# Stellar mass and nebular metallicity relation for galaxies

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## ABSTRACT

This study explores the mass-metallicity relation in galaxies using data from the Sloan Digital Sky Survey (SDSS). Analyzing star-forming galaxies, we investigate the correlation between stellar mass and nebular metallicity. Our results demonstrate a positive relationship between these parameters, indicating a connection between galaxy mass and metal enrichment. We also find that high-mass star formation and supernova feedback play significant roles in metal enrichment. By considering gas exchange processes, our study contributes to understanding galaxy formation and evolution. By initially making simple assumptions and employing both closed box and open box approaches, we derive insights that contribute to a deeper understanding of the underlying mechanisms driving galaxy formation and evolution. These findings deepen our knowledge of the cosmic ecosystem and provide insights into the interplay between star formation, gas inflow/outflow, and heavy element enrichment. The results of our study imply that open box model provides a more accurate representation of the complex processes involved in the development of galaxies.


**Key words:** galaxy formation – mass-metallicity

## 1 INTRODUCTION

Galaxies, colossal systems composed of billions of stars, are the basic units of the universe, and their composition, structure, and behavior provide valuable information about the physical laws of the cosmos.

Understanding the mechanisms underlying the formation and evolution of galaxies is one of the grand challenges of modern astrophysics. The nature of this task requires us to be in the known of physical processes, such as star formation, accretion or ejection of gas, etc. However, amidst these complexities, the study of galactic properties has provided valuable empirical relationships, such as the mass-metallicity relation observed in galaxies, a relationship that offers crucial insights into the cosmic lifecycle of matter and the physical processes shaping the formation and evolution of galaxies.

Sandra Faber(1973) examined the metallicity-luminosity relation in elliptical galaxies, highlighting the connection between metallicity and the overall brightness of these galaxies, Mannucci & et al. (2010). Then Tremonti & et al. (2004) analyzed the relationship between stellar mass and nebular metallicity in the star-forming galaxies. They find a clear correlation between these two quantities, with more massive galaxies exhibiting higher metallicities. More work on this has been done by Zahid & et al. (2019), Maiolino & Mannucci (2019) ,Maier & et al. (2019).

The main objectives of this study were to investigate the relationship between stellar mass and metallicity in galaxies using data from the Sloan Digital Sky Survey (SDSS). Specifically, we aimed to determine the correlation between stellar mass and metallicity. Which processes and the variables involved in this relation. We conduct an in-depth analysis of star-forming galaxies, honing in on their stellar mass, metallicity, and star formation rates.

To ensure the applicability of our model to a wide variety of galaxies, we start by having a look at closed box model and then the open box approach in our analysis. These approaches have their basis in differing assumptions about the degree of interaction between a galaxy and its surrounding environment. While a closed box model assumes no inflow or outflow of gas, an open box model considers the continual exchange of material between a galaxy and its surroundings. By exploring these two models, we seek not only to construct a comprehensive model of galaxy formation but also to elucidate the complex interplay of various processes involved in the development of galaxies.

## 2 METHODS

We now proceed to have a look at our data and the models we build to compare with observational data.

### 2.1 DATA

We utilized the data provided by the Sloan Digital Sky Survey (SDSS) as the starting point. The SDSS is an international collaboration aimed at constructing detailed three-dimensional imagery of the Universe.

In this study, our main objective was to investigate the relationship between stellar mass and nebular metallicity in galaxies. To accomplish this, we utilized the SDSS MPA-JHU catalogue, which which provides important parameters for galaxies, including the best estimate for stellar mass (`Log_stellar_mass_Msun`), star formation rate (`Log_SFR_Msun_yr`), and nebular metallicity (`Metallicity_nebular`). To be precise we accessed this `"sdss_mpajhu_catalogue.fits"` data file from the SDSS DR8.

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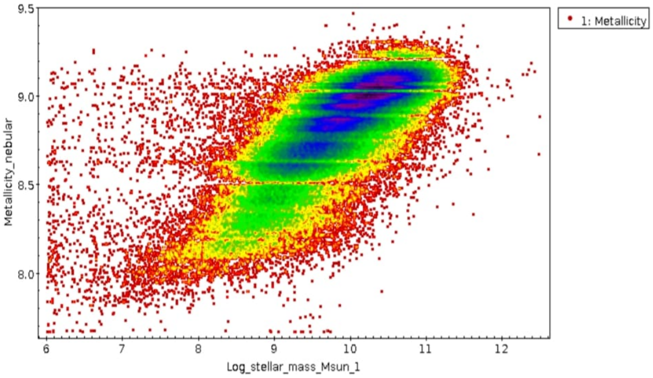


**Figure 1.** stellar mass - Nebular metallicity relation, X-axis:stellar mass in log scale ,Y-axis: The metallicity is given by $Z = 12 + log(O/H)$ at redshift$\sim 0$ image created using TOPCAT.

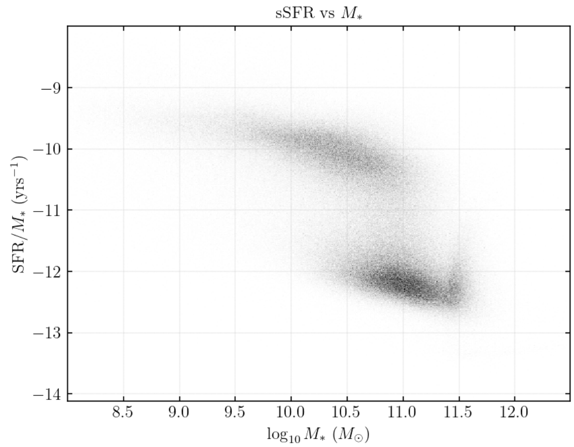


**Figure 2.** Specific star formation rate SFR/$M_*$ vs Stellar mass

We utilized the TOPCAT software and python to perform data matching and explore the relationships between various parameters.

In Figure.1, which displays the relationship between the logarithm of stellar mass and nebular metallicity, made by utilizing our observational data from sdss catalogue, at Redshift nearly $\sim 0$. We observe a continuous trend where nebular metallicity increases as stellar mass increases. The majority of the objects in the plot lie within the stellar mass range of $10M_\odot$ to $11.5M_\odot$. Notably, the relationship initially exhibits a steep slope and then gradually flattens around a stellar mass of 10.5 solar masses. It is evident that there is an overall positive correlation between nebular metallicity and stellar mass.

## 2.2 Model

We will now develop a simple model of the build up of metals in a galaxy over time. Consider a galaxy, initially we have very low metallicity. Then stars are born from the gravitational collapse of interstellar gas and dust, and during their lifetime, they undergo nuclear fusion reactions in their cores, converting lighter elements into heavier ones. As stars evolve, they release energy in the form of radiation. When stars reach the end of their life cycle, they undergo various processes depending on their mass. High-mass stars, for example, end their lives in dramatic supernova explosions, ejecting enriched material into the surrounding interstellar medium (ISM). This ejected material contains heavier elements synthesized within the star through nuclear processes.

*Starting Assumptions*

The system starts as purely primordial gas, We will assume only death of high mass stars are responsible for the enrichment of galaxies with elements heavier than H, He. All formation is from a single burst. Star Formation Rate (SFR) Relationship: We assume a single burst of star formation, where the SFR is related to the gas mass ($M_{\rm gas}$) through the equation

$$SFR = \epsilon \cdot (M_{\rm gas})^{\alpha} \tag{1}$$

In our simplified model, we adopt the specific values $\alpha = 1$ and $\epsilon$ as a constant. This assumption is driven by the Jeans criterion, which considers the equilibrium between gravity and kinetic pressure in the collapsing gas. According to the Jeans criterion, gas collapses if its mass exceeds the Jeans mass ($M_J$). Considering the relation between the stellar mass and the star formation rate (SFR), we can express the stellar mass as an integral over time:

$$M_*(t) = \int_{t_0}^{t} \mathrm{SFR}(t')dt' = \int_{t_0}^{t} \epsilon M_{\rm gas}(t')dt'$$

So, now remembering that our goal is metallicity, $M_z$ = mass of metals in galaxy. $M_g$ = gas mass = mass(hydrogen). $Z_{\rm neb}$ is the fraction od metals which we assume is equal to fraction of mass(oxygen) to mass(hydrogen).

$$Z_{\rm neb} \propto \frac{M_{\rm O}}{M_{\rm gas}} \propto \frac{M_z}{M_{\rm gas}} \tag{2}$$

To better understand the evolution of the metal mass ($M_{\rm Z}$), we need to determine its rate of change with respect to time:

$$\frac{dM_{\rm Z}}{dt} = ?$$

For this purpose, we make an assumption known as the closed box approximation.

