AIAA Guidance, Navigation, and Control Conference and Exhibit
15 - 18 August 2005, San Francisco, California

AIAA 2005-6282

An Input-to-State Stabilizing Control Lyapunov Function for Autonomous Guidance and Control

J. Willard Curtis
Air Force Research Laboratory
Eglin AFB, FL

Abstract

This paper presents a novel control Lyapunov function (clf) for guiding an autonomous munition to a mobile and evasive target. The scenario is modeled as a planar engagement with discrete time, and the target's motion is modeled as an exogenous input disturbance. The guidance/control law is based on Sontag's concept of input-to-state stability (ISS), and an ISS control Lyapunov function is constructed to guarantee this brand of robust stability subject to realistic evasion capabilities.

The state evolution of the munition is modeled kinematically via the well-known (non-holonomic) unicycle model, and the realistic assumptions of limited turn rate and limited forward velocity are explicitly modeled and naturally admitted into the Lyapunov framework.

The advantage of using this clf-based strategy is that an entire family of stabilizing feedback laws is produced. This paper shows how any particular member of this family can be chosen at any time step (through a complete point-wise parameterization); thus, the munition might choose from a variety of distinct control laws depending on the particular demands of its current situation.

This flexibility can be utilized by autonomous systems to better address higher-level goals and commands that are passed down from, e.g. a mode selection algorithm. This paper demonstrates, through simulation, a realistic scenario where an autonomous munition switches (in mid-course) to a different guidance law in response to a mode command change (from a fast-attack mode to a time-coordinated strike mode).

1 Introduction

The guidance and control of wide-area search munitions (WASMs [6]) is an active area of reasearch due to the ongoing improvements in cooperative algorithms [3] and decentralized command and control modalities. In this paper, the concept of input-to-state stability [13, 7, 8] (ISS), where regulation to the origin is weakened to regulation to some region of the origin dependent on the strength of disturbance inputs, is applied to a realistic WASM model.

Specifically, we model the WASM with a non-holonomic kinematic model which includes a stochactic disturbance input whose structure captures the evasive maneuver potential of ground targets as well as possible wind or other aerodynamic disturbances. Our approach is to construct a numerical object that will mimic an input-to-state control Lyapunov function (ISS-clf). This will take the form of a large look-up table that stores the Lyapunov value of a tesselation of small regions in the state space (similar to a gain scheduling approach). The benefit of using an ISS-clf is that ISS is guaranteed for a well-defined class of disturbances (target maneuvers), and the numerical approach to constructing a quasi-clf entity is inspired by [12], where a clf was used to estimate the cost-to-go in a receding horizon optimization framework.

Essentially, we address the problem of a vehicle with nonlinear actuator constraints in the form of a constant positive air-speed and limited turn-rate, coupled with a dynamic target whose evasive maneuvers are unkown and thus modeled as an exogenous disturbance. ISS-clfs are an attractive tool for generating a control signals because the theory exists for explicitly dealing with actuator constraints and random input disturbances even for nonlinear systems.

Specifically, extensions to the theory of clf-based control has emerged for nonlinear systems that are subject to input constraints [9, 10, 11, 15, 14]. Also we address the need for different operational modes and mission-level goals by parameterizing the class of control laws that will guarantee Lyapunov stability: it has previously been shown [5, 4] that the point-wise Lyapunov stability requirement, $\dot{V} < 0$, can be interpreted as adding another bounding hyperplane to the input constraint polytope. From this half-space representation we can exploit the *vertex enumeration* algorithm introduced in [2, 1] to find a vertex representation of a stabilizing polytope.

We first introduce some preliminary theory in Section 2, where ISS is reviewed and the concept of an ISS-clf is precisely defined. Then, in Section 2.2, the method of vertex enumeration is explained in terms of constraint-generated bounding hyper-planes in the control space. Section 3 describes our model of the WASM and how we generated our quasi-ISS-clf object, as well as how two distinct mission-level modes of operation can be implemented via a vertex enumeration of control values. Finally Section 5 presents some results from simulation and directions for future research.

2 Preliminary Theory

Input-to-state stability describes a method for attenuating an exogenous disturbance and analytically bounding its effects on the system state. The following results briefly present the results that will be relevent to this paper.

