* Conversion of Finite Automata to RE :-

1) Arden's Lemma (Theorem)

2) State/loop Elimination Method.

1) Arden's Lemma :-

This method only used for DFA & NFA and can not be used for ϵ-NFA.

If P, Q, R be the three regular expression on Σ such that

$$R = Q + RP$$ when P doesn't contain 'ϵ'

① $R = Q + RP$ which has unique solution if P is from 'ϵ'

i.e. $R = Q + RP \Rightarrow R = QP^*$

② If P contains ϵ then equation $R = Q + RP$ has infinitely many solutions.

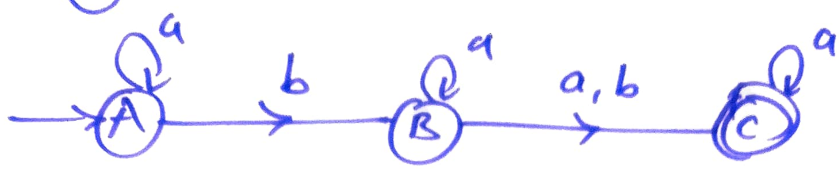
Proof :- $R = Q + RP$

$R = Q + (QP^*)P$ ($\because$ substitute $R = QP^*$)

$R = Q(\epsilon + P^*P)$ ($\because \epsilon + rr^* = r^*$)

$R = QP^*$

Ex:- (1)



$\rightarrow$ $A = Aa + \epsilon$ $\Rightarrow$ Transition coming to the states

$$\underset{R}{A} = \underset{Q}{\epsilon} + \underset{RP}{Aa}$$

$$A = \epsilon a^* = a^*$$

$$B = Ab + Ba$$

$$\underset{R}{B} = \underset{Q}{a^*b} + \underset{RP}{Ba}$$

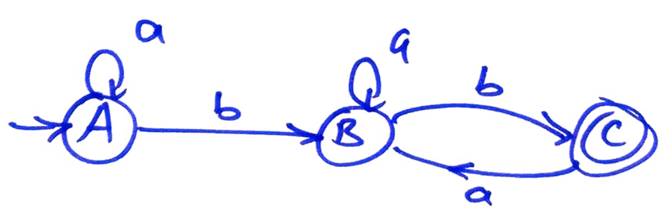
$$B = a^*ba^*$$

$$C = Ba + Bb + Ca = B(a+b) + Ca$$

$$\underset{R}{C} = \underset{Q}{a^*ba^*(a+b)} + \underset{RP}{Ca}$$

$$C = a^*ba^*(a+b)a^* \Rightarrow \text{R.E.}$$

(2)



$$A = Aa + \epsilon$$

$$\underset{R}{A} = \underset{Q}{\epsilon} + \underset{RP}{Aa}$$

$$A = a^*$$

$$B = Ba + Ab + Ca$$

$$B = Ab + Ba + Ca$$

$$B = a^*b + Ba + Bba$$

$$\underset{R}{B} = \underset{Q}{a^*b} + \underset{R}{B}\underset{P}{(a+ba)}$$

$$B = a^*b(a+ba)^*$$

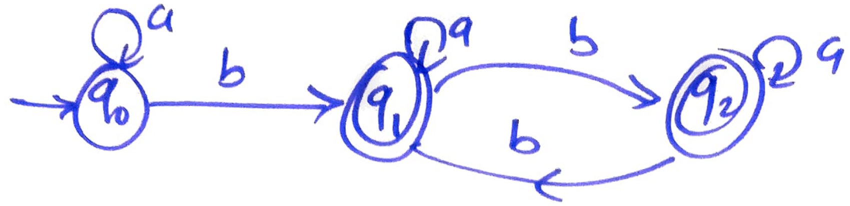
$$C = Bb$$

$$C = a^*b(a+ba)^*b$$

RE =

$$a^*b(a+ba)^*b$$

③



→ $\underset{R}{q_0} = \underset{Q}{\epsilon} + \underset{R}{q_0}\,\underset{P}{a} = \underline{a^*}$

