

with (*LinearAlgebra*) :

$$dJ := \begin{bmatrix} 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ J_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$dJ := \begin{bmatrix} 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ J_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

$dJ_inv := \text{MatrixInverse}(dJ)$

$$dJ_inv := \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{J_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{J_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{J_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

$$dw := \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ 0 \\ v_1 \\ v_2 \\ v_3 \\ 0 \end{bmatrix}$$

$$dw := \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ 0 \\ v_1 \\ v_2 \\ v_3 \\ 0 \end{bmatrix} \tag{3}$$

$$rf := \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$$rf := \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{4}$$

$$Ft := \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$Ft := \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{5}$$

$$T := CrossProduct(rf, Ft)$$

$$T := \begin{bmatrix} -F_2\,r_3 + F_3\,r_2 \\ F_1\,r_3 - F_3\,r_1 \\ -F_1\,r_2 + F_2\,r_1 \end{bmatrix} \tag{6}$$

$$dF := \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ T_1 \\ T_2 \\ T_3 \\ 0 \end{bmatrix}$$

$$dF := \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ -F_2\,r_3 + F_3\,r_2 \\ F_1\,r_3 - F_3\,r_1 \\ -F_1\,r_2 + F_2\,r_1 \\ 0 \end{bmatrix} \tag{7}$$

$$dw2 := \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ 0 \\ V_1 \\ V_2 \\ V_3 \\ 0 \end{bmatrix}$$

$$dw2 := \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ 0 \\ V_1 \\ V_2 \\ V_3 \\ 0 \end{bmatrix} \tag{8}$$

$$w_comb := dw + dw2$$

$$w_comb := \begin{bmatrix} w_1 + W_1 \\ w_2 + W_2 \\ w_3 + W_3 \\ 0 \\ v_1 + V_1 \\ v_2 + V_2 \\ v_3 + V_3 \\ 0 \end{bmatrix} \tag{9}$$

$$dw_omega := \begin{bmatrix} 0 & -w_3 & w_2 & w_1 & 0 & 0 & 0 & 0 \\ w_3 & 0 & -w_1 & w_2 & 0 & 0 & 0 & 0 \\ -w_2 & w_1 & 0 & w_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -v_3 & v_2 & v_1 & 0 & -w_3 & w_2 & w_1 \\ v_3 & 0 & -v_1 & v_2 & w_3 & 0 & -w_1 & w_2 \\ -v_2 & v_1 & 0 & v_3 & -w_2 & w_1 & 0 & w_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$dw_omega := \begin{bmatrix} 0 & -w_3 & w_2 & w_1 & 0 & 0 & 0 & 0 \\ w_3 & 0 & -w_1 & w_2 & 0 & 0 & 0 & 0 \\ -w_2 & w_1 & 0 & w_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -v_3 & v_2 & v_1 & 0 & -w_3 & w_2 & w_1 \\ v_3 & 0 & -v_1 & v_2 & w_3 & 0 & -w_1 & w_2 \\ -v_2 & v_1 & 0 & v_3 & -w_2 & w_1 & 0 & w_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

$$dw2_omega := \begin{bmatrix} 0 & -W_3 & W_2 & W_1 & 0 & 0 & 0 & 0 \\ W_3 & 0 & -W_1 & W_2 & 0 & 0 & 0 & 0 \\ -W_2 & W_1 & 0 & W_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -V_3 & V_2 & V_1 & 0 & -W_3 & W_2 & W_1 \\ V_3 & 0 & -V_1 & V_2 & W_3 & 0 & -W_1 & W_2 \\ -V_2 & V_1 & 0 & V_3 & -W_2 & W_1 & 0 & W_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$dw2_omega := \begin{bmatrix} 0 & -W_3 & W_2 & W_1 & 0 & 0 & 0 & 0 \\ W_3 & 0 & -W_1 & W_2 & 0 & 0 & 0 & 0 \\ -W_2 & W_1 & 0 & W_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -V_3 & V_2 & V_1 & 0 & -W_3 & W_2 & W_1 \\ V_3 & 0 & -V_1 & V_2 & W_3 & 0 & -W_1 & W_2 \\ -V_2 & V_1 & 0 & V_3 & -W_2 & W_1 & 0 & W_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

$$dw3_omega := \begin{bmatrix} [0, -(w_3 + W_3), (w_2 + W_2), (w_1 + W_1), 0, 0, 0, 0], \\ [(w_3 + W_3), 0, -(w_1 + W_1), (w_2 + W_2), 0, 0, 0, 0], \\ [-(w_2 + W_2), (w_1 + W_1), 0, (w_3 + W_3), 0, 0, 0, 0], \\ [0, 0, 0, 0, 0, 0, 0, 0], \\ [0, -(v_3 + V_3), (v_2 + V_2), (v_1 + V_1), 0, -(w_3 + W_3), (w_2 + W_2), (w_1 + W_1)], \\ [(v_3 + V_3), 0, -(v_1 + V_1), (v_2 + V_2), (w_3 + W_3), 0, -(w_1 + W_1), (w_2 + W_2)] \end{bmatrix}$$

$$\begin{bmatrix} -(v_2 + V_2), (v_1 + V_1), 0, (v_3 + V_3), -(w_2 + W_2), (w_1 + W_1), 0, (w_3 + W_3) \\ [0, 0, 0, 0, 0, 0, 0, 0] \end{bmatrix}$$

$dw3_omega :=$

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$$\begin{aligned} & [0, -w_3 - W_3, w_2 + W_2, w_1 + W_1, 0, 0, 0, 0], \\ & [w_3 + W_3, 0, -w_1 - W_1, w_2 + W_2, 0, 0, 0, 0], \\ & [-w_2 - W_2, w_1 + W_1, 0, w_3 + W_3, 0, 0, 0, 0], \\ & [0, 0, 0, 0, 0, 0, 0, 0], \\ & [0, -v_3 - V_3, v_2 + V_2, v_1 + V_1, 0, -w_3 - W_3, w_2 + W_2, w_1 + W_1], \\ & [v_3 + V_3, 0, -v_1 - V_1, v_2 + V_2, w_3 + W_3, 0, -w_1 - W_1, w_2 + W_2], \\ & [-v_2 - V_2, v_1 + V_1, 0, v_3 + V_3, -w_2 - W_2, w_1 + W_1, 0, w_3 + W_3], \\ & [0, 0, 0, 0, 0, 0, 0, 0] \end{aligned}$$

$$djw_om := \begin{bmatrix} 0 & -m \cdot v_3 & m \cdot v_2 & m \cdot v_1 & 0 & 0 & 0 & 0 \\ m \cdot v_3 & 0 & -m \cdot v_1 & m \cdot v_2 & 0 & 0 & 0 & 0 \\ -m \cdot v_2 & m \cdot v_1 & 0 & m \cdot v_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \cdot w_3 & J_2 \cdot w_2 & J_1 \cdot w_1 & 0 & -m \cdot v_3 & m \cdot v_2 & m \cdot v_1 \\ J_3 \cdot w_3 & 0 & -J_1 \cdot w_1 & J_2 \cdot w_2 & m \cdot v_3 & 0 & -m \cdot v_1 & m \cdot v_2 \\ -J_2 \cdot w_2 & J_1 \cdot w_1 & 0 & J_3 \cdot w_3 & -m \cdot v_2 & m \cdot v_1 & 0 & m \cdot v_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$djw_om := \begin{bmatrix} 0 & -m \cdot v_3 & m \cdot v_2 & m \cdot v_1 & 0 & 0 & 0 & 0 \\ m \cdot v_3 & 0 & -m \cdot v_1 & m \cdot v_2 & 0 & 0 & 0 & 0 \\ -m \cdot v_2 & m \cdot v_1 & 0 & m \cdot v_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \cdot w_3 & J_2 \cdot w_2 & J_1 \cdot w_1 & 0 & -m \cdot v_3 & m \cdot v_2 & m \cdot v_1 \\ J_3 \cdot w_3 & 0 & -J_1 \cdot w_1 & J_2 \cdot w_2 & m \cdot v_3 & 0 & -m \cdot v_1 & m \cdot v_2 \\ -J_2 \cdot w_2 & J_1 \cdot w_1 & 0 & J_3 \cdot w_3 & -m \cdot v_2 & m \cdot v_1 & 0 & m \cdot v_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{aligned}
djw2_om &:= \begin{bmatrix} 0 & -m \cdot V_3 & m \cdot V_2 & m \cdot V_1 & 0 & 0 & 0 & 0 \\ m \cdot V_3 & 0 & -m \cdot V_1 & m \cdot V_2 & 0 & 0 & 0 & 0 \\ -m \cdot V_2 & m \cdot V_1 & 0 & m \cdot V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \cdot W_3 & J_2 \cdot W_2 & J_1 \cdot W_1 & 0 & -m \cdot V_3 & m \cdot V_2 & m \cdot V_1 \\ J_3 \cdot W_3 & 0 & -J_1 \cdot W_1 & J_2 \cdot W_2 & m \cdot V_3 & 0 & -m \cdot V_1 & m \cdot V_2 \\ -J_2 \cdot W_2 & J_1 \cdot W_1 & 0 & J_3 \cdot W_3 & -m \cdot V_2 & m \cdot V_1 & 0 & m \cdot V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
djw2_om &:= \begin{bmatrix} 0 & -m \cdot V_3 & m \cdot V_2 & m \cdot V_1 & 0 & 0 & 0 & 0 \\ m \cdot V_3 & 0 & -m \cdot V_1 & m \cdot V_2 & 0 & 0 & 0 & 0 \\ -m \cdot V_2 & m \cdot V_1 & 0 & m \cdot V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -J_3 \cdot W_3 & J_2 \cdot W_2 & J_1 \cdot W_1 & 0 & -m \cdot V_3 & m \cdot V_2 & m \cdot V_1 \\ J_3 \cdot W_3 & 0 & -J_1 \cdot W_1 & J_2 \cdot W_2 & m \cdot V_3 & 0 & -m \cdot V_1 & m \cdot V_2 \\ -J_2 \cdot W_2 & J_1 \cdot W_1 & 0 & J_3 \cdot W_3 & -m \cdot V_2 & m \cdot V_1 & 0 & m \cdot V_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}
\end{aligned}$$

