Dynamic Modeling and Simulation of a Flexible Satellite

Shengjian Bai, Xinsheng Huang and Yuhao Liu

***Abstract*—A novel dynamic equation of a flexible satellite with flexible appendages is developed based on the coupling deformation field, in which the second-order coupling term of axial displacement caused by that of transverse is included. Lagrangian principle and assumed mode method are used to derive the first-order approximate model of the multibody system. Theoretical analysis and numerical simulations show that the coupling term can have significant effect on the dynamic characteristics of the flexible system undergoing large rigid-body motion and the dynamic stiffening is accounted for.**

I. Introduction

IN response to demanding mission requirements, modern space vehicles often carry lightweight deployable members (solar panels, booms, antennas), which are inherently flexible. The problem of modeling rotating multi-body systems have developed mainly in response to the needs of satellite attitude control; more particularly, to needs for numerical simulation of attitude behavior [1]. Dynamic modeling of such system involving interconnected rigid structures and flexible appendages is a difficult task to accomplish, as most of these systems generally involve complex dynamics characterized by nonlinearities and strong coupling between flexible and rigid modes. As the paper [2] pointed that, the increasing size, structural flexibility, and dynamic complexity, coupled with competing demands for greater precision and autonomy, continue to outstrip our ability to model these systems adequately. Moreover, modern engineering technology is leading to ever more demanding performance criteria, such as high rotating speed and large angular maneuvering, increasing precision and pointing accuracy, which have posed serious difficulties for all currently advocated control design methodologies [3]. Thus, it is a fundamental task to develop a dynamic model of a flexible satellite, which not only can describe the dynamic characteristics of flexible satellites under new working conditions(e.g., high speed rotation), but also has the simplicity for controller design.

The mechanical systems undergoing high-speed and long range motion can produce the phenomenon of dynamic stiffening [4, 5] due to the coupling between rigid motion and elastic deflection, and traditional dynamic analysis can hardly deal with this phenomenon. In most cases problems arise not because of a lack of available analytical/numerical design procedures but because of failure on our part to recognize and appreciate the mechanism of dynamic stiffening. Traditional modeling theory adopts small deformation assumption in structural dynamics, which assumes that the axial and transverse deformations at any point in the beam are uncoupled, namely linear deformation field of flexible beam is adopted. Therefore, modal characteristic changes due to high rotational speeds do not include in the traditional dynamic equations [6].

Dynamic modeling of flexible satellites depends on the development of multi flexible body dynamics. Different from the researches [7, 8] wanting to “capture” the dynamic stiffening terms, Hong et al [9-11] emphatically studied the mechanism of dynamic stiffening, and pointed that the coupling deformation field can explain the phenomenon. Studies [9, 11] indicate that the coupling term in axial displacement caused by that of the transverse, can have significant effect on the dynamic characteristics of the multibody system when it undergoes high speed motion, and the coupling deformation field is verified numerically and experimentally to be valid for dynamic description of the system.

In this paper, a novel satellite model is developed by taking the second order coupling term into account. Compared with finite element method utilized in the references [9, 11] for studying the dynamic characteristic, the assumed mode method (AMM) adopted here has the property of simplicity , especially applicable to controller design. As we know, the first five modes are adequate to describe the characteristic of the flexible system. A simplified first-order approximate model is also given for controller design.

In section II, the first-order approximate model (FOAM) based on the coupling deformation field is established. Next, in section III, numerical simulations and comparisons with traditional linear approximate model (TLAM) are presented to demonstrate the validity of the developed model (FOAM). Lastly, a concluding discussion is given in section IV.

II. Modeling of a Flexible Satellite

Configuration of the model satellite is presented in Fig.1. The satellite consists of center rigid body with two flexible structures attached. The flexile structures represent satellite structural elements such as on-board antenna and solar arrays. Although the modeling approaches of the present work can be applied to multi-axis maneuver, for simplicity only single-axis maneuver is considered. The satellite is controlled by a torquer on the rigid hub. When the satellite is maneuvered, the elastic member connecting the hub experiences structural deformation. The problem of interest here is to model the satellite experiencing high speed maneuvering. It is assumed that the rotational maneuver excite the two flexible appendages anti-symmetrically. For

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the configuration of the satellite, this assumption of anti-symmetric deformations is reasonable. As shown in the figure, define the OXY and oxy as the inertial frame and the reference frame. Denote $\vec{u}_p$ as the flexible deformation at point p with respect to the oxy frame. θ is considered as rigid body coordinate. $\vec{r}_{p_0}$ represents position vector of point p_0 in oxy frame, and $\vec{r}_p$ the position vector of point p in OXY frame.