$q_1 = q_0 b + q_1 a + q_2 b$

$q_1 = a^* b + q_1 a + q_2 b$

$q_1 = a^* b + q_1 a + q_1 b a^* b$

$\underset{R}{q_1} = \underset{Q}{a^* b} + \underset{R}{q_1}\,\underset{P}{(a + ba^* b)}$

$\underline{q_1 = a^* b\,(a + ba^* b)^*}$

$\underset{R}{q_2} = \underset{Q}{q_1 b} + \underset{R}{q_2}\,\underset{P}{a}$

$\underline{q_2 = q_1 b a^*}$

$q_2 = a^* b\,(a + ba^* b)^* b a^*$

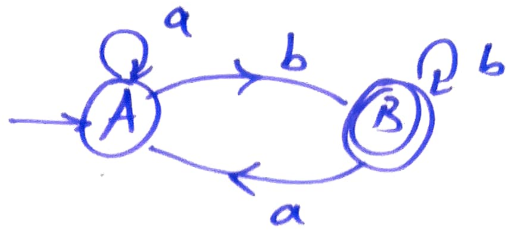
There are two final states; q_1 & q_2 so,

$RE = q_1 + q_2$

$r = a^* b(a + ba^* b)^* + a^* b(a + ba^* b)^* ba^*$

$\boxed{r = a^* b(a + ba^* b)^* [\epsilon + ba^*]}$

④ Find RE



→ $A = \epsilon + Aa + Ba$

$A = \epsilon + Aa + Ab^+a$

$\underset{R}{A} = \underset{Q}{\epsilon} + \underset{R}{A}\underset{P}{(a + b^+a)}$

$A = (a+b^+a)^*$

$\underset{R}{B} = \underset{Q}{Ab} + \underset{R}{B}\underset{P}{b}$

$B = Abb^*$

$B = Ab^+$ [$\because r.r^* = r^+$]

$B = (a+b^+a)^* b^+$

$RE = (a+b^+a)^* b^+$ ⇐

$= [(\epsilon + b^+)a]^* b^+$

$RE = [b^*a]^* b^+$

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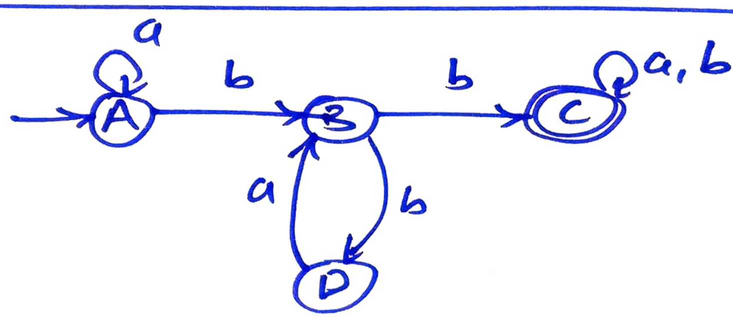
⑤



$A = \cancel{\epsilon} + Aa = a^*$

$B = Ab + Da$

$B = a^*b + Da$

$B = a^*b + Bba$

$B = a^*b(ba)^*$

$C = Bb + Ca + Cb$

$C = Bb + C(a+b)$

~~C = Bb~~

$\underset{R}{C} = \underset{Q}{a^*b(ba)^*b} + \underset{R}{C}\underset{P}{(a+b)}$

