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Experiment 8

Aim: To Learn Calculus with SageMath

7.1 The syntax for Evaluation of Integration is

$$integral(expression, v, a, b)$$

Return the indefinite integral with respect to the variable v, ignoring the constant of integration. Or, if endpoints a and b are specified, returns the definite integral over the interval [a,b].

Problem:

Evaluate $\int \frac{sin(x)}{\sqrt{cos(x)+1}}dx$

In [3]:
```
f(x) = sin(x)/sqrt(1+cos(x))
```

In [4]:
```
show(integrate(f(x),x))
```

Out[4]:

$$-2\sqrt{\cos(x)+1}$$

Evaluate $\int_0^{\frac{\pi}{2}} \frac{sin(x)}{\sqrt{cos(x)+1}}dx$

In [5]:
Out[5]:
```
show(integrate(f(x),x,0, pi/2))
```

–

Evaluate $\int_0^{\infty} e^{-x^2+1}dx$

In [10]:
```
integral(e^(-x^2+1),x,0, infinity)
```

Out[10]:
```
1/2*sqrt(pi)*e
```

Exercise 7.1

Evaluate following integrals

(1). $\int \frac{-4}{\sqrt{1-x^2}}dx$

(2) $\int sin^5(x)cos^2(x)dx$

(3) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{1+sin(x)-cos(x)}$

In [11]:
```
f(x) = -4/sqrt(1-x^2)
```

In [12]:
```
show(integrate(f(x),x))
```

Out[12]:

$$-4\arcsin(x)$$

In [14]:
```
f(x) = ((sin(x))^5)*((cos(x))^2)
```

In [15]:
```
show(integrate(f(x),x))
```

Out[15]:

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

In [16]:
```
f(x) = 1/(1+sin(x)-cos(x))
```

In [17]:
```
show(integrate(f(x),x,pi/3, pi/2))
```

Out[17]:

$$\frac{1}{2}\log(3)-\log(2)+\log\left(\frac{1}{3}\sqrt{3}+1\right)$$

7.2. Average Value of a function

Average value of function $f(x)$ on interval $[a, b]$ is given by $Avg = \frac{1}{b-a}\int_a^b f(x)dx$

Problem: A car travels with velocity $y = 4t + 10$ between $t = 0$ and $t = 5$. Find the average velocity.

In [18]:
```
var('t')
a,b = 0, 5
v(t) = 4*t+10
avg_vel = 1/(b-a)*integral(v(t),t,a,b)
print(avg_vel)
```

```
20
```

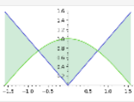
7.3 Applications of Integrals

7.3.1 Areas

Problem: Find the area enclosed between $|x|$ and $cos(x)$

In [19]:
```
## Area between two curves
f (x ) = abs (x)
g(x) = cos(x)
a, b = -pi/2,pi/2
h = plot ((f(x), g(x)),x, a,b, figsize =4, fill ={0:g,1:f})
show(h)
```

Out[19]:



In [20]:
```
c1 = find_root(f(x) -g(x),-1,0)
c2 = find_root(f(x)-g(x),0,1)
c1,c2
```

Out[20]:
```
(-0.7390851332151559, 0.7390851332151559)
```

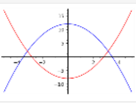
In [21]:
```
I1 = integral(f(x)-g(x) ,x ,a,c1)
I2 = integral(g(x)-f(x) ,x ,c1,c2)
I3 = integral (f(x)-g(x) ,x ,c2 , b)
(I1+I2+I3).n()
```

Out[21]:
```
2.06935554872585
```

Problem: Find the area enclosed between two curves $y = 12 - x^2$ and $y = x^2 - 8$.

