

$$qc_cross := \begin{bmatrix} q_4 & -q_3 & q_2 & -q_1 \\ q_3 & q_4 & -q_1 & -q_2 \\ -q_2 & q_1 & q_4 & -q_3 \\ q_1 & q_2 & q_3 & q_4 \end{bmatrix}$$

$$qc_cross := \begin{bmatrix} q_4 & -q_3 & q_2 & -q_1 \\ q_3 & q_4 & -q_1 & -q_2 \\ -q_2 & q_1 & q_4 & -q_3 \\ q_1 & q_2 & q_3 & q_4 \end{bmatrix} \quad (1)$$

$$qq := \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$qq := \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \quad (2)$$

$$TT := \begin{bmatrix} 0 & T_3 & -T_2 & T_1 \\ -T_3 & 0 & T_1 & T_2 \\ T_2 & -T_1 & 0 & T_3 \\ -T_1 & -T_2 & -T_3 & 0 \end{bmatrix}$$

$$TT := \begin{bmatrix} 0 & T_3 & -T_2 & T_1 \\ -T_3 & 0 & T_1 & T_2 \\ T_2 & -T_1 & 0 & T_3 \\ -T_1 & -T_2 & -T_3 & 0 \end{bmatrix} \quad (3)$$

$$p := \frac{1}{m} . \sim ((qc_cross \bullet TT) . qq)$$

$$p := \left[\left[\frac{1}{m} \left((T_1 q_1 + T_2 q_2 + T_3 q_3) q_1 + (-T_1 q_2 + T_2 q_1 + T_3 q_4) q_2 + (-T_1 q_3 - T_2 q_4 \right. \right. \right. \quad (4)$$

$$\begin{aligned}
& + T_3 q_1) q_3 + (T_1 q_4 - T_2 q_3 + T_3 q_2) q_4) \Big], \\
& \left[\frac{1}{m} ((T_1 q_2 - T_2 q_1 - T_3 q_4) q_1 + (T_1 q_1 + T_2 q_2 + T_3 q_3) q_2 + (T_1 q_4 - T_2 q_3 \right. \\
& \quad \left. + T_3 q_2) q_3 + (T_1 q_3 + T_2 q_4 - T_3 q_1) q_4) \Big], \\
& \left[\frac{1}{m} ((T_1 q_3 + T_2 q_4 - T_3 q_1) q_1 + (-T_1 q_4 + T_2 q_3 - T_3 q_2) q_2 + (T_1 q_1 + T_2 q_2 \right. \\
& \quad \left. + T_3 q_3) q_3 + (-T_1 q_2 + T_2 q_1 + T_3 q_4) q_4) \Big], \\
& \left[\frac{1}{m} ((-T_1 q_4 + T_2 q_3 - T_3 q_2) q_1 + (-T_1 q_3 - T_2 q_4 + T_3 q_1) q_2 + (T_1 q_2 - T_2 q_1 \right. \\
& \quad \left. - T_3 q_4) q_3 + (T_1 q_1 + T_2 q_2 + T_3 q_3) q_4) \Big] \right] \\
p &:= \text{simplify}(p) \\
p &:= \begin{bmatrix} \frac{(q_1^2 - q_2^2 - q_3^2 + q_4^2) T_1 + (2 T_2 q_2 + 2 T_3 q_3) q_1 - 2 q_4 (T_2 q_3 - T_3 q_2)}{m} \\ \frac{(-q_1^2 + q_2^2 - q_3^2 + q_4^2) T_2 + (2 T_1 q_2 - 2 T_3 q_4) q_1 + 2 q_3 (T_1 q_4 + T_3 q_2)}{m} \\ \frac{(-q_1^2 - q_2^2 + q_3^2 + q_4^2) T_3 + (2 T_1 q_3 + 2 T_2 q_4) q_1 - 2 q_2 (T_1 q_4 - T_2 q_3)}{m} \\ 0 \end{bmatrix} \tag{5}
\end{aligned}$$

with (VectorCalculus) :

