



~~Module - 2~~

Problem - Solving Methods

Problem Solving by search

→ Goal based agent are called as ego problem
 - Solving agent

A well defined problem

1) Initial State

→ Operator / Successor function - for any state x return $S(x)$

The set of states reachable from x with one action

→ State Space : all states are reachable

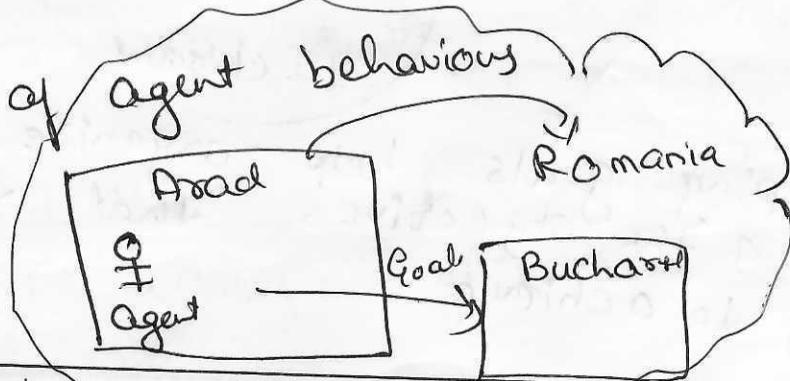
→ Path - Sequence

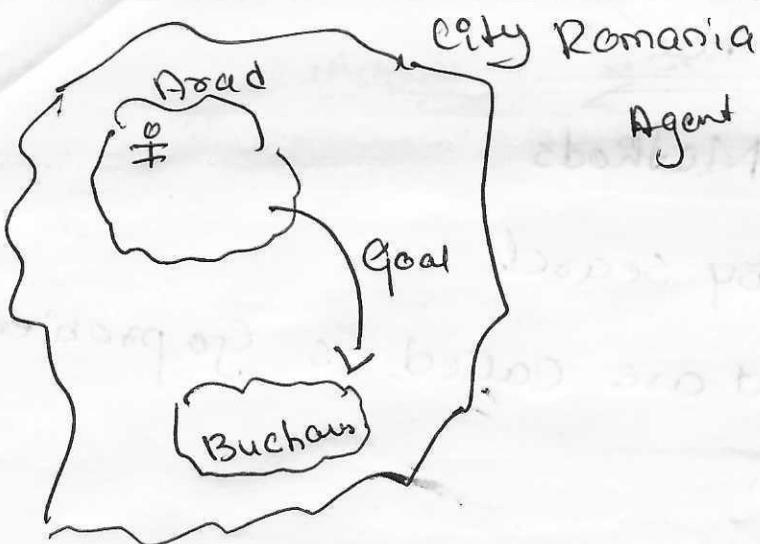
→ Goal Test : test to determine if a goal state

Problem - Solving Agents

Intelligent agents are supposed to maximize their performance

Example of agent behaviour



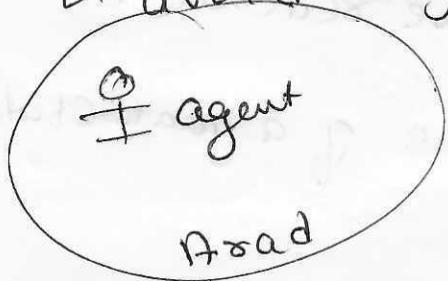


Agent is Enjoy the holiday

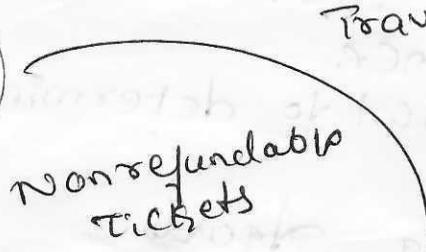
Goal formulation:

The agent performance measure contain many factors

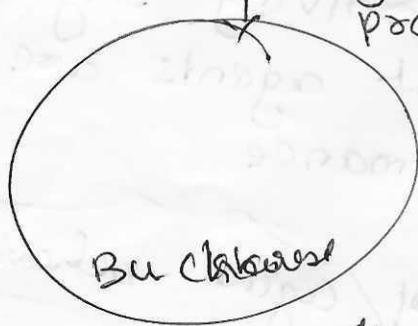
- ↳ To improve Sun tan
- ↳ avoid hangover etc



Road



Nonrefundable
Travel



Bucharest

Goal is to reach
on time
agent's decision
problem is great
simplified

- By keeping goals help organize behavior that the agent by limiting the objectives is trying to achieve

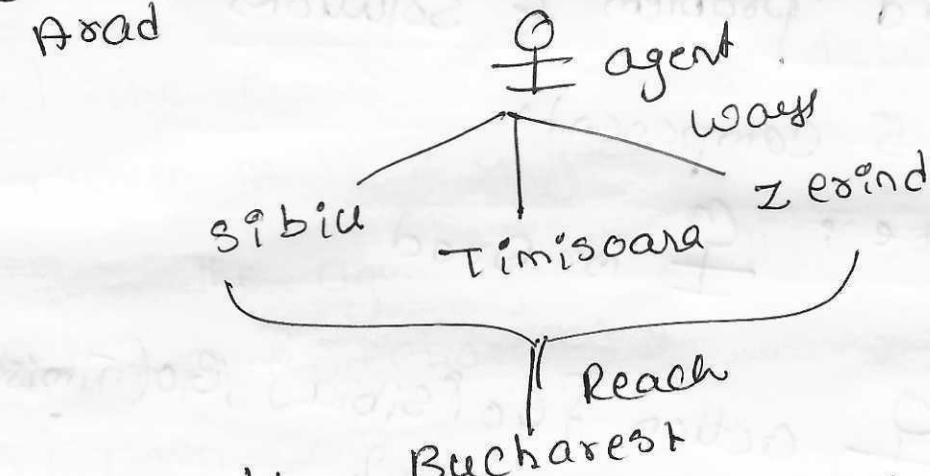


Goal formulation: problem solving

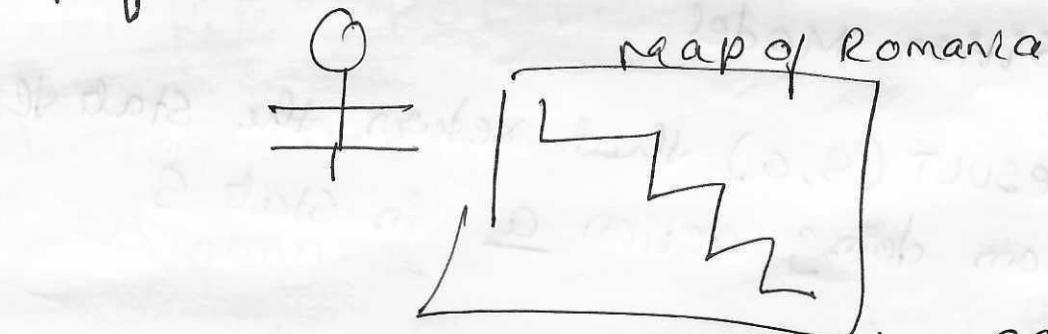
problem formulation - what action & process to be followed.

Agent adopted goal to travel by road or driving to Bucharest & considering were to go from Sibiu

road



Should be GPS map of Romania



The point of map is to provide agent with information about states & then take an action

→ think which route to move
find the best path

If found the goal achieved

To carry out this goal action is required

In general an agent with several immediate options of unknown value can decide what to do by first examining future actions that eventually lead to states of known value.

Well-defined problem & Solutions

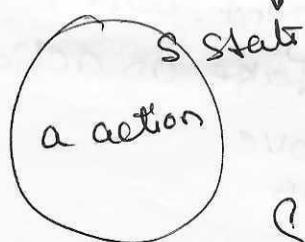
5 components

① Initial state : \emptyset is Arad

② Action \emptyset action $\{g_0(\text{Sibiu}), g_0(\text{Timisoara})\}$
 $g_0(\text{Zerind})\}$

③ Transition Model

RESULT(s, a) that returns the state that results from doing action a in state s



Successor \leftarrow refers to any state reachable from a given state by a single state action.