$$\begin{aligned}
qd_om &:= \begin{bmatrix} q_8 & q_7 & -q_6 & q_5 \\ -q_7 & q_8 & q_5 & q_6 \\ q_2 & -q_5 & q_8 & q_7 \\ -q_5 & -q_6 & -q_7 & q_8 \end{bmatrix} \\
qd_om &:= \begin{bmatrix} q_8 & q_7 & -q_6 & q_5 \\ -q_7 & q_8 & q_5 & q_6 \\ q_2 & -q_5 & q_8 & q_7 \\ -q_5 & -q_6 & -q_7 & q_8 \end{bmatrix} \tag{15}
\end{aligned}$$

$$rr := simplify \left(2. \left(qd_om. \begin{bmatrix} -q_1 \\ -q_2 \\ -q_3 \\ q_4 \end{bmatrix} \right) \right)$$

$$rr := \begin{bmatrix} -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4 \\ 2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4 \\ (-2. q_1 + 2. q_5) q_2 - 2. q_8 q_3 + 2. q_7 q_4 \\ 2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4 \end{bmatrix} \quad (16)$$

$$R := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4 \\ 2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4 \\ -2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4 \\ 2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4 \end{bmatrix}$$

$$R := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4 \\ 2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4 \\ -2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4 \\ 2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4 \end{bmatrix} \quad (17)$$

$$dw_dot := \text{Multiply}(dJ_inv, (dF - \text{Multiply}(dw3_omega, (\text{Multiply}(dJ, w_comb))) - \text{Multiply}(dJ, (\text{Multiply}(dw2_omega, dw)) - \text{Multiply}(dJ, \text{Multiply}(dw2_omega, (\text{Multiply}(dw2_omega, R)))))))$$

$$dw_dot := \left[\left[\frac{1}{J_1} (-F_2 r_3 + F_3 r_2 - (-v_3 - V_3) m (v_2 + V_2) - (v_2 + V_2) m (v_3 + V_3) - (-w_3 - W_3) J_2 (w_2 + W_2) - (w_2 + W_2) J_3 (w_3 + W_3) - J_1 (W_2 w_3 - W_3 w_2)) \right] \right. \quad (18)$$

$$\left. \left[\frac{1}{J_2} (F_1 r_3 - F_3 r_1 - (v_3 + V_3) m (v_1 + V_1) - (-v_1 - V_1) m (v_3 + V_3) - (w_3 + W_3) J_1 (w_1 + W_1) - (-w_1 - W_1) J_3 (w_3 + W_3) - J_2 (-W_1 w_3 + W_3 w_1)) \right] \right]$$

$$\begin{aligned}
& \left[\frac{1}{J_3} (-F_1 r_2 + F_2 r_1 - (-v_2 - V_2) m (v_1 + V_1) - (v_1 + V_1) m (v_2 + V_2) - (-w_2 \right. \\
& \quad \left. - W_2) J_1 (w_1 + W_1) - (w_1 + W_1) J_2 (w_2 + W_2) - J_3 (W_1 w_2 - W_2 w_1)) \right], \\
& \left[0. \right], \\
& \left[\frac{1}{m} (F_1 - (-w_3 - W_3) m (v_2 + V_2) - (w_2 + W_2) m (v_3 + V_3) - m (V_2 w_3 \right. \\
& \quad - V_3 w_2 + W_2 v_3 - W_3 v_2) - m (-W_3 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) \\
& \quad - W_1 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& \quad + 2. q_8 q_4)) + W_2 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 \\
& \quad - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \right], \\
& \left[\frac{1}{m} (F_2 - (w_3 + W_3) m (v_1 + V_1) - (-w_1 - W_1) m (v_3 + V_3) - m (-V_1 w_3 \right. \\
& \quad + V_3 w_1 - W_1 v_3 + W_3 v_1) - m (W_3 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) \\
& \quad + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& \quad + 2. q_8 q_4)) - W_1 (-W_2 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) + W_1 (2. q_7 q_1 \\
& \quad - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) + W_3 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \right], \\
& \left[\frac{1}{m} (F_3 - (-w_2 - W_2) m (v_1 + V_1) - (w_1 + W_1) m (v_2 + V_2) - m (V_1 w_2 \right. \\
& \quad - V_2 w_1 + W_1 v_2 - W_2 v_1) - m (-W_2 (-W_3 (2. q_7 q_1 - 2. q_8 q_2 - 2. q_5 q_3 + 2. q_6 q_4) \\
& \quad + W_2 (-2. q_2 q_1 + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_1 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 \\
& \quad + 2. q_8 q_4)) + W_1 (W_3 (-2. q_8 q_1 - 2. q_7 q_2 + 2. q_6 q_3 + 2. q_5 q_4) - W_1 (-2. q_2 q_1 \\
& \quad + 2. q_5 q_2 - 2. q_8 q_3 + 2. q_7 q_4) + W_2 (2. q_5 q_1 + 2. q_6 q_2 + 2. q_7 q_3 + 2. q_8 q_4))) \right], \\
& \left[0. \right]
\end{aligned}$$

with (VectorCalculus) :

$$\frac{\partial}{\partial F_1} dw_dot$$

$$(0)e_{x1} + \left(\frac{r_3}{J_2}\right)e_{x2} + \left(-\frac{r_2}{J_3}\right)e_{x3} + (0)e_{x4} + \left(\frac{1}{m}\right)e_{x5} + (0)e_{x6} + (0)e_{x7} + (0)e_{x8} \quad (19)$$

$$\frac{\partial}{\partial F_2} dw_dot$$

$$\left(-\frac{r_3}{J_1}\right)e_{x1} + (0)e_{x2} + \left(\frac{r_1}{J_3}\right)e_{x3} + (0)e_{x4} + (0)e_{x5} + \left(\frac{1}{m}\right)e_{x6} + (0)e_{x7} + (0)e_{x8} \quad (20)$$

$$\frac{\partial}{\partial F_3} dw_{_dot}$$

$$\left(\frac{r_2}{J_1}\right)e_{x1} + \left(-\frac{r_1}{J_2}\right)e_{x2} + (0)e_{x3} + (0)e_{x4} + (0)e_{x5} + (0)e_{x6} + \left(\frac{1}{m}\right)e_{x7} + (0)e_{x8} \quad (21)$$

$$phi_f := dJ_inv. \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_3 & r_2 & r_1 & 0 & 0 & 0 & 0 \\ r_3 & 0 & -r_1 & r_2 & 0 & 0 & 0 & 0 \\ -r_2 & r_1 & 0 & r_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$phi_f := \begin{bmatrix} 0 & -\frac{r_3}{J_1} & \frac{r_2}{J_1} & \frac{r_1}{J_1} & 0 & 0 & 0 & 0 \\ \frac{r_3}{J_2} & 0 & -\frac{r_1}{J_2} & \frac{r_2}{J_2} & 0 & 0 & 0 & 0 \\ -\frac{r_2}{J_3} & \frac{r_1}{J_3} & 0 & \frac{r_3}{J_3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

$$f_dot := \left\langle \frac{\partial}{\partial F_1} dw_{_dot} \middle| \frac{\partial}{\partial F_2} dw_{_dot} \middle| \frac{\partial}{\partial F_3} dw_{_dot} \right\rangle$$

$$f_dot := \begin{bmatrix} 0 & -\frac{r_3}{J_1} & \frac{r_2}{J_1} \\ \frac{r_3}{J_2} & 0 & -\frac{r_1}{J_2} \\ -\frac{r_2}{J_3} & \frac{r_1}{J_3} & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{m} \\ 0 & 0 & 0 \end{bmatrix} \tag{23}$$