

with (*LinearAlgebra*) :

$$dJ := \begin{bmatrix} 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ J_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$dJ := \begin{bmatrix} 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ J_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

$dJ_inv := \text{MatrixInverse}(dJ)$

$$dJ_inv := \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{J_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{J_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{J_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

$$dw := \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ 0 \\ v_1 \\ v_2 \\ v_3 \\ 0 \end{bmatrix}$$

$$dw := (w_1)e_{x1} + (w_2)e_{x2} + (w_3)e_{x3} + (0)e_{x4} + (v_1)e_{x5} + (v_2)e_{x6} + (v_3)e_{x7} + (0)e_{x8} \tag{3}$$

$$rf := \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$rf := (r_1)e_x + (r_2)e_y + (r_3)e_z \tag{4}$$

$$Ft := \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$Ft := (F_1)e_x + (F_2)e_y + (F_3)e_z \tag{5}$$

$$T := CrossProduct(rf, Ft)$$

$$T := \begin{bmatrix} -F_2\,r_3 + F_3\,r_2 \\ F_1\,r_3 - F_3\,r_1 \\ -F_1\,r_2 + F_2\,r_1 \end{bmatrix} \tag{6}$$

$$dF := \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ T_1 \\ T_2 \\ T_3 \\ 0 \end{bmatrix}$$

$$dF := (F_1)e_{x1} + (F_2)e_{x2} + (F_3)e_{x3} + (0)e_{x4} + (-F_2 r_3 + F_3 r_2)e_{x5} + (F_1 r_3 - F_3 r_1)e_{x6} \quad (7)$$

$$+ (-F_1 r_2 + F_2 r_1)e_{x7} + (0)e_{x8}$$

$$dw2 := \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ 0 \\ V_1 \\ V_2 \\ V_3 \\ 0 \end{bmatrix}$$

$$dw2 := (W_1)e_{x1} + (W_2)e_{x2} + (W_3)e_{x3} + (0)e_{x4} + (V_1)e_{x5} + (V_2)e_{x6} + (V_3)e_{x7} + (0)e_{x8} \quad (8)$$

$$w_comb := dw + dw2$$

$$w_comb := (w_1 + W_1)e_{x1} + (w_2 + W_2)e_{x2} + (w_3 + W_3)e_{x3} + (0)e_{x4} + (v_1 + V_1)e_{x5} \quad (9)$$

$$+ (v_2 + V_2)e_{x6} + (v_3 + V_3)e_{x7} + (0)e_{x8}$$

$$\begin{aligned}
dw_omega &:= \begin{bmatrix} 0 & -w_3 & w_2 & w_1 & 0 & 0 & 0 & 0 \\ w_3 & 0 & -w_1 & w_2 & 0 & 0 & 0 & 0 \\ -w_2 & w_1 & 0 & w_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -v_3 & v_2 & v_1 & 0 & -w_3 & w_2 & w_1 \\ v_3 & 0 & -v_1 & v_2 & w_3 & 0 & -w_1 & w_2 \\ -v_2 & v_1 & 0 & v_3 & -w_2 & w_1 & 0 & w_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
dw_omega &:= \begin{bmatrix} 0 & -w_3 & w_2 & w_1 & 0 & 0 & 0 & 0 \\ w_3 & 0 & -w_1 & w_2 & 0 & 0 & 0 & 0 \\ -w_2 & w_1 & 0 & w_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -v_3 & v_2 & v_1 & 0 & -w_3 & w_2 & w_1 \\ v_3 & 0 & -v_1 & v_2 & w_3 & 0 & -w_1 & w_2 \\ -v_2 & v_1 & 0 & v_3 & -w_2 & w_1 & 0 & w_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
dw2_omega &:= \begin{bmatrix} 0 & -W_3 & W_2 & W_1 & 0 & 0 & 0 & 0 \\ W_3 & 0 & -W_1 & W_2 & 0 & 0 & 0 & 0 \\ -W_2 & W_1 & 0 & W_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -V_3 & V_2 & V_1 & 0 & -W_3 & W_2 & W_1 \\ V_3 & 0 & -V_1 & V_2 & W_3 & 0 & -W_1 & W_2 \\ -V_2 & V_1 & 0 & V_3 & -W_2 & W_1 & 0 & W_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{10}$$

