ATTITUDE DYNAMICS AND CONTROL OF SATELLITES ORBITING ROTATING ASTEROIDS

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Abstract

The paper focuses on the attitude dynamics and control of satellites orbiting rotating asteroids. The general formulation of the satellite equations of motion in an equatorial eccentric orbit is obtained through Lagrangian method. The linearized system model is derived and the stability analysis is presented. The control laws for three-axis attitude control of satellites are developed and a closed-form solution of the system is derived. For an illustration of the linear analysis followed by the numerical simulation of the governing nonlinear equations of motion of the satellite, several asteroids (Eros, Castalia, Vesta, Ida, and Gaspra) are considered. Attitude resonances for satellites in retrograde orbits are found. The results of the linear system model compare well with the corresponding system nonlinear equations of motion. The proposed controllers are successful in stabilizing the attitude of the satellites even in presence of high attitude disturbances and orbital eccentricities.

Keywords: *Attitude dynamics, attitude control, satellites, asteroids.*

1. INTRODUCTION

Studying the motion of asteroids and their composition has been of interest to the scientific community for a long time. This study would lend insight into how our solar system was formed. In this respect, several studies have been conducted and some missions have been flown and planned [1]. An asteroid's irregular shape, mass distribution and the state of its rotation (rapid or slow) has significant effects on the evolution of satellite orbit and attitude motion. An asteroid's gravity field is represented by the second-degree and second-order gravity coefficients C_{20} and C_{22}. In earlier studies [2], the equatorial ellipticity of the primary body (i.e., C_{22} gravity term), is assumed to be small; that is appropriate for planetary bodies in the solar system. However, C_{22} gravity terms for asteroids are much larger than those for planetary bodies, and therefore these terms can not be neglected.

Several researchers have examined the problem of orbital dynamics about asteroids [3]. Attitude dynamics about asteroids has been investigated only by Riverin and Misra [4]. In this study [4], the satellite pitch motion was examined assuming the satellite is in an equatorial orbit. The satellite attitude control was not considered. In the present study, three-dimensional attitude motion of the satellite is examined. In addition, the satellite attitude control is also considered and attitude control laws are developed to stabilize the system. Additionally, detailed study is undertaken to examine the effects of various system parameters on the system response.

2. SYSTEM EQUATIONS OF MOTION

The system comprising of a rigid body satellite is assumed to be orbiting in an equatorial plane around an asteroid (Fig. 1). The coordinate frame $X_oY_oZ_o$ passing through the system center of mass S represents the orbital reference frame. Here the X_o-axis is taken along normal to the orbital plane, the Y_o-axis points along the local vertical and the Z_o-axis represents the third axis of this right handed frame. The orientation of the satellite is specified by a set of three successive rotations: α (pitch) about the X_0-axis, ϕ about the new roll axis (Z-axis, if α=0), and finally γ about the resulting yaw axis. The corresponding principal body-fixed coordinate axes for the satellite are denoted by S-XYZ.

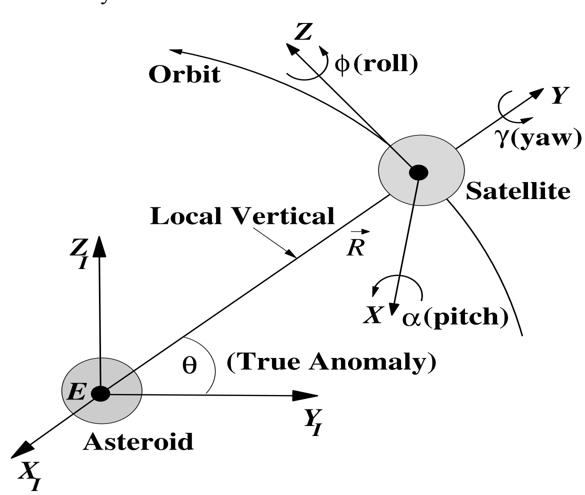

Fig. 1 Orbiting satellite around asteroid.

