

- (24) Design a TM for multiplication of two integers  
 (or) unary nos.
- ⇒ In this  $m \& n$  as  $\underline{0^m 1} \underline{0^n 1}$  on the input tape.
- ⇒ On first occurrence of '0' replace it by Blank & for each occurrence of '0' of second integer write '0' at the end.
- ⇒ Once all 0's of second integer are replaced with 'X'. Now modify all X's to 0's & move extreme left to find B.
- ⇒ Repeat this process until all 0's of first integer are replaced with B.
- ⇒ Now replace all 0's of second integer by Blank & Halt.

Ex:-  $2 \times 3$

$$\begin{array}{ccccccc}
 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
 & \downarrow & & \downarrow & & & \downarrow \\
 B & 0 & 1 & X & 0 & 0 & 1 & B \\
 & \downarrow & & \downarrow & & & \downarrow \\
 B & 0 & 1 & X & X & 0 & 1 & 0 \\
 & \downarrow & & \downarrow & & & \downarrow \\
 B & 0 & 1 & X & X & X & 1 & 0 & 0 \\
 & \downarrow & & \downarrow & & & \downarrow & & \downarrow \\
 B & 0 & 1 & X & X & X & 1 & 0 & 0 & B
 \end{array}$$

I<sup>st</sup>

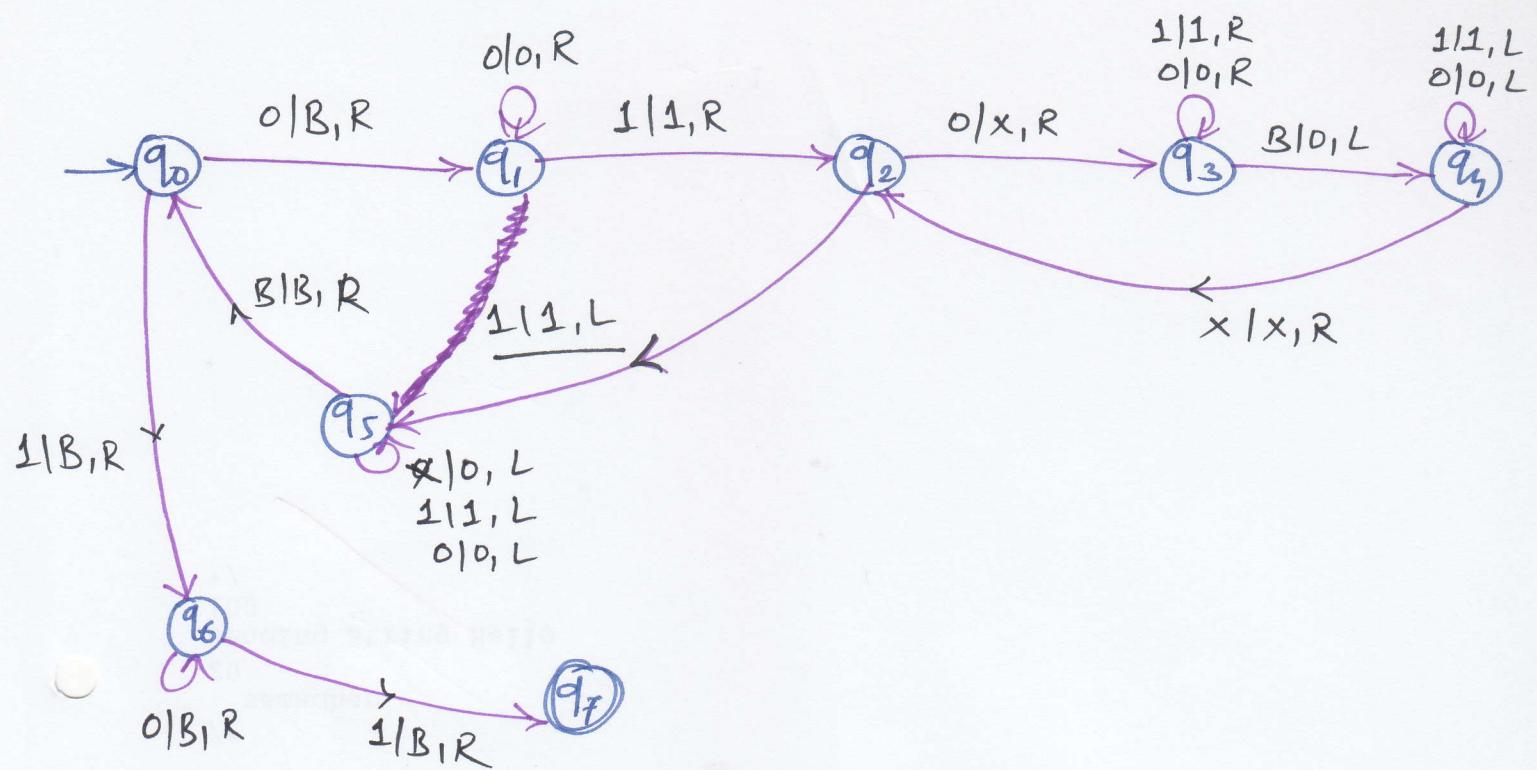
$$\begin{array}{ccccccc}
 & B & 0 & 1 & X & X & 1 & 0 & 0 & 0 & B \\
 & \downarrow & & \downarrow & & & \downarrow & & & \\
 B & 0 & 1 & X & 0 & 0 & 1 & 0 & 0 & 0 & B \\
 & \downarrow & & \downarrow & & & \downarrow & & & \\
 B & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & B \\
 & \downarrow & & \downarrow & & & \downarrow & & & \\
 B & B & 1 & X & 0 & 0 & 1 & 0 & 0 & 0 & B
 \end{array}$$

