0/1 Knapsack using LC

LC gives u faster result

$P_1 - P_4$ 10 10 12 18

$w_1 - w_4$ 2 4 6 9

$u = \sum_{i=1}^{n} P_i x_i \le m$ (without fraction)

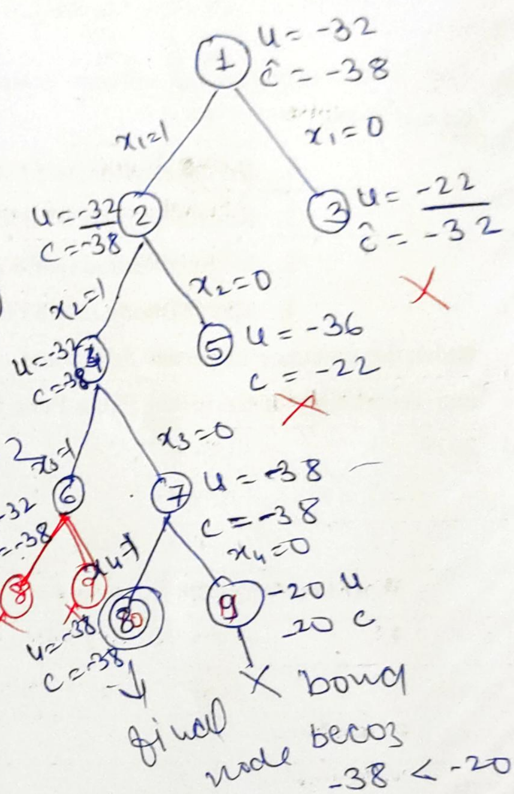
$c = \sum_{i=1}^{n} P_i x_i$ (with fraction)

upper = ~~∞~~ ~~-32~~ -38

P_i / x_i = ~~1/5~~ ~~1/3~~ ~~∅~~ 3

4	$18 \times 3/9$	u
3	12	
2	10	c
1	10	~~15~~

$x_1 = 0$



$\hat{c}(1) = -(10+10+12+\frac{18}{9} \times 3)$

$= -38$

$u(1) = -(10+10+12) = -32$

$\hat{c}(3) = -(10+12+18 \times \frac{5}{9})$

$= -32$

$u(3) = -(10+12) = -22$

$-32 < -22$

↳ branch ~~start~~ from (2) node

explore that node whose cost is min

$u(5) = -(10+12)\cancel{*} = -22$

$\hat{c}(5) = -(10+12+18 \times \frac{7}{9})$

$= -36$

$u = -32, u = -36$

$-32 < \cancel{u} -36$

↳ start branch from (4) node

$\hat{c}(7) = -(10+10+18)$
$= -38$

$u(7) = -(10+10+18)$
$= -38$

for node 6 & 7
$u = -32$, $u = -38$
so min $-32 > -38$
↳ branch

$\hat{c}(8) = -(10+10+18)$
$= -38$

$u(9) = -38$

$\hat{c}(9) = -(10+10)$
$= -20$

$u(9) = -20$

$(1,1,0,1)$
10 10 0 18 = 38.

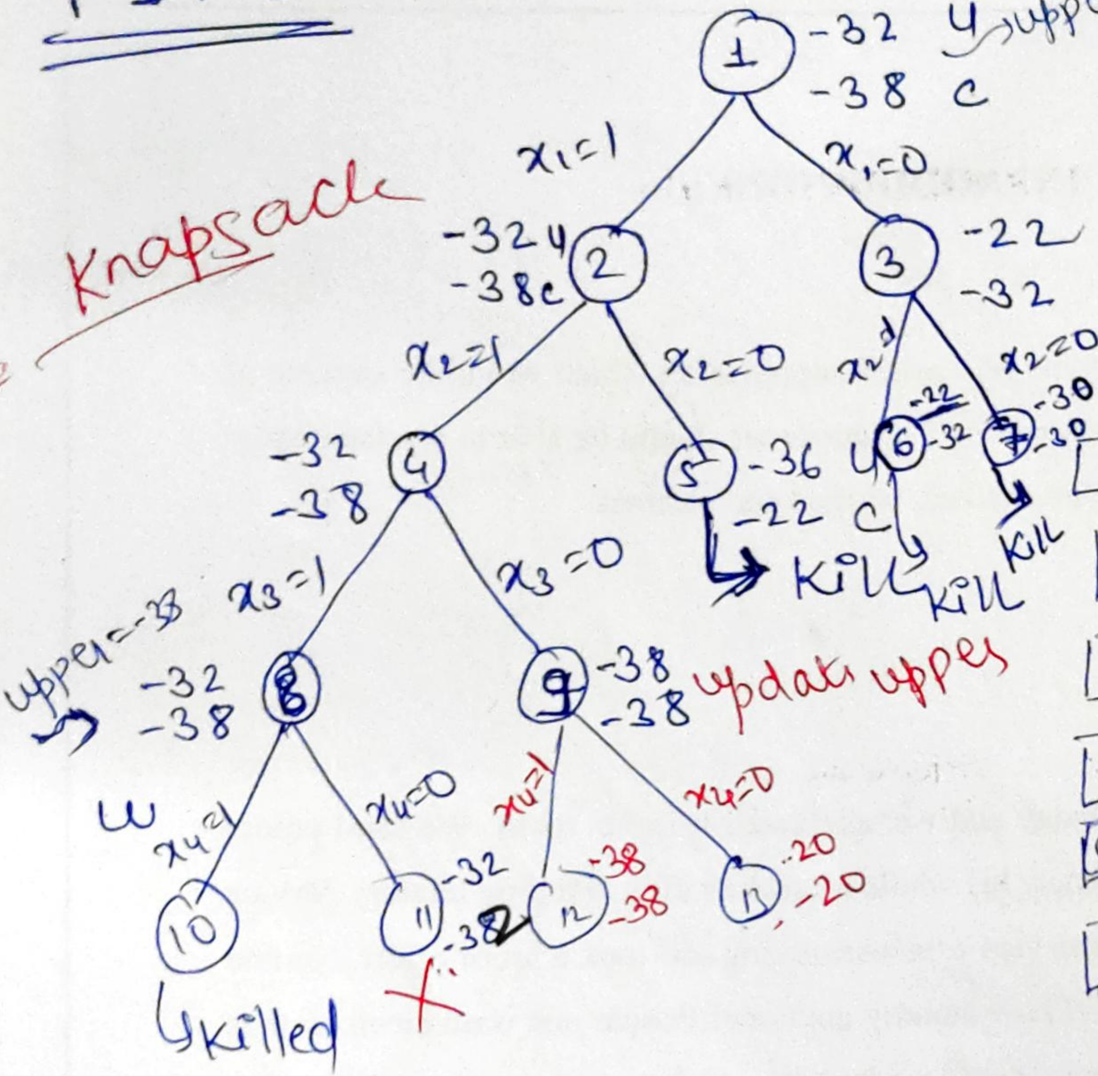
FIFO

upper = ∞

= $-\cancel{32}$

= -38

Knapsack



2	3	

1

2	3

3	4	~~5~~

4	~~5~~	~~6~~	7

6	7	8	9

10	11	12	13

upper < c

↳ kill

upper > u

update upper

$-32 < -22$

upper u

$-32 > -36$

upper $\cancel{-36} < -22$

c(5) $-22 > -32$

kill

c(7)

$-30 > -32$

update upper

$\hat{c}(11) = x_1\ x_2\ x_3$

$= 10 + 10 + 12 + \frac{18}{9} \times 3$

$2 + 4 + 6$

$= -38$

$\hat{u}(11) = -32$

Travelling Salesman prob using B&B

$G = \{V, E\}$

Let C_{ij} equal to cost of edge i, j.