Result (In Arad, to Zerind) = To (Zerind)

∴ Initial state, action & transition model

A path in the state space is a sequence of states connected by a sequence of actions

(4) The goal test → To determines whether a given state is a goal state.

The agent goal in Romania is the singleton set {In (Bucharest)}

(5) path cost

EXAMPLE PROBLEMS

1 TOY PROBLEMS

Vacuum World Example.

(1) States: the state is defined by the agent's location & dirt locations.

In 2×2 grid example

↓ each location is either dirt / clean
agent in 2 locations

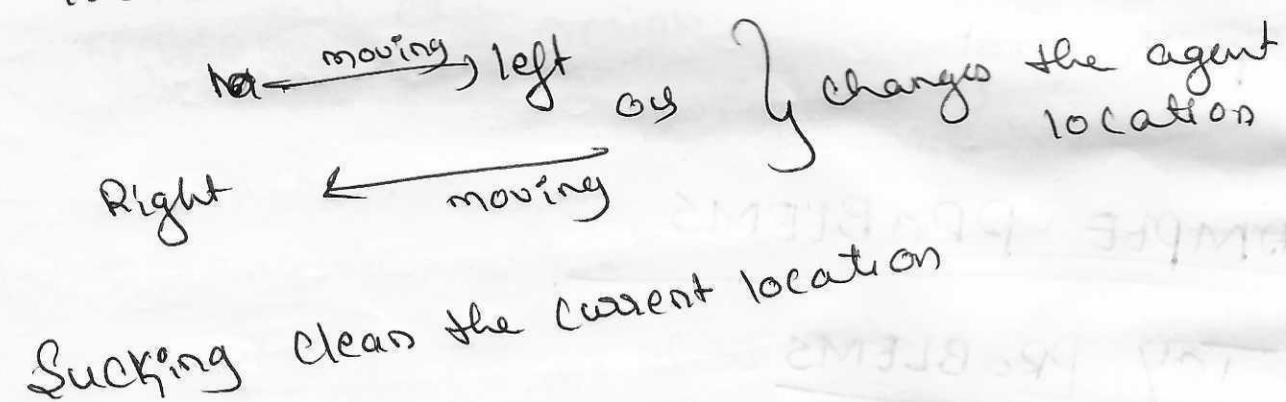
$$2 \times 2^2 = 2 \times 4 = 8 \text{ possible states}$$

{

- Initial State: Any state can be designated as initial state.
- Actions \rightarrow agent has just 3 states
 - Left \rightarrow move left
 - Right \rightarrow move right
 - Suck \rightarrow clean the current square

In larger environment might include up & down

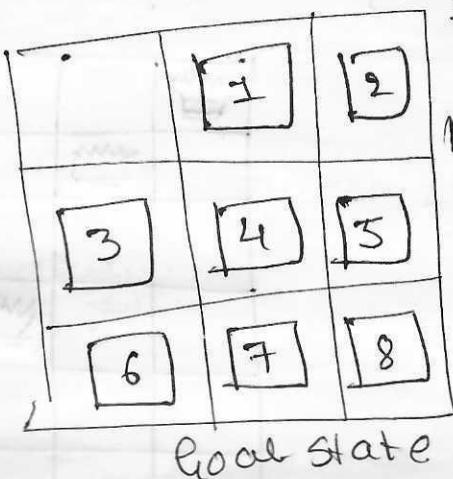
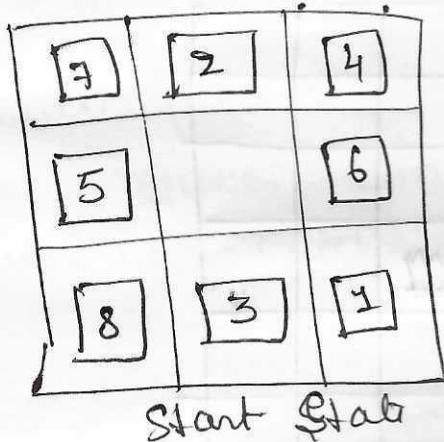
- Transition Model



- Goal path test: The goal is achieved when all squares are clean
- Path cost: Each step costs 1, so the path cost is the number of steps in the path.