II<sup>nd</sup>

$$\begin{array}{ccccccc}
 & B & B & 1 & X & 0 & 0 & 1 & 0 & 0 & 0 & 0 & B \\
 & \downarrow & & \downarrow & & & \downarrow & & & \\
 B & B & 1 & X & X & 0 & 1 & 0 & 0 & 0 & 0 & B \\
 & \downarrow & & \downarrow & & & \downarrow & & & \\
 B & B & 1 & X & X & X & 1 & 0 & 0 & 0 & 0 & 0 & B
 \end{array}$$

$$\begin{array}{ccccccc}
 & B & B & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & B \\
 & \downarrow & & \downarrow & & & \downarrow & & & \\
 B & B & B & q_5 & B & B & B & q_6 & B & 0 & 0 & 0 & 0 & 0 & B
 \end{array}$$

2x3



(25) Design a TM to recognize an arbitrary string divisible by 4.  $\Sigma = \{0, 1, 2\}$ .

→ As input string is  $\Sigma = \{0, 1, 2\}$ , any arbitrary string of i/p should be a ternary string & we have to check such ternary input is divisible by 4 (or) not.

$$\begin{aligned} (121)_3 &= 1 \times 3^2 + 2 \times 3^1 + 1 \times 3^0 \\ &= 9 + 6 + 1 = (16)_{10} \Rightarrow \underline{16/4 = 0} \end{aligned}$$

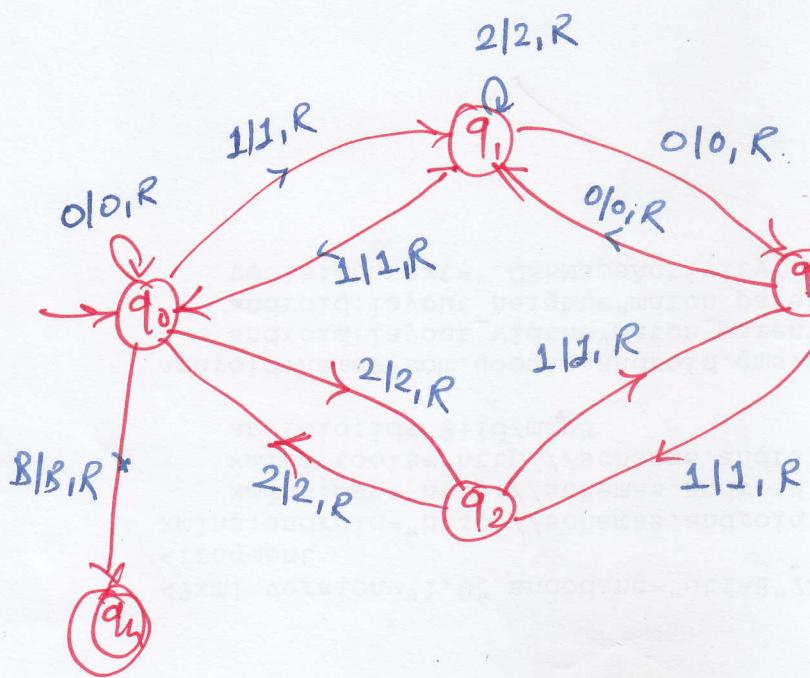
$$\begin{aligned} (120)_3 &= 1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0 \\ &= 9 + 6 + 0 = (15)_{10} \Rightarrow \underline{15/4 = 3} \end{aligned}$$