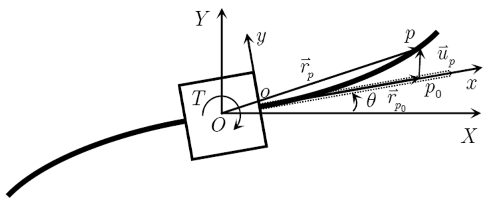


Fig. 1. A satellite model with flexible structures

As shown in Fig.1, the position vector from O to p can be expressed as

$$\vec{r}_p = \vec{r}_o + \vec{r}_{p_0} + \vec{u}_p \tag{1}$$

where $\vec{r}_o = \overline{Oo}$, $\vec{r}_0 = \overline{op_0}$, and $\vec{u}_p = \overline{p_0 p}$.

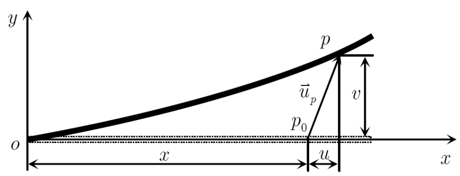


Fig. 2. Beam description of deformation

The coordinate of the deformation vector $\vec{u}_p$ can be represented as

$$\boldsymbol{u}_p = (u \quad v)^T = (w_1 + w_c \quad w_2)^T \tag{2}$$

where u `1and v are the deformation quantities of the point p_0 in x direction and y direction in the oxy frame, respectively; w_1 represents the pure axial deformation, w_2 represents the transverse deformation along the y-axis of the reference frame oxy. The *second order term* w_c is the deformation associated with the foreshortening quantity due to w_2, represented [9, 11] as

$$w_c = -\frac{1}{2}\int_0^x \left(\frac{\partial w_2}{\partial x}\right)^2 \mathrm{d}x \tag{3}$$

The coordinates of $\vec{r}_0$ and $\vec{r}_{p_0}$ are represented by $\boldsymbol{r}_o$ and $\boldsymbol{r}_{p_0}$, respectively. And the Eq.(1) may be rewritten as the coordinate form

$$\boldsymbol{r}_p = \boldsymbol{r}_o + \Theta(\boldsymbol{r}_{p_0} + \boldsymbol{u}_p) = \Theta(r_o\mathbf{e} + \boldsymbol{r}_{p_0} + \boldsymbol{u}_p) \tag{4}$$

where $\boldsymbol{r}_{p_0} = (x \quad 0)^T$, $\boldsymbol{u}_p = (u \quad v)^T$, $\boldsymbol{e} = (1 \quad 0)^T$, and $\boldsymbol{r}_o = r_o(\cos\theta \quad \sin\theta)^T$, as shown in Fig.2, the variable x is the coordinate of the point p_0 in the oxy frame, the parameter Θ is a direction cosine matrix which is the oxy frame with respect to the OXY frame, given by

$$\boldsymbol{\Theta} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{5}$$

where θ is the angular displacement of the hub.

The first-order derivative of $\boldsymbol{r}_P$ may be expressed as

$$\dot{\boldsymbol{r}}_P = \boldsymbol{\Theta}\mathbf{I}(r_o\mathbf{e} + \boldsymbol{r}_{p_0} + \boldsymbol{u}_p) + \boldsymbol{\Theta}\dot{\boldsymbol{u}}_p \tag{6}$$

where

$$\mathbf{I} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \dot{\boldsymbol{u}}_p = \begin{pmatrix} \dot{w}_1 - \dot{w}_c \\ \dot{w}_2 \end{pmatrix} \tag{7}$$

