AIAA Guidance, Navigation and Control Conference and Exhibit
20 - 23 August 2007, Hilton Head, South Carolina

AIAA 2007-6355

# Autonomous rendezvous: Guidance and Control in elliptical orbit

N. Silva[*], M. Ganet[†], E. Gogibus[‡], P. Delpy[§] and P. Augros[**]
*Astrium Space Transportation, Les Mureaux, France*

**This paper deals with a spacecraft autonomous rendezvous in a targeted elliptical orbit. Spacecrafts like European ATV or Japanese HTV are automated but not fully autonomous. One focuses in particular on guidance and control algorithms whose main guidelines are based on ATV ones. The implemented techniques presented hereafter were developed in the frame of in-house studies. This paper presents alternatives to classical Clohessy and Wiltshire to deal with elliptical orbits, consequent guidance and its performances. Robust autonomous control algorithms are also identified and analyzed for each phase and sub-phase of the rendezvous mission scenario in both circular and elliptical orbits.**



## Nomenclature

| | | |
|---|---|---|
| *AMM* | = | Autonomous Mission Management |
| *AR&C* | = | Automated Rendezvous and Capture |
| *AR&D* | = | Automated Rendezvous and Docking |
| *ATV* | = | Automated Transfer Vehicle |
| *AVGS* | = | Advanced Video Guidance Sensor |
| *CAM* | = | Collision Avoidance Maneuver |
| *CDR* | = | Critical Design Review |
| *CoM* | = | Center of Mass |
| *DART* | = | Demonstration of Autonomous Rendezvous Technology |
| *EA* | = | Eigenstructure Assignment |
| *GC* | = | Guidance and Control |
| *GNC* | = | Guidance Navigation and Control |
| *HTV* | = | H-II Transfer Vehicle |
| *IMU* | = | Inertial Measurement Unit |
| *ISS* | = | International Space Station |
| *LEM* | = | Lunar Exploration Module |
| *LEO* | = | Low Earth Orbit |
| *MCI* | = | Mass, Centering and Inertia |
| *MIMO* | = | Multi Input Multi Output |
| *QFT* | = | Quantitative Feedback Theory |
| *PID* | = | Proportional, Integral and Derivative control |
| *SISO* | = | Single Input Single Output |
| *WRT* | = | With Respect To |

[*] Guidance and Control engineer, Guidance & Control Systems, nuno.silva@astrium.eads.net.
[†] Control expert, Guidance & Control Systems, martine.ganet@astrium.eads.net
[‡] Guidance and Control engineer, Guidance & Control Systems, emmanuel.gogibus@astrium.eads.net
[§] Space Vehicles Attitude&Orbital Control manager, Guidance & Control Systems, patrick.delpy@astrium.eads.net
[**] Guidance expert, GNC & Functional Engineering, philippe.augros@astrium.eads.net

# I. Introduction

THIS document focus on the autonomous guidance and control algorithms for an autonomous rendezvous, namely in an elliptical orbit. An historical perspective of space rendezvous can be found in [1] and [2].

Russian Space Agency was the first to develop an AR&D capability. The first unmanned Cosmos spacecrafts rendezvous took place in 1967. Then they have used it extensively to re-supply their MIR space station and part of ISS. Unmanned Progress vehicle and manned Soyuz vehicle AR&D are similar using IMU, rate gyrometer, accelerometer, rendezvous radar and optical devices for final docking.

NASA has not yet developed an AR&C capability. Main experience is based on Gemini and Apollo programs that developed the initial concept for rendezvous and capture with crewman participation to the GC rendezvous system. Shuttle orbiter GNC system for a more general LEO mission is also similar to LEM for rendezvous and docking. Apollo rendezvous planning was performed on ground but the on-board system was able to target and automatically control burns. The final capture/docking phase was controlled manually by the crew. For Shuttle, the crew still manually performs the final maneuvers using visual aids. GNC systems use IMU, rate gyrometer, optical equipment and radar.

DART mission fails in April 2006. Collision Avoidance Maneuver was engaged after over-consumption during proximity operations. Investigation explanation is proposed today: AVGS navigation never passes the pre-programmed waypoint from relative attitude information to relative attitude and position information as errors between velocity measurement and estimates causes regular computational reset.

ESA and JAXA do not have AR&D capability as yet but both have independent on going programs for developing it. ATV is to be launched in the coming months and HTV is scheduled in 2008 (berthing with Canadian arm).

JAXA 1998-1999 ETS-VII successfully achieves a number of fully autonomous low impact docking and docking/berthing maneuvers. The system consisted of two cooperative spacecrafts that employed a set of sensors including GPS navigation and a data link to accomplish rendezvous and docking.

The MIT Space Systems Laboratory has developed the SPHERES Test bed to provide an experimental laboratory to develop and test formation flight and autonomous docking algorithms. It allows the development of relative position and attitude control algorithms, CAM algorithms, propellant balancing algorithms, etc… for various types of docking maneuvers.

Presented guidance algorithm is based on ATV fundamental ideas. The main difference is in the heart of the algorithm: the relative motion model. For ATV (and others space vehicles whose mission includes a rendezvous), there is a linear relative motion model inside the onboard guidance algorithm. Its purpose is to give an accurate estimation of the boosts effects that the spacecraft has to perform in order to get from a state (position and velocity) to another.

First researches on relative motion model were conducted by Clohessy and Wiltshire[1], in 1960. They developed a method to assess the relative position and velocity of a chaser in a relative coordinate system whose origin is the target and whose directions are given by the local vertical and local horizontal directions. Given the initial position, velocity and true anomaly on a circular orbit, the final position and velocity at a given true anomaly are obtained by solving a linear stationary system of equations. This solution is only accurate when the chaser is close to the target (when the distance target-chaser is small in comparison with the distance target-Earth's centre) and when the eccentricity of the target's orbit is small ($e < 0.01$).