In [22]:
```
f(x)=12-x^2
g(x) = x^2-8
plot(f(x),-5,5)+plot(g(x),-5,5,color='red',figsize=4)
```

Out[22]:

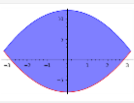


In [23]:
```
S = solve(f(x)==g(x),x,solution_dict=True)
a,b = S[0][x],S[1][x]
a,b
```

Out[23]:
```
(-sqrt(10), sqrt(10))
```

In [24]:
```
p1 = plot(f(x), (x,a,b),fill=g(x),fillcolor='blue')
p2 = plot(g(x), (x,a,b),color='red')
show(p1+p2,figsize=4)
```

Out[24]:



In [25]:
```
A = integral((f(x)-g(x)),x,a,b)
show(A.n())
```

Out[25]:

$$84.3274042711568$$

7.3.2. Arc Length

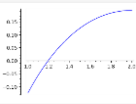
If the curve is given by $y = f(x), a \le x \le b$.Then arc length of curve is given by

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$$

Problem: Find arc length of the curve $y = log(x) - \frac{x^2}{8}, 1 \le x \le 2$.

In [26]:
```
f(x)=ln(x)-x^2/8
plot(f(x), (x,1,2),figsize=4)
```

Out[26]:



In [27]:
```
integral(sqrt(1+derivative(f,x)^2),x,1,2)
```

Out[27]:
```
log(2) + 3/8
```

7.3.3. Areas of Surfaces of Surface of Revolution

- The surface area of the surface of revolution obtained by rotating the curve $y = f(x), a \le x \le b$, about the x-axis is given by

$$S = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx.$$

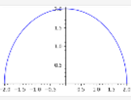
- The surface area of the surface of revolution obtained by rotating the curve $x = f(y), c \le y \le b$, about the y-axis is given by

$$S = \int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}dy.$$

Problem: Find the surface area of the Sphere $x^2 + y^2 + z^2 = r^2$

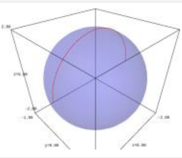
In [28]:
```
var('x,y,r')
r=2
f(x) = sqrt(r^2-x^2)
plot(f(x),-r,r,figsize=4,aspect_ratio='automatic')
```

Out[28]:



In [29]:
```
revolution_plot3d(f,     (x,-r,r),show_curve=True,opacity=0.3,parallel_axis='x',aspect_ratio='automatic')
```

Out[29]:



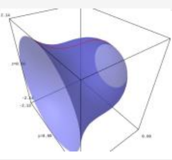
In [30]:
```
## Surface area of a sphere of radius r
var('r')
f(x)=sqrt(r^2-x^2)
integral(2*pi*f(x)*sqrt(1+derivative(f,x)^2),x,-r,r)
```

Out[30]:
```
4*pi*r^2
```

Problem: Find the surface area of the surface of revolution obtained be rotation $f(x) = x + cos(x)$ between $x = 0$ and $x = \pi$ about the x-axis

In [31]:
```
f(x)=x+cos(x)
a,b = 0, pi
surf = revolution_plot3d(f,  (x,a,b),show_curve=True,opacity=0.6,parallel_axis='x')
show(surf)
```

Out[31]:



In [32]:
```
A = integral(2*pi*f(x)*sqrt(1+f.diff()(x)^2),(x,a,b))
A.n()
```

Out[32]:
```
34.1375871765462
```

7.3.4. Volumes of solid of revolution

- The volume of the solid of revolution about the x-axis of the solid obtained by revolving a region between $y = f(x), a \le x \le b$, about the x-axis is given by

$$V = \int_a^b \pi f(x)^2 dx$$

- The volume of the solid of revolution about the x-axis of the solid obtained by revolving a region between $x = f(y), c \le x \le d$, about the x-axis is given by

$$V = \int_c^d \pi f(y)^2 dy$$

- If the region bounded by two curves $y = f(x), y = g(x)$ and the lines $x = a, x = b$ with $g(x) \le f(x)$, the volume of the solid of revolutn obtained by revolving a region about the x-axis is given by

$$V = \int_a^b \pi[f(x)^2 - g(x)^2]dx$$

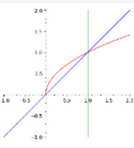
In [33]:
```
## Volume of sphere of radius r
var('x,r')
f(x) = sqrt(r^2-x^2)
V = integral(pi*f(x)^2,x,-r,r)
V
```

Out[33]:
```
4/3*pi*r^3
```

Problem: Find the volume of the solid obtained by rotation the region bounded by $y + x, y = \sqrt{x}$ about the line $x = 1$.

