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THE DYNAMICS OF FLEXIBLE MULTIBODY SYSTEMS: A FINITE SEGMENT APPROACH—I. THEORETICAL ASPECTS

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Abstract—Modelling the dynamics of flexible multibody systems is a difficult problem. There has been much discussion about how to approach this problem. This paper presents a method in which the flexibility may be included in the dynamical analysis. The method has distinct advantages over other proposed methods. The dynamical formulation is based on Kane's equations. The flexibility is modelled by using springs and dampers at the joints of the system. The stiffness and damping coefficients of these are found using the physical properties of the members. These are then incorporated into the equations of motion. Part I of this paper discusses the theoretical aspects of this type of analysis, while part II shows two example problems.

1. INTRODUCTION

Recently there has been increasing interest in studying flexible multibody systems—that is, multibody systems containing elastic or flexible members. Examples of such systems are composite structures, rotorcraft blades, antennas, trusses and large frames.

The dynamic analysis of flexible systems is a difficult problem. Most of the dynamical techniques are applicable to rigid bodies only. In reality there is no such thing as a rigid body. All bodies deflect to some extent when forces are applied. In many cases though, these deflections are so small that they may be neglected. But in some cases, the deflections are not small. This is where the standard dynamical formulations will not be accurate. Composite structures, rotorcraft blades, cables and space structures are examples of these systems. The majority of these will actually deflect under their own weight. A method needs to be developed to model the dynamics of these systems.

There have been many analyses and studies of flexible multibody systems as documented in the literature [1–29]. However, there is disagreement about acceptable and optimal methods for studying flexible multibody systems. The disagreement generally arises over the approximation methods used to model component mode methods and modified finite segment methods.

Procedures have been developed to analyze multibody systems. These procedures are ideally suited for studying the flexibility effects. These procedures might be characterized as 'finite segment' modelling. In this sense, the method is analogous to the finite element method of structural analysis.

There has been a great deal of discussion about how to approach this problem. There are modified finite element methods. Special elements are developed to take into account the flexibility effects. These elements are 'add-ons' and are difficult to use. Also, the finite element is not well suited to large motion dynamic systems. Another way of looking at this problem is to use a dynamical formulation in conjunction with the mode shapes of the system. This method is difficult to use. How are the mode shapes found? In some cases, it is very difficult to determine the mode shapes. A third method is to use a dynamical formulation modified in another way. The flexibility is simulated by adding springs and dampers to the rigid elements. The stiffness and damping coefficients are calculated according to the physical characteristics of the elements. These coefficients are then used in the analysis to simulate the flexibility of the elements. This method could have distinct advantages over the other methods of analysis. No special elements are needed. This new method is not as cumbersome as the others. Also, the mode shapes are not needed for this method.

The finite segment method is one in which a system is broken down into a number of finite, rigid members. Using formulations derived from Kane's equations of motion the dynamical equations may be developed and analyzed. This type of analysis has been developed for rigid bodies. The next logical step is to incorporate flexibility into this type of analysis.

This method has already been proven on simple beam structures. A rigid body finite segment analysis program was modified to include flexibility effects. The beams were broken down into segments and torsional springs were added at the joints of the

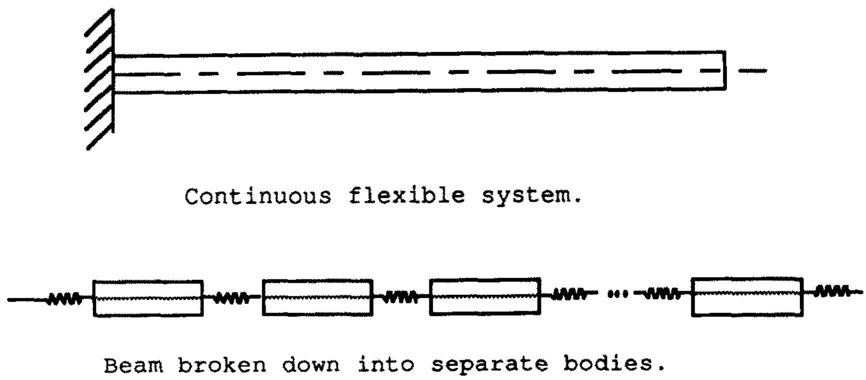

Fig. 1. Finite segment modelling.