When the target is in eccentric orbit, the relative motion of the chaser is described by nonlinear differential equations with periodic coefficients. The linearized equations are known as the Tschauner-Hempel equations. In that direction, another important research was driven by Carter[2] to change the coordinate system in order to admit non-null eccentricity. The equations of motion are, however, more difficult to solve. Carter proposed a solution that requires integrating some functions and uses the eccentric anomaly. The result is not simple to use in an engineering work and has a singularity for a circular orbit ($e = 0$).

Note that other developments exist for relative motion model and associated state transition matrix and/or tensor: linear model with inclusion of J2 effects[14], and non linear model[9].

The solution chosen in this document is based on the developments by Yamanaka and Ankersen[3] in 2002. It uses the same inertial frame but modifies the integration term expression. Consequently it can be easily computed by using the variable time instead of the eccentric anomaly. Assessment of position and velocity at a given time/true anomaly, from position and velocity at another given time/true anomaly, is now easier to adapt to a guidance algorithm and faster to compute.

Robust autonomous control algorithms are identified and analyzed for each phase and sub-phase of the rendezvous mission scenario in both circular and elliptical orbits. Selection criteria are identified and compiled taking into account representative requirements for trade off purposes.

From control function point of view, it is more judicious to select the control mode related to the more stringent requirements associated to the more constrained flight phase and to analyze the trade off of the control methods on this specific phase. Analysis is complete and the resulting robust control trade off on this specific complex case is also applicable and robust for all the other flight phases.

## II. Linear relative motion model

### A. Hypothesis

The whole relative motion model is based on a single hypothesis: the relative distance between the chaser and the target is small when compared to the distance between the target and the Earth centre. This assumption will help us to simplify the chaser's equations of motion. One recalls that only the gravitational force is taken into account by this model. Neither the $J_2$ disturbance, nor the atmospheric drag, nor the vehicle thrust, are taken onto account.

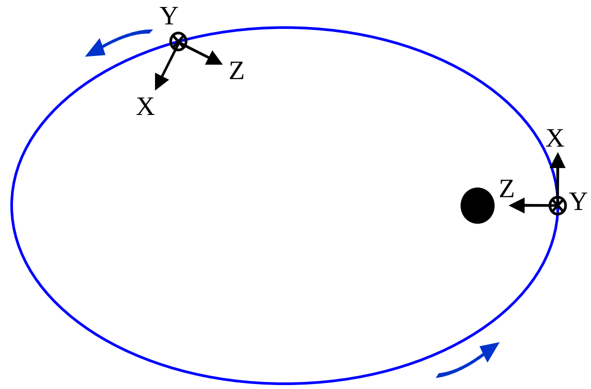


**Figure II.1. The LVLH (Local Vertical / Local Horizontal) frame**

One notes $\vec{R}$ the absolute position of the target in the equatorial frame and $\vec{r}$ the relative position of the chaser wrt the target, in the LVLH frame based on the target MCI. The gravitational force uses the ratio $\dfrac{\vec{R}+\vec{r}}{\left\|\vec{R}+\vec{r}\right\|^3}$ to be determined. This term is estimated linearly wrt the relative position of the chaser:

$$\frac{\vec{R}+\vec{r}}{\left\|\left(\vec{R}+\vec{r}\right)\right\|^3} \approx \frac{\vec{R}+\vec{r}}{R^3}\left(1+2\frac{\vec{R}.\vec{r}}{R^2}+\frac{r^2}{R^2}\right)^{-\frac{3}{2}}$$

$$\frac{\vec{R}+\vec{r}}{\left\|\left(\vec{R}+\vec{r}\right)\right\|^3} \approx \frac{1}{R^3}\left(\vec{R}+\vec{r}-3\frac{\vec{R}.\vec{r}}{R^2}\vec{R}\right)$$

This approximation allows us to get simple linear equations for the chaser's relative motion, linking the chaser's relative position, velocity and acceleration as follows:

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix} -k\omega^{\frac{3}{2}}x + 2\omega\dot{z} + \dot{\omega}z + \omega^2 x \\ -k\omega^{\frac{3}{2}}y \\ 2k\omega^{\frac{3}{2}}z - 2\omega\dot{x} - \dot{\omega}x + \omega^2 z \end{bmatrix} \qquad \text{(II.1)}$$

where $k=\dfrac{\mu}{C^{\frac{3}{2}}}$, $C$ is the orbital angular momentum of the target, $\omega$ is the target orbital rate and $\vec{r}$ is replaced by its components $x$, $y$ and $z$.

### B. Equations simplification

These equations cannot be simply solved. Time cannot be used as the reference variable (used to place the target on its orbit and to derive the position to get the velocity, and the velocity to get the acceleration). One needs a variable giving the target position on its orbit without ambiguity. By ambiguity, one means leading to an indetermination due to the target's movement periodicity, like the target's radius would do, for instances. The chosen variable is the target's true anomaly θ, the angle between the Perigee vector and the position vector:

$$\begin{cases} \dot{x} = \omega x' \\ \ddot{x} = \omega^2 x'' + \omega\omega' x' \\ \omega' = -2k^2 e \sin(\theta)\rho \end{cases} \text{, where } \rho = \left(1 + e\cos(\theta)\right)$$

Deriving wrt the true anomaly the new system is:

$$\begin{cases} \rho x'' - 2e\sin(\theta)x' - e\cos(\theta)x = 2\rho z' - 2e\sin(\theta)z \\ \rho y'' - 2e\sin(\theta)y' = -y \\ \rho z'' - 2e\sin(\theta)z' - (3 + e\cos(\theta))z = -2\rho x' + 2e\sin(\theta)x \end{cases} \tag{II.2}$$

As it remains difficult to solve, a new variable change is performed: $\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \rho \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

The final solvable system is:

$$\begin{cases} \tilde{x}'' = 2\tilde{z}' \\ \tilde{y}'' = -\tilde{y} \\ \tilde{z}'' = \dfrac{3\tilde{z}}{\rho} - 2\tilde{x}' \end{cases} \tag{II.3}$$

### C. Linear problem solution

The system's solution for the out-of-plane motion is simple and corresponds to the $y$ component - its solution is periodical. Given the satellite state at a moment $t_0$, one can get the satellite's state at a moment $t$ as follows:

$$\begin{bmatrix} \tilde{y} \\ \tilde{v}_y \end{bmatrix} = \frac{1}{\rho(\theta - \theta_0)} \begin{bmatrix} \cos(\theta - \theta_0) & \sin(\theta - \theta_0) \\ -\sin(\theta - \theta_0) & \cos(\theta - \theta_0) \end{bmatrix} \begin{bmatrix} \tilde{y}_0 \\ \tilde{v}_{y0} \end{bmatrix} \tag{II.4}$$

The solution of the linear system on the XoZ plane is not trivial. One starts by integrating once $\tilde{x}''$ and one replaces $\tilde{x}'$ in $\tilde{z}''$ equation.

Now one looks for the homogeneous second-order differential equation. One focuses on the F. Ankersen and K. Yamanaka[5] contribution to the J. Tschauner and P. Hempel[4] first solution. Their solution has no singularity:

$$\varphi = C_1 \rho(\theta)\sin(\theta) \int_{\theta_0}^{\theta} \frac{1}{\rho(\tau)^2} d\tau - \rho(\theta)\cos(\theta) + C_2 \tag{II.5}$$

The great interest of such a formulation is on the integral part because it allows a fast computation of its value. Indeed, it can be assessed by using the time separating the two limit true anomalies:

$$J(\theta) = \int_{\theta_0}^{\theta} \frac{1}{\rho(\tau)^2} d\tau = k^2 (t - t_0) \tag{II.6}$$

The final equation form is:

$$\begin{bmatrix}\tilde{x}\\ \tilde{x}'\\ \tilde{z}\\ \tilde{z}'\end{bmatrix}=\begin{bmatrix}1 & -c(1+1/\rho) & s(1+1/\rho) & 3\rho^2 J\\ 0 & s & c & (2-3esJ)\\ 0 & 2s & 2c-e & 3(1-2esJ)\\ 0 & s' & c' & -3e\left(s'J+s/\rho^2\right)\end{bmatrix}_\theta\begin{bmatrix}K_1\\ K_2\\ K_3\\ K_4\end{bmatrix} \tag{II.7}$$

where $c=\cos(\theta)$, $s=\sin(\theta)$, $c'=-[\sin(\theta)+e\sin(2\theta)]$, $s'=\cos(\theta)+e\cos(2\theta)$, $J=k^2(t-t_0)$ and $K_1$, $K_2$, $K_3$ and $K_4$ are constants.

## D. Final expression

One transforms the obtained variables in order to get the solution with the initial variables.

The chaser's state at a true anomaly θ, knowing its state at the true anomaly $\theta_0$, is:

$$\begin{bmatrix}x\\ \dot{x}\\ z\\ \dot{z}\end{bmatrix}=Mat_{xz}\begin{bmatrix}1 & -c(1+1/\rho) & s(1+1/\rho) & 3\rho^2 J\\ 0 & s & c & (2-3esJ)\\ 0 & 2s & 2c-e & 3(1-2esJ)\\ 0 & s' & c' & -3e\left(s'J+s/\rho^2\right)\end{bmatrix}_\theta\begin{bmatrix}1 & -c(1+1/\rho) & s(1+1/\rho) & 0\\ 0 & s & c & 2\\ 0 & 2s & 2c-e & 3\\ 0 & s' & c' & -3es/\rho^2\end{bmatrix}_{\theta_0}^{-1} Mat_{x_0z_0}^{-1}\begin{bmatrix}x_0\\ \dot{x}_0\\ z_0\\ \dot{z}_0\end{bmatrix} \tag{II.8}$$

Where:

$$Mat_{xz}=\begin{bmatrix}1/\rho & 0 & 0 & 0\\ 0 & 1/\rho & 0 & 0\\ \alpha & 0 & \beta & 0\\ 0 & \alpha & 0 & \beta\end{bmatrix},\qquad M_{x_0z_0}^{-1}=\begin{bmatrix}\rho_0 & 0 & 0 & 0\\ 0 & \rho_0 & 0 & 0\\ \gamma_0 & 0 & \delta_0 & 0\\ 0 & \gamma_0 & 0 & \delta_0\end{bmatrix},\qquad \alpha=\frac{\mu^2 e\sin(\theta)}{C^3},\qquad \beta=\frac{\mu^2\rho}{C^3},$$

$$\rho_0=1+e\cos(\theta_0),\ \gamma_0=-e\sin(\theta_0),\ \delta_0=\frac{1}{\rho_0\mu^2C^3}$$

For simplicity sake, one defines:

$$X_1=M(\theta_1,\theta_0)X_0$$

Where M is the product of the different matrices participating on the final state computation, $X_t$ is the chaser state (position and velocity) at a give true anomaly $\theta_t$.

## III. Control

### A. Autonomous rendezvous control specificities

A key ingredient in AR&C capabilities for exploration is the application of rendezvous techniques to various mission scenarios, including integration of the vehicle constraints with the sensor field of view and lighting constraints. Another key element is the ability to perform AR&C unique design analysis and integrated testing for each mission. Lessons learned from the past and further autonomy must be employed to provide routine and cost affordable AR&C operations. Finally AR&C should be adequate for all exploration missions.

The mastering of such systems is the compulsory step towards a European capability for autonomous orbital rendezvous and docking.

In this frame, one can suppose two missions categories: Mars sample return missions (so called C1) and in-orbit servicing (so called C2). Due to the lag time in communication between the Earth and the Martian system, real time control during C1 mission maneuvers will not be possible. Therefore, a highly autonomous GNC system must be developed to ensure feasibility of such a mission.

Control should be a highly adaptable system that has the ability to perform a safe rendezvous including:

1) Proximity operations with a high level of autonomy,
2) Autonomous docking or capture,
3) Elliptical orbit (circular being a particular case),
4) And cooperative or non-cooperative target.

### B. Autonomous control model

Robust autonomous control algorithms are identified and analyzed for each phase and sub-phase of the rendezvous mission scenario in both circular and elliptical orbits. Selection criteria are identified and compiled taking into account all the requirements for trade off purposes.

