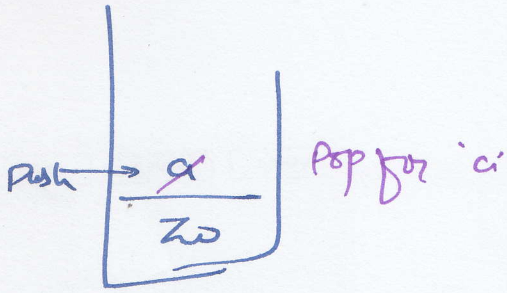
① $w = \underline{aba}$

$w = a\ b\ a$
Push ↳ pop

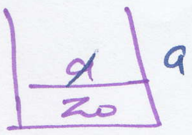
$\delta(q_0, a, z_0) = (q_0, a z_0)$ Push

$\delta(q_0, b, a) = (q_1, a)$ Read

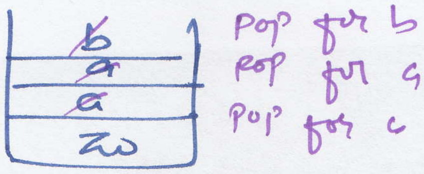
$\delta(q_1, a, a) = (q_1, \epsilon)$ Pop



② $w = aa = a\epsilon a$

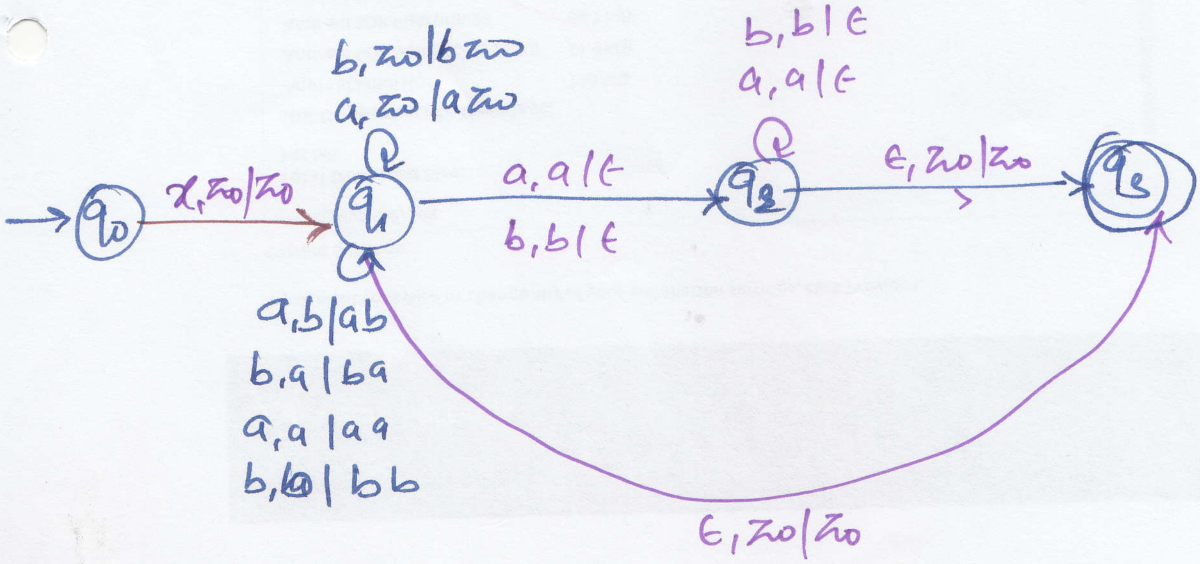


③ aabbaa



④ $L = \{x w w^R \mid w \in (a,b)^*\}$

→ $L = \{\epsilon, x, xabba, \dots\}$



$w = xabba \cdot \epsilon$, $w = x \cdot \epsilon$

$q_0 \to q_1 \to q_1 \to q_2 \to q_2 \to q_3$

$q_0 \to q_1 \to q_3$

* Equivalence of PDA & CFG

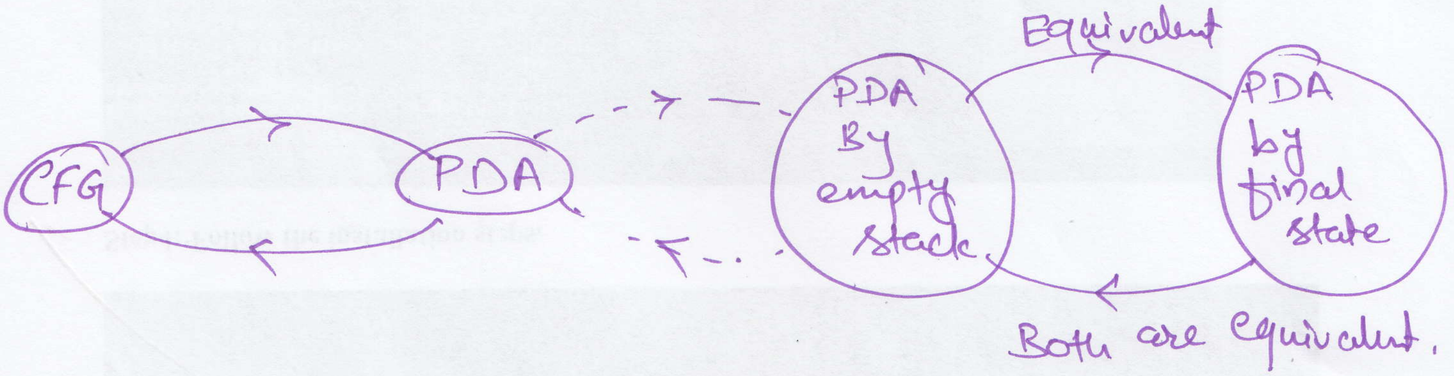
* CFG to PDA Conversion*:-

CFG

$G = \{V, T, P, S\}$

PDA

$M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$



⇒ If CFG is equivalent to PDA & there is grammar equivalent to PDA then there is equivalence between them.

⇒ The class of languages accepted by PDA is exactly the class of CFG languages.

P:- $A \rightarrow \alpha$ is production

$A \in V$, $\alpha \in (V \cup T)^*$

$\Rightarrow$ PDA accepting L(G) by empty stack is given by —

$M = \quad Q \quad \Sigma \quad \Gamma \quad \delta \quad q_0 \quad z_0 \quad F$

$M = \{ \{q\}, T, \underbrace{V \cup T}_{\Gamma}, \delta_P, q, S, \phi \}$

$\{q\}$ — No. of States

T — I/P Symbol by Terminal T

q — State

S — Start Symbol

ϕ — Nothing in stack

Where,

δ : ① $\delta(q, \epsilon, \underline{A}) \Rightarrow (q, \underline{\alpha})$

A — Top of stack

A replaced by α i.e. Right side prodn of CFG

② For every Terminal $\underline{a \in T}$ (Pop opn)

$\delta(q, \underline{a}, \underline{a}) \Rightarrow (q, \underline{\epsilon}) \Rightarrow \underline{Pop}$

