③ $L = \{a^m b^n \mid m \geq n\}$

$\Rightarrow \quad n = 0 \Rightarrow a^m \mid m \geq 1$

$n \neq 0 \Rightarrow aab, aaabb, \ldots$

$S \rightarrow aSb \mid aS \mid a$

$S \rightarrow aSb$
$\rightarrow aaSbb$
$\rightarrow aaabb$

$S \rightarrow aS$
$\rightarrow aa$

---

4) $L = \{a^m b^n \mid m = 2n\}$

$L = \{\epsilon, aab, aaaabb, \ldots\}$

$S \rightarrow aaSb \mid \epsilon$

$S \rightarrow aaSb$
$\rightarrow aab$

$S \rightarrow aaSb$
$\rightarrow aa\,aaSbb$
$\rightarrow aaaabb$

⑤ $L = \{a^m b^n \mid m \neq n\}$

$m < n$ → S_1

$m > n$ → S_2

$S \rightarrow S_1$
$\rightarrow aS_1b$
$\rightarrow aS_1bb$
$\rightarrow abbb$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow aS_1b \mid S_1b \mid b$

$S_2 \rightarrow aS_2b \mid aS_2 \mid a$

(or) $S \rightarrow aSb \mid A \mid B$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

Q.12) Construct the grammar for the following language -

1) $L = \{a^m b^n c^p \mid m, n, p \geq 0\}$ RL

$\rightarrow$ $\underbrace{a^*}_{A}\ \underbrace{b^*}_{B}\ \underbrace{c^*}_{C}$

$S \rightarrow ABC$
$\rightarrow aA\,bB\,cC$
$\rightarrow abc$

$S \rightarrow ABC$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bB \mid \epsilon$
$C \rightarrow cC \mid \epsilon$

2) $L = \{a^m b^n c^p \mid m = n\}$ NRL

$\Rightarrow$ $\underbrace{a^m b^m}_{A}\ \underbrace{c^p}_{B}$ $\underline{\underline{m=n}}$

$S \rightarrow AB$
$\rightarrow aAbcB$
$\rightarrow abccB$
$\rightarrow abcc$

$S \rightarrow AB$
$A \rightarrow aAb \mid \epsilon$
$B \rightarrow cB \mid \epsilon$

3) $L = \{a^m b^n c^p \mid n = p\}$ NRL

$\rightarrow$ $\underbrace{a^m}_{A}\ \underbrace{b^n c^n}_{B}$ $\underline{\underline{n=p}}$

$S \rightarrow AB$
$\rightarrow aAbBc$
$\rightarrow abbBcc$
$\rightarrow abbcc$

$S \rightarrow AB$
$A \rightarrow aA \mid \epsilon$
$B \rightarrow bBc \mid \epsilon$

4) $L = \{ a^m b^n c^p \mid m = p \}$

$\rightarrow$ $a^m b^n c^p$ $\quad$ $a^m b^n c^m$ $\quad$ $m = p$

$S \rightarrow aSc \mid A$

$A \rightarrow bA \mid \epsilon$

$S \rightarrow aSc$
$\rightarrow aaScc$
$\rightarrow aacc$

$S \rightarrow aSc$
$\rightarrow aAc$
$\rightarrow abAc$
$\rightarrow abbAc$
$\rightarrow abbc$

---

5) $L = \{ a^m b^n c^p \mid m = n \text{ (or) } n = p \}$

$m = n$ $\quad$ $n = p$

$\underbrace{a^m b^m}_{A} \underbrace{c^p}_{B}$ $\quad$ $\underbrace{a^m}_{C} \underbrace{b^n c^n}_{D}$

