QUIZ 4

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1 PROBLEM 1

1. The scalar product of the vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$ with a unit vector along the sum of vectors $\begin{pmatrix}2\\4\\-5\end{pmatrix}$ and $\begin{pmatrix}\lambda\\2\\3\end{pmatrix}$ is equal to one. Find the value of λ.

2 SOLUTION:

Let,

$$\mathbf{a} = \begin{pmatrix}1\\1\\1\end{pmatrix} ; \mathbf{b} = \begin{pmatrix}2\\4\\-5\end{pmatrix} ; \mathbf{c} = \begin{pmatrix}\lambda\\2\\3\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix}2\\4\\-5\end{pmatrix} + \begin{pmatrix}\lambda\\2\\3\end{pmatrix} \tag{2.0.2}$$

$$= \begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix} \tag{2.0.3}$$

$$\text{and,} \, \|\mathbf{b} + \mathbf{c}\| = \sqrt{(2+\lambda)^2 + 6^2 + 2^2} \tag{2.0.4}$$

$$= \sqrt{(2^2 + 2 \times 2 \times \lambda + \lambda^2) + 36 + 4} \tag{2.0.5}$$

$$= \sqrt{\lambda^2 + 4\lambda + 44} \tag{2.0.6}$$

Here, unit vector along $\mathbf{b} + \mathbf{c}$ is given by,

$$\mathbf{v} = \frac{\mathbf{b} + \mathbf{c}}{\|\mathbf{b} + \mathbf{c}\|} \tag{2.0.7}$$

$$= \frac{\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}}{\sqrt{\lambda^2 + 4\lambda + 44}} \tag{2.0.8}$$

It is given that,

$$\mathbf{a}\mathbf{v} = 1 \tag{2.0.9}$$

$$\implies \begin{pmatrix}1\\1\\1\end{pmatrix} \begin{pmatrix}\frac{2+\lambda}{\sqrt{\lambda^2+4\lambda+44}}\\ \frac{6}{\sqrt{\lambda^2+4\lambda+44}}\\ \frac{-2}{\sqrt{\lambda^2+4\lambda+44}}\end{pmatrix} = 1 \tag{2.0.10}$$

$$\implies \frac{2+\lambda+6-2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1 \tag{2.0.11}$$

$$\implies 2 + \lambda + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 44} \tag{2.0.12}$$

$$\implies \lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 44} \tag{2.0.13}$$

squaring both sides, we get,

$$(\lambda + 6)^2 = \lambda^2 + 4\lambda + 44 \tag{2.0.14}$$

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \tag{2.0.15}$$

$$8\lambda = 8 \tag{2.0.16}$$

$$\lambda = 1 \tag{2.0.17}$$