$$qq := \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$qq := (q_1)e_{x1} + (q_2)e_{x2} + (q_3)e_{x3} + (q_4)e_{x4} \tag{1}$$

$$ww := \begin{bmatrix} 0 & w_3 & -w_2 & w_1 \\ -w_3 & 0 & w_1 & w_2 \\ w_2 & -w_1 & 0 & w_3 \\ -w_1 & -w_2 & -w_3 & 0 \end{bmatrix}$$

$$ww := \begin{bmatrix} 0 & w_3 & -w_2 & w_1 \\ -w_3 & 0 & w_1 & w_2 \\ w_2 & -w_1 & 0 & w_3 \\ -w_1 & -w_2 & -w_3 & 0 \end{bmatrix} \tag{2}$$

$$dq := \frac{1}{2} \cdot \sim ww \bullet qq$$

$$dq := \left(\frac{w_3\, q_2}{2} - \frac{w_2\, q_3}{2} + \frac{w_1\, q_4}{2} \right) e_{x1} + \left(-\frac{w_3\, q_1}{2} + \frac{w_1\, q_3}{2} + \frac{w_2\, q_4}{2} \right) e_{x2} + \left(\frac{w_2\, q_1}{2} - \frac{w_1\, q_2}{2} + \frac{w_3\, q_4}{2} \right) e_{x3} + \left(-\frac{w_1\, q_1}{2} - \frac{w_2\, q_2}{2} - \frac{w_3\, q_3}{2} \right) e_{x4} \tag{3}$$

$$\frac{\partial}{\partial w_1} dq$$

$$\left(\frac{q_4}{2} \right) e_{x1} + \left(\frac{q_3}{2} \right) e_{x2} + \left(-\frac{q_2}{2} \right) e_{x3} + \left(-\frac{q_1}{2} \right) e_{x4} \tag{4}$$

$$\frac{\partial}{\partial w_2} dq$$

$$\left(-\frac{q_3}{2} \right) e_{x1} + \left(\frac{q_4}{2} \right) e_{x2} + \left(\frac{q_1}{2} \right) e_{x3} + \left(-\frac{q_2}{2} \right) e_{x4} \tag{5}$$

$$\frac{\partial}{\partial w_3} dq$$

$$\left(\frac{q_2}{2} \right) e_{x1} + \left(-\frac{q_1}{2} \right) e_{x2} + \left(\frac{q_4}{2} \right) e_{x3} + \left(-\frac{q_3}{2} \right) e_{x4} \tag{6}$$

$$\frac{\partial}{\partial q_1} dq$$

$$(0)e_{x1} + \left(-\frac{w_3}{2}\right)e_{x2} + \left(\frac{w_2}{2}\right)e_{x3} + \left(-\frac{w_1}{2}\right)e_{x4} \tag{7}$$

$$\frac{\partial}{\partial q_2}dq$$

$$\left(\frac{w_3}{2}\right)e_{x1} + (0)e_{x2} + \left(-\frac{w_1}{2}\right)e_{x3} + \left(-\frac{w_2}{2}\right)e_{x4} \tag{8}$$

$$\frac{\partial}{\partial q_3}dq$$

$$\left(-\frac{w_2}{2}\right)e_{x1} + \left(\frac{w_1}{2}\right)e_{x2} + (0)e_{x3} + \left(-\frac{w_3}{2}\right)e_{x4} \tag{9}$$

$$\frac{\partial}{\partial q_4}dq$$

$$\left(\frac{w_1}{2}\right)e_{x1} + \left(\frac{w_2}{2}\right)e_{x2} + \left(\frac{w_3}{2}\right)e_{x3} + (0)e_{x4} \tag{10}$$