1) **Theory Problem 1**

1) Solution $\qquad C(X, Y, Z)$

(a) Normalized chromacity co-ordinates for 3 primaries

For P_1

$$(x_1, y_1, z_1) = \left(\frac{X_1}{X_1+Y_1+Z_1}, \frac{Y_1}{X_1+Y_1+Z_1}, \frac{Z_1}{X_1+Y_1+Z_1} \right)$$

For P_2

$$(x_2, y_2, z_2) = \left(\frac{X_2}{X_2+Y_2+Z_2}, \frac{Y_2}{X_2+Y_2+Z_2}, \frac{Z_2}{X_2+Y_2+Z_2} \right)$$

For P_3

$$(x_3, y_3, z_3) = \left(\frac{X_3}{X_3+Y_3+Z_3}, \frac{Y_3}{X_3+Y_3+Z_3}, \frac{Z_3}{X_3+Y_3+Z_3} \right)$$

b) Express the normalized chromacity co-ordinates of the color C in terms of the chromacity co-ordinates of P_1, P_2, P_3.

Soln:- Normalized chromacity co-ordinates for color C is

$$(x, y, z) = \left(\frac{X}{X+Y+Z}, \frac{Y}{X+Y+Z}, \frac{Z}{X+Y+Z} \right)$$

And we also know that color C can be expressed as linear combination of P_1, P_2, P_3, we get

$$X = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3,$$
$$Y = \alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3,$$
$$Z = \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3.$$

Therefore chromaticity co-ordinates of color C in terms of P_1, P_2, P_3 would be.

$$(x, y, z) = \left(\frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{(\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3) + (\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3) + (\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3)}, \right.$$
$$\frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{(\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3) + (\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3) + (\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3)},$$
$$\left. \frac{\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}{(\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3) + (\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3) + (\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3)} \right)$$

Further simplifying it gives

$(x, y, z) =$

$$\left(\frac{\alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)}, \frac{\alpha_1 Y_1 + \alpha_2 Y_2 + \alpha_3 Y_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)}, \right.$$

$$\left. \frac{\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3}{\alpha_1(X_1+Y_1+Z_1) + \alpha_2(X_2+Y_2+Z_2) + \alpha_3(X_3+Y_3+Z_3)} \right)$$

(c) Hence prove that chromaticity co-ordinates of any color C can be expressed/represented also as a linear combination of the chromaticity co-ordinates of the respective primaries.

i.e $C(X, Y, Z) = \alpha_1 P_1(X_1, Y_1, Z_1) + \alpha_2 P_2(X_2, Y_2, Z_2) + \alpha_3 P_3(X_3, Y_3, Z_3)$

$\boxed{\text{To prove: } x = \alpha_1' x_1 + \alpha_2' x_2 + \alpha_3' x_3 \text{ same for } y \text{ and } z}$

For (a) we have the representation of x co-ordinate for 3 primaries as

$$x_1 = \frac{X_1}{X_1+Y_1+Z_1}, \quad x_2 = \frac{X_2}{X_2+Y_2+Z_2}, \quad x_3 = \frac{X_3}{X_3+Y_3+Z_3}$$

for y co-ordinates

$$y_1 = \frac{Y_1}{X_1+Y_1+Z_1}, \quad y_2 = \frac{Y_2}{X_2+Y_2+Z_3}, \quad y_3 = \frac{Y_3}{X_3+Y_3+Z_3}$$

for z co-ordinates

$$z_1 = \frac{Z_1}{X_1+Y_1+Z_1}, \quad \frac{Z_2}{X_2+Y_2+Z_2}, \quad \frac{Z_3}{X_3+Y_3+Z_3}$$

So, for the above norms, divide and multiply by $\alpha_1, \alpha_2, \alpha_3$ respectively.