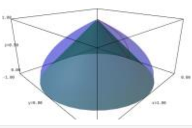
In [34]:
```
f(x)=x
g(x)=sqrt(x)
plot(f(x), (x,-1,2))+plot(g(x),0,2,color='red')+parametric_plot((1,y), (y,-1,2),color='green')
```

Out[34]:



In [35]:
```
S1 = revolution_plot3d(f,  (x,0,1),show_curve=True,opacity=0.5,parallel_axis='z',color='green',axis=(1,0))
S2 = revolution_plot3d(g,  (x,0,1),show_curve=True,opacity=0.3,parallel_axis='z',color='blue',axis=(1,0))
show(S1+S2,figsize=3)
```

Out[35]:



In [36]:
```
A(y)=(1-y^2)^2-(1-y)^2
V = integral(pi*A(y),y,0,1)
V
```

Out[36]:
```
1/5*pi
```

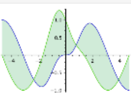
Exercise 5.6

1. Find the area between $f(x) = sin(x) - xe^{-x^2}$ and $g(x) = cos(x) - xe^{-x^2}$ and $x = 1$ and $x = 3.5$.

2. Graph the curve $y = (1 + x^2)^{\frac{3}{2}}$ for $0 \le x \le 4$ and hence find its arc length.
3. Find the volume of solid revolution of the curve $y = cos(x), 0 \le x \le \pi$ revolve about the axis $y = 1$.

In [46]:
```
## Area between two curves
f (x ) = sin(x)-x*(e)^(-x*x)
g(x) = cos(x)-x*(e)^(-x*x)
a, b = -3*pi/2,3*pi/2
h = plot ((f(x), g(x)),x, a,b, figsize =4, fill ={0:g,1:f})
show(h)
```

Out[46]:



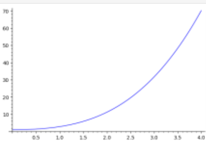
In [48]:
```
A=integral((f(x)-g(x)),x,1,3.5)
show(A.n())
```

Out[48]:

$$2.66901320565645$$

In [50]:
```
f(x)=(1+x^2)^(3/2)
plot(f(x), (x,0,4),figsize=6)
```

Out[50]:



In [52]:
```
show(integral(sqrt(1+derivative(f,x)^2),x,0,4))
```

Out[52]:

$$\int_0^4 \sqrt{9(x^2+1)x^2+1}\,dx$$

In [54]:
```
g(x)=sqrt(9*(x^2+1)*x^2+1)
```

In [55]:
```
show(integrate(f(x),x,0,4))
```

Out[55]:

$$\frac{37}{2}\sqrt{17}+\frac{3}{8}\operatorname{arsinh}(4)$$

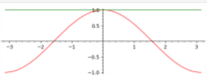
In [60]:
```
## Volume of sphere of radius r
var('x')
f(x) = cos(x)
V = integral(pi*f(x)^2,x,0,pi)
V
```

Out[60]:
```
1/2*pi^2
```

In [0]:
```
S1 = revolution_plot3d(f,  (x,0,1),show_curve=True,opacity=0.5,parallel_axis='z',color='green',axis=(1,0))
S2 = revolution_plot3d(g,  (x,0,1),show_curve=True,opacity=0.3,parallel_axis='z',color='blue',axis=(1,0))
show(S1+S2,figsize=3)
```

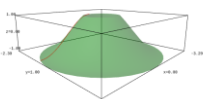
In [64]:
```
f(x)=cos(x)
plot(f(x),(x,-pi,pi),color='red')+parametric_plot((x,1),(x,-pi,pi),color='green')
```

Out[64]:



In [67]:
```
S1 = revolution_plot3d(f,  (x,0,pi),show_curve=True,opacity=0.5,parallel_axis='z',color='green',axis=(0,1))
show(S1,figsize=3)
```

Out[67]:



Learning:(Write in details about your Learnings from the activity)

1.Evaluation of definite and indefinite integration

2.How to find average value of a function

3.Calculate:

(a)Areas

(b)Arc Length

(c)Areas of Surfaces of Surface of Revolution

(d)Volumes of solid of revolution

between two or more curves

In [0]: