Optimal Control on a Plug-in Hybrid Vehicle

by

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Abstract

This report presents an attempt to apply the Discrete time Pontryagin Maximum Principle (DPMP) to the objective of minimizing fuel consumption in the Chevrolet Volt, a plug-in hybrid vehicle. First, a general introduction to optimal control is given and the Pontryagin's Maximum Principle (PMP) is stated. After this, a brief introduction to convex sets and the method of tents along with the DPMP is presented. Then, a description of the plug-in hybrid vehicle (PHEV) along with its 4 different modes of operation and the system dynamics for each mode is explained. Lastly, three double integrator system problems with different kinds of state constraints are solved using the DPMP and the results are compared with the solution generated using a standard optimization solver (CasAdi, [1]) to the optimization objective. We further plan to use the same algorithm to solve the hybrid vehicle problem that we have studied in detail. Our work differs from previous attempts in the following sense: We discretize the differential equations governing the battery charge and the dynamics of the vehicle upfront, and then incorporate all inequality constraints to obtain the necessary conditions for optimality. Constrained optimization of this type are, in general, difficult to solve, and the DPMP is one of the few tools that incorporates all the complexities of the problem.

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Chapter 1

Optimal Control and Pontryagin's Maximum Principle

1.1 The Optimal Control Problem

Much of this chapter is taken from [2].

The optimal control problem can be stated as finding a control input for a particular control system that minimizes the cost functional over all admissible controls:

$$\min_u J(u) := \int_{t_0}^{t_f} L(t, x(t), u(t))dt + K(t_f, x_f) \quad \text{subject to} \quad \begin{cases} \dot{x} = f(t, x, u) \\ x(t_0) = x_0 \\ x \in R^n \\ u \in U \subset R^m \end{cases} \tag{1.1.1}$$

Where x is the state taking values in R^n, u is the control input taking values in some control set $U \in R^m$, t is time, t_0 is the initial time, and x_0 is the initial state. A system is well-posed when for every choice of the initial data (t_0, x_0) and every admissible control $u(t)$, the system (2.2) has a unique solution $x(t)$ on some time interval $[t_0, t_1]$. To ensure this, we need to impose some regularity conditions on the right-hand side f and on the admissible controls u: f is continuous in t and u and $C1$ in x; f_x is continuous in t and u; and $u(t)$ is piecewise continuous as a function of t [2].

For a given initial data (t_0, x_0), the behaviors of the system are parameterized by control function u. The cost functional J assigns a cost value to each possible behaviour, and thus to each admissible control. Thus, L and K are given functions (running cost and terminal cost, respectively), t_f is the final (or terminal) time which is either free or fixed, and $x_f := x(t_f)$ is the final (or terminal) state which is either free or fixed or belongs to some given target set. The objective is to find a control u that minimizes $J(u)$ over all admissible controls (or at least over nearby controls). The set in which the controls u take values may be constrained by some practical considerations, which would make it difficult to solve the above problem using calculus of variations. In the optimal control formulation, such constraints are incorporated very naturally by working with an appropriate control set.

1.2 The Maximum Principle

The PMP is used to solve such optimal control problems by providing necessary conditions for optimality for a certain problem.

The basic fixed-endpoint problem is described by the optimal control problem described above with a few additional specifications: the dynamics of the system and the lagrangian or running cost are both time-independent, both the system dynamics and lagrangian satisfy stronger regularity conditions (f, f_x, L, L_x are continuous), and (x_f, t_f) belong to the target set $S = [t_0, \text{inf}] \times \{x_1\}$ where x_1 is a fixed point in R^n. The PMP [2] for a basic fixed-endpoint problem is stated below:

Theorem 1. *Let $u^*(t) : [t_0, t_f] \to U$ be an optimal control (in the global sense) and let $x^*(t) : [t_0, t_f] \to R^n$ be the corresponding optimal state trajectory. Then there exists a function $p^*(t) : [t_0, t_f] \to R^n$ and a constant $p_0^* \leq 0$ satisfying $(p_0^*, p^*(t)) \neq (0, 0)$ for all $t \in [t_0, t_f]$ and having the following properties:*

1. *x^* and p^* satisfy the canonical equations:*

$$\dot{x^*} = H_p(x^*, u^*, p^*, p_0^*) \tag{1.2.1}$$
$$\dot{p^*} = H_x(x^*, u^*, p^*, p_0^*) \tag{1.2.2}$$

   with the boundary conditions $x^*(t_0) = x_0$ and $x^*(t_f) = x_1$, where the Hamiltonian $H : R^n \times U \times R^n \times R \to R$ is defined as

$$H(x, u, p, p_0) := \langle p, f(x, u)\rangle + p_0 L(x, u) \tag{1.2.3}$$

2. *For each fixed t, the function $u \to H(x^*(t), u, p^*(t), p_0^*)$ has a global maximum at $u = u^*(t)$, i.e.,*

$$H(x^*(t), u^*(t), p^*(t), p_0^*) \geq H(x^*(t), u, p^*(t), p_0^*)\ \forall\ t \in [t_0, t_f],\ \forall\ u \in U \tag{1.2.4}$$

3.
$$H(x^*(t), u^*(t), p^*(t), p_0^*) = 0\ \forall\ t \in [t_0, t_f] \tag{1.2.5}$$

A more general problem would be the variable-endpoint problem, and our fuel minimization problem for a PHEV can be defined as such a problem. The difference in this problem is that the boundary condition for the final state x_f changes from a fixed point in R^n to a surface in R^n. Here the target set for (x_f, t_f) is not defined as a fixed point in R^n, and is instead defined as $S = [t_0, \inf] \times S_1$, where S_1 is a k-dimensional surface in R^n for some nonnegative integer $k \leq n$. The PMP [2] for a basic variable-endpoint problem is stated below:

Theorem 2. *Let $u^*(t) : [t_0, t_f] \to U$ be an optimal control (in the global sense) and let $x^*(t) : [t_0, t_f] \to R^n$ be the corresponding optimal state trajectory. Then there exists a function $p^*(t) : [t_0, t_f] \to R^n$ and a constant $p_0^* \leq 0$ satisfying $(p_0^*, p^*(t)) \neq (0, 0)$ for all $t \in [t_0, t_f]$ and having the following properties:*

1. *x^* and p^* satisfy the canonical equations:*

$$\dot{x^*} = H_p(x^*, u^*, p^*, p_0^*) \tag{1.2.6}$$

$$\dot{p^*} = H_x(x^*, u^*, p^*, p_0^*) \tag{1.2.7}$$

   with the boundary conditions $x^*(t_0) = x_0$ and $x^*(t_f) \in S_1$, where the Hamiltonian $H : R^n \times U \times R^n \times R \to R$ is defined as

$$H(x, u, p, p_0) := \langle p, f(x, u)\rangle + p_0 L(x, u) \tag{1.2.8}$$

2. *For each fixed t, the function $u \to H(x^*(t), u, p^*(t), p_0^*)$ has a global maximum at $u = u^*(t)$, i.e.,*

$$H(x^*(t), u^*(t), p^*(t), p_0^*) \geq H(x^*(t), u, p^*(t), p_0^*) \ \forall\, t \in [t_0, t_f],\ \forall\, u \in U \tag{1.2.9}$$

3.

$$H(x^*(t), u^*(t), p^*(t), p_0^*) = 0 \ \forall\, t \in [t_0, t_f] \tag{1.2.10}$$

4. *The vector $p^*(t_f)$ is orthogonal to the tangent space to S_1 at $x^*(t_f)$:*

$$\langle p^*(t_f), d\rangle = 0 \ \forall\, d \in T_{x^*(t_f)} S_1 \tag{1.2.11}$$

   This is called the transversality condition and is equivalent to stating that $p^*(t_f)$ is a linear combination of the gradient vectors at $x^*(t_f)$ of the equality conditions forming the surface S_1.

The maximum principle provides necessary conditions for optimality, and it thus helps us single out optimal control candidates which need to be further analyzed to determine if they are optimal. In many cases the conditions provided by the PMP are strong enough to reach the optimal solution. It is also possible that for some problems, the optimal control may not exist.

Chapter 2

Discrete Time PMP and Multiple Shooting

2.1 Introduction

Pontryagin's maximum principle in continuous time attracted researchers to obtain similar conditions for discrete time optimal control. The article [3] presents the necessary conditions for an optimal control problem in discrete time using method of tents, used for solving extremal problems. In this chapter a brief introduction to method of tents will be given, the necessary conditions arising from the DPMP will be stated, and the multiple shooting algorithm that uses the DPMP to give a solution to optimal control problems in discrete time will be explained.

