ASSIGNMENT

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1 PROBLEM

1.If $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}$ are respectively the mid-points of the sides of a Parallelogram ABCD, show that area of Area of Parallelogram EFGH = $\frac{1}{2}$ Area of Parallelogram ABCD.
SOLUTION: Let,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.0.1}$$

$\therefore$ mid-points will be,

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{1.0.2}$$

$$= \frac{\begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix}}{2} \tag{1.0.3}$$

$$= \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{1.0.4}$$

$$\mathbf{F} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{1.0.5}$$

$$= \frac{\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{2} \tag{1.0.6}$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{1.0.7}$$

$$\mathbf{G} = \frac{\mathbf{D} + \mathbf{C}}{2} \tag{1.0.8}$$

$$= \frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}}{2} \tag{1.0.9}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.0.10}$$

$$\mathbf{H} = \frac{\mathbf{D} + \mathbf{A}}{2} \tag{1.0.11}$$

$$= \frac{\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}}{2} \tag{1.0.12}$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{1.0.13}$$

$$\tag{1.0.14}$$

Thus, Area of Parallelogram ABCD is given by,

$$= (\mathbf{D} - \mathbf{A}) \times (\mathbf{D} - \mathbf{C}) \tag{1.0.15}$$

$$= \begin{vmatrix} 0 & -2 \\ -4 & 0 \end{vmatrix} \tag{1.0.16}$$

$$= -8 \tag{1.0.17}$$

And, Area of Parallelogram EFGH is given by,

$$= (\mathbf{G} - \mathbf{H}) \times (\mathbf{G} - \mathbf{F}) \tag{1.0.18}$$

$$= \begin{vmatrix} 1 & -1 \\ -2 & -2 \end{vmatrix} \tag{1.0.19}$$

$$= -4 \tag{1.0.20}$$

$$= \frac{1}{2}\text{Area of Parallelogram ABCD} \tag{1.0.21}$$

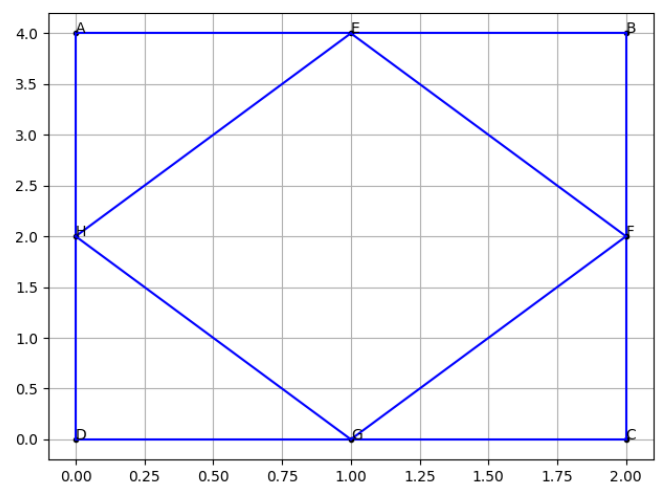

Fig. 0: Paralleogram according to the given vectors