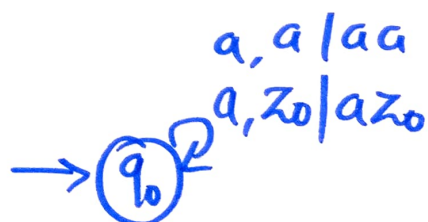


Q) Construct PDA for following language -

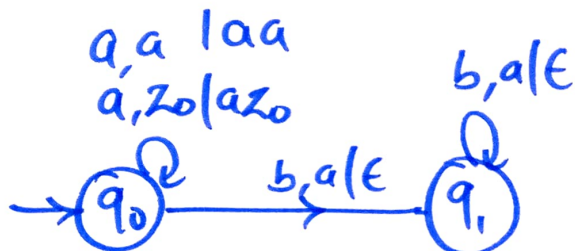
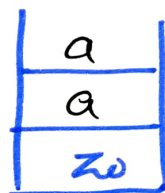
(i) $L = \{a^n b^n / n \geq 1\}$

→ then count b
→ first count no. of a's

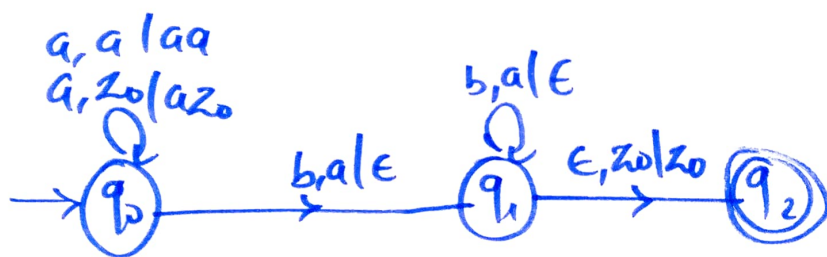
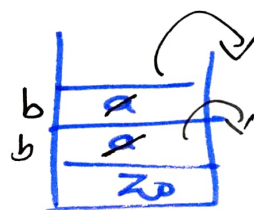
$L = \underline{aabb} \in$
Push Pop



⇒ Push

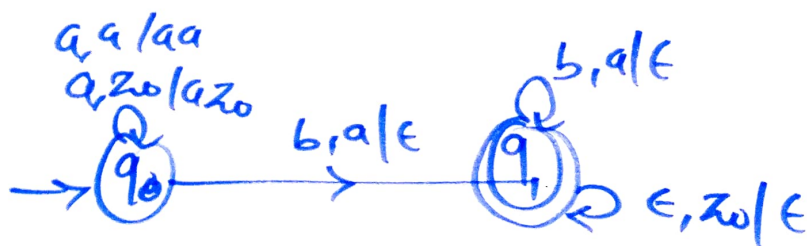
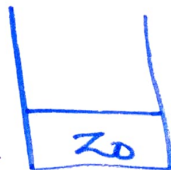


⇒ POP



Acceptance by final State.

Final State



Acceptance by empty stack

$$\textcircled{2} L = \{ w \mid n_a(w) = n_b(w) \}$$

\Rightarrow Equal no. of a's & b's i.e. count no. of a's equal to no. of b's count.

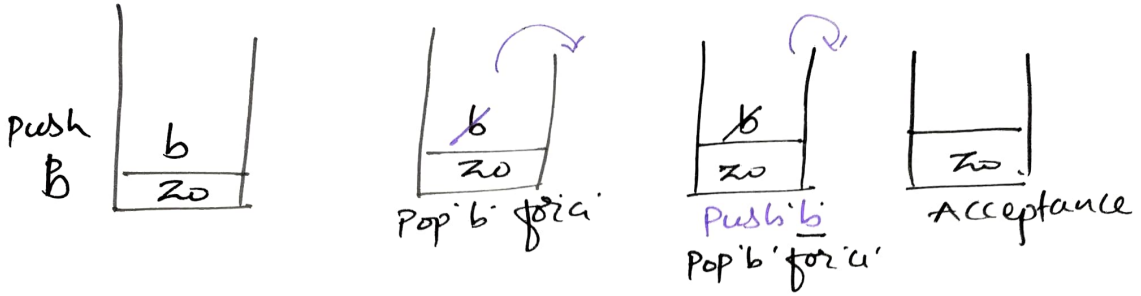
This problem is diff from $a^n b^n$ $| n \geq 1$ because in this you have to read first a & then b but in this given problem string can be like this $w = ab, ba$.

$$\therefore L = \{ \epsilon, \underline{ab}, \underline{ba}, \underline{aabb}, \underline{abab}, \underline{baba}, \underline{bbaa}, \dots \}$$

