Q4 Construct the grammar that generates the grammar of all strings of a's & b's
ⓐ including 'ε' ⓑ Excluding 'ε'

→

$\Sigma = \{a, b\}$

ⓐ $L = \Sigma^* = (a+b)^*$

$S \to XS \mid \epsilon$
$X \to a \mid b$

(or)

$S \to aS \mid bS \mid \epsilon$

ⓑ $L = \Sigma^+ = (a+b)^+$

$S \to aS \mid bS \mid a \mid b$

Q.5 Construct the grammar that generates the grammar of a's & b's where every string

1> Starts with a
2> Starts with ab
3> Ends with b
4> ends with ba
5> Contained the substring ab.

$\rightarrow \Sigma = \{a, b\}$

① $w = aX$

a b

$S \rightarrow aX$

$X \rightarrow a \mid bX \mid \epsilon$

④ $w = Xba$

a b

$S \rightarrow Xba$

$X \rightarrow aX \mid bX \mid \epsilon$

(or)

$S \rightarrow Ya$

$Y \rightarrow Xb$

$X \rightarrow aX \mid bX \mid \epsilon$

② $w = abX$

a b

$S \rightarrow abX$

$X \rightarrow aX \mid bX \mid \epsilon$

③ $w = Xb$

$S \rightarrow Xb$

$X \rightarrow aX \mid bX \mid \epsilon$

⑤ $w = \underbrace{Xa}_{A}\underbrace{bX}_{B}$

$S \rightarrow AB$

$A \rightarrow Xa$

$B \rightarrow bX$

$X \rightarrow aX \mid bX \mid \epsilon$

Q.6 Construct the grammar that generates all the strings of a's & b's where every string —

① Starts & ends with a
② Starts & ends with same symbol
③ Starts & ends with diffn symbol.

→ $\Sigma = \{a, b\}$

① $w = a x a, | a$
x: a b

$S \rightarrow axa | a$
$X \rightarrow aX | bX | \epsilon$

$w = aaba$
$S \rightarrow axa$
$\rightarrow aaxa$
$\rightarrow aabxa$
$\rightarrow aaba$

② $w = axa, a \quad ; \quad bxb, b$
x: a b ; x: a b

$S \rightarrow axa | a | bxb | b$
$X \rightarrow aX | bX | \epsilon$

③ $w = axb, bxa$

$S \rightarrow axb | bxa$
$X \rightarrow aX | bX | \epsilon$

$w = abab$
$S \rightarrow axb$
$\rightarrow abxb$
$\rightarrow abaxb$
$\rightarrow abab$

Q.7 Construct the grammar that generates all the strings of a's & b's where –

① 3^{rd} symbol from left end is 'a'

② 4^{th} symbol from right end is 'b'