## 2.3 Closed box model

This model assumes that the gas inflow ($M_{\rm gas}^{\rm in}$) and outflow ($M_{\rm gas}^{\rm out}$) are both negligible, resulting in the only process responsible for the change in gas mass being star formation:

$$\frac{dM\,\mathrm{gas}}{dt} = -\mathrm{SFR} \implies M_{\rm gas} = M_0 e^{-\epsilon t} \tag{3}$$

$$\therefore M_*(t) == \int_{t_0}^{t} \epsilon M_{\rm gas}(t')dt' = M_0(1 - e^{\epsilon(t-t_0)})$$

$M_{\rm tot} = M_0$, Continuing with the analysis, y = yield due to supernovae explosion (refers to the metal yield, which represents the amount of new metals produced by stellar processes (such as supernovae) per unit of star formation).

$$\frac{dM_{\rm Z}}{dt} = \mathrm{y.SNR} - Z.\mathrm{SFR}$$

**Another assumption:** we only have type II supernovae $\implies$ SNR = SFR. (This assumption is based on the understanding that main sequence stars are primarily responsible for the production and release of heavy elements through their death as supernovae. In particular,

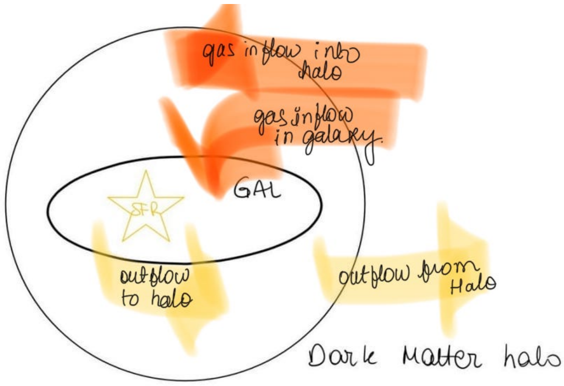


**Figure 3.** Open box model

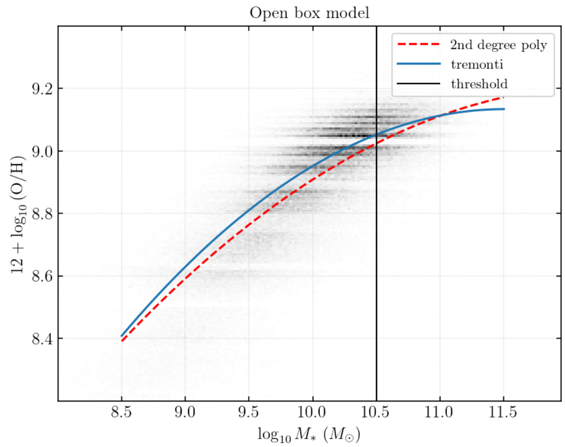


**Figure 4.** metallicty vs stellar mass plot for open box model, comparing with tremonti.et al, we put the threshold at 10.5 solar masses and we can see that this model fits our SDSS data better.

Type II supernovae occur as a result of the death of high-mass stars, which are known to release significant amounts of oxygen and other metals. By considering only Type II supernovae, we focus on the specific channel through which metals are injected into the interstellar medium, thereby simplifying our analysis and assumptions regarding the metallicity of the galaxy.) putting $Z = \frac{M_z}{M_{\rm gas}}$ we get:

$$\frac{dM_z}{dt} = (\mathrm{y} - \frac{M_z}{M_{\rm gas}}).\mathrm{SFR}$$

Now, recall that $\epsilon = \frac{\rm SFR}{M_{\rm gas}}$ finally giving us, first order differential equation

$$\frac{dM_z}{dt} + \epsilon.M_z = \mathrm{SFR}$$

Solving this we get our final result,

$$M_z(t) = M_z^{closed} = y.M_0.\epsilon(t - t_0)e^{-\epsilon(t-t_0)} \tag{4}$$

giving us the following equation for metallicity :

$$Z^{closed} = \frac{M_z}{M_{gas}}^{closed} = y\epsilon(t - t_0) \tag{5}$$

Now we want to express metallicity as a function of the stellar mass, Since we already have $\frac{M_*}{M_0} = (1 - e^{\epsilon(t-t_0)})$, we can use this to get

$$Z^{closed} = y.log(\frac{1}{1 - M_*/M_0})$$

which kind of linear for low masses.

