5) $L = \{a^{m+n} b^m c^n \mid m, n \geq 1\}$

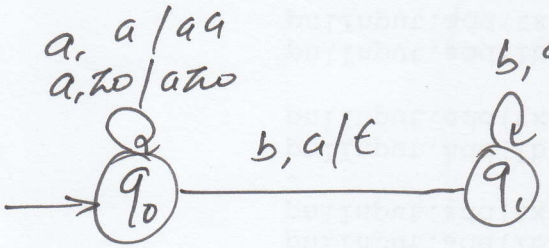
$\Rightarrow$ $a^m a^n$ $b^m c^n$

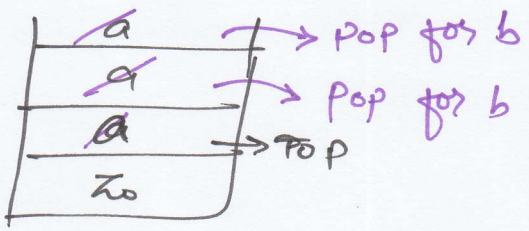
Push 'a' | Pop for 'a'

Here count all 'a' & match with b and c

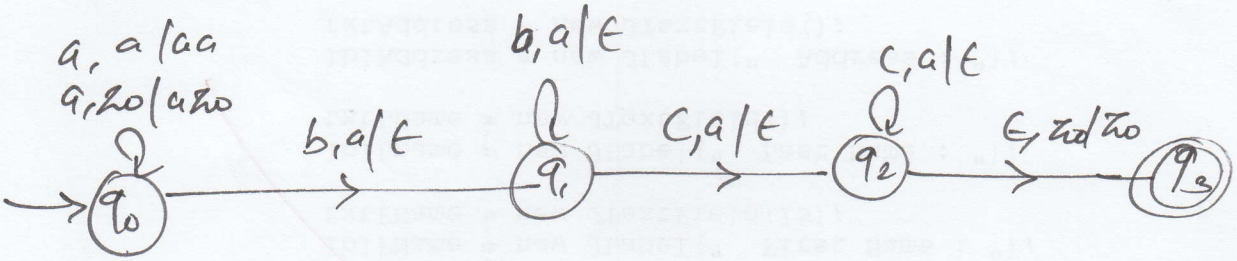
$w = \underbrace{a}_{m}\underbrace{a}_{n}\underbrace{a}_{}\underbrace{b}_{m}\underbrace{b}_{}\underbrace{c}_{n}$

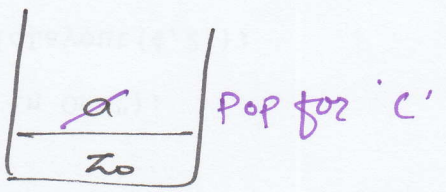
Push | Pop





~~b = a~~





So, here for 'a', push it on the stack & after this 'b' is there then pop 'a' for all 'b' & change the state i.e. q_1. For C we can't do the Pop operation at q_1 state itself because after b, C will come and after C b will come, so order will change

For this we have to change the state for 'C'

⑥ $L=\{a^n b^{m+n} c^m \mid m,n \geq 1\}$

$\Rightarrow a^n b^m b^n c^m \Rightarrow a^n b^n b^m c^m$

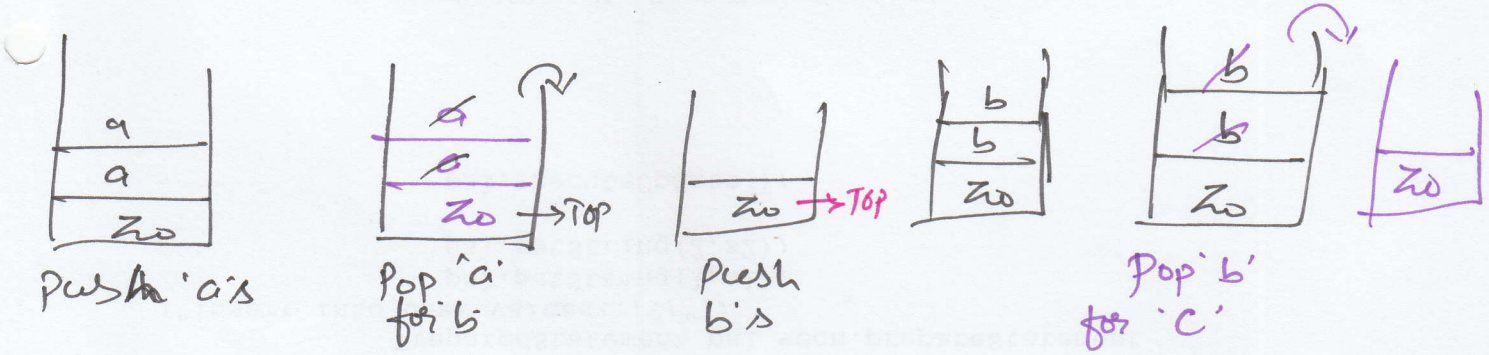
Here, a^n push on stack, whenever you see b's you have to match them with a's & Remaining b's should pushed on the stack & match them against 'c'