8-puzzle / belongs to family ~~NP~~ of Sliding block puzzles



Consists of a 3×3 board

→ with 8 numbered tiles

→ one blank space

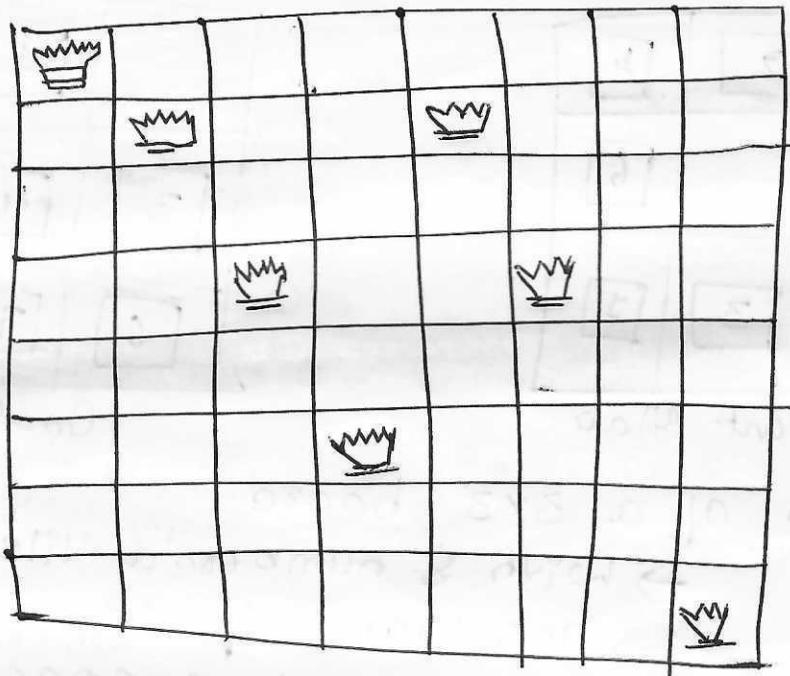
→ A tile adjacent to blank space can slide into space.

- State → The puzzle is represented by 8 numbered tiles & 1 blank space on 3×3 board
- Initial state: any configuration of the tiles can be the starting point.
- Actions: movement of the blank space left, down, up & Right
- Transition Model: The goal is to reach a specified configuration as shown in figure
- Path Cost: Each step costs 1, so the path cost is number of steps in the path.



8 - Queens Problem [N - Queens problem]

- States :



State : Any arrangement of 0 to 8 queen ($N=8$)

Initial State : No queens are placed initially

Actions : A queen is placed initially is added to any empty square

Transition Model : The system returns the updated board after placing a queen.

Goal Test : All 8 queen should be on board & not as to attack each other

Formulation

Incremental formulation : Add queens one at a time.



Complete - State formulation : place all queens & move them around.

Challenges :

A Queen can attack another Queen if they are in same row, column or diagonal

Understanding the 8-Queen

- 8 queens on a 8x8 Chessboard
- no two queens attack each other
- a queen can attack in any row, columns or diagonal

Board size - 8 rows x 8 columns

State formulation - 64 possible squares [1st queen, 2nd queen]

Once the queen take place, for next queen the possible will reduce

Total possible sequence = $64 \times 63 \times 62 \times \dots \times 57$

$\frac{64!}{(64-8)!}$ which is large numbers

$1.8 \times 10^{14} \leftarrow$ Total number of possible ways to place 8 queens on a chessboard



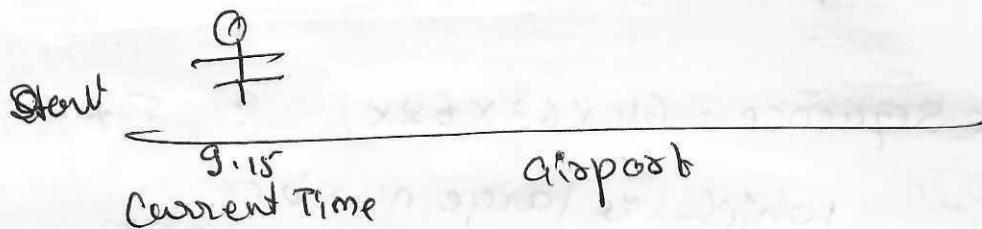
REAL - WORLD PROBLEMS

Route-finding Problem

- Defined in terms of specified location & transitions along links, between them
- Routing in CN, military operation planning, airline travel planning systems

Airline Travelling problem

- State : Each state represents a location (airport) & current state



- Initial state : This is specified by the problem [where you are & the time you are there]
- Successor function : This function generates possible next states based on scheduled flights. A flight can only be taken if-



- The departure time is later than the current time
- you are at the current airport to board

Goal Test: The goal test is to check if you have reached the destination airport within specific time

- Path Cost → This depends upon the monetary cost, waiting time, flight time, customs & immigration, seat quality, time of day & so on

TOURING PROBLEMS

→ Closely related to route-finding problems
Visit every city atleast once

- Initial State: The agent starts in Bucharest & has not yet visited
- Intermediate state: The agent might be in other city visiting A, B & C as well.
- Goal: The goal is to check if all cities have been visited, meaning the agent has traveled through each one atleast once.



THE TRAVELLING SALESPERSON PROBLEM(TSP)

- IS a touring problem in which each city must be visited exactly once.
- The aim is to find shortest tour.
- The problem is called NP-hard

VLSI layout

- Layout problem requires positioning millions of components & connections on a chip to minimize area
- ↳ 2 problems
 - Cell layout
 - Channel routing

Robot Navigation

- Robot navigation is a generalization of the route find problem.
- Rather than a discrete set of routes a robot can move in continuous space with an infinite set of possible actions & states.

Automatic Assembly Sequencing

- assembly electric motors
- aim in "pblm" is to find the order in which to assemble the parts of some objects.



UNINFORMED SEARCH STRATEGIES

Strategies that know whether one non goal state is "more promising" than another are

Informed Search / heuristic Search

5 - Uninformed

- 1) Breadth - first
- 2) Uniform - cost Search
- 3) Depth - first Search
- 4) Depth - limited search
- 5) Iterative deepening Search



Searching for Solutions

TREE SEARCH & GRAPH - SEARCH

(1) TREE SEARCH : fundamental Search algorithms in AI

1. Search Tree : Search tree is created starting with initial state (root) & Expands all possible actions (Branches) → Each action leads to a new state.

Ex Arad to Bucharest, each state (city) is connected by actions (roads & flights)

(2) Frontier (or Open List)