2.1 Input-to-state Stability and ISS-clfs

Consider the affine nonlinear system

$$\dot{x} = F(x) + g_1(x)d, \tag{1}$$

where $x \in \mathbb{R}^n$, $F : \mathbb{R}^n \to \mathbb{R}^n$, $g_1 : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ and $d \in \mathbb{R}^r$.

Definition 2.1 *([8]) The system given in Equation* (1) *is* **input-to-state stable** *if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function χ, such that, for any $x(t_0)$ and for any input $d(\cdot)$ continuous on $[0,\infty)$ a solution exists for all $t \geq 0$ and satisfies:*

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0) + \chi(\sup_{t_0 \leq \tau \leq t} |d(\tau)|). \tag{2}$$

Now consider the autonomous system

$$\dot{x} = f(x) + g(x)u + g_1(x)d, \tag{3}$$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}^n$, $g_1 : \mathbb{R}^n \to \mathbb{R}^{n\times r}$, $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$, with control input $u \in \mathbb{R}^m$, and exogenous input $d \in \mathbb{R}^r$. We will assume throughout the paper that f, g, and g_1 are locally Lipschitz functions and that $f(0) = 0$. System (3) will be called input-to-state stabilizable (ISS) if there exists a continuous control law $u = q(x)$ with $q(0) = 0$, such that the closed-loop system is input-to-state stable with respect to the disturbance d.

Definition 2.2 *A continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ is said to be an* **input-to-state stabilizing control Lyapunov function (ISS-clf)** *for system* (1) *if V is positive definite, radially unbounded, and if there exists a class $\mathcal{K}_\infty$ function ρ such that the following implication holds $\forall x \neq 0$ and $\forall\ d \in \mathbb{R}^r$:*

$$|x| \geq \rho(|d|) \Rightarrow \inf_{u\in\mathbb{R}^m} V_x^T(f + gu + g_1 d) < 0. \tag{4}$$

Lemma 2.3 *([8]) A pair (V, ρ), where $V(x)$ is positive definite and radially unbounded, and $\rho \in \mathcal{K}_\infty$ satisfies Equation*(4) *if and only if*

$$V_x^T g = 0 \Rightarrow V_x^T f + |V_x^T g_1|\rho^{-1}(|x|) < 0\ \forall\ x \neq 0. \tag{5}$$

Definition 2.4 *([8]) The ISS-clf V is said to satisfy the* **small control property** *if there exists a control law $\alpha_c(x)$ continuous in $\mathbb{R}^n$ such that*

$$V_x^T f + V_x^T g\alpha_c + |V_x^T g_1|\rho^{-1}(|x|) < 0, \quad \forall x \neq 0.$$

where ρ satisfies the requirements of (4).

Theorem 2.5 *([8]) System* (3) *is input-to-state stabilizable if and only if there exists an ISS-clf with the small control property.*

Finally, we can state the fundamental result that will be used in the remainder of this paper to ensure input-to-state stability.

Theorem 2.6 *Given an ISS-clf V satisfying the small control property, system* (3) *will be input-to-state stable if and only if $u = k$ is a continuous control law with $k(0) = 0$ and such that*

$$V_x^T f + V_x^T gk + |V_x^T g_1|\rho^{-1}(|x|) < 0 \quad \forall x \neq 0. \tag{6}$$

Proof: Follows directly from Lemma 2.3.

■

Definition 2.7 *We say that a control law is* input-to-state stabilizing with respect to V (ISS_V), *if it satisfies the requirements of Theorem 2.6.*

These results imply that when the size of the disturbance is small compared to the size of the state, the Lyapunov measure must decrease. This differs from a traditional notion of Lyapunov assymptotic stability because the state is likely to converge to some region around the origin due to the possible Lyapunov measure increase when the size of the state becomes small.

2.2 The Stabilization Constraint

Now we present some theory involving interpreting Lyapunov stability and input constraints in a common geometrical framework. We begin by defining a polytope and then show how input constraint and ISS can be interpreted as sides of an m-dimensional polytope.

