CPSC 4820/6820
Spring 2021

Midterm Exam

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Instructions

1. Grading
   The exam is worth a total of 100 points, but questions <u>are not</u> weighted equally.

2. Rules
   This is an closed-book, closed-laptop test. Read each question carefully.

Score

(Please leave this area blank.)

1. (linear algebra: 20 points)
Solve the following 3-space vector expressions (the · operator signifies dot product).

a) $\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix} = 7$ b) $\begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}3\\-1\\2\end{pmatrix} = \begin{pmatrix}4\\1\\5\end{pmatrix}$ c) $\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}3\\-1\\2\end{pmatrix} = \begin{pmatrix}-2\\3\\1\end{pmatrix}$

a.) $\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}3\\-1\\2\end{pmatrix} = 1(3)+2(-1)+3(2)$

$= 3-2+6$

$= 9-2 = 7$

b.) $\begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}3\\-1\\2\end{pmatrix} = \begin{pmatrix}1+3\\2-1\\3+2\end{pmatrix} = \begin{pmatrix}4\\1\\5\end{pmatrix}$

c.) $\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}3\\-1\\2\end{pmatrix} = \begin{pmatrix}1-3\\2+1\\3-2\end{pmatrix} = \begin{pmatrix}-2\\3\\1\end{pmatrix}$

2. (quadrature mirror filters: 20 points)
The expression for deriving a quadrature mirror of the filter $\{h_k\}$ is given by:

$$g_k = (-1)^k h_{1-k}, \quad k \in \mathbf{Z}.$$

Suppose

$$h_k = \begin{bmatrix} \frac{1+\sqrt{3}}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & \frac{3-\sqrt{3}}{4\sqrt{2}} & \frac{1-\sqrt{3}}{4\sqrt{2}} \end{bmatrix}$$

Derive the coefficients for g_k.

$$h_k = \begin{bmatrix} \frac{1+\sqrt{3}}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & \frac{3-\sqrt{3}}{4\sqrt{2}} & \frac{1-\sqrt{3}}{4\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-\sqrt{3}}{4\sqrt{2}} & \frac{3-\sqrt{3}}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & \frac{1+\sqrt{3}}{4\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-\sqrt{3}}{4\sqrt{2}} & -\left(\frac{3-\sqrt{3}}{4\sqrt{2}}\right) & \frac{3+\sqrt{3}}{4\sqrt{2}} & -\left(\frac{1+\sqrt{3}}{4\sqrt{2}}\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-\sqrt{3}}{4\sqrt{2}} & \frac{\sqrt{3}-3}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & \frac{-1-\sqrt{3}}{4\sqrt{2}} \end{bmatrix}$$

$$\therefore g_k = \begin{bmatrix} \frac{1-\sqrt{3}}{4\sqrt{2}} & \frac{\sqrt{3}-3}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & \frac{-1-\sqrt{3}}{4\sqrt{2}} \end{bmatrix}$$

3. (separable filters: 20 points)
Given the two 1D separable filters below, derive the equivalent 2D filter.

$$\frac{1}{4}\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}, \quad \frac{1}{4}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\frac{1}{4}\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}, \quad \frac{1}{4}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}, \quad \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} \quad \frac{1}{2}\begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} \quad \frac{1}{4}\begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

4. (orthogonal filters: 20 points)
Show that the Haar filters h_k and g_k are orthonormal:

$$h_k = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad g_k = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Hint: first show both are orthogonal to each other, then show each is normal.

First, when $i \neq j$

$$\langle h_k, g_k \rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2} - \frac{1}{2} = 0$$

$\therefore$ there is no overlap. Hence, orthogonal.

Now, when $i = j$

$$\langle h_k, h_k \rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

= Normal.

$$\langle g_k, g_k \rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

= Normal.

$\therefore$ Since, h_K and g_K are orthogonal to each other and each of them are normal, h_K and g_K are orthonormal.

5. (DWT: 20 points)
Trace through 1 level of the DWT given the 4×4 image I below (no need to convert to $[0, 1]$ range) and $h_k = \{1/2, 1/2\}$, $g_k = \{1/2, -1/2\}$.
Hint: you should end up with four 2×2 matrices.

$$I = \begin{bmatrix} 2 & 0 & 8 & 12 \\ 2 & -4 & 8 & 4 \\ 12 & -2 & 7 & 13 \\ 0 & -2 & 7 & -3 \end{bmatrix}$$

$$h_K(I) = \begin{bmatrix} \frac{1}{2}(2)+\frac{1}{2}(0) & \frac{1}{2}(8)-\frac{1}{2}(12) \\ \frac{1}{2}(2)+\frac{1}{2}(-4) & \frac{1}{2}(8)+\frac{1}{2}(4) \\ \frac{1}{2}(12)+\frac{1}{2}(2) & \frac{1}{2}(7)+\frac{1}{2}(13) \\ \frac{1}{2}(0)+\frac{1}{2}(-2) & \frac{1}{2}(7)+\frac{1}{2}(-3) \end{bmatrix} \quad g_K(I) = \begin{bmatrix} \frac{1}{2}(2)-\frac{1}{2}(0) & \frac{1}{2}(8)-\frac{1}{2}(12) \\ \frac{1}{2}(2)-\frac{1}{2}(-4) & \frac{1}{2}(8)-\frac{1}{2}(4) \\ \frac{1}{2}(12)-\frac{1}{2}(-2) & \frac{1}{2}(7)-\frac{1}{2}(13) \\ \frac{1}{2}(0)-\frac{1}{2}(-2) & \frac{1}{2}(7)-\frac{1}{2}(-3) \end{bmatrix}$$

$$h_K(I) = \begin{bmatrix} 1 & 10 \\ -1 & 6 \\ 5 & 10 \\ -1 & 2 \end{bmatrix} \quad g_K(I) = \begin{bmatrix} 1 & -2 \\ 3 & 2 \\ 7 & -3 \\ 1 & 5 \end{bmatrix}$$

$$h_K h_K(I) = \begin{bmatrix} \frac{1}{2}(1)+\frac{1}{2}(-1) & \frac{1}{2}(10)+\frac{1}{2}(6) \\ \frac{1}{2}(5)+\frac{1}{2}(-1) & \frac{1}{2}(10)+\frac{1}{2}(2) \end{bmatrix} \quad g_K h_K(I) = \begin{bmatrix} \frac{1}{2}(1)+\frac{1}{2}(3) & \frac{1}{2}(-2)+\frac{1}{2}(2) \\ \frac{1}{2}(7)+\frac{1}{2}(1) & \frac{1}{2}(-3)+\frac{1}{2}(5) \end{bmatrix}$$

$$h_K g_K(I) = \begin{bmatrix} \frac{1}{2}(1)-\frac{1}{2}(-1) & \frac{1}{2}(10)-\frac{1}{2}(6) \\ \frac{1}{2}(5)-\frac{1}{2}(-1) & \frac{1}{2}(10)-\frac{1}{2}(2) \end{bmatrix} \quad g_K g_K(I) = \begin{bmatrix} \frac{1}{2}(1)-\frac{1}{2}(3) & \frac{1}{2}(-2)-\frac{1}{2}(2) \\ \frac{1}{2}(7)-\frac{1}{2}(1) & \frac{1}{2}(-3)-\frac{1}{2}(5) \end{bmatrix}$$

$$h_K h_K(I) = \begin{bmatrix} 0 & 8 \\ 2 & 6 \end{bmatrix} \quad g_K h_K(I) = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$h_K g_K(I) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad g_K g_K(I) = \begin{bmatrix} -1 & -2 \\ 3 & -4 \end{bmatrix}$$