2.2 Optimal Control in Discrete Time and Theory of Tents

In the discrete setting, the classical optimal control problem is of the form

$$\min\,(u_t)_{t=0}^{N-1} \ : \ \sum_{t=0}^{N-1} c_t(x_t, u_t) \quad \text{subject to} \quad \begin{cases} x_{t+1} = f_t(x, u) \\ x_t \in M_t \subset R^d \ \forall \ t = 0, ..., N-1 \\ u_t \in U_t \subset R^m \ \forall \ t = 0, ..., N-1 \end{cases} \tag{2.2.1}$$

where, $c_t : R^d \times R^m \to R \forall t = 0, ..., N-1$, are smooth functions that accounts for cost per stage,
$f_t : R^d \times R^m \to Rd \forall t = 0, ..., N-1$, are smooth functions that govern system dynamics,
M_t is a set of admissible state trajectories at time t, $\forall t = 0, ..., N$,
U_t is a set of admissible control inputs at time t, $\forall t = 0, ..., N-1$.

Thus, the objective is to obtain sequence of control actions $(u_t)_{t=0}^{N-1}$ that minimizes the total cost (eq 2.2.1), satisfying system dynamics, state and input constraints. We can formulate this optimal control problem as an optimization problem.

Let us define a new variable

$$z := (x_0, ..., x_N, u_0, ..., u_{N1}) \in R^k, \ where\ k = N \times (d+m) + d \tag{2.2.2}$$

Let us define projection map on $R^k \ni z = (z_0^x, z_1^x, ..., z_N^x, z_0^u, ..., z_{N1}^u)$ *on* R^d *and* R^m

$$\pi_t^x : R^k \to R^d \ such\ that\ R^k \ni \eta \to \pi_t^x(\eta) = \eta_t^x \in R^d, \ for\ t = 0, ..., N \tag{2.2.3}$$

$$\pi_t^u : R^k \to R^m \ such\ that\ R^k \ni \eta \to \pi_t^u(\eta) = \eta_t^u \in R^m, \ for\ t = 0, ..., N-1 \tag{2.2.4}$$

the cost function and constraints can now be written as

$$C(z) = \sum_{t=0}^{N-1} c_t(\pi_t^x(z), \pi_t^u(z)) \tag{2.2.5}$$

$$\Sigma = (\bigcap_{t=0}^{N-1} \Omega_t) \bigcap (\bigcap_{t=0}^{N} \Omega_t^*) \bigcap (\bigcap_{t=0}^{N-1} \Xi_t) \tag{2.2.6}$$

where,

$$\Omega_t := \{z \in R^k \mid F_t(z) = 0\}, \ F_t(z) := -\pi_{t+1}^x(z) + f_t(\pi_t^x(z), \pi_t^u(z)) \tag{2.2.7}$$

$$\Omega_t^* := \{z \in R^k \mid \pi_t^x(z) \in M_t\} \tag{2.2.8}$$

$$\Xi_t := \{z \in R^k \mid \pi_t^u(z) \in U_t\} \tag{2.2.9}$$

Now, the main optimal control problem can be written as an optimization problem given by

$$\min_z C(z) \quad \text{subject to} \quad z \in \Sigma \tag{2.2.10}$$

Assume for the above optimization problem the solution exists and let $z^* \in argmin\ C(z)$, then the following proposition gives the necessary and sufficient condition for optimality of z^*:

$$z^* \in argmin\ C(z) \iff \Sigma \cap \Omega_c = z^*, \ \ where,\ \Omega_c := \{z \in R^k | C(z^*) > C(z)\} \cup \{z^*\} \tag{2.2.11}$$

Thus, the optimal control problem reduces to finding the condition under which the above intersection holds. The basic idea behind the theory of tents is to closely approximate the sets $(\Omega_t)_{t=0}^{N-1}$, $(\Omega_t^*)_{t=0}^{N}$, $(\Xi_t)_{t=0}^{N-1}$ by the convex cones or tangent cones called as tents. Once the convex cones(tents) at z^* are obtained for each set, then geometric necessary condition is obtained in terms of these tents.

Some basic definitions will now be discussed.

- Affine Set: Set S is affine if and only if the line passing through any two distinct points of a set is in the set. Every affine set takes the form $S = x \in R^n | Ax = b$ for some matrix A and vector b.

- Convex Set: Set S is a convex set if and only if for every $x, y \in S, ax+(1-a)y \in S$, where $a \in [0, 1]$

- Cone: $S \subset R^n$, is a cone with vertex x_0 iff $\lambda(x - x_0) \in S \ \forall \ x \in S$ and $\lambda \geq 0$.

- Dual Cone: If set S is a cone with vertex x_0, then $D(S) = \{y \in R^n \mid y^T(x - x_0) \geq 0\}$ is the dual of S.

Definition 2.2.1. *Let Ω be a subset of R^k and $z_0 \in \Omega \subset R^k$ be the convex cone with vertex at z_0. Then Q is a tent of Ω at z_0 if there is a smooth map defined in the neighbourhood of z_0 such that*

$$\phi(z) = z + o(z - z_0) \tag{2.2.12}$$

$$\phi(z) \in \Omega \ for \ z \in Q \cap B_\epsilon(z_0) \tag{2.2.13}$$

where, $B_\epsilon(z_0)$ is a ball of ϵ length with centre at z_0.

The set Ω_c (from 2.2.11) near z^* is closely approximated by the half-space K_c that consists of those points z for which the inner product $\left\langle \frac{\partial C(z^*)}{\partial z}, z - z^* \right\rangle$ (from 2.2.10) is non-positive. Here, K_c is a convex cone with vertex z^*. If $\frac{\partial F_i(z^*)}{\partial z} \neq 0$ (from 2.2.7), then the set Ω_i is "closely approximated" near z^* by the convex cone $K_i = z \in R^k \mid \left\langle \frac{\partial F_i(z^*)}{\partial z}, z - z^* \right\rangle = 0$.

Thus, for every i = 0, 1, . . . ,N 1 the sets Ω_i, Ω_c has has tangent convex cone or tent. Each of the tents $K_0, K_1, ..., K_{N1}$ is a plane (of dimension $K1$), whereas K_c is not a plane.
The geometric necessary condition for optimality is defined for the tents using the concept of separability. A family of convex cones $K_0, K_1, ..., K_s$ in R^d with a common vertex x^* is separable if and only if there are vectors not all of them zero, that satisfy the condition:

$$a_i \in D(K_i), \ for \ i \ = \ 1, 2, ..., s \tag{2.2.14}$$

where, $D(K_i) = \{y \in R^d \mid \langle y, (x - x^*) \rangle \geq 0\}$ is a dual cone of K_i, and

$$a_0 \ + \ a_1 ... \ + \ a_s = 0 \tag{2.2.15}$$

The geometric necessary condition for optimality can now be stated as:

Theorem 3. *Let* $\Omega_1, ..., \Omega_s,\ \Xi_1, ..., \Xi_l$ *be subsets of* R^d *and let* $c(x)$ *be a smooth function whose domain contains the set* $\Sigma = \Omega_1 \cap ... \cap \Omega_s \cap \Xi_1 ... \Xi_l$*. Further, let* $x^* \in \Sigma$ *and let* K_i *(or* Q_j*) be a local tent of* Ω_i *(or* Ξ_j*) at* x^* *for* $i = 1, ..., s$, $j = 1, ..., l$*. Suppose that the family of cones* $Q_1, ..., Q_l$ *is not separable in* R^d*. For* $c(x)$ *to attain its minimum at* x^* *relative to* Σ*, it is necessary that there are vectors* $a_1 \in D(K1), ..., a_s \in D(K_s)$ *and a number* ϕ_0 *such that:*

1.
$$\phi_0 \leq 0,\ and\ if\ \phi_0 = 0 \tag{2.2.16}$$
then atleast one of the vectors $a_1, ..., a_s$ *is not zero, and*

2.
$$\left\langle (\phi_0(\frac{\partial c(x^*)}{\partial x}) + a_1 + ... + a_s), \delta x \right\rangle \leq 0\ \forall \delta x\ s.t. \tag{2.2.17}$$
$$x^* + \delta x \in Q_1 \cap ... \cap Q_l \tag{2.2.18}$$

The discrete Pontryagin's Maximum Principle is thus stated below:

Theorem 4. *Given discrete time control system and cost function as in (2.2.1). Let* $(u_t^*)_{t=0}^{N-1}$ *be the sequence optimal control and* $(x_t^*)_{t=0}^{N}$ *be the sequence of corresponding optimal trajectories. Further, let* Q_t *be the tent of* U_t *at* u_t^* *for* $t = 0, ..., N-1$ *and* K_t *be the tent of* M_t *at* x_t^* *for* $t = 0, ..., N$*. We define the Hamiltonian:*

$$R^d \times R \times R^d \times R^m \ni (p, s, z, v) \to H^\eta(p, s, z, v) := \langle p, f_s(z, v) \rangle + \eta c(z, v) \tag{2.2.19}$$

then, there exists a constant $\eta \leq 0$ *a sequence of vectors* $(\phi_t^*)_{t=0}^{N-1} \subset R^d$, $(a_t^*)_{t=0}^{N} \subset (D(K_t))_{t=0}^{N}$ *not all zero for* $\eta = 0$*, such that*

$$\phi_{t-1} = a_t + \frac{\partial H^\eta(p, s, z, v)}{\partial x}\ for\ t = 0, ..., N-1,\ with\ H_N = 0\ and \phi_{-1} = 0,\ and \tag{2.2.20}$$
$$H^\eta(\phi_t, t, x_t^*, u_t^*) = \max_v H^\eta(\phi_t, t, x_t^*, v)\ for\ t = 0, ..., N-1 \tag{2.2.21}$$

2.3 The Multiple Shooting Algorithm

Shooting methods were mainly developed for solving ordinary differential equations or difference equations with given boundary conditions. An initial guess is taken for the unknown initial values of the differential or difference equation variables. Then, the variables are computed at the terminal time and compared with a known value of the variables at the boundaries. Then, the initial guess is improved at each iteration to match the known boundary values. In multiple shooting methods, the time domain is divided into subintervals (time domain decomposition), and the boundary value problems are solved for each subinterval with the condition that the boundary values at the common points of the adjacent intervals are the same. Multiple shooting method is a generalization of the single shooting method in which the two point boundary value problem is solved at each iteration for the subintervals of time domain simultaneously. Let:

$$\dot{x} = f(x,t) \quad x \in R^n \quad \text{with boundary conditions,} \tag{2.3.1}$$

$$B : R^n \times R^n \rightarrow R^n \quad \text{such that} \quad B(x(a), x(b)) = 0 \tag{2.3.2}$$

Let the time domain be decomposed into N subintervals as follows:

$$t_f = \tau_N \geqslant ... \geqslant \tau_0 = t_0 \tag{2.3.3}$$

and let us consider $(N+1)$ variables $s_0, s_1, ..., s_n$ known as the multiple shooting variables. These multiple shooting variables are the guessed initial values of the dependent variable x defined in (2.3.1) at the specified time instants. Now we define initial value problem for each sub-interval as follows:

$$\dot{x^k} = f(x^k, t) \quad such\ that\ x^k : [\tau_k, \tau_{k+1}] \rightarrow R^n\ with \tag{2.3.4}$$

$$X^k(\tau_k) = s^k \quad for\ k = 0, 1, ..., (N-1) \tag{2.3.5}$$

Now, we can notice that the solution $x^k(\dot{,} s^k) : \ k = 0, 1, ..., (N-1)$ to the initial value problems can be a solution to the boundary value problem (2.3.1) only if the solution x^k of the interval $[\tau_k; \tau_{k+1}]$ matches with the initial condition for the next interval i.e. $x^k(\tau_{k+1}, s^k) = s^{k+1}$. This condition is known as the matching condition. These matching conditions for each interval can be combined together and can be represented in the mathematical form as follows:

$$F(s) := \begin{bmatrix} s^1 - x^0(\tau_1, s^0) \\ s^2 - x^1(\tau_2, s^1) \\ \cdot \\ \cdot \\ \cdot \\ s^N - x^{N-1}(\tau_N, s^{N-1}) \\ B(x(a), x(b)) \end{bmatrix} = 0 \tag{2.3.6}$$

with $x^k(\dot{,} s^k)$ be the solution of IVP and $s := (s^0, s^1, ..., s^N)$. The system of algebraic equations (2.3.6) can be solved using Newton Raphson algorithm for multivariable functions. Newton's iterates can be defined as follows:
Let $s_0 \rightarrow R^n$ be the initial guess, then

$$update: \ s_{m+1} = s_m + \Delta s_m \quad for\ m = 0, 1, 2... \tag{2.3.7}$$

where, Δs_m is the solution of the system $DF(s_m)\Delta s_m = -F(s_m)$. Terminate the algorithm once the difference between the two consecutive updates is less than the pre-specified tolerance.

Chapter 3

The PHEV model and 4 modes of operation

3.1 Introduction

The work presented in this article stems from four sources: We have used the paper [4] for pointers on how to design an algorithm for minimizing fuel consumption using Pontryagin's Maximum Principle. For modelling of the plugin hybrid vehicle, we've used [5] for inputs on how to build our SIMULINK model, the vehicle and battery data given in [4], and the detailed description of the powertrain and the 4 modes of operation of the vehicle provided in [6]. We have also used [2] for our understanding of the continuous time PMP, and [3] for applying the DPMP on the problem. Constrained optimal control problems continue to pose a challenge to the control community. One of the tools available to solve such problems is the Pontryagin Maximum Principle (PMP) [2]. However, incorporating state and control inequality constraints in the continuous time case makes the solution procedure quite intractable. An alternate route is the DPMP [3], which permits incorporation of inequality constraints, and thus makes a suitable framework for the automotive optimal control problem under study.

3.2 The GM Voltec Powertrain and 4 modes of operation

In this chapter we describe in detail the four modes of operation of the Chevrolet Volt. Much of the content in this section is taken from [4], [6], and [5]. The powertrain of the Chervolet Volt consists of an electric motor, a generator, a gasoline engine and a battery connected to the wheels through a planetary gear system and a differential axle, a schematic of which is provided in Figure 3.1. Three clutches as placed so that the vehicle can be operated in four different modes according to the state of the clutches. The electric motor develops a power up to 140 kW and

370 Nm of torque, the generator 55 kW and 110 Nm, and the engine 63 kW and 130 Nm. The battery has a capacity of 16 kWh, a maximum power 110 kW and a maximum recharging power of 60 kW. The planetary gear set, or PGS, of the

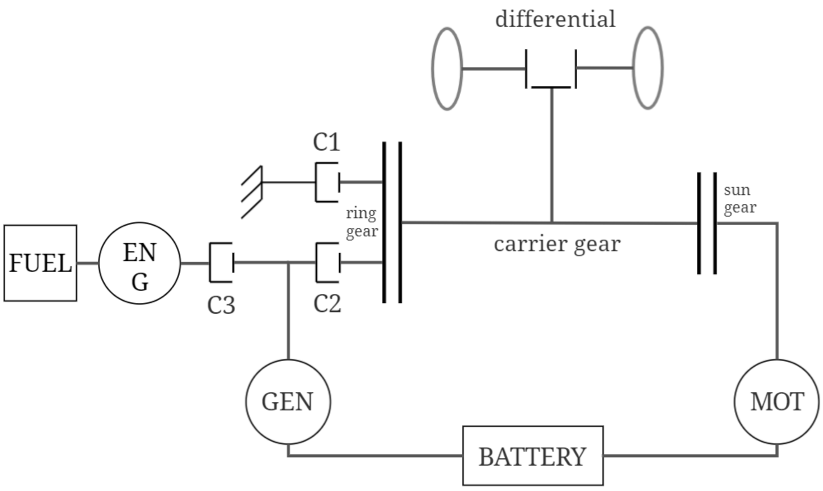


Figure 3.1: Schematic of the GM Voltec powertrain

Chevrolet Volt connects the generator and engine (ring gear) to the motor (sun gear) and the differential (carrier gear) as depicted in the figure. The ring gear may also be disconnected from the generator and locked, connected to only the generator, or connected to both the engine and the generator based on the clutches. The torque relations imposed by the planetary gear set are:

$$T_s = \frac{T_r}{\rho} = \frac{T_c}{\rho + 1} \tag{3.2.1}$$

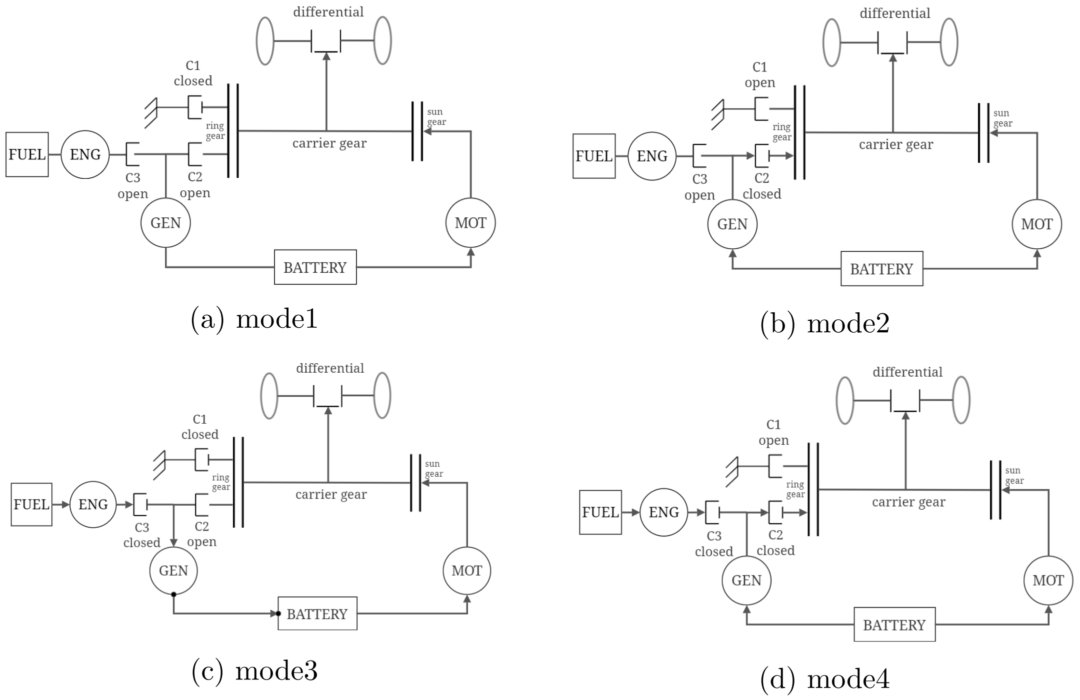
Where T_s is the torque at sun gear, T_r is torque at the ring gear, and T_c is torque at the carrier gear. $\rho = N_r/N_s = 2.24$ represents the ratio between the number of teeth of the ring and the sun gear. By conservation of energy and using torque relations, we get the Willis kinematic relation between ω_r angular speed of ring gear, ω_s angular speed of sun gear, and ω_c angular speed of carrier gear:

$$\rho\omega_r + \omega_s = \omega_c(\rho + 1) \tag{3.2.2}$$

The carrier gear is connected to the wheels through a differential, with a final gear ratio of $R_d = 2.16$. Thus the relation between the torque and angular speed of the carrier gear and that of the wheel is:

$$T_c = \frac{T_{wh}}{R_d} \tag{3.2.3}$$

$$\omega_c = \omega_{wh} R_d \tag{3.2.4}$$



(a) mode1

(b) mode2

(c) mode3

(d) mode4

Figure 3.2: Schematic of each mode

We now present the four modes:

Mode 1: This mode lets the PHEV operate as a fully electric vehicle, with clutch C1 locked, C2 and C3 being open and the engine being off. The motor derives power from the battery that is then transmitted to the wheel via the planetary gear system and differential. Thus the torque and angular speed of the motor is:

$$T_{mot} = \frac{T_{wh}}{R_d(\rho + 1)} \tag{3.2.5}$$

$$\omega_{mot} = \omega_{wh} R_d(\rho + 1) \tag{3.2.6}$$

Here T_{mot} is mechanical torque of the motor, and T_{wh} is the torque at the wheels. The power demanded from the battery will thus be:

$$P_{batt} = \begin{cases} \frac{T_{mot}\omega_{mot}}{\eta_{mot}}, & \text{if } T_{mot} > 0 \\ T_{mot}\omega_{mot}\eta_{mot}, & \text{if } T_{mot} < 0 \end{cases} \tag{3.2.7}$$

Here η_{mot} is efficiency of the motor, which varies with the torque and angular speed of the motor.

Mode 2: This mode allows the PHEV to operate both the motor and the generator using only battery power. In this case C1 and C3 are open, C2 is locked and the engine is off. The generator is connected to the PGS via the ring gear and the power delivered to the wheels is thus mix of power from the motor and the generator. Thus the torque of the ring gear and angular speed of ring gear is equal to that of the generator. The torque and angular speeds of the sun gear is that of the motor. It can be seen from efficiency maps that the efficiency of the motor decreases at high angular speeds, which is required when the velocity of the car is high. In this mode, the power delivered at the wheels can be divided between the two electric machines while still operating as a fully electric vehicle. This would allow us to decrease the motor speed and improve overall efficiency. The power demanded from the battery in this case would be:

$$P_{batt} = \frac{P_{mot}}{(\eta_{mot})^{\gamma}} + \frac{P_{gen}}{(\eta_{gen})^{\beta}} \tag{3.2.8}$$

where:

$$\gamma = sgn(T_{mot}), \beta = sgn(T_{gen}) \tag{3.2.9}$$

This is because both electrical machines may behave as generators or motors depending on the torque demanded at the wheels from the powertrain. This would thus effect the relation between the efficiency of the machine and the power demanded from or given to the vehicle. By convention, the torque of both machines is taken as positive when operating as an electric motor, and negative when operating as a generator. Here, the torque of the motor and generator would usually be positive unless the car is braking. The angular speeds of the motor, wheel and the generator are related by the Willis-Kinematic equation:

$$(\rho + 1)R_d\omega_{wh} = \rho\omega_{gen} + \omega_{mot} \tag{3.2.10}$$

The torques of generator and motor would depend on the torque of the wheel through the torque constraints:

$$\frac{T_{gen}}{\rho} = T_{mot} = \frac{T_{wh}}{(\rho + 1)R_d} \tag{3.2.11}$$

Minimizing battery power consumption can thus be interpreted as an optimization problem:
minimize the following:

$$\frac{T_{wh}}{R_d(\rho + 1)}(\frac{\rho\omega_{gen}}{\eta_{gen}} + \frac{\omega_{mot}}{\eta_{mot}}) \tag{3.2.12}$$

with the constraints:

$$R_d(\rho + 1)\omega_{wh} = \rho\omega_{gen} + \omega_{mot} \tag{3.2.13}$$

$$110kW > P_{batt} > -60kW \tag{3.2.14}$$

This would give a particular value of ω_{gen} and ω_{mot} that minimize the battery power and follow the kinematic relation imposed by the PGS.

Mode 3: This mode allows the PHEV to operate as a series hybrid vehicle. Here, C1 and C3 are locked, C2 is open and the engine is on. The engine and generator are connected to each other, while the generator is connected to the battery which then powers the motor. Thus the generator torque and angular speeds are equal to that of the engine in this mode. Since the ring gear is locked, only motor power is transmitted to the wheels via the PGS and the differential. The generator produces battery power in this case and is not connected to the PGS. The power generated by the generator recharges the battery:

$$P_{gen} = P_{charging} = \eta_{gen} T_{gen} \omega_{gen} \tag{3.2.15}$$

Here η_{gen} is the efficiency of the generator that depends on the mechanical torque and angular speed of the generator. The brake-specific fuel consumption factor of the engine gives us the mass of fuel consumed per unit mechanical energy produced by the engine at a particular torque and angular velocity. It can also be written in terms of time rate of mass consumption and power delivered by the engine:

$$\dot{m}_{bsfc}(T_{eng}, \omega_{eng}) = \frac{\dot{m_{fuel}}}{T_{eng}\omega_{eng}} \tag{3.2.16}$$

Thus, the mass of fuel at any point of time would be:

$$m_{fuel} = m^0_{fuel} + \int_{t^0}^{t} \dot{m}_{bsfc}(T_{eng}, \omega_{eng}) T_{eng} \omega_{eng} dt \tag{3.2.17}$$

mode 4 This mode allows the engine, generator and motor to act on the wheels via the PGS. Here, C1 is open, C2 and C3 are locked and the engine is on. Thus, the power at the wheels is divided between the engine, generator and motor allowing the motor to operate at better efficiency compared to mode 3 for higher speeds. Here the ring gear torque is the sum of both generator and engine torques since they both act on the gears together. The relations thus become:

$$T_r = T_{eng} + T_{gen}, \omega_r = \omega_{mot} = \omega_{gen} \tag{3.2.18}$$

The torque constraints would then give the following relations:

$$\frac{T_{gen} + T_{eng}}{\rho} = T_{mot} = \frac{T_{wh}}{(\rho + 1)R_d} \tag{3.2.19}$$

Using the above equations and using energy conservation, power at the wheels would be:

$$P_{wh} = T_{wh}\omega_{wh} = P_c = P_s + P_r \tag{3.2.20}$$

$$P_{wh} = T_{mot}\omega_{mot} + \omega_{eng}(T_{eng} + T_{gen}) = P_{mot} + P_{gen} + P_{eng} \tag{3.2.21}$$

The power drained from the battery in this case would only depend on the mechanical power and efficiency of the two electrical machines, and would be given by equation 3.2.8.

