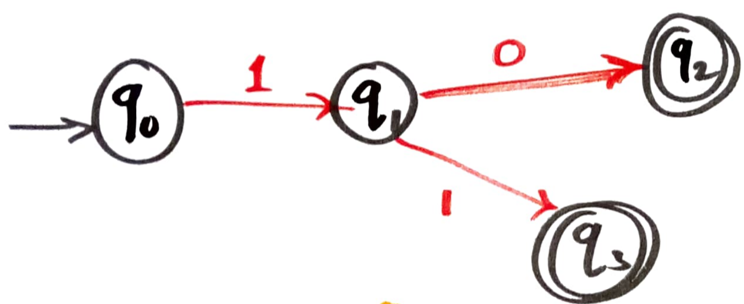
22> Construct the minimal FA that accepts 0's & 1's where a) The 2nd symbol from right end is 1
b) The 3rd symbol from right end is 0

→ $\Sigma = \{0, 1\}$

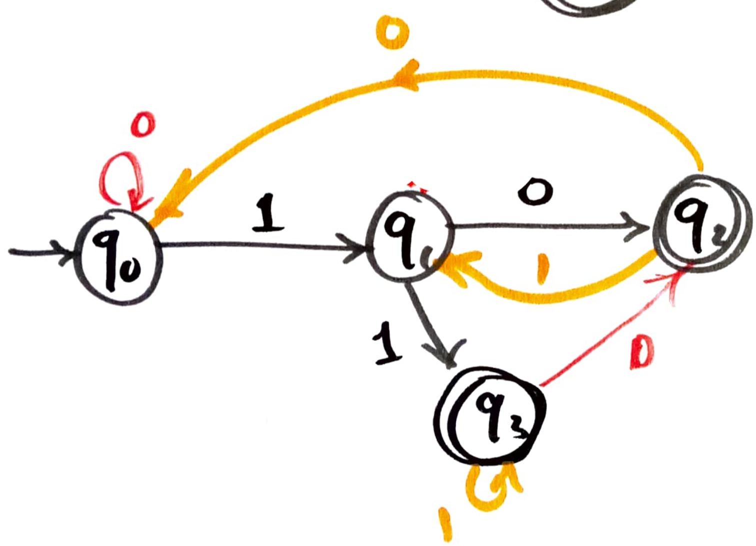
$L = \{10, 11, 1010, 1111, 0110, 110, 1110, 010,$

(a) w = ...1x

0 , 1



- 10
- 11



- 10
- 11
- 010
- 0110
- 110
- 1110

- 10010
- 1010
- 10110

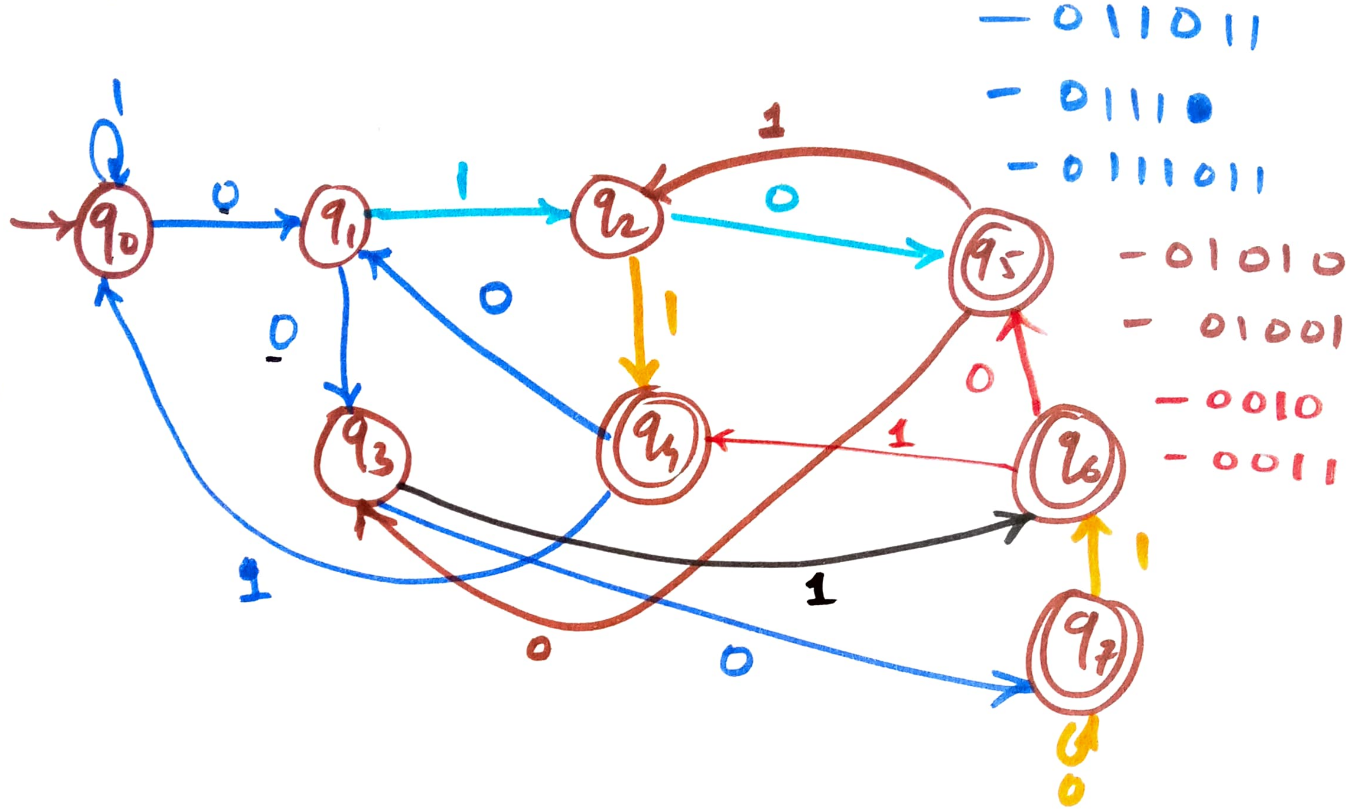
δ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_3

