→ If $r_1, r_2$ be the regular expression then $r_1 + r_2$, $r_1 \cdot r_2$ are also RE

$$L(r_1 + r_2) = L(r_1) + L(r_2)$$
$$L(r_1 \cdot r_2) = L(r_1) \cdot L(r_2)$$

→ If $r_1$ & $r_2$ be the two RE's then

$$(r_1 + r_2)^* = (r_1^* + r_2^*)^*$$
$$= (r_1 + r_2^*)^*$$
$$= (r_1^* + r_2)^*$$
$$(r_1 + r_2)^* = (r_1^* \cdot r_2^*)^*$$

→ If $r_1, r_2$ be the two RE then,

$$(r_1 \cdot r_2)^* r_1 = r_1 (r_2 \cdot r_1)^*$$

→ Two RE $r_1$ & $r_2$ are equal iff $L(r_1) = L(r_2)$

→ RE is just like NFA & generates only strings of Regular language (only valid)

→ Every RE generates only one regular language but a regular language can be generated by more than one form of RE i.e. RE is not unique

→ In general both $r^*$, $r^+$ represents infinite languages for $r = \phi$ (or) $\epsilon$.

- If $r = \epsilon$ or $\phi$ then $r^*, r^+$ represents finite language

Ex: $r = \epsilon$

$r^* = \{\epsilon\}$

$r^+ = \{\epsilon\}$

$r = \phi$

$r^* = \{\phi\}^* = \{\epsilon\}$

$r^+ = \phi$

**Note:-**

1) $\Sigma = \{a, b\} = a + b$ → Strings of length 1

$\Sigma^2 = \{a, b\}^2 = (a+b)^2$ → — " — 2

$\Sigma^3 = \{a, b\}^3 = (a+b)^3$ → — " — 3

⋮

$\Sigma^k = \{a, b\}^k = (a+b)^k$ → — — length k

2) $\Sigma^* = \{a, b\}^* = \{a+b\}^* \Rightarrow \epsilon, a+b, (a+b)^2 \ldots$

3) $\Sigma^+ = \{a, b\}^+ = \{a+b\}^+ \Rightarrow a+b, (a+b)^2, \ldots$

1) Construct the RE that generate all strings of a's & b's

ⓐ Including '$\epsilon$'  ⓑ Excluding '$\epsilon$'.

→ $\Sigma = \{a, b\}$

ⓐ $L = \Sigma^* \Rightarrow r = (a+b)^*$ ; $L = \{\epsilon, a+b, (a+b)^2, \ldots\}$

ⓑ $L = \Sigma^+ \Rightarrow r = (a+b)^+$ ; $L = \{a+b, (a+b)^2, \ldots\}$

2) Construct the RE that generates all the the strings of a's & b's where every string-

1) Starts with 'a'  2) Starts with 'ba'

3) ends with 'b'  4) ends with 'ab'

5) Contain the substring 'ab'

→ $\Sigma = \{a, b\}$

1) $L = \{a, aa, aaa, ab, abb, \ldots\}$

$w = a\,x$

a  b

$r = a(a+b)^*$

$a \cdot \epsilon = a$

$a \cdot a = aa$

$a \cdot b = ab$

$a \cdot aa = aaa$

2) $L = \{ ba, baa, bab, baaa, babb, \dots \}$

$w = ba\,x$

a    b

$r = ba(a+b)^*$

$ba.\epsilon = ba$

$ba.a = baa$

$ba.b = bab.$

3) $L = \{ b, ab, bb, abb, aab, \dots \}$

$w = x\,b$

a    b

$r = (a+b)^*b$

$\epsilon.b = b$

$a.b = ab$

$b.b = bb$

4) $L = \{ ab, aab, aaab, bab, bbab, \dots \}$

$w = x\,ab$

a    b

$r = (a+b)^*ab$

$\epsilon.ab = ab$

$a.ab = aab$

$b.ab = bab$

$aa.ab = aaab.$

5) $L = \{ab, aab, abb, babb, aabb, \ldots\}$

$w = x\,ab\,x$

a b a b

$r = (a+b)^* ab (a+b)^*$

$\epsilon . ab . \epsilon = ab$

$a . ab . \epsilon = aab$

$\epsilon . ab . b = abb$

---

(3) Construct the RE that generates all the strings of a's & b's where every string

(i) starts & ends with 'a'.

(ii) Starts & ends with different symbol

(iii) Starts & ends with same symbol.