### 2.4 Open box model

Now, Instead we a look at what would happen when the galactic volume has an inflow and outflow of material.

**assumptions**:Initially we have pristine Gas meaning it has no pre-existing metallicity at the beginning of our study ($z(t_0) = 0$), homogeneous metal distribution in our galaxy lastly, $\dot{M}_{in} = \dot{M}_{out} \implies M_g = \epsilon.$SFR, This means the otflow of gas $\dot{M}_{out}$ is only due to supernovae feedback(we will assume this feedback rate ($\eta$) to be constant). A dark matter halo + IGM surround the galaxies. From this galaxy to the dark matter halo now you can have inflow and outflow of the gas mass.

So, here we have opened up the galaxy for possible interaction with the neighbouring space that consists of IGM gas(mostly pristine), we have basically given given free access to gas mass of galaxy to change over time via different processes. So for inflow we go simple by assuming only pristine inflow this leads to increase in $M_g$ which in turn decreases the metallicity($M_z/M_g$) , whereas for outflow we have the Supernovae. These occur at the end of the life cycle of massive stars and release large amounts of energy and enriched material.

With these assumptions, we can derive the evolution equation for metallicity:

$$\dot{M}_z = (y - Z).\mathrm{SFR} - Z.\eta.\mathrm{SFR}$$

So what have we done? we have again assumed (SNR=SFR), and $\eta$ here is the supernovae feedback rate, which influences the outflow of gas and metals from the galaxy.

$$\dot{M}_z + Z.(1 + \eta).\mathrm{SFR} = y.\mathrm{SFR}$$

This equation represents the evolution of metallicity in our galaxy, taking into account the metal yield, metallicity, and the effects of supernova feedback.

Comparing this with the standard form of a linear first-order ordinary differential equation $\dot{M}_z + P(t)M_z = Q(t)$. we get $I(t) = e^{\int_{t_0}^{t}(1+\eta)\epsilon dt} = e^{(1+\eta)\epsilon(t-t_0)}$.

$$e^{(1+\eta)\epsilon(t-t_0)}M_z = \int_{t_0}^{t} y \cdot \mathrm{SFR} \cdot e^{(1+\eta)\epsilon(t-t_0)}dt$$

giving the following equation for mass of metallicity in the open boxy model,we put eq. 3 in eq. 1 and use it to get,

$$M_z(t) = M_z^{open} = \frac{1}{\epsilon \cdot \eta} \cdot y.M_0.\epsilon e^{-\epsilon(t-t_0)} \tag{6}$$

we notice that

$$M_z^{open} = \frac{1}{\epsilon \cdot \eta} \frac{M_z^{closed}}{(t - t_0)}$$

And since mass of the gas, $M_g$ is the same for the open and the closed box, we can find meatllicity as

$$Z^{open} = \frac{M_z}{M_{gas}}^{open} = \frac{M_z{}^{closed}}{M_{gas} \cdot \epsilon\eta(t - t_0)} = \frac{Z^{closed}}{\epsilon\eta(t - t_0)}$$

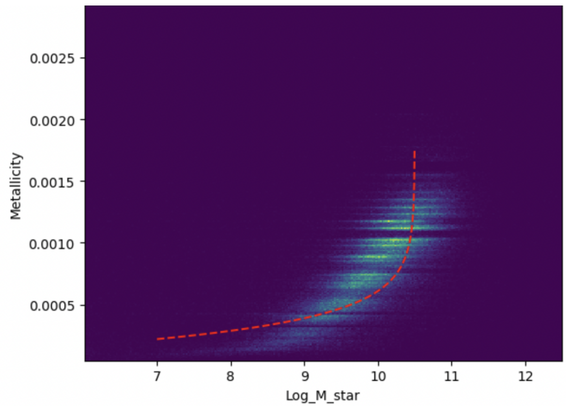


**Figure 5.** metallicity vs stellar mass plot for the closed box model, red dotted line is our model. We use y = .0002 and $M_0$ = 10.5 our model $Z_{pred} = y * np.log(1/(1 - x/M_0))$