No. of states $= 2^n$

$= 2^2 = \underline{4}$

(b) $w = \ldots\ldots 0\underset{0,1}{x}\underset{0,1}{x}$

$L = \{\underline{000}, \underline{010}, \underline{001}, \underline{011}, \ldots\ldots\}$



δ	0	1
→ q_0	q_1	q_0
q_1	q_3	q_2
q_2	q_5	q_4
q_3	q_7	q_6
q_4	q_1	q_0
q_5	q_3	q_2
q_6	q_5	q_4
q_7	q_7	q_6

Note:-

The minimal FA that accepts all the strings of 0's & 1's where nth symbol from ~~f~~ right end is fixed. Contains exactly 2^n states & 2^{n-1} final states

$= 2^3 = \underline{8}$ States

$= 2^{n-1} = 2^{3-1} = 2^2 = \underline{4}$ final States

23> Construct the FA that accept all the strings of 0's & 1's where the no. of 0's in a string is —

ⓐ Exactly 2 ⓑ atmost 2 ⓒ atleast 2

ⓓ even ⓔ odd

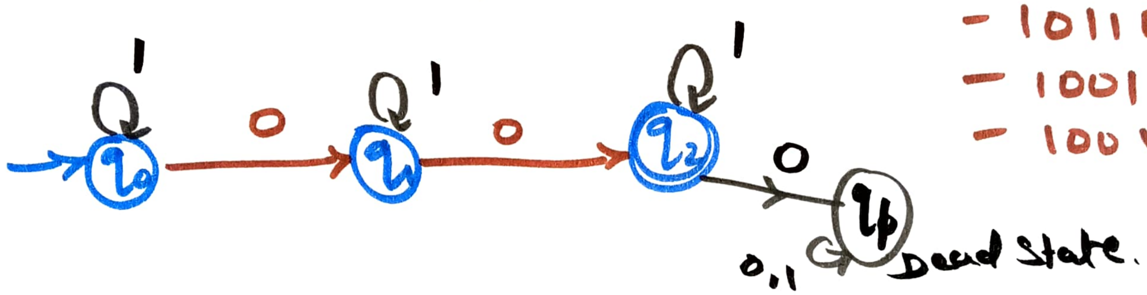
ⓕ 2 (mod 3)

→ $\Sigma = \{0, 1\}$

ⓐ Exactly 2 0's $= |W|_0 = 2$

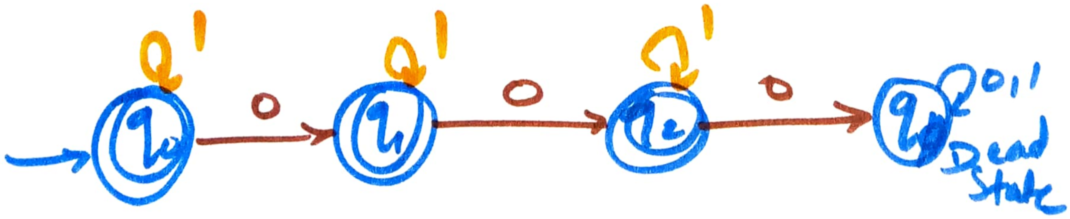
$L = \{$
- 00
- 100
- 10110, 101
- 1001
- 10011