The AMM is used to discretize the elastic beam, then the deformation u and v can be represented as

$$\begin{aligned} u(x,t) &= \sum_{i=1}^{n} \phi_i^{(1)}(x) q_i^{(1)}(t) \\ v(x,t) &= \sum_{i=1}^{n} \phi_i^{(2)}(x) q_i^{(2)}(t) \end{aligned} \tag{8}$$

where $\phi_i^{(1)}(x)$ and $\phi_i^{(2)}(x)$ are the admissible functions; $q_i^{(1)}(t)$ and $q_i^{(2)}(t)$ are the mode generalized coordinates, and n refers to the number of included modes. In the sequel, $\boldsymbol{\phi}_1(x)$, $\boldsymbol{\phi}_2(x)$, $\boldsymbol{q}_1(t)$ and $\boldsymbol{q}_2(t)$ are adopted to represent the vectors of $\phi_i^{(1)}(x)$, $\phi_i^{(2)}(x)$, $q_i^{(1)}(t)$ and $q_i^{(2)}(t)$ respectively.

Using AMM with n assumed modes, the kinetic energy and potential energy of the flexible satellite can be written as

$$\begin{aligned} T = \dot{\theta}^2 \Bigg(& \frac{1}{2}J_h + \frac{1}{2}J_b + \boldsymbol{V}_1\boldsymbol{q}_1 + \frac{1}{2}\boldsymbol{q}_1^T\boldsymbol{M}_1\boldsymbol{q}_1 + \frac{1}{2}\boldsymbol{q}_2^T\boldsymbol{M}_2\boldsymbol{q}_2 \\ & -\frac{1}{2}\boldsymbol{q}_2^T\boldsymbol{D}\boldsymbol{q}_2 \Bigg) + \dot{\theta}\left(\boldsymbol{V}_2\dot{\boldsymbol{q}}_2 + \boldsymbol{q}_1^T\boldsymbol{R}\dot{\boldsymbol{q}}_2 - \boldsymbol{q}_2^T\boldsymbol{R}^T\dot{\boldsymbol{q}}_1\right) \\ & + \frac{1}{2}\dot{\boldsymbol{q}}_1^T\boldsymbol{M}_1\dot{\boldsymbol{q}}_1 + \frac{1}{2}\dot{\boldsymbol{q}}_2^T\boldsymbol{M}_2\dot{\boldsymbol{q}}_2 \end{aligned} \tag{9}$$

$$U = \frac{1}{2}\boldsymbol{q}_1^T\boldsymbol{K}_1\boldsymbol{q}_1 + \frac{1}{2}\boldsymbol{q}_2^T\boldsymbol{K}_2\boldsymbol{q}_2 \tag{10}$$

where J_h is the hub moment of inertia; J_b is the rotational inertia of the beam about the hub centre; the matrices $\boldsymbol{K}_1 \in R^{n\times n}$ and $\boldsymbol{K}_2 \in R^{n\times n}$ are the conventional stiffness matrices; $\boldsymbol{M}_i \in R^{n\times n}$, $i = 1,2$ are generalized elastic mass matrix; the matrix $\boldsymbol{D}$ results from the second order term(3) of the deformation field; the matrix $\boldsymbol{R}$ results from the gyroscopic effects. All the parameters are given as follows

$$J_b = 2\int_0^L \rho A(r_o + x)^2\, dx \tag{11}$$

$$\boldsymbol{K}_1 = 2\int_0^L EA\left(\frac{\partial\boldsymbol{\phi}_1(x)}{\partial x}\right)^T \frac{\partial\boldsymbol{\phi}_1(x)}{\partial x}\, dx \tag{12}$$

$$\boldsymbol{K}_2 = 2\int_0^L EI\left(\frac{\partial^2\boldsymbol{\phi}_2(x)}{\partial x^2}\right)^T \frac{\partial^2\boldsymbol{\phi}_2(x)}{\partial x^2}\, dx \tag{13}$$

$$\boldsymbol{M}_i = 2\int_0^L \rho A \phi_i^T \phi_i dx, \qquad i=1,2 \tag{14}$$

$$\boldsymbol{V}_i = 2\int_0^L \rho A\left(r_o + x\right)\phi_i dx \qquad i=1,2 \tag{15}$$

$$\boldsymbol{D} = 2\int_0^L \rho A\left(r_o + x\right)\boldsymbol{S}(x) dx \tag{16}$$

$$\boldsymbol{R} = 2\int_0^L \rho A \phi_1^T \phi_2 dx \tag{17}$$

where $\boldsymbol{S}(x)$ results from w_c and is represented as

$$\boldsymbol{S}(x) = \int_0^x \frac{\partial \phi_2^T(\xi)}{\partial \xi}\frac{\partial \phi_2(\xi)}{\partial \xi} \mathrm{d}\xi \tag{18}$$

It is important to note that, the matrix $\boldsymbol{D}$ is *non-negative definite* because $\boldsymbol{S}(x)$ is a non-negative definite matrix.

