ASSIGNMENT

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1 PROBLEM 1

1.If $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$ are respectively the mid-points of the sides of a EFGH Parallelogram ABCD, show that area of Area of Parallelogram EFGH = $\frac{1}{2}$ Area of Parallelogram ABCD.
SOLUTION: Let,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.0.1}$$

$\therefore$ mid-points will be,

$$\mathbf{E} = \frac{A + B}{2} \tag{1.0.2}$$

$$= \frac{\begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{2} \tag{1.0.3}$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{1.0.4}$$

$$\mathbf{F} = \frac{B + C}{2} \tag{1.0.5}$$

$$= \frac{\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{2} \tag{1.0.6}$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{1.0.7}$$

$$\mathbf{G} = \frac{D + C}{2} \tag{1.0.8}$$

$$= \frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{2} \tag{1.0.9}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.0.10}$$

$$\mathbf{H} = \frac{D + A}{2} \tag{1.0.11}$$

$$= \frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}}{2} \tag{1.0.12}$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{1.0.13}$$

$$\tag{1.0.14}$$

Thus, Area of Parallelogram ABCD is given by,

$$= DA \times DC \tag{1.0.15}$$

$$= \begin{pmatrix} 0 \\ -4 \end{pmatrix} \times \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1.0.16}$$

$$= 0 - 8 \tag{1.0.17}$$

$$= -8 \tag{1.0.18}$$

Similarly, Area of Parallelogram EFGH will be given by,

$$= GH \times GF \tag{1.0.19}$$

$$= \begin{pmatrix} 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \end{pmatrix} \tag{1.0.20}$$

$$= -2 - 2 \tag{1.0.21}$$

$$= -4 \tag{1.0.22}$$

$$= \frac{1}{2}\text{Area of Parallelogram ABCD} \tag{1.0.23}$$

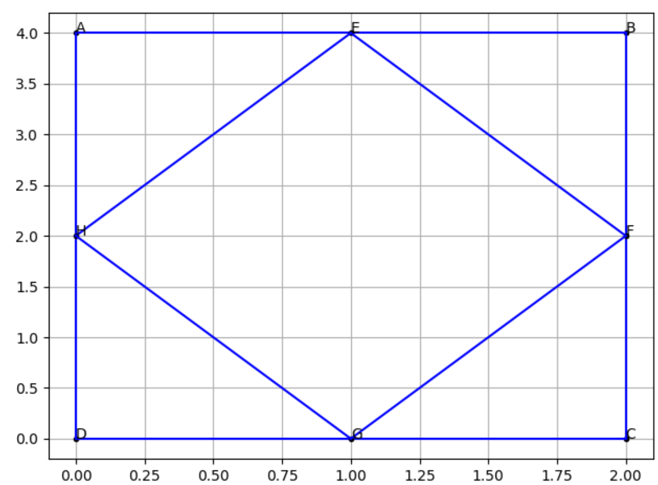

Fig. 0: Paralleogram according to the given vectors