Definition 2.8 *A* **polytope***, $P \subset \mathbb{R}^m$, is the bounded intersection of a finite number of closed half-spaces:*

$$P \triangleq \left\{z \in \mathbb{R}^m : \; z^T h_i \leq b_i, 2m \leq i \leq k < \infty\right\}$$

We assume that the control values are constrained to lie in some polytope, $\mathcal{U}$, which contains the origin. This input constraint can be written compactly as follows:

$$\mathcal{U} = \{u \in \mathbb{R}^m : \; Mu \leq b\}, \tag{7}$$

where the matrix $M \in \mathbb{R}^{k \times m}$ and the vector $b \in \mathbb{R}^k$ define the polytope. For the ubiquitous special case when $\mathcal{U}$ is a hyper-box, $M \in \mathbb{R}^{2m \times m}$ can be represented as

$$M \triangleq \begin{bmatrix} I \\ -I \end{bmatrix},$$

and the vector b is of length $2m$ with the i^{th} element of b containing the magnitude of the control constraint along the i^{th} semi-axis. (In other words $\mathcal{U}$ is a hyper-box which encloses the origin and whose faces are perpendicular to the axes. The i^{th} face of $\mathcal{U}$ lies a distance $b(i)$ from the origin.)

We assume that there is a minimum desired rate of decrease, $\dot{V} = V_x^T(f + gu + g_1 d) \leq -\epsilon(\|x\|)$, where $\epsilon(\|x\|)$ is a positive-definite function and $V_x(x)$ is the gradient of V with respect to x; this assumption will ensure a closed control value set.

Definition 2.9 *The* **Stabilizing Set***, denoted $\mathcal{S}(x)$ is a state-dependent control value set containing all the points in $\mathbb{R}^m$ which satisfy:*

$$\mathcal{S}(x) \triangleq \left\{u \in \mathcal{U}\text{: } V_x^T(f + gu + g_1 d) \leq -\epsilon(\|x\|)\right\}, \tag{8}$$

Definition 2.7 guarantees that $\mathcal{S}(x)$ is always non-empty if $\epsilon(\|x\|)$ is chosen to be sufficiently small. S is thus a closed state-dependent half-space in $\mathbb{R}^m$.

Definition 2.10 *Given a constraint polytope, $\mathcal{U} \subset \mathbb{R}^m$, A control law $u(t)$ is* **feasible** *if $u(\cdot) \in \mathcal{L}^\infty$ and $u(t) \in \mathcal{U}$ for all $t \geq t_0$.*

It can be trivially shown that any continuous selection from S will be a feasible control and will render the closed loop system asymptotically stable (since $\dot{V} < 0 \;\; \forall \;\; x \neq 0$ and $u \in S \Longrightarrow u \in \mathcal{U}$).

The next theorem shows how the stability constraint embodied in (8) can be folded into the input constraints described by (7).

Theorem 2.11 $\mathcal{S}(x) = \left\{u \in \mathbb{R}^m : \overline{M}u < \overline{b}\right\}$*, where $\overline{M}$ and $\overline{b}$ are defined as follows:*

$$\overline{M} = \begin{bmatrix} M \\ V_x^T g \end{bmatrix},$$

$$\overline{b} = \begin{bmatrix} b \\ -V_x^T f - |V_x^T g_1|\rho^{-1}(|x|) - \epsilon(\|x\|) \end{bmatrix}.$$

Proof: The matrix inequality

$$\overline{M}u \leq \overline{b} \tag{9}$$

represents a system of inequalities. The first k rows ensure that $u \in \mathcal{U}$. The last $(k+1)$ row, $V_x^T gu < -V_x T f - |V_x^T g_1|\rho^{-1}(|x|) - \epsilon(\|x\|)$ is simply a restatement of the Lyapunov stability requirement.

Note that Theorem (2.11) reveals an important result: asymptotic stability at any fixed state can be viewed as a *linear constraint* on the input (see Figure 1). This reduces the design of a feasible, stabilizing control to performing a point-wise selection from the well-defined set $\mathcal{S}(x)$.