$W = x\,0\,x\,0\,x$



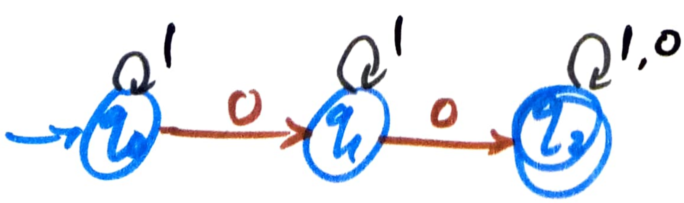
ⓑ atmost 2 ≤ 2 $|W|_0 = \{0, 1, 2\}$

- ε
- 0
- 00, 000
- 10, 100
- 1010
- 10011

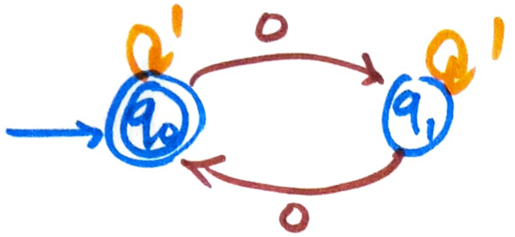


ⓒ atleast 2, $|W|_0 = 2, 3, \ldots$

- 00, 000,
- 100
- 1010
- 010100

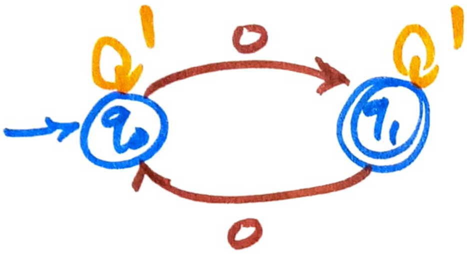


(d) even $|w|_0$ = even = 0, 2, 4, 6 ...



L = { ε, 0, 00, 100, 010, ... }

(e) odd $|w|_0$ = odd = 1, 3, 5, 7, ...



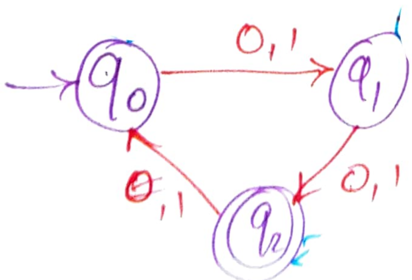
L = {0, 000, 10, 101, 1001, ...}

Lang.	No. of States.
$\|w\|_0 = n$	n+2
$\|w\|_0 \leq n$	n+2
$\|w\|_0 \geq n$	n+1
$\|w\|_0 = r(\text{mod } n)$	n

(f) 2(mod 3) 2, 5, 8, 11, ...

L 0 1 2

q_0 q_1 q_2



4 2 1

0 0 0 0

2 0 1 0

5 - 0 1 0 1

8 - 1 0 0 0

1 - 0 0 1

1 0 1

24) Construct the minimal FA that accept all the strings of 0 & 1 where the length of the

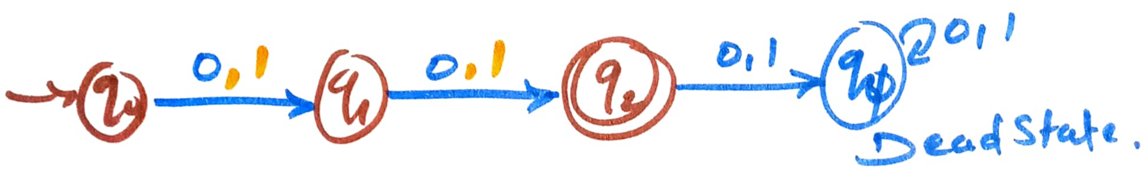
(a) string is exactly two (b) atmost 2

(c) atleast 2 (d) even (e) odd.

