INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Department of Chemical Engineering

CH5350 : Applied Time Series Analysis (Jul-Nov 2018)
Solutions to Quiz 2

Marks Distribution

	Question 1	Question 2	Question 3
(a)	10	20	15
(b)	18	10	10
(c)	17	–	–

Question 1

(a)

Given

$$w[k] = \nabla v[k]$$
$$w[k] = v[k] - v[k-1]$$
$$Also\ \sigma_{vv}[0] = 255.24, \sigma_{vv}[1] = 253.59, \sigma_{vv}[2] = 249.74$$

Assuming $E[v[k]]$ to be zero

Variance of w[k]

$$
\begin{aligned}
\sigma_{ww}[0] &= E[(v[k]-v[k-1])(v[k]-v[k-1])] \\
&= \sigma_{vv}[0] - 2\sigma_{vv}[1] + \sigma_{vv}[0] \\
&= 2(255.24 - 253.59) \\
&= 3.3
\end{aligned}
$$

(b)

$$
\begin{aligned}
\sigma_{ww}[1] &= E[(v[k]-v[k-1])(v[k-1]-v[k-2])] \\
&= 2\sigma_{vv}[1] - \sigma_{vv}[2] - \sigma_{vv}[0] \\
&= 2(253.59) - 255.24 - 249.74) \\
&= 2.2
\end{aligned}
$$

He fits AR(1) model to $w[k]$. Therefore, $w[k] = -cw[k-1] + e[k]$

Writing Yule-Walker equations

$$
\begin{aligned}
\sigma_{ww}[0] &= -c\,\sigma_{ww}[1] + \sigma_e^2 \\
\sigma_{ww}[1] &= -c\,\sigma_{ww}[0] \\
\text{We get, } c &= -0.67, \sigma_e^2 = 1.83
\end{aligned}
$$

(c)

Given

$$
\begin{aligned}
\gamma_{vv}(f) &= 16.125 \text{ at } f = 0.1 \text{ cycles/sample} \\
w[k] &= (1-q^{-1})v[k] \\
\text{Therefore, } \gamma_{ww}(f) &= \mid (1-e^{-j2\pi f}) \mid^2 \gamma_{vv}(f) \\
\text{For f} = 0.1 \text{ cycles/sample }, \gamma_{ww}(f=0.1) &= 6.159 \\
\text{Using AR(1) model,} \gamma_{ww}(f) = \mid \frac{1}{(1-0.67e^{-j2\pi f})} \mid^2 \gamma_{ee}(f) &= 4.791
\end{aligned}
$$

Therefore AR(1) is inadequate

Question 2

From the PSD estimate we can fit an AR(2) model, with the poles on the unit circle
$\omega_0 = 0.2$
$H(z^{-1}) = \frac{1}{(1-aq^{-1})(1-bq^{-1})}$ from the definition,
$PSD = |H(e^{j\omega})|^2 \frac{\sigma^2 e}{2\pi}$
as $ab = 1$
and $a + b = -2cos(\omega_0)$ The co efficients of the model are $a = 0.98$
Additional phase information is needed to validate the model.
Note: If the student has rejected the AR model then 50 % of the marks is reduced.

Question 3

$$Given, \;\; y[k] = -a_1 y[k-1] + b_D u[k-D] + e_y[k]$$
$$y[k] = \frac{b_D q^{-D}}{1 + a_1 q^{-1}} u[k] + \frac{1}{1 + a_1 q^{-1}} e_y[k]$$
$$\text{Therefore, } \;\; \gamma_{yu}(\omega) = G(e^{-j\omega})\gamma_{uu}(\omega) \;\; \text{where} \;\; G(e^{-j\omega}) = \frac{b_D e^{-j\omega D}}{1 + a_1 e^{-j\omega}}$$
$$\text{Similarly, } \;\; \gamma_{ye}(\omega) = H(e^{-j\omega})\gamma_{ee}(\omega) \;\; \text{where} \;\; H(e^{-j\omega}) = \frac{1}{1 + a_1 e^{-j\omega}}$$
$$\text{Therefore equating the phase we get D and equating the magnitude we get } \;\; a_1, b_D$$