Assignment 9.10.5.11

Shristy Sharma (EE22BNITS11001)

1 Problem

ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

2 Solution

We will use vectors to prove that $\angle CAD = \angle CBD$. Let's assume that A, B, C, and D are the vertices of the two right triangles.
We can represent the position vectors of points A, B, C, and D as $\mathbf{a}, \mathbf{b}, \mathbf{c}, and \mathbf{d}$, respectively. Then we have:

$$AB = \mathbf{b} - \mathbf{a} \tag{2.0.1}$$
$$AC = \mathbf{c} - \mathbf{a} \tag{2.0.2}$$
$$CD = \mathbf{d} - \mathbf{c} \tag{2.0.3}$$

Since ABC and ADC are right triangles, we know that:

$$AB \cdot AC = 0 \tag{2.0.4}$$
$$CD \cdot AC = 0 \tag{2.0.5}$$

substituting 2.0.1 and 2.0.3 in 2.0.5

$$CD \cdot AC = 0 \tag{2.0.6}$$
$$(\mathbf{d} - \mathbf{c}) \cdot AC = 0 \tag{2.0.7}$$
$$\mathbf{d} \cdot AC - \mathbf{c} \cdot AC = 0 \tag{2.0.8}$$

Since AC is non-zero, we can divide both sides by $||\mathbf{AC}||^2$ to get:

$$\left(\frac{\mathbf{d} \cdot AC}{||AC||^2}\right) - \left(\frac{\mathbf{c} \cdot AC}{||AC||^2}\right) = 0 \tag{2.0.9}$$

But $\frac{\mathbf{d} \cdot AC}{||AC||^2} = \cos \angle CAD$ and $\frac{\mathbf{c} \cdot AC}{||AC||^2} = \cos \angle CBD$. Therefore, we have:

$$\cos \angle CAD - \cos \angle CBD = 0 \tag{2.0.10}$$
$$\implies \angle CAD = \angle CBD \tag{2.0.11}$$

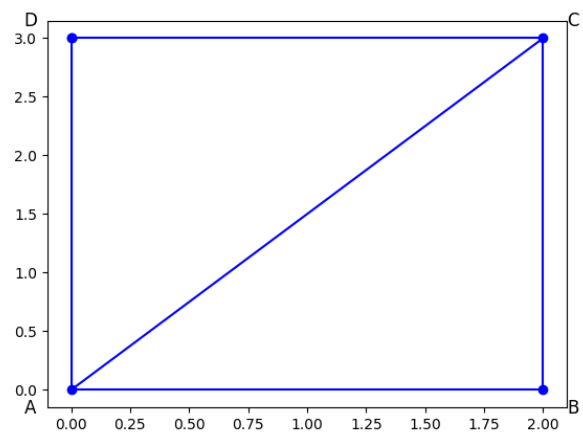

Fig. 0: Plot of $ABCD$