→ (i) $\Sigma = \{a, b\}$

$w = a\,x\,a \,,\, a$

a b

$r = a(a+b)^* a + a$

(ii) $w = a\,x\,b \;;\; b\,x\,a$

$r = a(a+b)^* b + b(a+b)^* a$

(iii) $w = a \times a \;,\; b \times b \;,\; a \;,\; b$

$$r = a(a+b)^* a + b(a+b)^* b + a + b$$

④ Construct the RE that generates all the strings of a's & b's where –

i) 3rd symbol from left end is 'a'

ii) 4th symbol from right end is 'b'

→ $\Sigma = \{a, b\}$

i) $w = \underbrace{x}_{(a+b)} \underbrace{x}_{(a+b)} a \underbrace{\cdots\cdots}_{(a+b)^*}$

$$r = (a+b)^2 a (a+b)^*$$

ii) $w = \underbrace{\cdots\cdots}_{(a+b)^*} b \underbrace{x}_{a+b} \underbrace{x}_{a+b} \underbrace{x}_{a+b}$

$$r = (a+b)^* b (a+b)^3$$

5) Construct RE that generates all the strings of a's & b's where the length of the string is —

i) Exactly 2   ii) atmost 2   iii) atleast 2

iv) even   v) odd   vi) 2(mod 3)

→ $\Sigma = \{a, b\}$

i) $|w| = 2$ ; $w = x\,x$

$r = (a+b)(a+b) = (a+b)^2$

ii) $|w| \leq 2$ ; $|w| = 0, 1, 2$

$r = \underline{\epsilon + (a+b) + (a+b)^2}$

(or)

$r = \underline{(a+b+\epsilon)^2}$

$r = (a+b+\epsilon).(a+b+\epsilon)$

$= \epsilon, aa, bb, ab, ba, a, b.$

iii) $|w| = x\,x \cdots\cdots$

$r = (a+b)^2 . (a+b)^*$

iv) $|w| = \text{even} = 0(\text{mod } 2)$

$r = [(a+b)^2]^*$

v) $|w| = \text{odd} = 1 \pmod 2$

$$r = (a+b)\left[(a+b)^2\right]^*$$

vi) $|w| = 2 \pmod 3$

$$r = (a+b)^2\left[(a+b)^3\right]^*$$

6) Construct RE that generates all the strings of a's & b's where,

1) If the string start with a then the length of the string is even.

2) If the string start with 'b' then the length of the string is odd.

→ $\Sigma = \{a, b\}$

- 1) $w = aX \Rightarrow |w| = \text{even}$

$X$: a, b

| | | | |
|---|---|---|---|
| a | $\epsilon$ | a+b | = aa, ab |
| a | aa | a+b | = aaaa, aaab |
| a | bb | a+b | = abba, abbb |

- $r = a\left[(a+b)^2\right]^*(a+b)$

| | | | |
|---|---|---|---|
| a | $\epsilon$ | (a+b) | = aa, ab |
| a | aa | (a+b) | = aaaa, aaab |
| a | bbbb | (a+b) | = abbbba, abbbbb |

2) $w = bX \Rightarrow |w| = \text{odd}$.

b . $\epsilon$ ; b.bb

b . aa ; baaaa

$r = b\left[(a+b)^2\right]^*$

b . $\epsilon$ = b

b aa = baa

b. bb = bbb

b. aaaa = baaaa

7) Construct the RE. for the following:-

1) $L = \{a^m \mid m \geq 0\} \Rightarrow a^*$

2) $L = \{a^m \mid m \geq 1\} \Rightarrow a^+$

3) $L = \{a^m b^n / m, n \geq 0\} \Rightarrow a^* b^*$

4) $L = \{a^m b^n / m, n \geq 1\} \Rightarrow a^+ b^+$

5) $L = \{a^m b^n \mid m \geq 0, n \geq 1\} \Rightarrow a^* b^+$

6) $L = \{a^m b^n \mid m \geq 1, n \geq 0\} \Rightarrow a^+ b^*$

7) $L = \{a^m b^n c^p \mid m, n, p \geq 0\} \Rightarrow a^* b^* c^*$

8) $L = \{a^m b^n c^p \mid m, n, p \geq 1\} \Rightarrow a^+ b^+ c^+$

8) Construct R.E. for the language.

