

Assignment 4

Due on November 7 (by 11:59pm)

Part A: Questions: (50%)

(1) (12%) Use the definition of Dilation to prove " $A \oplus B = B \oplus A$ "

The image shows a handwritten proof on lined paper. It starts with the definition of dilation $(A)_b = \{a+b \mid a \in A\}$ (1). Then it defines dilation $A \oplus B = \bigcup_{b \in B} (A)_b$ (2). The proof then shows that $A \oplus B = \bigcup_{b \in B} \{a+b \mid a \in A\}$. This is followed by the definition of dilation $(B)_a = \{b+a \mid b \in B\}$ (3). Then it defines dilation $B \oplus A = \bigcup_{a \in A} (B)_a$ (4). The proof then shows that $B \oplus A = \bigcup_{a \in A} \{b+a \mid b \in B\}$. Finally, it concludes that $B \oplus A = b+a$, with a note that $a+b = b+a$ given $b \in B$ and $a \in A$.

i) $(A)_b = \{a+b \mid a \in A\} \quad - (1)$
we know that,
 $A \oplus B = \bigcup_{b \in B} (A)_b \quad - (2)$
put (1) in (2),
 $A \oplus B = \bigcup_{b \in B} \{a+b \mid a \in A\}$
 $\therefore A \oplus B = a+b$
 $(B)_a = \{b+a \mid b \in B\} \quad - (3)$
we know that,
 $B \oplus A = \bigcup_{a \in A} (B)_a \quad - (4)$
put (3) in (4),
 $\therefore B \oplus A = \bigcup_{a \in A} \{b+a \mid b \in B\}$
 $\therefore B \oplus A = b+a$
 $a+b = b+a$
condition, given $b \in B$ and $a \in A$

(2) (12%) Binary image A and structuring element B are defined below. Show the result of opening operation ($A \circ B$).

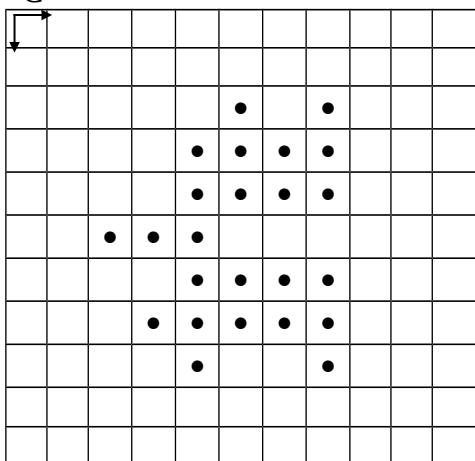
A

A 19x19 Go board diagram. The board features black dots at various intersections representing stones. A black arrow points from the top-left corner towards the center of the board.

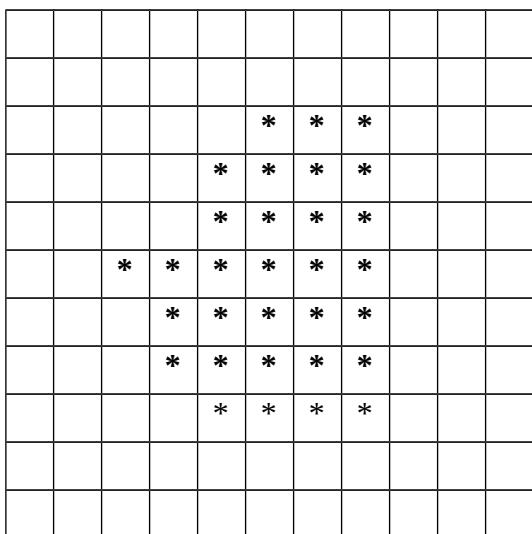
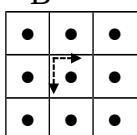
B

(2) (12%) Binary image C and structuring element B are defined below. Show the result of closing operation ($C \bullet B$).

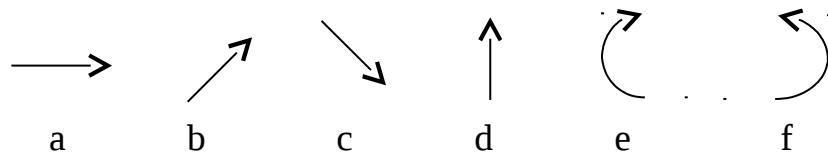
C



B



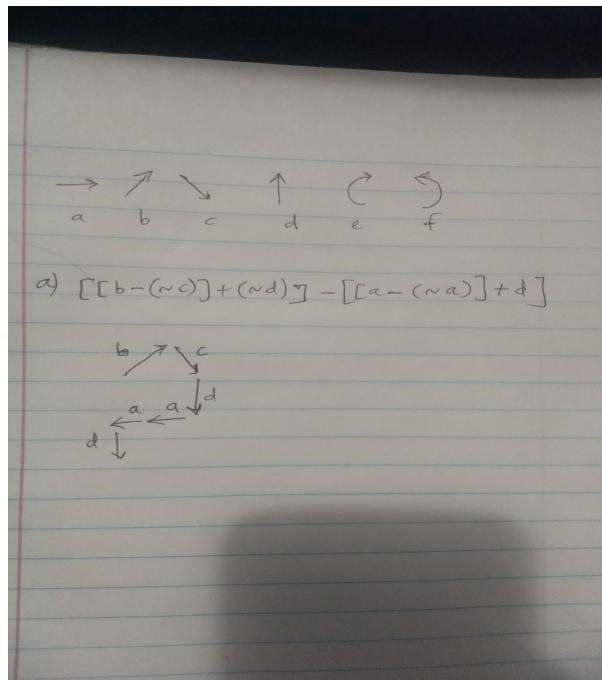
(3) (14%) Use the following primitives:



and use the structure relations given in class ($+, -, \times, *, \sim$)

- (a) (7%) Sketch the structure whose PDL (program description language) structural description is:

$$[[b - (\sim c)] + (\sim d)] - [[a - (\sim a)] + d]$$



(b) (7%) Give PDL structural description of the following two structures:

