Pergamon

Computers & Structures Vol. 50, No. 2, pp. 259–262, 1994

0045-7949/94 $6.00 + 0.00

THE DYNAMICS OF FLEXIBLE MULTIBODY SYSTEMS: A FINITE SEGMENT APPROACH—II. EXAMPLE PROBLEMS

J. D. CONNELLY and R. L. HUSTON
Department of Mechanical Engineering, University of Cincinnati, Cincinnati, OH 45221-0072, U.S.A.

(*Received* 7 *September* 1992)

Abstract—Modelling the dynamics of flexible multibody systems is a difficult problem. There has been much discussion about how to approach this problem. This paper presents a method in which the flexibility may be included in the dynamical analysis. The method has distinct advantages over other proposed methods. The dynamical formulation is based on Kane's equations. The flexibility is modelled by using springs and dampers at the joints of the system. The stiffness and damping coefficients of these are found using the physical properties of the members. These are then incorporated into the equations of motion. Part I of this paper discusses the theoretical aspects of this type of analysis, while part II shows two example problems.

1. INTRODUCTION

Recently there has been increasing interest in studying flexible multibody systems—that is, multibody systems containing elastic or flexible members. Examples of such systems are composite structures, rotorcraft blades, antennas, trusses and large frames.

Procedures have been developed to analyze multibody systems [1–29]. These procedures are ideally suited for studying the flexibility effects. These procedures might be characterized as 'finite segment' modelling. In this sense, the method is analogous to the finite element method of structural analysis.

The finite segment method is one in which a system is broken down into a number of finite, rigid members. Using formulations derived from Kane's equations of motion [1, 14, 15], the dynamical equations may be developed and analyzed. This type of analysis has been developed for rigid bodies. The next logical step is to incorporate flexibility into this type of analysis.

This method has been proven on simple beam structures. A rigid body finite segment analysis program was modified to include flexibility effects. The beams were broken down into segments and torsional springs were added at the joints of the segements. Two example cases were analyzed. The first was a vibrating beam. The equation was derived and solved. The beam was also modelled using the 'flexible finite segment' method. The motion compared very well between the two analyses. The second case was a rotating beam that was subjected to a 'spin up', where its angular velocity increased over time to a constant value. This problem was originally analyzed using another technique to incorporate the flexibility effects. The problem was reanalyzed using the technique discussed here and the results compared to the original analysis. Again, the results compared very well. These two cases show the method may be used to model flexibility effects in dynamical systems.

2. EXAMPLE 1: VIBRATING BEAM

The beam (as shown in Fig. 1) is given an initial deflection on five degrees. Figure 2 shows the initial conditions of the beam.

The results of the computer simulation will be compared to the analytical solution of the problem. The unforced rotational vibration of a beam may be described by the following differential equation

$$I_0\ddot{\theta} + K\theta = 0.$$

Assuming that the solution of eqn (1) is equal to

$$\theta = A_1\cos(\omega t) + A_2\sin(\omega t).$$

Then, the derivative is equal to

$$\dot{\theta} = -A_1\omega\sin(\omega t) + A_2\omega\cos(\omega t).$$

The initial conditions are

$$t = 0.0, \quad \theta(0.0) = 0.0$$

$$t = 0.0, \quad \theta(0.0) = \theta_0, \quad (\theta_0 = 5°).$$

Solving for the constants A_1, A_2, ω, and θ_0, the equation of motion that describes the vibration of the beam is equal to

$$\theta(t) = 0.0873\cos 0.5916(t).$$

Fig. 1. Example 1: vibrating beam.

This equation is used to calculate the moment on the beam. The analysis simulates the clamped end by using a pinned end joint and a moment on the beam. Figure 3 shows the analysis configuration.

The following is a description of how the beam is analyzed using the modified finite segment technique. The beam was broken down into ten separate elements. All of the joints were spring–pin joints except for the joint to ground. This joint is allowed to rotate with respect to the Z axis. The angle is calculated using the following

$$\text{ANGLE}(3,2) = 0.0873\cos(0.5916(\text{TIME})).$$

The moment is calculated using

$$\text{IMOMNT}(3,2) = \frac{\text{BG1} \times \text{ANGLE}(3,2)\cdot\pi}{\text{BFF} \times 360.0}.$$

These moments are included in the equations of motion. The angles are then found. The beam should rotate back and forth. The angles of the vibrating beam may be calculated separately from the results from the computer analysis. The analysis was initially run for a period of 6 sec with a printout interval of 1/10 sec. A longer analysis was also performed. Its duration was 20 sec, with a printout time of 1 sec. Table 1 shows the results of these two analyses. The theoretical results are also included for comparison.

Examining the results, the angles match very closely for the first second of time. After that, the angles are further apart, but the percentage difference increases and decreases. The results do not diverge. The angle of rotation is not greater than five degrees. This shows that the basic flexibility theory is correct.

3. EXAMPLE 2: BEAM SPIN-UP

The second example problem is a rotational dynamics problem. It is basically the same beam as in Example 1 (see Fig. 4).

The main difference between this example and the first example, is the first joint of the beam. In this example, the first joint is pinned rather than clamped. This allows the beam to rotate freely. This problem was first analyzed by Ryan. This beam was subjected

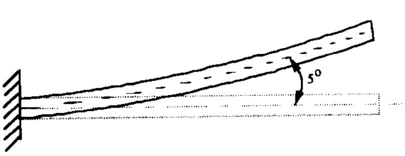

Fig. 2. Initial condition of the beam.

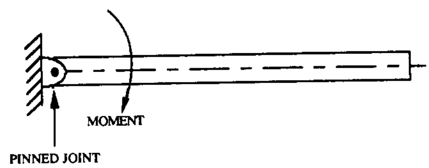

Fig. 3. Analysis configuration.

to a velocity spin up. This velocity can be written as follows:

$$\Omega(t) = \left(\frac{2}{5}\right)\left[t - \left(\frac{7.5}{\pi}\right)\sin\left(\frac{\pi t}{7.5}\right)\right]\text{rad/sec.}$$

Also, the software requires the angular position profile and the angular acceleration profile. These may be found by integrating and differentiating the above function

$$\theta(t) = \int \Omega(t)\,\mathrm{d}t = \int \left(\frac{2}{5}\right)\left[t - \left(\frac{7.5}{\pi}\right)\sin\left(\frac{\pi t}{7.5}\right)\right]\mathrm{d}t$$

$$\theta(t) = \left(\frac{2}{5}\right)\left[\frac{t^2}{2} + \left(\frac{7.5}{\pi}\right)^2\cos\left(\frac{\pi t}{7.5}\right)\right] + C_1,$$

where C_1 is a constant. Solving for the constant, using the boundary conditions, results in the following equation

$$\theta(t) = 0.4\left(\frac{t^2}{2} + 5.6993\cos(0.4189t)\right) - 2.2797.$$

Now the equation may be differentiated to find the equation for the angular acceleration profile

Table 1. Comparison of analysis results

Time	θ_{THER}	θ_{FLEX}
0.0	5.000	5.000
0.125	4.9711	4.988
0.250	4.9303	4.9522
0.375	4.8625	4.8928
0.500	4.7628	4.8100
0.750	4.5021	4.5764
1.000	4.1376	4.2655
1.500	3.1475	3.3877
2.000	1.8840	2.2790
3.000	−1.009	−0.2601
4.000	−3.5610	−2.4981
5.000	−4.9010	−3.6741
6.000	−4.5760	−3.3885
7.000	−2.6950	—
8.000	0.1017	0.71362
9.000	2.8638	—
10.000	4.1376	4.7005
11.000	4.8597	—
12.000	3.1415	3.6185
13.000	0.8098	—
14.000	−2.0708	−1.1837
15.000	−4.2476	—
16.000	−4.9805	−3.7294
17.000	−4.0207	—
18.000	−1.6941	0.84913
19.000	1.208	—
20.000	3.700	3.8769

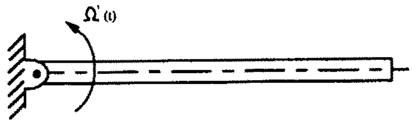

Fig. 4. Example 2: rotating beam.

$$\Omega(t) = 0.4\left(t - 2.3873\sin\left(\frac{7.5}{\pi}\right)t\right)$$

$$\dot{\Omega}(t) = 0.4(1.0 - \cos(0.4189t)).$$

This is the equation for the angular acceleration profile.