Therefore for x-co-ordinates we get

$$x_1 = \frac{\alpha_1 X_1}{\alpha_1(X_1+Y_1+Z_1)}, \quad x_2 = \frac{\alpha_2 X_2}{\alpha_2(X_2+Y_2+Z_2)}, \quad x_3 = \frac{\alpha_3 X_3}{\alpha_3(X_3+Y_3+Z_3)}$$

the same holds good for y and z coordinates.
Further simplification gives us

$$\alpha_1 X_1 = \alpha_1 (X_1 + Y_1 + Z_1)\, x_1$$
$$\alpha_2 X_2 = \alpha_2 (X_2 + Y_2 + Z_2)\, x_2$$
$$\alpha_3 Y_3 = \alpha_3 (X_3 + Y_3 + Z_3)\, x_3$$

Substituting this in the equation obtained in part (b) we get

$$x = \frac{\alpha_1 (X_1+Y_1+Z_1)\, x_1 + \alpha_2 (X_2+Y_2+Z_2)\, x_2 + \alpha_3 (X_3+Y_3+Z_3)\, x_3}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$$y = \frac{\alpha_1 (X_1+Y_1+Z_1)\, y_1 + \alpha_2 (X_2+Y_2+Z_2)\, y_2 + \alpha_3 (X_3+Y_3+Z_3)\, y_3}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$$z = \frac{\alpha_1 (X_1+Y_1+Z_1)\, Z_1 + \alpha_2 (X_2+Y_2+Z_2)\, Z_2 + \alpha_3 (X_3+Y_3+Z_3)\, Z_3}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

The above equation can be written as

$$\cancel{x} \quad \alpha_1' = \frac{\alpha_1 (X_1 + Y_1 + Z_2)}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

Same for α_2', i.e (that is)

$$\alpha_2' = \frac{\alpha_2 (X_2 + Y_2 + Z_2)}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

$$\alpha_3' = \frac{\alpha_3 (X_3 + Y_3 + Z_2)}{\alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3)}$$

Therefore

$$x = \alpha_1' x_1 + \alpha_2' x_2 + \alpha_3' x_3$$
$$y = \alpha_1' y_1 + \alpha_2' y_2 + \alpha_3' y_3$$
$$z = \alpha_1' z_1 + \alpha_2' z_2 + \alpha_3' z_3$$

$\boxed{\text{$x_1, x_2, x_3$ are x co-ordinates of P_1, P_2, P_3 respectively. Same holds true for y \& z}}$

This shows that chromaticity co-ordinates of ~~an~~ color C can ~~be~~ be expressed as linear combination of chromaticity co-ordinates of Primaries P_1, P_2, P_3.

(Note :- x and X are different representation of co-ordinate
x
X is co-ordinate)

2) Theory Problem 2:

2) a)

	Level
0.25 →	0
0.5 →	1
0.75 →	2
1.00 →	3
1.25 →	4
1.50 →	5
1.75 →	6
2.00 →	7
2.25 →	8
2.50 →	9
2.75 →	10
3.00 →	11
3.25 →	12
3.50 →	13
3.75 →	14
4.00 →	15
4.25 →	16
4.50 →	17
4.75 →	18
5.00 →	19
5.25 →	20
5.50 →	21
5.75 →	22
6.00 →	23
6.25 →	24
6.50 →	25
6.75 →	26
7.00 →	27
7.25 →	28
7.50 →	29
7.75 →	30
8.00 →	31

