ASSIGNMENT 12.11.3.6

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1 Problem 1

1. Find the equations of the planes that passes through three points.

1) $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$
2) $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$

2 Solution for 1

Equation of plane is given by,

$$\mathbf{n}^\top \mathbf{x} = c \tag{2.0.1}$$

$$\implies \begin{pmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{pmatrix}^\top \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.2}$$

$$\implies \begin{pmatrix} 1 & 6 & -4 \\ 1 & 4 & -2 \\ -1 & -5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2.0.3}$$

$$\tag{2.0.4}$$

the augmented matrix is given by,

$$\left(\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 1 & 4 & -2 & 1 \\ -1 & -5 & 3 & 1 \end{array}\right) \tag{2.0.5}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 + R_1]{R_2 \leftarrow R_2 - R_1} \tag{2.0.6}$$

$$\left(\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 1 & -2 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_3 \leftarrow 2R_3 + R_2} \tag{2.0.8}$$

$$\left(\begin{array}{ccc|c} 1 & 6 & -4 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.9}$$

$$\tag{2.0.10}$$

Since, all the elements in one of the row is 0. Therefore, there will be no solution. This signifies that the given points are collinear.

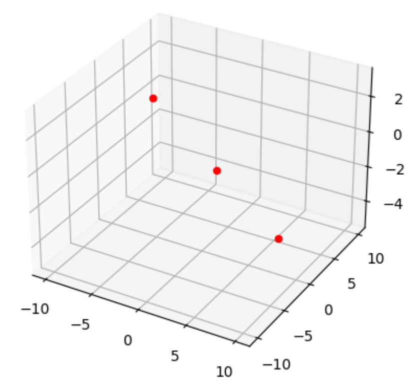

Fig. 2: The figure shows that the given points are collinear

3 Solution for 2

Equation of plane is given by,

$$\mathbf{n}^\top \mathbf{x} = c \tag{3.0.1}$$

$$\implies \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{pmatrix}^\top \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{3.0.2}$$

$$\implies \begin{pmatrix} 1 & 1 & -2 \\ 1 & 2 & 2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{3.0.3}$$

$$\tag{3.0.4}$$

the augmented matrix is given by,

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 1 & 2 & -2 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right) \tag{3.0.5}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_2]{R_2 \leftarrow R_2 - R_1} \tag{3.0.6}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -5 & 1 \end{array}\right) \tag{3.0.7}$$

$$\xleftrightarrow[R_1 \leftarrow 5R_1 - 2R_3]{R_2 \leftarrow 5R_2 + 4R_3} \tag{3.0.8}$$

$$\left(\begin{array}{ccc|c} 5 & 5 & 0 & 3 \\ 0 & 5 & 0 & 4 \\ 0 & 0 & -5 & 1 \end{array}\right) \tag{3.0.9}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \tag{3.0.10}$$

$$\left(\begin{array}{ccc|c} 5 & 0 & 0 & -1 \\ 0 & 5 & 0 & 4 \\ 0 & 0 & -5 & 1 \end{array}\right) \tag{3.0.11}$$

Now, the equation of the plane will be,

$$\begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \tag{3.0.12}$$

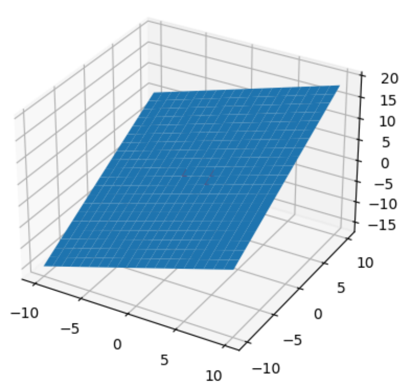

Fig. 2: Plane passing through the given points