3.3 Modelling for simulation

The energy management strategy is implemented on a detailed model of the vehicle in SIMULINK/MATLAB given in figure 3.3 and the modelling of each component of the vehicle ensures that the fuel consumption estimates are reliable. The simulation model consists of three blocks, the driver module, the energy management strategy module or the controller, and the vehicle module or the plant.

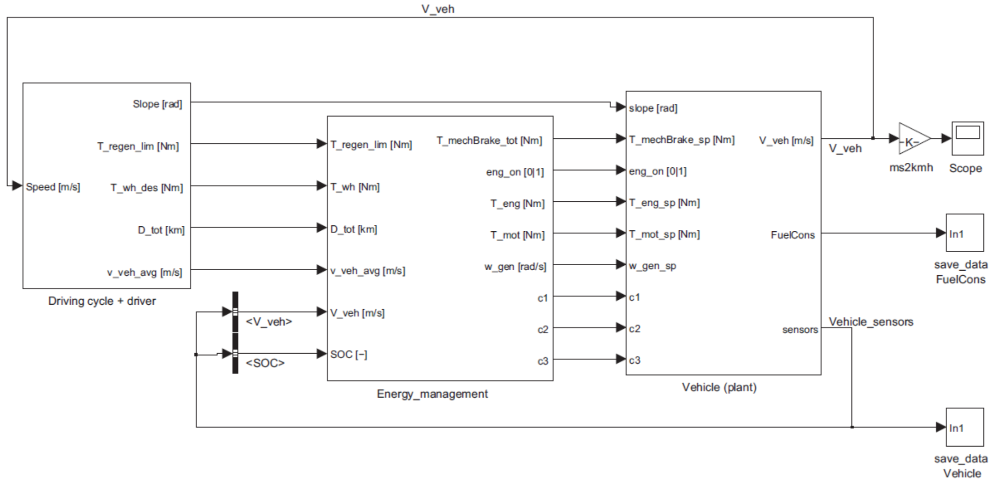


Figure 3.3: Simulation model, [5]

The driver model: The driver block contains the road grade profile and velocity profile for the entire duration of the drive. It calculates and gives out the desired torque at the wheels and desired angular velocity of the wheels using the profiles to the controller block.

The energy management strategy block: The energy management strategy block takes in the following parameters:

- Battery state of charge, from vehicle block
- Velocity of vehicle, from vehicle block
- Desired torque at the wheels, from driver block
- Desired angular velocity of the wheels, from driver block

It then calculates the vehicle setpoints based on its optimization strategy. The setpoints are comprise of:

- Torque of the motor
- Torque of the engine

- Angular speed of the wheel
- State of the three clutches or the mode of operation

The vehicle block: The vehicle block models each component of the vehicle and based on the setpoints given into it, calculates the real value of velocity and state of charge of the vehicle, and the fuel consumption of the vehicle It models the battery and the longitudinal dynamics of the vehicle using dynamical equations, models the electric motor, generator and internal combustion engine using quasi-static models and models the planetary gear system and clutches to incorporate the four modes of the powertrain. Although vehicle propulsion systems involve several dynamic phenomena, for the purpose of fuel economy estimation we can use quasi static models to describe various components. The models for longitudinal dynamics, the electric machines and the internal combustion engine implemented are based on published GM data, and the battery model dynamics is based on an experimentally validated model of the battery. The modelling of each component is discussed below:

Transmission Model

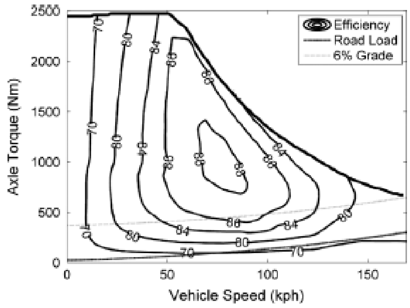
The powertrain model or the transmission model takes in setpoint values for torque of the engine and motor, the setpoint values of angular velocity of wheel and generator, and the mode decided by the energy management block and gives out the generator and wheel torques, as well as the angular velocity of the engine and the motor. It calculates these values based on the Willis Kinematic relation for the three elements of the planetary gear set, and the torque relations imposed by the planetary gear set. The relation between the angular velocity and torque of the gears and that of the electrical machines and ICE depends on the mode of operation, these relations are tabulated below:

Mode	**Relation**
Mode 1	$w_r = 0, \;\; w_e = 0$ $T_g = 0$
Mode 2	$w_r = w_g, \;\; w_e = 0$ $T_g = \rho \cdot T_m$
Mode 3	$w_r = 0, \;\; w_e = w_g$ $T_g = -T_e$
Mode 4	$w_r = w_g, \;\; w_e = w_g$ $T_g = \rho \cdot T_m - T_e$
All modes	$w_m = w_s, \;\; w_c = R_d \cdot w_{wh}$ $T_m = T_s, \;\; T_{wh} = R_d \cdot T_c$

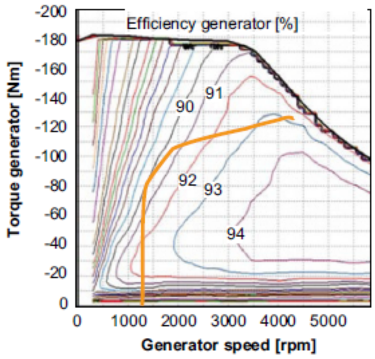
Table 3.1: PSG relations [5]

Electric machines and internal combustion engine

The motor and generator are modelled by efficiency maps, as shown in figure 3.4 which give the efficiency of the electric machine based on the torque and angular velocity of the electric machine. By convention, positive torque indicates the electrical machine is consuming electric power and giving mechanical torque and angular velocity as output, and negative torque indicates the electrical machine takes mechanical power as input and gives electric power as output.



(a) Electric motor efficiency map



(b) Generator efficiency map

Figure 3.4: Efficiency maps of electrical machines [5]

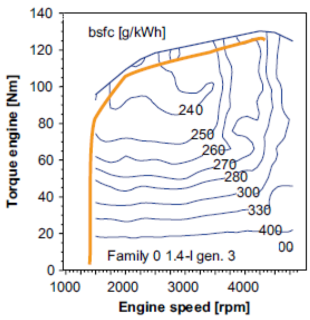


Figure 3.5: Fuel consumption map of ICE [5]

Thus, based on the mechanical torque and angular speed demanded of each electric machine, the modelled block gives the power demanded from the battery. The same map is used for calculating efficiency if the generator has negative torque and is thus acting as a generator. The engine is modelled by a fuel consumption map which gives the brake specific fuel consumption in kilograms per kilowatt hour based on the torque and angular velocity of the engine.

Battery

The battery pack used in the GM Chevrolet Volt is composed of 288 LG Chem R1S3 Pouch Li-Ion cells, in a 96S 3P pack configuration. Each cell has a nominal capacity of 15 Ah and nominal voltage of 3.85 V. A set of experimental tests were conducted to characterize a cell over different temperatures and all values of state of charge from 0 to 100.

Parameters	**Value**
Total energy capacity	16 KWh
Total nominal voltage	360 V
SOC range	65%
Number of cells in series	96
Number of strings in parallel	3
Peak current	400 A
Peak power (charge)	110 KW
Peak power (discharge)	-60 KW

Table 3.2: Battery parameters

It is observed that the nominal open circuit voltage and internal resistance of a cell varies with change in SOC, and this has been incorporated into the dynamical model of the battery by calculating these quantities through functions of SOC obtained through polynomial interpolation of experimental data. The experimental results along with the interpolated polynomials are shown in Figure 3.6 The zero order electrical circuit model shown in Figure 3.7 has been used to model every cell of the battery, which are then arranged in three 96-cell series, in parallel arrangement. An equivalent circuit for the entire battery pack is thus made by calculating net open circuit voltage, Voc and internal resistance Ro. Using these quantities, the state of charge dynamics can be written as follows:

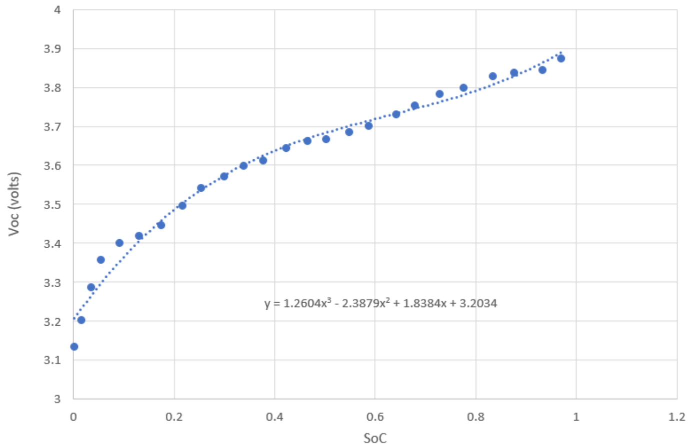
$$V_L = V_{OC}(SoC) - I * R_O(SoC) \tag{3.3.1}$$