$$dw2_omega := \begin{bmatrix} 0 & -W_3 & W_2 & W_1 & 0 & 0 & 0 & 0 \\ W_3 & 0 & -W_1 & W_2 & 0 & 0 & 0 & 0 \\ -W_2 & W_1 & 0 & W_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -V_3 & V_2 & V_1 & 0 & -W_3 & W_2 & W_1 \\ V_3 & 0 & -V_1 & V_2 & W_3 & 0 & -W_1 & W_2 \\ -V_2 & V_1 & 0 & V_3 & -W_2 & W_1 & 0 & W_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

$$dw3_omega := \begin{bmatrix} [0, -(w_3 + W_3), (w_2 + W_2), (w_1 + W_1), 0, 0, 0, 0], \\ [(w_3 + W_3), 0, -(w_1 + W_1), (w_2 + W_2), 0, 0, 0, 0], \\ [-(w_2 + W_2), (w_1 + W_1), 0, (w_3 + W_3), 0, 0, 0, 0], \\ [0, 0, 0, 0, 0, 0, 0, 0], \\ [0, -(v_3 + V_3), (v_2 + V_2), (v_1 + V_1), 0, -(w_3 + W_3), (w_2 + W_2), (w_1 + W_1)], \\ [(v_3 + V_3), 0, -(v_1 + V_1), (v_2 + V_2), (w_3 + W_3), 0, -(w_1 + W_1), (w_2 + W_2)], \\ [-(v_2 + V_2), (v_1 + V_1), 0, (v_3 + V_3), -(w_2 + W_2), (w_1 + W_1), 0, (w_3 + W_3)], \\ [0, 0, 0, 0, 0, 0, 0, 0] \end{bmatrix}$$

$$dw3_omega := \begin{bmatrix} [0, -w_3 - W_3, w_2 + W_2, w_1 + W_1, 0, 0, 0, 0], \\ [w_3 + W_3, 0, -w_1 - W_1, w_2 + W_2, 0, 0, 0, 0], \\ [-w_2 - W_2, w_1 + W_1, 0, w_3 + W_3, 0, 0, 0, 0], \\ [0, 0, 0, 0, 0, 0, 0, 0], \\ [0, -v_3 - V_3, v_2 + V_2, v_1 + V_1, 0, -w_3 - W_3, w_2 + W_2, w_1 + W_1], \\ [v_3 + V_3, 0, -v_1 - V_1, v_2 + V_2, w_3 + W_3, 0, -w_1 - W_1, w_2 + W_2], \\ [-v_2 - V_2, v_1 + V_1, 0, v_3 + V_3, -w_2 - W_2, w_1 + W_1, 0, w_3 + W_3], \\ [0, 0, 0, 0, 0, 0, 0, 0] \end{bmatrix} \quad (12)$$

$$\begin{aligned}
d_{jw_om} &:= \begin{bmatrix} 0 & -m \cdot v_3 & m \cdot v_2 & m \cdot v_1 & 0 & 0 & 0 & 0 \\ m \cdot v_3 & 0 & -m \cdot v_1 & m \cdot v_2 & 0 & 0 & 0 & 0 \\ -m \cdot v_2 & m \cdot v_1 & 0 & m \cdot v_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \cdot w_3 & J_2 \cdot w_2 & J_1 \cdot w_1 & 0 & -m \cdot v_3 & m \cdot v_2 & m \cdot v_1 \\ J_3 \cdot w_3 & 0 & -J_1 \cdot w_1 & J_2 \cdot w_2 & m \cdot v_3 & 0 & -m \cdot v_1 & m \cdot v_2 \\ -J_2 \cdot w_2 & J_1 \cdot w_1 & 0 & J_3 \cdot w_3 & -m \cdot v_2 & m \cdot v_1 & 0 & m \cdot v_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
d_{jw_om} &:= \begin{bmatrix} 0 & -m \, v_3 & m \, v_2 & m \, v_1 & 0 & 0 & 0 & 0 \\ m \, v_3 & 0 & -m \, v_1 & m \, v_2 & 0 & 0 & 0 & 0 \\ -m \, v_2 & m \, v_1 & 0 & m \, v_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \, w_3 & J_2 \, w_2 & J_1 \, w_1 & 0 & -m \, v_3 & m \, v_2 & m \, v_1 \\ J_3 \, w_3 & 0 & -J_1 \, w_1 & J_2 \, w_2 & m \, v_3 & 0 & -m \, v_1 & m \, v_2 \\ -J_2 \, w_2 & J_1 \, w_1 & 0 & J_3 \, w_3 & -m \, v_2 & m \, v_1 & 0 & m \, v_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
d_{jw2_om} &:= \begin{bmatrix} 0 & -m \cdot V_3 & m \cdot V_2 & m \cdot V_1 & 0 & 0 & 0 & 0 \\ m \cdot V_3 & 0 & -m \cdot V_1 & m \cdot V_2 & 0 & 0 & 0 & 0 \\ -m \cdot V_2 & m \cdot V_1 & 0 & m \cdot V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \cdot W_3 & J_2 \cdot W_2 & J_1 \cdot W_1 & 0 & -m \cdot V_3 & m \cdot V_2 & m \cdot V_1 \\ J_3 \cdot W_3 & 0 & -J_1 \cdot W_1 & J_2 \cdot W_2 & m \cdot V_3 & 0 & -m \cdot V_1 & m \cdot V_2 \\ -J_2 \cdot W_2 & J_1 \cdot W_1 & 0 & J_3 \cdot W_3 & -m \cdot V_2 & m \cdot V_1 & 0 & m \cdot V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{13}$$

