ASSIGNMENT 12.10.5.13

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1 PROBLEM 1

1. The scalar product of the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ with a unit vector along the sum of vectors $\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$ is equal to one. Find the value of λ.

2 SOLUTION:

Let,

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} ; \mathbf{c} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} ; \mathbf{d} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

According to the question,

$$\frac{\mathbf{a}^\top (\mathbf{b} + \mathbf{c} + \lambda\mathbf{d})}{||\mathbf{b} + \mathbf{c} + \lambda\mathbf{d}||} = 1 \tag{2.0.2}$$

squaring both sides we get,

$$\left(\frac{\mathbf{a}^\top (\mathbf{b} + \mathbf{c} + \lambda\mathbf{d})}{||\mathbf{b} + \mathbf{c} + \lambda\mathbf{d}||}\right)^2 = 1 \tag{2.0.3}$$

$$(||\mathbf{b} + \mathbf{c} + \lambda\mathbf{d}||)^2 = \left(\left\|\begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix}\right\|\right)^2 \tag{2.0.4}$$

$$= (2+\lambda)^2 + 6^2 + 2^2 \tag{2.0.5}$$

$$= 2^2 + 2 \times 2 \times \lambda + \lambda^2 + 36 + 4 \tag{2.0.6}$$

$$= \lambda^2 + 4\lambda + 44 \tag{2.0.7}$$

$$\text{and, } (\mathbf{a}^\top (\mathbf{b} + \mathbf{c} + \lambda\mathbf{d}))^2 = \left(\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix}\right)^2 \tag{2.0.8}$$

$$= (2 + \lambda + 6 - 2)^2 \tag{2.0.9}$$

$$= (6+\lambda)^2 \tag{2.0.10}$$

$$= 36 + 12\lambda + \lambda^2 \tag{2.0.11}$$

Substituting 2.0.7 and 2.0.11 in 2.0.3, we get

$$\frac{\lambda^2 + 12\lambda + 36}{\lambda^2 + 4\lambda + 44} = 1 \tag{2.0.12}$$

$$\implies \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \tag{2.0.13}$$

$$\implies \lambda^2 - \lambda^2 + 12\lambda - 4\lambda = 44 - 36 \tag{2.0.14}$$

$$\implies 8\lambda = 8 \tag{2.0.15}$$

$$\implies \lambda = 1 \tag{2.0.16}$$