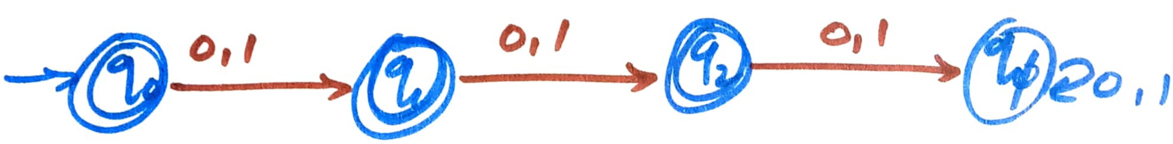
→ (a) $\Sigma = \{0, 1\}$

$|w| = 2$; $w = \underset{0,1}{x}\ \underset{0,1}{x}$

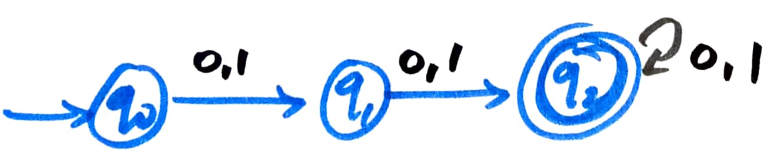
L = { 00, 01, 10, 11 }



(b) $|w| \leq 2$; $|w| = \{0, 1, 2\}$

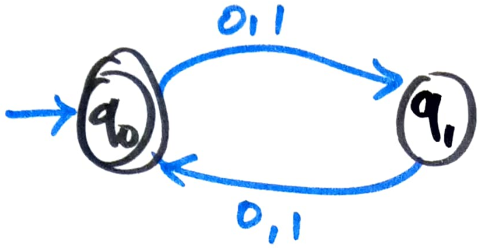


(c) $|w| \geq 2$, $|w| = 2, 3, 4, \dots$



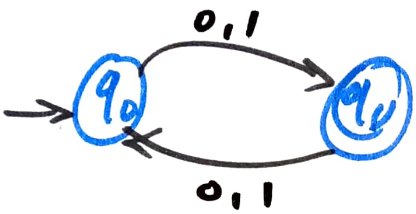
(d) $|w| = \text{even} = 0, 2, 4, 6 = 0 \pmod 2$

└ q_0 q_1



(e) $|w| = 1, 3, 5 = 1 \pmod 2$

└ 0, 1



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25) Design a DFA that accepts all the strings with atmost 3 a's $\Sigma = \{a, b\}$

26) DFA with prefix ab with alphabets $\Sigma = \{a, b\}$

27) DFA accepts even no. of a's $\Sigma = \{a, \}$

28) DFA that contains 001 as substring in all strings over $\Sigma = \{0, 1\}$

29) Construct the minimal FA that accepts all the strings of 0's & 1's where every string starts with '0' and length is 2(mod 3)

→ $\Sigma = \{0,1\}$; $W = 0X$

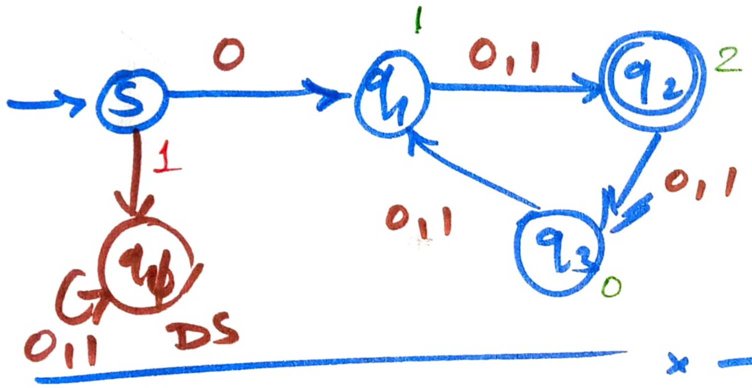
2, 5, 8, ...