	Nearest value	Nearest level	d(n)
5.8 →	5.75	22	
6.2 →	6.25	24	2
6.2 →	6.25	24	0
6.2 →	7.25	28	4
7.2 →	7.25	28	0
7.3 →	7.25	28	0
7.3 →	6.50	25	-3
6.5 →	6.75	26	+1
6.8 →	6.75	26	0
6.8 →	6.75	26	0
6.8 →	5.50	21	-5
5.5 →	5.00	19	-2
5.0 →	5.25	20	1
5.2 →	5.25	20	0
5.2 →	5.75	22	2
5.8 →	6.25	24	+2
6.2 →	6.25	24	0
6.2 →	6.25	24	0
6.2 →	6.00	23	-1
5.9 →	6.25	24	+1
6.3 →	5.25	20	-4
5.2 →	4.25	16	-4
4.2 →	2.75	10	-6
2.8 →	2.75	10	0
2.8 →	2.25	8	-2
2.3 →	3.00	11	3
2.9 →	1.75	6	-5
1.8 →	2.50	9	3
2.5 →	2.50	9	0
2.5 →	3.25	12	3
3.3 →	4.00	15	3
4.1 →	5.00	19	4
4.9 →			

a) Quantization sequence.

22, 24, 24, 28, 28, 28, 25, 26, 26, 26, 21, 19

20, 20, 22, 24, 24, 24, 23, 24, 20, 16, 10, 10,

8, 11, 6, 9, 9, 12, 15, 19

b) Number of bits needed to transmit

32 level $32 = 2^5$, we need 5 bits per level/signal

so $5 \times 32 = 160$ bits

c) Successive difference between values.

$d(n) = y(n) - y(n-1)$

2, 0, 4, 0, 0, −3, +1, 0, 0, −5, −2, 1, 0, 2, +2
0, 0, −1, +1, −4, −4, −6, 0, −2, 3, −5, 3, 0, 3, 3, 4

Max value = 4
Min value = −6

$[-6, 4] \rightarrow$ range

−6, −5, −4, −3, −2, −1, 0, 1, 2, 3, 4

11 levels.

No of bits required to encode the sequence

$11 = 2^x \Rightarrow$ taking natural logarithm on both the side

$\ln(11) = \ln(2^x)$

$2.3978 = x \ln(2)$

$2.3978 = x * 0.6931$

$x = \frac{2.3978}{0.6931} = 3.4595 \approx 4$ bits

∴ total number of bits = $4*31$ = 124 bits

(If we consider even the first value, the total number of bits would be $4*31+5 = 129$ bits (5 because 22 is represented using 5 bits)) ⟶ $\boxed{\text{Note}}$

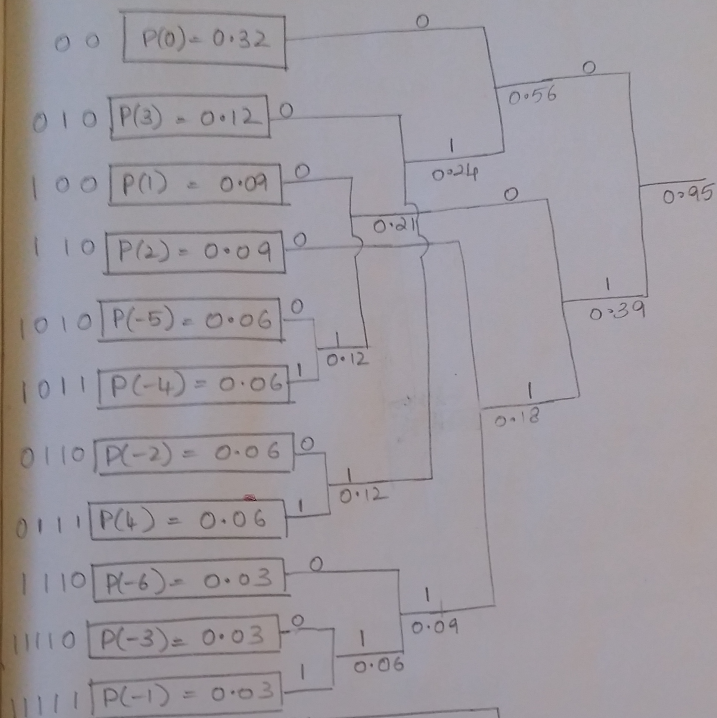
d) Compression ratio $= \frac{5}{4} = 1.25$

Compression ratio considering total number of bits.