① $S \rightarrow aSb \mid ab$

$\Rightarrow$ $V = \{S\}$, $T = \{a, b\}$

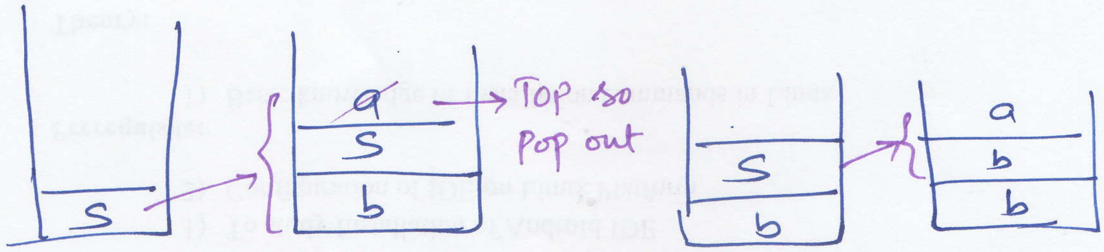
$S \rightarrow aSb \Rightarrow$

① $\delta(q, \epsilon, S) \Rightarrow (q, aSb) \Rightarrow$ variable

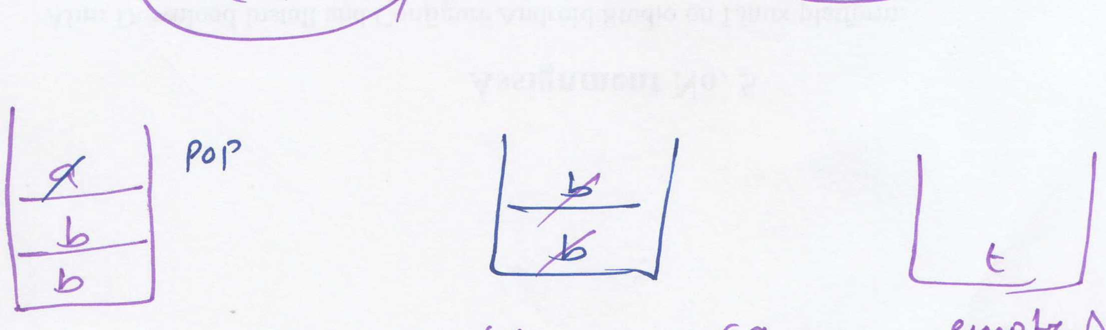
S is top of stack & replaced by right side production.

② $\delta(q, \epsilon, S) \Rightarrow (q, ab) \Rightarrow$ variable

③ $\delta(q, a, a) \Rightarrow (q, \epsilon)$
④ $\delta(q, b, b) \Rightarrow (q, \epsilon)$ $\Big\}$ Two terminals in example



① (q, ϵ, S) $\quad S \rightarrow aSb$ $\quad (q, \epsilon, aSb)$ $\quad$ ② $(q, \epsilon, S) \Rightarrow (q, ab)$



empty stack

③ $\delta(q, a, a) = (q, \epsilon)$ $\quad (q, b, b) \Rightarrow (q, \epsilon)$

$S \to aSb \mid ab$ Simulation of string $\underline{\underline{aabb}}$

$(q, aabb, S) \xrightarrow{\delta(q,\epsilon,S)\to(q,aSb)} (q, \underline{a}abb, \underline{a}Sb)$

$\xrightarrow{\delta(q,a,a)\Rightarrow(q,\epsilon)} (q, \epsilon abb, Sb)$

$\xrightarrow{\delta(q,\epsilon,S)\Rightarrow(q,ab)} (q, abb, \underline{\underline{ab}}b)$

$\xrightarrow{\delta(q,a,a)\Rightarrow(q,\epsilon)} (q, bb, bb)$

$\xrightarrow{\delta(q,b,b)\Rightarrow(q,\epsilon)} (q, \underline{b}, \underline{b})$

$\xrightarrow{\delta(q,b,b)\Rightarrow(q,\epsilon)} (q, \epsilon\epsilon)$

$\epsilon.aabb \; , \; S$ Replace 'S' by $\to aSb$

$aabb, aSb$ } P

$\epsilon.abb, Sb \;\to$ Pop

abb, abb Pop

$\epsilon.bb, bb$ Pop

b, b Pop

ϵ, ϵ

2. $S \to aSb \mid \epsilon$

$\Longrightarrow \quad \delta(q, \epsilon, S) \Rightarrow (q, aSb)$ – ✓

$\delta(q, \epsilon, S) \Rightarrow (q, \epsilon)$

$\left.\begin{array}{l} \delta(q, a, a) \Rightarrow (q, \epsilon) \\ \delta(q, b, b) \Rightarrow (q, \epsilon) \end{array}\right\} T$

$S \to aSb \mid \epsilon$

$(q, aabb, S) \xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,aSb)} (q, aabb, aSb)$

