88-4255-CP

AN OPTIMAL MANEUVER CONTROL METHOD FOR THE SPACECRAFT WITH FLEXIBLE APPENDAGES

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Abstract

When a spacecraft with flexible appendages is accomplishing maneuver or certain purpose movement under applied forces, the resultant motion will, generally, consist of two kinds of motions : desired motion superposed by series of harmonics which are harmful and unexpected. The authors of this paper have suggested a mothod [4] to suppress one of the vibration modes. In this paper, another approach : two constant amplitude control strategy are proposed to meet the same purpose.

Introduction

In many operational modes of spacecraft the large angle attitude maneuver or orbital correction maneuver are necessarily required in order to point onboard devices and stationkeeping thrustes, or to accomplish rendezvous and docking mission of two or more vehicles. To investigate the motion performances and control laws of maneuver movement, particularly, taking spacecraft flexibility into account is of extreme importance. The existing major approaches [1], [2] for solving this problem lies in applying variational calculus and leads to a two point boundary value (TPBV) problem. The most significant difficulties of TPBV problem are that it needs a large amount of memories of onboard processor and more computer time. Moreover, they are applicable, in fact, only to the linear spacecraft system. Having this in mind, authors of [3] have suggested a method to overcome these problems. The baseline of their method lies in that : The motion of rigid body mode is controlled by an open loop optimal controller and the appendage vibration is regulated by the optimal LQ regulator.

The principle of method in this paper is quite different from above methods. It can suppress effectively elastic vibration mode, but needs not to solve the TPBV problem and the applied forces are very close to that obtained from the Pontryagin's maximum principle[5].

Principle of Proposed Method

To demonstrate the principle of the method we proposed, a simple example of two degree of freedom is given first. That is a simple pendulum of mass, m ,with moving support of mass, M, oxy is a reference frame. Let us denote by, ρ, the horizontal displacement of moving support subjected to the force, F(t), and by, φ, the angluar displacement of the weightless rod of length, r. Neglecting the friction and aerodynamic force, one can obtain the motion equations of system

$$(M+m)\ddot{\rho} - mr\cos\varphi\,\ddot{\varphi} + mr\dot{\varphi}^2\sin\varphi = F(t) \tag{1}$$
$$mr^2\ddot{\varphi} - mr\ddot{\rho}\cos\varphi + mrg\sin\varphi = 0 \tag{2}$$

Where, g is gravitational acceleration. Assuming that φ and $\dot{\varphi}$ are small values and its products can be ignored and solving for φ and ρ, we have

$$\ddot{\rho} = [F(t) - mg\varphi] / M \tag{3}$$
$$\ddot{\varphi} + \omega^2\varphi = F(t) / (mr) \tag{4}$$

Where the frequence of free vibration

$$\omega = \sqrt{g(M+m) / (rM)} \tag{5}$$

Eq.(4) is a typical second order differential equation of forced vibration, its solution under initial conditions, $\varphi(0)$ and $\dot{\varphi}(0)$, are

$$\varphi(t) = \dot{\varphi}(0)\sin\omega t / \omega + \varphi(0)\cos\omega t + \int_0^t F(\xi)\sin[\omega(t-\xi)]d\xi / (\omega r M) \tag{6}$$

Introducing Eq.(6) into Eq.(3), one can obtain corresponding solution for the horizontal displacement, $\rho(t)$. As we have proven in [4], a four-impulse control strategy can make the motion of moving pendulum under zero initial conditions over a given distance, L, with only two half-period vibrations of simple pendulum, one in the very beginning and end, respectively. And the motion in midcourse is free from vibration of pendulum, in fact, whole system moves with uniform velocity (Fig. 2). Here, I_i (i=1,2,3,4) are impulses

$$I = \int_0^{\Delta t} F(\xi)\, d\xi$$

Where, Δt is a small interval. The four impulses I_1 to I_4 should act at suitable instants with suitable amplitudes, they are refered to as optimal control strategy. According to [4], the time spaces, T_1 and T_3, between first and last two impulses are very critical and should be equal to the half of period π/ω of vibration, however,

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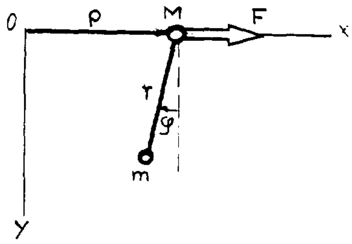