$\frac{160}{129} = 1.24$

... the difference, you use

2 a)



Symbol	Probability	Huffman code
0	0.32	00
3	0.12	010
1	0.09	100
2	0.09	110
-5	0.06	1010
-4	0.06	1011
-2	0.06	0110
4	0.06	0111
-6	0.03	1110
-3	0.03	11110
-1	0.03	11111

$L = \sum P_i l_i$

$= (2\times0.32)+(3\times0.12)+(3\times0.09)+(3\times0.09)+(4\times0.06)$
$+(4\times0.06)+(4\times0.06)+(4\times0.06)+(4\times0.03)+(5\times0.03)$
$+(5\times0.03)$

$= 0.64+0.36+0.27+0.27+0.24+0.24+0.24+0.24+$
$0.12+0.15+0.15$

$= 2.92 \text{ bits} \approx \underline{\underline{3 \text{ bits}}}$

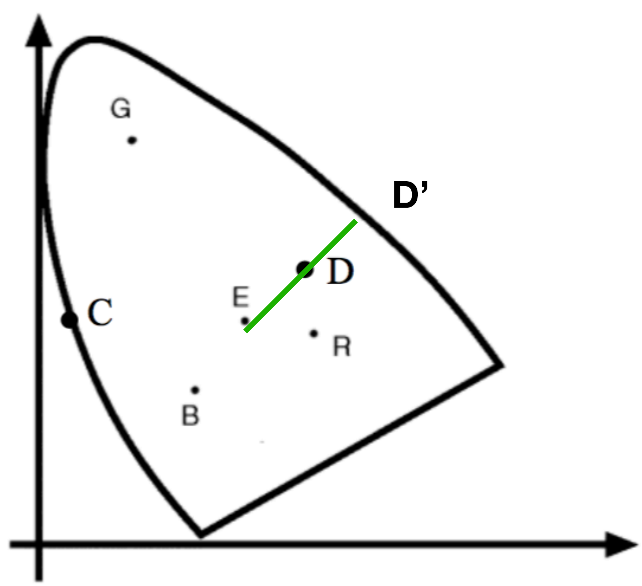
Total no of bits $= 31\times3 = \underline{\underline{93 \text{ bits}}}$

2(f) Compression ratio =

$$\frac{\text{Number of bits in original image}}{\text{Number of bits in compressed image}} = \frac{5}{3} = \underline{\underline{1.66}}$$

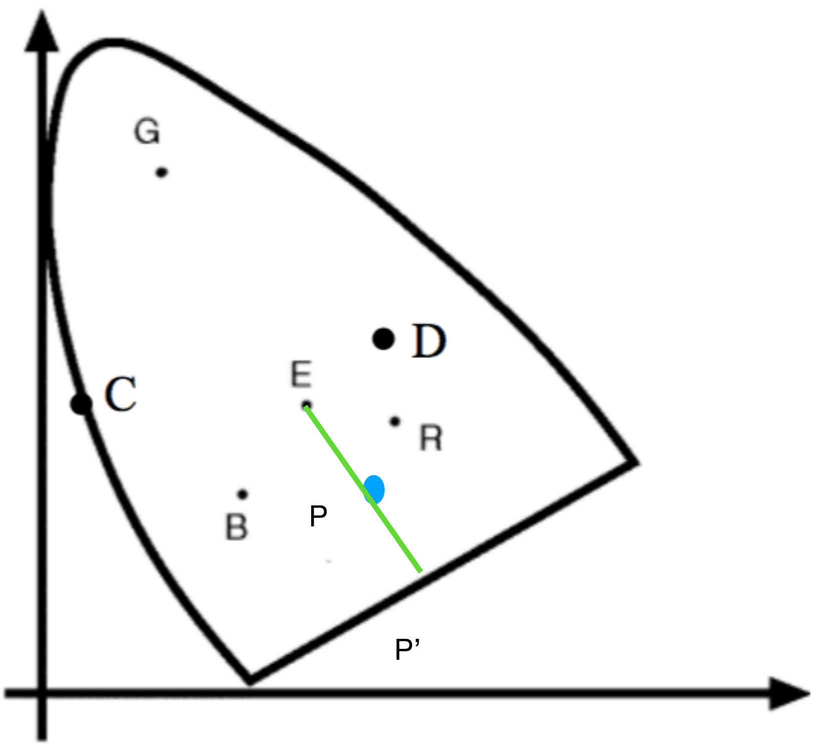
3) Theory Problem 3:

a) The dominant wavelength of color is D’. It is the wavelength of the color (Pure spectral color) to the nearest color D



b)

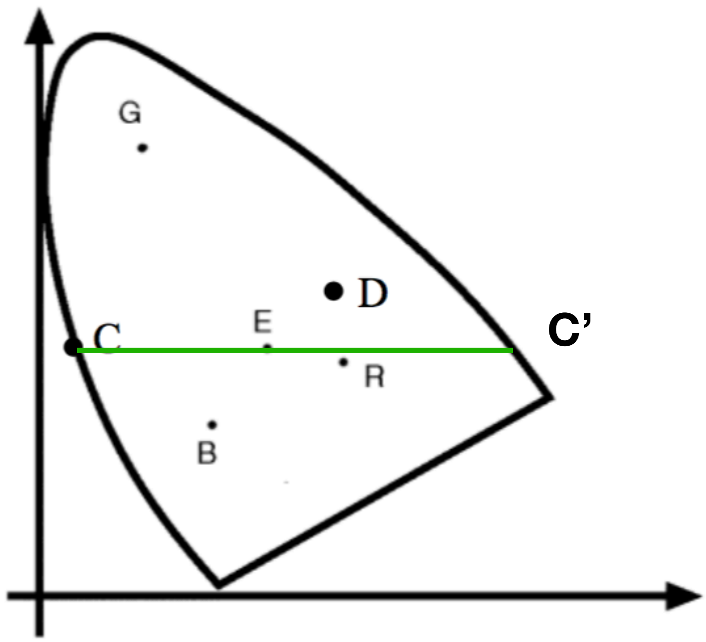
No, not all colors have dominant wavelength.
Purple color doesn’t have a dominant wavelength because of which purple is not a pure spectral color.



From the plot above its is clear that color P (color that is perceived as Purple) doesn’t have a dominant wavelength. P’ is not a dominant wavelength for P.

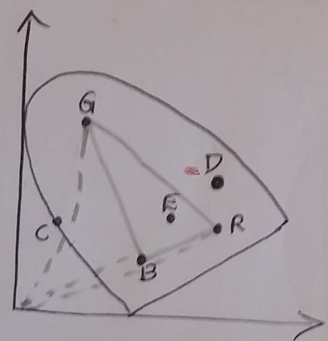
C)

Complementary color for Color C is C’ because it is the point of intersection in the opposite side of color space.



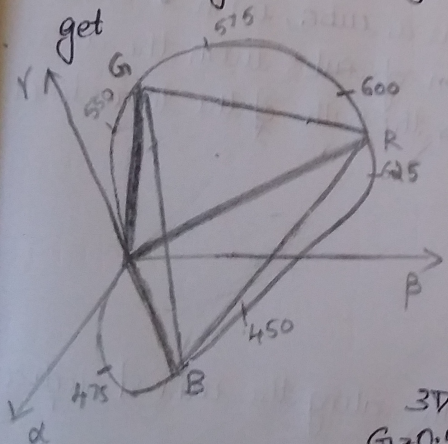
(D)

3 (d)



To find all the point which have value of G = 0.5

Representing chromacity diagram in 3D color space, we get



→ line represents ~~prima~~ axis.

When 1.0 along red, green and blue axis implies that axis touches the spectral locus termed as primaries.

