ASSIGNMENT 12.10.5.13

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1 PROBLEM 1

1. The scalar product of the vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$ with a unit vector along the sum of vectors $\begin{pmatrix}2\\4\\-5\end{pmatrix}$ and $\begin{pmatrix}\lambda\\2\\3\end{pmatrix}$ is equal to one. Find the value of λ.

2 SOLUTION:

Let,

$$\mathbf{a}=\begin{pmatrix}1\\1\\1\end{pmatrix};\mathbf{b}=\begin{pmatrix}2\\4\\-5\end{pmatrix};\mathbf{c}=\begin{pmatrix}0\\2\\3\end{pmatrix};\mathbf{d}=\begin{pmatrix}1\\0\\0\end{pmatrix} \tag{2.0.1}$$

(2.0.2)

According to the question,

$$\frac{\mathbf{a}^{\top}(\mathbf{b}+\mathbf{c}+\lambda\mathbf{d})}{\|\mathbf{b}+\mathbf{c}+\lambda\mathbf{d}\|}=1 \tag{2.0.3}$$

$$\|\mathbf{b}+\mathbf{c}+\lambda\mathbf{d}\|=\left\|\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}\right\| \tag{2.0.4}$$

$$=\sqrt{(2+\lambda)^2+6^2+2^2} \tag{2.0.5}$$

$$=\sqrt{(2^2+2\times2\times\lambda+\lambda^2)+36+4} \tag{2.0.6}$$

$$=\sqrt{\lambda^2+4\lambda+44} \tag{2.0.7}$$

$$\text{and, }\mathbf{a}^{\top}(\mathbf{b}+\mathbf{c}+\lambda\mathbf{d})=\begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix} \tag{2.0.8}$$

$$=2+\lambda+6-2 \tag{2.0.9}$$

$$=6+\lambda \tag{2.0.10}$$

Substituting 2.0.7 and 2.0.10 in 2.0.3, we get

$$\frac{\lambda+6}{\sqrt{\lambda^2+4\lambda+44}}=1 \tag{2.0.11}$$

$$\implies \lambda+6=\sqrt{\lambda^2+4\lambda+44} \tag{2.0.12}$$

squaring both sides, we get,

$$\implies (\lambda+6)^2=\lambda^2+4\lambda+44 \tag{2.0.13}$$

$$\implies \lambda^2+12\lambda+36=\lambda^2+4\lambda+44 \tag{2.0.14}$$

$$\implies 8\lambda=8 \tag{2.0.15}$$

$$\implies \lambda=1 \tag{2.0.16}$$