The governing equations of motion can now be obtained through application of the Lagrangian principle

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\eta}_i}\right) - \frac{\partial T}{\partial \eta_i} + \frac{\partial U}{\partial \eta_i} = Q_i \qquad i = 1,2,\cdots,n+1 \tag{19}$$

where η_i are the system generalized coordinates, and Q_i the nonconservative generalized forces due to environmental effects and actuators.

Substituting Eqs.(9) and (10) into Eq.(19), we derive the equations of motion of the flexible system in compact form as

$$\begin{bmatrix} M_{\theta\theta} & \boldsymbol{M}_{\theta q_1} & \boldsymbol{M}_{\theta q_2} \\ \boldsymbol{M}_{q_1\theta} & \boldsymbol{M}_{q_1q_1} & \boldsymbol{0} \\ \boldsymbol{M}_{q_2\theta} & \boldsymbol{0} & \boldsymbol{M}_{q_2q_2} \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{\boldsymbol{q}}_1 \\ \ddot{\boldsymbol{q}}_2 \end{bmatrix} + 2\dot{\theta}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{G}_{q_1q_2} \\ 0 & \boldsymbol{G}_{q_2q_1} & 0 \end{bmatrix}\begin{bmatrix} \dot{\theta} \\ \dot{\boldsymbol{q}}_1 \\ \dot{\boldsymbol{q}}_2 \end{bmatrix}$$
$$+\begin{bmatrix} 0 & \boldsymbol{0} & \boldsymbol{0} \\ 0 & \boldsymbol{K}_{q_1q_1} & \boldsymbol{0} \\ 0 & \boldsymbol{0} & \boldsymbol{K}_{q_2q_2} \end{bmatrix}\begin{bmatrix} \theta \\ \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \end{bmatrix} = \begin{bmatrix} Q_\theta \\ \boldsymbol{Q}_{q_1} \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \tau \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \tag{20}$$

where $M_{\theta\theta} \in R^1$ is the rotary inertia of the system; $\boldsymbol{M}_{q_1q_1} \in R^{n\times n}$ and $\boldsymbol{M}_{q_2q_2} \in R^{n\times n}$ are the beam generalized elastic mass matrices; $\boldsymbol{M}_{\theta q_1} \in R^{1\times n}$, $\boldsymbol{M}_{\theta q_2} \in R^{1\times n}$, $\boldsymbol{M}_{q_1\theta} \in R^{n\times 1}$ and $\boldsymbol{M}_{q_2\theta} \in R^{n\times 1}$ represent the nonlinear inertia coupling between the motion of the reference frame and the elastic deformations; $\boldsymbol{K}_{q_1q_1} \in R^{n\times n}$ and $\boldsymbol{K}_{q_2q_2} \in R^{n\times n}$ are generalized elastic stiffness matrices that are shown to be affected by both the motion of the reference frame and the elastic deformations; Q_θ represents inertia forces; τ is the rotational external torque. All the parameters in Eq.(20) are given as follows

$$M_{\theta\theta} = J_h + J_b + \boldsymbol{q}_1^T\boldsymbol{M}_1\boldsymbol{q}_1 + \boldsymbol{q}_2^T\boldsymbol{M}_2\boldsymbol{q}_2 + 2\boldsymbol{V}_{11}\boldsymbol{q}_1 - \underline{\boldsymbol{q}_2^T\boldsymbol{D}\boldsymbol{q}_2} \tag{21}$$

$$\boldsymbol{M}_{q_1\theta} = \boldsymbol{M}_{\theta q_1}^T = -\boldsymbol{R}\boldsymbol{q}_2 \tag{22}$$

$$\boldsymbol{M}_{\theta q_2} = \boldsymbol{M}_{q_2\theta}^T = \boldsymbol{V}_{12} + \boldsymbol{q}_1^T\boldsymbol{R} \tag{23}$$