i) $L = \{a^m b^n \mid m+n = \text{even}\}$

$m+n = \text{even}$

Both a & b should be even

$\text{even} = m = 2x$

$\text{even} = n = 2x$

$\downarrow$

$a^m b^n$

$a^{2x} b^{2x}$

$(aa)^x (bb)^x$

$(aa)^* (bb)^*$

Both should be odd

$\left.\begin{array}{l} \text{odd} = m = 2x+1 \\ \text{odd} = n = 2x+1 \end{array}\right\} x \geq 0$

$\downarrow$

$a^m b^n$

$a^{2x+1} b^{2x+1}$

$a^{2x}.a \quad b^{2x}.b$

$(aa)^x a \ (bb)^x b$

$(aa)^* a \ (bb)^* b$

$r = (aa)^* (bb)^* + (aa)^* a \ (bb)^* b$

$\epsilon \quad \epsilon \quad + \quad \epsilon \quad a \quad \epsilon \quad b = ab$

$(aa) \quad (bb) \quad + \quad \epsilon \quad a \quad \epsilon \ b = aabbab$

$\epsilon \quad \epsilon \quad + \quad (aa) \quad a(bb) \ b = aaabbb$

$aa \quad \epsilon \quad + \quad aa \quad a \quad \epsilon \ b = aaaab$

ii) $L = \{ a^m b^n \mid m+n = \text{odd} \}$

$m+n = \text{odd}$

even $= m = 2x$
odd $= n = 2x+1$

$\downarrow$

$a^m \; b^n$

$a^{2x} b^{2x+1}$

$(aa)^x (bb)^x b$

$(aa)^* (bb)^* b$

even $= n = 2x$
odd $= m = 2x+1$ $\Big\}$ $x \geq 0$

$\downarrow$

$a^m b^n$

$a^{2x+1} b^{2x}$

$(aa)^x a \; (bb)^x$

$(aa)^* a (bb)^*$

$$\boxed{r = (aa)^* (bb)^* b + (aa)^* a (bb)^*}$$

$\epsilon \quad \epsilon \quad b$ ———— $= b$

———— $\epsilon \quad a \quad \epsilon = a$

$aa \quad \epsilon \quad b$ ———— $= aab$

———— $aa \quad a \quad \epsilon = aaa$

$aa \quad bb \quad b$ ———— $= aabbb$

———— $aa \quad a \quad bb = aaabb$

9) Construct the RE that generates all the strings of a's & b's where every string starts with 'a' and does not contains two consecutive b's

→ $\Sigma = \{a, b\}$

$a^+ = \{a, aa, aaa, \ldots\}$

$(ab)^+ = \{ab, abab, \ldots\}$

$r = (a + ab)^+$   $L = \{a, aa, ab, abab, \ldots\}$

10) Construct the RE that generates all the strings of a's & b's where every string do not contain two consecutive a's (or) two consecutive b's.

→ $\Sigma = \{a, b\}$

$(ab)^* = \epsilon, ab, abab, \ldots$

$(ba)^* = \epsilon, ba, baba, \ldots$

$r = (b + \epsilon)\ (ab)^*\ (a + \epsilon)$

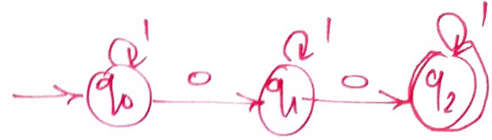
(or)

$r = (a + \epsilon)\ (ba)^*\ (b + \epsilon)$

11> Construct RE $\Sigma = \{0,1\}^*$

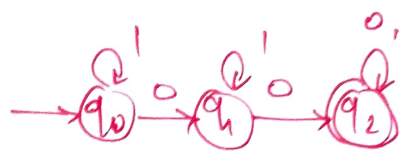
i> E = Exactly two 0's => x 0 x 0 x

$r = 1^* 0 1^* 0 1^*$



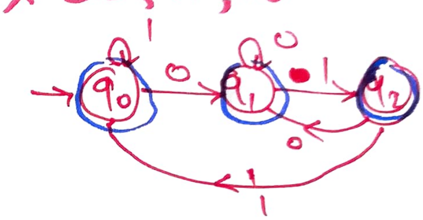

ii> Atleast two 0's

$r = 1^* 0 1^* 0 (1+0)^*$




iii> String not ending in 01 => x 00, 11, 10

$r = (0+1)^* (00 + 11 + 10)$




iv> All string starting with 11 => 11 x

$r = 11 (0+1)^*$


12> $L = \{x \mid x \in (a,b)^*$ & $x$ is any string that begins with "abb" or "a" $\}$

→ $\Sigma = \{a,b\}$

$r = (abb + a)(a+b)^*$