* Regular Expression *

Defⁿ:- An expression over the alphabet 'Σ' using the operators $*, +, \cdot$ is called as regular expression (RE)

(OR)

An expression that generates the regular language is called as RE.

→ RE describes the language accepted by finite automata.

Ex:- 1) $r = 0+1$ 2) $r = 0.1$

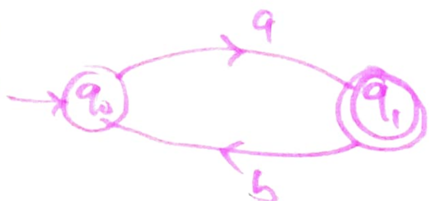
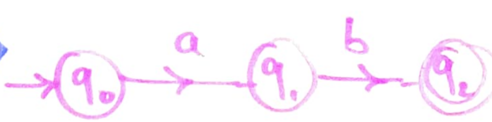
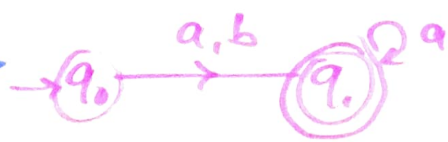
3) $r = 0^*1$ 4) $r = (1+0^*)^*10$

5) ~~(or~~ $r = (10^*)01$

Note:- 1) RE is a pattern to generate a set of strings.

2) If r is a RE then $L(r)$ is a language generated by r.

ex:-

FA	RE	RL
1) 	$r = \phi$	$L = \{ \ \}$
2) 	$r = \epsilon$	$L = \{\epsilon\}$
3) 	$r = a$	$L = \{a\}$
4) 	$r = a+b$ $r = a+b+c$	$L = \{a, b\}$ $L = \{a, b, c\}$
5) 	$r = a.b$	$L = \{ab\}$
6) 	$r = (a+b).a^*$	$L = \{aa, ba\}$
7) 	$r = a^*$	$L = \{\epsilon, a, aa, \cdots\}$
8) 	$r = a.a^*$ (or) a^+	$L = \{a, aa, \cdots\}$

* **Regular Operator :-** The operator $*$, $+$, $\bullet$ is called as regular operator.

$*$ $\rightarrow$ Kleen Closure operator

$+$ $\rightarrow$ Union Operator

$\bullet$ $\rightarrow$ Concatenation operator.

Operator precedence :-

$*$ $\rightarrow$ 1 $\rightarrow$ highest priority

$\bullet$ $\rightarrow$ 2 $\rightarrow$ next -"-

$+$ $\rightarrow$ 3 $\rightarrow$ comes last in precedence

$$r = (a \underset{3}{+} b \underset{2}{\cdot} a)^{*\,\leftarrow 1}$$

*Building RE :-

- Every finite language is regular language.

Proof: Let L is any finite language.

Let $L = \{w_1, w_2, w_3, \dots w_n\}$

→ To prove L is regular it is enough to prove that L is generated by some RE.

Now, $L = \{w_1, w_2, w_3, \dots w_n\}$

then RE for the language L can be obtain by inserting '+' operator between the string.

$\therefore r = w_1 + w_2 + w_3 + \dots + w_n$ is a RE generated the language L.

$\therefore$ L is regular language.

* Every finite language is regular.

Ex:- $r = ab \Rightarrow \{a.b\} = L$

$r^* = (ab)^* \Rightarrow L = \{\epsilon, ab, \cdots\}$

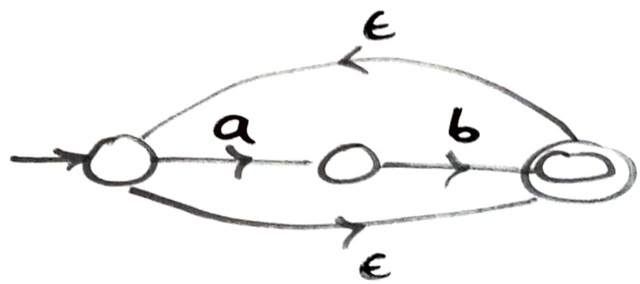
$r^+ = (ab)^+ \Rightarrow L = \{ab, abab, \cdots\}$

$r = (0+1)^* = \{\epsilon, 0, 1, 00, 01, \cdots\} = L$

$r = (0+1) = \{0, 1\} = L$

→ If r is any regular expression then both r^*, r^+ are also regular expression

$r^* = (ab)^*$



$r^+ = (ab)^+$

$r^* = \{\epsilon, r, rr, \cdots\}$

$r^+ = \{r, rr, rrr, \cdots\}$

→ Basic properties of Regular Expression:-

1) $\phi + r = r$

2) $\phi . r = \phi$

3) $\epsilon . r = r . \epsilon = r$

4) $\epsilon^* = \epsilon$

5) $\phi^* = \epsilon$

6) $r + r = r$

7) $r^* . r^* = r^*$

8) $r^* + \epsilon = r^*$

9) $r^+ + \epsilon = r^*$

10) $r^+ + r^* = r^*$

11) $r^* . r^+ = r^+$

12) $(r^*)^* = r^*$

13) $(r^+)^* = r^*$

14) $(r^*)^+ = r^*$

15) $r^* . r = r^+$

16) $\epsilon + r r^* = r^*$

17) $r^+ \cap r^* = r^+$

18) $(r^+)^+ = r^+$ ↳ doesn't contain ϵ

19) $(r^+)^* = (r^*)^*$ ⇒ both contains $\underline{\underline{\epsilon}}$

20) $(((r^+)^*)^+)^* = r^*$