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Robust Integrated Translation and Rotation Finite-Time Maneuver of a Rigid Spacecraft Based on Dual Quaternion

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This paper deals with simultaneous position and attitude robust maneuver problem of a rigid spacecraft. Based on a novel mathematical tool, i.e., dual quaternion, an integrated translational and rotational tracking error model of a rigid spacecraft is derived, whose form holds effective and compact. A robust finite-time control strategy is then proposed using backstepping technique to enable the spacecraft to track reference position and rotation motions in finite time. Within the Lyapunov framework, the finite-time stability of the closed-loop system is guaranteed in the presence of system uncertainties and bounded external disturbances; meanwhile, the bound of settling time can be derived as well. A scenario numerical simulation demonstrates the effect of the designed control law.

Nomenclature

$\hat{\mathbf{d}}$	= Dual disturbance force
$\mathbf{f}$	= Control force acting on the spacecraft
$\mathbf{f}_d$	= External perturbation force acting on the spacecraft
$\mathbf{f}_g^p$	= Gravitational force acting on the spacecraft in the frame **P**
$\mathbf{F}^p$	= Total force acting on the spacecraft in the frame **P**
$\hat{\mathbf{h}}$	= Dual momentum of a rigid body
I	= Inertial coordinate frame
$\mathbf{J}_p$	= Moment of inertia matrix of the spacecraft in the body-fixed frame **P**
m_p	= Mass of the spacecraft
$\hat{\mathbf{M}}$	= Dual inertial operator
P	= Body-fixed frame of the spacecraft
$\mathbf{p}_{pt}^p$	= Error position between the body frame **P** and reference frame **T** represented in the frame **P**
$\dot{\mathbf{p}}_{pt}^p$	= Derivative of the vector $\mathbf{p}_{pt}^p$ with respect to time
q_p	= Attitude quaternion of the spacecraft
q_t	= Reference attitude quaternion
q_e	= Error attitude quaternion between the frames **P** and **T**
q_{e0}	= Scalar part of the quaternion q_e
$\mathbf{q}_{ev}$	= Vector part of the quaternion q_e
$\hat{q}$	= Dual quaternion
$\hat{q}^*$	= Conjugation of the dual quaternion $\hat{q}$
q	= Real part of the dual quaternion $\hat{q}$
q'	= Dual part of the dual quaternion $\hat{q}$
q^*	= Conjugation of the quaternion q
$\hat{q}_p$	= Dual quaternion of the spacecraft
$\hat{q}_t$	= Reference dual quaternion

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$\hat{q}_e$ = Error dual quaternion between the frames **P** and **T**
$\mathbf{r}_p^p$ = Inertial position of the spacecraft in the frame **P**
$\mathbf{r}_t^t$ = Reference inertial position of the spacecraft in the frame **T**
τ = Control torque acting on the spacecraft
τ_d = External disturbance torque acting on the spacecraft
τ_g^p = Gravity gradient torque acting on the spacecraft represented in the frame **P**
$\mathbf{T}^p$ = Total torque acting on the spacecraft in the frame **P**
T_m = Upper bound of the settling time of the spacecraft maneuvering
$\hat{\mathbf{u}}$ = Dual control force
$\hat{\mathbf{u}}_g^p$ = Dual gravitational force
$\mathbf{v}_p^p$ = Velocity of the spacecraft in the frame **P**
$\mathbf{v}_t^t$ = Reference velocity in the frame **T**
$\mathbf{v}_e^p$ = Quasi-error velocity in the frame **P**
ω_p^p = Angular velocity of the spacecraft in the frame **P**
ω_t^t = Reference angular velocity in the frame **T**
ω_e^p = Error angular velocity between the frames **P** and **T** represented in the frame **P**
$\hat{\omega}_p^p$ = Twist of the spacecraft in the frame **P**
$\hat{\omega}_t^t$ = Reference twist represented in the frame **T**
$\hat{\omega}_e$ = Error twist represented in the frame **P**
μ = Geocentric gravitational constant

I. Introduction

With the leap in requirements of modern space activities, fast and precise maneuver of spacecraft has become increasingly important in recent years. Traditionally, orbital and attitude motions of spacecraft were usually separately modeled and controlled for early space missions, in which their mutual couplings were neglected. In contrast to this, integrated modeling and control for translation and rotation of a rigid spacecraft is capable of achieving higher maneuverability and control accuracy, which is of utmost importance for future spatial tasks, especially proximity operations, involving space debris removal, in-orbit maintenance, spacecraft formation flying (SFF) and space station installation. Accompany with these advantages, come various challenges including high dimensional, nonlinear and parameter uncertainty problems. Even though, this interesting problem has been gradually dealt with in some literature.[1–14]

To enable simultaneous position and attitude maneuvers of a single rigid spacecraft, Terui[1] utilized sliding mode control technique to handle it; while Stansbery et al.[2] used state dependent Riccati equation method to propose a fuel suboptimal feedback control strategy. Besides, for multi-spacecraft missions like SFF, in-orbit spacecraft servicing and space debris capture, a variety of control methods were designed to enable relative position and attitude of two spacecraft, e.g., pursuer and target, to track desired trajectories. These control strategies involved adaptive nonlinear tracking control,[3,4] output feedback tracking control,[5,6] chattering-free sliding mode robust control,[7] θ-D fuel suboptimal control,[8–10] passivity-based PD+ control,[12] backstepping control,[12] adaptive learning synchronization control,[13] and decentralized tracking control.[14]

Note that, although translation and rotation motions of rigid spacecraft in these works were considered in a unit framework, the position and attitude dynamics modeling were still separately developed in essence. Specifically, the translation of a spacecraft is represented by two 3-dimensional vectors: position vector and velocity vector; while the rotation is represented by a 3-dimensional angular velocity vector and attitude parameters like Euler angles, unit quaternion or Rodrigues parameters. Unlike this method, the compact and effective integrated translation and rotation modeling of rigid spacecraft has received growing attention these years.[15–20] One of effective ways is based on a new mathematic tool, i.e., dual quaternion, which provides an efficient global representation for rotation and translation of rigid body simultaneously and performs more compact and effective than other tools.[15] Although applied to various areas, the first utilization of dual quaternion for spacecraft was proposed in spatial navigation missions,[18] where the kinematic motion of a rigid spacecraft is described using dual quaternion; based on this kinematic model, Wang et al.[16] designed an attitude and position regulator. Nevertheless, few studies utilize dual quaternion to deal with the dynamics modeling of spacecraft for the integrated translation and rotation maneuver mission. Although a dual number-based coupled relative dynamic model of two spacecraft was given in Ref. 17, yet the relationship between the dual numbers and the usual Cartesian vectors was not fully derived, which led that traditional methods of control system design and stability analysis were hard to apply directly; consequently, the closed-loop stability analysis was absent, though an incremental PID control scheme was proposed. Therefore, motivated by Refs. 18 and 21, one of contributions of this paper is to derive a dual quaternion-based error kinematics and dynamics of a rigid spacecraft as the foundation of the integrated translation and rotation maneuvering; furthermore, the relationship between dual quaternion and Cartesian vectors is developed so as to facilitate the subsequent dual quaternion-based control scheme design and Lyapunov theory-based stability analysis.

Besides the modeling, control strategies mentioned above[1–14] only ensured asymptotic closed-loop stability, which means position and attitude error converge to the equilibrium as time goes to infinity. Control strategy with fast convergence is immensely needed to achieve higher maneuverability of spacecraft. To this end, recent years have witnessed the growing development of finite-control theory,[22–25] which enabled system states to converge to equilibrium in finite time. Due to high-precision performance and finite-time convergence, its application has been widely studied on robotic manipulators.[26–31] Furthermore, space activities have gradually become another application area of finite-time control technique in recent years. Ding et al.[32] employed it to solve the attitude stabilization of a rigid spacecraft with external disturbances; furthermore, Jin et al.[33] proposed finite-time attitude tracking control law to ensure a rigid spacecraft track the desired attitude trajectory in the presence of model uncertainties and disturbances. Liu et al.[34] designed a relative potion control of SFF using terminal sliding mode technique to get rapid formation reconfiguration. Zhou et al.[35] obtained a guidance law of a missile to guarantee the finite time convergence of the line-of-sight angular rate. However, little research is done on finite-time integrated translational and rotational maneuver of a rigid spacecraft.

Another problem should not be ignored is system uncertainty and external disturbances. Specifically, system uncertainty mainly comes from mass and moment of inertia matrix uncertainties; while disturbances largely results from spatial perturbation forces and torques, such as gravitational forces, attractions from celestial bodies, atmospheric drag, solar radiation pressure, magnetic and gravity gradient torques.[36] It is extremely necessary to guarantee the system robustness in controller design.

In this paper, a rigid spacecraft is required to be maneuvered to simultaneously satisfy position and attitude requirements in finite time. To do so, motivated by Refs. 18 and 21, an integrated translational and rotational tracking error dynamic model of a rigid spacecraft is derived by using dual quaternion, where the mutual couplings of the orbital and attitude systems are pointed out; meanwhile, the system uncertainties are also analyzed and can be bounded by several state-dependent functions. Moreover, the relationships between dual quaternion and Cartesian vectors are addressed such that Lyapunov theory can be applied to the dual quaternion-based control system design. According to the finite-time stability theory,[22] a robust finite-time controller is proposed using backstepping technique. The following stability analysis guarantees the finite-time stability of the closed-loop system in spite of system uncertainties and external disturbances; meanwhile, the bound of the settling time of system convergence is given as well, which is significant for the real time control of spacecraft in practice. A scenario simulation is given to demonstrate the effect of the proposed controller; meanwhile, the effect of control parameter on the system is also analyzed.

The rest of this paper is organized as follows. In section 2, the integrated translational and rotational dynamic model of a rigid spacecraft is stated by using dual quaternion. Then, a robust finite-time backstepping control scheme is proposed in section 3. Next, numerical simulation results applying the proposed control law to a scenario in section 4. At last, section 5 draws the conclusions.

II. Problem Formulation

In this section, a brief review of dual quaternion is given firstly. Subsequently, the integrated translation and rotation tracking error model of a rigid spacecraft is derived using dual quaternion; meanwhile, the tracking control problem is stated as well. More details on dual quaternion can be found in Ref. 18.

A. Dual Quaternion

A dual quaternion can be in fact regarded as a dual number

$$\hat{q} = q + \varepsilon q' \tag{1}$$

where the real part q and the dual part q' are both usual quaternion; ε is a nilpotent satisfying

$$\varepsilon^2 = 0, \varepsilon \neq 0. \tag{2}$$

A dual vector can be considered as a special dual quaternions composed of two usual quaternion with zero scalar part

$$\hat{\mathbf{x}} = \mathbf{x} + \varepsilon \mathbf{x}' \tag{3}$$

or equivalently, it can be seen as a composition of two usual Cartesian vectors $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^3$.

Basic operations of dual quaternion yield

$$\hat{q}_1 + \hat{q}_2 = (q_1 + q_2) + \varepsilon(q_1' + q_2') \tag{4}$$

$$\lambda \hat{q} = \lambda q + \varepsilon \lambda q', \ \forall \lambda \in \mathbb{R} \tag{5}$$

$$\hat{q}_1 \circ \hat{q}_2 = q_1 \circ q_2 + \varepsilon(q_1 \circ q_2' + q_1' \circ q_2) \tag{6}$$

where the sign "$\circ$" denotes quaternion multiplication. The conjugation of dual quaternion $\hat{q}$ is defined as

$$\hat{q}^* = q^* + \varepsilon q'^* \tag{7}$$

where q^*, q'^* are conjugations of the corresponding quaternion q, q', respectively; moreover, the conjugation of dual vector $\hat{\mathbf{x}}$ can be obtained as

$$\hat{\mathbf{x}}^* = -\mathbf{x} - \varepsilon \mathbf{x}'. \tag{8}$$

B. Integrated Error Kinematics

Before proceeding with the development, several frames are introduced which will be used in the sequel. As Fig. 1 shown, let **I** be an inertial frame. The spacecraft is considered as a rigid body; meanwhile, a body frame **P** is defined with origin at the center of mass and three mutually perpendicular axes coincident with the principle axis of inertia.

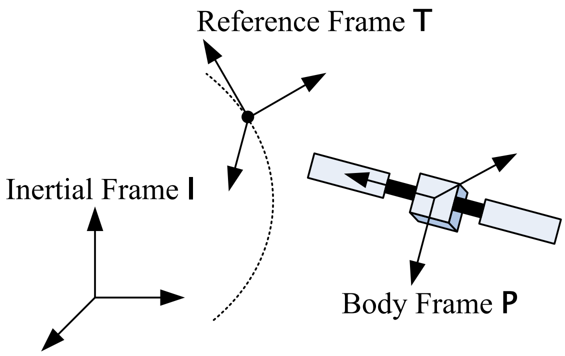

Figure 1. Coordinate frames

According to Chasles Theorem in Ref. 37, rigid body displacement can be produced by a translation along a line followed or preceded by a rotation about that line. Suppose that there is a rotation represented by a unit quaternion q, succeeded by a translation $\mathbf{p}^b \in \mathbb{R}^3$, then the whole transformation can be described by a unit dual quaternion $\hat{q}$ as[18]

$$\hat{q} = q + \varepsilon \frac{1}{2} q \circ \mathbf{p}^b \tag{9}$$

where $\mathbf{p}^b$ is the translation vector represented in a frame b. Thus, the displacement of the spacecraft can be described as[18]

$$\hat{q}_p = q_p + \varepsilon \frac{1}{2} q_p \circ \mathbf{r}_p^p \tag{10}$$

where the unit quaternion q_p represents the orientation of the body frame **P** with respect to the inertial frame **I**; the vector $\mathbf{r}_p^p$ denotes the inertial position of the spacecraft expressed in the frame **P**.

The command attitude and position motions are also given in form of a virtual reference body frame T represented by a corresponding dual quaternion $\hat{q}_t$ as

$$\hat{q}_t = q_t + \varepsilon \frac{1}{2} q_t \circ \mathbf{r}_t^t \tag{11}$$

where the unit quaternion q_t denotes the rotation from I to T, which is deemed as the desired attitude for the spacecraft; meanwhile, $\mathbf{r}_t^t$ is the desired inertial position represented in the frame T.

According to Ref. 18, the spacecraft kinematics described by dual quaternion yields

$$\dot{\hat{q}}_p = \frac{1}{2} \hat{q}_p \circ \hat{\omega}_p^p \tag{12}$$

where $\hat{\omega}_p^p$, namely twist, is essentially a dual vector represented in the frame P, yielding

$$\hat{\omega}_p^p = \omega_p^p + \varepsilon(\dot{\mathbf{r}}_p^p + \omega_p^p \times \mathbf{r}_p^p) = \omega_p^p + \varepsilon \mathbf{v}_p^p, \tag{13}$$

in which $\omega_p^p, \mathbf{v}_p^p$ are in fact the usual angular velocity vector and velocity vector of the spacecraft in the frame P, respectively. Likewise, the reference kinematics is governed by

$$\dot{\hat{q}}_t = \frac{1}{2} \hat{q}_t \circ \hat{\omega}_t^t \tag{14}$$

where $\hat{\omega}_t^t$ is the desired twist, yielding

$$\hat{\omega}_t^t = \omega_t^t + \varepsilon(\dot{\mathbf{r}}_t^t + \omega_t^t \times \mathbf{r}_t^t) = \omega_t^t + \varepsilon \mathbf{v}_t^t. \tag{15}$$

Also, $\omega_t^t, \mathbf{v}_t^t$ are the command angular velocity vector and velocity vector in the frame T, respectively.

Error attitude quaternion q_e between P and T can be given as

$$q_e = q_t^* \circ q_p \tag{16}$$

which leads to

$$\mathbf{x}_t^p = q_e^* \circ \mathbf{x}_t^t \circ q_e, \tag{17}$$

for an arbitrary vector $\mathbf{x} \in \mathbb{R}^3$. Define $\mathbf{p}_{pt}^p$ as the error position between frame P and T represented in the frame P, yielding

$$\mathbf{p}_{pt}^p = \mathbf{r}_p^p - q_e^* \circ \mathbf{r}_t^t \circ q_e = \mathbf{r}_p^p - \mathbf{r}_t^p. \tag{18}$$

Analogous to Eq. (16), define error dual quaternion $\hat{q}_e$ as

$$\hat{q}_e = \hat{q}_t^* \circ \hat{q}_p, \tag{19}$$

and one can thus obtain using Eqs. (10) and (11) that

$$\begin{aligned} \hat{q}_e &= \left(q_t^* - \varepsilon \frac{1}{2} \mathbf{r}_t^t \circ q_t^*\right) \circ \left(q_p + \varepsilon \frac{1}{2} q_p \circ \mathbf{r}_p^p\right) \\ &= q_e + \varepsilon \frac{1}{2} q_e \circ \left(\mathbf{r}_p^p - q_e^* \circ \mathbf{r}_t^t \circ q_e\right) \\ &= q_e + \varepsilon \frac{1}{2} q_e \circ \mathbf{p}_{pt}^p. \end{aligned} \tag{20}$$

Further, error angular velocity in the frame P is given as

$$\omega_e^p = \omega_p^p - q_e^* \circ \omega_t^t \circ q_e = \omega_p^p - \omega_t^p. \tag{21}$$

Similarly, define error twist in the frame P as

$$\hat{\omega}_e = \hat{\omega}_p^p - \hat{q}_e^* \circ \hat{\omega}_t^t \circ \hat{q}_e = \hat{\omega}_p^p - \hat{\omega}_t^p, \tag{22}$$

which also leads by using Eqs. (17) and (20) that

$$\begin{aligned} \hat{\omega}_e &= \omega_p^p + \varepsilon \mathbf{v}_p^p - \left(q_e^* - \varepsilon \frac{1}{2} \mathbf{p}_{pt}^p \circ q_e^*\right) \circ \left(\omega_t^t + \varepsilon \mathbf{v}_t^t\right) \circ \left(q_e + \varepsilon \frac{1}{2} q_e \circ \mathbf{p}_{pt}^p\right) \\ &= \omega_p^p - \omega_t^p + \varepsilon \left(\mathbf{v}_p^p - \mathbf{v}_t^p - \omega_t^p \times \mathbf{p}_{pt}^p\right). \end{aligned} \tag{23}$$

Note that

$$\left.\frac{d\mathbf{p}_{pt}}{dt}\right|_i^p = \mathbf{v}_p^p - \mathbf{v}_t^p = \dot{\mathbf{p}}_{pt}^p + \omega_p^p \times \mathbf{p}_{pt}^p \tag{24}$$

and substituting Eq. (24) into Eq. (23), we has

$$\hat{\omega}_e = \omega_e^p + \varepsilon\left(\dot{\mathbf{p}}_{pt}^p + \omega_e^p \times \mathbf{p}_{pt}^p\right) = \omega_e^p + \varepsilon \mathbf{v}_e^p \tag{25}$$

where $\mathbf{v}_e^p$, namely quasi-error velocity, can be written as

$$\mathbf{v}_e^p = \dot{\mathbf{p}}_{pt}^p + \omega_e^p \times \mathbf{p}_{pt}^p \tag{26}$$

$$= \mathbf{v}_p^p - \mathbf{v}_t^p - \omega_t^p \times \mathbf{p}_{pt}^p. \tag{27}$$

Hence, taking the derivative of Eq. (19) with respect to time, and viewing Eqs. (12), (14) and (25), we can derive the integrated translation and rotation error kinematics of the spacecraft as

$$\dot{\hat{q}}_e = \dot{\hat{q}}_t^* \circ \hat{q}_p + \hat{q}_t^* \circ \dot{\hat{q}}_p = \frac{1}{2}\hat{q}_e \circ \hat{\omega}_e. \tag{28}$$

C. Integrated Error Dynamics

According to Ref. 21, the dual number representation of momentum for a rigid body is given by

$$\hat{\mathbf{h}} = \hat{\mathbf{M}}\hat{\omega} \tag{29}$$

where $\hat{\omega}$ is the twist of the rigid body; $\hat{\mathbf{M}}$ is the dual inertial operator composed of the mass m and the inertial matrix $\mathbf{J}$ of the rigid body, yielding

$$\hat{\mathbf{M}} = m\frac{d}{d\varepsilon}\mathbf{I} + \varepsilon\mathbf{J} \tag{30}$$

$$= \begin{bmatrix} m\frac{d}{d\varepsilon} + \varepsilon J_{xx} & \varepsilon J_{xy} & \varepsilon J_{xz} \\ \varepsilon J_{xy} & m\frac{d}{d\varepsilon} + \varepsilon J_{yy} & \varepsilon J_{yz} \\ \varepsilon J_{xz} & \varepsilon J_{yz} & m\frac{d}{d\varepsilon} + \varepsilon J_{zz} \end{bmatrix} \tag{31}$$

which leads to

$$\hat{\mathbf{M}}\hat{\omega} = \left(m\frac{d}{d\varepsilon}\mathbf{I} + \varepsilon\mathbf{J}\right)(\omega + \varepsilon\mathbf{v}) = m\mathbf{v} + \varepsilon\mathbf{J}\omega. \tag{32}$$

Thus, it can be seen that, the real part $m\mathbf{v}$ and the dual part $\mathbf{J}\omega$ of dual vector $\hat{\mathbf{h}}$ represent the usual linear and angular momentum, respectively. Then, using dual number, Newton-Euler's dynamic equation of a rigid body is governed by

$$\hat{\mathbf{F}} = \dot{\hat{\mathbf{h}}} = \hat{\mathbf{M}}\dot{\hat{\omega}} + \hat{\omega} \times \hat{\mathbf{M}}\hat{\omega} \tag{33}$$

where $\hat{\mathbf{F}}$ is the dual force composed of the force vector $\mathbf{F} \in \mathbb{R}^3$ and the torque vector $\mathbf{T} \in \mathbb{R}^3$ acting on the body

$$\hat{\mathbf{F}} = \mathbf{F} + \varepsilon\mathbf{T}. \tag{34}$$

Define the dual force acting on the spacecraft represented in the frame **P** as

$$\hat{\mathbf{F}}^p = \mathbf{F}^p + \varepsilon\mathbf{T}^p \tag{35}$$

where the force $\mathbf{F}^p$ is mainly composed of three parts: control force $\mathbf{f}$, gravitational force $\mathbf{f}_g^p$, and external perturbation $\mathbf{f}_d$; similarly, the torque $\mathbf{T}^p$ comprises control torque τ, gravity gradient torque τ_g^p, and external disturbance torque τ_d. Hence, the dual force $\hat{\mathbf{F}}^p$ can be rewritten as

$$\hat{\mathbf{F}}^p = \hat{\mathbf{u}} + \hat{\mathbf{u}}_g^p + \hat{\mathbf{d}} \tag{36}$$

where $\hat{\mathbf{u}}, \hat{\mathbf{u}}_g^p, \hat{\mathbf{d}}$ are dual control force, dual gravitational force and dual disturbance force, respectively, satisfying

$$\hat{\mathbf{u}} = \mathbf{f} + \varepsilon\tau \tag{37}$$

$$\hat{\mathbf{u}}_g^p = \mathbf{f}_g^p + \varepsilon\tau_g^p \tag{38}$$

$$\hat{\mathbf{d}} = \mathbf{f}_d + \varepsilon\tau_d. \tag{39}$$

Meanwhile, the disturbance is bounded and yields

$$\|\mathbf{f}_d\| \leq d_f,\ \|\tau_d\| \leq d_\tau, \tag{40}$$

and the gravitational force and torque are submitted to[36]

$$\mathbf{f}_g^p = -\frac{\mu m_p}{r_p^3}\mathbf{r}_p^p,\ \ \tau_g^p = \frac{3\mu}{r_p^5}(\mathbf{r}_p^p \times \mathbf{J}_p\mathbf{r}_p^p) \tag{41}$$

where m_p, $\mathbf{J}_p$ are the mass and inertia matrix of the rigid spacecraft, respectively.

Then, according to Eq. (33), the dual number representation of dynamics of the spacecraft yields

$$\hat{\mathbf{M}}_p\dot{\hat{\omega}}_p^p + \hat{\omega}_p^p \times \hat{\mathbf{M}}_p\hat{\omega}_p^p = \hat{\mathbf{u}} + \hat{\mathbf{u}}_g^p + \hat{\mathbf{d}} \tag{42}$$

where

$$\hat{\mathbf{M}}_p = \frac{d}{d\varepsilon}m_p\mathbf{I} + \varepsilon\mathbf{J}_p. \tag{43}$$

Next, to get the error dynamics, taking the derivative of Eq. (22) with respect to time as

$$\dot{\hat{\omega}}_e = \dot{\hat{\omega}}_p^p - \dot{\hat{q}}_e^* \circ \hat{\omega}_t^t \circ \hat{q}_e - \hat{q}_e^* \circ \dot{\hat{\omega}}_t^t \circ \hat{q}_e - \hat{q}_e^* \circ \hat{\omega}_t^t \circ \dot{\hat{q}}_e \tag{44}$$

and noticing Eq. (28) result in

$$\dot{\hat{\omega}}_e = \dot{\hat{\omega}}_p^p - \hat{q}_e^* \circ \dot{\hat{\omega}}_t^t \circ \hat{q}_e + \hat{\omega}_e \times \hat{\omega}_t^p, \tag{45}$$

and thus one can obtain by combining Eqs. (45) and (42) that

$$\begin{aligned}
\hat{\mathbf{M}}_p\dot{\hat{\omega}}_e &= \hat{\mathbf{M}}_p\dot{\hat{\omega}}_p^p - \hat{\mathbf{M}}_p(\hat{q}_e^* \circ \dot{\hat{\omega}}_t^t \circ \hat{q}_e) + \hat{\mathbf{M}}_p(\hat{\omega}_e \times \hat{\omega}_t^p) \\
&= \hat{\mathbf{u}} + \hat{\mathbf{u}}_g^p + \hat{\mathbf{d}} - \hat{\omega}_p^p \times \hat{\mathbf{M}}_p\hat{\omega}_p^p - \hat{\mathbf{M}}_p(\hat{q}_e^* \circ \dot{\hat{\omega}}_t^t \circ \hat{q}_e) + \hat{\mathbf{M}}_p(\hat{\omega}_e \times \hat{\omega}_t^p).
\end{aligned} \tag{46}$$

Taking Eq. (46) into account, notice that

$$\hat{\omega}_p^p \times \hat{\mathbf{M}}_p\hat{\omega}_p^p = -(\hat{\mathbf{M}}_p(\hat{\omega}_e + \hat{\omega}_t^p))^\times\hat{\omega}_e + (\hat{\omega}_t^p)^\times\hat{\mathbf{M}}_p\hat{\omega}_e + \hat{\omega}_t^p \times \hat{\mathbf{M}}_p\hat{\omega}_t^p, \tag{47}$$

and

$$\hat{\mathbf{M}}_p(\hat{\omega}_e \times \hat{\omega}_t^p) = -\hat{\mathbf{M}}_p(\hat{\omega}_t^p)^\times\hat{\omega}_e, \tag{48}$$

hence

$$-\hat{\omega}_p^p \times \hat{\mathbf{M}}_p\hat{\omega}_p^p + \hat{\mathbf{M}}_p(\hat{\omega}_e \times \hat{\omega}_t^p) = -\hat{\mathbf{C}}\hat{\omega}_e - \hat{\omega}_t^p \times \hat{\mathbf{M}}_p\hat{\omega}_t^p \tag{49}$$

where $\hat{\mathbf{C}}$ is the skew-symmetric operator composed of two skew-symmetric matrices $\mathbf{C}_1, \mathbf{C}_2 \in \mathbb{R}^{3\times3}$

$$\begin{aligned}
\hat{\mathbf{C}} &= \mathbf{C}_1\frac{d}{d\varepsilon} + \varepsilon\mathbf{C}_2 && (50)\\
\mathbf{C}_1 &= m_p(\omega_e^p)^\times + 2m_p(\omega_t^p)^\times && (51)\\
\mathbf{C}_2 &= (\omega_t^p)^\times\mathbf{J}_p + \mathbf{J}_p(\omega_t^p)^\times - (\mathbf{J}_p(\omega_t^p + \omega_e^p))^\times. && (52)
\end{aligned}$$

Meanwhile, let

$$\begin{aligned}
\hat{\mathbf{n}} &= \hat{\mathbf{u}}_g^p - \hat{\mathbf{M}}_p(\hat{q}_e^* \circ \dot{\hat{\omega}}_t^t \circ \hat{q}_e) - \hat{\omega}_t^p \times \hat{\mathbf{M}}_p\hat{\omega}_t^p && (53)\\
&= \mathbf{n}_1 + \varepsilon\mathbf{n}_2 && (54)
\end{aligned}$$

where $\mathbf{n}_1, \mathbf{n}_2$ can be further obtained using Eq. (41) as

$$\begin{aligned}
\mathbf{n}_1 &= -\frac{\mu m_p}{r_p^3}\mathbf{r}_p^p - m_p q_e^* \circ \dot{\mathbf{v}}_t^t \circ q_e - m_p(q_e^* \circ \dot{\omega}_t^t \circ q_e) \times \mathbf{p}_{pt}^p - m_p\omega_t^p \times \left(\mathbf{v}_t^p + \omega_t^p \times \mathbf{p}_{pt}^p\right) && (55)\\
\mathbf{n}_2 &= \frac{3\mu}{r_p^5}(\mathbf{r}_p^p \times \mathbf{J}_p\mathbf{r}_p^p) - \mathbf{J}_p(q_e^* \circ \dot{\omega}_t^t \circ q_e) - \omega_t^p \times \mathbf{J}_p\omega_t^p. && (56)
\end{aligned}$$

Consequently, in view of Eqs. (49) and (53), we can obtain the integrated translation and rotation error dynamics of the rigid spacecraft from (46) as

$$\hat{\mathbf{M}}_p\dot{\hat{\omega}}_e = \hat{\mathbf{u}} - \hat{\mathbf{C}}\hat{\omega}_e + \hat{\mathbf{n}} + \hat{\mathbf{d}}. \tag{57}$$

Furthermore, Eqs. (28) and (57) constitute the whole integrated translation and rotation error dynamic model of the rigid spacecraft.

Remark 1. *As can be seen from Eqs. (28) and (57), either equation is capable of simultaneously representing the translation and rotation tracking errors. Compared with other works,*[1–14] *the model shows a more compact form. Meanwhile, considering Eqs. (20) and (25), components of error dual quaternion* $\hat{q}_e$ *and error twist* $\hat{\omega}_e$ *have a clear physical interpretation. Specifically, on one hand, the real parts of* $\hat{q}_e$ *and* $\hat{\omega}_e$*, i.e.,* q_e *and* ω_e*, denote the usual relative rotation motion from the spacecraft attitude to the reference one, as formulated by many literature. On the other hand, different from rotation, for dual parts of* $\hat{q}_e$ *and* $\hat{\omega}_e$*, i.e.,* $\frac{1}{2}q_e \circ \mathbf{p}_{pt}^p$ *and* $\mathbf{v}_e^p$*, not only the usual relative translation motion* $\mathbf{p}_{pt}^p, \dot{\mathbf{p}}_{pt}^p$ *is involved, but also the coupling effect from the translation and rotation, i.e.,* $q_e \circ \mathbf{p}_{pt}^p$*,* $\omega_e^p \times \mathbf{p}_{pt}^p$ *can be clearly shown.*

Remark 2. *As pointed out above, the translation and rotation motion of the spacecraft can be simultaneously described by dual error variables* $\hat{q}_e$*,* $\hat{\omega}_e$*; moreover, relationships between dual error variables and traditional error states* $\mathbf{p}_{pt}^p, \dot{\mathbf{p}}_{pt}^p, q_e, \omega_e^p$ *are also stated in the above subsections in order to make traditional design and analysis methods of control systems still apply, which will be seen in the following development.*

D. Control Problem

For integrated translation and rotation error dynamic model described by Eqs. (28) and (57), it should be noted that system parameters $m_p, \mathbf{J}_p$ exist uncertainties in applications. These parameters can be in fact deemed as sums of nominal parts and uncertainties

$$m_p = \bar{m}_p + \Delta m_p,\ \mathbf{J}_p = \bar{\mathbf{J}}_p + \Delta \mathbf{J}_p \tag{58}$$

where $\bar{m}_p, \bar{\mathbf{J}}_p$ are nominal mass and inertia matrix of the spacecraft, respectively; $\Delta m_p, \Delta\mathbf{J}_p$ represent the corresponding uncertainties which can be bounded by known scalars $b_1, b_2 > 0$, respectively, i.e.,

$$\|\Delta m_p\| \leq b_1,\ \ \|\Delta \mathbf{J}_p\| \leq b_2. \tag{59}$$

Consequently, terms $\hat{\mathbf{M}}_p, \hat{\mathbf{C}}, \hat{\mathbf{n}}$ can be also regarded as one consisted of nominal parts and uncertainties, respectively, i.e.,

$$\begin{aligned} \hat{\mathbf{M}}_p &= \hat{\bar{\mathbf{M}}}_p + \Delta\hat{\mathbf{M}}_p & (60)\\ \hat{\mathbf{C}} &= \hat{\bar{\mathbf{C}}} + \Delta\hat{\mathbf{C}} & (61)\\ \hat{\mathbf{n}} &= \hat{\bar{\mathbf{n}}} + \Delta\hat{\mathbf{n}} & (62) \end{aligned}$$

where $\hat{\bar{\mathbf{M}}}_p$, $\hat{\bar{\mathbf{C}}}$, $\hat{\bar{\mathbf{n}}}$ can be obtained from substituting $\bar{m}_p, \bar{\mathbf{J}}_p$ for $m_p, \mathbf{J}_p$ in $\hat{\mathbf{M}}_p, \hat{\mathbf{C}}, \hat{\mathbf{n}}$, respectively; in the same way, $\Delta\hat{\mathbf{M}}_p$, $\Delta\hat{\mathbf{C}}$, $\Delta\hat{\mathbf{n}}$ can be obtained from substituting $\Delta m_p, \Delta\mathbf{J}_p$ for $m_p, \mathbf{J}_p$ in $\hat{\mathbf{M}}_p, \hat{\mathbf{C}}, \hat{\mathbf{n}}$, respectively; furthermore, let

$$\hat{\bar{\mathbf{C}}} = \bar{\mathbf{C}}_1 \frac{d}{d\varepsilon} + \varepsilon \bar{\mathbf{C}}_2, \hat{\bar{\mathbf{n}}} = \bar{\mathbf{n}}_1 + \varepsilon \bar{\mathbf{n}}_2 \tag{63}$$

$$\Delta\hat{\mathbf{C}} = \Delta\mathbf{C}_1 \frac{d}{d\varepsilon} + \varepsilon \Delta\mathbf{C}_2, \Delta\hat{\mathbf{n}} = \Delta\mathbf{n}_1 + \varepsilon \Delta\mathbf{n}_2 \tag{64}$$

where $\bar{\mathbf{C}}_1, \bar{\mathbf{C}}_2, \bar{\mathbf{n}}_1, \bar{\mathbf{n}}_2$ can be obtained from substituting $\bar{m}_p, \bar{\mathbf{J}}_p$ for $m_p, \mathbf{J}_p$ in $\mathbf{C}_1, \mathbf{C}_2, \mathbf{n}_1, \mathbf{n}_2$, respectively; as such, $\Delta\mathbf{C}_1, \Delta\mathbf{C}_2, \Delta\mathbf{n}_1, \Delta\mathbf{n}_2$ can be obtained from substituting $\bar{m}_p, \bar{\mathbf{J}}_p$ for $m_p, \mathbf{J}_p$ in $\mathbf{C}_1, \mathbf{C}_2, \mathbf{n}_1, \mathbf{n}_2$, respectively;

Besides the uncertainties, the equilibrium of system in tracking problems is also in need of determination in advance. For error states $\mathbf{p}_{pt}^p$, $\dot{\mathbf{p}}_{pt}^p$, ω_e^p, it is clear that $\mathbf{0} \in \mathbb{R}^3$ is their desired equilibrium. However, it is worth mentioning that, error attitude quaternion $q_e = \begin{bmatrix} q_{e0} & \mathbf{q}_{ev}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ has two equilibrium points, i.e., $\begin{bmatrix} \pm 1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$, representing the same physical orientation. According to Refs. 12 and 38, to minimize the path length, the selection of the equilibrium point can be determined by the given initial condition. Specifically, choose $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ as equilibrium point when $q_{e0}(0) \geq 0$, and $\begin{bmatrix} -1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ is chosen for $q_{e0}(0) < 0$; meanwhile, it is further assumed that the scalar part of the error quaternion does not change sign, i.e.,

$$\mathrm{sgn}(q_{e0}(0)) = \mathrm{sgn}(q_{e0}(t)),\ \forall t > 0 \tag{65}$$

Without loss of generality, this paper only considers the case $q_{e0}(0) \geq 0$, which means $\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ is the equilibrium of q_e and $q_{e0}(t) \geq 0$. Furthermore, noticing that

$$\mathbf{v}_e^p = 0 \Longleftrightarrow \dot{\mathbf{p}}_{pt}^p = 0, \tag{66}$$

the equilibrium of system (28) and (57) in the form of dual quaternion can thus be defined as

$$\hat{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} + \varepsilon \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{67}$$

$$\hat{O} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} + \varepsilon \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}. \tag{68}$$

Then, the tracking control problem can be summarized as follows.

Problem 1. *For system described by (28) and (57), synthesize a dual control law $\hat{\mathbf{u}} = \mathbf{f} + \varepsilon\tau$ such that $\{\hat{q}_e(t) = \hat{E},\ \hat{\omega}_e(t) = \hat{O}\}$ is reached in finite time in the presence of model uncertainties (58) and external disturbance (39).*

III. Controller Design

In this section, a robust finite-time control strategy is proposed based on backstepping method[39] for *Problem 1.* Before we tackle the problem of controller design, the definition of finite-time stability and two relative lemmas are stated as follows, which will be used later.

Definition 1. [25]*Consider a nonlinear system*

$$\dot{\mathbf{x}} = f(\mathbf{x}),\ \ f(\mathbf{0}) = 0,\ \mathbf{x} \in \mathbb{R}^n \tag{69}$$

where $f : D \to \mathbb{R}^n$ is continuous on an open neighborhood D of the origin $\mathbf{x} = \mathbf{0}$. The system equilibrium $\mathbf{x} = \mathbf{0}$ is said to be (locally) finite-time stable if, for any given initial state $\mathbf{x}(0) = \mathbf{x}_0 \in D$, there exists a settling time $T_r > 0$, such that every solution of system (69), $\mathbf{x}(t, \mathbf{x}_0) \in D/\{\mathbf{0}\}$, satisfies

$$\begin{cases} \lim\limits_{t \to T_r} \mathbf{x}(t, \mathbf{x}_0) = 0, & \textit{if } t \in [0, T_r) \\ \mathbf{x}(t, \mathbf{x}_0) = 0, & \textit{if } t \geq T_r. \end{cases} \tag{70}$$

Moreover, if $D = \mathbb{R}^n$, the equilibrium $\mathbf{x} = \mathbf{0}$ is globally finite-time stable.

Lemma 1. [22]*For a C^1 smooth function $V : U \to \mathbb{R}$, given $\sigma > 0$ and $\beta \in (0, 1)$, if there exists a neighborhood $U_0 \subset U$ such that V is positive definite and $\dot{V} + \sigma V^\beta$ is negative semidefinite on U_0, where $\dot{V} = \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}^{\mathrm{T}}} f(\mathbf{x})$. Then the origin of the system (69) is a finite-time stable equilibrium. Moreover, the settling time T_r satisfies*

$$T_r \leq \frac{1}{\sigma(1-\beta)} V^{1-\beta}. \tag{71}$$

Lemma 2. [27]*Suppose $x_1, x_2, ..., x_n$ and $0 < p < 2$ are all positive numbers; then, the following inequality holds*

$$(x_1^2 + x_2^2 + \cdots + x_n^2)^p \leq (x_1^p + x_2^p + \cdots + x_n^p)^2. \tag{72}$$

For simplicity but not losing generality, following notations are introduced

$$\mathbf{sgn}^\rho(\mathbf{x}) = \begin{bmatrix} |x_1|^\rho \operatorname{sgn}(x_1) & |x_2|^\rho \operatorname{sgn}(x_2) & \cdots & |x_n|^\rho \operatorname{sgn}(x_n) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^n, \tag{73}$$

$$\operatorname{diag}\{|\mathbf{x}|^\rho\} = \operatorname{diag}\{\ |x_1|^\rho \quad |x_2|^\rho \quad \cdots \quad |x_n|^\rho\ \} \in \mathbb{R}^{n \times n}, \tag{74}$$

$$|\mathbf{x}|^\rho = \begin{bmatrix} |x_1|^\rho & |x_2|^\rho & \cdots & |x_n|^\rho \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^n, \tag{75}$$

for an arbitrary vector $\mathbf{x} \in \mathbb{R}^n$ and a scalar $\rho \in \mathbb{R}$; moreover, for a dual vector $\hat{\mathbf{x}} = \mathbf{x}_1 + \varepsilon \mathbf{x}_2$, $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^n$, let

$$\mathbf{sgn}^\rho(\hat{\mathbf{x}}) = \mathbf{sgn}^\rho(\mathbf{x}_1 + \varepsilon \mathbf{x}_2) = \mathbf{sgn}^\rho(\mathbf{x}_1) + \varepsilon \mathbf{sgn}^\rho(\mathbf{x}_2). \tag{76}$$

According to standard backstepping technique,[39] in the light of error kinematics (28), the desired virtual control is firstly designed as

$$\hat{\alpha} = \alpha_1 + \varepsilon\alpha_2 = -\hat{\mathbf{K}}_1\mathbf{sgn}^\rho(\hat{\mathbf{g}}) \tag{77}$$

where ρ is a parameter to be designed and $0 \le \rho < 1$; dual vector $\hat{\mathbf{g}}$ is defined as

$$\hat{\mathbf{g}} = \mathbf{p}_{pt}^p + \varepsilon\mathbf{q}_{ev}, \tag{78}$$

$\hat{\mathbf{K}}_1$ is gain operator defined with positive definite matrices $\mathbf{K}_p, \mathbf{K}_q \in \mathbb{R}^{3\times3}$, yielding

$$\hat{\mathbf{K}}_1 = \mathbf{K}_q\frac{d}{d\varepsilon} + \varepsilon\mathbf{K}_p, \tag{79}$$

and thus the real part and the dual part of Eq. (77) are governed by

$$\alpha_1 = -\mathbf{K}_q\mathbf{sgn}^\rho(\mathbf{q}_{ev}),\ \alpha_2 = -\mathbf{K}_p\mathbf{sgn}^\rho(\mathbf{p}_{pt}^p). \tag{80}$$

Furthermore, the derivative of $\hat{\alpha}$ with respect to time is $\dot{\hat{\alpha}} = \dot{\alpha}_1 + \varepsilon\dot{\alpha}_2$, whose components are

$$\dot{\alpha}_1 = -\rho\mathbf{K}_q\mathrm{diag}\{\,|\mathbf{q}_{ev}|^{\rho-1}\,\}\dot{\mathbf{q}}_{ev},\ \dot{\alpha}_2 = -\rho\mathbf{K}_p\mathrm{diag}\{\,\left|\mathbf{p}_{pt}^p\right|^{\rho-1}\,\}\dot{\mathbf{p}}_{pt}^p. \tag{81}$$

Since $\rho - 1 < 0$, to avoid the singularity problem, the following definition is given

$$|x|^{\rho-1} = \begin{cases} |x|^{\rho-1} & x \neq 0 \\ 0 & x = 0 \end{cases},\ \forall x \in \mathbb{R}. \tag{82}$$

Moreover, defining

$$\hat{\mathbf{e}} = \hat{\omega}_e - \hat{\alpha} = \mathbf{e}_1 + \varepsilon\mathbf{e}_2, \tag{83}$$

and noticing Eqs. (25) and (77), one has

$$\mathbf{e}_1 = \omega_e^p - \alpha_1,\ \mathbf{e}_2 = \mathbf{v}_e^p - \alpha_2. \tag{84}$$

Then, in view of *Problem 1*, a robust finite-time backstepping (RFTB) controller is proposed as

$$\hat{\mathbf{u}} = \hat{\hat{\mathbf{C}}}\hat{\alpha} - \hat{\hat{\mathbf{n}}} + \hat{\hat{\mathbf{M}}}_p\dot{\hat{\alpha}} - \hat{\mathbf{g}} - \hat{\mathbf{K}}_2\mathbf{sgn}^\rho(\hat{\mathbf{e}}) - \hat{\eta}\mathbf{sgn}(\hat{\mathbf{e}}) \tag{85}$$

where $\hat{\mathbf{K}}_2 = \mathbf{K}_v\frac{d}{d\varepsilon} + \varepsilon\mathbf{K}_\omega$, $\mathbf{K}_v, \mathbf{K}_\omega \in \mathbb{R}^{3\times3}$ are positive matrices to be designed; $\hat{\eta}$ is defined as

$$\begin{aligned}
\hat{\eta} &= \eta_1\frac{d}{d\varepsilon} + \varepsilon\eta_2 && (86)\\
\eta_1(\alpha_2, \dot{\alpha}_2, \omega_e^p, \mathbf{p}_{pt}^p) &= b_1\left\|\omega_e^p \times \alpha_2 + 2\omega_t^p \times \alpha_2 + (\mu/r_p^3)\mathbf{r}_p^p\right. \\
&\quad \left. + q_e^* \circ \dot{\mathbf{v}}_t^t \circ q_e + (q_e^* \circ \dot{\omega}_t^t \circ q_e) \times \mathbf{p}_{pt}^p + \omega_t^p \times \left(\mathbf{v}_t^p + \omega_t^p \times \mathbf{p}_{pt}^p\right) + \dot{\alpha}_2\right\| + d_f && (87)\\
\eta_2(\alpha_1, \dot{\alpha}_1, \omega_e^p) &= b_2\left\|\omega_t^{p\times}\right\|\|\alpha_1\| + b_2\left\|\omega_t^p + \omega_e^p\right\|\left\|\alpha_1^\times\right\| + \frac{3b_2\mu}{r_p^4}\left\|\mathbf{r}_p^{p\times}\right\| \\
&\quad + b_2\left\|\omega_t^{p\times}\right\|\left\|\omega_t^p\right\| + b_2\left\|\omega_t^p \times \alpha_1 + q_e^* \circ \dot{\omega}_t^t \circ q_e + \dot{\alpha}_1\right\| + d_\tau. && (88)
\end{aligned}$$

Theorem 1. *For integrated translation and rotation error system composed of kinematics (28) and dynamics (57) in the presence of model uncertainty (58) and external disturbance (39), the RFTB control law (85) can guarantee that $\hat{q}_e(t) = \hat{E}$, $\hat{\omega}_e(t) = \hat{O}$ in a finite time.*

Proof. First, take the derivative of Eq. (20) with respect to time, and we can hence derive from real parts of both sides of (28) that

$$\dot{q}_e = \frac{1}{2}\begin{bmatrix} -\mathbf{q}_{ev}^T \\ q_{e0}\mathbf{I} + \mathbf{q}_{ev}^\times \end{bmatrix}\omega_e^p. \tag{89}$$

Moreover, in view of the integrated error dynamics (57), comparing both sides of the real part and the dual part, respectively, and meanwhile noting the definitions (25), (39), (50), and (54), lead to

$$\begin{aligned}
m_p\dot{\mathbf{v}}_e^p &= \mathbf{f} - \mathbf{C}_1\mathbf{v}_e^p + \mathbf{n}_1 + \mathbf{f}_d && (90)\\
\mathbf{J}_p\dot{\omega}_e^p &= \tau - \mathbf{C}_2\omega_e^p + \mathbf{n}_2 + \tau_d. && (91)
\end{aligned}$$

As such, the control law (85) can be divided into

$$
\begin{aligned}
\mathbf{f} &= \bar{\mathbf{C}}_1\alpha_2 - \bar{\mathbf{n}}_1 + \bar{m}_p\dot{\alpha}_2 - \mathbf{p}_{pt}^p - \mathbf{K}_v\mathbf{sgn}^\rho(\mathbf{e}_2) - \eta_1\mathbf{sgn}(\mathbf{e}_2) &(92)\\
\tau &= \bar{\mathbf{C}}_2\alpha_1 - \bar{\mathbf{n}}_2 + \bar{\mathbf{J}}_p\dot{\alpha}_1 - \mathbf{q}_{ev} - \mathbf{K}_\omega\mathbf{sgn}^\rho(\mathbf{e}_1) - \eta_2\mathbf{sgn}(\mathbf{e}_1). &(93)
\end{aligned}
$$

Subsequently, a Lyapunov function is constructed as

$$
\begin{aligned}
V &= \mathbf{q}_{ev}^{\mathrm{T}}\mathbf{q}_{ev} + (1-q_{e0})^2 + \frac{1}{2}(\mathbf{p}_{pt}^p)^{\mathrm{T}}\mathbf{p}_{pt}^p + \frac{1}{2}\mathbf{e}_1^{\mathrm{T}}\mathbf{J}_p\mathbf{e}_1 + \frac{1}{2}m_p\mathbf{e}_2^{\mathrm{T}}\mathbf{e}_2 &(94)\\
&= 2(1-q_{e0}) + \frac{1}{2}(\mathbf{p}_{pt}^p)^{\mathrm{T}}\mathbf{p}_{pt}^p + \frac{1}{2}\mathbf{e}_1^{\mathrm{T}}\mathbf{J}_p\mathbf{e}_1 + \frac{1}{2}m_p\mathbf{e}_2^{\mathrm{T}}\mathbf{e}_2. &(95)
\end{aligned}
$$

Differentiating the function V with respect to time, and using Eq. (26) result in

$$
\begin{aligned}
\dot{V} &= \mathbf{q}_{ev}^T\omega_e^p + (\mathbf{p}_{pt}^p)^{\mathrm{T}}\dot{\mathbf{p}}_{pt}^p + \mathbf{e}_1^{\mathrm{T}}\mathbf{J}_p\dot{\mathbf{e}}_1 + m_p\mathbf{e}_2^{\mathrm{T}}\dot{\mathbf{e}}_2\\
&= \mathbf{q}_{ev}^T\omega_e^p + (\mathbf{p}_{pt}^p)^{\mathrm{T}}(\mathbf{v}_e^p - \omega_e^p\times\mathbf{p}_{pt}^p) + \mathbf{e}_1^{\mathrm{T}}\mathbf{J}_p\dot{\mathbf{e}}_1 + m_p\mathbf{e}_2^{\mathrm{T}}\dot{\mathbf{e}}_2. &(96)
\end{aligned}
$$

Furthermore, substituting Eqs. (84), (90), and (91) into Eq. (96), and viewing $\mathbf{C}_1, \mathbf{C}_2$ are skew-symmetric matrices, we can obtain that

$$
\begin{aligned}
\dot{V} &= \mathbf{q}_{ev}^T\alpha_1 + (\mathbf{p}_{pt}^p)^{\mathrm{T}}\alpha_2 + \mathbf{e}_1^{\mathrm{T}}(\tau - \mathbf{C}_2\alpha_1 + \mathbf{n}_2 - \mathbf{J}_p\dot{\alpha}_1 + \mathbf{q}_{ev} + \tau_d) + \mathbf{e}_2^{\mathrm{T}}(\mathbf{f} - \mathbf{C}_1\alpha_2 + \mathbf{n}_1 - m_p\dot{\alpha}_2 + \mathbf{p}_{pt}^p + \mathbf{f}_d)\\
&= \mathbf{q}_{ev}^T\alpha_1 + (\mathbf{p}_{pt}^p)^{\mathrm{T}}\alpha_2 + \mathbf{e}_1^{\mathrm{T}}(\tau - \bar{\mathbf{C}}_2\alpha_1 + \mathbf{q}_{ev} + \bar{\mathbf{n}}_2 - \bar{\mathbf{J}}_p\dot{\alpha}_1 + \Delta_2)\\
&\quad + \mathbf{e}_2^{\mathrm{T}}(\mathbf{f} - \bar{\mathbf{C}}_1\alpha_2 + \bar{\mathbf{n}}_1 - \bar{m}_p\dot{\alpha}_2 + \mathbf{p}_{pt}^p + \Delta_1) &(97)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_1 &= -\Delta\mathbf{C}_1\alpha_2 + \Delta\mathbf{n}_1 - \Delta m_p\dot{\alpha}_2 + \mathbf{f}_d &(98)\\
\Delta_2 &= -\Delta\mathbf{C}_2\alpha_1 + \Delta\mathbf{n}_2 - \Delta\mathbf{J}_p\dot{\alpha}_1 + \tau_d. &(99)
\end{aligned}
$$

Meanwhile, one can derive that

$$
\begin{aligned}
\|\Delta_1\| &\leq \|\Delta\mathbf{C}_1\alpha_2 - \Delta\mathbf{n}_1 + \Delta m_p\dot{\alpha}_2\| + \|\mathbf{f}_d\|\\
&\leq \eta_1(\alpha_2, \dot{\alpha}_2, \omega_e^p, \mathbf{p}_{pt}^p), &(100)
\end{aligned}
$$

$$
\begin{aligned}
\|\Delta_2\| &\leq \|\Delta\mathbf{C}_2\alpha_1 - \Delta\mathbf{n}_2 + \Delta\mathbf{J}_p\dot{\alpha}_1\| + \|\tau_d\|\\
&\leq \left\|\omega_t^{p\times}\right\|\|\Delta\mathbf{J}_p\|\|\alpha_1\| + \|\Delta\mathbf{J}_p\|\|\omega_t^p + \omega_e^p\|\left\|\alpha_1^\times\right\|\\
&\quad + \frac{3\mu}{r_p^5}\left\|\mathbf{r}_p^{p\times}\right\|\|\Delta\mathbf{J}_p\|\left\|\mathbf{r}_p^p\right\| + \left\|(\omega_t^p)^\times\right\|\|\Delta\mathbf{J}_p\|\|\omega_t^p\|\\
&\quad + \|\Delta\mathbf{J}_p\|\left\|\omega_t^p\times\alpha_1 + q_e^*\circ\dot{\omega}_t^t\circ q_e + \dot{\alpha}_1\right\| + \|\tau_d\|\\
&\leq \eta_2(\alpha_1, \dot{\alpha}_1, \omega_e^p). &(101)
\end{aligned}
$$

After inserting the control law (92), (93) and virtual control (80) into Eq. (97), one has

$$
\begin{aligned}
\dot{V} &= -\mathbf{q}_{ev}^T\mathbf{K}_q\mathbf{sgn}^\rho(\mathbf{q}_{ev}) - (\mathbf{p}_{pt}^p)^{\mathrm{T}}\mathbf{K}_p\mathbf{sgn}^\rho(\mathbf{p}_{pt}^p) - \mathbf{e}_2^{\mathrm{T}}\mathbf{K}_v\mathbf{sgn}^\rho(\mathbf{e}_2)\\
&\quad - \mathbf{e}_1^{\mathrm{T}}\mathbf{K}_\omega\mathbf{sgn}^\rho(\mathbf{e}_1) + \mathbf{e}_2^{\mathrm{T}}(\Delta_1 - \mathbf{sgn}(\mathbf{e}_2)\eta_1) + \mathbf{e}_1^{\mathrm{T}}(\Delta_2 - \mathbf{sgn}(\mathbf{e}_1)\eta_2). &(102)
\end{aligned}
$$

Then, we can easily verify that

$$
\mathbf{e}_2^{\mathrm{T}}(\Delta_1 - \mathbf{sgn}(\mathbf{e}_2)\eta_1) \leq 0,\ \mathbf{e}_1^{\mathrm{T}}(\Delta_2 - \mathbf{sgn}(\mathbf{e}_1)\eta_2) \leq 0, \tag{103}
$$

and by using Eq. (65), one has

$$
\mathbf{q}_{ev}^{\mathrm{T}}\mathbf{q}_{ev} = 1 - q_{e0}^2 \geq 1 - q_{e0},\ \forall q_{e0} > 0. \tag{104}
$$

Meanwhile, let

$$
\sigma_q = \sigma_{\min}(\mathbf{K}_q), \sigma_p = \sigma_{\min}(\mathbf{K}_p), \sigma_v = \sigma_{\min}(\mathbf{K}_v), \sigma_\omega = \sigma_{\min}(\mathbf{K}_\omega) \tag{105}
$$

where $\sigma_{\min}(\mathbf{Q})$ is the minimum singular value of a matrix $\mathbf{Q} \in \mathbb{R}^{n\times n}$; hence

$$\dot{V} \leq -\sigma_q \left|\mathbf{q}_{ev}\right|^{\mathrm{T}} \left|\mathbf{q}_{ev}\right|^{\rho} - \sigma_p \left|\mathbf{p}_{pt}^p\right|^{\mathrm{T}} \left|\mathbf{p}_{pt}^p\right|^{\rho} - \sigma_v \left|\mathbf{e}_2\right|^{\mathrm{T}} \left|\mathbf{e}_2\right|^{\rho} - \sigma_\omega \left|\mathbf{e}_1\right|^{\mathrm{T}} \left|\mathbf{e}_1\right|^{\rho}.$$

Recalling Eqs. (58) and (59), let

$$m_1 = \bar{m}_p - b_1 \leq m_p \leq \bar{m}_p + b_1 = m_2 \tag{106}$$

$$J_1 = \left\|\bar{\mathbf{J}}_p\right\| - b_2 \leq \left\|\mathbf{J}_p\right\| \leq \left\|\bar{\mathbf{J}}_p\right\| + b_2 = J_2, \tag{107}$$

and we can thus derive using *Lemma 2* that

$$\dot{V} \leq -2^{-\beta}\sigma_q \left(2(1-q_{e0})\right)^{\beta} - 2^{\beta}\sigma_p \left(\frac{1}{2}(\mathbf{p}_{pt}^p)^{\mathrm{T}}\mathbf{p}_{pt}^p\right)^{\beta} - 2^{\beta} m_1^{-\beta}\sigma_v \left(\frac{1}{2}m_1\mathbf{e}_2^{\mathrm{T}}\mathbf{e}_2\right)^{\beta} - 2^{\beta} J_1^{-\beta}\sigma_\omega \left(\frac{1}{2}J_1\mathbf{e}_1^{\mathrm{T}}\mathbf{e}_1\right)^{\beta} \tag{108}$$

where $\beta = \frac{\rho+1}{2} < 1$.

Define

$$\sigma = \min\{2^{-\beta}\sigma_q, 2^{\beta}\sigma_p, 2^{\beta}m_1^{-\beta}\sigma_v, 2^{\beta}J_1^{-\beta}\sigma_\omega\}, \tag{109}$$

and using *Lemma 2* again, Eq. (108) can be further derived into

$$\dot{V} \leq -\sigma V^{\beta}. \tag{110}$$

Then, based on *Lemma 1*, one can conclude that

$$q_{e0} = 1, \mathbf{q}_{ev} = 0, \mathbf{p}_{pt}^p = 0, \mathbf{e}_1 = 0, \mathbf{e}_2 = 0, \text{ for } t \geq T_r, \tag{111}$$

and the settling time T_r can be bounded by T_m as

$$T_r \leq \frac{V(0)^{1-\beta}}{\sigma(1-\beta)} \leq T_m = \frac{V_m^{1-\beta}}{\sigma(1-\beta)} \tag{112}$$

where

$$V_m = \mathbf{q}_{ev}^{\mathrm{T}}(0)\mathbf{q}_{ev}(0) + (1-q_{e0}(0))^2 + \frac{1}{2}(\mathbf{p}_{pt}^p(0))^{\mathrm{T}}\mathbf{p}_{pt}^p(0) + \frac{J_2}{2}\mathbf{e}_1^{\mathrm{T}}(0)\mathbf{e}_1(0) + \frac{m_2}{2}\mathbf{e}_2^{\mathrm{T}}(0)\mathbf{e}_2(0). \tag{113}$$

We further derive from Eqs. (80) and (111) that

$$\mathbf{v}_e^p = 0, \omega_e^p = 0, \text{ for } t \geq T_r. \tag{114}$$

Hence, $\{\hat{q}_e(t) = \hat{E}, \hat{\omega}_e(t) = \hat{O}\}$ is achieved in settling time T_r. □

Remark 3. *As can be seen, the system uncertainties and external disturbances are analyzed and summarized into functions Δ_1, Δ_2, which can be bounded by state-dependent functions η_1, η_2; what's more, the coefficients in η_1, η_2 can be obtained rather than directly assumed like treatment for robotic manipulators.*[26, 31]

Remark 4. *It is worthy of noting that, term $-\hat{\eta}\mathbf{sgn}(\hat{\mathbf{e}})$ in RFTB control law (85) is derived to deal with system uncertainties and disturbances. If only the nominal system is considered, i.e., $m_p = \bar{m}_p, \mathbf{J}_p = \bar{\mathbf{J}}_p$, and $\hat{\mathbf{d}} = \hat{O}$, then the finite-time control law can be proposed as*

$$\hat{\mathbf{u}} = \hat{\bar{\mathbf{C}}}\hat{\alpha} - \hat{\bar{\mathbf{n}}} + \hat{\bar{\mathbf{M}}}_p\dot{\hat{\alpha}} - \hat{\mathbf{g}} - \hat{\mathbf{K}}_2\mathbf{sgn}^{\rho}(\hat{\mathbf{e}}). \tag{115}$$

Remark 5. *From Theorem 1, not only the finite-time stability of the closed loop system is guaranteed, but also the bound of settling time T_m can be estimated from (112), which is greatly beneficial for real time space missions or fast maneuver tasks. Furthermore, we can conclude that the settling time of system T_r can be adjusted by proper selections of the gain operators $\hat{\mathbf{K}}_1, \hat{\mathbf{K}}_2$ and the scalar ρ.*

Remark 6. *RFTB control law (85) involves vectors $\mathbf{p}_{pt}^p, \mathbf{q}_{ev}, \omega_e^p, \mathbf{v}_e^p, \dot{\mathbf{p}}_{pt}^p, \dot{\mathbf{q}}_{ev}$, where $\omega_e^p, \mathbf{v}_e^p$ can be directly obtained from the twist $\hat{\omega}_e$ for the definition (25). For vectors $\mathbf{p}_{pt}^p, \mathbf{q}_{ev}$, it can also be easily derived from a dual quaternion $\hat{q}_e = q_e + \varepsilon q'$. On one hand, $\mathbf{q}_{ev}$ is the vector part of the unit quaternion q_e; on the other hand, $\mathbf{p}_{pt}^p$ is the vector part of $2q_e^* \circ q'$ due to definition (20). Furthermore, $\dot{\mathbf{p}}_{pt}^p, \dot{\mathbf{q}}_{ev}$ can be derived from $\hat{q}_e$ and $\hat{\omega}_e$, using Eqs. (26) and (89). Therefore, the RFTB controller can be easily obtained from the dual variables $\hat{q}_e$ and $\hat{\omega}_e$.*

Remark 7. *Notice that the signum function sgn(·) in Eq. (85) may incur chattering of control input. To avoid this phenomenon, for a variable $x \in \mathbb{R}$, the function sgn(x) can be replaced by the following continuous function sgnc(x).*

$$sgnc(x) = \frac{x}{|x| + \delta} \tag{116}$$

where δ is a small positive scalar.

IV. Numerical Simulation

To demonstrate the effect of the RFTB controller, a scenario is considered in this section. Our aim is to make a spacecraft maneuver to motion in a circular orbit and become nadir pointing. Thus, without loss of generality, assume the reference body frame **T** is the LVLH frame with the x_t-axis points towards zenith; the y_t-axis is perpendicular to the x_t-axis in the direction of the velocity direction; and the x_t-axis completes the right-hand system. The orbit height is assumed to be $r_t = 250$ km, where gravity gradient torque τ_g can not be ignored. Then, the orbit angular velocity can be obtained from $\omega_0 = \sqrt{\mu/r_t^3}$; meanwhile, notice that on the circular orbit, one has

$$\dot{\mathbf{v}}_t^t + \omega_t^t \times \mathbf{v}_t^t = -\frac{\mu}{r_t^3}\mathbf{r}_t^t, \tag{117}$$

and thus the term $\mathbf{n}_1$ in Eq. (55) can be transformed into

$$\mathbf{n}_1 = -\frac{\mu m_p}{r_p^3}\mathbf{r}_p^p + \frac{\mu m_p}{r_t^3}\mathbf{r}_t^p - m_p(q_e^* \circ \dot{\omega}_t^t \circ q_e) \times \mathbf{p}_{pt}^p - m_p\omega_t^p \times \left(\omega_t^p \times \mathbf{p}_{pt}^p\right). \tag{118}$$

Moreover, we can derive

$$\eta_1 = b_1 \left\|\omega_e^p \times \alpha_2 + 2\omega_t^p \times \alpha_2 + (\mu/r_p^3)\mathbf{r}_p^p - (\mu/r_t^3)\mathbf{r}_t^p + (q_e^* \circ \dot{\omega}_t^t \circ q_e)_t^p \times \mathbf{p}_{pt}^p + \omega_t^p \times \left(\omega_t^p \times \mathbf{p}_{pt}^p\right) + \dot{\alpha}_2\right\| + d_f. \tag{119}$$

Meanwhile,

$$\mathbf{r}_t^t = \begin{bmatrix} r_t \\ 0 \\ 0 \end{bmatrix}, \omega_t^t = \begin{bmatrix} 0 \\ 0 \\ \omega_0 \end{bmatrix}, \mathbf{v}_t^t = \omega_t^t \times \mathbf{r}_t^t, \dot{\omega}_t^t = \mathbf{0}.$$

In the light of the spacecraft, assume that

$$\bar{m}_p = 20 \text{ kg}, \ \Delta m_p = 2 \text{ kg},$$

$$\bar{\mathbf{J}}_p = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{kg m}^2, \ \Delta\mathbf{J}_p = \begin{bmatrix} 1 & 1.5 & 0.5 \\ 1.5 & 1 & 1.5 \\ 0.5 & 1.5 & 10 \end{bmatrix} \text{kg m}^2,$$

and the initial error position, attitude, velocity and angular velocity are

$$\mathbf{p}_{pt}^p(0) = \begin{bmatrix} 50 & -50 & 40 \end{bmatrix}^{\mathrm{T}} \text{m}, \ q_e(0) = \begin{bmatrix} 0.8 & 0.4 & 0.2 & -0.4 \end{bmatrix}^{\mathrm{T}},$$

$$\dot{\mathbf{p}}_{pt}^p(0) = \begin{bmatrix} 2 & 4 & 18 \end{bmatrix}^{\mathrm{T}} \text{m/s}, \ \omega_e^p(0) = \begin{bmatrix} 0.1 & 0.2 & -0.1 \end{bmatrix}^{\mathrm{T}} \text{rad/s}.$$

Therefore, the initial dual quaternion and initial dual velocity vector can be derived from Eq. (20) as

$$\hat{q}_e(0) = \begin{bmatrix} 0.8 & 0.4 & 0.2 & -0.4 \end{bmatrix}^{\mathrm{T}} + \varepsilon \begin{bmatrix} -3 & 14 & -38 & 1 \end{bmatrix}^{\mathrm{T}},$$

$$\hat{\omega}_e = \begin{bmatrix} 0.1 & 0.2 & -0.1 \end{bmatrix}^{\mathrm{T}} + \varepsilon \begin{bmatrix} 5 & -5 & 3 \end{bmatrix}^{\mathrm{T}}.$$

The external disturbances are viewed as

$$\hat{\mathbf{d}} = \begin{bmatrix} 0.01 & 0.01 & 0.01 \end{bmatrix}^{\mathrm{T}} \sin\omega_0 t + \varepsilon \begin{bmatrix} 0.001 & 0.001 & 0.001 \end{bmatrix}^{\mathrm{T}} \sin\omega_0 t.$$

For the RFTB controller, to investigate the effect of scalar ρ on the system, choose $\rho = 0.4, 0.6, 0.9$, respectively, and gain operators are selected as

$$\hat{\mathbf{K}}_1 = 0.3\mathbf{I}\frac{d}{d\varepsilon} + \varepsilon\mathrm{diag}\{0.2, 0.6, 0.2\},\ \hat{\mathbf{K}}_2 = \mathbf{I}\frac{d}{d\varepsilon} + \varepsilon 0.4\mathbf{I}.$$

Then, for the function (116), set $\delta = 0.008$.

To give a clear physical meaning, in stead of dual states $\hat{q}_e, \hat{\omega}_e$, variations of equivalent states $\mathbf{p}_{pt}^p(t)$, $\dot{\mathbf{p}}_{pt}^p(t)$, $q_e(t)$, $\omega_e^p(t)$ are shown in Figs. 2-5, respectively; Figs. 6 and 7 show control force $\mathbf{f}$ and control torque τ histories. As can be seen, every error state arrives at the equilibrium in finite time with a good performance; further, by comparison, one can straightforwardly conclude effects of the parameter ρ on the system response are that 1) different values of ρ cause the translation and the rotation tend to change in the opposite direction; specifically, larger value of ρ contributes to a faster convergence of translation error state but a slower rotation motion; and 2) smaller value of ρ is liable to induce larger overshoot in error state variations and requires higher control input.

Table 1 shows detailed simulation results including settling time of error states $T(\mathbf{p}_{pt}^p)$, $T(\dot{\mathbf{p}}_{pt}^p)$, $T(q_e)$, $T(\omega_e^p)$, bound of system settling time, T_m, the peak Euclidean norm of the control force, $\max(\|\mathbf{f}\|)$, and the peak Euclidean norm of the control torque, $\max(\|\tau\|)$. In fact, the real settling time of the system, T_r, can be regarded as the largest value of $T(\mathbf{p}_{pt}^p)$, $T(\dot{\mathbf{p}}_{pt}^p)$, $T(q_e)$, $T(\omega_e^p)$, i.e., $T_r = \max\{T(\mathbf{p}_{pt}^p), T(\dot{\mathbf{p}}_{pt}^p), T(q_e), T(\omega_e^p)\}$. From table 1, we can conclude that 1) the real settling times of system with different ρ are less than the corresponding bound T_m. This conservatism is mainly due to repeated inequalities transformation in stability analysis; 2) the minimum value of T_m may exist since they do not change monotonously with the parameter ρ as mentioned above.

Now that the initial translation and rotation error, gain operators are selected, the bound T_m only depends on the parameter ρ in view of Eq. (112). In the sequel, the relationship between bound T_m and scalar ρ is investigated in order to get the minimum of T_m and reduce conservatism. Thus the following optimization problem is taken into account and further solved by using the sequential quadratic programming (SQP):

$$\min_{0 \leq \rho < 1} \|T_m\| \tag{120}$$

As a result, the minimum T_m can be obtained with $T_m^* = 172.2$ s, and the corresponding parameter $\rho^* = 0.7125$, the system response results with ρ^* are illustrated in the final row of Table 1. Besides, the variation of T_m with respect to the parameter ρ is shown in Fig. 8, where one can stress that when ρ tends to 1, the bound T_m changes greatly. In fact, when $\rho = 1$, T_m will be infinity, which is just in accord with the asymptotic convergence owing to Eq. (110).

Meanwhile, it is noteworthy that although the bound of system settling time T_m can be minimized to 172.2 s, yet still the control input shows higher than ones in the case of $\rho = 0.9$. In a summary, to reduce control input, large value of parameter ρ is preferable; however, this will leads to a longer convergence of system states. Therefore, in applications, it is necessary to choose a proper parameter ρ for the trade-off between the system performance and control energy according to control requirements.

Table 1. Simulation results

ρ	$T(\mathbf{p}_{pt}^p)$ (s)	$T(\dot{\mathbf{p}}_{pt}^p)$ (s)	$T(q_e)$ (s)	$T(\omega_e^p)$ (s)	T_m (s)	$\max(\|\mathbf{f}\|)$ (N)	$\max(\|\tau\|)$ (N m)
0.4	77	76	11	18	240.7	227.5	1.9
0.6	54	55	13	22	182.3	153.6	1.8
0.9	43	42	43	48	258.6	121.3	1.6
ρ^*	40	38	20	24	172.2	138.4	1.7

V. Conclusion

This paper considers simultaneous translation and rotation finite-time maneuver of a rigid spacecraft. An effective and compact integrated position and attitude dynamic model of the rigid spacecraft is firstly

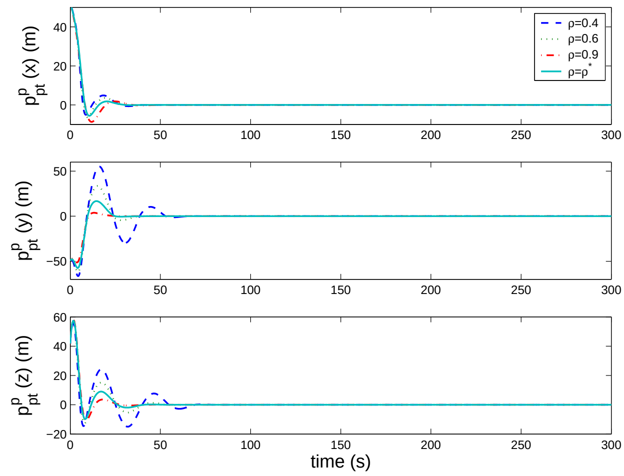

Figure 2. Position error

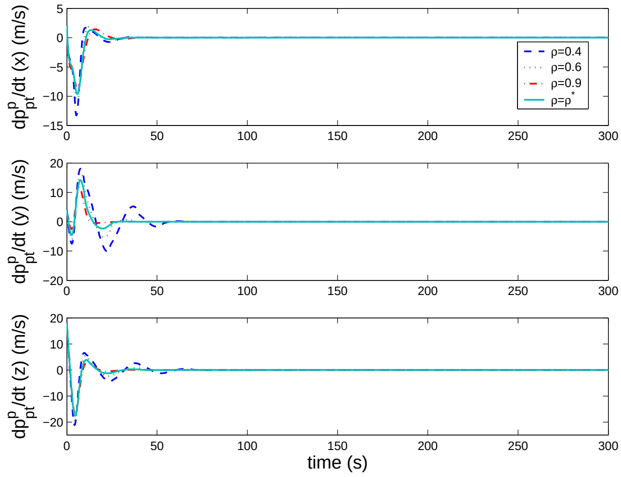

Figure 3. Velocity error

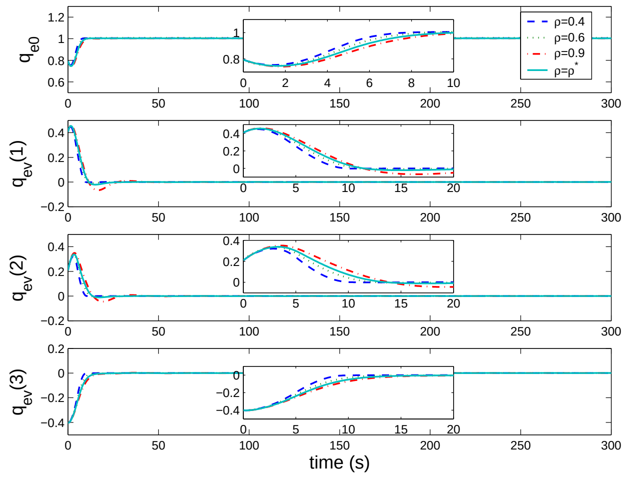

Figure 4. Error attitude quaternion

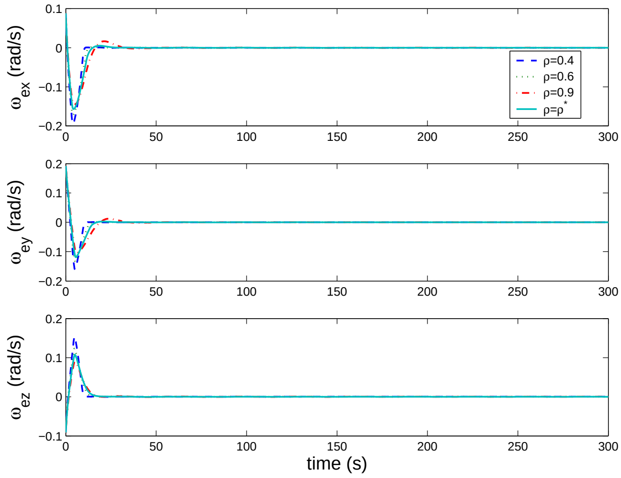

Figure 5. Angular velocity error

derived using dual quaternion, where the couplings can be clearly shown. The relationship between system states in the form of dual numbers and widely known Cartesian vectors are developed, which makes the traditional stability analysis methods still apply. After that, a robust finite-time backstepping controller is proposed to guarantee the finite-time stability of the closed-loop system in the presence of the system uncertainties and external disturbances within the Lyapunov framework; besides, the bound of settling time is also given. Although conservatism exists to some extent, this settling time bound is significant for real time control of spacecraft in practice. A scenario simulation shows the effect of the proposed RFTB controller; meanwhile, further analysis concludes that controller parameter ρ affects both the system response and the control input. As a result, the parameter should be properly selected for the trade-off between the system performance and control energy according to control requirements.

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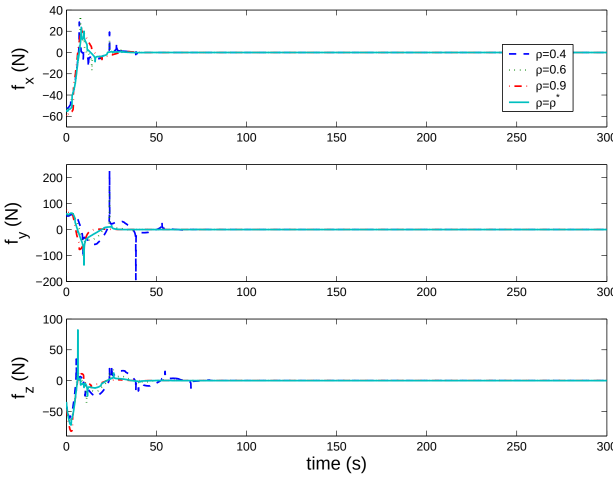

Figure 6. Control force

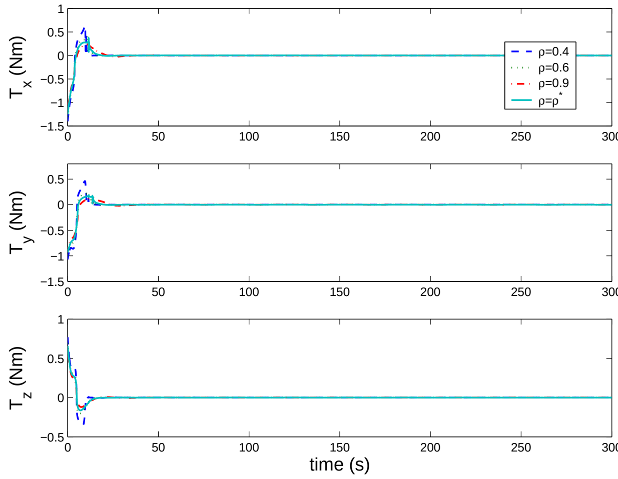

Figure 7. Control torque

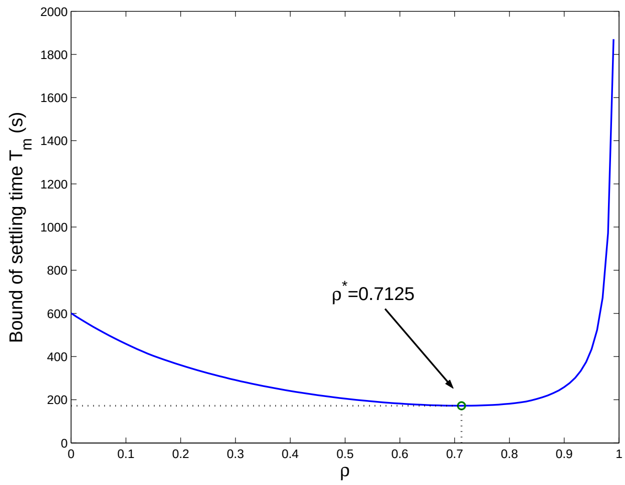

Figure 8. Bound of settling time T_m relative to parameter ρ

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