The asteroid is assumed to be a rotating tri-axial body with a constant rate of rotation about an axis perpendicular to the equatorial plane. The orbital motion of the satellite is fully described as a closed, planar and periodic orbit, and is considered to be negligibly affected by attitude dynamics. The Lagrangian equations of motion corresponding to the generalized coordinate $q=\alpha,\phi,\gamma$ indicated earlier with respect to the true anomaly are obtained as follows:

Pitch (α)

$$
\begin{aligned}
&(\cos^2\gamma+K_{zx}\sin^2\gamma)\{\cos^2\phi\,[e_1\alpha''-e_2(1+\alpha')] \\
&-e_1(1+\alpha')\,\phi'\sin2\phi\} \\
&-(1-K_{zx})\sin\gamma\cos\gamma\,\{\cos\phi\,[e_1\phi''-e_2\phi']+e_1\phi'^2\sin\phi\} \\
&-K_{yx}\sin\phi\,[e_1P_\gamma'-e_2P_\gamma]-K_{yx}e_1P_\gamma\phi'\cos\phi \\
&-e_1(1-K_{zx})\gamma'\cos\phi\,[(1+\alpha')\cos\phi\sin2\gamma+\phi'\cos2\gamma] \\
&-(3/2)(1+t_1)\{K_{yx}\sin2\alpha\cos^2\phi+(1-K_{zx})[(1+\sin^2\phi)\sin\alpha\cos\alpha\cos2\gamma \\
&-\cos2\alpha\sin\phi\sin2\gamma]-(1+K_{zx})\sin\alpha\cos\alpha\cos^2\phi\}+t_{2\alpha}=u_\alpha
\end{aligned}
\tag{1}
$$

Roll (ϕ)

$$
\begin{aligned}
&e_1(-1+K_{zx})\cos\phi\cos\gamma\sin\gamma\,\alpha''+e_1(\sin^2\gamma+K_{zx}\cos^2\gamma)\,\phi'' \\
&-e_2[-(1+\alpha')(1-K_{zx})\cos\phi\sin\gamma\cos\gamma+\phi'(\sin^2\gamma+K_{zx}\cos^2\gamma)] \\
&+e_1\{(1-K_{zx})\gamma'[-(1+\alpha')\cos\phi\cos2\gamma+\gamma'\sin2\gamma] \\
&+(1+\alpha')^2(\cos^2\gamma+K_{zx}\sin^2\gamma-K_{yx})\sin\phi\cos\phi \\
&+K_{yx}(1+\alpha')\gamma'\cos\phi\}-(3/2)(1+t_1)[(K_{yx}-K_{zx}\sin^2\gamma-\cos^2\gamma) \\
&\times\cos^2\alpha\sin2\phi+\sin2\alpha\sin2\gamma\cos\phi]+t_{2\phi}=u_\phi
\end{aligned}
\tag{2}
$$

Yaw (γ)

$$
\begin{aligned}
&K_{yx}[e_1P_\gamma'-e_2P_\gamma]+e_1(1-K_{zx})[(1+\alpha')\cos\phi\sin\gamma \\
&+\phi'\cos\gamma][(1+\alpha')\cos\phi\cos\gamma-\phi'\sin\gamma] \\
&-3(1+t_1)(1-K_{zx})(\cos\alpha\sin\phi\cos\gamma+\sin\alpha\sin\gamma) \\
&\times(\cos\alpha\sin\phi\sin\gamma-\sin\alpha\cos\gamma)+t_{2\gamma}=u_\gamma
\end{aligned}
\tag{3}
$$

where $e_1=1+e\cos\theta$, $e_2=2e\sin\theta$, $P_\gamma=\gamma'-(1+\alpha')\sin\phi$,

$t_1=-5(R_e/R)^2\left[0.5C_{20}-C_{22}(4+7\cos2\lambda_c)\right]$, and

$$
\begin{aligned}
t_{2q}=&-1.5(R_e/R)^2C_{20}[(K_{zx}+1-K_{yx})b_{10}b_{10q} \\
&+(1-K_{zx}+K_{yx})b_{11}b_{11q}+(-1+K_{zx}+K_{yx})b_{12}b_{12q}] \\
&-3(R_e/R)^2C_{22}[(K_{zx}+1-K_{yx})(2b_7b_{7q}+b_{10}b_{10q}) \\
&+(1-K_{zx}+K_{yx})(2b_8b_{8q}+b_{11}b_{11q}) \\
&+(-1+K_{zx}+K_{yx})(2b_9b_{9q}+b_{12}b_{12q})] \\
&+30(R_e/R)^2C_{22}\cos\lambda_c[(K_{zx}+1-K_{yx})(b_{1q}b_7+b_1b_{7q}) \\
&+(1-K_{zx}+K_{yx})(b_{2q}b_8+b_2b_{8q}) \\
&+(-1+K_{zx}+K_{yx})(b_{3q}b_9+b_3b_{9q})],\quad q=\alpha,\phi,\gamma
\end{aligned}
$$

C_{n0}=zonal harmonic coefficients of order 0; $K_{zx}=I_z/I_x$; $K_{yx}=I_y/I_x$; I_k=mass moment of inertia of the satellite about k-axis, k=x,y,z; R_e =equatorial radius or characteristic length of the asteroid; λ_c=longitude of satellite center of mass in the asteroid-fixed frame;