where $|W| = 2 (\bmod 3)$ q_1 q_2 q_3

L 0 1 2

q_0 q_1 q_2

8 4 2 1
0 – 0 0 0
1 – 0 0 1
2 – 0 1 0
0 1 1
2 – 0 0 0 1 0
5 – 0 0 1 0 1



No. of States = 2+3 = 5

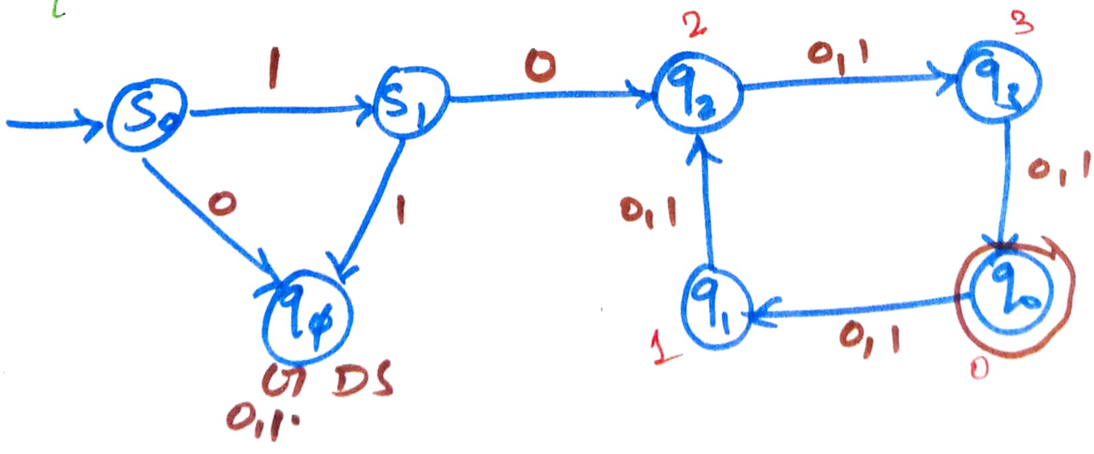
30) $\Sigma = \{0,1\}$ where every string starts with 10 and the length of the string is 0(mod 4)

→ $\Sigma = \{0,1\}$; $W = 10X$; $|W| = 0 (\bmod 4)$

L 0 1 2 3
q_0 q_1 q_2 q_3

0, 4, 8, 12, 16, ...

L = {1000,



– 1000
– 1011
– 1001
– 1010
– 1000000
– 10101001

1) $W = 0X$ & $|W| = 2 \pmod 3$ q_1 q_2 q_3 2, 5, 8, 11 ....

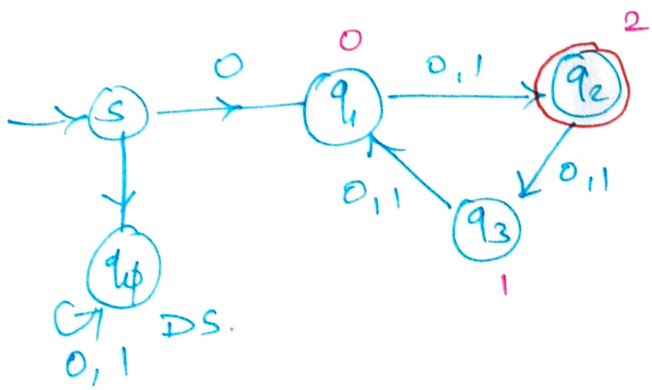
0 1

L 0 1 2

q_0 q_1 q_2

0 -	00000
1 -	00001
2 -	00010
3 -	00011
4 -	00100
5 -	00101
8 -	01000
11 -	01011

$L = \{ \underbrace{00}_{2}, \underbrace{01}_{2}, \underbrace{00100}_{5}, \underbrace{01111}_{5}, \underbrace{00110}_{5}, \ldots \}$



	R	
- 0 ⇒	1	— q_1
- 00 —	2	q_2
- 01 —	2	
- 000 —	0	q_3
- 011 —	0	
- 010 —	0	
- 001 —	0	
- 0000 —	1	q_1
- 0011 —	1	
- 0101 —	1	
- 0010 —	1	

---

2) $W = 10x$; $|W| = 0 \pmod 4$

	R
⇒ 10 —	2
- 100	3
- 101	
- 1000	-0
- 1010	
- 1001	
- 1011	

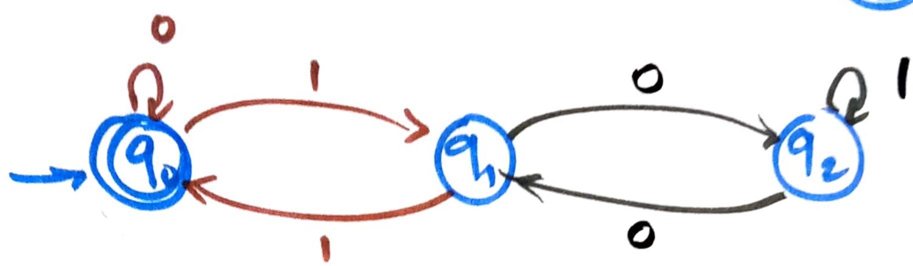
10000	-1
10100	
10011	
10101	

31) Construct the minimal FA that accept all the binary strings whose equivalent integer is

ⓐ Divisible by 3

→ $\Sigma = \{0, 1\}$

0 (mod 3) → q_0 (0), q_1 (1), q_2 (2)



	8	4	2	1
0 -	0	0	0	0
1 -	0	0	0	1
2 -	0	0	1	0
3 -	0	0	1	1
4 -	0	1	0	0
6 -	0	1	1	0
9 -	1	0	0	1
	0	1	0	1

δ	0	1
→ q_0	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

$q_0 \neq q_1 \neq q_2$ ⇐ Minimal DFA

ⓑ 1 (mod 4) BS to int.