Figure 1 shows a two dimensional example of the the state-dependent set S at some state where $V_x^T f + |V_x^T g_1|\rho^{-1}(|x|) + \epsilon(\|x\|) = 2$, $V_x^T g = (1 \;\; 1)^T$, and $\mathcal{U}$ is rectangular. Note that S is always closed, always convex, and always a polytope.

2.3 Parameterizing $\mathcal{S}$

In order to put the constraints in a form amenable to a convenient parameterization, it will first be necessary to find the vertices of the convex polytope defined by (9). This problem can be solved via the *vertex enumeration* algorithm introduced in [2]. We will not repeat the details of this algorithm, but its function is explained below.

The Minkowski-Weyl Theorem [16, p. 29] states that every polytope $\mathcal{P}$ can be described as the intersection of a finite set of half spaces, $\mathcal{P} = \{\xi \in \mathbb{R}^m : A\xi \leq d\}$, called the $\mathcal{H} - representation$, or described equivalently in terms of the convex hull of its vertices $\mathcal{V} = \{v_1, v_2, ...v_N\}, \;\; v_i \in \mathbb{R}^m, \;\;$ and $N < \infty$, termed its $\mathcal{V} - representation$.

The vertex enumeration algorithm transforms a polytope's $\mathcal{H} - representation$ into its $\mathcal{V} -$

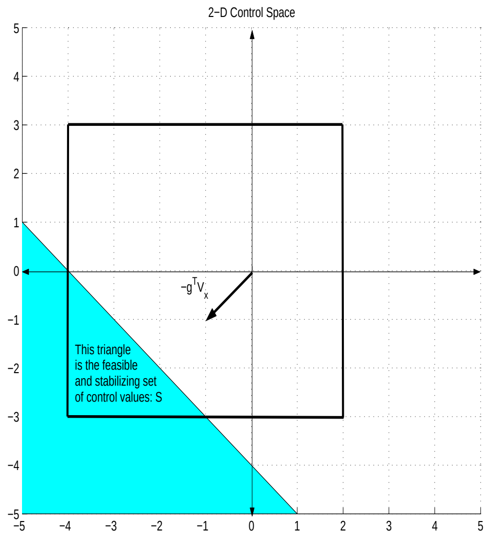

Figure 1: Stabilizing Polytope

representation. More specifically, given (9) which has been pruned of its redundant inequalities, the algorithm generates a set of points $\mathcal{V} = \{v_1, v_2, ...v_N\}$, $v_i \in \mathbb{R}^m$ corresponding to the (exterior) vertices of the polytope $S \subset \mathbb{R}^m$.

The constrained stabilizing control value set, $\mathcal{S}(x)$, can be completely parameterized in terms of its vertices.

Definition 2.12 *Given a polyhedron's $\mathcal{V}-$ representation, we define the $m \times N$* **vertex matrix** *V_m and the $N \times 1$* **weighting vector** *w as follows:*

$$V_m = \begin{bmatrix} v_1 & v_2 & \cdots & v_N \end{bmatrix},$$
$$w^T = \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix},$$

where $\sum_{i=1}^{N} w_i = 1$ and $w_i \geq 0$.

Note that all control values in S can be written as $u = V_m w$, since S is a closed, convex polytope and since Vw is a convex weighting of points on the surface of S. The weighting vector w therefore defines a particular convex combination of the vertices, resulting in a point on the interior or on the surface of $\mathcal{S}$. Thus, if we know the $\mathcal{V} - representation$ of $\mathcal{S}$ at some state, then S is completely parameterized by the weighting vector w at that state. We have now reduced the problem of designing a stabilizing controller in the presence of input constraints to the point-wise choice of an $N-$dimensional weighting vector.

This parameterization of S allows us to incorporate team goals in the form of mode commands: each mode command will be associated with a unique policy for choosing the weight vector w.

3 Scenario Model

We assume the existence of a mission-level planner that will generate at all time one of two mode commands (fast attack, time-coordinated strike), and that the WASM will instantaneously know this mode command as well as the time target for the coordinated strike mode. Also, we assume that the munition has at all times a target way-point (a point in $\mathbb{R}^2$). In the event that the WASM achieves its way-point (or comes within some epsilon ball of its way-point), we assume that a new way-point will be supplied by the mission-level planner. Our goal in designing controllers for the agents is to guarantee regulation to their way-points while simultaneously acting in a manner that accomplishes the mission-mode command.

