

Problem set 1

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Linear algebra

1. Projection Matrices:

Consider the following matrices $\mathbf{P} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ and $\mathbf{M} = \mathbf{I}_n - \mathbf{P}$. Using only the equation $y = \mathbf{X}\beta + e$ and $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'y$. Show the following:

- a. \mathbf{P} is idempotent ($\mathbf{P}\mathbf{P} = \mathbf{P}$)
- b. \mathbf{M} is idempotent
- c. $\mathbf{P}y = \hat{y}$
- d. $\mathbf{M}y = \hat{e}$
- e. $\mathbf{P}y + \mathbf{M}y = y$
- f. $\hat{y} \perp \hat{e}$

Programming

2. Generalized hailstone:

Recall in class that we discussed the hailstone numbers where $x_n = 3x_{n-1} + 1$ if x_{n-1} is odd and $x_n = x_{n-1}/2$ if x_{n-1} is even. The hailstone numbers converge to a sort of holding pattern $4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1$. We call this kind of pattern a convergence because it repeats.

Write a program to test generalized hailstone numbers where $x_n = ax_{n-1} + b$ if x_{n-1} is odd and $x_n = x_{n-1}/2$ if x_{n-1} is even where a and b are positive integers. For each of the a, b pairs less than or equal to 10, find whether or not the sequence seems to converge. Once you can do that, tell me how many different holding patterns the sequence converges to. For instance, when $a = 3$ and $b = 5$ then $1 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ this is holding pattern. However, if we start with a different number, say 5 we get a different holding pattern $5 \rightarrow 20 \rightarrow 10 \rightarrow 5$.