$$djw2_om := \begin{bmatrix} 0 & -m V_3 & m V_2 & m V_1 & 0 & 0 & 0 & 0 \\ m V_3 & 0 & -m V_1 & m V_2 & 0 & 0 & 0 & 0 \\ -m V_2 & m V_1 & 0 & m V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 W_3 & J_2 W_2 & J_1 W_1 & 0 & -m V_3 & m V_2 & m V_1 \\ J_3 W_3 & 0 & -J_1 W_1 & J_2 W_2 & m V_3 & 0 & -m V_1 & m V_2 \\ -J_2 W_2 & J_1 W_1 & 0 & J_3 W_3 & -m V_2 & m V_1 & 0 & m V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$$qd_om := \begin{bmatrix} q_8 & q_7 & -q_6 & q_5 \\ -q_7 & q_8 & q_5 & q_6 \\ q_2 & -q_5 & q_8 & q_7 \\ -q_5 & -q_6 & -q_7 & q_8 \end{bmatrix}$$

$$qd_om := \begin{bmatrix} q_8 & q_7 & -q_6 & q_5 \\ -q_7 & q_8 & q_5 & q_6 \\ q_2 & -q_5 & q_8 & q_7 \\ -q_5 & -q_6 & -q_7 & q_8 \end{bmatrix} \quad (15)$$

$$rr := simplify \left(2. \left(qd_om. \begin{bmatrix} -q_1 \\ -q_2 \\ -q_3 \\ q_4 \end{bmatrix} \right) \right)$$

$$rr := (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) e_{x1} + (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) e_{x2} + ((-2. q_1 + 2. q_5) q_2 - 2. q_8 q_3 + 2. q_7 q_4) e_{x3} + (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4) e_{x4} \quad (16)$$

$$R := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4 \\ 2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4 \\ -2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4 \\ 2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4 \end{bmatrix}$$

$$R := (0)e_{x1} + (0)e_{x2} + (0)e_{x3} + (0)e_{x4} + (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4)e_{x5} \quad (17)$$

$$+ (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4)e_{x6} + (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4)e_{x7} + (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)e_{x8}$$

$$dw_dot := \text{Multiply}(dJ_inv, (dF - \text{Multiply}(dw3_omega, (\text{Multiply}(dJ, w_comb))) - \text{Multiply}(dJ, (\text{Multiply}(dw2_omega, dw))) - \text{Multiply}(dJ, \text{Multiply}(dw2_omega, (\text{Multiply}(dw2_omega, R))))))$$

$$dw_dot := \left[\left[\frac{1}{J_1} (-F_2 r_3 + F_3 r_2 - (-v_3 - V_3) m (v_2 + V_2) - (v_2 + V_2) m (v_3 + V_3) - (-w_3 - W_3) J_2 (w_2 + W_2) - (w_2 + W_2) J_3 (w_3 + W_3) - J_1 (W_2 w_3 - W_3 w_2) \right) \right. \right. \quad (18)$$

