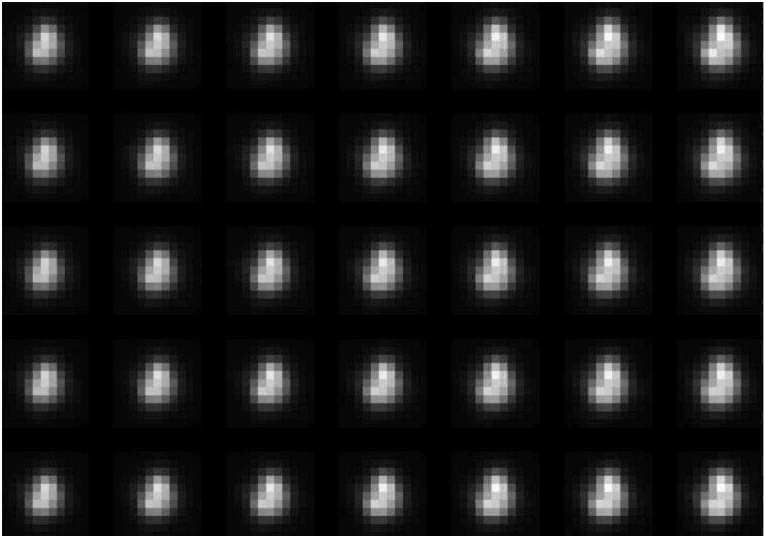
Karhunen-Loeve Transform For PSF Modeling

29 January 2021

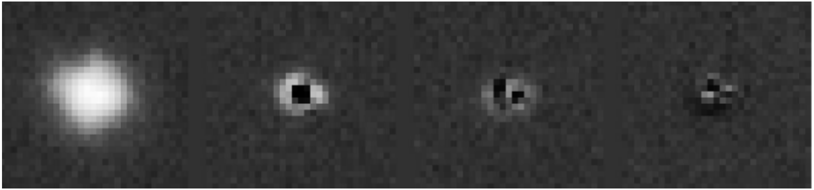


Methods:

1. Each star cutout(21 X 21) is flattened to 1D array.
2. x_i^k is ith pixel for kth star cutout.
3. Covariance matrix(441 X 441) is formed.

$$\hat{\sigma}_{ij}^2 = \frac{1}{K}\sum_{k=1}^{K}\left(x_i^{(k)} - \hat{\mu}_i\right)\left(x_j^{(k)} - \hat{\mu}_j\right) = \frac{1}{K}\sum_{k=1}^{K} x_j^{(k)} x_j^{(k)} - \hat{\mu}_i\hat{\mu}_j$$

4. Covariance matrix is Hermitian, so eigen vectors are orthogonal.
5. These eigen vectors(ϕ_r) can be reshaped to eigen images.
6. Any star can be written as sum of these eigen images.



1. KLT completely decorrelates the signal
2. KLT maximally compacts the energy (information) contained in the signal into first few eigen vectors.

1. PSF can be written as

$$P_{(i)}(u,v) = \sum_{r=1}^{r=n} a^r_{(i)} \phi_r(u,v)$$

2. coefficients *a* can be obtained by dot product of ϕ_r and P_i.
3. Coefficient maps can also be generated.

$$a^r_{(i)} \approx \sum_{l=m=0}^{l+m \leq N} b^r_{lm} x^l_{(i)} y^m_{(i)}$$

4. Oversampling and interpolation can be a problem.

References

[1] Excellent lectures. http://fourier.eng.hmc.edu/e161/lectures/klt/node3.html

[2] https://www.engr.colostate.edu/ECE513/SP09/lectures/lectures13_14.pdf

[3] PCA for PSF https://arxiv.org/abs/0710.5560

[4] SDSS Image Processing II: The Photo Pipelines https://www.astro.princeton.edu/~rhl/photo-lite.pdf

[5] Jupyter Notebook https://github.com/sukhbinder/Notebooks/blob/master/Karhunen%20Loeve%20Transform.ipynb

[6] Wolfram https://reference.wolfram.com/language/ref/KarhunenLoeveDecomposition.html