Where the open circuit voltage load is V_L, and I is the current flowing in an out of the battery terminal, it is positive when the battery is discharging and negative when it is recharging. The power generated by the battery would be open circuit voltage load multiplied by current:

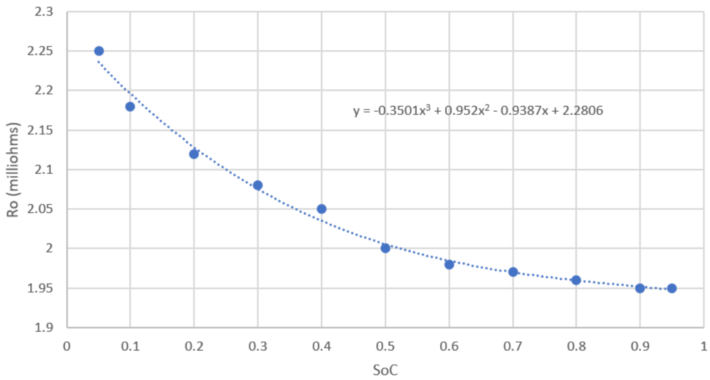
$$P_{batt} = V_L * I = V_{OC} * I - I^2 * R_O(SoC) \tag{3.3.2}$$

Solving the above equation for current we get:

$$I = \frac{V_{OC}(SoC) - \sqrt{V_{OC}(SoC)^2 - 4P_{batt}R_O(SoC)}}{2R_O(SoC)} \tag{3.3.3}$$



(a) Voltage of cell vs SoC



(b) Internal resistance of cell vs SoC

Figure 3.6: Experimentally measured values of open circuit voltage and internal resistance of cell varying with state of charge [4]

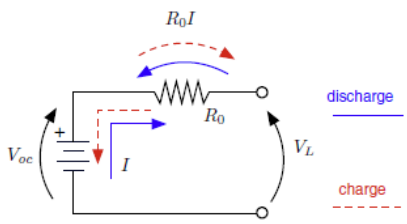


Figure 3.7: Zeroth order electrical circuit model of one cell in the battery unit [4]

The variation of the battery SOC is defined by:

$$S\dot{o}C = -\eta_c * \frac{I}{Q_{nom}} \tag{3.3.4}$$

Q_{nom} is the nominal battery capacity and η_c is the coulombic efficiency of the battery. Hence the dynamics of the SoC becomes: 5

$$\dot{SoC} = -\eta_c * \frac{V_{OC}(SoC) - \sqrt{V_{OC}(SoC)^2 - 4P_{batt}R_O(SoC)}}{2R_O(SoC)Q_{nom}} \tag{3.3.5}$$

Thus the battery is modelled using this differential equation to give the actual state of charge of the battery based on the current state of charge and the power demanded from the battery.

Vehicle Dynamics

The longitudinal vehicle dynamics are implemented via a separate block for propagating the actual velocity of the vehicle based on the torque applied at the wheel generated through the powertrain of the vehicle.

Attributes	**Value**
Curb weight m_v	1812 Kg
Road law coefficient c_0	105.95 N
Road law coefficient c_1	0.01 N s m^{-1}
Road law coefficient c_2	0.4340 N s^2 m^{-2}
Wheel radius r_wh	33 cm
Vehicle inertia J_v	207 kg m^2
Wheelbase	2685 mm
Center of gravity height	550 mm
Front/rear static weight distribution	49%/51%

Table 3.3: Vehicle Parameters [4]

The dynamics of the velocity of the vehicle are as follows:

$$\dot{v} = \frac{r_{wh}}{J_v} * [T_{wh} + T_{br} - m_v r_{wh} g \sin\alpha - r_{wh}(c_0 + c_1 v + c_2 v^2)] \tag{3.3.6}$$

The constants c_0, c_1, and c_2 are road grade coefficients, J_v is the vehicle inertia, T_{wh} is wheel torque, T_{br} is brake torque, and α is the slope of the road.

Chapter 4

Problem Formulation and Results

4.1 Problem Formulation

Since our goal is to pose the Chevrolet Volt engine management problem as a discrete time optimal control problem, we present the formulation of the four modes in this chapter. The four modes of operation of the PHEV can be described as discrete time optimal control problems as follows:

4.1.1 Mode 1

We define our state as a two dimensional vector comprising of battery state of charge ξ and velocity v of the vehicle, and our control input as the torque on the wheels T_{wh}. Since in mode 1, the energy demand is entirely on the battery, the dynamics would be:

$$\dot{x} = \begin{bmatrix} \dot{v} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} f_v(v, T_{wh}) \\ f_\xi(\xi, P_{batt}(v, T_{wh})) \end{bmatrix} \tag{4.1.1}$$

$$P_{batt}(v, T_{wh}) = \frac{T_{wh} \times v}{r_{wh} \times \eta_{mot}(T_{mot}, w_{mot})} = \frac{T_{wh} \times v}{r_{wh} \times \eta_{mot}(\frac{T_{wh}}{R_d(\rho+1)}, \frac{vR_d(\rho+1)}{r_{wh}})} \tag{4.1.2}$$

where, η_{mot} is the motor efficiency characterized by the efficiency map of the motor, that we've interpolated as a function in two variables: motor angular speed, and torque delivered by the motor. Thus, in this mode, η_{mot} can be written as a function of wheel torque and velocity of vehicle.
Boundary conditions and constraints for the problem would be:

$$v(t) = v_{desired}(t) \ \forall \ t \tag{4.1.3}$$

$$\xi(0) = \xi_0, \ \xi_0 \in R \tag{4.1.4}$$

$$P_{max} \geqslant P_{batt}(v, T_{wh}) \geqslant P_{min} \tag{4.1.5}$$

With the intention of minimizing battery use for this mode, the cost will be defined as:

$$\sum_{t=0}^{T-1} f_\xi(\xi(t), P_{batt}(v(t), T_{wh}(t)))dt \tag{4.1.6}$$

4.1.2 Mode 2

We define our state as a two dimensional vector comprising of battery state of charge ξ and velocity v of the vehicle, and our control input as a two dimensional vector comprising of the torque on the wheels T_{wh} and angular speed of the motor w_{mot}. In mode 2, although the energy demand is entirely on the battery, both the electric machines are used. The dynamics in this case would be:

$$\dot{x} = \begin{bmatrix} \dot{v} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} f_v(v, T_{wh}) \\ f_\xi(\xi, P_{batt}(v, T_{wh}, w_{mot})) \end{bmatrix} \tag{4.1.7}$$

$$u = \begin{bmatrix} w_{mot} \\ T_{wh} \end{bmatrix} \tag{4.1.8}$$

$$P_{batt}(v, T_{wh}, w_{mot}) = \frac{T_{wh} \times w_{mot}}{R_d(\rho+1) \times \eta_{mot}(\frac{T_{wh}}{R_d(\rho+1)}, w_{mot})} + \frac{T_{wh}\rho \times w_{gen}}{R_d(\rho+1) \times \eta_{gen}(\frac{T_{wh}\rho}{R_d(\rho+1)}, w_{gen})} \tag{4.1.9}$$

The angular speed of the generator is dependent on the motor angular speed and the vehicle speed through the willis kinematic relation:

$$\rho w_{gen} + w_{mot} = \frac{v R_d(\rho+1)}{r_{wh}} \tag{4.1.10}$$

where, η_{gen} is the generator efficiency characterized by the efficiency map of the motor, that we've interpolated as a function in two variables: motor angular speed, and torque delivered by the motor.
Boundary conditions and constraints for the problem would be:

$$v(t) = v_{desired}(t) \ \forall \ t \tag{4.1.11}$$

$$\xi(0) = \xi_0, \ \ \xi_0 \in R \tag{4.1.12}$$

$$P_{max} \geqslant P_{batt}(v, T_{wh}) \geqslant P_{min} \tag{4.1.13}$$

With the intention of minimizing battery use for this mode, the cost will be defined as:

$$\sum_{t=0}^{T-1} f_\xi(\xi(t), P_{batt}(v(t), T_{wh}(t)))dt \tag{4.1.14}$$

4.1.3 Mode 3

In mode 3, the vehicle behaves as a series hybrid vehicle. In this case, we take a three dimensional vector comprising of torque at the wheel T_{wh}, angular speed of the generator w_{gen} and the torque of the generator T_{gen} as the control input.