$a_1=0.5C_{20}(R_e/R)^2$; $a_2=3(R_e/R)^2C_{22}$;

$b_1=\cos\alpha\cos\phi$; $b_2=\cos\alpha\sin\phi\sin\gamma-\sin\alpha\cos\gamma$;

$b_3=\cos\alpha\sin\phi\cos\gamma+\sin\alpha\sin\gamma$; $b_4=\sin\alpha\cos\phi$;

$b_5=\sin\alpha\sin\phi\sin\gamma+\cos\alpha\cos\gamma$;

$b_6=\sin\alpha\sin\phi\cos\gamma-\cos\alpha\sin\gamma$; $b_7=b_1\cos\lambda-b_4\sin\lambda$;

$b_8=b_2\cos\lambda-b_5\sin\lambda$; $b_9=b_3\cos\lambda-b_6\sin\lambda$;

$b_{10}=-\sin\phi$; $b_{11}=\cos\phi\sin\gamma$; $b_{12}=\cos\phi\cos\gamma$;

3. CONTROLLER DESIGN

With a view to understand the system dynamics, we linearize the equations of motion of the system (Eqs. (1) - (3)) about the system equilibrium state given by $\mathbf{q}''=\mathbf{q}'=0$. The resulting linear equations of motion of the system are as follows:

$$
\begin{aligned}
\alpha''+3k_2\left[1-\left(\frac{R_e}{R}\right)^2\left(\frac{5}{2}C_{20}-19C_{22}\cos\nu\theta\right)\right]\alpha& \\
-24k_2C_{22}\left(\frac{R_e}{R}\right)^2\sin\nu\theta-2e\sin\theta=0&
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
\phi''+k_1\left[4-3\left(\frac{R_e}{R}\right)^2\left(3.5C_{20}-17C_{22}\cos\nu\theta\right)\right]\phi& \\
+(1-k_1)\gamma'-24k_3\left[C_{22}\left(\frac{R_e}{R}\right)^2\sin\nu\theta\right]\gamma=0&
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\gamma''+k_3\left[1-3\left(\frac{R_e}{R}\right)^2\left(C_{20}+2C_{22}\cos\nu\theta\right)\right]\gamma& \\
-(1-k_3)\phi'-24k_3\left[C_{22}\left(\frac{R_e}{R}\right)^2\sin\nu\theta\right]\phi=0&
\end{aligned}
\tag{6}
$$

where $k_2=(k_1-k_3)/(1-k_1k_3)$, $k_1=(I_x-I_y)/I_z$, $k_2=(I_z-I_y)/I_x$ and $\nu=2[1\pm(\omega/\Omega)]$, with ω as the asteroid's spin rate and Ω as the orbital rate. Here, (+) sign denotes prograde orbits or direct orbits while (-) sign signifies retrograde orbits. Note that the pitch motion is decoupled from the roll and yaw motions, and this fact is similar to the case of a satellite orbiting symmetrically mass distributed planetary bodies. Assuming that the satellite is orbiting in a circular orbit (i.e., e=0), neglecting the product of $\sin\nu\theta$ or $\cos\nu\theta$ and attitude angle (as the average $\sin\nu\theta$ or $\cos\nu\theta$ over an orbit is zero) and the attitude angles are considered small, the characteristics equations of the system are:

$$
s^2+3k_2\left[1-2.5\left(\frac{R_e}{R}\right)^2C_{20}\right]=0
\tag{7}
$$

$$s^4+\left[1+3k_1+k_1k_3-3\left(\frac{R_e}{R}\right)^2 C_{20}\left(3.5k_1+k_3\right)\right]s^2$$
$$+k_1k_3\left[4-22.5\left(\frac{R_e}{R}\right)^2 C_{20}+31.5\left(\frac{R_e}{R}\right)^4 C_{20}^2\right]=0 \tag{8}$$

Applying Routh-Hurwitz criterion, the conditions of stability can be expressed as

$$(k_1-k_3)\left[1-\frac{5}{2}\left(\frac{R_e}{R}\right)^2 C_{20}\right]>0 \tag{9}$$

$$1+3k_1+k_1k_3>3\left(\frac{R_e}{R}\right)^2 C_{20}\left(3.5k_1+k_3\right) \tag{10}$$

$$k_1k_3\left[4-22.5\left(\frac{R_e}{R}\right)^2 C_{20}+31.5\left(\frac{R_e}{R}\right)^4 C_{20}^2\right]>0 \tag{11}$$

$$\left[1+3k_1+k_1k_3-3\left(\frac{R_e}{R}\right)^2 C_{20}\left(3.5k_1+k_3\right)\right]^2$$
$$-4k_1k_3\left[4-22.5\left(\frac{R_e}{R}\right)^2 C_{20}+31.5\left(\frac{R_e}{R}\right)^4 C_{20}^2\right]>0 \tag{12}$$