$$\left. \left[\frac{1}{J_2} (F_1 r_3 - F_3 r_1 - (v_3 + V_3) m (v_1 + V_1) - (-v_1 - V_1) m (v_3 + V_3) - (w_3 + W_3) J_1 (w_1 + W_1) - (-w_1 - W_1) J_3 (w_3 + W_3) - J_2 (-W_1 w_3 + W_3 w_1) \right) \right. \right.$$

$$\left. \left[\frac{1}{J_3} (-F_1 r_2 + F_2 r_1 - (-v_2 - V_2) m (v_1 + V_1) - (v_1 + V_1) m (v_2 + V_2) - (-w_2 - W_2) J_1 (w_1 + W_1) - (w_1 + W_1) J_2 (w_2 + W_2) - J_3 (W_1 w_2 - W_2 w_1) \right) \right. \right.$$

$$\left. \left[0. \right] \right]$$

$$\left[\frac{1}{m} (F_1 - (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) - m (V_2 w_3 - V_3 w_2 + W_2 v_3 - W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) + W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \right]$$

$$\begin{aligned}
& \left[\frac{1}{m} (F_2 - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 + V_3) - m (-V_1 w_3 \right. \\
& + V_3 w_1 - W_1 v_3 + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) \\
& + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& + 2. q_8 q_4)) - W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 \\
& - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \Big], \\
& \left[\frac{1}{m} (F_3 - (-w_2 - W_2) m (v_1 + V_1) - (w_1 + W_1) m (v_2 + V_2) - m (V_1 w_2 \right. \\
& - V_2 w_1 + W_1 v_2 - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) \\
& + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& + 2. q_8 q_4)) + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \Big], \\
& \left[0. \right] \\
dw_dotl := & \left[\left[\frac{1}{J_1} (-F_2 r_3 + F_3 r_2 - (v_3 + V_3) m (v_2 + V_2) - (v_2 + V_2) m (v_3 + V_3) - (-w_3 \right. \right. \\
& - W_3) J_2 (w_2 + W_2) - (w_2 + W_2) J_3 (w_3 + W_3) - J_1 (W_2 w_3 - W_3 w_2)) \Big], \\
& \left[\frac{1}{J_2} (F_1 r_3 - F_3 r_1 - (v_3 + V_3) m (v_1 + V_1) - (-v_1 - V_1) m (v_3 + V_3) - (w_3 \right. \\
& + W_3) J_1 (w_1 + W_1) - (-w_1 - W_1) J_3 (w_3 + W_3) - J_2 (-W_1 w_3 + W_3 w_1)) \Big], \\
& \left[\frac{1}{J_3} (-F_1 r_2 + F_2 r_1 - (v_2 + V_2) m (v_1 + V_1) - (v_1 + V_1) m (v_2 + V_2) - (-w_2 \right. \\
& - W_2) J_1 (w_1 + W_1) - (w_1 + W_1) J_2 (w_2 + W_2) - J_3 (W_1 w_2 - W_2 w_1)) \Big], \\
& \left[0. \right], \\
& \left[\frac{1}{m} (F_1 - (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) - m (V_2 w_3 - V_3 w_2 \right. \\
& + W_2 v_3 - W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (- \\
& -2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& + W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 \\
& + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \Big], \\
& \left[\frac{1}{m} (F_2 - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 + V_3) - m (-V_1 w_3 + V_3 w_1 \right. \\
& - W_1 v_3 + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1
\end{aligned}$$

