QUIZ 4

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1 PROBLEM 1

1. The scalar product of the vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$ with a unit vector along the sum of vectors $\begin{pmatrix}2\\4\\-5\end{pmatrix}$ and $\begin{pmatrix}\lambda\\2\\3\end{pmatrix}$ is equal to one. Find the value of λ.

2 SOLUTION:

Let,

$$\mathbf{a}=\begin{pmatrix}1\\1\\1\end{pmatrix}\ ;\mathbf{b}=\begin{pmatrix}2\\4\\-5\end{pmatrix}\ ;\mathbf{c}=\begin{pmatrix}\lambda\\2\\3\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b}+\mathbf{c}=\begin{pmatrix}2\\4\\-5\end{pmatrix}+\begin{pmatrix}\lambda\\2\\3\end{pmatrix} \tag{2.0.2}$$

$$=\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix} \tag{2.0.3}$$

$$\text{and, } \|\mathbf{b}+\mathbf{c}\|=\sqrt{(2+\lambda)^2+6^2+2^2} \tag{2.0.4}$$

$$=\sqrt{(2^2+2\times2\times\lambda+\lambda^2)+36+4} \tag{2.0.5}$$

$$=\sqrt{\lambda^2+4\lambda+44} \tag{2.0.6}$$

Here, unit vector along $\mathbf{b}+\mathbf{c}$ is given by,

$$\mathbf{v}=\frac{\mathbf{b}+\mathbf{c}}{\|\mathbf{b}+\mathbf{c}\|} \tag{2.0.7}$$

$$=\frac{\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}}{\sqrt{\lambda^2+4\lambda+44}} \tag{2.0.8}$$

It is given that,

$$\mathbf{a}.\mathbf{v}=1 \tag{2.0.9}$$

$$\implies \begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}\frac{2+\lambda}{\sqrt{\lambda^2+4\lambda+44}}\\\frac{6}{\sqrt{\lambda^2+4\lambda+44}}\\\frac{-2}{\sqrt{\lambda^2+4\lambda+44}}\end{pmatrix}=1 \tag{2.0.10}$$

$$\implies \frac{2+\lambda+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1 \tag{2.0.11}$$

$$\implies 2+\lambda+6-2=\sqrt{\lambda^2+4\lambda+44} \tag{2.0.12}$$

$$\implies \lambda+6=\sqrt{\lambda^2+4\lambda+44} \tag{2.0.13}$$

squaring both sides, we get,

$$(\lambda+6)^2=\lambda^2+4\lambda+44 \tag{2.0.14}$$

$$\lambda^2+12\lambda+36=\lambda^2+4\lambda+44 \tag{2.0.15}$$

$$8\lambda=8 \tag{2.0.16}$$

$$\lambda=1 \tag{2.0.17}$$