Fig.1

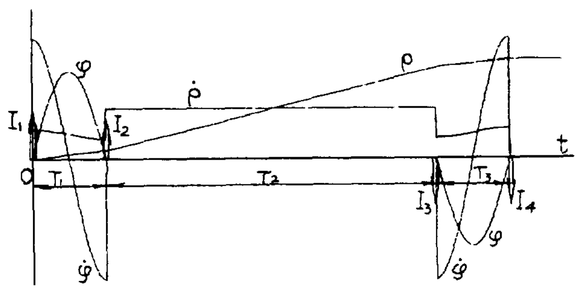

Fig.2

the duration $,T_2$, is not sensitive to the vibration of pendulum , it is determined only by the specified moving distance , L. The optimal control strategy can then be written as (Fig 3)

$$I=I_1\ \delta(0)+I_2\ \delta(t-T_1)+I_3\ \delta(t-T_1-T_2)+ + I_4\ \delta(t-T_1-T_2-T_3) \qquad (7)$$

Where , $\delta(x)$are Kronecker delta

$$\delta(x) = \begin{cases} 0 & x=0 \\ 1 & x\neq 0 \end{cases}$$

and

$$I_1=I_2=-I_3=-I_4$$
$$T_1=T_3=\pi/\omega$$

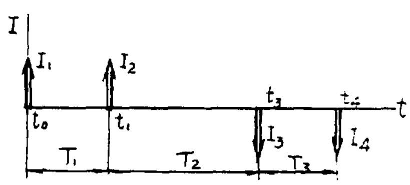

Fig.3

The kernel of four-impulse optimal control strategy is the set of two impulses, I_1 and I_2, or I_3 and I_4 , which can be used repeatedly .

Some times the capability of actuators (say thrusters) restricts the values of impulses , therefore , instead of four impulse optimal control , a two constant amplitude control is proposed in this paper . Here , the applied forces are constant ones , the equations for evaluating their amplitudes , F_0 , and acting times, T (see Fig. 4) should be deduced as follows.

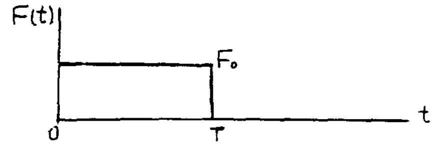

Fig.4

(1) In the interval $,0\leqslant t\leqslant T$.
The integral in Eq.(6) becomes

$$F_0(1-\cos\omega t)\ /\ (\omega^2 rM)$$

And the solutions for $\varphi(t)$ and $\rho(t)$ are

$$\varphi(t) =[\varphi(0)-F_0/a]\cos\omega t + + \dot{\varphi}(0)\sin\omega t/\omega +F_0/a \qquad (9)$$

$$\rho(t) =\rho(0)+\dot{\rho}(0)t+F_0 t^2/2(M+m)+ +mg[\varphi(0)-F_0/a]\cos\omega t/(M\omega^2)+ +mg\,\dot{\varphi}(0)\sin\omega t/(M\omega^3) \qquad (10)$$

the corresponding velocities, therefore, are

$$\dot{\varphi}(t)=-[\varphi(0)-F_0/a]\ \sin\omega t+\dot{\varphi}(0)\cos\omega t \qquad (11)$$
$$\dot{\rho}(t)=\dot{\rho}(0)+F_0 t/(M+m)-mg[\varphi(0)-F_0/a]\sin\omega t+ +mg\,\dot{\varphi}(0)\cos\omega t\ /(M\omega^3) \qquad (12)$$

In Eqs.(9) to (12)

$$a = \omega^2 rM$$

(2) In the interval , $t>T$
The integral in Eq.(6) is

$$F_0(\cos\omega T\ \cos\omega t+\sin\omega T\sin\omega t-\cos\omega t)\ /a$$

Hence, the displacements and velocities are

$$\varphi(t)=[\dot{\varphi}(0)/\omega+F_0\sin\omega T/a]\sin\omega t+ +[\varphi(0)-F_0(1-\cos\omega t)/\ a]\cos\omega t \qquad (14)$$
$$\rho(t)=\rho(0)+\dot{\rho}(0)t+F_0 t^2/2(M+m)+ +mg[\varphi(0)-F_0/a]\cos\omega t/(M\omega^2)+ +mg\dot{\varphi}(0)\sin\omega t\ /(M\omega^3) \qquad (15)$$
$$\dot{\varphi}(t)=[\dot{\varphi}(0)/\omega+F_0\sin\omega T/a]\omega\cos\omega t- -[\varphi(0)-F_0(1-\cos\omega T)]\omega\sin\omega t \qquad (16)$$
$$\dot{\rho}(t)=\rho(0)+F_0 t/(M+m)-mg[\varphi(0)-F_0/a] \times\sin\omega t\ /(M\omega)+mg\dot{\varphi}(0)\cos\omega t/(M\omega^2) \qquad (17)$$