Next we analyze the existence of parametric resonance. Referring to the pitch equation of motion given by Eq. (4), when the satellite is in a retrograde orbit, the parametric pitch resonance exists [4] and the condition for the same for the satellite in a circular orbit is as follows [4]:

$$R=\frac{\mu^{1/3}}{\omega^{2/3}}\left[\frac{j\mp(3k_2)^{1/2}}{j}\right]^{2/3}, \quad j=1,2,3,\ldots \tag{13}$$

Taking ω_q and ξ_q, $q=\alpha,\phi,\gamma$ as closed-loop frequency and damping ratio, respectively and final desired final attitude angles and the corresponding rates are assumed to be null, the following control laws are obtained:

$$u_\alpha=3k_2\left[1-\left(\frac{R_e}{R}\right)^2\left(\frac{5}{2}C_{20}-19C_{22}\cos\nu\theta\right)\right]\alpha-\omega_\alpha^2\alpha$$
$$-2\xi_\alpha\omega_\alpha\alpha'+24k_2C_{22}\left(\frac{R_e}{R}\right)^2\sin\nu\theta-2e\sin\theta \tag{14}$$

$$u_\phi=k_1\left[4-3\left(\frac{R_e}{R}\right)^2\left(3.5C_{20}-17C_{22}\cos\nu\theta\right)\right]\phi-\omega_\phi^2\phi$$
$$-2\xi_\phi\omega_\phi\phi'+(1-k_1)\gamma'-24k_3\left[C_{22}\left(\frac{R_e}{R}\right)^2\sin\nu\theta\right]\gamma \tag{15}$$

$$u_\gamma=k_3\left[1-3\left(\frac{R_e}{R}\right)^2\left(C_{20}+2C_{22}\cos\nu\theta\right)\right]\gamma-\omega_\gamma^2\gamma$$
$$-2\xi_\gamma\omega_\gamma\gamma'-(1-k_3)\phi'-24k_3\left[C_{22}\left(\frac{R_e}{R}\right)^2\sin\nu\theta\right]\phi \tag{16}$$

Note that in the preceding equations, the satellite attitude angles and the corresponding rates are assumed to be measured by onboard attitude sensors.

5. Results and Discussion

In order to study the system dynamics and control, the detailed system response is numerically simulated using Eqs. (1) – (3). The numerical integration is carried out using International Mathematical and Statistical Library (IMSL) routine DDASPG based on the Petzoid-Gear BDF method. The desired satellite attitude angles are null and the attitude control response settling time (1% of steady-state value) is assumed to be 0.5 orbit; this corresponds to controller parameters of ω_q =1.5 and ξ_q =1, $q=\alpha,\phi,\gamma$. The physical parameters of asteroids have been taken from Refs. [3,4]. The asteroids are assumed to be bodies of constant density ρ and triaxial ellipsoids with major semiaxes of *a*, *b*, and *c*.

We first study the attitude motion of the satellite in the absence of control torques (Fig. 2). This unstable response is because of parametric resonance at certain orbital radii as per Eq. (13) for a satellite in retrograde orbits. Note that such resonant radii do not exist when satellites orbit in a prograde or direct orbit. Next, we examine the efficacy of controllers developed in Section 3. Figure 3 shows the new controller performance as affected by satellite motion around various asteroids (Vesta, Ida, and Gaspra). Even considering resonant orbital radius, the proposed controllers are highly successful in stabilizing satellite attitude with desired requirements. The steady state pitch control inputs are $|u_\alpha|_{max}$=0.01 (for Vesta), $|u_\alpha|_{max}$=0.08 (for Ida), and $|u_\alpha|_{max}$=0.04 (for Gaspra) while the corresponding periods of these inputs are 1 orbit (for Vesta), 0.6 orbit (for Ida) and 0.6 orbit (for Gaspra), respectively. These differences in the periods arise from the fact that the period of control input is $1/|\nu|$ (referring to Eqs. (14-16)) while $|\nu|$=0.824 for Vesta and $|\nu|$=1.667 for Ida and Gaspra.