Basically, from a control function point of view, it is more judicious to select the control mode related to the more stringent requirements associated to the more constrained flight phase and to analyze the trade off of the control methods on this specific phase. The analysis will be more complete, and the resulting robust control trade-off on this specific complex case, will also be applicable and robust for all the other flight phases.

The selection of these specific flight phase and models are presented hereunder.

As long as the distance between the two vehicles is large enough, each DoF rotation and translation may be controlled independently by a SISO control system. During final approach, when the docking mechanism has to be aligned with the docking port of the target vehicle, all the motions are coupled. In this case a MIMO control system may be necessary.

During final approach, accuracy requirements are also very stringent, thus this flight phase is the most suitable for control function trade-off purposes. C1 rendezvous is a capture scenario, while C2 servicing missions use a docking scenario. In the docking case some additional coupling effects (wrt capture) appear as one controls the attitude around the centre of gravity and the position of the docking point that differs from centre of gravity. Docking scenario is more complex from control point of view.

Control actuators may be linear with the use of reaction wheels or continuous propulsion thrusters, or non linear with classical RCS pulse width modulation. RCS model with minimal and maximal opening durations will be selected.

Finally, elliptical and circular orbits have to be considered which is very important for guidance aspects. However, from relative control aspects during rendezvous a sizing circular model could be used for design and in a first analysis. The elliptical model will be used for the final control function validation. The models used are described in next section.

The selected model for control trade-off is:

1) the 6 degrees of freedom control function
2) applied to the vehicle during final approach of the rendezvous,
3) with docking scenario
4) on circular orbit

One has reused our experience on ATV docking models without sloshing disturbances and with ISS sinusoidal motion.

## C. Equations of motion for control design

### *1. Relative position*

Clohessy Wiltshire that describes the proximity relative motion of circular orbits are valid *if the relative distance is small with respect to the distance of the centre of Earth and if the orbit is near circular*. These equations can be solved analytically and are typically used for control algorithm validation. In this case, the relative equation of motion expressed in target local orbital frame (z towards earth and y out of plane) correspond to the (II.1) system with acceleration and:

$$\left\{ \dot{\omega} = 0; \omega = \omega_0 = \sqrt{\frac{\mu}{R^3}} \right. \tag{III.1}$$

For future rendezvous missions, target orbit may be not circular and it is well known that the Clohessy Wiltshire equations cause significant error if they are applied to fairly elliptical orbits.

In [8] a new and simple solution to differential equations of relative motion on an arbitrary elliptical orbit is developed and described in a convenient state space transition matrix form. However, for control validation, one may directly use the equations of proximity relative motion in the target orbital coordinate system that depends on the angular rate (maximum value at perigee).

In this case, the relative equation of motion on elliptical orbit expressed in target local orbital frame (z towards earth and y out of plane) are as (II.1) with acceleration terms.

The difference between relative motion on circular and elliptical orbits is linked to one varying parameter: the orbital pulsation $\omega$ (equation (III.1)).

The orbital pulsation derivative considered for missions C1 and C2 are lower than $10^{-6}$ rad/s$^2$ which is very small wrt orbital pulsation of about $10^{-3}$ rad/s. Thus, for control design, one considers the right hand terms of (II.1) as disturbances, one neglects these derivative terms and take as conservative hypothesis the maximal value of the orbital pulsation. Finally, for control design, one still uses the Clohessy Wiltshire equations with this maximal orbital pulsation.

Furthermore, the in plane coupling terms in the Clohessy Wiltshire equations correspond to Coriolis and gravity forces. For a closed loop controller design, these terms may be considered as low frequency disturbances and for this reason, no longer need to appear in the design model. Consequently a double integrator model independent for each axis can be applied for the position controller design.

For control validation purposes during final approach, one proposes to first consider a simplified simulator using acceleration integration then, Clohessy Wiltshire equations for circular orbits and finally similar equations for elliptical orbits. With this approach, the impact of elliptical orbits on control performance will be assessed.

For GC validation, one may also use a mission analysis tool that simulates both the chaser and target motions.

Once more, for trade-off purposes, comparison of control methods could be achieved with circular orbit hypothesis without loss of generality and the performance evaluation of the selected controller will be done for elliptical orbit in complete design studies.

### *2. Kinematics equations*

Euler angles are defined by successive rotations from local orbital frame to vehicle frame ($\theta$/z, $\psi$/y and $\phi$/x).

p, q and r are the components of the absolute angular rate expressed in vehicle frame.

For small angles, one obtains the following linear model:

$$\begin{cases} p = \dot{\phi} - \omega\phi \\ q = \dot{\theta} - \omega \\ r = \dot{\psi} + \omega\psi \end{cases} \tag{III.2}$$

*Remark: For large angles the equations are non linear.*

If one uses the quaternion formulation $Q=(q_1\ q_2\ q_3\ q_4)$, one directly obtains the linear relation (no small angle hypothesis). This model is used for control function definition and validation, however Euler angles are still used in design approach for better results interpretation.

$$\dot{Q} = 1/2Q \times Q_\Omega$$
$$Q_\Omega = \begin{bmatrix} 0 \\ p+\omega(2q_1q_2+\lambda q_3) \\ q+\omega(\lambda^2-q_1^2+q_2^2-q_3^2) \\ r+\omega(2q_1q_3-\lambda q_2) \end{bmatrix} \tag{III.3}$$

For circular orbits, one uses the above equation with constant orbital pulsation $\omega$ (equal to $\omega_0$).

For control design, considering the orbital pulsation terms of (III.2) to (III.3) as disturbances, one may take as conservative hypothesis the maximal value of the orbital pulsation noted $\omega_0$.

For trade-off purposes, comparison of control methods could be achieved with circular orbit hypothesis without loss of generality and the performance evaluation of the selected controller will be done with elliptical orbit equations (III.2) or (III.3) in complete design studies.

*3. Euler dynamic equations*

Liberalized Euler equations of motion are described in [7] for circular orbits.