$= \frac{4}{2} = 2$

$= 2 + 1 = 3$ MFA

→ q_0 (0), q_1 (1), q_2 (2), q_3 (3)

DFA

δ	0	1
→ q_0	q_0	q_1
q_1	q_2 //	q_3
q_2	q_0	q_1
q_3	q_2	q_3

$q_0 = q_2$

Minimal FA

δ	0	1
→ q_0	q_0	q_1
q_1	q_0 //	q_3
q_3	q_0 //	q_3

No. of states in Minimal FA = $\underline{3}$

(c) BS → int = 2 (mod 5)

L	0	1	2	3	4
	q_0	q_1	q_2	q_3	q_4

δ	0	1
$\to q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

No of States in minimal FA = 5

(d) BS → int = 4 (mod 6)

L	0	1	2	3	4	5
	q_0	q_1	q_2	q_3	q_4	q_5

δ	0	1
$\to q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_0	q_1
q_4	q_2	q_3
q_5	q_4	q_5

$q_0 = q_3$

$q_2 = q_5$

δ	0	1
$\to q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_4	q_2
q_4	q_2	q_0

Replace q_3 by q_0 & q_5 by q_2

32) BS → int = 3 (mod 8)

L $q_0\ q_1\ q_2\ q_3\ q_4\ q_5\ q_6\ q_7$

$= \frac{8}{2} = \frac{4}{2} = \frac{2}{2} = 1$

$= 1 + 3$

$= 4$ states.

DFA

δ	0	1
→ q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_7
q_4	q_0	q_1
q_5	q_2	q_3
q_6	q_4	q_5
q_7	q_6	q_7

$q_0 = q_4$

$q_1 = q_5$

$q_2 = q_6$

$q_0 = q_4$
$q_1 = q_5$
$q_2 = q_6$

Minimal DFA

δ	0	1
→ q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_0	q_1
q_3	q_2	q_7
q_7	q_2	q_7

$q_0 = q_2$

δ	0	1
→ q_0	q_0	q_1
q_1	q_0	q_3
q_3	q_0	q_7
q_7	q_0	q_7

$q_0 = q_2$

33) 0 (mod 10)

L 0 1 2 3 4 5 6 7 8 9

q_0 q_1 q_2 q_3 q_4 q_5 q_6 q_7 q_8 q_9

$= \frac{10}{2} = 5+1 = 6$ states.

DFA

δ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_7
q_4	q_8	q_9
q_5	q_0	q_1
q_6	q_2	q_3
q_7	q_4	q_5
q_8	q_6	q_7
q_9	q_8	q_9

$q_1 = q_6$

$q_2 = q_7$

$q_3 = q_8$

$q_4 = q_9$

$q_1 = q_6$
$q_2 = q_7$
$q_3 = q_8$
$q_4 = q_9$

Minimal DFA

δ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_1	q_2
q_4	q_3	q_4
q_5	q_0	q_1

34) BS → int $= 1 \pmod{12}$

$$= 12 = \frac{12}{2} = \frac{6}{2} = \underline{\underline{3}}$$

Minimal DFA $= 3 + 1 + 1 = \underline{\underline{5}}$ States

35) $10 \pmod{16}$

$$16 = \frac{16}{2} = \frac{8}{2} = \frac{4}{2} = \frac{2}{2} = 1$$

Minimal DFA $= 1 + 4 = 5$ states.

(OR)

$$16 = 2^4 = 4 + 1 = 5$$

Note:-

BS → int $= \varepsilon \pmod{n}$

n is odd

↓

No. of States $= \underline{n}$

n is even

No. of States = odd no. + No. of steps.

$n = 2^m$

No. of States $= \underline{\underline{m+1}}$

36> Construct the minimal FA that accept all the nos. of base 3 (or) ternary which are divisible by 4.

→ $\Sigma = \{0,1,2\}$ ⇒ Ternary (or) base 3 no.

0(mod 4)

↳	0	1	2	3
	q_0	q_1	q_2	q_3

Ternary No.

