

24) Design a TTM for multiplication of two integers & (or) unary num.

⇒ In this  $m$  &  $n$  as  $0^m 1 0^n 1$  on the input tape.

⇒ on first occurrence of '0' replace it by Blank & for each occurrence of '0' of second integer write '0' at the end.

⇒ once all 0's of second integer are replaced with 'X'. Now renotify all X's to 0's & move extreme left to find B.

⇒ Repeat this process until all 0's of first integer are replaced with B.

⇒ Now replace all 0's of second integer by Blank & Halt.

Ex:-  $2 \times 3$



$0010001B$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $B01X001OB$   
 $\rightarrow \quad \rightarrow \quad \rightarrow \quad \leftarrow \quad \rightarrow$   
 $B01XX01OB$   
 $\rightarrow \quad \rightarrow \quad \rightarrow \quad \leftarrow \quad \rightarrow$   
 $B01XX100OB$   
 $\rightarrow \quad \rightarrow \quad \rightarrow \quad \leftarrow \quad \rightarrow$

I<sup>st</sup>

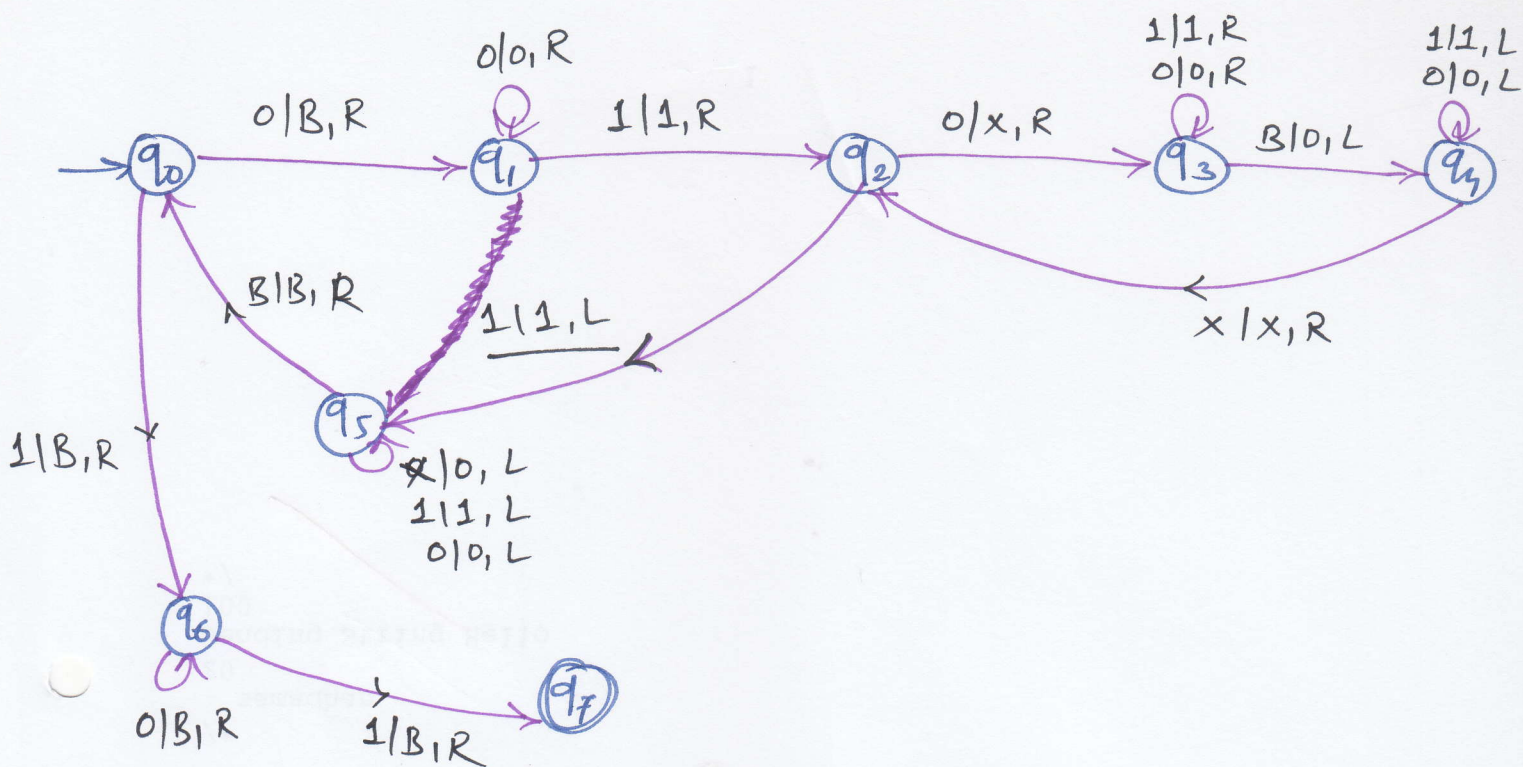
$B01XX01000B$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $B01X001000B$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $B010001000B$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$B010001000B$   
 $\rightarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $BB1X001000OB$   
 $\rightarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $BB1XX01000OB$   
 $\rightarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $BB1XXX100000OB$   
 $\rightarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