$$\dot{x} = \begin{bmatrix} \dot{v} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} f_v(v, T_{wh}) \\ f_\xi(\xi, P_{batt}(v, T_{wh}, w_{gen}, T_{gen})) \end{bmatrix} \tag{4.1.15}$$

$$u = \begin{bmatrix} T_{wh} \\ w_{gen} \\ T_{gen} \end{bmatrix} \tag{4.1.16}$$

$$P_{batt}(v, T_{wh}, w_{gen}, T_{gen}) = \frac{T_{wh} \times v}{r_{wh} \times \eta_{mot}(\frac{T_{wh}}{R_d(\rho+1)}, \frac{vR_d(\rho+1)}{r_{wh}})} + T_{gen} w_{gen} \eta_{gen}(T_{gen}, w_{gen}) \tag{4.1.17}$$

where, T_{gen} will be negative, as the generator is always charging the battery. Boundary conditions and constraints for the problem would be:

$$v(t) = v_{desired}(t) \ \forall \ t \tag{4.1.18}$$

$$\xi(0) = \xi_0, \ \ \xi_0 \in R \tag{4.1.19}$$

$$P_{max} \geqslant P_{batt}(v, T_{wh}, w_{gen}, T_{gen}) \geqslant P_{min} \tag{4.1.20}$$

$$T_{gen} \leq 0 \ \forall \ t \tag{4.1.21}$$

With the intention of minimizing fuel consumption while maintaining some final state of charge for this mode, the cost will be defined as:

$$\sum_{t=0}^{T-1} m_{fuel}(w_{gen}, T_{gen}) + K\xi(T), \quad K \in R \tag{4.1.22}$$

where, m_{fuel} is the rate of change of mass of fuel remaining with respect to time (kg/s), considering rate of increase as positive and that of decrease as negative.

4.1.4 Mode 4

In mode 3, the vehicle behaves as a series hybrid vehicle. In this case, we take a three dimensional vector comprising of torque at the wheel T_{wh}, angular speed of the generator w_{gen} and the torque of the generator T_{gen} as the control input.

$$\dot{x} = \begin{bmatrix} \dot{v} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} f_v(v, T_{wh}) \\ f_\xi(\xi, P_{batt}(v, T_{wh}, w_{gen}, T_{gen})) \end{bmatrix} \tag{4.1.23}$$

$$u = \begin{bmatrix} T_{wh} \\ w_{gen} \\ T_{gen} \end{bmatrix} \tag{4.1.24}$$

$$P_{batt}(v, T_{wh}, w_{gen}, T_{gen}) = \frac{T_{wh} \times w_{mot}}{R_d(\rho+1) \times \eta_{mot}(\frac{T_{wh}}{R_d(\rho+1)}, w_{mot})} + T_{gen} w_{gen} \eta_{gen}(T_{gen}, w_{gen}) \tag{4.1.25}$$

here, T_{gen} may be negative or positive, since the engine is acting directly on the gearset and not via the generator. The angular speed of the motor is related to the vehicle velocity, torque at the wheel, torque of the generator and angular speed of the generator thus:

$$T_{wh} \times (v/r_{wh}) = (\frac{T_{wh}\rho}{R_d(\rho+1)} - T_{gen}) w_{gen} + \frac{T_{wh}}{R_d(\rho+1)} w_{mot} + T_{gen} w_{gen} \tag{4.1.26}$$

Boundary conditions and constraints for the problem would be:

$$v(t) = v_{desired}(t) \ \forall \ t \tag{4.1.27}$$

$$\xi(0) = \xi_0, \ \xi_0 \in R \tag{4.1.28}$$

$$P_{max} \geqslant P_{batt}(v, T_{wh}, w_{gen}, T_{gen}) \geqslant P_{min} \tag{4.1.29}$$

With the intention of minimizing fuel consumption while maintaining some final state of charge for this mode, the cost will be defined as:

$$\sum_{t=0}^{T-1} m_{fuel}(w_{gen}, T_{gen}) + K\xi(T), \quad K \in R \tag{4.1.30}$$

For the four modes above a simple strategy for choosing a mode based on vehicle velocity and battery state of charge is devised. For each mode the control input is to be calculated by pursuing the optimal solution to each problem using the discrete maximum principle via the multiple shooting algorithm.

4.2 A representative problem solved using the DPMP

Consider a simple double integrator system as following.

$$\ddot{x} = u \tag{4.2.1}$$

where, x is the state and u is the control input.
The system dynamics can be written in the discrete time as following.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{i+1} = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_i + \begin{pmatrix} 0 \\ h \end{pmatrix} u_i \tag{4.2.2}$$

$$i = 0, 1, 2, \ldots, N-1$$

where $x_1 = x$, $x_2 = \dot{x_1}$ and h is the discretization step size.
The cost function is defined in following manner.

$$J = \sum_{i=0}^{N-1} u_i^2 \tag{4.2.3}$$

Objective is to find an optimal control in discrete time, which minimizes the cost given by equation 4.2.3 under following constraints.

$$\text{subject to} \quad \begin{cases} \text{equation 4.2.2} \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_0 = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_N = \begin{pmatrix} \gamma \\ \delta \end{pmatrix} \text{where } \alpha, \beta, \gamma, \delta \in \mathbb{R} \\ u_i^2 \leq 1 \qquad\qquad\qquad \forall i \in \{0, 1, \ldots, N-1\} \end{cases} \tag{4.2.4}$$

Using the DPMP, via multiple shooting, the results obtained are as follows:

Case 1:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x_N = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \tag{4.2.5}$$

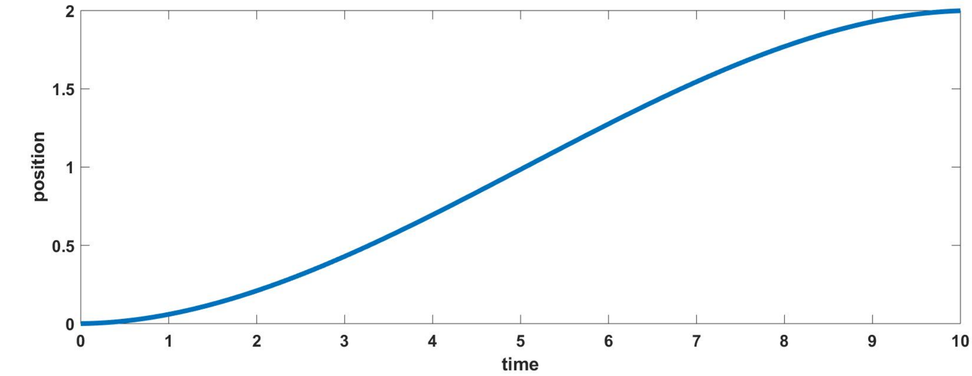
Case 2: Imposed constraint on control input can be seen

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x_N = \begin{bmatrix} 200 \\ 0 \end{bmatrix} \tag{4.2.6}$$

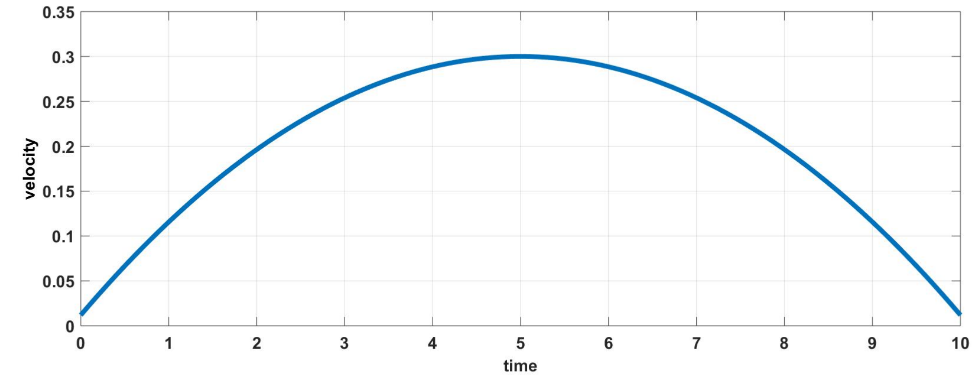
Case 2: Different boundary conditions are imposed

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x_N = \begin{bmatrix} 20 \\ 1 \end{bmatrix} \tag{4.2.7}$$

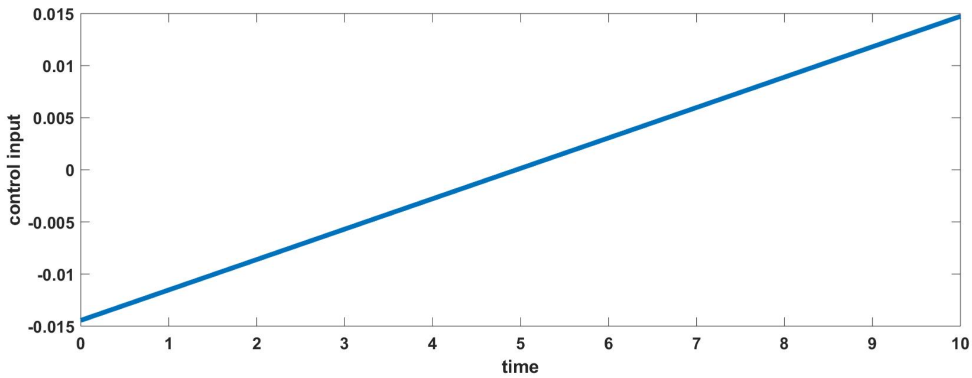
Using the same algorithm, the four optimal control problems formulated are now to be solved.