segments. Two example cases were analyzed. The first was a vibrating beam. The equation was derived and solved. The beam was also modelled using the 'flexible finite segment' method. The motion compared very well between the two analyses. The second case was a rotating beam that was subjected to a 'spin-up', where its angular velocity increased over time to a constant value. This problem was originally analyzed using another technique to incorporate the flexibility effects. The problem was reanalyzed using the technique discussed here and the results compared to the original analysis. Again, the results compared very well. These two cases show the method may be used to model flexibility effects in dynamical systems.

2. IMPLEMENTATION OF FLEXIBILITY

This section will discuss the procedure that will be used to add flexibility into a rigid body dynamics software package. This has already been done in a simplified form and has worked successfully.

The first step to model a flexible system is to break down the system into separate bodies (see Fig. 1).

The orientation of these bodies are specified in the software. The material and physical properties are also specified. The flexibility is modelled in a subroutine within the software package. This software package has been developed at the University of Cincinnati, and is called UCIN-DYNCOMBS. The subroutine takes the displacement and the velocity and the physical properties of a body, and calculates both forces and moments that will act on the body. These forces and moments are then added into the dynamical equations of motion of the system. This is done for every time step that is specified. The flowchart in Fig. 2 shows the basic process of incorporating flexibility into the software. POSVQB (3, NB) and ANGLE (3, NB) are the relative translations and rotations between two bodies; Fig. 3 shows this. These are used in conjunction with the stiffness of the body to calculate the forces and moments that simulate the flexibility. If damping is important in the system, the translational and angular velocities may be used with damping coefficients to calculate forces and moments to be incorporated into the system. These factors and moments are stored in the arrays IFORQK (3, NB) and IMOMNT (3, NB).

3. INCORPORATION OF FLEXIBILITY INTO THE EQUATIONS OF MOTION

This section will discuss the theoretical aspects of incorporating flexibility effects into the equations of motion. As an example consider the finite segment model of a rod as depicted in Fig. 4.

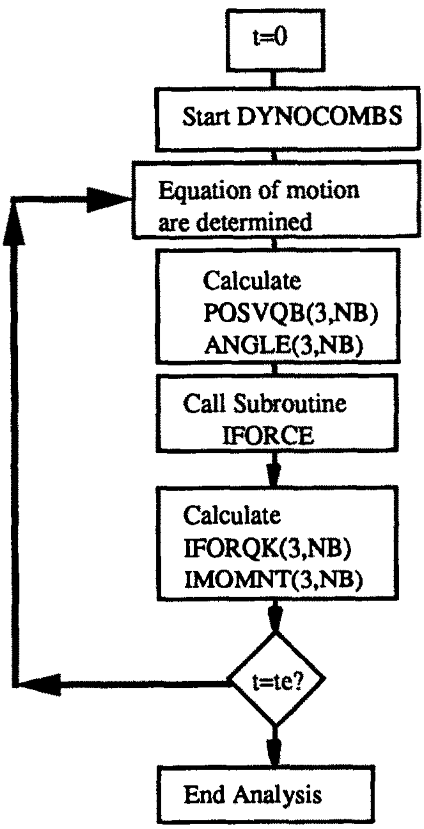

Fig. 2. Flowchart for the incorporation of flexibility effects.

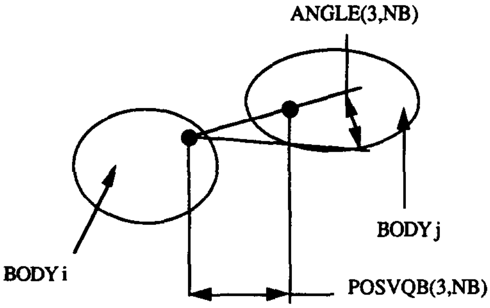

Fig. 3. Relative translations and rotations.