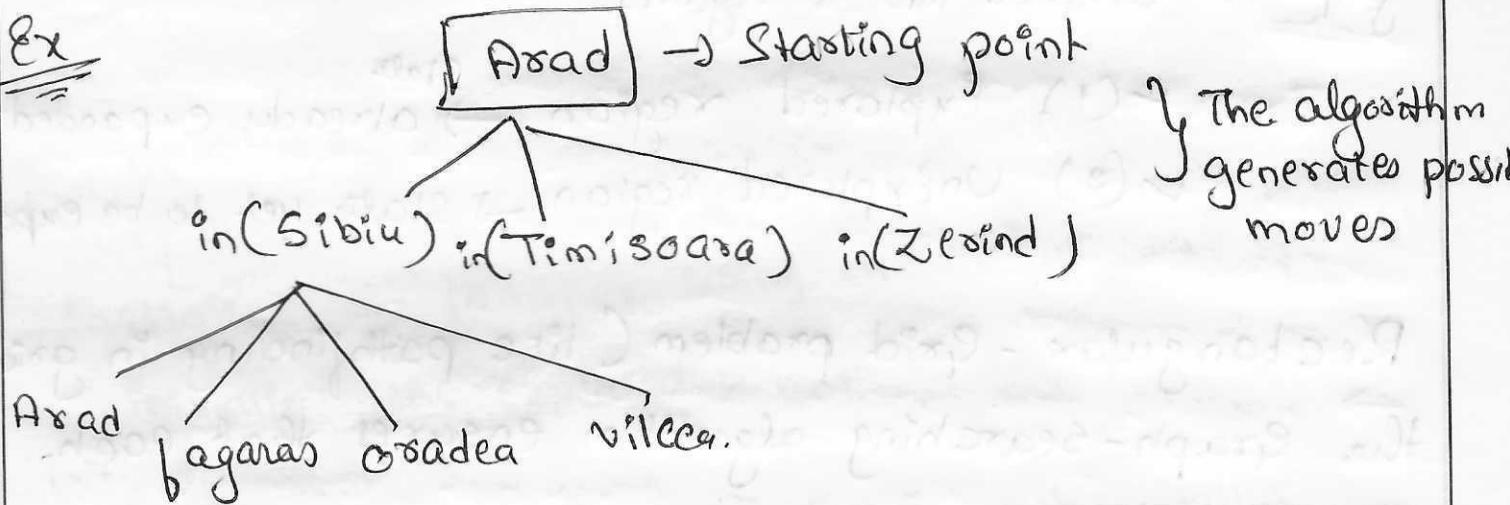
The frontier is a set of all leaf nodes in the search tree, set of nodes that have been generated but not yet expanded

At each step the algorithm selects a node from the frontier, expands it & add its child nodes to the frontier.

Key Steps of TREE - SEARCH

- Initialize the frontier { Start with the initial state

- Loop until the frontier is empty
 - ✓ If Empty → No more nodes to explore
 - If node contains the goal state → The algorithm terminates successfully.

Ex

Function TREE - SEARCH (problem) → return a solution or failure

initialize the frontier using the initial state of problem

loop do

if the frontier is empty then return failure

choose a leaf node & remove it from the frontier

if the node contains a goal state then return the corresponding solution

expand the chosen node, adding the resulting nodes to the frontier



GRAPH - SEARCH ALGORITHM

In the Graph - Search algorithm, the State - space graph is divided into 2 regions,

- ~ (1) Explored region \rightarrow ^{state} already expanded
- ~ (2) Unexplored region \rightarrow states yet to be expanded

Rectangular - Grid problem (like pathfinding in grid)
the Graph - searching algorithm ensures that each state is only expanded once.

Illustration

Consider a rectangular grid where the goal is to find a path from the top-left corner to the bottom-right corner

1. The start state is placed in the frontier
2. The first state from frontier is expanded & moved into explored region
3. After expanding a state, its neighbors are added to the frontier, which grows as more states are generated



The process continues until the algorithm finds the goal state.

Figure 3.9

function GRAPH-SEARCH (problem) returns a solution or failure
initialize the frontier using the initial state of problem
initialize the explored set to be empty
loop do
 if the frontier is empty then return failure
 choose a leaf node & remove it from frontier
 if the node contains a goal state then return
 the corresponding solution
 add the node to the explored set
 expand the chosen node, adding the resulting nodes to the frontier
 only if not in the frontier or explored set



