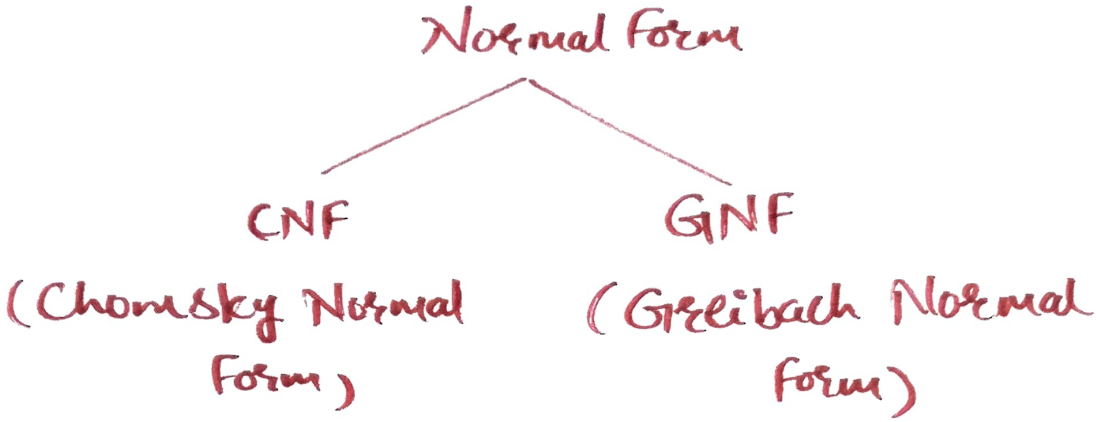
* Normal Forms :-



⇒ To convert into Normal form the grammar must be free from ∈-production.

1) CNF := Chomsky Normal Form.

In CNF, we have restriction on the length of RHS; which is; elements in RHS should either be two variables or a Terminal.

A CFG is in CNF if the productions are in the following forms:

$$\begin{cases} A \to BC & \text{where,} \\ A \to a & A, B, C \in V \ \& \ a \in T \end{cases}$$

⇒ The grammar should have no useless symbols

1) ⇒ If the RHS have production in the form $A \to aB$ where a is terminal & A & B are non-terminals, then the production is replaced by $A \to XB$ & $X \to a$. Repeat this step for every production which is of the form $A \to aB$

2) ⇒ Replace each production $A \to B_1 \dots B_n$ where $n > 2$ [$S \to ABC$ where $S \to AD$ & $D \to BC$]
Repeat this step for all productions having two or more symbols on the right side.

3) ⇒ Remove Null & Unit productions.

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① Convert given CFG to CNF

$S \to aSb \mid \epsilon$

$S \to aSb$
$\to a\epsilon b$
$\to ab$

⇒ i) $S \to ASb \mid Ab$
$A \to a$

ii) $S \to ASB \mid AB$
$A \to a$
$B \to b$

iii) $S \to AC \mid AB$
$A \to a$
$B \to b$
$C \to SB$

CNF

② $S \to aA \mid bB$

$A \to aAb \mid bB \mid a$

$B \to bBa \mid Ba \mid b$

$\Rightarrow$

$S \to XA \mid YB$

$A \to \underbrace{XAY}_{C} \mid YB \mid a$

$B \to \underbrace{YBX}_{D} \mid bBX \mid b$

$X \to a$

$Y \to b$

$\Downarrow$

$S \to XA \mid YB$

$A \to XC \mid YB \mid a$

$C \to AY \mid$

$B \to YD \mid BX \mid b$

$D \to BX$

$X \to a$

$Y \to b$

12 productions

CNF

③ $S \to AbA \mid Bb$

$A \to ab \mid Ba \mid b$

$B \to bBa \mid BA \mid a$

$X \to a$

$Y \to b$

$\Rightarrow$ $S \to AYA \mid BY$ (C)

$A \to XY \mid BX \mid b$

$B \to YBX \mid BA \mid a$ (D)

$X \to a$

$Y \to b$

$\Downarrow$

$S \to AC \mid BY$

$C \to AYA$

$A \to XY \mid BX \mid b$

$B \to YD \mid BA \mid a$

$D \to BX$

$X \to a$

$Y \to b$

CNF

12 productions in CNF

④ $S \to ASA \mid aB$

$A \to B \mid S$

$B \to b \mid \epsilon$

2) Greibach Normal Form (GNF) :-

The grammar G is said to be in GNF if every production is in the form

$$A \rightarrow a\alpha$$

where, $a \in T$

$\alpha \in V^*$

$(\alpha \rightarrow \epsilon \Rightarrow A \rightarrow a)$

⇒ Right side should start with terminal followed by non-terminals of length Zero (or) more

1) ⇒ Remove units or Null productions.

2) ⇒ Check whether the CFG is already in Chomsky Normal form & Convert it into CNF if it is not.

3) ⇒ Change the names of non-terminals / variables in

① $S \rightarrow aSb \mid \epsilon$

⇒ $S \rightarrow aSb \mid \epsilon$

$\Downarrow$

$S \rightarrow aSb \mid ab$ (Removed Null symbols)

$\Downarrow$

$S \rightarrow aSB \mid aB$ ($\because A \rightarrow a\alpha$)

$B \rightarrow b$

② $S \rightarrow aSb \mid bSa \mid \epsilon$

$\Rightarrow$ $S \rightarrow aSb \mid bSa \mid \epsilon$

$\Downarrow$

$S \rightarrow aSb \mid bSa \mid ab \mid ba$ { $A \rightarrow a$, $B \rightarrow b$ }

$\Downarrow$

$S \rightarrow aSB \mid bSA \mid aB \mid bA$

$A \rightarrow a$

$B \rightarrow b$

} GNF

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③ $S \rightarrow Ab \mid Bb$

$A \rightarrow aA \mid b$

$B \rightarrow bBa \mid a$

Remove the left recursion

$\Rightarrow$ $S \rightarrow \underbrace{aAb}_{A} \mid \underbrace{bBab}_{B} \mid bb \mid ab$

$A \rightarrow aA \mid b$

$B \rightarrow bBa \mid a$

$\Downarrow$

$S \rightarrow aAY \mid bBXY \mid bY \mid aY$

$A \rightarrow aA \mid b$

$B \rightarrow bBX \mid a$

$X \rightarrow a$

$Y \rightarrow b$

} GNF

④ $S \to aA \mid bB$

$A \to Aa \mid b$

$B \to AaB \mid b$

$\Rightarrow$

$S \to aA \mid bB$

$A \to ba \mid b$

$B \to baB \mid b$

$\Downarrow$

$S \to aA \mid bB$

$A \to bX \mid b$

$B \to bXB \mid b$

$X \to a$

} GNF

Note:- $S \to aSb \mid \epsilon$

CNF

$S \to AC \mid AB$

$C \to SB$

$A \to a$

$B \to b$

$S \to AC$
$\to ASB$
$\to aSB$
$\to aABB$
$\to aaBB$
$\to aabB$
$\to aabb$

$2n-1$ productions

$= 2 \times 4 - 1$
$= 8 - 1$
$= 7$ //

GNF

$S \to aSB \mid aB$

$B \to b$

$S \to aSB$
$\to aaBB$
$\to aabB$
$\to aabb$

n productions

$|w| = 4 = n$

$w = aabb$

$|w| = 4$