$$\begin{aligned}
s &:= \left\langle \frac{(q_1^2 - q_2^2 - q_3^2 + q_4^2) T_1 + (2 T_2 q_2 + 2 T_3 q_3) q_1 - 2 q_4 (T_2 q_3 - T_3 q_2)}{m}, \right. \\
& \quad \frac{(-q_1^2 + q_2^2 - q_3^2 + q_4^2) T_2 + (2 T_1 q_2 - 2 T_3 q_4) q_1 + 2 q_3 (T_1 q_4 + T_3 q_2)}{m}, \\
& \quad \left. \frac{(-q_1^2 - q_2^2 + q_3^2 + q_4^2) T_3 + (2 T_1 q_3 + 2 T_2 q_4) q_1 - 2 q_2 (T_1 q_4 - T_2 q_3)}{m}, 0 \right\rangle \\
s &:= \left(\frac{(q_1^2 - q_2^2 - q_3^2 + q_4^2) T_1 + (2 T_2 q_2 + 2 T_3 q_3) q_1 - 2 q_4 (T_2 q_3 - T_3 q_2)}{m} \right) e_{x1} \\
& + \left(\frac{(-q_1^2 + q_2^2 - q_3^2 + q_4^2) T_2 + (2 T_1 q_2 - 2 T_3 q_4) q_1 + 2 q_3 (T_1 q_4 + T_3 q_2)}{m} \right) e_{x2} \\
& + \left(\frac{(-q_1^2 - q_2^2 + q_3^2 + q_4^2) T_3 + (2 T_1 q_3 + 2 T_2 q_4) q_1 - 2 q_2 (T_1 q_4 - T_2 q_3)}{m} \right) e_{x3} \\
& + (0) e_{x4} \tag{6}
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial q_1} s \\
& \left(\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m} \right) e_{x1} + \left(\frac{2 T_1 q_2 - 2 T_2 q_1 - 2 T_3 q_4}{m} \right) e_{x2} \\
& + \left(\frac{2 T_1 q_3 + 2 T_2 q_4 - 2 T_3 q_1}{m} \right) e_{x3} + (0) e_{x4}
\end{aligned} \tag{7}$$

$$\begin{aligned}
& \frac{\partial}{\partial q_2} s \\
& \left(\frac{-2 T_1 q_2 + 2 T_2 q_1 + 2 T_3 q_4}{m} \right) e_{x1} + \left(\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m} \right) e_{x2} \\
& + \left(\frac{-2 T_1 q_4 + 2 T_2 q_3 - 2 T_3 q_2}{m} \right) e_{x3} + (0) e_{x4}
\end{aligned} \tag{8}$$

$$\begin{aligned}
& \frac{\partial}{\partial q_3} s \\
& \left(\frac{-2 T_1 q_3 - 2 T_2 q_4 + 2 T_3 q_1}{m} \right) e_{x1} + \left(\frac{2 T_1 q_4 - 2 T_2 q_3 + 2 T_3 q_2}{m} \right) e_{x2} \\
& + \left(\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m} \right) e_{x3} + (0) e_{x4}
\end{aligned} \tag{9}$$

$$\begin{aligned}
& \frac{\partial}{\partial q_4} s \\
& \left(\frac{2 T_1 q_4 - 2 T_2 q_3 + 2 T_3 q_2}{m} \right) e_{x1} + \left(\frac{2 T_1 q_3 + 2 T_2 q_4 - 2 T_3 q_1}{m} \right) e_{x2} \\
& + \left(\frac{-2 T_1 q_2 + 2 T_2 q_1 + 2 T_3 q_4}{m} \right) e_{x3} + (0) e_{x4}
\end{aligned} \tag{10}$$

$$\begin{aligned}
& \frac{\partial}{\partial T_1} s \\
& \left(\frac{q_1^2 - q_2^2 - q_3^2 + q_4^2}{m} \right) e_{x1} + \left(\frac{2 q_1 q_2 + 2 q_3 q_4}{m} \right) e_{x2} + \left(\frac{2 q_1 q_3 - 2 q_2 q_4}{m} \right) e_{x3} + (0) e_{x4}
\end{aligned} \tag{11}$$

$$\begin{aligned}
& \frac{\partial}{\partial T_2} s \\
& \left(\frac{2 q_1 q_2 - 2 q_3 q_4}{m} \right) e_{x1} + \left(\frac{-q_1^2 + q_2^2 - q_3^2 + q_4^2}{m} \right) e_{x2} + \left(\frac{2 q_4 q_1 + 2 q_2 q_3}{m} \right) e_{x3} + (0) e_{x4}
\end{aligned} \tag{12}$$