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \omega_0 \mathbf{I}^{-1} \begin{bmatrix} I_{zx} & 2I_{zy} & I_{zz}-I_{yy} \\ -I_{zy} & 0 & I_{xy} \\ I_{yy}-I_{xx} & -2I_{xy} & -I_{xz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \mathbf{I}^{-1}(C_c + C_{gg}) \tag{III.4}$$

These equations can be extended to the elliptical case as follows:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \omega \mathbf{I}^{-1} \begin{bmatrix} I_{zx} & 2I_{zy} & I_{zz}-I_{yy} \\ -I_{zy} & 0 & I_{xy} \\ I_{yy}-I_{xx} & -2I_{xy} & -I_{xz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \mathbf{I}^{-1}(C_c + C_{gg}) + \begin{bmatrix} 0 \\ \dot{\omega} \\ 0 \end{bmatrix} \tag{III.5}$$

As in paragraph §C-1, the orbital pulsation derivative considered for C1 and C2 missions is lower than $10^{-6}$ rad/s$^2$ which is very small wrt orbital pulsation of about $10^{-3}$ rad/s. Thus, for control design, considering the right hand terms of (III.5) as disturbances, one may neglect this derivative terms and take as conservative hypothesis the maximal value of the orbital pulsation.

For the three axis earth pointing vehicle rotational motion, Euler's moment equations can be linearized so that a double integrator dynamic model can also be used for the attitude controller design.

Again, for trade-off purposes, comparison of control methods could be achieved with circular orbit hypothesis without loss of generality and the performance evaluation of the selected controller will be done with elliptical orbit equations III.2 or III.3 in complete design studies.

*4. Geometric attitude position coupling*

In the phases prior to the final approach, position and attitude control are related to the CoM of the vehicle.

In the last part of a final approach to docking, position and attitude of the chaser's docking port have to be aligned with the target one. Consequently, the vehicle has to be controlled wrt its docking frame which requires modeling the geometric relations between the docking frame and the spacecraft body frame centred in the CoM.

Such a control problem typically calls for a multivariable control design method.

For control design these geometric relations may be taken into account or neglected, however for stability or performance analysis the coupling terms have to be taken into account.

## D. Criteria for design of autonomous control function

One is looking for a highly adaptable system control that has the ability to perform a safe rendezvous including proximity operations with a high level of autonomy, autonomous docking or capture, circular or elliptical orbit and cooperative or non-cooperative target. Therefore a highly autonomous GNC system must be developed to ensure the feasibility of such a mission.

The final selection criteria that will be used for control selection purposes, highlights the specificities of an autonomous system.

The criteria are listed hereafter with decreasing importance:

1) Stability: closed loop stability shall be guaranteed with sufficient margins in nominal and off-nominal conditions (3 dB frequency margin, 1 control period delay margin, 6 dB bending mode attenuation)
2) Performances: tracking accuracy requirement shall be respected in nominal and off-nominal conditions to insure mission success (see [8])
3) Autonomy: a highly autonomous GNC system must be developed to ensure feasibility of explanatory mission.
4) Versatility and robustness of the design techniques with respect to changes in the control problem statement, sensors, actuators, vehicle characteristics, cooperative or non cooperative target, and mode transition. The GNC shall be modular to be adapted to future missions.
5) Computation load shall be minimized.
6) Optimization: Control commands shall minimize the fuel consumption.
7) Safety shall be insured wrt target collision.

Criteria 1 and 2 represent control design objectives, if they are not respected the control method is not applicable. Criteria 3 4 and 5 are the most important qualitative criteria that will be used for trade off when 1 and 2 are respected. Criteria 6 and 7 are secondary selection criteria.

## E. Control function trade-off

The selected robust control concepts are: phase plane, EA plus band pass filter, H∞/PID (current ATV design) and QFT plus band pass filter are compared in this section. Phase plane control method may not be applied during the whole rendezvous phase from far range approach until docking and was not selected to be deeply analyzed in these Control studies.

The final selection criteria applied for control selection purposes, based on the specificities of an autonomous system, are expressed in Table III.1 with decreasing importance.

The stability and performance of EA, H∞ / PID and QFT controller have been evaluated through analysis in frequency domain and time domain simulations for ATV model.

Stability analysis show that MIMO systems are always stable for the three control laws and that the worst case SISO stability margins for position and attitude loops respect margin objectives. They are compared on Figure III.1. Stability margins obtained for translation control are similar for all the methods with a smaller gain margin linked to a better control of bending modes of $H_\infty$ control law. Stability margins of PID rotation control ($H_\infty$/PID) are not as large as EA or QFT margins. They just respect the requirement but this was a design choice; they could be improved as it was done after CDR for ATV control.

Regarding time-domain validations, the main performance results are expressed in terms of final accuracy at docking: lateral position and lateral orientation. One can see on Figure III.2 and III.3, 50 cases Monte Carlo simulation results are plotted for position and attitude deviation at docking. The results are well within the accuracy requirements and they are similar for EA, $H_\infty$ /PID and QFT ATV controller.

As stability and performances are similar with both controllers, and autonomy aspects are managed by mission management, the control method selection relies on versatility and computation load criteria.

$H_\infty$ and QFT are well suited for ATV control that requires compromise between accurate tracking and bending modes filtering. However, both present the disadvantage of being designed using frequency weighting functions dedicated to a specific problem. They cannot be easily adapted on board to varying parameters. Moreover, $H_\infty$ computation load is very important and QFT design is based on "ad-hoc" rules that are not generic.

EA control presents the advantage of being very adaptive to flight conditions and varying parameters: controller gains depend only on eigenvalue objective and state space matrices and can be recomputed on board if necessary.

Risks of discontinuity at mode transition are also reduced. In addition, EA rigid mode controller order is zero and the final order of the controller depends only on additional filtering for bending modes or consumption purpose.

Thus, EA method is selected for control design during 6 degrees of freedom rendezvous approach. It is also easily applicable during 3 degrees of freedom drift attitude control and boost control.