$$\boldsymbol{M}_{q_iq_i} = \boldsymbol{M}_i \qquad i=1,2 \tag{24}$$

$$\boldsymbol{G}_{q_1q_2} = -\boldsymbol{G}_{q_2q_1}^T = -\boldsymbol{R} \tag{25}$$

$$\boldsymbol{K}_{q_1q_1} = \boldsymbol{K}_1 - \dot{\theta}^2\boldsymbol{M}_1 \tag{26}$$

$$\boldsymbol{K}_{q_2q_2} = \boldsymbol{K}_2 - \dot{\theta}^2\boldsymbol{M}_2 + \underline{\dot{\theta}^2\boldsymbol{D}} \tag{27}$$

$$Q_\theta = -2\dot{\theta}\left[\left(\boldsymbol{q}_1^T\boldsymbol{M}_1\dot{\boldsymbol{q}}_1 + \boldsymbol{q}_2^T\boldsymbol{M}_2\dot{\boldsymbol{q}}_2\right) + \boldsymbol{V}_1\dot{\boldsymbol{q}}_1 - \underline{\boldsymbol{q}_2^T\boldsymbol{D}\dot{\boldsymbol{q}}_2}\right] \tag{28}$$

$$\boldsymbol{Q}_{q_1} = \dot{\theta}^2\boldsymbol{V}_1^T \tag{29}$$

In Eq.(20), it is easily seen the nonlinear coupling between the rigid-body motion and the elastic deformations. The underlined terms in Eqs.(21), (27) and (28) are resulting from the second-order term in coupling deformation field. The newly established Eqs.(20)-(29) are called the first-order approximate model (FOAM) of the flexible satellite. The equations without the underlined terms are called traditional linear approximate model (TLAM).

Actually, the transverse displacement is much larger than that of the axial displacement [9, 11]. Thus, a simplified equation of motion of hub-beam system can be obtained from Eq.(20) by deleting the elements related to $\boldsymbol{q}_1$ and $\dot{\boldsymbol{q}}_1$

$$\begin{bmatrix} M_{\theta\theta} & \boldsymbol{M}_{\theta q_2} \\ \boldsymbol{M}_{q_2\theta} & \boldsymbol{M}_{q_2q_2} \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{\boldsymbol{q}}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{q_2q_2} \end{bmatrix}\begin{bmatrix} \theta \\ \boldsymbol{q}_2 \end{bmatrix} = \begin{bmatrix} Q_\theta \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \tau \\ \boldsymbol{0} \end{bmatrix} \tag{30}$$

where $M_{\theta\theta}$, $\boldsymbol{M}_{\theta q_2}$, $\boldsymbol{M}_{q_2\theta}$, $\boldsymbol{M}_{q_2q_2}$, $\boldsymbol{K}_{q_2q_2}$ and Q_θ can also be obtained by deleting the elements related to $\boldsymbol{q}_1$ and $\dot{\boldsymbol{q}}_1$ in(21), (23), (24), (27)and (28). It is noted that, the simplified model can be used for controller design.

III. Simulations and Results

The physical parameters of the flexible satellite model are shown in Table I. The number of assumed mode n is 5.

TABLE I
PHYSICAL PARAMETERS

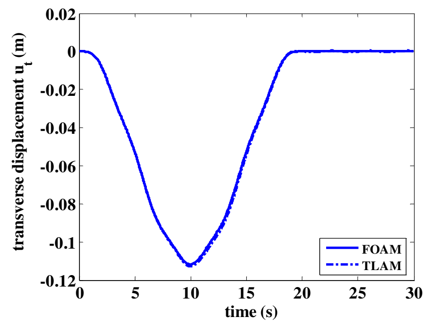
Property	Symbol	Value
Beam length	L	10m
Cross-Section	A	7.3025×10^{-5}m^2
Beam area moment of inertia	I	8.0265×10^{-9}m^4
Mass per unit volume	ρ	2.6538×10^{3}kg/ m^3
Young's modulus	E	6.9268×10^{10}N/m^2
Hub moment of inertia	J_h	500kg/m^2
Length of $\vec{r}_o$	r_o	1m