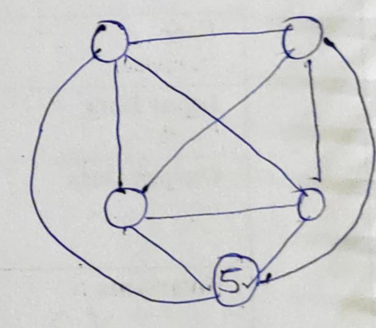
$C_{ij} = \infty$ if $(i,j) \notin E$

$\hat{c}(x)$ can be obtained by reduced cost Matrix

A row (column) is said to be reduce iff it contains atleast one zero and all remaining entries are non ~~zero~~ negative.

A matrix is reduce if every row & column is reduce

$$M = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$



$$M_1 = \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ 4 \\ \hline 21 \end{array}$$

→ min value from every row

Now Minus this min value with Matrix

$$\begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

min from column $\quad 1 \quad 0 \quad 3 \quad 0 \quad 0 = 4$

reduce

$$M_{11} = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

cost of reduction is 25.

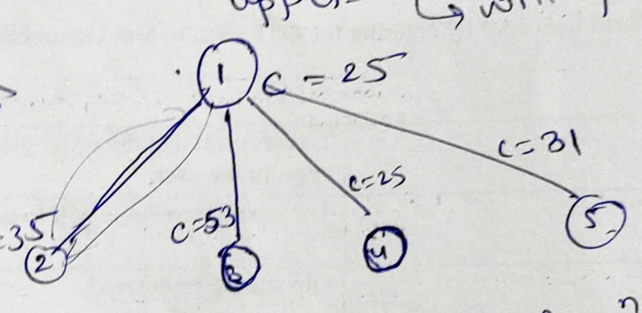
$21 + 4 = 25$ cost of reduction is 25

↳ min cost of tour may be 25

it may be > greter than > 25

upper = ∞ ↳ will update only in leaf node

LC Approach



1 to 2 make 1^{st} row & 2^{nd} column & ∞
& 2 to 1 is also ∞
in M_{11}

$$M_{12} = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \\ \begin{array}{ccccc} 0 & & 0 & 0 & 0 \end{array} \end{array}$$

reduced

$c(1,2) + r + \hat{r}$

$10 + 25 + 0 = 35$

(1 to 3) $1^{st}\ r = \infty$ $3c = \infty$ $3 \to 1 = \infty$

$$M_{13} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & 0 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

11 0 0 0

not reduce

so reduce it

$$M_{13R} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & 0 \end{bmatrix}$$

$\hat{c}(1,3) = c(1,3) + r + \hat{r}$

$= 17 + 25 + 11$

$= 53$

1 to 4 (1, 4) 1^{st} row $= \infty$ 6^{th} column $= \infty$

$4 \to 1 = \infty$

$$M_{14} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} \infty \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

0 0 0 ∞ 0 reduce Matrix

$\hat{c}(1,4) = c(1,4) + r + \hat{r}$

$= 0 + 25 + 0 = 25$

1 to 5 $\quad 1R = \infty \quad 5C = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{matrix} \infty \\ 2 \\ 0 \\ 3 \\ 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & \infty \end{matrix} \qquad 5$$

$$= \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{matrix} \infty \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$$\hat{c}(1,5) = c(1,5) + r + \hat{r}$$
$$= 1 + 25 + 5 = 31$$

it is LC Approach select node(4) as it has least cost

from Matrix (1, 4)