The above three equations are required to run the simulation. They are input into subroutine STARTTR of the program SUPER. The program SUPER was used instead of DYNOCOMBS because of computational difficulties.

The model of the beam was broken down into two elements. The first element is 1 m long and the second is 9 m longer. These two elements are attached by a hinge joint. Using the theory discussed in Sec. 2, a torsional spring was connected to this hinge joint (see Figs 5 and 6).

The stiffness for this spring was calculated to be the following

$$k = \frac{EI}{L} = \frac{70{,}000{,}000.0 \times 0.0000002}{9.0}$$

$$k = 1556.$$

The moment is then calculated

$$\text{CM}(3, 2) = -1556(X(9)),$$

where CM(3, 2) is the moment applied in the Z direction of body 2, $X(9)$ is the Z rotation of body 2, and 1156 is the stiffness of member 2.

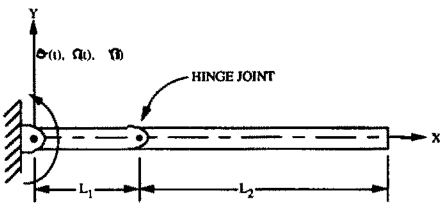

Fig. 5. Beam model.

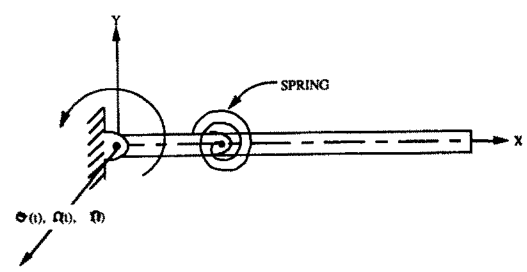

Fig. 6. Beam model with torsional spring.

Table 2. Results of the analysis

Time (sec)	$\theta_{(t)\text{RELATIVE}}$ (in degrees)
0.0	0.0
2.0	−0.8588E − 10
4.0	−0.5083E − 04
4.5	−0.6274E − 02
5.0	−0.2883
5.125	−0.4745E − 01
6.0	−0.1385
7.0	−0.1043E − 01
8.9	−0.3803E − 01
9.0	−0.568E − 01
9.938	−0.2384E − 01
11.0	−0.672E − 01
12.0	−0.276E − 01
14.0	−0.4228E − 04
15.0	−0.2369E − 02

The moment CM(3, 2) was input into the program SUPER using subroutine EXPRESSR. The following variables were determined by the above equations

$$X(6) = OA(T^2/2 + 5.6993\cos(0.4189t)) - 2.2797$$

$$X(39) = 0.4(T - 2.3973\sin(0.4189t))$$

$$\text{DDX}(6) = 0.4(1.0 - \cos(0.4189t)).$$

These three equations determine the angular position, velocity and accelerations. Table 2 gives the results of the analysis.

4. SUMMARY

The analysis of flexible multibody systems is a difficult problem. There are many physical systems where the normal 'rigid-body' techniques will not give correct results. There has been much discussion on how to approach these problems. This paper presents one of these approaches. This technique is a modified finite segment method. This technique has advantages over other techniques. This paper discusses two examples that demonstrate the technique. The first example compares the analysis technique with a theoretical analysis and the second compares with a second computational technique. Both of these examples correlate very well with the other techniques. This shows that the technique is a viable alternative.

REFERENCES

1. F. M. L. Amirouche and R. L. Huston, Dynamics of large constrained flexible structures. *J. Dynamic Systems Measurement and Control* **110,** 78–83 (1988).
2. P. M. Bainum and V. K. Kumar, Dynamics of orbiting flexible beams and platforms in the horizontal orientation. *Acta Astronautica* **9,** 119–127 (1982).
3. C. E. Benedict and D. Tesar, Dynamic response analysis of quasi-rigid mechanical systems using kinematic influence coefficients. *J. Mechanisms* **6,** 383–403 (1971).
4. P. Boland, J. C. Samin and P. Y. Willems, Stability analysis of interconnected deformable bodies in a topological tree. *AIAA Jnl* **12,** 1025–1030 (1974).