We model the kinematics of the WASM in discrete-time, as follows:

$$\begin{aligned} x_{k+1} &= x_k + v\cos(\theta + u_k) + d_k^1, \\ y_{k+1} &= y_k + v\sin(\theta + u_k) + d_k^2, \\ \theta_{k+1} &= \theta_k + u_k; \end{aligned} \tag{10}$$

where x and y are the spatial coordinates, v (linear velocity) is a positive constant, θ is the WASM's heading angle, and where the control variable is u the target maneuver is the disturbance function $d(k) \in \mathbb{R}^2$. We will assume that u, the angular rate of the vehicle, is limited as follows:

$$|\omega| \leq \bar{\omega}.$$

Asymptotic stabilization in the usual sense is impossible, however our motivation in studying this problem is related to a real-world need to autonomously control a WASM, we thus re-pose the regulation problem as follows: we desire to drive the spatial state of the agent to within an epsilon-ball of its way-point in finite-time. We assume, without loss of generality, that the target way-point is the origin and that the target maneuver thus affects the relative distance between munition and target by modifying the WASM state through the disturbance d.

We denote this objective as “partial input-to-state stabilization” since we are not concerned with controlling angular orientation and though we are also unconcerned with the state of the agent as time goes to infinity we do require regulation to within some region of the origin.

This system's input constraints are clearly rectangular and can be written in the form of (7) as follows:

$$Mu \le b;$$
$$M \triangleq \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad b \triangleq \begin{bmatrix} \bar{\omega} \\ \bar{\omega} \end{bmatrix}.$$

It remains to find an ISS-clf for system (3) in order to use the vertex enumeration technique for stabilization. Since (3) is nonholonomic and discrete-time, we will search for a non-smooth quasi ISS-clf function, whose purpose will be similar to that of a regular ISS-clf: at every state it will indicate a direction in the control space, $V_x g$, along which increasing magnitudes result in increasingly quick regulation to the origin, despite point-wise disturbances.

To accomplish the construction of this quasi-ISS-clf, we started with an open-loop algorithm that generates a flyable path from any point in $\mathbb{R}^2$ to the origin: Dubin's algorithm of traveling along circular arcs and straight line segments. The arcs are constructed with the minimum turn radius of the vehicle (derived from the turn rate and velocity). An optimization over this algorithm was performed over a large region of the state space and the length of the resulting path was stored in a two-dimensional array (V), indexed by range from the origin and initial vehicle heading angle relative to the range vector. Thus, this matrix returns a (always positive except at the origin) path length given a range from the origin and a heading. In order to incorporate the target's potential for maneuver, the magnitude of a computed δV was computed and an upper bound on target maneuver (at any point in a region of the state space) was derived. Thus, this matrix can function as an estimate of an ISS-clf – a mathematical (or numerical) object which provides an estimate of the "cost-to-go" at any (range, heading) state of the WASM. This matrix V, will generate a vector $V_x^T g$ at any point in the state space (actually indicating whether the WASM should turn left or right and with what angular rate to ensure ISS) via a finite difference algorithm. Thus, the theory outlined in Section 2.2 can be brought to bear on the partial-stabilization problem described above.

4 Cooperative Modes

At any state, the weapon's control law can be expressed as:

$$u(x) = V_m(x)w(x),$$

where we have explicitly shown state-dependence to emphasize the (possibly) dynamic nature of the weighting vector, w. The choice of weighting vector gives great flexibility and adaptability to the final control laws, and the only requirement on $w = [w_1 \cdots w_i \cdots w_N]$ is that $\sum_{i=1}^{N} w_i = 1$ and $w_i \ge 0$.

By defining two policies for choosing w at every state, corresponding to two possible mode commands, we enable responsiveness to multiple high level objectives. We do this by defining two cost functions, and setting the weight-selection policies as optimizations of these cost functions.