$q_0$  - Remainder '0'

$q_2$  - Remainder '2'

$q_1$  - Remainder '1'

$q_3$  - Remainder '3'



$$q_0 = (121)_3 = (16)_{10} = R = 0$$

$$q_3 = (120)_3 = (15)_{10} = R = 3$$

$$q_3 = (12021)_3 = (142)_{10}$$

$$R = 2$$

$$(120211)_3 = (427)_{10}$$

$$R = 3$$

$$q_0 = (110)_3 = (12)_{10} = 0$$

$$q_2 = (112)_3 = (14)_{10} = 2$$

$$q_2 = (1122)_3 = (44)_{10} = 2$$

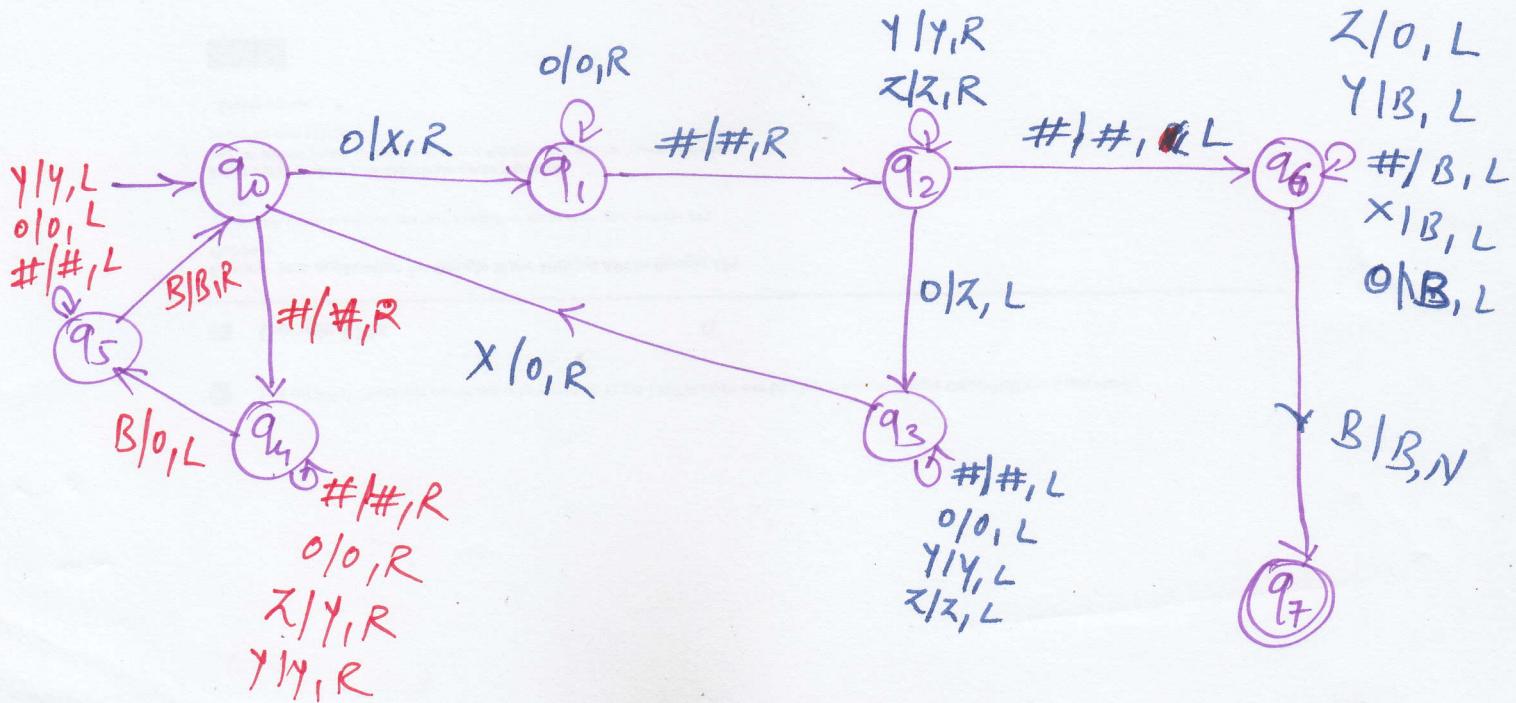
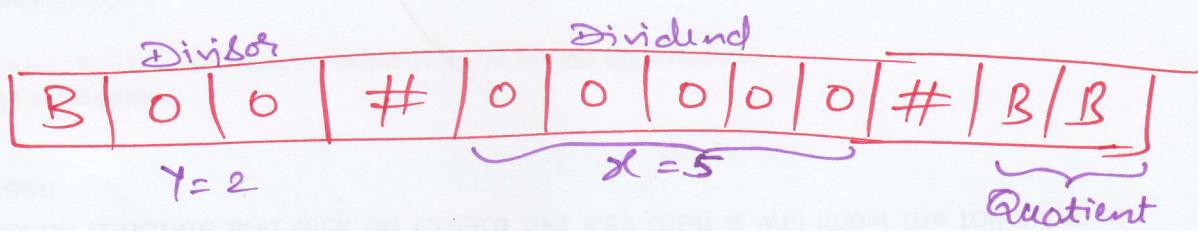
(26) Design a TM to recognize the divisibility

$$5 \overline{)2} \Rightarrow B \underline{\text{00}} \# \underline{\text{00000}} \# \frac{BBB}{\text{Remainder Quotient}}$$

$\Rightarrow$  Division can be performed through repeated subtractions.

⇒ To perform  $\frac{x}{y}$  both  $x$  &  $y$  are represented in unary system  $y$  is divisor &  $x$  is dividend.

$\Rightarrow$  y is repeatedly subtracted from x as long as  $x > y$  each time y is subtracted from x, quotient is incremented by 1.



00 # 000000# B  
 ↓ ↓  
 X 0 # Z 0 000# B  
 q<sub>1</sub> ← q<sub>2</sub>  
 0 0 # Z 0 000# B  
 → ← ↓  
 0 X # Z Z 0 00# B  
 → ←  
 0 0 # Z Z 0 00# B  
 → q<sub>10</sub> ↓ ↓  
 00 # Y Y 0 00# B  
 → → q<sub>11</sub> ← B

1st cycle

2 is subtracted from 5

Quotient is incremented by 1

B 0 0 # Y Y 0 0 0 # O B  
95 ↓ ← ↓

The diagram illustrates a Deterministic Finite Automaton (DFA) with two states. The top state is shown with transitions: X leads to a final state (double circle), O leads to another state, # leads to another state, Y leads to a final state, Y leads to another state, and Z leads to another state. The bottom state is shown with transitions: O leads to a final state, X leads to another state, # leads to another state, Y leads to a final state, Y leads to another state, and Z leads to another state. Red arrows point from the top state to the bottom state for each corresponding transition (X, O, #, Y, Y, Z).

#<sup>nd</sup> cycle

2 is subtracted from 3

it is incremented  
by 1

B x o # y y y y ~~x~~ # o o B

$B \downarrow 0^{\circ} \# \overbrace{Y Y Y Y}^1 Z \# 0^{\circ} B \Rightarrow$  2 can't be 15 from 1

A diagram illustrating a division problem. On the left, there is a sequence of numbers: B, B, B, B, B, B, B. To the right of this sequence is a box containing the digit 0. Above the box, the number 100 is written with a red hash symbol (#) in front of it. Below the box, the word "Quotient" is written in red. Below the sequence of B's, the word "Reminder" is written in red, with an arrow pointing towards the sequence.