Analysing Eqs (14) to (17) , one can obtain the conditions under which the system in Fig. 1 moves for interval $t > T$ with uniform velocity , and simple pendulum does not vibrate . They are stated as :

(a) For zero initial conditions of pendulum vibration , i.e. $\varphi(0)=\dot{\varphi}(0)=0$, the acting time of force should be equal to n times period of vibration,

$$T=2\pi n/\omega \quad , \quad n=1,2, \ldots \qquad (18)$$

Where, n is positive integer number . The amplitude, F_0, is determined by following expression , depending on specified velocity, $\dot{\rho}(t)$,at and after moment , T , as

$$F_o = [\dot{\rho}(T) - \dot{\rho}(0)](m+M)/T \qquad (19)$$

(b) For non-zero initial conditions, we have

$$T = \arctan\{\dot{\varphi}(0)\omega/[\varphi(0)\omega^2 - F_o/(Mr)]\} \qquad (20)$$

$$F_o = [\dot{\varphi}^2(0) + \omega^2\varphi^2(0)]Mr/[2\varphi(0)] \qquad (21)$$

Eqs (18) and (19) can be used as well for determining the required T and F_o of constant amplitude force to stop moving of the system shown in Fig. 1 while it moves with uniform velocity , $\dot{\rho}(T)$. To this end , it needs only to put $\dot{\rho}(0)=0$ in Eq. (19) and to exchange into minus sign , however , Eq. (18) remains changeless.

The constant amplitude forces determined by Eqs (18) to (21) are refered to as optimal control. Fig. 5 shows an example of movement of moving pendulum system of Fig. 1 over a distance , L , with optimal control. System parameters for Fig.5 are M=100 kg ,m=20 kg, r=1 m and L=44.27 m. The system free frequency ω = 3.431 sec^{-1} , therefore from Eqs (18) and (19) one can get the force amplitudes, F_o , and acting times , T , for zero initial conditions and for $\dot{\rho}(T)$= 7.95 m/s. they are

$T = 2\pi/\omega = 1.83$ sec.
F = 518 N

As we can see from Fig. 5a , the pendulum oscillates only in the intervals , $t0 \leqslant t \leqslant t1$ and $t2 \leqslant t \leqslant t3$ with only one vibration cycle in each, the motion in the interval , $t1<t<t2$, however , is of uniform one and system stops moving immediately at $t=t_3$. The function of the first force applied in period $T=t_1-t_0$ is to accelerate system , and so it can be called as accelerating force , the period between t_0 and t_1 , therefore, accelerating phase . Conversely, the function of the second force acted in period $T=t_3-t_2$ is to decelerate the system, and so this force and its acting period are referred to as decelerating force and decelerating phase , respectively . The duration between accelerating and decelerating phases is called as transfer phase.