As the value increases the intensity decreases.

This diagram to the left shows 3D representation of color space with G = 0.5.

When G = 0.5, R = 0.25, B = 0.25 (one case)

G + R + B = 0.5 + 0.25 + 0.26 = 1.0

The wavelength for G = 560nm, B = 470nm, R = 610nm.

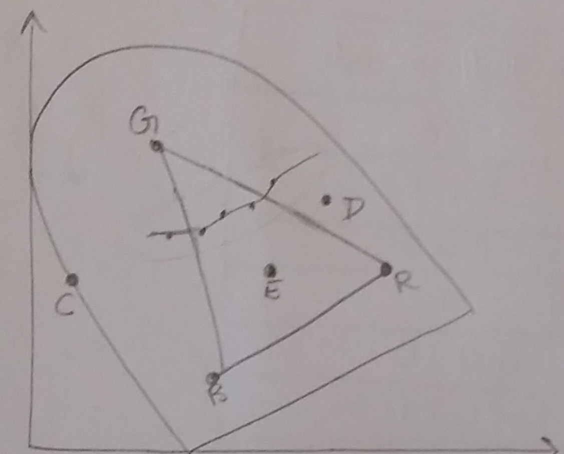
The top diagram shows all the points that have G = 0.5

- - - - (dashed lines) represent the axes. And triangle represents color gamut.

These are the value of R and B for which G=0.5. Plotting this one a curve while joining all the points will give a some what straight line

R	G	B
0.24	0.5	0.26
0.25	0.5	0.25
0.39	0.5	0.12
0.36	0.5	0.14
0.09	0.5	0.41

Table (1)

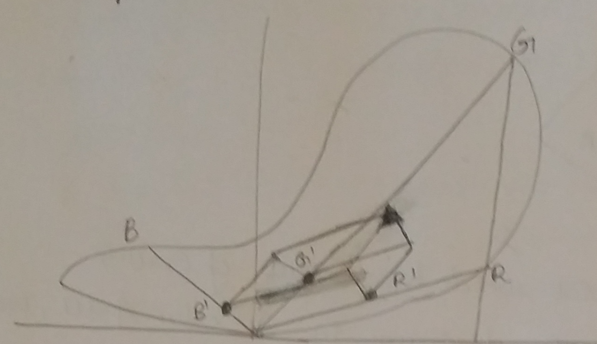


(The point represented in the table 1 are values for which G=0.5. Plotting this in chromaticity space give the above diagram.)

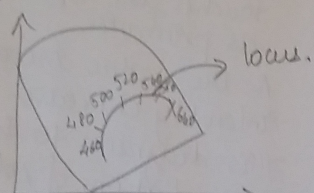
We are getting somewhat straight line because of the additive property of R, G, B i.e R+B+G=1 it cannot exceed 1. So when G=0.5, we get different point of R & B for which G=0.5.

(E)

3 (e) Take G = 0.5 locus above, how does this line map in the RGB space. Explain



The RGB space is represented as a cube, with • black dot representing the intersection of cube with the axes. In chromaticity diagram, locus is the path of the incandescent black body.



In RGB space locus is the path along the curved surface in the above diagram (first, top)

The line in the previous question maps as a line parallel to BR plane

The shaded area in the represent the line in RGB space.

Given the RGB coordinates of a color, its (r,g,b) coordinates can be computed as (R/(R+G+B),G/(R+G+B),B/(R+G+B)). The division projects the points in RGB on to R+G+B=1 plane.

Using transformation matrix we get

XYZ to sRGB' - apply transformation matrix

$$\begin{bmatrix} sR' \\ sG' \\ sB' \end{bmatrix} = \begin{bmatrix} 3.2404542 & -1.5371385 & -0.4985314 \\ -0.9692660 & 1.8760108 & 0.0415560 \\ 0.0556434 & -0.2040259 & 1.0572252 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

XYZ to RGB space conversion.