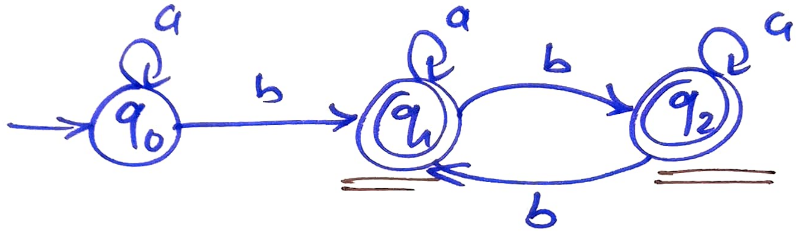
$C = a^*b(ba)^*b(a+b)^*$

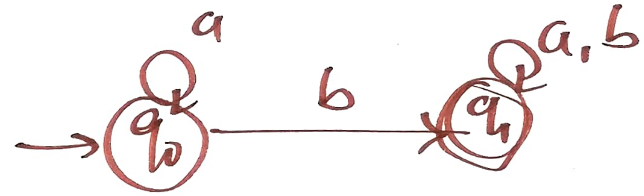
$D = Bb$

$D = a^*b(ba)^*b$

$RE = a^*b(ba)^*b(a+b)^*$

Note:- If DFA contains nonproductive states like dead states, unreachable states & equal states then remove them from DFA and find RE from the rest of the states.

① 

→ 

$q_0 = \epsilon + q_0 a$

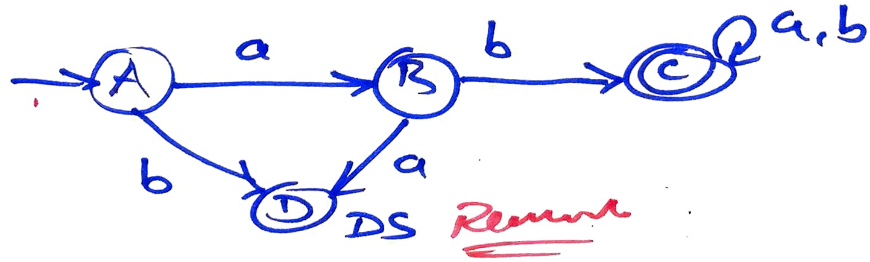
$q_0 = a^*$

$q_1 = q_0 b + q_1 a + q_2 b$

$q_1 = q_0 b + q_1 (a+b)$

$q_1 = a^* b + q_1 (a+b)$

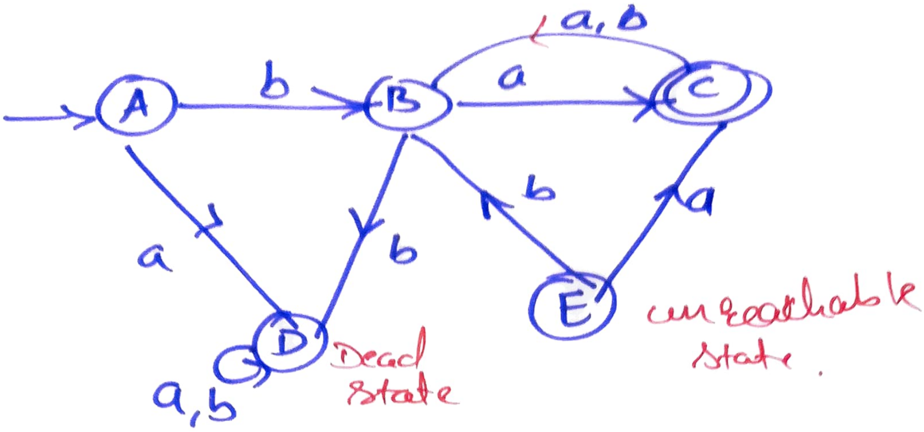
$q_1 = a^* b (a+b)^*$

② 

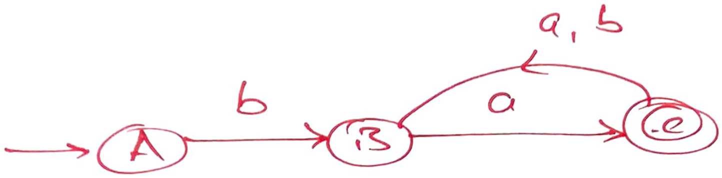
→ 

RE = $ab(a+b)^*$

(3)



$\Rightarrow$



$$A = \epsilon$$

$$B = Ab + C(a+b)$$

$$\underset{R}{B} = \underset{Q}{b} + \underset{R}{B}\underbrace{a(a+b)}_{P}$$

$$B = b[a(a+b)]^*$$

$$C = Ba$$

$$\underline{C = b[a(a+b)]^* a}$$

$$RE = b[a(a+b)]^* a$$