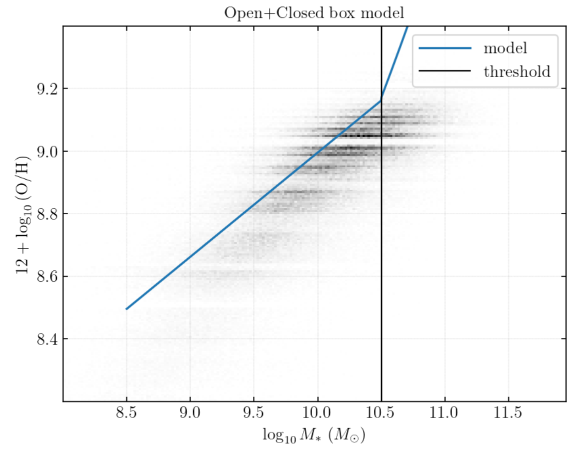


**Figure 6.** In the metallicity vs stellar mass plot, we combine the predictions of the closed box model and the open box model. However, for the open box model, we will only consider the mass range up to $10^{10.5}M_\odot$ above which we conside the closed box model

$$Z^{open} = \frac{y}{\eta} \tag{7}$$

Again we want to express metallicity as a function of the stellar mass, for which we get

$$Z^{open} = y.\eta^{-1}(M_*) \propto \eta^{-1}(M_*)$$

## 3 RESULTS

So, We have our two models ready that give us their respective relations for mass-metallicity. We then compared both the models with our obsevational data to check which one is better. Our SDSS data gives us the following quantity

$$\tilde{Z} = 12 + \log\left(\frac{\mathrm{O}}{\mathrm{H}}\right)$$

Next, we compare the data-set with our prediction models. by comparing with the closed box approach we get figure. 6 The epsilon is given by the Kennycutt et al. paper and it points to a value of $\epsilon$ = 0.25 Gyr$^{-1}$ putting in our $M_z$ for closed box , we see linear relationsip for $M_z$ with time. Whereas, the metallicty given by this model is

$$Z(t) = \mathrm{y}\ \log\left(\frac{1}{1 - M_*/M_0}\right)$$

The closed box model for the metallicity depends only on the metal yield and the total mass of the galaxy. We can see that this model works only above $10^{10.5}M_\odot$. it gives higher metallicity value for masses less than this. This goes to show that our fit wasn't so bad but at around $M < 10^{10.5}M_\odot$ considering the inflow of presteen gas. So we needed a model that could better fit this low stellar mass region.

Open box model approach gives us fig. 4

$$Z^{open} = y.\eta^{-1}(M_*) \propto \eta^{-1}(M_*)$$

We can see in fig. 4 that the open box model fits for lower as well as higher mass galaxies.The open box model takes into account the inflow of external gas and outflow of material through processes like the supernovae, which seems to be a plausible assumption considering that galaxies can accrete gas from their surroundings.

To gain deeper insights and assess the accuracy of these assumptions, further discussions and analysis are necessary. Factors such as the efficiency of gas accretion, the role of external influences, and the impact of different feedback mechanisms (e.g., supernovae, active galactic nuclei) need to be considered.

## 4 DISCUSSION

We started our research in order to derive a model that can predict the metallicity trend in galaxies. Let's review the specific SFR plot, here we can identify two populations. We notice in Fig. 2 the two regions are split at the value of SFR/$M_*$ = $10^{-11}$, which corresponds to $1/10^{11}$ yrs.
We started from the following relation , which describes the evolution of gas mass in the galaxy over time:

$$\frac{dM_{\mathrm{gas}}}{dt} = \dot{M}^{in}_{\mathrm{gas}}(t) - \dot{M}^{out}_{\mathrm{gas}}(t) - \mathrm{SFR}(t)$$

A similar analogy can be made for the evolution of metals in galaxy. The metallicity of gas in galaxies is commonly quantified by the oxygen to hydrogen abundance ratio. Oxygen has emerged as the preferred element for studying interstellar medium (ISM) compositions due to several factors. Firstly, oxygen is the most abundant element in the ISM, allowing for accurate measurements of its abundance relative to hydrogen. Additionally, oxygen is relatively resistant to depletion onto dust grains, meaning it largely remains in a visible state within the gas. This characteristic makes it a reliable tracer of metallicity.