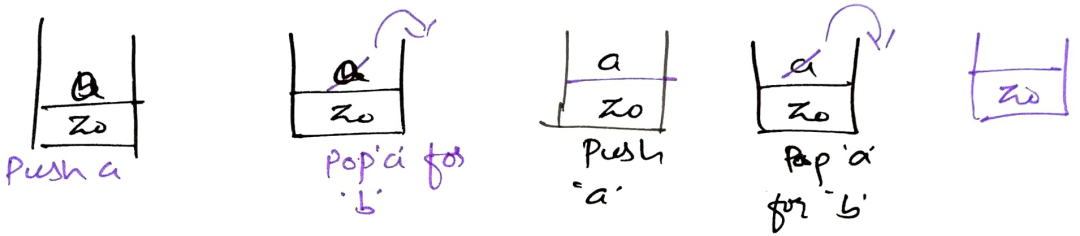
So, we can't say that for all 'a' push 'a' & for all 'b' pop 'a' (or) vice versa there should be other approach

Note: Push the symbols if you don't have chance of popping & if you have chance of popping then pop the symbols.

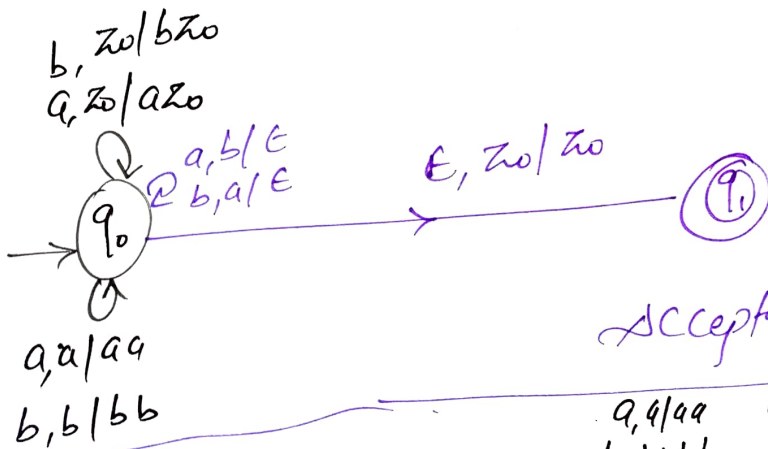
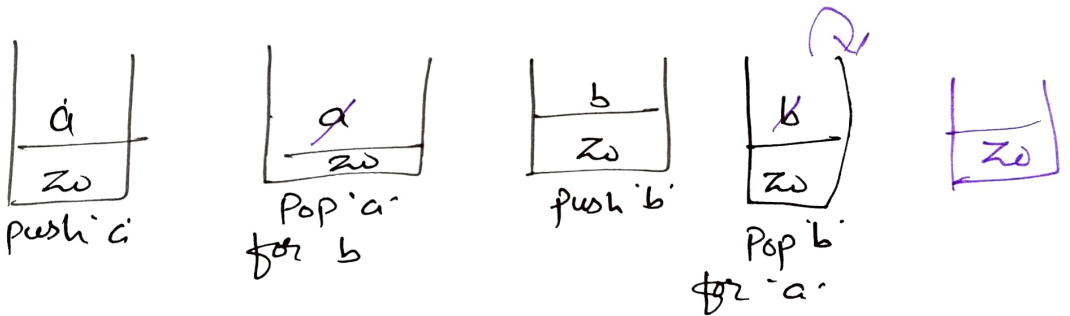
i) $w = baba$
 $\uparrow \uparrow \uparrow \uparrow$



ii) $w = abab$
 $\uparrow \uparrow$

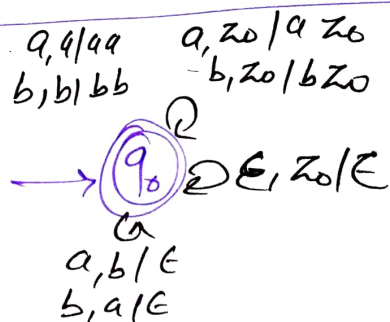


iii) $w = abba$



Acceptance by final state.

Acceptance by empty stack

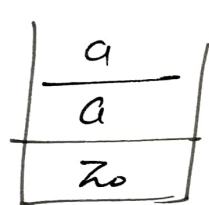


$$\textcircled{3} L = \{ a^n b^n c^m \mid n, m \geq 1 \}$$

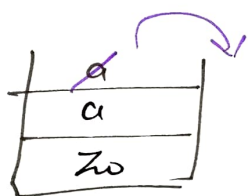
\Rightarrow Here, first we compare a & b first & that should be equal. Here for every a, we are pushing 'a' & for every 'b' pop 'a' And finally simply read 'c' (minimum one 'c')

$$w = \underline{a} \underline{a} \underline{b} \underline{b} \underline{c} \overset{\epsilon}{\uparrow} \text{No-op}$$