A. Free Vibration of the Flexible Satellite

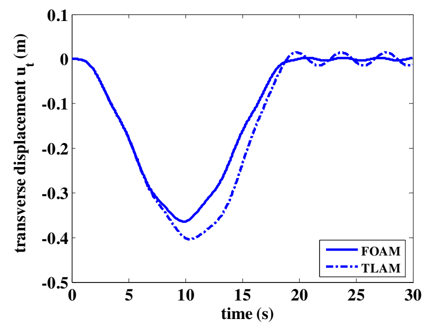
The response of the flexible motion is simulated by assuming that the slewing motion follows a prescribed trajectory, and the maneuver profile is given by [5]:

$$\dot{\theta} = \begin{cases} \dfrac{w_f}{t_f} - \dfrac{w_f}{2\pi}\sin\left(\dfrac{2\pi}{t_f}t\right), & 0 \le t \le t_f \\ w_f\,, & t > t_f \end{cases} \tag{31}$$

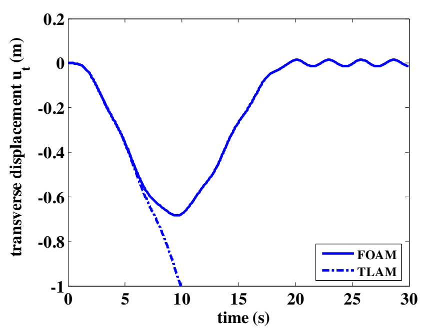
where w_f and $t_f = 20s$ represent the velocity of the hub at the end of the maneuver, and the time to reach the maximum velocity, respectively.



(a) $\dot{\theta} = 0.3rad/s$



(b) $\dot{\theta} = 1rad/s$



(c) $\dot{\theta} = 2rad/s$

Fig. 3. Transverse response of the tip of the beam

Fig.3 shows simulation results of FOAM and TLAM for different velocities. In case of low velocity, as shown in (a) when the velocity is 0.3rad/s, the difference is insignificant between the transverse displacements of the two models. As in low speed case, the general elastic stiffness matrices of the two models (FOAM, TLAM) are dominated by $\boldsymbol{K}_2$, the conventional stiffness matrix. When the velocity reaches 1rad/s, as shown in (b), significant difference is observed. The maximum amplitude of TLAM is much larger than that of FOAM, so is the amplitude of residual vibration. Obvious difference is shown in (c), when angular velocity is 2rad/s. The vibrational response of TLAM goes infinite, which is not true actually. It should be noted that, the resulting tip displacement of TLAM has exceed the assumption of small deformation. So a conclusion can be made that, TLAM is invalid in describing the deformation of flexible beam undergoing high-speed maneuvering. However, FOAM can predict valid results.

Because the second order term in deformation is not included, the generalized elastic stiffness matrix in the TLAM is expressed as $\boldsymbol{K}_{q_2q_2} = \boldsymbol{K}_2 - \dot{\theta}^2\boldsymbol{M}_2$. From this expression, we can see that the stiffness matrix may be negative definite when the angular velocity surpasses a critical value. Actually, it can be calculated from the expression that the critical angular velocity is 1.9rad/s. In fact, it is the first order natural circle frequency of the beam, see Table II. The frequencies evaluated with TLAM are 'softening' compared with the natural frequencies. On the other hand, the generalized elastic stiffness matrix in FOAM is expressed as $\boldsymbol{K}_{q_2q_2} = \boldsymbol{K}_2 - \dot{\theta}^2\boldsymbol{M}_2 + \underline{\dot{\theta}^2\boldsymbol{D}}$, in which the underlined term $\dot{\theta}^2\boldsymbol{D}$ is non-negative definite, and can make $\boldsymbol{K}_{q_2q_2}$ definite positive.

As shown in Table II, the natural vibration frequency is larger than that evaluated with TLAM, but less than that evaluated with FOAM. That is, the second order term has a 'stiffening' effect on the frequency.

From the above, it is seen that the second order term in deformation field have significant effect on dynamics behaviour of flexible multibody systems in high-speed case, and the developed model based on the coupling deformation field can predict valid results.

