INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering
CH 5350 Applied Time-Series Analysis

Assignment 6

Due: Thursday, November 22, 2018

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1. [MLE and Least Squares]

   (a) Given two observations $x[1]$ and $x[2]$ of a series, fit an AR(1) model using the MLE method (ITSM, Brockwell and Davis).

   (b) For the process above, arrive at the Least Squares solution and compare it with the MLE.

2. [Hannan-Rissanen algorithm]

   (a) Write an R function file to implement the H-R algorithm for the estimation of the ARMA model parameters. The user is expected to supply the orders of the respective components.

   (b) Test your code on two different processes: (i) MA(2) process with $c_1 = 1,\ c_2 = 0.21$ and (ii) ARMA(1,2) process with $d_1 = 0.4$, $c_1 = 0.7,\ c_2 = 0.12$.

   (c) Compare the results of your routine with the `arma` routine of the `tseries` package in R.

3. [Spectral Densities]

   (a) Determine the theoretical power spectral density of the series formed by

   $$x[k] = e[k-2] + 2e[k-1] + 4e[k] \quad e[k] \sim \mathcal{N}(0,1)$$

   (b) Generate 2000 samples of $x[k]$. Estimate the power spectrum using different estimators, namely, (i) periodogram (ii) smoothed periodogram using Daniell's smoother (iii) Welch's averaged periodogram method and (iv) parametric method. Compare the estimates with the theoretical one obtained in part (3a) above.

4. Fit a suitable time-series model for the `tcm1yd` data set in the `TSA` package. Follow each step of the systematic procedure that is used in the development of a time-series model.

5. [Predictions]
   For the MA(1) process $x[k] = e[k] - e[k-1]$ where $e[k]$ is the usual zero-mean WN. Suppose we consider the problem of predicting $x[k+1]$ using only $\{x[1],\ x[2],\ \cdots,\ x[k]\}$.

   (a) Arrive at the best BLP of $x[k+1]$ using the Projection Theorem.

   (b) Show that the mean square error in the prediction of $x[k+1]$ is

   $$E((x[k+1] - \hat{x}[k+1|k])^2) = \frac{k+2}{k+1}\sigma_e^2$$