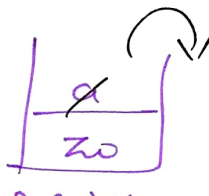
Push Pop



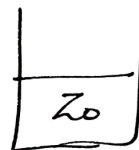
push 'a'



pop 'a'
for b



pop 'a'
for 'b'



No-operation
for 'c'

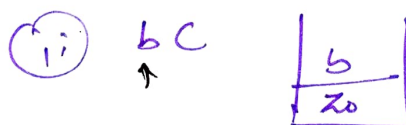
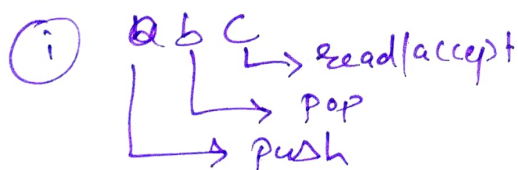
Push { a, a / a a
a^n { a, z₀ / a z₀

pop 'a'
b, a / t

c, z₀ / z₀



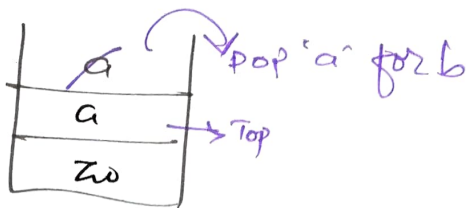
Note:- In DFA for every state there will be transitions but ~~here~~ here for every state there is no transition.



\Rightarrow This is dead configuration
 so for this PDA will halt &
 it is not accepted.

(i) (ii)

$aabcc$
 Push Pop Read
 (C, z_0)

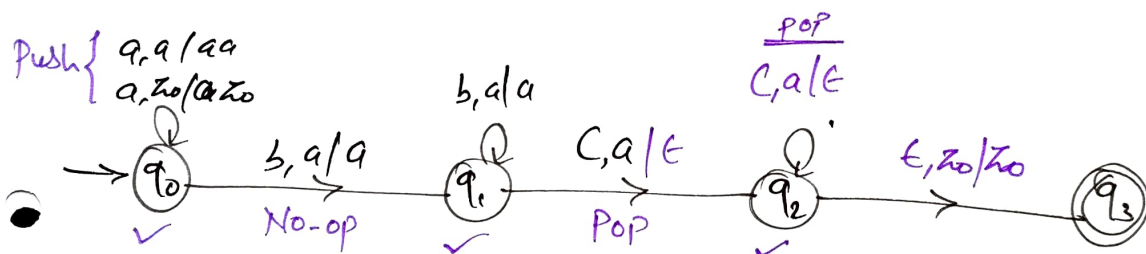


but here (C, a) so it is dead configuration.
 halt there.

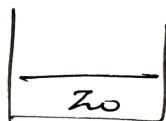
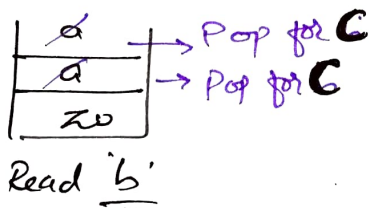
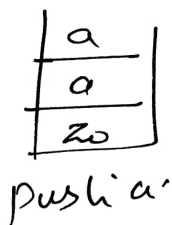
* Halting is at final (or) non-final state; no infinite loop here.

4) $L = \{ a^n b^m c^n \mid n, m \geq 1 \}$

⇒ Here 'a' should match with 'c' & 'b' should be left alone for accepting/reading.



$aabcc$
 Push Pop
~~Read~~

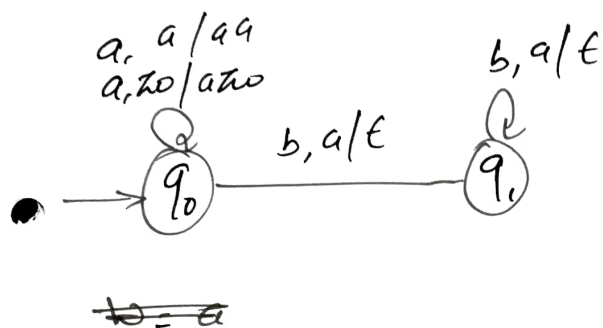


Note:- Three states are compulsory (q_0, q_1, q_2) because after seeing 'b' you should go to next state & after 'b' no 'a's are allowed & after seeing 'c' go to next state & remember that you have seen 'c' so no 'b' is allowed.