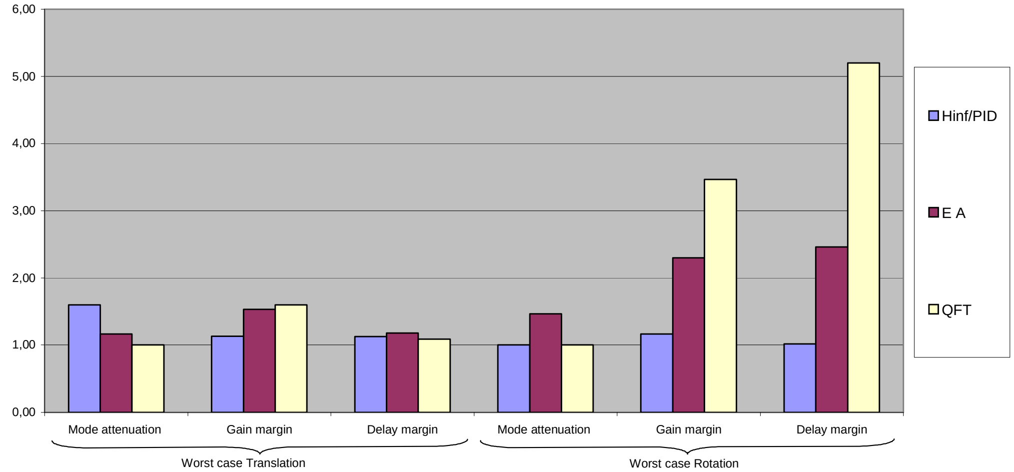


**Figure III.1: Normalized stability margins wrt requirements**

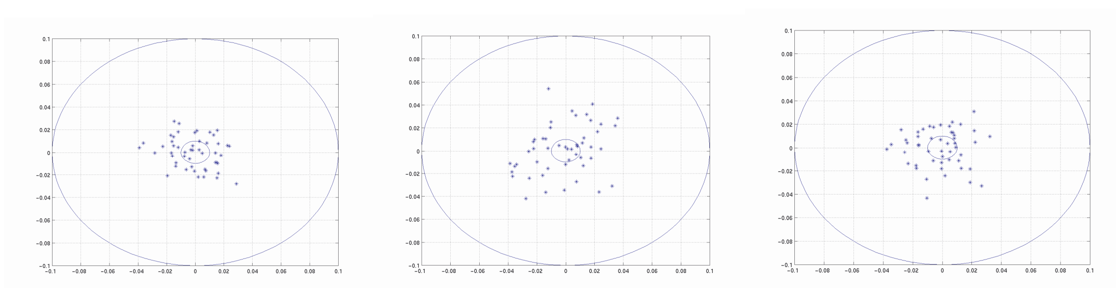

**Figure III.2: EA,H∞/PID and QFT docking accuracy**
**Docking lateral position Z(m) / Y (m)**

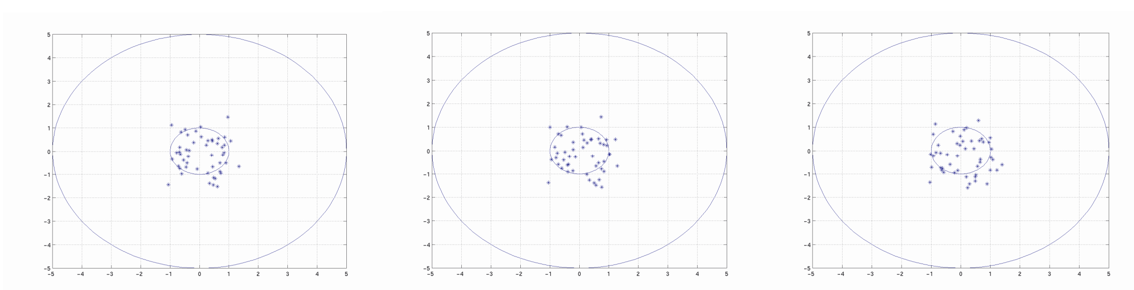

**Figure III.3: EA,H∞/PID and QFT docking accuracy**
**Lateral attitude Psi (°) / Theta (°)**

| | **EA** | **H∞/PID** | **QFT** |
|---|---|---|---|
| **Stability** | Stability margin requirement OK | Stability margin requirement OK | Stability margin requirement OK |
| **Performance** | Docking accuracy OK | Docking accuracy OK | Docking accuracy OK |
| **Autonomy** | Mission Management role | Mission Management role | Mission Management role |
| **Versatility** | **Yes**<br>**Direct state matrices dependency onboard computation** | No Complex specific shaping | No<br>Complex shaping & "ad-hoc" design rules |
| **Computation load** | **Order 4 per axis** | Order 11 to 13 per axis | Order 3 per axis |
| **Optimization** | No overconsumption | No overconsumption | No overconsumption |
| **Safety** | TBD | TBD | TBD |
| **MIMO/SISO** | MIMO | MIMO/SISO | SISO |
| **Discrete time implementation** | Discrete design | Discrete design | Discrete design |

**Table III.1: Control trade-off criteria**

# IV. Guidance

One assumes that it is possible to drive a satellite from a state to another, in space, by using only two boosts: one performed when leaving a first position, and the other, when arriving at the aimed position. The linear relative motion model is used by the guidance algorithm in order to get boosts directions and duration's estimations, able to drive the chaser.

## A. Background

The general on board guidance problem may be generically described as follows:

i. Find a control ***u*** (driving force orientation)
ii. starting from the current estimated state
iii. to reach a targeted state
iv. under a set of constraints, including:
   a. The dynamics of the vehicle
   b. Intermediate path constraints
v. and possibly also allowing minimizing a criterion.

For launchers the general problem for the on board guidance algorithm has been solved under the particular case of no criterion considered ([10]). The solution uses a segmentation of the trajectory and a time parameterization of the sought control. This scheme is currently part of the Ariane 5 launch vehicle exo-atmospheric guidance.

Following this development, in house advanced studies have also been performed to extend these basic elements through application of collocation and pseudospectral ([11] and [12]) type methods to on board guidance in the mid nineties. Indeed, this allows to cope with the general problem including both the path constraints and the criterion.

However the parametric problem obtained after transcription of the optimal control setting remains to be solved by a reliable NLP solver, and one knows that for on board application additional research is needed based on sparsity and structure of the problem at hand to get positive convergence results and full mastering of optimizer behavior.