0 – 0	10 – 3	20 – 6	100 – 9	110 – 12
1 – 1	11 – 4	21 – 7	101 – 10	111 – 13
2 – 2	12 – 5	22 – 8	102 – 11	112 – 14

120 – 15	200 – 18	
121 – 16	201 – 19	....
122 – 17	202 – 20	

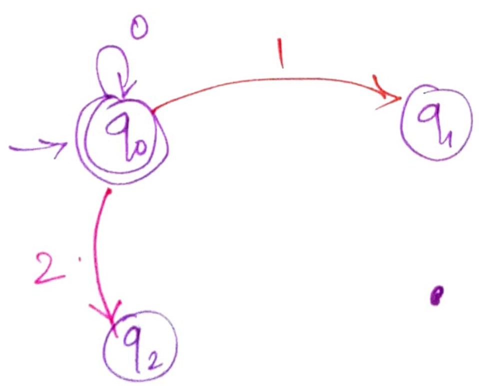
$= (121)_3 = (16)_{10}$

$= 1\times 3^0 + 2\times 3^1 + 1\times 3^2$

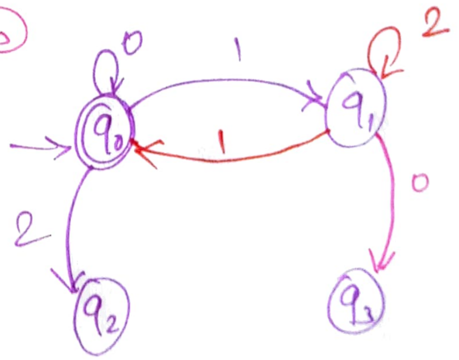
$= 1 + 6 + 9$

$= 16$

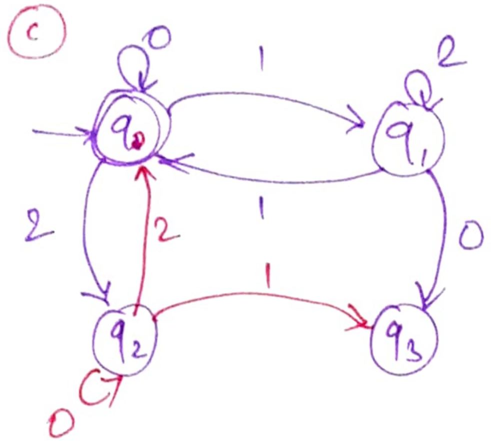
(a)



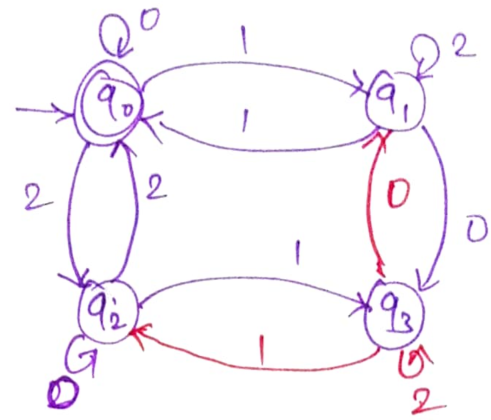
(b)



(c)



(d)



δ	0	1	2
→ q_0	q_0	q_1	q_2
q_1	q_3	q_0	q_4
q_2	q_2	q_3	q_0
q_3	q_4	q_2	q_3

0(mod 4)

⇐ Minimal FA

---

37) Construct min. FA. accepts base 3 nos. which are divisible by 5

→ $\Sigma = \{0, 1, 2\}$; 0(mod 5)

L 0 1 2 3 4

q_0 q_1 q_2 q_3 q_4

δ	0	1	2
→ q_0	q_0	q_1	q_2
q_1	q_3	q_4	q_0
q_2	q_1	q_2	q_3
q_3	q_4	q_0	q_1
q_4	q_2	q_3	q_4

38) Construct minimal FA that accept all the positive integer no. which are divisible by 3.