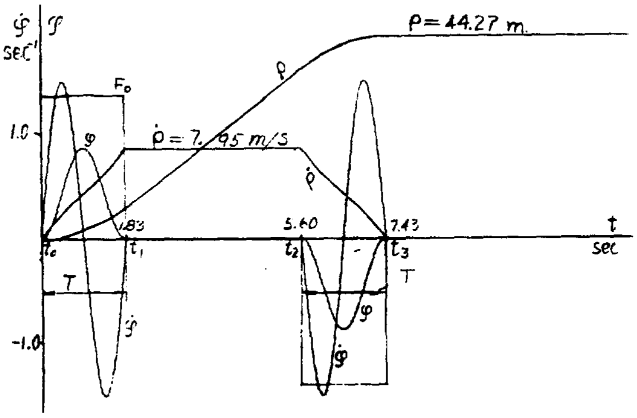

Fig.5a

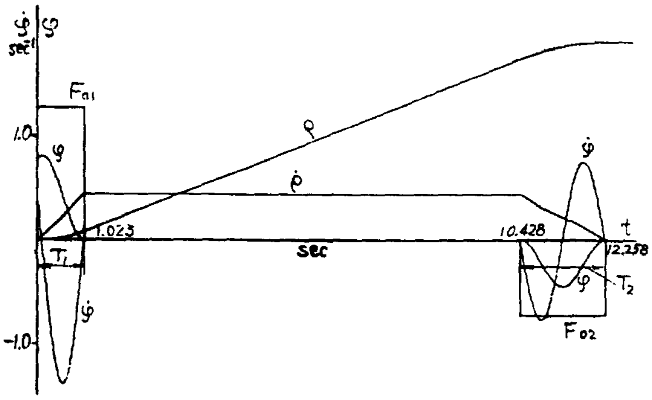

Fig.5b

The example in Fig. 5b has the same motion features , but with partial nonzero initial conditions , i.e. $\varphi(0)=\pi/4$ and $\dot{\varphi}(0)=0.5$ sec^{-1} . The parameters of accelerating force determined from Eqs (20) and (21) are F_{o1} = 478.2 N and T_1 = 1.023 sec , and those of decelerating force determined from Eqs (18) and (19) are F_{o2} = 261.6 N and T_2 =T=1.83 sec. Here , F_{o1} is solely determined by the initial conditions , consequently , it is much more less than F_o in Fig. 5a , that results in a lower velocity in transfer phase and longer motion time . In order to make the motion faster , we can insert several accelerating forces with arbitrary needed amplitudes into transfer phase , each one of which acts just as accelerating force in Fig.5a ,i.e. adds certain increment of velocity to moving support , but does not cause any additional effects to pendulum oscillation.

We have demonstrated through simple moving pendulum examples that for the system with one vibration degree of freedom , one can suppress the vibration in transfer phase and stop moving of whole system immediately after application of second force of optimal control . For the system with two or more vibration degrees of freedom , we may choose one of the vibration modes and make it suppressed using the optimal control of two constant amplitude forces , acting time of which should be evaluated with the frequency of this mode.

Large Angle Maneuver of Flexible Spacecrafts

Assume that the spacecraft consists of central rigid body , Bo , i.e. a sphere of radius , b, and several flexible uniform cantilever-like appendages , Sp (p=1,2,...), with a tip mass,m , on each one (Fig. 6) . oXYZ and oxyz are the inertial and local reference frames , with the origins at mass center of spacecraft and at the joint of appendage , Sp , with central body,respectively . ox coincides with undeformed cantilever , Sp . For simplicity, only the attitude motion in plane oXY will be analysed. Let I_b be the moment of inertia of Bo about oZ axis,l_p , σ,and EJ_p the length, mass per unit length and bending stiffness of cantilever, Sp , Wp(x,t) the transverse displacement of point , x , in cantilever and $\theta(t)$, the attitude angle of spacecraft.

The basic equations contain motion equation of spacecraft

$$I\ddot{\theta} + \sum_p \int_0^{l_p} \ddot{W}_p \sigma (b+x)dx + \sum_p [\ddot{W}_p(l_p,t) + (b+l_p)\ddot{\theta}] m(b+l_p) = Tr \tag{22}$$

and pth appendage equation

$$W_p^{IV} + \sigma \frac{\partial^2}{\partial t^2}[W_p + (b+x)\theta]/(EJ_p) = 0 \quad p=1,2,\ldots \tag{23}$$

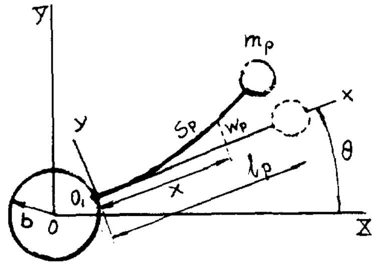

Fig.6

Where Tr is control moment of force, $I = I_b + \sum_p I_{Ap}$ is moment of inertia of spacecraft, and

$$I_{Ap} = \int_0^{l_p} \sigma (b+x)^2 dx + m(b+l_p)^2 \tag{24}$$

The boundary conditions for Eq. (23) are

$$W_p(0,t) = W_p'(0,t) = W_p''(l_p,t) = 0 \quad , \; p=1,2\ldots \tag{25}$$

$$m_p[\ddot{W}_p(l_p,t) + (b+l_p)\ddot{\theta}] = EJ_p W_p'''(l_p,t) \quad p=1,2,\ldots \tag{26}$$

Where , W_p^{i}(i=', ",''', IV) are the ith partial derivatives of W_p with respect to x, while $\ddot{W}_p$ and $\ddot{\theta}$ partial derivatives with espect to t.

Constrained Mode

The solution of Eqs (22) and (23) in conjunction with the boundary conditions (25) and (26) can be obtained by constrained modal analysis . Hence , putting $\theta = 0$, one can get from Eqs (23), (25) and (26) the constrained equation

$$W_{pc}^{IV} + \sigma \ddot{W}_{pc}/(EJ_p) = 0 \; , \quad p=1,2,\ldots \tag{27}$$

and boundary conditions

$$W_{pc}(0,t) = W_{pc}'(0,t) = W_{pc}''(l_p,t) = 0 \; , \; p=1,2.. \tag{28}$$

$$m_p \ddot{W}_{pc}(l_p,t) = EJ_p W_{pc}'''(l_p,t) \quad , \; p=1,2,\ldots. \tag{29}$$

Where subscript , c , denotes constrained mode. Substituting

$$W_{pc} = \varphi_p(x) q_p(t) \tag{30}$$

into Eq. (25), one can obtain

$$\varphi_p^{IV} - k^4 \varphi_p(x) = 0 \tag{31}$$

$$\ddot{q}_p + p_p^2 q_p(t) = 0 \tag{32}$$

Where , p_p is free frequency of appendage, Sp, and

$$k^4 = \sigma p_p^2 / (EJ_p) \tag{33}$$

Solution of Eqs(31) and (32) are

$$q_p(t) = \dot{q}(0)\sin(p_p t)/p_p + q(0)\cos(p_p t) \tag{34}$$

$$\varphi_p(x) = Ay_1(kx) + By_2(kx) + Cy_3(kx) + Dy_4(kx) \tag{35}$$

Where , q(0), $\dot{q}(0)$ are initial conditions , A,B,C and D constants and

$$\begin{aligned} y_1(kx) &= [Ch(kx)+\cos(kx)]/2 \\ y_2(kx) &= [Sh(kx)+\sin(kx)]/2 \\ y_3(kx) &= [Ch(kx)-\cos(kx)]/2 \\ y_4(kx) &= [Sh(kx)-\sin(kx)]/2 \end{aligned} \tag{36}$$

Considering the boundary conditions (28) and (29), one can obtain modal shape functions

$$\varphi_{pn}(x) = C[y_3(k_n x) - y_4(k_n x) y_1(k_n l_p)/y_2(k_n l_p)] \quad p=1,2,\ldots, \; n=1,2,\ldots \tag{37}$$

The parameter k_n should be evaluated from following charasteristic equation

$$m_p[th(kl_p)\cos(kl_p) - \sin(kl_p) + EJ_p[\cos(kl_p) + 1/Ch(kl_p)] = 0 \tag{38}$$

The condition of orthogonality of normalized modal shape functions is

$$\sigma \int_0^{l_p} \varphi_{pn}(x)\varphi_{pm}(x)dx + m\,\varphi_{pn}(l_p)\varphi_{pm}(l_p) = \begin{cases} 0, & n \neq m \\ 1, & n=m \end{cases} \tag{39}$$

Solution of Basic Equations

The solution of Eqs (22) and (23) can be formed by the series of constrained modal shape functions

$$W_p(x,t) = \sum_{n=1}^{\infty} \varphi_{pn}(x) \psi_{pn}(t) \qquad p=1,2,\ldots \tag{40}$$

$\psi_{pn}(t)$ are unknown time functions called normal coordinates and remained to be determined . Inserting Eq.(40) into Eq. (22) , it becomes

$$I\ddot{\theta} + \sum_p \sum_n F_{pn} \ddot{\psi}_{pn}(t) = Tr \tag{41}$$

where,

$$F_{pn} = \int_0^{l_p} \sigma \varphi_{pn}(x)(b+x)dx + m\varphi_{pn}(l_p)(b+l_p) \tag{42}$$

Inserting, again, Eq. (40) into Eq.(23) and Eq.(26), employing Eqs (29) and (31) and orthogonality condition , Eq. (39), the appendage equation (23) turns into

$$\ddot{\psi}_{pn} + p_{pn}^2 \psi_{pn} + F_{pn}\ddot{\theta} = 0 \; , \quad p=1,2,\ldots, \; n=1,2,\ldots \tag{43}$$

Solving Eqs (41) and (43) and using Eq. (40) , the attitude angle $\theta(t)$ and displacement $W_p(x,t)$ can ,then, be found. However, in order to use the principle of above mentioned optimal control strategy , one does not need to solve them , only the free frequencies of unconstrained system must be evaluated prior. To this end , letting Tr in Eq. (41) be zero and, then, taking the Fourier transform of Eqs (41) and (43) , the following equations are obtained

$$I\ddot{\bar{\theta}} + \sum_p \sum_n F_{pn}\ddot{\bar{\psi}}_{pn} = 0$$

$$(p_{pn}^2 - \omega_i^2)\bar{\psi}_{pn} - \omega_i^2 F_{pn}\bar{\theta} = 0 \qquad p=1,2,\ldots,n=1,2,\ldots$$

Overbars denote values deduced by Fourier transformation . The characteristic equation of above two equations

$$I/\omega_i + \sum_p \sum_n F_{pn}(p_{pn}^2 - \omega_i^2) = 0 \qquad i=1,2,\ldots \tag{44}$$

can be used for evaluating the free frequencis , ω_i (i=1,2,...) of unconstrained system .

For numerical simulation , the state equations should be deduced . To this end ,an useful relationship is given, first , i.e. the sum of $\sum_{n=1}^{\infty} F_{pn}$ is

$$\sum_{n=1}^{\infty} F_{pn}^2 = I_{Ap} \tag{45}$$

Taking this feature into account and considering $I_b = I - \sum_p I_{Ap} = I - \sum_p \sum_n F_{pn}^2$, we can obtain the state equations from Eqs (41) and (43) . For simplicity, but lossing no generality , we may assume that there are R identical flexible appendages on the spacecraft . Under this condition , the subscript p can be dropped and Eqs (41) and (43) will become

$$I\ddot{\theta} + R\sum_n \ddot{\psi}_n = Tr \tag{46}$$

$$\ddot{\psi}_n + p_n^2\psi_n + F_n\ddot{\theta} = 0 \quad , \quad n=1,2\ldots \tag{47}$$

Defining the state variables as

$$x_1 = \theta \quad , \quad x_2 = \dot{\theta}$$
$$x_{2i+1} = \psi_i \quad , \quad x_{2i+2} = \dot{\psi}_i \quad i=1,2,\ldots \tag{48}$$

and

$$x = [\ x_1,\ x_2,\ \ldots\ ]^T \tag{49}$$

the state equations can be written as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \sum_{n=1}^{\infty} f_n x_{2n+1} + Tr/I_b$$

$$\dot{x}_{2i+1} = x_{2i+2}$$

$$\dot{x}_{2i+2} = -p_i^2 x_{2i+1} - F_i\sum_{n=1}^{\infty} f_n x_{2n+1} - F_i Tr/I_b$$

$$i=1,2,\ldots \tag{50}$$

Where , $f_i = RF_i p_i^2 / I_b$

Numerical Example

For illustration of the optimal control strategy proposed in this paper , the simple example in Fig . 6 will be examined . Assume the parameters of spacecraft are

$$EJ_p = 625\ \text{kgm}^3\text{s}^{-2}\ ,\ l_p = 6\text{m}\ ,\ b = 0.6\text{m}$$

$$\sigma = 1\ \text{kgm}^{-1}\ ,\quad m = 1\ \text{kg}\ ,\quad I_b = 121.36\ \text{kgm}^2$$

Solving Eq.(38) and taking Eq.(33) into account , one can find frequencies , p_n (n=1,2,...) , of constrained modes. They are

1.89, 12.85, 37.52,.... sec−1

Then, F_n (n=1,2,...) and frequencies, ω_i(i=1,2. ..) of unconstrained modes can be determined through Eqs (42) and (44), respectively

F_n = 11.52 ,2.23 , 0.93 ,... kg
ω_i =3.25 , 13.36 , 37.80 , ... sec^{-1}

Now , the due parameters of optimal control forces can be evaluated . Suppose the 1st vibration mode is decided to be suppressed , the acting times and amplitudes of optimal control ,in this case,should be determined by Eqs (18) to (21) using ω_1 and other values required by these equations.

Fig. 7 shows the numerical simulation result of attitude angle , angular velocity , normal coordinate and its velocity of first harmonic,when the spacecraft completes large angle attitude maneuver with zero initial conditions . Owing to the smallness the other harmonics are not shown . The acting time of accelerating and decelerating forces is $T = 2\pi/\omega_1$ = 1.93 sec and their amlitude Tr= 5 Nm . For the sake of contrast a set of corresponding plots is illustrated in Fig. 8 for non-optimal controls. The amplitudes are enlarged by 7/5 times, but acting times are decreased by 5/7 ,so that the total impulse remains changeless. As we can see,the motion in Fig. 7 is much more smooth than that in Fig. 8 , particularly, in Fig. 8 the appendages vibrate intensely and a large fluctuation is bound to follow the attitude maneuver even in transfer phase and after the end of decelerating force.

In fact , if we retain only first mode in Eq. (47) and take Laplace tranform of Eqs (46) and (47), we can get

$$\theta(s) = Tr(s)/[Is^2 - F_1^2 s^2/(s^2+p_1^2)]$$
$$\psi_1(s) = -Tr(s)F_1/[I(s^2+p_1^2) - F_1^2]$$

Assume Tr is a control force as shown in Fig. 4 with unit amplitude and acting time , τ_o, its Laplace transform is ,then, $Tr(s) = (1-e^{-\tau_o s})/s$. Inserting it into above two equations and taking inverse Laplace transfotm for zero initial conditions , one can obtain the responses of $\theta(t)$ and $\psi_1(t)$ in the interval $t > \tau_o$.

$$\theta(t) = \tau_o t/I - \tau_o^2/(2I) - F_1^2\{\cos[p_1 t/\sqrt{1-F_1^2/I}\ ]$$

$$-\cos[p_1(t-\tau_o)/\sqrt{1-F_1^2/I}\ ]\}/(I^2 p_1^2) \quad ,(t>\tau_o) \tag{51}$$

$$\psi_1(t) = F_1\{\cos[p_1 t/\sqrt{1-F_1^2/I}] - \cos[p_1(t-\tau_o)$$

$$/\sqrt{1-F_1^2/I}]\}\ /(Ip_1^2) \quad , \quad (t>\tau_o) \tag{52}$$

Where , $p_1/\sqrt{1-F_1^2/I}$ is just the frequency , ω_1 , of 1st unconstrained mode determined from Eq.(44) . Inspecting Eqs (51) and (52), one can find that if

$$\tau_o = 2\pi/\omega_1 = 2\pi\sqrt{1-F_1^2/I}\ /p_1 \tag{53}$$

equations (51) and (52) will become

$$\theta(t) = \tau_0 t / I - \tau_0^2/(2I) \quad , \quad (t > \tau_0) \tag{54}$$

$$\psi_1(t) = 0 \quad , \quad (t > \tau_0) \tag{55}$$

This just is the same result of Fig. 7 in the tranfer phase.

If we want to reduce the vibration of appendages but keep the attitude angular velocity changeless , we may apply the optimal control with decreased amplitude twice or more times such as shown in Fig. 9. Each of the accelerating and decelerating control forces has half the amplitude and same acting time of those in Fig. 7. As we can see the resultant angular velocity, $\dot{\theta}(t)$, in transfer phase is as large as in Fig. 7, but the amplitude of the 1st mode of appendage vibration is only one half of the former. Therefore , if the total impulse is kept constant , the more times we apply optimal controls to the system ,the weaker appendage vibration will be.

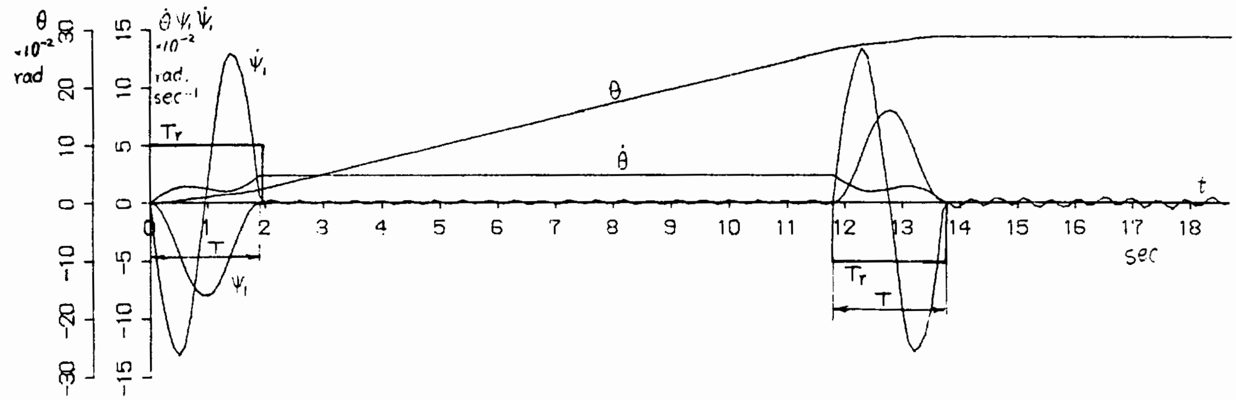

Fig.7

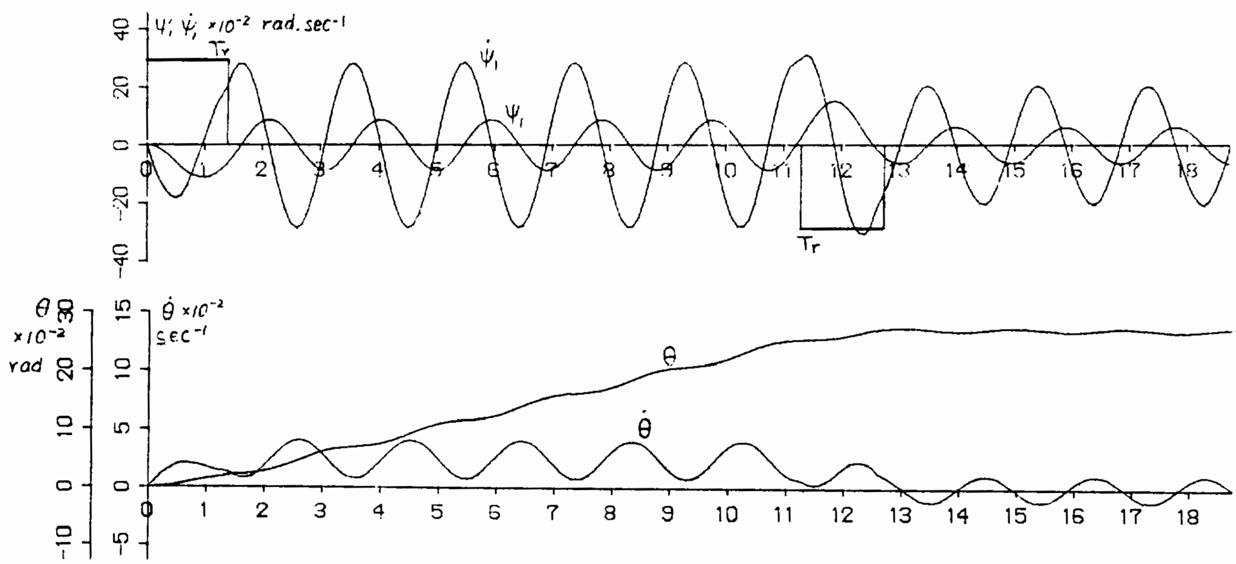

Fig.8

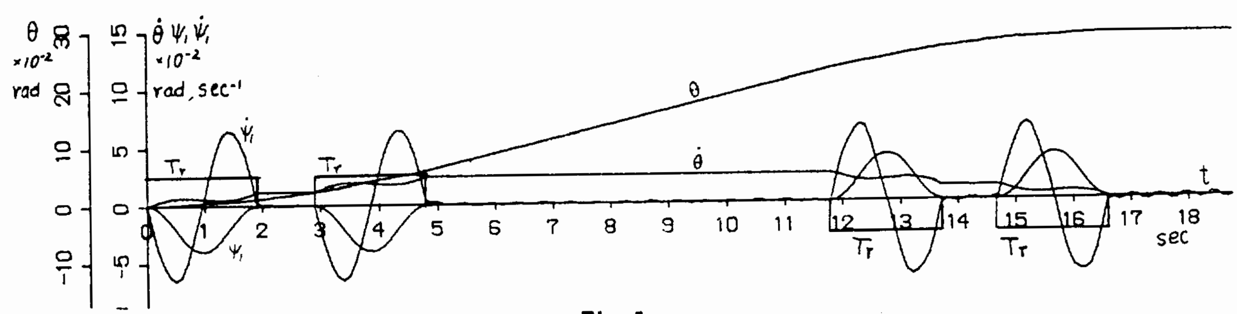

Fig.9

Conclusion

1. Two constant amplitude optimal control proposed in this paper as well as the four impulse optimal control put forward in [4] all can effectively suppress one of the harmonics when they are applied to the system consisted of rigid and flexible bodies. This principle can be used in many fields ,for instance,to restrict or eliminate elastic vibration, liquid sloshing, pendulum oscillation and so on of spacecrafts, cranes , flexible manipulatops and other machines.

2. This appoach is easy to accomplish , in sense of calculating algorithm and means for realization . There is no need to solve the TPBV problem , and so a great deal of computer time can be saved . However , this appoach requires identification or determination of vibration parameters before application of optimal controls to the system , as the amplitudes and acting times are sensitive to initial conditions of vibration (say $\omega(0)$ and $\dot{\omega}(0)$, or $\psi_i(0)$ and $\dot{\psi}_i(0)$).

3. This approach can be developed into such a method that two of the specified vibrations will be suppressed . The demand for suppressing simultaneously two vibrations is often necessary , for example, one of them may be the elastic vibration, the another liquid sloshing.

This work is sponsored by National Natural Science Foundation of China under contract No.Ji 85016.

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