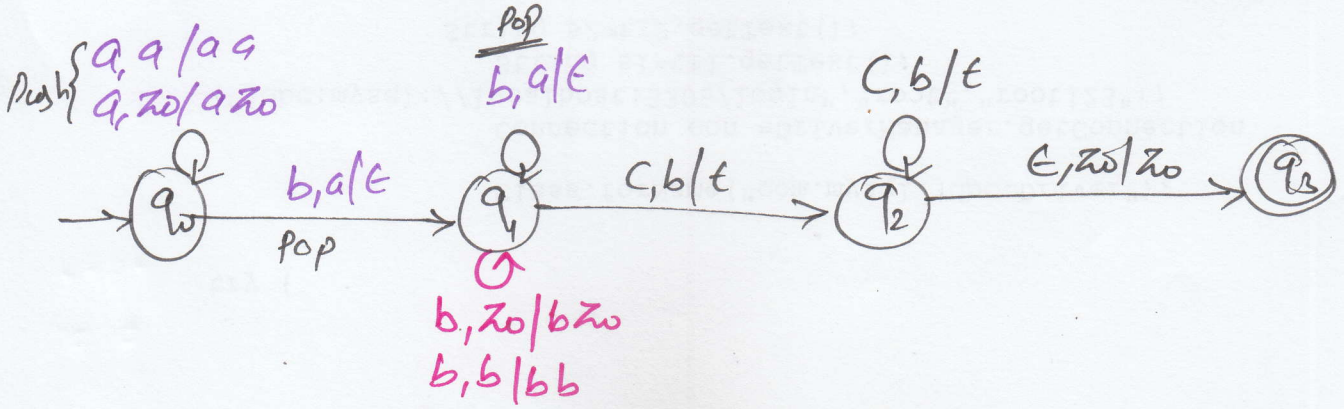
Rewrite as $\{a^n b^n \mid b^m c^m \mid n,m \geq 1\}$

∴ For all a push 'a' & for all b pop 'a' & for all remaining b's keep on pushing & match them against 'c's

$w = aabbbbcc$

Push pop Push Pop



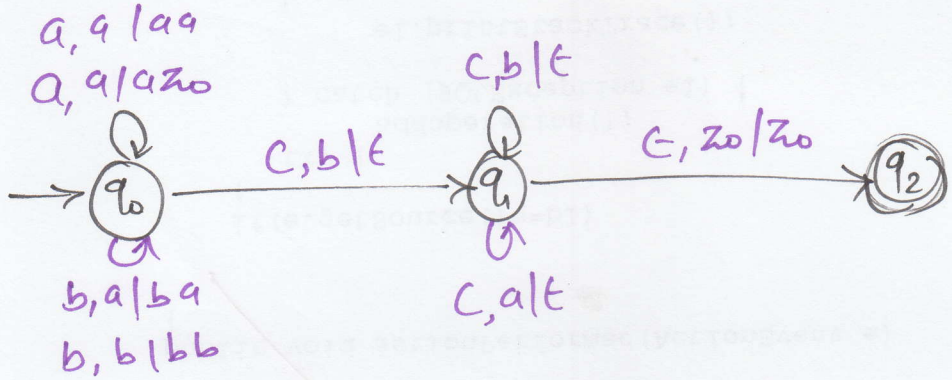


(7) $L = \{a^n b^m c^{n+m} \mid n, m \geq 1\}$

$a^n b^m \; c^n c^m$

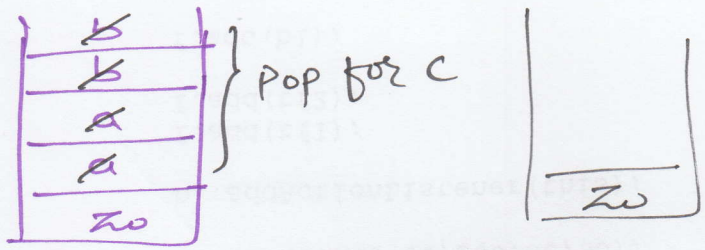
push pop

$\Rightarrow$ Here, we have to push 'a' then push 'b'. After this for every 'c' we have to pop a & b