6. CONCLUSIONS

The present paper examines the attitude dynamics and control aspects of satellites orbiting rotating asteroids. The equations of motion of the system are derived using Lagrangian approach with the assumption that orbital motion is not affected by attitude motion. The linearized system model is developed and stability analysis is presented. The control laws are derived and the efficacy of the controller performance is tested via numerical simulation of the nonlinear governing equations of motion of the system. Results of the numerical simulation match with closed-form solutions and these results along with the linear system analysis provide several insights into the dynamics and control of satellites orbiting asteroids. Attitude resonances occur for satellites in retrograde orbits. These resonances for several asteroids ((Eros, Castalia, Vesta, Ida, and Gaspra) are numerically found and compared with analytical results. The shape of the asteroid predominantly affects the satellite pitch motion. As a consequence, the controllers based on the assumption of a spherical symmetrical body of the asteroid, result in drastic deterioration of the satellite attitude performance. However, the proposed controllers accounting for the asteroid shape result in remarkable attitude accuracies. Even in the presence of high attitude disturbances, orbital eccentricities, and changes in satellite mass moment of inertia parameters, the controller performance remains almost unaffected for all asteroids considered in the present study.

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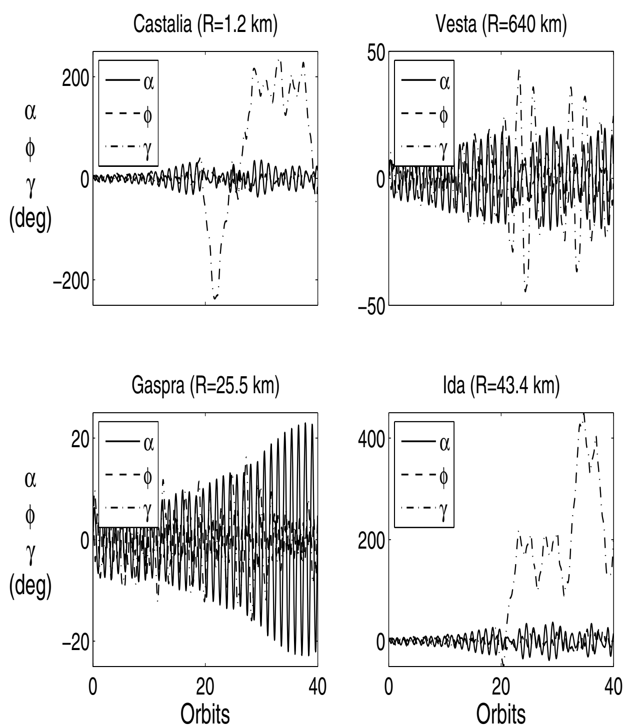

Fig. 2 Attitude response as affected by satellite motion in retrograde orbit around asteroids: e=0, k_1=0.5, k_3=0.3, $\alpha_0 = \phi_0 = \gamma_0$ =0, $\alpha'_0 = \phi'_0 = \gamma'_0$ =0.1, u_q =0, q=α,φ,γ.

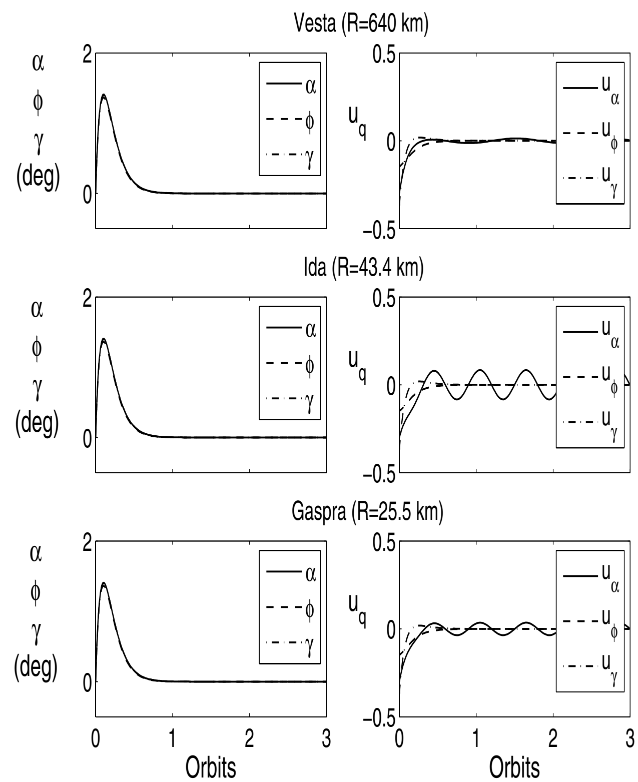

Fig. 3 Controller performance as affected by satellite motion around asteroids: e=0, k_1=-0.5, k_3=0.3, $\alpha_0 = \phi_0 = \gamma_0$ =0, $\alpha'_0 = \phi'_0 = \gamma'_0$ =0.1.