ASSIGNEMNT 9.10.4.5

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1 Problem

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

2 Solution

Let Reshma, Salma and Mandip be standing at **A**, **B** and **C** respectively, where the center the of the circle **O**. Let

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{O} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.2}$$

Therefore, the equation of the cicle is given by

$$\|\mathbf{x} - \mathbf{O}\|^2 = 25 \tag{2.0.3}$$

$$\|\mathbf{x}\|^2 - 2\mathbf{O}^\top \mathbf{x} + \|\mathbf{O}\|^2 - 25 = 0 \tag{2.0.4}$$

$$\|\mathbf{x}\|^2 - 2\begin{pmatrix} 5 & 0 \end{pmatrix}\mathbf{x} = 0 \tag{2.0.5}$$

Since, **A** and **C** are equidistant (6m) from **B**, we can say that they lie on the circle having **B** as center and radius 6m. Equation of this circleis given by,

$$\|\mathbf{x}\|^2 - 2\mathbf{B}^\top \mathbf{x} + \|\mathbf{B}\|^2 - 36 = 0 \tag{2.0.6}$$

$$\|\mathbf{x}\|^2 = 36 \tag{2.0.7}$$

$$\implies \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and } f = -36 \tag{2.0.8}$$

line passing through **A** and **C** from (2.0.5) and (2.0.8),

$$\begin{pmatrix} 5 & 0 \end{pmatrix}\mathbf{x} = 18 \tag{2.0.9}$$

$$\mathbf{x} = \begin{pmatrix} \frac{18}{5} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{h} = \begin{pmatrix} \frac{18}{5} \\ 0 \end{pmatrix} \text{ and } \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.11}$$

For circle,$\mathbf{V} = \mathbf{I}$

$$\mu_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{(\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}))^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})}\right) \tag{2.0.12}$$

where,

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \tag{2.0.13}$$

here,

$$\mu_i = \pm\frac{24}{5} \tag{2.0.14}$$

Therefore,

$$\mathbf{A} = \begin{pmatrix} \frac{18}{5} \\ \frac{24}{5} \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{C} = \begin{pmatrix} \frac{18}{5} \\ -\frac{24}{5} \end{pmatrix} \tag{2.0.16}$$

Therefore,the distance between Reshma and Mandip will be,

$$\|\mathbf{A} - \mathbf{C}\| = \left\|\begin{pmatrix} 0 \\ \frac{48}{5} \end{pmatrix}\right\| \tag{2.0.17}$$

$$= \frac{48}{5} \tag{2.0.18}$$

$$= 9.6 \tag{2.0.19}$$

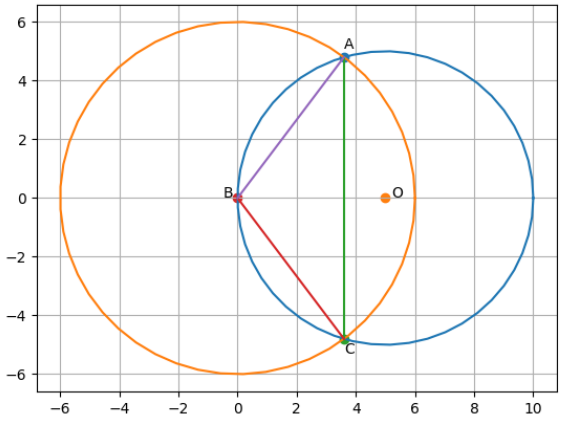

Fig. 0: Figure 1