II<sup>nd</sup>

$BB10001000000B$   
 $\rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow$   
 $BBB BBB 0000000B$   
 $\rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow \quad \rightarrow$

2x3





(25) Design a TM to recognize an arbitrary string divisible by 4  $\Sigma = \{0, 1, 2\}$ .

→ As input string is  $\Sigma = \{0, 1, 2\}$ , any arbitrary string of i/p should be a ternary string & we have to check such ternary input is divisible by 4 (or) not.

$$\begin{aligned} \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix}_3 &= 1 \times 3^2 + 2 \times 3^1 + 1 \times 3^0 \\ &= 9 + 6 + 1 = (16)_{10} \Rightarrow \underline{16/4 = 0} \end{aligned}$$

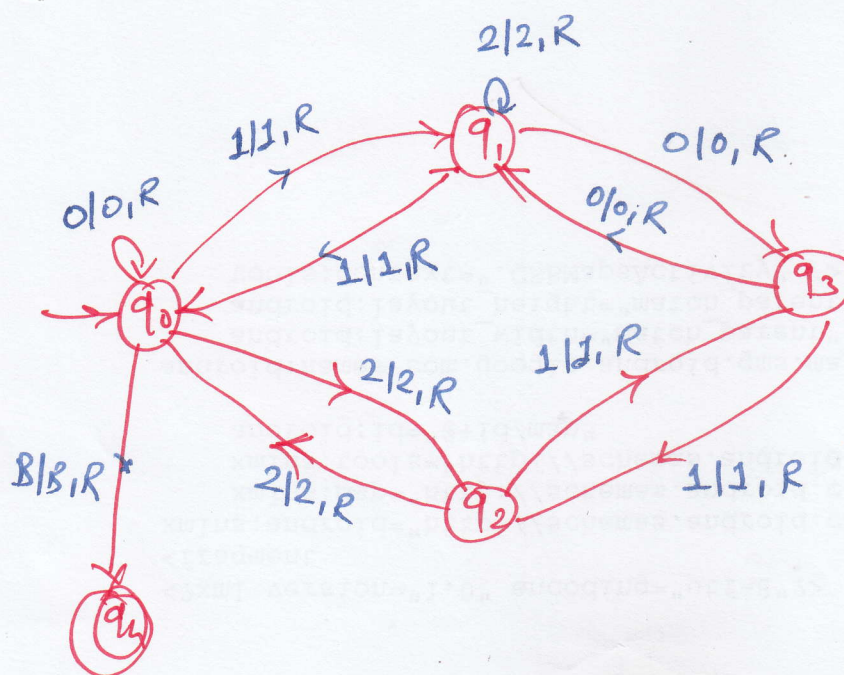
$$\begin{aligned} \underline{(120)}_3 &= 1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0 \\ &= 9 + 6 + 0 = (15)_{10} \Rightarrow \underline{15/4 = 3} \end{aligned}$$

$q_0$  - Remainder '0'

$q_2$  - Remainder '2'

$q_1$  - Remainder '1'

$q_3$  - Remainder '3'