To see how the stiffness can be incorporated into the governing equations, consider two typical segments B_j and B_k and the connections between them. Let the coordinates of the segment centers measured along the rod axes be x_j and x_k. Let s_k measure the separation between the segments as shown in Fig. 5. Then s_k is a measure of the elongation (or contraction) of the rod at the joint connecting B_j and B_k. Expressed another way, s_j is the contribution to the rod elongation (or contraction) from end 2 (right end) portion of B_j and the end 1 (left end) portion of B_k.

For simplicity, let an individual segment have a uniform cross-section along its length. Then from Fig. 5, s_k may be expressed as

$$s_k = [x_k - [(L_k/2)] - [x_j + (L_j/2)].$$

Let the force in the connecting springs be f_k. Then f_k and s_k are related by the simple expression

$$f_k = k_{jk} s_k,$$

where k_{jk} is the equivalent stiffness of the connecting springs between B_i and B_j. That is

$$k_{jk} = \frac{k_{j2}k_{k1}}{k_{j2}+k_{k1}}.$$

Let δ_{j2} and δ_{k1} represent the elongation (or contraction) of the respective joint springs. Then we have the relations

$$s_k = \delta_{j2} + \delta_{k1}$$

and

$$\delta_{j2} = \frac{f_k}{k_{j2}} \quad \text{and} \quad \delta_{k1} = \frac{f_k}{k_{k1}}.$$

Hence, by substituting, we have

$$s_k = (f_k/k_{j2}) + f_k/k_{k1} = f_k[(1/k_{j2}) + (1/k_{k1})]$$

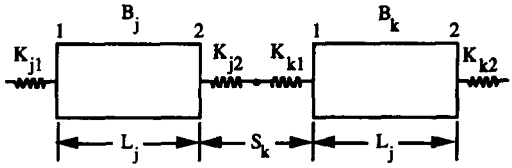

Fig. 5. Joint separation and geometry of two segments.

or

$$s_k = f_k\left(\frac{k_{j2}+k_{k1}}{k_{j2}k_{k1}}\right).$$

Next consider a free-body diagram of a typical segment, say B_k, as shown in Fig. 6. Let f_k^* represent the inertia force on B_k. Hence we have

$$f_k + f_l + f_k^* = 0$$

Let m_k be the mass of B_k. Then f_k^* may be expressed as

$$f_k^* = -m_k \ddot{x}_k.$$

By substituting, we have

$$f_k + f_l = m_k \ddot{x}_k \quad \text{(no sum)}.$$

The above equations may be used to obtain a governing differential equation for the mass center displacements. That is

$$k_{jk}s_k + k_{kl}s_l = m_k \ddot{x}_k$$

or

$$\left(\frac{k_{j2}k_{k1}}{k_{j2}+k_{k1}}\right)\{[x_k - (l_k/2)] - [x_j + (l_j/2)]\}$$
$$+\left(\frac{k_{k2}k_{L1}}{k_{k2}+k_{L1}}\right)\{[x_l - (l_L/2)]$$
$$- [x_k + (l_k/2)]\} = m_k \ddot{x}_k.$$

The development of the above equation illustrates the incoporation of stiffness effects into the dynamical equations.

4. DISCUSSION

The analysis of flexible multibody systems is a difficult problem. There are many physical systems where the normal 'rigid-body' techniques will not give correct results. There has been much discussion on how to approach these problems. This paper presents

Fig. 4. A finite segment model of a rod.

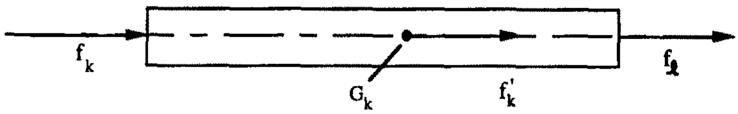

Fig. 6. Free body diagram of B_k.

one of these approaches. This technique is a modified finite segment method. This technique has advantages over other techniques. This paper discusses theoretical aspects and part II of this paper discusses two examples that demonstrate the technique.

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