(a) Optimal position (x_1) trajectory

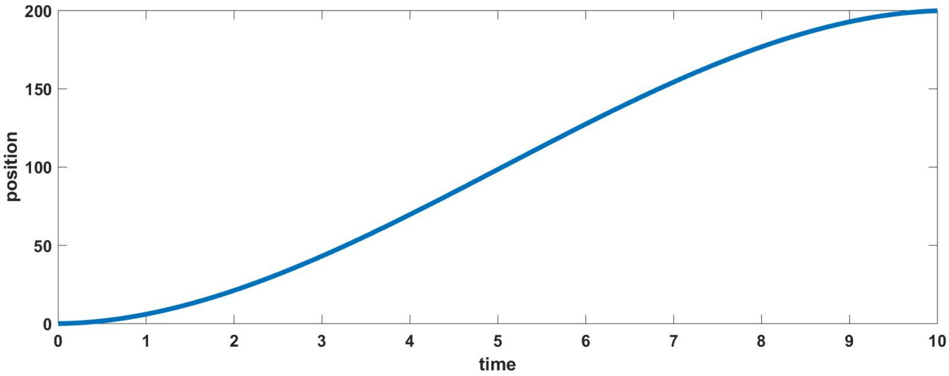


(b) Optimal velocity (x_2) trajectory

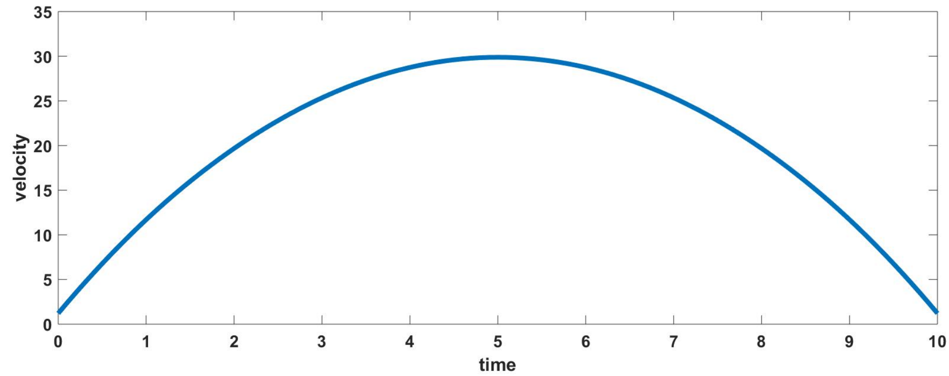


(c) Optimal control input (u)

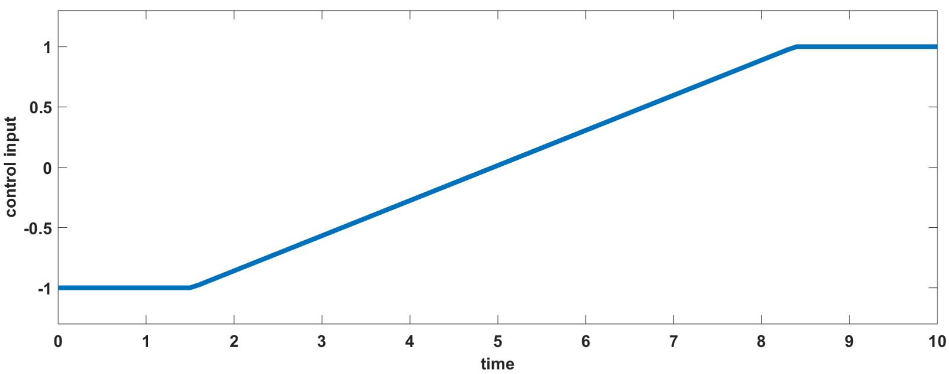
Figure 4.1: Simulation 1



(a) Optimal position (x_1) trajectory

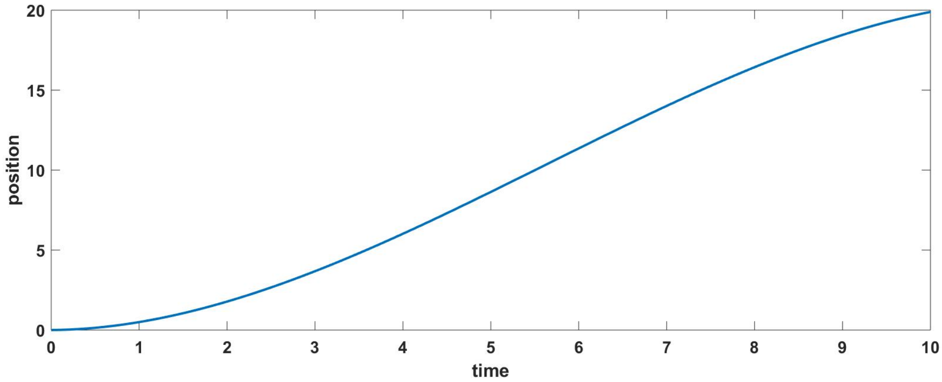


(b) Optimal velocity (x_2) trajectory

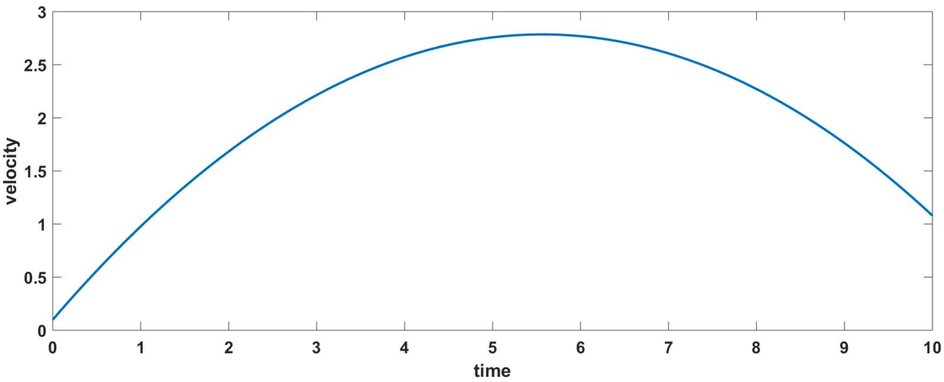


(c) Optimal control input (u)

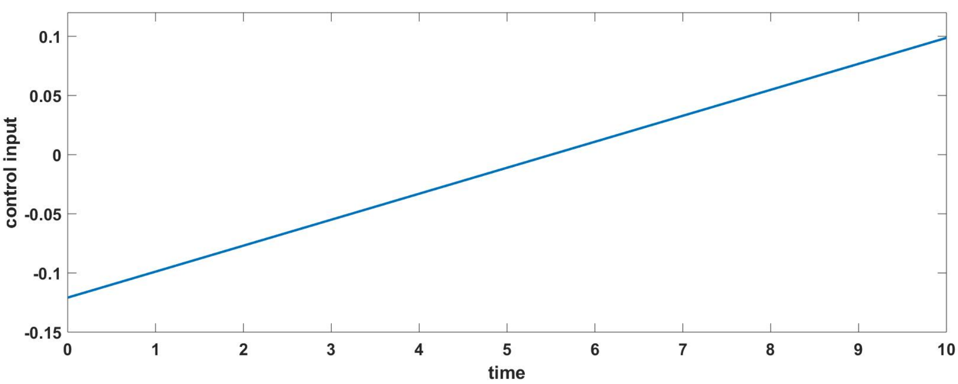
Figure 4.2: Simulation 2



(a) Optimal position (x_1) trajectory



(b) Optimal velocity (x_2) trajectory



(c) Optimal control input (u)

Figure 4.3: Simulation 3

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