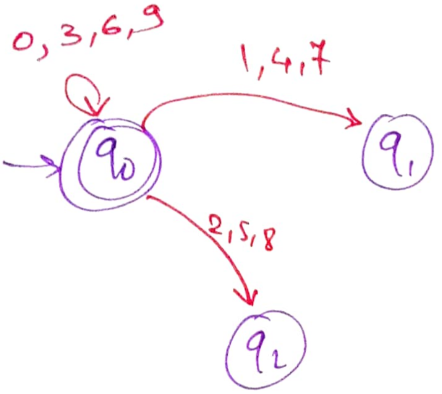
→ $\Sigma = \{0, 1, \ldots\ldots 9\}$

$0 \pmod 3$

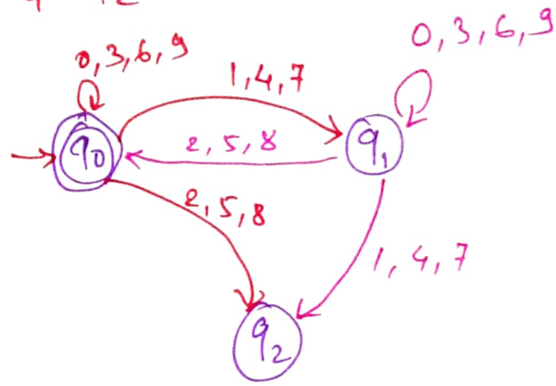
$\llcorner$ 0 1 2

q_0 q_1 q_2

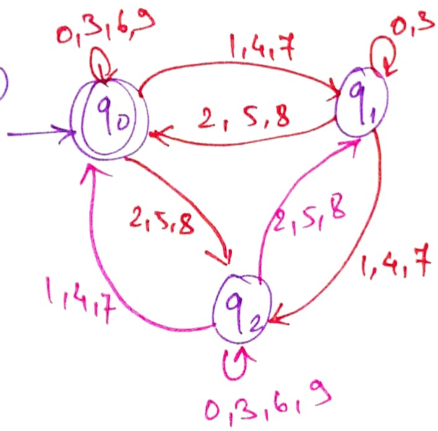
(a)



(b)



(c)



δ:

δ	0	1	2	3	4	5	6	7	8	9
→ q_0	q_0	q_1	q_2	q_0	q_1	q_2	q_0	q_1	q_2	q_0
q_1	q_1	q_2	q_0	q_1	q_2	q_0	q_1	q_2	q_0	q_1
q_2	q_2	q_0	q_1	q_2	q_0	q_1	q_2	q_0	q_1	q_2

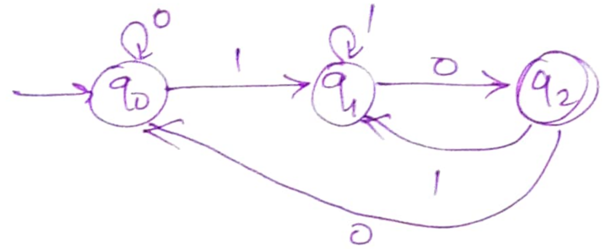
$$M = \left\{ \begin{array}{l} Q = \{q_0, q_1, q_2\}, \\ \Sigma = \{0,1,2,3,4,5,6,7,8,9\}, \; q_0 = \{q_0\} \\ F = \{q_0\}, \delta \end{array} \right\}$$

39) Construct minimal FA that accept all the strings of 0's & 1's where every string do not ends with 10

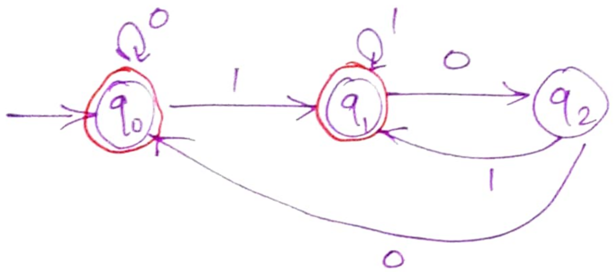
→ $\Sigma = \{0, 1\}$ $\quad w \neq x10$

$L = \{10, 010, 110, 1010, 11010, 10010, \ldots\}$

F.A.



Now, complement the F.A.



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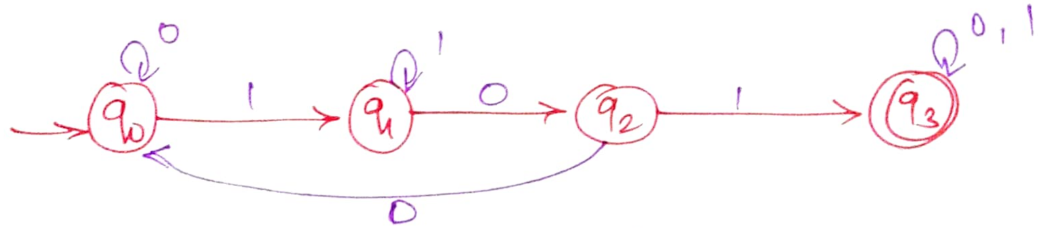
40) Construct minimal FA that accept all the strings of 0's & 1's where every string can not contain substring 101

→



Now take complement of FA