Q.9 Construct the grammar that generates all the strings of a's & b's where the length of the string is —

1) Exactly two 2) almost two 3) atleast 2
4) even 5) odd 6) 2(mod 3)

→ ① $|w| = 2$

$w = x\,x$

$S \to xx$

$x \to a|b$

$S \to xx$
$\to ab$

② $|w| \leq 2 \Rightarrow 0, 1, 2$

$S \to xx$

$x \to a|b|\epsilon$

$S \to xx$
$\to ax$
$\to aa$

$S \to xx$
$\to b.\epsilon$
$\to b$

③ $|w| \geq 2 \Rightarrow 2, 3, \ldots$

$w = x\,x \ldots\ldots$

$a+b$ $a+b$ $(a+b)^*$

A A

$w = \underbrace{(a+b)^2}_{x}\underbrace{(a+b)^*}_{y}$

x { $S \to xx$
$x \to ax|bx|a|b$ (a)

$S \to xy$
$\to AAy$
$\to aaay$
$\to aaa$

$S \to xy$
$x \to AA$
$A \to a|b$
$y \to ay|by|\epsilon$

④ $|w| = \text{even}$
$= 0 \pmod 2$

$q = [\underbrace{(a+b)^2}_{A}]^*$

$S \to xS|\epsilon \Rightarrow x^*$

$x \to AA$
$A \to a|b$

$S \to xS$
$\to AAxS$
$\to aaAAS$
$\to aabb.\epsilon$
$\to aabb.$

5) $|w| = odd$

$= 1 \pmod 2$

$(a+b)\,[(a+b)^2]^*$

$\underbrace{(a+b)}_{X}\ \underbrace{[(a+b)^2]^*}_{Y}$

$\Rightarrow$

$S \to XY$

$\to aAY$

$\to a\,xxY$

$\to aaa.\epsilon$

$\to aaa$

$S \to xy$

$x \to a \mid b$

$Y \to AY \mid \epsilon$

$A \to xx$

$\Rightarrow$

$S \to xYy$

$x \to a \mid b$

$Y \to aY \mid bY \mid \epsilon$

$S \to xYy$

$\to a\underline{Y}y$

$\to a\underline{aY}y$

$\to aa\epsilon.\epsilon.$

$\to aa$

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6) $|w| = 2 \pmod 3$

$\underbrace{(\underbrace{a+b}_{A})^2}_{X}\ \underbrace{[(a+b)^3]^*}_{Y}$

$S \to xy$

$x \to AA$

$A \to a \mid b$

$Y \to BY \mid \epsilon$

$B \to AAA$

$S \to xy$

$\to AAy$

$\to aaBy$

$\to aaAAAy$

$\to aabbb.\epsilon$

$S \to xy$

$\to AAy$

$\to bb.\epsilon$

$S \to xy$

$\to AAy$

$\to aaBy$

$\to aaAAABy$

$\to aabbbAAA.\epsilon$

$\to aabbbaaa$

Q.10) Construct the grammar for the following language –

① $L=\{a^m b^n \mid m,n \geq 0\}$ ② $L=\{a^m b^n \mid m,n \geq 1\}$

③ $L=\{a^m b^n \mid m \geq 0, n \geq 1\}$ ④ $L=\{a^m b^n \mid m \geq 1, n \geq 0\}$

⑤ $L=\{a^m b^n \mid m+n = \text{even}\}$ ⑥ $L=\{a^m b^n \mid m+n = \text{odd}\}$

→ ① $L=\{a^m b^n \mid m,n \geq 0\}$

$a^* b^*$
A B

$S \to AB$
$A \to aA \mid \epsilon$
$B \to bB \mid \epsilon$

$S \to AB$
$\to aAbB$
$\to aaAbbB$
$\to aa.\epsilon\, bb.\epsilon$
$\to aabb$

② $L=\{a^m b^n \mid m,n \geq 1\}$

$a^+ b^+$
A B

$S \to AB$
$A \to aA \mid a$
$B \to bB \mid b$

$S \to AB$
$\to aAbB$
$\to aaAbbB$
$\to aaabbb$

③ $L = \{a^m b^n \mid m \geq 0, n \geq 1\}$

$\underset{A}{a^*}\ \underset{B}{b^+}$

$S \to AB$

$A \to aA \mid \epsilon$

$B \to bB \mid b$

$S \to AB$
$\to aAB$
$\to aaAbB$
$\to aa.\epsilon\, bb$
$\to aabb$

④ $L = \{a^m b^n \mid m \geq 1, n \geq 0\}$

$\underset{A}{a^+}\ \underset{B}{b^*}$

$S \to AB$

$A \to aA \mid a$

$B \to bB \mid \epsilon$

$S \to AB$
$\to aAB$
$\to aabB$
$\to aab$

$S \to AB$
$\to aAB$
$\to aa.\epsilon$
$\to aa$

⑤ $L = \{a^m b^n \mid m+n = \text{even}\}$

$0 + 0$
$1 + 1$
$2 + 2$

$\underset{A}{(aa)^*}\ \underset{B}{(bb)^*} + \underset{A}{(aa)^*}\, a\, \underset{B}{(bb)^*}\, b$

$S \to AB \mid AaBb$

$A \to aaA \mid \epsilon$

$B \to bbB \mid \epsilon$

$S \to AB$
$\to aaAB$
$\to aa\epsilon.\epsilon$
$\to aa$

$S \to AB$
$\to aaAbbB$
$\to aabb$

$S \to AaBb$
$\to aaAabbBb$
$\to aaabbb$

$S \to AaBb$
$\to \epsilon.abbB.b$
$\to abbb$

⑥ $L = \{a^m b^n \mid m+n = \text{odd}\}$

$\underbrace{(aa)^*}_{A}\ a\ \underbrace{(bb)^*}_{B} + \underbrace{(aa)^*}_{A}\ b\ \underbrace{(bb)^*}_{B}$

$S \to AaB \mid AbB$

$A \to aaA \mid \epsilon$

$B \to bbB \mid \epsilon$

$S \to AaB$
$\to aaAabbB$
$\to aaabb$

$S \to AbB$
$\to aaAbbbB$
$\to aabbb$

Q. 11 Construct the grammar for the following language (NRL)

→ ① $L = \{a^m b^n \mid m = n\}$

$L = \{\epsilon, ab, aabb, \ldots\}$

$S \to aSb \mid \epsilon$

$S \to aSb$
$\to aaSbb$
$\to aabb$

② $L = \{a^m b^n \mid m < n\}$

$m = 0 \Rightarrow b^n \quad n \geq 1$

$m \neq 0 \Rightarrow abb, aabbb, aaabbbb$

$S \to aSb \mid Sb \mid b$

$S \to Sb$
$\to bb$

$S \to aSb$
$\to asbb$
$\to abbb$

③ $L = \{a^m b^n \mid m > n\}$

$S \to aSb$
$\to aaSbb$
$\to aaaabb$

$S \to aS$
$\to aa$

$\Rightarrow \quad n = 0 \Rightarrow a^m \mid m \geq 1$

$n \neq 0 \Rightarrow aab, aaaabb, \ldots$

$S \to aSb \mid aS \mid a$

④ $L = \{a^m b^n \mid m = 2n\}$

$S \to aaSb$
$\to aab$

$L = \{\epsilon, aab, aaaabb, \ldots\}$

$S \to aaSb \mid \epsilon$

$S \to aaSb$
$\to aa\,aaSbb$
$\to aaaabb$

⑤