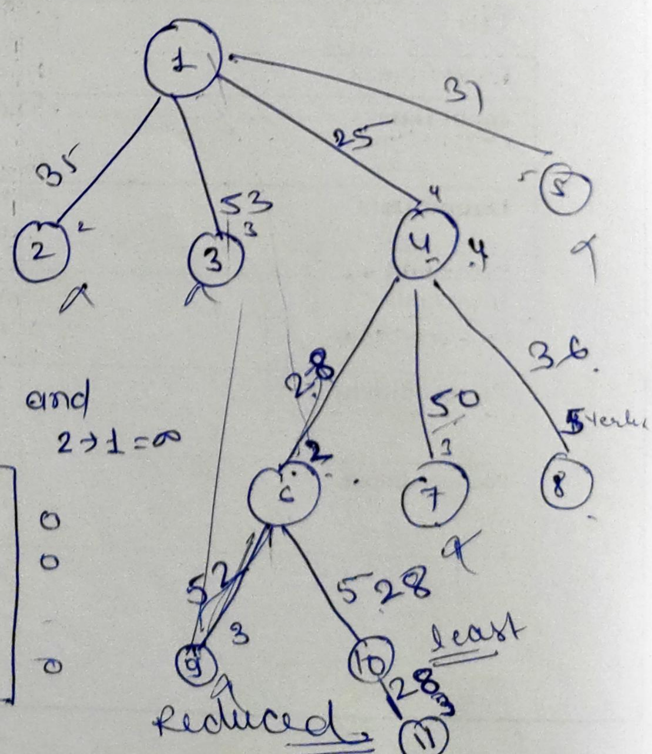
$$M_{14.3} \quad \begin{array}{c|ccccc} & 1 & 2 & & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & 12 & \infty & 11 & \infty & 0 \\ 3 & 0 & 3 & \infty & \infty & 2 \\ 4 & \infty & 3 & 12 & \infty & 0 \\ 5 & 11 & 0 & 0 & \infty & \infty \end{array}$$

$4^{th} r = \infty \quad 2^{nd} c = \infty$ and $2 \to 1 = \infty$

M

$$\underline{M42} \quad \begin{array}{c|ccccc} 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \cancel{12}\infty & \infty & 11 & \infty & 0 \\ 3 & 0 & \infty & \infty & \infty & 2 \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & 11 & \infty & 0 & \infty & \infty \end{array} \begin{matrix} \\ 0 \\ 0 \\ \\ 0 \end{matrix}$$

$$\begin{matrix} 0 & & 0 & & 0 \end{matrix}$$



from Matrix M14

$\hat{C}(4,2) = C(4,2) + \hat{r} + r$

$= 3 + 25 + 0 = 28$

for $(4,3) = (4 \text{ to } 3)$

$$M_{14} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Now for 4 to 3 $\quad 4^{th} R = \infty \quad 3^{rd} C = \infty$

$\& \ (3,1) = \infty$

$$M_{43} \quad \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ 2 \\ \\ \\ \end{array} \\ \begin{array}{ccccc} 11 & 0 & & & 0 \end{array} \end{array}$$

Reduce 11 & 2 $\quad$ total reduct^n = 13

$$M_{43} \quad \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{array}{c} \\ 0 \\ 0 \\ \\ \\ \end{array} \\ \begin{array}{ccccc} 0 & 0 & & & \end{array} \end{array}$$

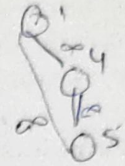
$\hat{C}(4,3) = C(4,3) + r + \hat{r}$

from M_{14}

$= 12 + 25 + 13 = 50$

node
for (4, 8) for edge (4 to 5)

$$M_{14} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$



4 to 5 means set $4^{th}\ r = \infty$ $5^{th}\ c = \infty$ & 5 to ~~4~~ 1 $= \infty$

$$M_{45} = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} \\ 11 \\ 0 \\ - \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} \end{array}$$

Reduce 11

$$M_{45} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

$$\hat{C}(4,5) = C(4,5) + r^{1-4} + \hat{r}$$
$$= 0 + 25 + 11 = 36$$

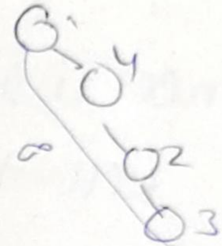
out of 2, 3, 5, 6, 7, 8 Node select Node 6 to explore as it has less cost

for node (6 to 9) with edge (2,3)

$$M_{42} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

...major pain point for the users.

