

QUIZ 4

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1 PROBLEM 1

1. Find the slope of a line, which passes through the origin, and the mid-point of the line segment joining the points $P = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$ and $B = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$

SOLUTION:

$$\text{Given, } \mathbf{P} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

The mid-point is given by,

$$\mathbf{Q} = \frac{P+B}{2} = \frac{1}{2}\mathbf{P} + \frac{1}{2}\mathbf{B} = \frac{1}{2}\begin{pmatrix} 0 \\ -4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$\text{Therefore, } \mathbf{Q} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

The slope of the line is given by,

$$m = \mathbf{Q} - \mathbf{0} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

2 PROBLEM 2

2. Without using distance formula, show that points $\mathbf{A} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ are the vertices of a parallelogram.

SOLUTION:

$$\text{Given, } \mathbf{A} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \mathbf{D} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

To show that the given points are the vertices of a parallelogram:

$$1) \mathbf{A} - \mathbf{B} = \mathbf{D} - \mathbf{C}$$

$$2) \mathbf{A} - \mathbf{D} = \mathbf{B} - \mathbf{C}$$

So,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ -1 \end{pmatrix}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

Therefore, we can now say that the given points are the vertices of a parallelogram since both the sides of the quadrilateral are parallel to each other.