TABLE II
The Fist Five Vibration Frequencies (Hz)

Modal order	1	2	3	4	5
Natural frequency	0.3008	1.8793	5.2622	10.312	17.046
TLAM (1 rad/s)	0.2541	1.8726	5.2598	10.311	17.045
FOAM (1 rad/s)	0.3141	1.9216	5.3086	10.362	17.097

B. *Maneuver of the Flexible Satellite*

It is assumed that, a moment generator is located on the rigid centre-body, and the moment profile is given by

$$\tau(t) = \begin{cases} \tau_0 \sin\left(\dfrac{2\pi}{T}t\right), & 0 \le t \le T \\ 0, & t > T \end{cases} \quad (32)$$

where T=15s, τ_0 is the maximum torque.

Figs.4 and 5 show the tip response of the two models (FOAM, TLAM) applied the torque 50Nm and 300Nm respectively. In Fig.4, we can see that no substantial differences occur between the simulation results, as the angular velocity is low under the small torque. Fig.5 shows that, the resulting tip displacement of TLAM is much larger than that of FOAM, and even larger than the beam length. This is because the peak angular velocity reaches 1.9rad/s(the first order natural circle frequency) when τ =300Nm, the TLAM is invalid in describing the deformation of the flexible appendages.

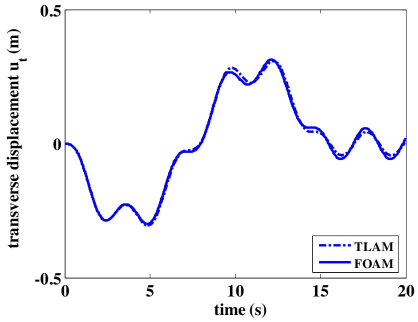


Fig. 4. Tip response of the beam when τ =50Nm

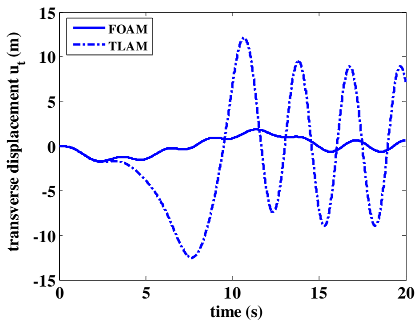
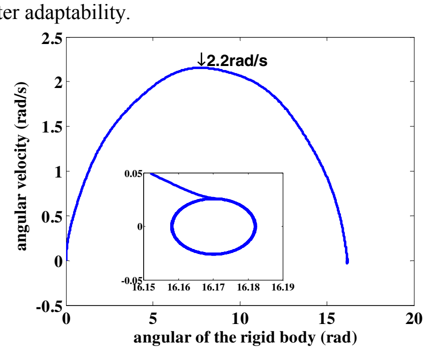
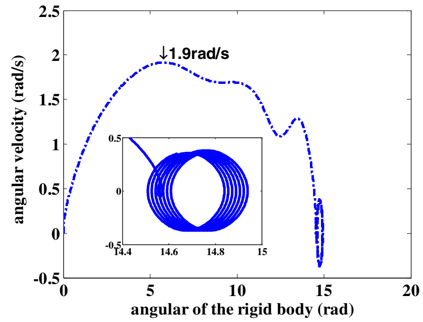


Fig. 3. Tip response of the beam when τ =300Nm

Phase plane of attitude is illustrated in Fig.6. As shown in (a), the peak angular velocity of FOAM is 2.2rad/s，which is larger than that of TLAM (1.9rad/s, i.e., the first order natural circle frequency). The angular velocity of TLAM can not surpass 1.9rad/s, though the torque is still acted on the rigid body. Then the control energy is stored as elastic deformation energy during the maneuvering. Conversely, the tip deflection affects the attitude angular through the inertia force Q_θ . Hence, the attitude controller and structural flexibility interacts and deserve attention, especially when it undergoes high-speed rotation. However, FOAM of the flexible satellite system can predict valid results and takes on better adaptability.



(a) FOAM



(b) TLAM

Fig.6 phase plane of satellite attitude when τ =300Nm

IV. CONCLUSION

A novel flexible satellite dynamic model has been established based on the coupling deformation field, in which the second-order coupling term is taken into account. A simplified first-order model has also been given for controller design. Theoretical analysis and simulation results indicated that the second order term in deformation field has significant effect on dynamics behavior of flexible multibody systems in high-speed case and the dynamic stiffening is accounted for. The FOAM takes on better adaptability, especially in high rotating speed cases.

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