

$$6) \quad E \rightarrow E + E$$

$$E \rightarrow E * E$$

$$E \rightarrow id$$

$$w = id + id = LMD$$

$$w = id + id * id = RMD$$

\Rightarrow LMD

RMD

$$E \rightarrow E + E$$

$$\rightarrow id + E$$

$$\rightarrow id + id$$

$$E \rightarrow E + E$$

$$\rightarrow E + E * E$$

$$\rightarrow E + E * id$$

$$\rightarrow E + id * id$$

$$\rightarrow id + id * id$$



Q.2. Find the language generated by following grammar.

$$1) A \rightarrow \alpha A \mid \beta$$

$$\begin{aligned} \rightarrow A &\rightarrow \beta && \alpha^0 \beta \\ A &\rightarrow \alpha \underline{A} && \alpha^1 \beta \\ &\rightarrow \alpha \underline{\alpha A} && \alpha^2 \beta \\ &\rightarrow \alpha \alpha \underline{\alpha A} && \alpha^3 \beta \\ &\vdots \end{aligned}$$

$$\boxed{L = \alpha^* \beta}$$

$$A \rightarrow \alpha A \mid \beta \Leftrightarrow A \rightarrow \alpha^* \beta$$

$$2) A \rightarrow A\alpha \mid \beta$$

$$\begin{aligned} \rightarrow A &\rightarrow A\alpha \\ &\rightarrow \beta \alpha \end{aligned}$$

$$\begin{aligned} A &\rightarrow \underline{A\alpha} \\ &\rightarrow \blacktriangle \underline{A\alpha} \\ &\rightarrow \blacktriangle \underline{A\alpha\alpha} \\ &\rightarrow \beta \alpha \alpha \end{aligned}$$

$$\boxed{L = \{ \beta \alpha^* \}}$$

$$3) A \rightarrow A\alpha \mid \alpha$$

$$\begin{aligned} A &\rightarrow \alpha \alpha^* && [\because \alpha \cdot \alpha^* = \alpha^+] \\ &\rightarrow \alpha^+ \end{aligned}$$

$$\boxed{L = \alpha^+}$$

$$4) A \rightarrow \alpha A \mid \alpha$$

$$\rightarrow \underline{A \rightarrow \alpha}$$

$$A \rightarrow \alpha A$$

$$A \rightarrow \alpha \alpha A$$

$$A \rightarrow \alpha \alpha \alpha$$

$$A \rightarrow \alpha \cdot \alpha^*$$

$$\rightarrow \alpha^+$$

$$\therefore \underline{L = \alpha^+}$$

$$5) A \rightarrow A\alpha \mid \epsilon$$

$$\rightarrow \underline{A \rightarrow \epsilon \cdot \alpha}$$

$$A \rightarrow \alpha$$

$$A \rightarrow A\alpha$$

$$\rightarrow A\alpha\alpha$$

$$\rightarrow A\alpha\alpha\alpha$$

$$\rightarrow \epsilon \cdot \alpha\alpha\alpha$$

$$\rightarrow \epsilon \cdot \alpha^*$$

$$\rightarrow \alpha^*$$

$$\therefore \underline{L = \alpha^*}$$

$$6) S \rightarrow AB$$

$$A \rightarrow aA | \epsilon \rightarrow a^*$$

$$B \rightarrow bB | \epsilon \rightarrow b^*$$

$$S \rightarrow AB \\ \rightarrow a^* b^*$$

$$S \rightarrow AB$$

$$\rightarrow aAB$$

$$\rightarrow aAbB$$

$$\rightarrow aaAbbB$$

$$\rightarrow aa\epsilon b b \epsilon$$

$$\rightarrow aabb$$

$$\rightarrow a^* b^*$$

$$\therefore L = \{ a^m b^n \mid m, n \geq 0 \}$$

$$7) S \rightarrow aA$$

$$A \rightarrow bA | \epsilon \Rightarrow b^*$$

$$S \rightarrow aA$$

$$\rightarrow ab^*$$

$$L = ab^*$$

$$S \rightarrow aA$$

$$\rightarrow abA$$

$$\rightarrow abbA$$

$$\rightarrow abbb\epsilon$$

$$\rightarrow abbb$$

$$\rightarrow ab^*$$

$$\therefore L = \{ ab^n \mid n \geq 0 \}$$

$$8) S \rightarrow aSb | \epsilon$$

$$\rightarrow S \rightarrow \epsilon$$

$$S \rightarrow aSb = ab$$

$$\rightarrow aasbb = a^2 b^2$$

$$\rightarrow \underline{aaasbb} = a^3 b^3$$

$$\rightarrow aaasbb$$

$$\Rightarrow aaabbb$$

$$\vdots \\ a^n b^n$$

$$\therefore L = a^n b^n$$

$$L = \{ a^n b^n \mid n \geq 0 \}$$

$$9) S \rightarrow aS \mid bS \mid \epsilon$$

$$\Rightarrow \begin{aligned} S &\rightarrow aS \\ &\rightarrow aaaS \\ &\rightarrow aaas \\ &\rightarrow aaa \end{aligned}$$

$$\begin{aligned} S &\rightarrow bS \\ &\rightarrow bbs \\ &\rightarrow bbbS \\ &\rightarrow bbb \end{aligned}$$

$$\begin{aligned} S &\rightarrow (a+b)S \mid \epsilon \\ \Rightarrow & \\ &\rightarrow (a+b)(a+b)S \\ &\rightarrow (a+b)^* \end{aligned}$$

$$\therefore \underline{\underline{L = (a+b)^*}}$$



Q.3 Construct the grammar for the following language.

$$1) L = \{a, ab\}$$

$$\Sigma = \{a, b\}$$

$$\Rightarrow \begin{aligned} S &\rightarrow aX \\ X &\rightarrow \epsilon \mid b \end{aligned}$$

$$w = aX$$

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graph TD; w["w = aX"] --- e["ε"]; w --- b["b"]
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$$2) L = \{\underline{ab}_x, \underline{aba}_x, \underline{bab}_x\}$$

$$\rightarrow \begin{aligned} S &\rightarrow X \mid Xa \mid bX \\ X &\rightarrow AB \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

$$3) L = \{\underline{aab}_x, \underline{aba}_x, \underline{abb}_x\}$$

$$\rightarrow \begin{aligned} S &\rightarrow aX \mid Xa \mid Xb \\ X &\rightarrow AB \\ A &\rightarrow a \\ B &\rightarrow b \end{aligned}$$

$$4) L = \{ \underline{a} \underline{b} a, b \underline{a} b, \underline{b} \underline{b} a, \underline{a} \underline{b} a \underline{b}, \underline{b} a \}$$

\underline{x}
 \underline{x}
 \underline{y}
 \underline{x} \underline{x}
 \underline{y}

⇒

$$S \rightarrow xa | bx | by | xx | y$$

$$X \rightarrow AB$$

$$Y \rightarrow BA$$

$$A \rightarrow a$$

$$B \rightarrow b$$

$$5) L = \{ \underline{2} \underline{@} \underline{3}, \underline{2} - \underline{3}, \underline{3} \# \underline{2}, \underline{3} * \underline{2}, \underline{2} \$ \underline{3} \}$$

\underline{x}
 \underline{y}
 \underline{x} \underline{y}
 \underline{y} \underline{x}
 \underline{y} \underline{x}
 \underline{x} \underline{y}

→

$$S \rightarrow xAy | yBx$$

$$xA \rightarrow 2$$

$$yB \rightarrow 3$$

$$A \rightarrow @ | - | \$$$

$$B \rightarrow \# | *$$