \in \underline{\Gamma}\}$$

$$S \rightarrow [P \times q] \quad \text{so } S \rightarrow A \underline{a} B$$

ex:- If $Q = \{q_0, q_1\}$ & $\Gamma = \{a, b, z\}$ then the possible set of variables in the corresponding CFG is given by -



- 1) $S \rightarrow$ Start symbol
- 2) $[q_0^a q_0], [q_0^a q_1], [q_1^a q_0], [q_1^a q_1]$
- 3) $[q_0^b q_0], [q_0^b q_1], [q_1^b q_0], [q_1^b q_1]$
- 4) $[q_0^z q_0], [q_0^z q_1], [q_1^z q_0], [q_1^z q_1]$

* To Convert PDA to CFG, we use the following three rules —

1) Add the following production for the start symbol 'S'.

$$S \rightarrow [q_0 \underline{x} q_i] ; \text{ for each } \underline{q_i} \in Q, \text{ where } \underline{x} \text{ is start symbol.}$$

$Q = \{q_0, q_1, q_2, \dots\}$

2) For each transition of the form | Popping symbol from stack

Popping

$$\delta(q_i, \underline{a}, B) \Rightarrow (q_j, \underline{C})$$

$$\delta(q_i, \underline{\epsilon}, B) \Rightarrow (q_j, \underline{\epsilon}) \Rightarrow \underline{\text{Popping}}.$$

$$\textcircled{1} q_i, q_j \in Q ; \textcircled{2} a \in \{\epsilon \cup \Sigma\} \textcircled{3} B, C \in \{\Gamma \cup \Sigma\}$$

Popping can be done

Then for each $q \in Q$ we add the production

$$[q_i \textcircled{B} q_j] \rightarrow \underline{a} [\underline{q_j C q}] \text{ (or) } \underline{a} \quad q_j \text{ in } Q$$

3) For each transition of the form $C_1 C_2$

Pushing

$$\delta(q_i, a, B) \Rightarrow (q_j, C_1 C_2)$$

~~Doesn't~~ Doesn't Pop symbol from the stack (pushing)

where,

$$\textcircled{a} q_i, q_j \in Q \textcircled{b} a \in \{\epsilon \cup \Sigma\} \textcircled{c} B, C_1 \& C_2 \in \{\Gamma\} \text{ No popping.}$$

Then for each $P_1, P_2 \in Q$, we add the productions.

$$[q_i^B P_1] \Rightarrow P a [q_j^G P_2] [P_2^{C_2} P_1]$$

OR

$$[q_i^B P_m] \Rightarrow a [q_j^G P_2] [P_2^{C_2} P_3] [P_3^{C_3} P_4] \dots \dots$$

$$[P_{m-1}^{C_m} P_m]$$

* After defining all the rules, apply simplification of grammar to get reduced grammar.

① Convert PDA to CFG -

$$M = \{ (p, q), (q, 0, 1), (x, z), \delta, p, q, z \}$$

transition function δ is defined by -

$$\delta(\overset{q_0}{q}, 1, z) \Rightarrow \{q, xx\}$$

$$\delta(q, 1, x) \Rightarrow \{q, xx\}$$

$$\delta(q, \epsilon, x) \Rightarrow \{q, \epsilon\}$$

$$\delta(q, 0, x) \Rightarrow \{p, x\}$$

$$\delta(\cancel{q}, p, 1, x) \Rightarrow \{p, \epsilon\}$$

$$\delta(p, 0, z) \Rightarrow \{q, z\}$$

$$q_0 = \{q\}$$

$$Q = \{p, q\}$$

$$\Sigma = \Gamma = \{0, 1\}$$

$$VUT = \{x, z\} = \Gamma$$

$$S = \{z\}$$

⇒ ① Add the productions for start symbol 'S'

Rule ① $S \rightarrow [q_0^z q_i]$ for each $\underline{q_i} \in Q$.

$$Q = \{p, q\} = q_i$$

$$S \rightarrow [q^z \underline{q}]$$

$$S \rightarrow [q^z \underline{p}]$$