Considering the state of the art with regard to collocation and pseudo spectral methods, (despite recent progress on the latter approach[13]), a guidance scheme without explicit on board management of intermediate path constraints has been selected, i.e. choice has been made to use the specific dynamics of rendezvous but to rely on off-line mission analysis work to tackle the intermediate constraints (forbidden volume for the trajectory for instance).

## B. Principle

In a guidance algorithm, the interest of the relative motion model may by to predict a future state of the chaser knowing a previous one, or to get the Lambert's problem solution. For the reasons explained in the previous paragraph one is interested in getting the Lambert's problem solution.

One recalls that this problem aims at getting the transfer orbit linking two states in space in a given time duration. Its solution is the first and most important step to deal with. Indeed, the comparison between the velocities on the transfer orbit, the initial velocity and the final aimed velocity gives us the differences of velocity to perform at the beginning and at the end of the transfer in order to reach the final aimed state. These differences of velocity are recorded as accelerations and durations to be ordered.

However, boosts cannot be performed instantaneously. If the last computed values were commanded, the chaser would not be driven to the correct final state. Therefore, they have to be computed again, taking into account their durations. This is the second step of the guidance algorithm which may now compute the spread boosts effects on the chaser's state. The trajectory is not simulated by the linear motion model but by a gravity model simulator. The dispersions due to the $J_2$ term of the Earth gravitational field or the atmospheric drag will not be inserted into the simulator.

Thus, by iteratively modifying the boosts directions, norm and durations, one will reach a solution theoretically driving the chaser, as exactly as wished, to the final aimed state.

## C. Tests

One performs a first series of tests. They show us the guidance algorithm accuracy on elliptical orbits, since this is its design purpose. One considers a maneuver on a highly elliptical orbit: a GTO (Geosynchronous Transfer Orbit), whose eccentricity is 0.73. One also considers maneuvers starting far from the target. Based on ATV maneuvers, one chooses a transfer lasting half an orbital period and whose initial and final states are set 25 km away from the target. Since the guidance algorithm is to take into account an ideal gravitational field and nothing more, one tests it by adding the disturbances due to the atmospheric drag and to the $J_2$ term of the Earth gravitational field.

### *1. Open-loop scheme*

One first run tests following an open-loop scheme. By open-loop, one means that the boosts direction, norm and duration are computed once, at the beginning of the transfer, when the chaser is in its initial state. This will show us how far our model is from the reality.

One runs 20 tests. For each of them, the initial position and velocity are randomly chosen according to a Gaussian law, around a given position and velocity. One sets the 3-σ dispersion at 3000 m on position and 0.5 m/s on velocity. This shows the initial state influence on the final state dispersion.

One obtains the following results:

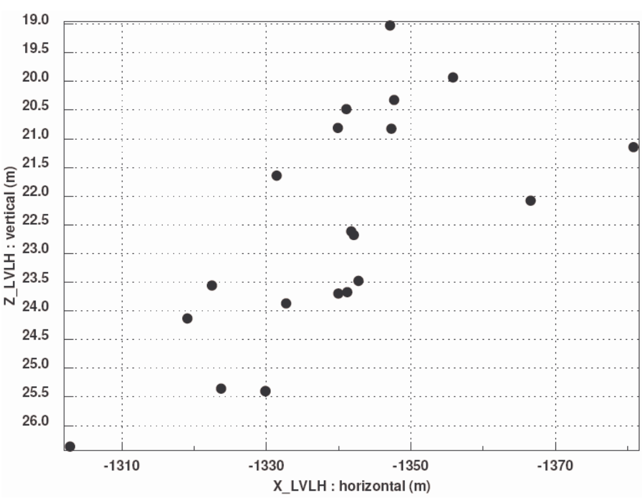


**Figure IV.1 : Position accuracy (Open-loop)**

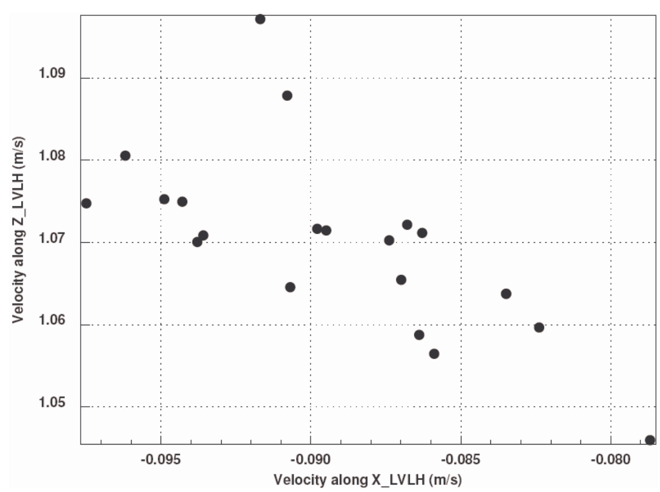


**Figure IV.2 : Velocity accuracy (Open-loop)**

One observes (Figure IV.1 and IV.2) that the dispersion due to the atmospheric drag and to the $J_2$ term, may drive the chaser more than 1 km away from the targeted position and more than 1 m/s away from the targeted velocity. These results are not acceptable when compared to the ATV GNC requirements (30 m and 0.2 m/s). In spite of not seeking better accuracy than ATV GNC requirements, they are used as a reference for the guidance scheme in order to establish a first conclusion on its possible use on board a chaser.

Concerning the effects of the initial state on the final accuracy, one concludes that it has a small impact on the final velocity, but a strong one on the final position.

### *2. Closed- loop scheme*

This mode is able to deal with the disturbances occurring during the achievement of the 1st boost and the maneuver free-drift phase. During this mode, boosts are regularly computed and performed if their magnitude is high enough.

Now one observes (Figure IV.3 and IV.4) that position and velocity accuracy allows concluding that the chaser is correctly driven to the final aimed state. The position accuracy is close to 15 m and on velocity is close to 0.08 m/s. Space environment disturbances are correctly treated, as well as dispersions on initial position and velocity.