$$\frac{\partial}{\partial T_3} s$$

$$\left(\frac{2 q_1 q_3 + 2 q_2 q_4}{m} \right) e_{x1} + \left(\frac{-2 q_4 q_1 + 2 q_2 q_3}{m} \right) e_{x2} + \left(\frac{-q_1^2 - q_2^2 + q_3^2 + q_4^2}{m} \right) e_{x3} + (0) e_{x4} \quad (13)$$

$$\frac{\partial}{\partial m} s$$

$$\left(- \frac{(q_1^2 - q_2^2 - q_3^2 + q_4^2) T_1 + (2 T_2 q_2 + 2 T_3 q_3) q_1 - 2 q_4 (T_2 q_3 - T_3 q_2)}{m^2} \right) e_{x1} + \left(\right. \quad (14)$$

$$- \frac{(-q_1^2 + q_2^2 - q_3^2 + q_4^2) T_2 + (2 T_1 q_2 - 2 T_3 q_4) q_1 + 2 q_3 (T_1 q_4 + T_3 q_2)}{m^2} \left. \right) e_{x2} + \left(\right.$$

$$- \frac{(-q_1^2 - q_2^2 + q_3^2 + q_4^2) T_3 + (2 T_1 q_3 + 2 T_2 q_4) q_1 - 2 q_2 (T_1 q_4 - T_2 q_3)}{m^2} \left. \right) e_{x3}$$

$$+ (0) e_{x4}$$

$$JJ := \left[\left[\left(\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m} \right), \left(\frac{-2 T_1 q_2 + 2 T_2 q_1 + 2 T_3 q_4}{m} \right), \right. \right.$$

$$\left. \left(\frac{-2 T_1 q_3 - 2 T_2 q_4 + 2 T_3 q_1}{m} \right), \left(\frac{2 T_1 q_4 - 2 T_2 q_3 + 2 T_3 q_2}{m} \right) \right],$$

$$\left[\left(\frac{2 T_1 q_2 - 2 T_2 q_1 - 2 T_3 q_4}{m} \right), \left(\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m} \right), \right.$$

$$\left(\frac{2 T_1 q_4 - 2 T_2 q_3 + 2 T_3 q_2}{m} \right), \left(\frac{2 T_1 q_3 + 2 T_2 q_4 - 2 T_3 q_1}{m} \right) \right],$$

$$\left[\left(\frac{2 T_1 q_3 + 2 T_2 q_4 - 2 T_3 q_1}{m} \right), \left(\frac{-2 T_1 q_4 + 2 T_2 q_3 - 2 T_3 q_2}{m} \right), \right.$$

$$\left. \left(\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m} \right), \left(\frac{-2 T_1 q_2 + 2 T_2 q_1 + 2 T_3 q_4}{m} \right) \right],$$

$$\left[0, 0, 0, 0 \right] \right]$$

$$JJ := \left[\left[\frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m}, \frac{-2 T_1 q_2 + 2 T_2 q_1 + 2 T_3 q_4}{m}, \right. \right. \quad (15)$$

$$\frac{-2 T_1 q_3 - 2 T_2 q_4 + 2 T_3 q_1}{m}, \frac{2 T_1 q_4 - 2 T_2 q_3 + 2 T_3 q_2}{m} \left. \right],$$

$$\left[\frac{2 T_1 q_2 - 2 T_2 q_1 - 2 T_3 q_4}{m}, \frac{2 T_1 q_1 + 2 T_2 q_2 + 2 T_3 q_3}{m}, \right.$$

$$\frac{2 T_1 q_4 - 2 T_2 q_3 + 2 T_3 q_2}{m}, \frac{2 T_1 q_3 + 2 T_2 q_4 - 2 T_3 q_1}{m} \left. \right],$$

$$\left[\frac{2 T_1 q_3 + 2 T_2 q_4 - 2 T_3 q_1}{m}, \frac{-2 T_1 q_4 + 2 T_2 q_3 - 2 T_3 q_2}{m}, \right.$$

$$\frac{2\,T_1\,q_1+2\,T_2\,q_2+2\,T_3\,q_3}{m},\frac{-2\,T_1\,q_2+2\,T_2\,q_1+2\,T_3\,q_4}{m}\Bigg],$$

$$\Bigg[0,0,0,0\Bigg]$$