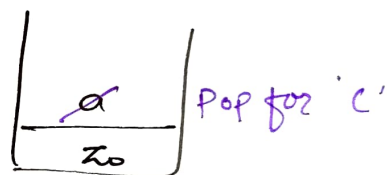
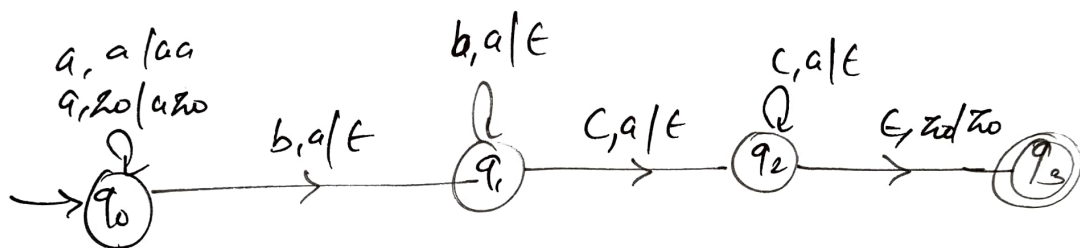
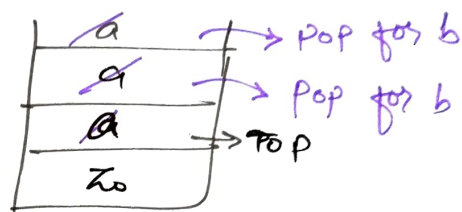
$$5) L = \{a^{m+n} b^m c^n \mid m, n \geq 1\}$$

$$\Rightarrow \frac{a^m a^n b^m c^n}{\begin{array}{cc} \text{Push} & \text{Pop for} \\ 'a' & 'a' \end{array}}$$

Here Count all 'a' & match with b and c



$$w = \frac{a^m a^n b^m c^n}{\begin{array}{cc} \text{Push} & \text{Pop} \end{array}}$$



So, here for 'a', push it on the stack & after this 'b' is there then pop 'a' for all 'b' & change the state i.e. q_1 . For c we can't do the POP operation at q_1 state itself because after b , c will come and after c b will come, so order will change. For this we have to change the state for ' c '

$$\textcircled{6} L = \{a^n b^m + c^m \mid m, n \geq 1\}$$

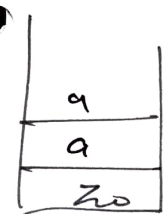
$$\Rightarrow a^n b^m b^n c^m \Rightarrow \underline{a^n b^n} \underline{b^m c^m}$$

Here, a^n push on stack, whenever you see b 's you have to match them with a 's & Remaining b 's should pushed on the stack & match them against c

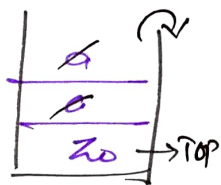
Rewrite as $\{a^n b^n \mid b^m c^m \mid m, n \geq 1\}$

\therefore For all a push a & for all b pop a & for all remaining b 's keep on pushing & match them against c 's

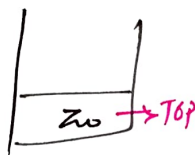
$w = \underline{a a b b b b c c}$
 Push Pop Push Pop



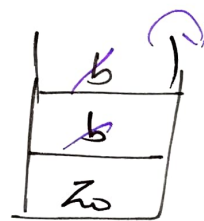
push a 's



pop a for b



push b 's



pop b for c

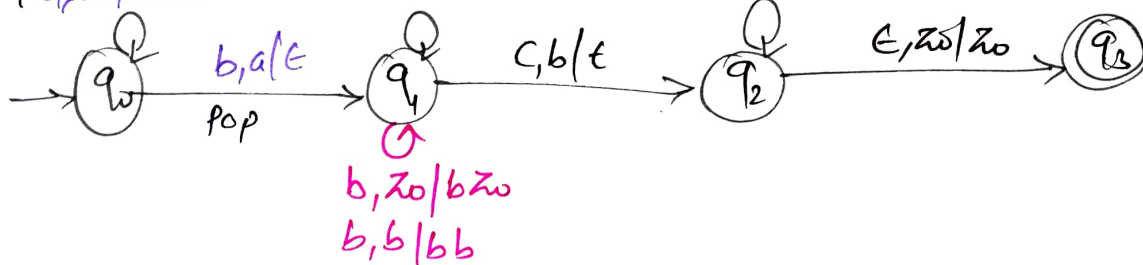


Push $\begin{cases} a, a/a a \\ a, z_0/a z_0 \end{cases}$

Pop $\begin{cases} b, a/\epsilon \\ b, z_0/b z_0 \\ b, b/b b \end{cases}$

$c, b/\epsilon$

$\epsilon, z_0/z_0$



$$\textcircled{7} \quad L = \{ a^n b^m c^{n+m} \mid n, m \geq 1 \}$$

$$a^n b^m c^n c^m$$

\Rightarrow Here, we have to push 'a' then push 'b'.
 After this for every 'c' we have to pop ~~a~~ & b.