Nevertheless, this kind of guidance scheme shall not be used on-board without additional design constraints. Indeed, boost timeline shall be predictable by the ground segment. In its present form, the guidance prototype is a useful tool to determine where correction boosts should be placed and if the guidance algorithm is stable when the chaser gets close to the final aimed state. Its performances are currently being assessed using predefined boost timeline.

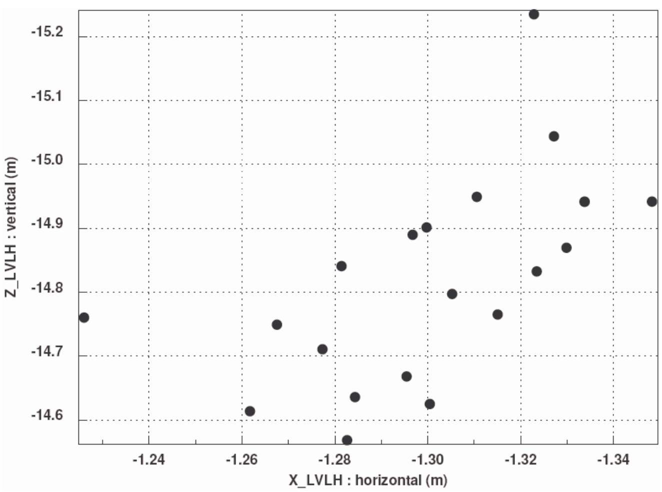


**Figure IV.3 : Position accuracy (Closed-loop)**

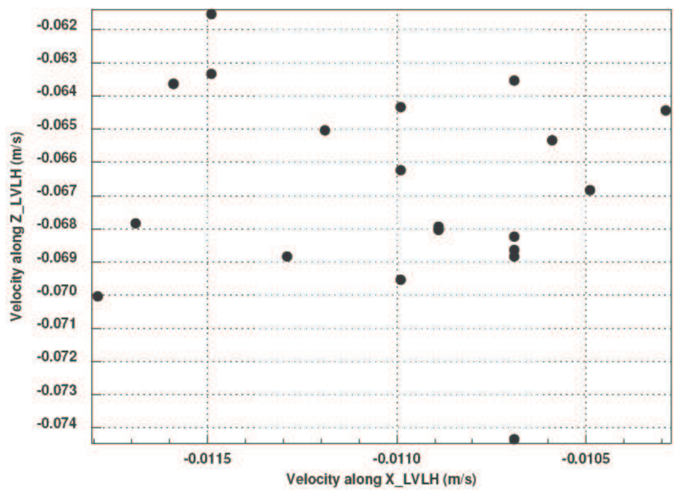


**Figure IV.4 : Velocity accuracy (Closed-loop)**



## V. Conclusion

During this study, robust autonomous control algorithms have been identified and analyzed. Finally, EA method has been selected for autonomous control design for each phase and sub-phase of the rendezvous mission scenario, in both circular and elliptical orbits.

A guidance algorithm working on elliptical orbits has equally been designed and tested. It is a useful tool to analyze and manage the atmospheric drag and the Earth gravitational field $J_2$ term disturbances effects, as well as thruster's malfunctions or navigation errors. The preliminary performances are encouraging and one will also assess in a near future derived schemes with alternate state transition matrix/tensor of relative motion[14].

## References


[1]Polites M.E.E., "An assessment of the Technology of Automated Rendezvous and Capture in Space", NASA/TP-1998-208528, Marshall Space Flight Centre, Alabama.

[2]Zimpfer D. & al., "Autonomous Rendezvous, Capture and In-space Assembly: Past Present and Future", AIAA 2005-2523.

[3]Clohessy, W. H., and Wiltshire, R. S., "Terminal Guidance System for Satellite Rendezvous", *Journal of Aerospace and Science*, Vol. 27, No. 9, Sept. 1960, pp. 653-658, 674

[4]Carter, T. E., "State Transition Matrices for Terminal Rendezvous Studies: Brief Survey and New Example", *Journal of Guidance, Control and Dynamics*, Vol. 21, No. 1, 1998, pp. 148-155

[5]Yamanaka, K., and Ankersen, F., "New State Transition Matrix for Relative Motion on an Arbitrary Elliptical Orbit", *Journal of Guidance, Control and Dynamics*, Vol. 25, No. 1, Jan.-Feb. 2002, pp. 60-66

[6]Tschauner, J., and Hempel, P., "Optimale Beschleunigungsprogramme für das Rendezvous-Manöver", *Astronautica Acta*, Vol. 10, No. 296, 1964, pp. 339-343

[7]Fehse W., "Automated Rendezvous and Docking of Spacecraft", Cambridge University Press 2003.

[8]Ganer M., Quinquis I., Bourdon G., Delpy P., "ATV GNC during rendezvous with ISS", DCSSS Conference, Cambridge, July 2002.
[9]Dunning R. S., "The orbital mechanics of flight mechanics", Scientific and Technical Information Offcie, NASA 1973, SP-325
[10]Demessant-Garnero A., "Explicit guidance law: a unique algorithm for multiple type of missions"; Proceedings of the ESA International Conference on GNC held at ESTEC, Nordwijk, 12-15 April 1994 (ESA WPP-071, April 1994)
[11]Delpy P., Ferreira E., "Explicit guidance using collocation methods for reentry vehicles"; AIAA Guidance, Navigation and Control Conf., Scottsdale, AZ, 1-3 August 1994, AIAA paper n°94-3564
[12]Delpy P., Meloux E., "Long range guidance for a homing missile"; AGARD Mission Systems Panel 1st Symposium on Guidance and Control for Future Air Defence Systems, 17-20 May 1994, Copenhagen, published in AGARD CP-555
[13]Yan H. and Alfriend K. T., "Approximate Minimum Energy Control Laws for Low-thrust Formation Reconfiguration", Journal of Guidance, Control, and Dynamics, Vol. 30, No. 4, July-August 2007
[14]Gim D. and Alfriend K. T., "The state transition matrix of relative motion for the perturbed non-circular reference orbit", AAS/AIAA 2001-222