$\xrightarrow{\delta(q,a,a) \Rightarrow (q,\epsilon)} (q, abb, Sb)$

$\xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,aSb)} (q, abb\ aSbb)$

$\xrightarrow{\delta(q,a,a) \Rightarrow (q,\epsilon)} (q, bb\ Sbb)$

$\xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,\epsilon)} (q, bb, bb)$

$\xrightarrow{\delta(q,b,b) \Rightarrow (q,\epsilon)} (q, b, b)$

$\xrightarrow{\delta(q,b,b) \Rightarrow (q,\epsilon)} (q, \epsilon, \epsilon)$

③ $S \to aSa \mid bSb \mid c$

$\to$ $V = \{S\}$; $T = \{a, b, c\}$

$$\left.\begin{array}{l}\delta(q, \epsilon, S) \Rightarrow (q, aSa) \\ \delta(q, \epsilon, S) \Rightarrow (q, bSb) \\ \delta(q, \epsilon, S) \Rightarrow (q, c)\end{array}\right\} V$$

$$\left.\begin{array}{l}\delta(q, a, a) \Rightarrow (q, \epsilon) \\ \delta(q, b, b) \Rightarrow (q, \epsilon) \\ \delta(q, c, c) \Rightarrow (q, \epsilon)\end{array}\right\} T$$

Simulation (or) Processing of string $w = abbcbba$

$\delta(q, a\underline{bbcbb}a, S) \xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,aSa)} (q, \underline{a}bbcbba, \underline{a}Sa)$

$\xrightarrow{\delta(q,a,a) \Rightarrow (q,\epsilon)} (q, \epsilon bbcbba, Sa)$

$\xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,bSb)} (q, \underline{b}bcbba, \underline{b}Sba)$

$\xrightarrow{\delta(q,b,b) \Rightarrow (q,\epsilon)} (q, \epsilon bcbba, Sba)$

$\xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,bSb)} (q, \underline{b}cbba, \underline{b}Sbba)$

$\xrightarrow{\delta(q,b,b) \Rightarrow (q,\epsilon)} (q, \epsilon cbba, Sbba)$

$\xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,c)} (q, \underline{c}bba, \underline{c}bba)$

$\xrightarrow{\delta(q,c,c) \Rightarrow (q,\epsilon)} (q, \underline{b}ba, \underline{b}ba)$

$\xrightarrow{\delta(q,b,b) \Rightarrow (q,\epsilon)} (q, \underline{b}a, \underline{b}a)$

$\xrightarrow{\delta(q,b,b) \Rightarrow (q,\epsilon)} (q, \underline{a}, \underline{a})$

$\xrightarrow{q(q,a,a) \Rightarrow (q,\epsilon)} (q, \epsilon, \epsilon)$

(4) Find PDA for given Grammar:

$S \to 0S1 \mid 00 \mid 11$

$\Rightarrow \quad \delta(q, \epsilon, S) \Rightarrow \{(q, 0S1), (q, 00), (q, 11)\} \Rightarrow V$

$$\left.\begin{array}{l} \delta(q, 0, 0) \Rightarrow (q, \epsilon) \\ \delta(q, 1, 1) \Rightarrow (q, \epsilon) \end{array}\right\} T$$

$M = \{ \{q\}, \underbrace{\{0,1\}}_{T}, \underbrace{\{0,1,S\}}_{V \cup T}, \delta, q, S, \phi \}$

$W = 000111$ show simulation (or) processing for given string

---

(5) $S \to 0S1 \mid A$

May 12 5M

$A \to 1A0 \mid S \mid \epsilon$

$\Rightarrow \quad V = \{S, A\} \; ; \; T = \{0, 1\}$

$\delta(q, \epsilon, S) \Rightarrow \{(q, 0S1), (q, A)\}$

$\delta(q, \epsilon, A) \Rightarrow \{(q, 1A0), (q, S), (q, \epsilon)\}$

$\delta(q, 0, 0) \Rightarrow (q, \epsilon)$

$\delta(q, 1, 1) \Rightarrow (q, \epsilon)$

$M = \{ (q), \underbrace{\{0,1\}}_{T}, \underbrace{\{S, A, 0, 1\}}_{V \cup T}, \delta, q, S, \phi \}$

⑥ $S \to 0BB$

May 12,13 5M $B \to 0S \mid 1S \mid 0$ Test if 010^4 is in language

$\Rightarrow$ $M = \{ (q), \underbrace{(0,1)}_{T}, \underbrace{(0,1,S,B)}_{V \cup T}, \delta, q, S, \phi \}$