$$q_0 = (121)_3 = (16)_{10} = R = 0$$

$$q_3 = (120)_3 = (15)_{10} = R = 3$$

$$q_3 (12021)_3 = (142)_{10} \\ R = 2$$

$$(120211)_3 = (427)_{10} \\ R = 3$$

$$q_0 (110)_3 = (12)_{10} = 0$$

$$q_2 (112)_3 = (14)_{10} = 2$$

$$q_2 (1122)_3 = (44)_{10} = 2$$



(26) Design a TM to recognize the divisibility

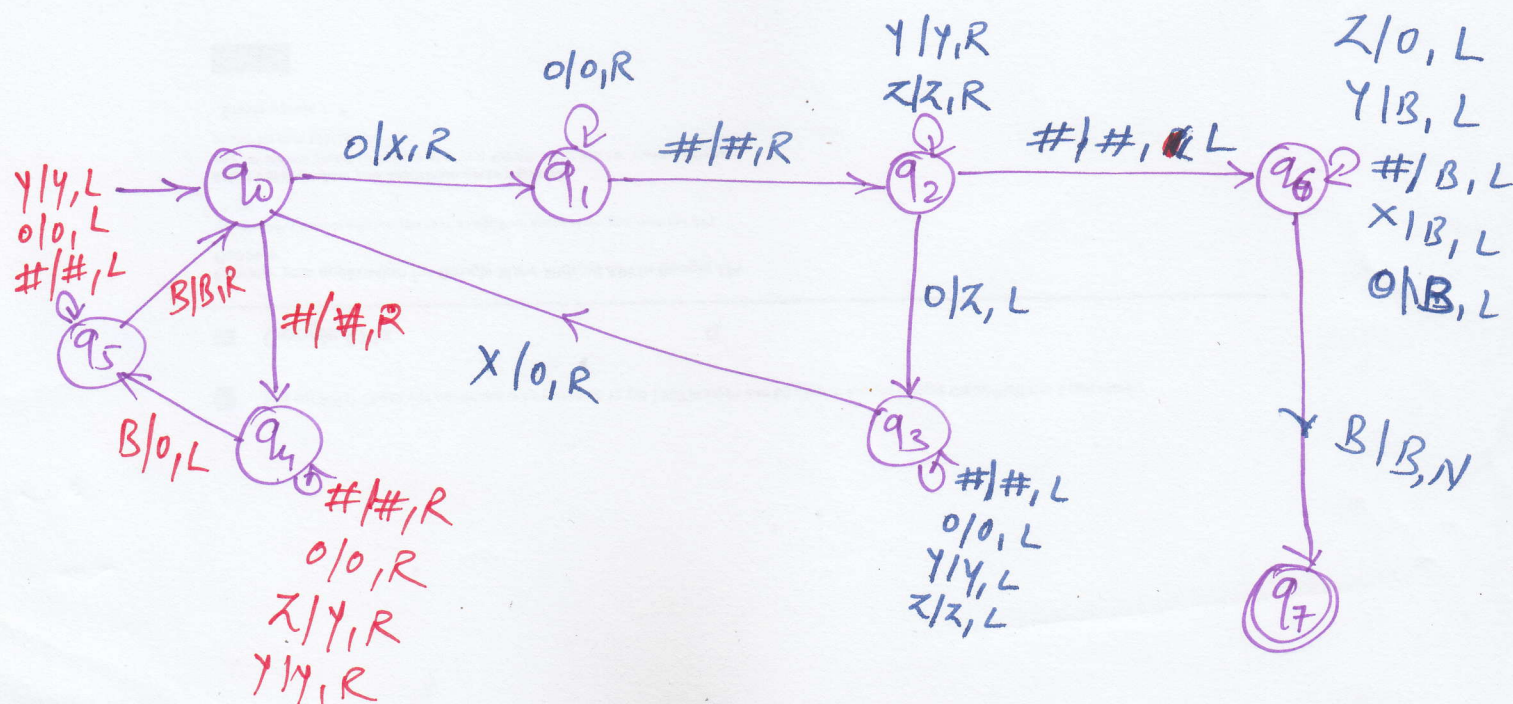
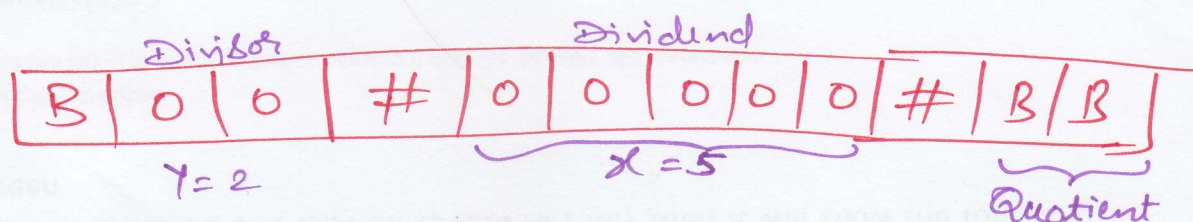
$5/2 \Rightarrow$

$B$	<u><math>00</math></u>	$\#$	<u><math>000000</math></u>	$\#$	<u><math>BBB</math></u>
$m/n$					Remainder Quotient

$\Rightarrow$  Division can be performed through repeated subtractions.

⇒ To perform  $x/y$  both  $x$  &  $y$  are represented  
in unary system  $y$  is divisor &  $x$  is dividend.

$\Rightarrow$  y is repeatedly subtracted from x as long as x  $\geq$  y each time y is subtracted from x, quotient is incremented by 1.





$$5/2$$

Handwritten notes showing the steps of the long division algorithm for binary numbers, with annotations in red and blue ink.

**1st cycle**

Annotations:  $2'$  is subtracted from 5, Quotient is incremented by 1.

**2nd cycle**

Annotations: 2 is subtracted from 3, Quotient is incremented by 1.

Annotations: 2 can't be subtracted from 1.

Annotations: Quotient, Remainder.