$$\begin{aligned}
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) - W_1 (\\
& - W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 \\
& + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)))], \\
& \left[\frac{1}{m} (F_3 - (-w_2 - W_2) m (v_1 + V_1) - (w_1 + W_1) m (v_2 + V_2) - m (V_1 w_2 - V_2 w_1 \right. \\
& + W_1 v_2 - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (\\
& - 2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 \\
& + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \left. \right) \left. \right] \\
& \left[0. \right] \\
dw_dotl := & \left(\frac{1}{J_1} (-F_2 r_3 + F_3 r_2 - (-v_3 - V_3) m (v_2 + V_2) - (v_2 + V_2) m (v_3 + V_3) - (\right. \quad (19) \\
& - w_3 - W_3) J_2 (w_2 + W_2) - (w_2 + W_2) J_3 (w_3 + W_3) - J_1 (W_2 w_3 - W_3 w_2)) \left. \right) e_{x1} \\
& + \left(\frac{1}{J_2} (F_1 r_3 - F_3 r_1 - (v_3 + V_3) m (v_1 + V_1) - (-v_1 - V_1) m (v_3 + V_3) - (w_3 \right. \\
& + W_3) J_1 (w_1 + W_1) - (-w_1 - W_1) J_3 (w_3 + W_3) - J_2 (-W_1 w_3 + W_3 w_1)) \left. \right) e_{x2} \\
& + \left(\frac{1}{J_3} (-F_1 r_2 + F_2 r_1 - (-v_2 - V_2) m (v_1 + V_1) - (v_1 + V_1) m (v_2 + V_2) - (-w_2 \right. \\
& - W_2) J_1 (w_1 + W_1) - (w_1 + W_1) J_2 (w_2 + W_2) - J_3 (W_1 w_2 - W_2 w_1)) \left. \right) e_{x3} \\
& + (0.) e_{x4} + \left(\frac{1}{m} (F_1 - (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) \right. \\
& - m (V_2 w_3 - V_3 w_2 + W_2 v_3 - W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 \\
& + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 \\
& + 2. q_7 q_3 + 2. q_8 q_4)) + W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) \\
& + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& + 2. q_8 q_4)))) \left. \right) e_{x5} + \left(\frac{1}{m} (F_2 - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 \right. \\
& + V_3) - m (-V_1 w_3 + V_3 w_1 - W_1 v_3 + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 \\
& - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 \\
& + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) - W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 \\
& + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2
\end{aligned}$$

$$\begin{aligned}
& + 2. q_7 q_3 + 2. q_8 q_4))))) e_{x6} + \left(\frac{1}{m} (F_3 - (-w_2 - W_2) m (v_1 + V_1) - (w_1 \right. \\
& + W_1) m (v_2 + V_2) - m (V_1 w_2 - V_2 w_1 + W_1 v_2 - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 \\
& - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) \\
& + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 \\
& + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 \\
& + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))))) e_{x7} + (0.) e_{x8}
\end{aligned}$$

$$dw_dot - dw_dot l$$

$$(0) e_{x1} + (0) e_{x2} + (0) e_{x3} + (0.) e_{x4} + (0) e_{x5} + (0) e_{x6} + (0) e_{x7} + (0.) e_{x8} \quad (20)$$

with (VectorCalculus) :

$$m_dot := \frac{\partial}{\partial m} dw_dot$$

$$m_dot := \left(\frac{-(-v_3 - V_3) (v_2 + V_2) - (v_2 + V_2) (v_3 + V_3)}{J_1} \right) e_{x1} \quad (21)$$

$$+ \left(\frac{-(v_3 + V_3) (v_1 + V_1) - (-v_1 - V_1) (v_3 + V_3)}{J_2} \right) e_{x2}$$

$$+ \left(\frac{-(-v_2 - V_2) (v_1 + V_1) - (v_1 + V_1) (v_2 + V_2)}{J_3} \right) e_{x3} + (0) e_{x4} + \left(-\frac{1}{m^2} (F_1 \right.$$

$$- (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) - m (V_2 w_3 - V_3 w_2 + W_2 v_3$$

$$- W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1$$

$$+ 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))$$

$$+ W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2$$

$$- 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))))) + \frac{1}{m} ($$

$$- (-w_3 - W_3) (v_2 + V_2) - (w_2 + W_2) (v_3 + V_3) - V_2 w_3 + V_3 w_2 - W_2 v_3 + W_3 v_2$$

$$+ W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2$$

$$- 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) - W_2 (-W_2 ($$

$$- 2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3$$

$$+ 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))))) e_{x5} + \left(-\frac{1}{m^2} (F_2 \right.$$

$$\begin{aligned}
& - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 + V_3) - m (-V_1 w_3 + V_3 w_1 - W_1 v_3 \\
& + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& - W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 \\
& - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) + \frac{1}{m} (\\
& - (w_3 + W_3) (v_1 + V_1) - (-w_1 - W_1) (v_3 + V_3) + V_1 w_3 - V_3 w_1 + W_1 v_3 - W_3 v_1 \\
& - W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) + W_1 (-W_2 (\\
& -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 \\
& + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) e_{x6} + \left(-\frac{1}{m^2} (F_3 \right. \\
& - (-w_2 - W_2) m (v_1 + V_1) - (w_1 + W_1) m (v_2 + V_2) - m (V_1 w_2 - V_2 w_1 + W_1 v_2 \\
& - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) + \frac{1}{m} (\\
& - (-w_2 - W_2) (v_1 + V_1) - (w_1 + W_1) (v_2 + V_2) - V_1 w_2 + V_2 w_1 - W_1 v_2 + W_2 v_1 \\
& + W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) - W_1 (W_3 (\\
& -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 \\
& + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) e_{x7} + (0) e_{x8}
\end{aligned}$$