$S \rightarrow AB \mid CD$

$A \rightarrow aAb \mid \epsilon$

$B \rightarrow cB \mid \epsilon$

$C \rightarrow aC \mid \epsilon$

$D \rightarrow bDc \mid \epsilon$

$S \rightarrow AB$
$\rightarrow aAbcB$
$\rightarrow abcB$
$\rightarrow abc$

6) $L = \{a^m b^n c^p \mid m+p = n\}$

$\rightarrow \quad a^m b^n c^p$

$a^m \, b^{m+p} c^p$

$\underbrace{a^m b^m}_{A} \; \underbrace{b^p c^p}_{B}$

$S \rightarrow AB$

$A \rightarrow aAb \mid \epsilon$

$B \rightarrow bBc \mid \epsilon$

$S \rightarrow AB$

$\rightarrow aAb\,bBc$

$\rightarrow abbc$

Q.13 Construct the grammar for the following language -

① $L = \{ww^R / w \in (a+b)^*\}$

$\Rightarrow$ $L = \{\epsilon, aa, bb, abba, abbbba, baab\}$

$S \to aSa \mid bSb \mid \epsilon$

$S \to aSa$
$\to aa$

$S \to bSb$
$\to bb$

$S \to aSa$
$\to abSba$
$\to abba$

$S \to bSb$
$\to baSab$
$\to baab$

② $L = \{wxw^R / w \in (a+b)^*\}$

$\to$ $S \to aSa \mid bSb \mid x$

$S \to aSa$
$\to axa$

$S \to bSb$
$\to bxb$

$S \to aSa$
$\to abSba$
$\to abxba$

$S \to bSb$
$\to baSab$
$\to baxab$

③ $L = \{w \in (a+b)^* \mid |w|_a = |w|_b\}$

$\Rightarrow$ $L = \{\epsilon, ab, ba, abab, baba, abba, \ldots\}$

$S \to SaSbS \mid SbSaS \mid \epsilon$

$S \to SaSbS$
$\to ab$

$S \to SbSaS$
$\to ba$

$S \to SaSbS$
$\to SbSaSaSbS$
$\to baab$

$S \to SbSaS$
$\to SaSbSbSaS$
$\to abba$

$S \to SaSbS$
$\to SaSbSaSbS$
$\to abab$

④ $L = \{ w \in (a+b)^* \mid |w|_a = 2|w|_b \}$

→ $S \to SbSaSaS \mid SaSbSaS \mid SaSaSbS \mid \epsilon$

$S \to SbSaSaS$
$\to baa$

$S \to SaSbSaS$
$\to aba$

$S \to SaSaSbS$
$\to aab$

⑤ $L = \{ (ab)^n \mid n \geq 1 \}$

ab, abab, ...

$S \to aA$
$A \to bS \mid b$ (or)

$S \to aA$
$\to ab$

$S \to aA$
$\to abS$
$\to abaA$
$\to abab$

$S \to abS \mid ab$

$S \to abS$
$\to abab$

⑥ $L = \{ (abc)^n \mid n \geq 1 \}$

$S \to abcS \mid abc$ (or)

$S \to aA$
$A \to bB$
$B \to cS \mid c$

$S \to abcS$
$\to abcabcS$
$\to abcabcabc$

$S \to aA$
$\to abB$
$\to abcS$
$\to abcaA$
$\to abcabB$
$\to abcabc$

* Chomsky Hierarchy :-

Based on the form of the production the grammar is classified into 4 types.—

① Type 0 (or) Recursive Enumerable Grammar (REG)

② Type 1 (or) Context Sensitive Grammar (CSG)

③ Type 2 (or) Context Free Grammar (CFG)

④ Type 3 (or) Regular Grammar (RG)

1) Type 0 (or) REG :-

The grammar G is said to be of type '0' (or) REG, if every production is in the form of $\alpha \to \beta$, where

$\alpha, \beta \in (V+T)^*$

Where atleast one variable must be there in LHS. of any production

Ex :- ① $S \to aA \mid \epsilon$

$aA \to abA \mid a$

$Bb \to ab \mid b \mid \epsilon$

② $S \to aSAc \mid abc \mid \epsilon$

$cA \to Ac$

$bA \to bb$

② Type 1 (or) CSG :-

In this, if every production is in the form -

$$\alpha \to \beta \;,\; |\alpha| \le |\beta| \;,\; \beta \ne \epsilon$$

where $\alpha, \beta \in (V+T)^{+}$

ex:- $S \to aSAC \mid abc$

$CA \to AC$

$bA \to bb$

Note:- ① CSG is also called as length increasing grammar.

② CSG doesn't contains any production to generate empty string ϵ.

③ Type 2 (or) CFG :-

If every production is in the form -

$A \to \alpha$ where $A \in V$, $\alpha \in (V+T)^{*}$

A → Variable

α → Variable/Terminal

Ex:- $S \to aSb \mid \epsilon$

④ Type '3' (or) RG :-

If every production is in the form of —

$A \to xB \mid y$ (or) $A \to Bx \mid y$

where, $A, B \in V$

$x, y \in T^*$

Ex:- ① $S \to 01S \mid 0$ ② $S \to 1010S \mid 00$

Note:-

Type 3 ⊂ Type 2 ⊂ Type 1 ⊂ Type 0