5. W. J. Brook, O. Maizza-Neto and D. E. Whitney, Feedback control of two beam, two joint systems with distributed flexibility. *J. Dynamic Systems Measurement and Control* 424–431 (1975).
6. H. Bremer, Dynamics of flexible hybrid structures. *J. Guidance and Control* **2,** 86–87 (1979).
7. S. Dubowsky and T. N. Gardner, Design and analysis of multilink flexible mechanisms with multiple clearance connections. *J. Engng Industry, ASME* **99,** 88–96 (1977).
8. A. G. Erdman, G. N. Sandor and R. G. Oakberg, A general method for kineto-elastodynamic analysis and synthesis of mechanisms. *J. Engng Industry, ASME* **94,** 1193–1205 (1972).
9. W. B. Gevarter, Basic relations for control of flexible vehicles. *AIAA Jnl* **8,** 666–678 (1970).
10. M. Gurgoze, On the vibrations of a rotating cantilever. *J. Sound Vibr.* **110,** 529–532 (1986).
11. J. Y. L. Ho, Direct path method for flexible multibody spacecraft dynamics. *J. Spacecraft and Rockets* **14,** 102–110 (1977).
12. J. Y. L. Ho and D. R. Herber, Development of dynamics and control simulations of large flexible space systems. *J. Guidance, Control and Dynamics* **8,** 374–383 (1985).
13. P. C. Hughes, Dynamics of a chain of flexible bodies. *J. Astronaut. Sci.* **XXVII,** 359–380 (1979).
14. R. L. Huston, Flexibility effects in multibody systems. *Mechanics Res. Commun.* **7,** 261–268 (1980).
15. R. L. Huston, Multi-body dynamics including the effects of flexibility and compliance. *Comput. Struct.* **14,** 443–451 (1981).
16. T. R. Kane, R. R. Ryan and A. K. Banerjee, Comprehensive theory for the dynamics of a general beam attached to a moving rigid base (Part 1). *J. Guidance, Control and Dynamics* **10,** No. 2 (1987).
17. P. W. Likins, Dynamic analysis of a system of hinge-connected rigid bodies with nonrigid appendages. *Int. J. Solids Struct.* **9,** 1473–1487 (1973).
18. K. W. Lips and V. J. Modi, Transient attitude dynamics of satellites with deploying flexible appendages. *Acta Astronautica* **5,** 797–815 (1978).
19. V. J. Modi, Attitude dynamics of satellites with flexible appendages—a brief review. *J. Spacecraft and Rockets* **11,** No. 11 (1974).
20. K. C. Park and K. J. Saczalski, Transient response of inelastically constrained rigid-body systems. *J. of Engng Industry, ASME* **96,** 1041–1047 (1974).
21. S. Rajaram and J. L. Junkins, Identification of vibrating flexible structures. *J. Guidance, Control and Dynamics* **8,** 463–470 (1985).
22. R. E. Robertson, A form of the translational dynamical equations for relative motion in systems of many non-rigid bodies. *Acta Mechanica* **14,** 297–308 (1972).
23. A. A. Shabana, Substructure synthesis methods for dynamic analysis of multi-body systems. *Comput. Struct.* **20,** 737–744 (1985).
24. W. Sunada and S. Dubowsky, The application of finite element methods to the dynamic analysis of flexible spatial and co-planar linkage systems. *J. Mech. Design* **10,** 643–651 (1981).
25. C. R. Wiley, Jr, *Advanced Engineering Mathematics*, 3rd Edn, Chap. 8. McGraw-Hill, New York (1966).
26. W. T. Thomson, *Theory of Vibration with Applications*, 3rd Edn, Chap 8. Prentice-Hall, Englewood Cliffs, NJ (1988).
27. L. Meirovitch, *Analytical Methods in Vibrations*, Chap. 5. Macmillan, New York (1967).
28. E. Volterra and E. C. Zachmanoglu, *Dynamics of Vibrations*, Chap. 4. C. E. Merrill, Columbus, OH (1965).
29. T. R. Kane, *Analytical Elements of Mechanics*, Vol. 1, pp. 150–152. Academic Press, New York (1959).