$$B := \text{map}(\text{diff}, dJ, m);$$

$$B := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

$$C := \text{map}(\text{diff}, dJ_{\text{inv}}, m);$$

$$C := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{m^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{m^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{m^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (23)$$

$$dd2 := B \bullet dw2$$

$$dd2 := (V_1)e_{x1} + (V_2)e_{x2} + (V_3)e_{x3} + (0)e_{x4} + (0)e_{x5} + (0)e_{x6} + (0)e_{x7} + (0)e_{x8} \quad (24)$$

$$dd_{om2} := \begin{bmatrix} 0 & -V_3 & V_2 & V_1 & 0 & 0 & 0 & 0 \\ V_3 & 0 & -V_1 & V_2 & 0 & 0 & 0 & 0 \\ -V_2 & V_1 & 0 & V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -V_3 & V_2 & V_1 \\ 0 & 0 & 0 & 0 & V_3 & 0 & -V_1 & V_2 \\ 0 & 0 & 0 & 0 & -V_2 & V_1 & 0 & V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$dd_om2 := \begin{bmatrix} 0 & -V_3 & V_2 & V_1 & 0 & 0 & 0 & 0 \\ V_3 & 0 & -V_1 & V_2 & 0 & 0 & 0 & 0 \\ -V_2 & V_1 & 0 & V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -V_3 & V_2 & V_1 \\ 0 & 0 & 0 & 0 & V_3 & 0 & -V_1 & V_2 \\ 0 & 0 & 0 & 0 & -V_2 & V_1 & 0 & V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (25)$$

$$phi_m1 := Multiply(C, (dF - Multiply(dw3_omega, (Multiply(dJ, w_comb))) - Multiply(dJ, (Multiply(dw2_omega, dw))) - Multiply(dJ, Multiply(dw2_omega, (Multiply(dw2_omega, R))))))$$

$$phi_m1 := \begin{bmatrix} 0. \end{bmatrix}, \quad (26)$$

$$\begin{bmatrix} 0. \end{bmatrix},$$

$$\begin{bmatrix} 0. \end{bmatrix},$$

$$\begin{bmatrix} 0. \end{bmatrix},$$

$$\begin{bmatrix} -\frac{1}{m^2} (F_1 - (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) - m (V_2 w_3 \\ - V_3 w_2 + W_2 v_3 - W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) \\ - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\ + 2. q_8 q_4)) + W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 \\ - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \end{bmatrix},$$

$$\begin{bmatrix} -\frac{1}{m^2} (F_2 - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 + V_3) - m (-V_1 w_3 \\ + V_3 w_1 - W_1 v_3 + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) \\ + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\ + 2. q_8 q_4)) - W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 \\ - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \end{bmatrix},$$

$$\begin{bmatrix} -\frac{1}{m^2} (F_3 - (-w_2 - W_2) m (v_1 + V_1) - (w_1 + W_1) m (v_2 + V_2) - m (V_1 w_2$$

$$\begin{aligned}
& -V_2 w_1 + W_1 v_2 - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) \\
& + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& + 2. q_8 q_4)) + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \Big], \\
& \Big[0. \Big]
\end{aligned}$$

$$\begin{aligned}
\text{phi_m2} := & \text{Multiply}(dJ_inv, (- \text{Multiply}(dw3_omega, (\text{Multiply}(B, w_comb))) - \text{Multiply}(B, \\
& (\text{Multiply}(dw2_omega, dw))) - \text{Multiply}(B, \text{Multiply}(dw2_omega, (\text{Multiply}(dw2_omega, \\
& R))))))
\end{aligned}$$

$$\text{phi_m2} := \left[\left[\frac{-(-v_3 - V_3)(v_2 + V_2) - (v_2 + V_2)(v_3 + V_3)}{J_1} \right], \right. \tag{27}$$