$w = a\,a\,b\,b\,c\,c\,c\,c\,\in$

push ↑ ↑ ↑ ↑



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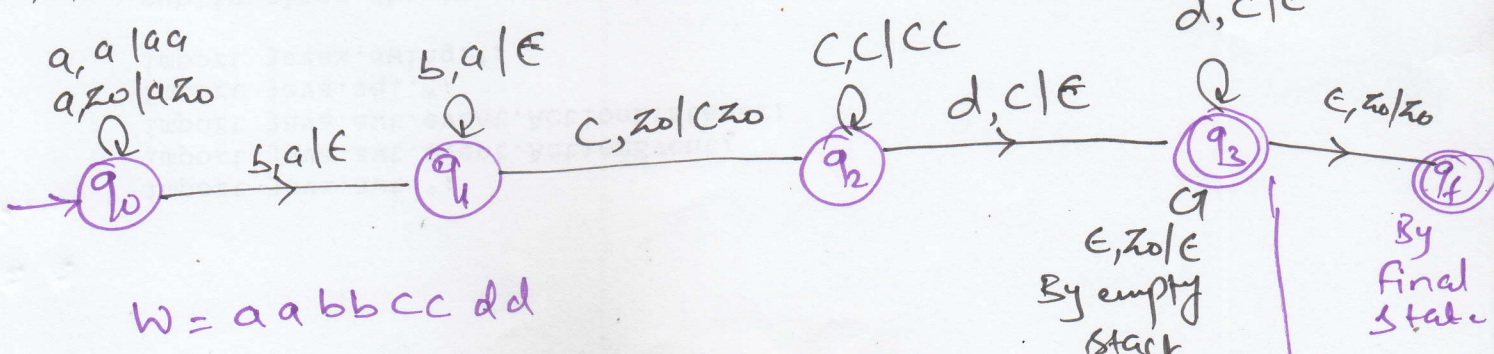
(8) $L = \{a^n b^n c^m d^m \mid n, m \geq 1\}$ find PDA SM

$\Rightarrow$ Here push 'a' & match them with 'b' with pop operation & then push 'c' & match them with 'd', with pop operation



$w = a\,a\,b\,b\,c\,c\,d\,d$

(9) $L = \{ a^n b^{2n} \mid n \geq 1 \}$

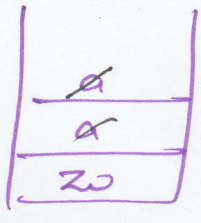
⇒ For, every 'a' there is 2 b's solution to this is for every 'a' we could push 2 a's (or)

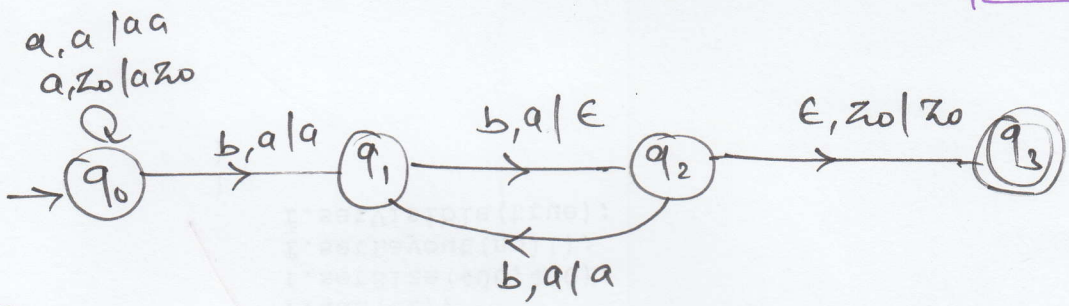
(i) push single 'a' on stack & whenever there is 'a' on top of stack pop it for 2nd 'b'. First 'b' should have to leave as it. is.

$w = a\ a\ b\ b\ b\ b\ \epsilon$

(No op) (No op) (Pop) (Pop)

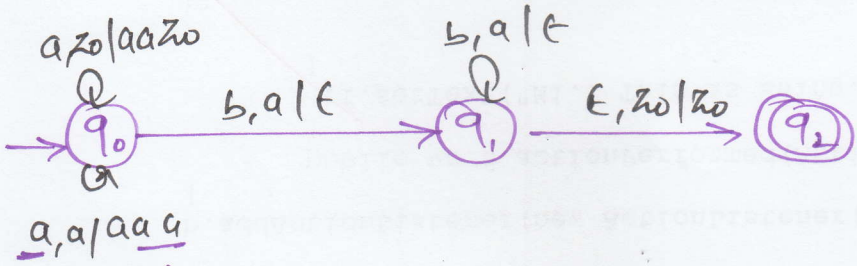
(Push)



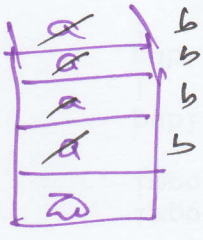


OR

(ii)



↳ For every 'a' pushing 2 a's on stack & for every b poping one a from stack



$w = a\ a\ b\ b\ b\ b\ \epsilon$.

(10) $L = \{a^n b^{2n+1} \mid n \geq 1\}$

$\Rightarrow$ $a^n b^{2n}.b \Rightarrow \underset{\text{Push}}{a}\ \underset{\text{Pop}}{bb}\ \underset{\text{No op}^n}{b}$

Here push single 'a' on stack & for every 2nd 'b' pop out 'a' from stack & for last single 'b' go for next state. & then accept the string.