

ASSIGNMENT 12.11.3.6

Shristy Sharma (EE22BNITS11001)

1 PROBLEM 1

1. Find the equations of the planes that passes through three points.

$$1) \mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$$

$$2) \mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

2 SOLUTION FOR 1

Equation of plane is given by,

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.1)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{pmatrix}^T \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.2)$$

$$\Rightarrow \begin{pmatrix} 1 & 6 & -4 \\ 1 & 4 & -2 \\ -1 & -5 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2.0.3)$$

(2.0.4) Fig. 2: The figure shows that the given points are collinear

the augmented matrix is given by,

$$\left(\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 1 & 4 & -2 & 1 \\ -1 & -5 & 3 & 1 \end{array} \right) \quad (2.0.5)$$

$$\begin{array}{l} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \\ \xleftrightarrow{R_3 \leftarrow R_3 + R_1} \end{array} \quad (2.0.6)$$

$$\left(\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 1 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right) \quad (2.0.7)$$

$$\xleftrightarrow{R_3 \leftarrow 2R_3 + R_2} \quad (2.0.8)$$

$$\left(\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (2.0.9)$$

(2.0.10)

Since, all the elements in one of the row is 0. Therefore, there will be no solution. This signifies that the given points are collinear.

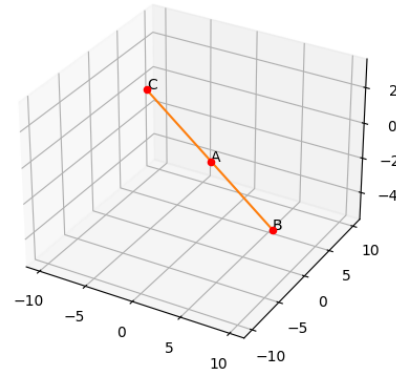
The direction vector of the line of collinear points will be,

$$\mathbf{m} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} 10 \\ 6 \\ -8 \end{pmatrix} \quad (2.0.11)$$

Since, the equation of a line is given by,

$$\mathbf{y} = \mathbf{m}\mathbf{x} + \mathbf{c} \quad (2.0.12)$$

$$\Rightarrow \mathbf{y} = \begin{pmatrix} 10 \\ 6 \\ -8 \end{pmatrix} \mathbf{x} + \mathbf{c} \quad (2.0.13)$$



3 SOLUTION FOR 2

Equation of plane is given by,

$$\mathbf{n}^T \mathbf{x} = c \quad (3.0.1)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{pmatrix}^T \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (3.0.2)$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & 2 \\ 0 & 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (3.0.3)$$

(3.0.4)

the augmented matrix is given by,

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 1 & 2 & -2 & 1 \\ 0 & 1 & -1 & 1 \end{array} \right) \quad (3.0.5)$$

$$\begin{array}{l} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \\ \xleftrightarrow{R_3 \leftarrow R_3 - R_2} \end{array} \quad (3.0.6)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -5 & 1 \end{array} \right) \quad (3.0.7)$$

$$\begin{array}{l} \xleftrightarrow{R_2 \leftarrow -5R_2 + 4R_3} \\ \xleftrightarrow{R_1 \leftarrow -5R_1 - 2R_3} \end{array} \quad (3.0.8)$$

$$\left(\begin{array}{ccc|c} 5 & 5 & 0 & 3 \\ 0 & 5 & 0 & 4 \\ 0 & 0 & -5 & 1 \end{array} \right) \quad (3.0.9)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \quad (3.0.10)$$

$$\left(\begin{array}{ccc|c} 5 & 0 & 0 & -1 \\ 0 & 5 & 0 & 4 \\ 0 & 0 & -5 & 1 \end{array} \right) \quad (3.0.11)$$

$$R_1 \leftarrow \frac{R_1}{5}; R_2 \leftarrow \frac{R_2}{5}; R_3 \leftarrow \frac{-R_3}{5} \quad (3.0.12)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{5} \\ 0 & 1 & 0 & \frac{4}{5} \\ 0 & 0 & 1 & -\frac{1}{5} \end{array} \right) \quad (3.0.13)$$

Now, the equation of the plane will be,

$$\begin{pmatrix} -\frac{1}{5} \\ \frac{4}{5} \\ -\frac{1}{5} \end{pmatrix}^T \mathbf{x} = 1 \quad (3.0.14)$$

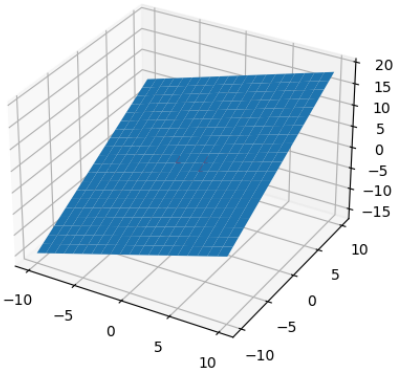


Fig. 2: Plane passing through the given points