(2,3) means 2^{nd} r = ∞ 3^{rd} c = ∞ & 3 to 1 = ∞



$$M_{23}\ \begin{array}{c} \left[\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{array}\right] \begin{array}{c} - \\ - \\ 2 \\ - \\ 11 \end{array} \\ \begin{array}{ccccc} 11 & & & & 2 \end{array} \end{array}$$

13 total Reduct = 11+2 = 13

~~RC~~ 1st R
2^{nd} C

$$M_{23}\ \left[\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{array}\right]$$

11+2
= 13

$$\hat{C}(2,3) = c(2,3) + r + \hat{r}$$
$$= 11 + 28 + 13$$
$$= 52$$

for node 6 to 10 edge (2,5)
by Making 2^{nd} r = ∞ 5^{th} c = ∞ 5 to 1 = ∞ in M_{42}

$$M_{25} = \left[\begin{array}{ccccc} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{array}\right] \begin{array}{c} - \\ - \\ 0 \\ - \\ - \end{array}$$

Reduced

$$(C_{25}) = c(25) + r + \hat{r}$$
$$= 0 + 28$$
$$= 28$$

0

for node (10 to 11) edge (5, 3)

$$M_{25} = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$ Reduced $= M_{53}$

5^{th} row ∞

3^{rd} c $= \infty$

~~$\hat{c}(2,5)$~~ $= c(2,5) + r + \hat{r}$

3 to 1 $\neq \infty \rightarrow$ dont do

M_{53}

$$\hat{c}(5,3) = c(5,3) + \underline{r} + \hat{r}$$
$$= 0 + 28 + 0$$
$$= 28$$

update upper = 28

Once u reach at last update upper = 28, node whose cost is > upper, kill that node

1-4-2-5-3-1

N Queen

Q.No.					2, ~~4~~, 6, ~~8~~

J = 1 to k-1

7 5 3 1)

q_1 q_2 q_3 q_4 q_5 q_6 q_7 q_8

7 5 3 1

7 5 3 1 ~~2~~

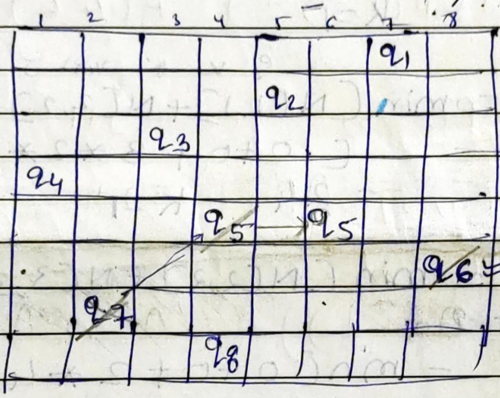
7 5 3 1 4

7 5 3 1 4 ~~6~~

7 5 3 1 4 8 ~~2~~

7 5 3 1 4 8 6

7 5 3 1 4 8 ~~6~~ ~~8~~



q_1 q_2 3 4 5 6 7 8

7 5 3 1 4 8 ~~2~~

backtrack 8.

7 5 3 1 4 ~~8~~

7 5 3 1 4

No acceptable position for q_{7} →
backtrack $q_{\cancel{5}}$

7 5 3 1 ~~4~~

7 5 3 1 6

7 5 3 1 6 8

7 5 3 1 6 8 2

7 5 3 1 6 8 2 4.

7-1=6
5-2=3
~~3-3=0~~
4+1=5
5+4=9
8-6=2
7+2=
7-6=1

5-1=3
4-1=3
6-6=3-3
=0

7+2=5+4
=9

7+6=13

2 ~~4~~ ~~6~~ ~~8~~
2, 6
7+2=9
5+4=9
2, 6, 8.

2, 4, 6, 8

2, 4, ~~8~~