# ASSIGNMENT 12.11.3.6

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## 1 PROBLEM 1

1. Find the equations of the planes that passes through three points.

1) $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix}$

2) $\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$

## 2 Solution for 1

Vector equation is given by,

$$(\mathbf{r} - \mathbf{A})^{\top} (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = 0 \tag{2.0.1}$$

$$\left(\mathbf{r} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\right)^{\top} \left(\begin{pmatrix} 6 \\ 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\right) \times \left(\begin{pmatrix} -4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\right) = 0 \tag{2.0.2}$$

$$\begin{pmatrix} \mathbf{r} - 1 & \mathbf{r} - 1 & \mathbf{r} + 1 \end{pmatrix} \left(\begin{pmatrix} 5 \\ 3 \\ -4 \end{pmatrix} \times \begin{pmatrix} -5 \\ -3 \\ 4 \end{pmatrix}\right) = 0 \tag{2.0.3}$$

$$\begin{pmatrix} \mathbf{r} - 1 & \mathbf{r} - 1 & \mathbf{r} + 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \tag{2.0.4}$$

$\implies$ The three points are collinear

Since, the above equation is satisfied for all values of $\mathbf{r}$.

$\therefore$ There will be infinite planes passing through the given 3 collinear points.

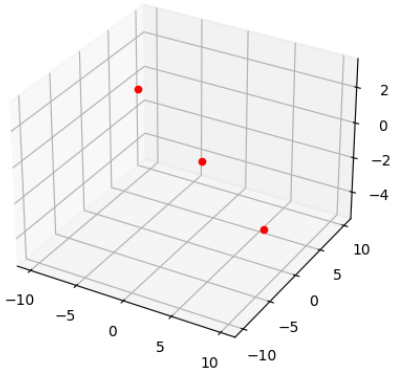


Fig. 2: The figure shows that the given points are collinear

## 3 Solution for 2

Vector equation is given by,

$$(\mathbf{r} - \mathbf{A})^{\top} (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = 0 \tag{3.0.1}$$

$$\left(\mathbf{r} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right)^{\top} \left(\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right) \times \left(\begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right) = 0 \tag{3.0.2}$$

$$\begin{pmatrix} \mathbf{r} - 1 & \mathbf{r} - 1 & \mathbf{r} \end{pmatrix} \left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}\right) = 0 \tag{3.0.3}$$

$$\begin{pmatrix} \mathbf{r} - 1 & \mathbf{r} - 1 & \mathbf{r} + 1 \end{pmatrix} \begin{pmatrix} -2 \\ -3 \\ 3 \end{pmatrix} = 0 \tag{3.0.4}$$

This is the Cartesian equation of the required plane.

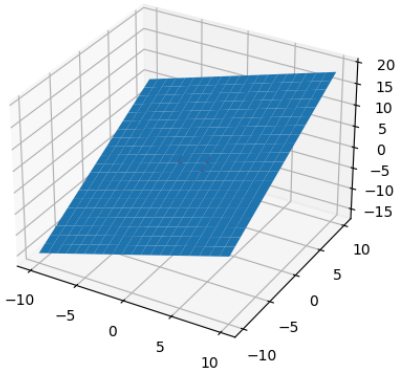


Fig. 2: Plane passing through the given points