Next, We imagine that what guides the metals formation is the SFR and the supernovae explosion rate $\frac{dM_z}{dt}$. we define $y$ as the metal yield, to account for the efficiency with which metals are released into the ISM and add this term for our metal equation. Our closed model ends with these few considerations for which we see that the metallicity increases linearly with time, but this linear relation still doesn't explain the sudden change in the two population. But, because our model has only two parameters the best it do was above $10^{10.5}M_\odot$. As we see in fig. 6, we tried to move the model by changing , y and $M_0$ but it just went above or below.

This happened because most galaxies don't work like that , to get a more complete picture we need to consider the inflow and outflow of materials from the galaxy.Just like our open model, the studies indicate that the observed mass-metallicity relation in galaxies, where more massive galaxies exhibit higher metallicities, can be better understood within the framework of an open box model. In an open box scenario, the inflow of gas can contribute to the metal content of galaxies, enriching them with heavy elements. On the other hand, outflows can remove metal-enriched gas, potentially lowering the metallicity of galaxies. It also accounts for the sudden flattening after a stellar mass of approximately $10^{10.5}$.

**Explanation for observed phenomenons**

The flattening or broadening of the mass-metallicity relation after a stellar mass of approximately $10^{10.5}$ solar masses is an interesting phenomenon observed in galaxies. This behavior is commonly referred to as the "mass-metallicity turnover" or "mass-metallicity plateau."

Tremonti & et al. (2004) Interpreted this flattening as evidence for metallicity saturation or a "metallicity floor" in massive galaxies. This means that once a galaxy reaches a certain stellar mass threshold, further increases in mass do not result in a significant increase in metallicity. The flattening of the mass-metallicity relation indicates that additional metal enrichment mechanisms, such as metal-poor gas accretion or metal-poor galaxy mergers, might be at play in massive galaxies to maintain their metallicity levels. The broadening indicates that there is increased scatter or dispersion in the metallicity measurements for galaxies with similar stellar masses.Galaxies with similar stellar masses may have experienced different merger histories, gas accretion rates, or star formation efficiencies, leading to variations in their metallicity levels. The scatter in metallicity could also be influenced by environmental factors, such as the density of the galactic environment or the presence of nearby galaxies.

Tremonti's analysis shows that y moves in a range , but on;y worked with smaller values of metal yield. They found that the observed mass-metallicity relation in galaxies is consistent with a metal yield that increases with galaxy mass. This suggests that more massive galaxies have a higher efficiency in producing and retaining metals, resulting in higher metallicities.

## 5 SUMMARY

The study investigated the mass-metallicity relation in galaxies to understand galaxy formation and evolution. The analysis of star-forming galaxies using SDSS data showed a clear correlation between stellar mass and nebular metallicity. Two models, the closed box model and the open box model, were employed to explain this relationship. The closed box model, which assumes no gas inflow or outflow, demonstrated a relationship in tune with the data between metallicity and stellar mass for high-mass galaxies.However, the open box model, which considers gas exchange and supernova feedback, offered a more realistic representation as it fit better with the data for low mass and high mass galaxies. The metallicity in the open box model is influenced by factors such as gas inflow rate, outflow rate, metal yield and supernovae feedback.

So we conclude that considering our galaxies as open box and accounting for the fact that galaxies interact with their neighbourhoods by exchanging material likes gas and other heavier elements, we get a better model and our understanding is more closer to actual observations. For future studies we must consider more complex parameters and revisit the assumptions, as these assumptions play very important role on the galaxies turn out. Additionally, we must consider that the metallicity of galaxies can be influenced by other factors such as mergers, active galactic nuclei (AGN) feedback, and interactions with the surrounding environment. More sophisticated models that incorporate these additional factors can lead to a more comprehensive understanding of galaxy formation and evolution. In conclusion, while the open box model and certain assumptions seem promising, a comprehensive and detailed analysis, combined with observational evidence, is essential to better understand the formation and evolution of galaxies.

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