

ASSIGNMENT

Shristy Sharma (EE22BNITS11001)

1 PROBLEM

1.If **E, F, G, H** are respectively the mid-points of the sides of a Parallelogram ABCD, show that area of Area of Parallelogram EFGH = $\frac{1}{2}$ Area of Parallelogram ABCD.

SOLUTION: Let,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1.0.1)$$

\therefore mid-points will be,

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (1.0.2)$$

$$= \frac{\begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{2} \quad (1.0.3)$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad (1.0.4)$$

$$\mathbf{F} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (1.0.5)$$

$$= \frac{\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{2} \quad (1.0.6)$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (1.0.7)$$

$$\mathbf{G} = \frac{\mathbf{D} + \mathbf{C}}{2} \quad (1.0.8)$$

$$= \frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{2} \quad (1.0.9)$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (1.0.10)$$

$$\mathbf{H} = \frac{\mathbf{D} + \mathbf{A}}{2} \quad (1.0.11)$$

$$= \frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}}{2} \quad (1.0.12)$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (1.0.13)$$

$$(1.0.14)$$

Thus, Area of Parallelogram ABCD is given by,

$$= (\mathbf{D} - \mathbf{A}) \times (\mathbf{D} - \mathbf{C}) \quad (1.0.15)$$

$$= \begin{vmatrix} 0 & -2 \\ -4 & 0 \end{vmatrix} \quad (1.0.16)$$

$$= -8 \quad (1.0.17)$$

And, Area of Parallelogram EFGH is given by,

$$= (\mathbf{G} - \mathbf{H}) \times (\mathbf{G} - \mathbf{F}) \quad (1.0.18)$$

$$= \begin{vmatrix} 1 & -1 \\ -2 & -2 \end{vmatrix} \quad (1.0.19)$$

$$= -4 \quad (1.0.20)$$

$$= \frac{1}{2} \text{Area of Parallelogram ABCD} \quad (1.0.21)$$

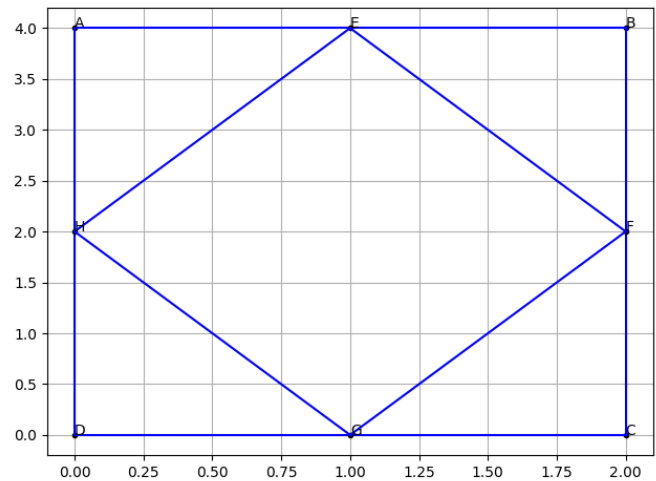


Fig. 0: Parallelogram according to the given vectors