$\delta :\Rightarrow$

$$\left.\begin{aligned} &\delta(q,\epsilon,S) \Rightarrow \{q, 0BB\} \\ &\delta(q,\epsilon,B) \Rightarrow \{(q,0S), (q,1S), (q,0)\} \end{aligned}\right\} V$$

$$\left.\begin{aligned} &\delta(q,0) \Rightarrow (q,\epsilon) \\ &\delta(q,1) \Rightarrow (q,\epsilon) \end{aligned}\right\} T$$

Acceptance of 010^4 by $M = 01\underline{0000}$

$\delta(q,010000,S) \xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,0BB)} (q,010000,0BB)$

$\xrightarrow{\delta(q,0,0) \Rightarrow (q,\epsilon)} (q,10000,BB)$

$\xrightarrow{\delta(q,\epsilon,B) \Rightarrow (q,1S)} (q,10000,1SB)$

$\xrightarrow{\delta(q,1,1) \Rightarrow (q,\epsilon)} (q,0000,SB)$

$\xrightarrow{\delta(q,\epsilon,S) \Rightarrow (q,0BB)} (q,0000,0BBB)$

$\xrightarrow{\delta(q,0,0) \Rightarrow (q,\epsilon)} (q,000,BBB)$

$\xrightarrow{\delta(q,\epsilon,B) \Rightarrow (q,0)} (q,000,0BB)$

$\xrightarrow{\delta(q,0,0) \Rightarrow (q,\epsilon)} (q,00,BB)$

$\xrightarrow{\delta(q,\epsilon,B) \Rightarrow (q,0)} (q,00,0B)$

$\xrightarrow{\delta(q,0,0) \Rightarrow (q,\epsilon)} (q,0,B)$

$\xrightarrow{\delta(q,\epsilon,B) \Rightarrow (q,0)} (q,0,0)$

$\therefore \underline{010^4 \in L}$ $\quad \delta(q,0,0) \Rightarrow (q,\epsilon)$ $\quad (q,\epsilon,\epsilon)$

⑦ $S \rightarrow aABB \mid aAA$

$A \rightarrow aBB \mid a$

$B \rightarrow bBB \mid A$

$\Longrightarrow \quad M = \{(q), (a,b), (a,b,S,A,B), \delta, q, S, \phi\}$

δ:

$$\left.\begin{array}{l} \delta(q, \epsilon, S) \Rightarrow \{(q, aABB), (q, aAA)\} \\ \delta(q, \epsilon, A) \Rightarrow \{(q, aBB), (q, a)\} \\ \delta(q, \epsilon, B) \Rightarrow \{(q, bBB), (q, A)\} \end{array}\right\} V$$

$$\left.\begin{array}{l} \delta(q, a, a) \Rightarrow (q, \epsilon) \\ \delta(q, b, b) \Rightarrow (q, \epsilon) \end{array}\right\} T$$

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⑧ $S \rightarrow SS \mid (S) \mid (\,)$ Aug 17 6 M Parenthis Matching

$\Rightarrow$

$\delta \Rightarrow \delta(q, \epsilon, S) \Rightarrow \{(q, SS), \{q, (S)\}, \{q, (\,)\}\} \Rightarrow V$

$$\left.\begin{array}{l} \delta(q, (, () \Rightarrow (q, \epsilon) \\ \delta(q,),)) \Rightarrow (q, \epsilon) \end{array}\right\} T$$

① $\delta(q, (()), S) \Rightarrow \xrightarrow{\delta(q, \epsilon, S) \Rightarrow (q, (S))} (q, (()), (S))$

$\xrightarrow{\delta(q, (, () \Rightarrow (q, \epsilon)} (q, \epsilon()), S))$

$\xrightarrow{\delta(q, \epsilon, S) \Rightarrow (q, ())} (q, ()), ()))$

$\xrightarrow{\delta(q, (, () \Rightarrow (q, \epsilon)} (q,)),)))$

$\xrightarrow{\delta(q,),)) \Rightarrow (q, \epsilon)} (q,),))$

$\xrightarrow{\delta(q,),)) \Rightarrow (q, \epsilon)} (q, \epsilon, \epsilon)$