→ $\Sigma = \{a, b\}$

① $W = \underbrace{XX}_{A}a\underbrace{\ldots\ldots}_{B}$

$XXa\ldots\ldots$

a b a b

$\underbrace{(a+b)^2}_{A} a \underbrace{(a+b)^*}_{B}$

$S \rightarrow AaB$

$A \rightarrow XX$

$X \rightarrow a \mid b$

$B \rightarrow aB \mid bB \mid \epsilon$

$S \rightarrow AaB$

$\rightarrow XXaB$

$\rightarrow aaaaB$

$\rightarrow aaaabB$

$\rightarrow aaaab$

---

② $W = \underbrace{\ldots\ldots}_{A}b\underbrace{XXX}_{B}$

$\underbrace{(a+b)^*}_{A} b \underbrace{(a+b)^3}_{B}$

$S \rightarrow AB$

$A \rightarrow aA \mid bA \mid \epsilon$

$B \rightarrow XXX$

$X \rightarrow a \mid b$

$S \rightarrow AB$

$\rightarrow aAB$

$\rightarrow abAB$

$\rightarrow abB$

$\rightarrow abXXX$

$\rightarrow abbbb$

Q.8 Construct grammar where no. of a's in string is —

① Exactly two ② atmost 2 ③ atleast two

④ even ⑤ odd ⑥ divisible by 3

→ $\Sigma = \{a, b\}$

① $|w|_a = 2$

$w = X a X a X$

$X = b^*$, b^*, b^*

$S \to XaXaX$

$X \to bX \mid \epsilon$

② $|w|_a \leq 2$

$|w|_a = 0, 1, 2$

$w = X \underbrace{(a+\epsilon)}_{A} X \underbrace{(a+\epsilon)}_{A} X$

$S \to XAXAX$

$A \to a \mid \epsilon$

$X \to bX \mid \epsilon$

$S \to XAXAX$

$\to bXabXabX$

$\to babab$

③ $|w|_a \geq 2$

$|w|_a = 2, 3, 4 \ldots$

$w = \underbrace{(a+b)^*}_{A} a \underbrace{(a+b)^*}_{A} a \underbrace{(a+b)^*}_{A}$

$S \to AaAaA$

$A \to aA \mid bA \mid \epsilon$

$S \to AaAaA$

$\to aAabAa.\epsilon$

$\to aaba$

(or) ⇒

$S \to XYX$

$X \to bX \mid \epsilon$

$Y \to aY \mid a$

$S \to XYX$

$\to bXYX$

$\to baYbX$

$\to baab$

④ $|w|_a$ = even

$0 \pmod 2 = 0, 2, 4, \ldots$

$$\underbrace{(\underset{B}{b^*}\, a\, \underset{B}{b^*}\, a\, \underset{B}{b^*})^*}_{X} \cdot \underset{B}{b^*}$$

A = {B a B a B}

(or)
$S \rightarrow XAX$
$A \rightarrow bX \mid \epsilon$
$X \rightarrow aa \mid \epsilon$

$\Rightarrow S \rightarrow XAX$
$\rightarrow aabXaa$
$\rightarrow aabaa$

$S \rightarrow XB$
$X \rightarrow AX \mid \epsilon$
$A \rightarrow BaBaB$
$B \rightarrow bB \mid \epsilon$

$S \rightarrow XB$
$\rightarrow AXB$
$\rightarrow BaBaB\cancel{X}B$
$\rightarrow BaBaB.bB$
$\rightarrow \epsilon.a.\epsilon.a\epsilon.b\epsilon$
$\rightarrow aab$

⑤ $|w|_a$ = odd

$1 \pmod 2 = 1, 3, 5, \ldots$

$$\underbrace{b^* a b^*}_{X}\, \underbrace{(\underset{B}{b^*} a b^* a b^*)^*}_{Y}$$

A

$S \rightarrow XY$
$X \rightarrow BaB$
$B \rightarrow bB \mid \epsilon$
$Y \rightarrow AY \mid \epsilon$
$A \rightarrow BaBaB$

$S \rightarrow XY$
$\rightarrow BaBY$
$\rightarrow bBaBY$
$\rightarrow baAY$
$\rightarrow baBaBaB$
$\rightarrow baaa$

~~S → XY~~
~~X → BaB~~
~~Y → BaBaB~~ (or)
~~B -~~

⇓

$S \rightarrow XaY$
$X \rightarrow bX \mid \epsilon$
$Y \rightarrow aaY \mid \epsilon$

⑥ $|w|_a = 0 \pmod 3$

$$\underbrace{b^*}_{B}\ \underbrace{A\{(b^* a b^* a b^* a b^*)}_{X}\ \underbrace{b^*}_{B}$$

$S \to XB$

$X \to AX \mid \epsilon$

$A \to BaBaBaB$

$B \to bB \mid \epsilon$

$S \to XB$

$\to AXB$

$\to BaBaBaBXB$

$\to aaabB$

$\to aaab$

(or)

$S \to XAX$

$X \to bX \mid \epsilon$

$A \to aB \mid a$

$B \to aaB \mid \epsilon$

$S \to XAX$

$\to bXaBbX$

$\to bXaaaBbX$

$\to baaab$