The first mode is "fast attack", where the munition should move as quickly as possible to its current way-point. One way of guaranteeing a timely trajectory toward a way-point is to choose the weight vector that will maximize the rate of Lyapunov decrease, $\dot{V}$. Thus, we define the "attack" cost function as follows:

$$J_2(w) = V_x^T g V_m w, \tag{11}$$

where the right hand side of equation (11) is seen to be equivalent to the Lyapunov rate associated with weight w, given vertex matrix V_m, system equation $g(x)$, and Lyapunov gradient V_x. Clearly minimization of J_2 will maximize the rate at which V decreases and thus move the WASM quickly toward the target way-point. The selection policy is therefore $w = \arg\min_w J_2$.

The second mode is "time-coordinated strike", where the munition should arrive at that way-point in at a certain time. One way of inducing this behavior is to have the munition be pulled toward a virtual guide, and then move this guide toward the waypoint such that the guide arrives at the desired moment in time. Thus, the cost function for the rendezvous mode becomes:

$$J_3(w) = \|x_{k+1}(w) - vg\|, \tag{12}$$

where the vg is the position of the virtual guide, and $x_{k+1}(w)$ is the position of WASM at the next time step as a function of the weight vector w. It should be noted that exactly coordinating the arrival time of a weapon is not always possible, and the efficacy of this mode is dependent on the turn radius of the WASM, as well as on the position of the weapon as it enters the "time-coordinated" mode. The selection policy for this mode is likewise $w = \arg\min_w J_3$.

We have now completely defined the control laws for the munition and it is guaranteed to achieve regulation to some region around its way-point as well as responsiveness to higher-level mode commands. It should be noted that more sophisticated high-level behaviors are easily accommodated by this vertex-enumeration framework.

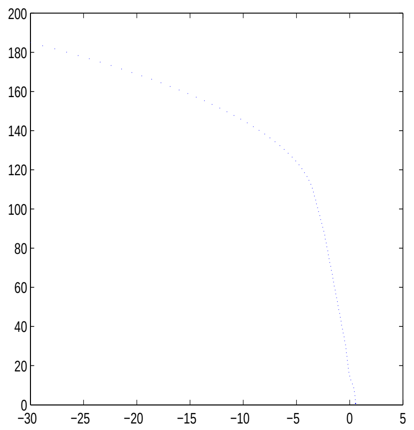

Figure 2: Fast Attack Mode

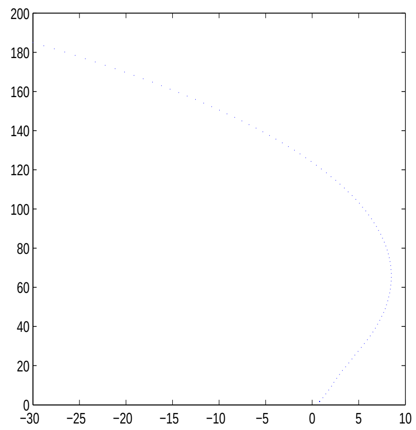

Figure 3: Time-Coordinated Mode

5 Conclusion

For our simulation we used SIMULINK, with the following parameters:

maximum turn rate = 14 degrees/sec
forward velocity = 40 m/sec
sample time = 20 samples/sec
max disturbance per sample = .5 m

Figure 2 shows a typical trajectory when the WASM is in fast attack mode, it follows the Lyapunov cues encoded in the matrix V to the origin to quickly move to the origin even when the system is disturbed by the target's random maneuvers.

Figure 3 shows a typical trajectory when the WASM is following a virtual guide so as to arrive at the target according to some mission-mediated time schedule. The mission-level planner was responsible for passing the WASM the coordinates of the virtual guide at every time step in this simulation.

This paper introduced a novel technique of encoding guidance information into a quasi-ISS-clf array, whose values act as indicators of the "distance" to the origin from any given state. This information was made sensitive to the actuation constraints of the munition as well as the maneuver capacity of the target, and two distinct optimizations were performed online in order to leverage the encoded Lyapunov information into tailor-made feedback control laws.

Future work is needed to analyze predicted circular-error probable (CEP) when munitions guide toward targets using ISS-clf-based control laws. This paper was a first step in applying the theory of input-to-state stability to a real world application, and we expect to show future benefits of using vertex-enumeration in contexts where actuation constraints and nonlinear plant dynamics make traditional linear-type control methodologies futile.

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