$$\left[\frac{-(v_3 + V_3)(v_1 + V_1) - (-v_1 - V_1)(v_3 + V_3)}{J_2} \right],$$

$$\left[\frac{-(-v_2 - V_2)(v_1 + V_1) - (v_1 + V_1)(v_2 + V_2)}{J_3} \right],$$

$$\left[0. \right],$$

$$\begin{aligned}
& \left[\frac{1}{m} (-(-w_3 - W_3)(v_2 + V_2) - (w_2 + W_2)(v_3 + V_3) - V_2 w_3 + V_3 w_2 - W_2 v_3 \right. \\
& + W_3 v_2 + W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& - W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 \\
& - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \Big],
\end{aligned}$$

$$\begin{aligned}
& \left[\frac{1}{m} (- (w_3 + W_3)(v_1 + V_1) - (-w_1 - W_1)(v_3 + V_3) + V_1 w_3 - V_3 w_1 + W_1 v_3 \right. \\
& - W_3 v_1 - W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& + W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 \\
& - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \Big],
\end{aligned}$$

$$\begin{aligned}
& \left[\frac{1}{m} (-(-w_2 - W_2)(v_1 + V_1) - (w_1 + W_1)(v_2 + V_2) - V_1 w_2 + V_2 w_1 - W_1 v_2 \right. \\
& + W_2 v_1 + W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& - W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2
\end{aligned}$$

$$\begin{aligned}
& -2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)))]], \\
& \left[0. \right] \\
\text{phi_m} &:= \text{phi_m1} + \text{phi_m2} \\
\text{phi_m} &:= \left(\frac{-(-v_3 - V_3) (v_2 + V_2) - (v_2 + V_2) (v_3 + V_3)}{J_1} \right) e_{x1} \\
& + \left(\frac{-(v_3 + V_3) (v_1 + V_1) - (-v_1 - V_1) (v_3 + V_3)}{J_2} \right) e_{x2} \\
& + \left(\frac{-(-v_2 - V_2) (v_1 + V_1) - (v_1 + V_1) (v_2 + V_2)}{J_3} \right) e_{x3} + (0.) e_{x4} + \left(-\frac{1}{m^2} (F_1 \right. \\
& - (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) - m (V_2 w_3 - V_3 w_2 + W_2 v_3 \\
& - W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& + W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 \\
& - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) + \frac{1}{m} (\\
& -(-w_3 - W_3) (v_2 + V_2) - (w_2 + W_2) (v_3 + V_3) - V_2 w_3 + V_3 w_2 - W_2 v_3 + W_3 v_2 \\
& + W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) - W_2 (-W_2 (\\
& -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 \\
& + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) e_{x5} + \left(-\frac{1}{m^2} (F_2 \right. \\
& - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 + V_3) - m (-V_1 w_3 + V_3 w_1 - W_1 v_3 \\
& + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& - W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 \\
\end{aligned}
\tag{28}$$

$$\begin{aligned}
& -2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) + \frac{1}{m} (\\
& - (w_3 + W_3) (v_1 + V_1) - (-w_1 - W_1) (v_3 + V_3) + V_1 w_3 - V_3 w_1 + W_1 v_3 - W_3 v_1 \\
& - W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) + W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 \\
& + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) e_{x6} + \left(-\frac{1}{m^2} (F_3 \right. \\
& - (-w_2 - W_2) m (v_1 + V_1) - (w_1 + W_1) m (v_2 + V_2) - m (V_1 w_2 - V_2 w_1 + W_1 v_2 \\
& - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 \\
& + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) \\
& + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) + \frac{1}{m} (\\
& - (-w_2 - W_2) (v_1 + V_1) - (w_1 + W_1) (v_2 + V_2) - V_1 w_2 + V_2 w_1 - W_1 v_2 + W_2 v_1 \\
& + W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_2 (-2. q_2 q_1 + 2. q_5 q_2 \\
& - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4)) - W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 \\
& + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) e_{x7} + (0.) e_{x8}
\end{aligned}$$

$$check1 := m_dot - phi_m$$

$$check1 := (0) e_{x1} + (0) e_{x2} + (0) e_{x3} + (0.) e_{x4} + (0) e_{x5} + (0) e_{x6} + (0) e_{x7} + (0.) e_{x8} \quad (29)$$