UNINFORMED SEARCH STRATEGIES

① Breadth - first Search

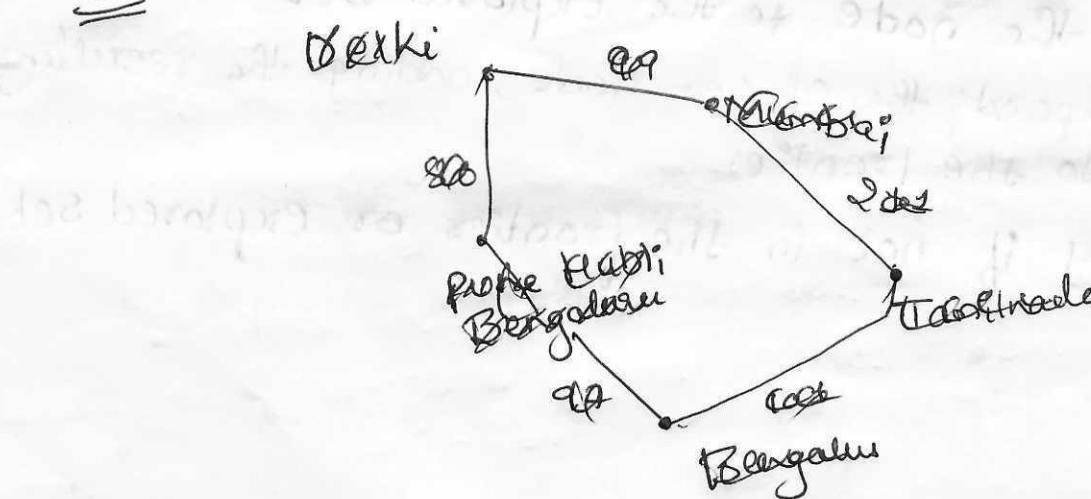
Def²

Breadth - first Search is a simple strategy in which the root node is expanded first, then all the successors of the root node are expanded next then their successors.

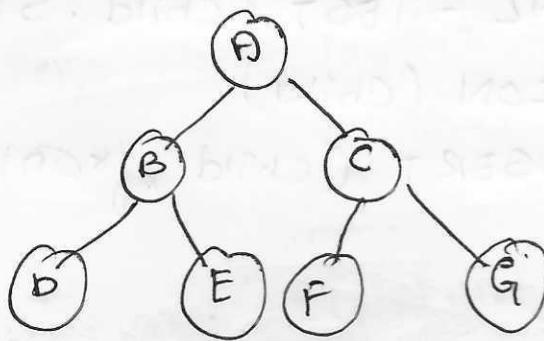
→ Explore the graph/ tree level - by - level.

- uses a FIFO (First-in, First out) queue to keep track of nodes
- visit nodes at each level before moving to next
- ensure the shortest path is found first

Ex



- (I) Delhi - Hubli - Bangalore - Tamilnadu = ~~278~~
- (II) Delhi - Mumbai - Tamilnadu = ~~310~~



Breadth - first Search on a graph \rightarrow Function

Function BREADTH - FIRST - SEARCH (problem) return
a solution, or failure

node \leftarrow a node with STATE = problem. INITIAL
-STATE, PATH - COST = 0
if problem. GOAL - TEST (node. STATE) then return
SOLUTION (node)

frontier \leftarrow a FIFO queue with node as the only
element

explored \leftarrow an empty set

loop do

if EMPTY ? (frontier) then return failure

node \leftarrow POP (frontier)

add node. STATE to explored

for each action in problem. ACTIONS
(node. STATE) do

child \leftarrow STATE is not in explored / frontier
then

* $\text{frontier} \rightarrow$ Set of nodes that are waiting waiting to be expanded.

Assignment No.



Date:

if child.STATE is not in explored / frontiers
then
 if problem.GOAL-TEST (child.STATE) then
 select SOLUTION (child)
 frontier \leftarrow INSERT (child, frontiers)

Expl²

1. Initialize
 - Start from initial state with cost = 0
 - If this node is the goal, select it as the solution
 - Use FIFO Queue called frontier to store nodes to be explored
 - Maintain an explored set to track visited nodes & avoid cycle
2. Loop until solution is found / frontier is empty
 - Remove (POP) the shallowest node from frontier
 - Add its state to the explored set
 - Generate child nodes from available action
 - If a child node is not in the explored set or frontier, insert it into the queue



Use of BFS

- Guaranteed to find the shortest path
 - Works well for unweighted graphs
 - Complete (always finds a solution if one exists)
- Not memory efficient (store all nodes in memory)

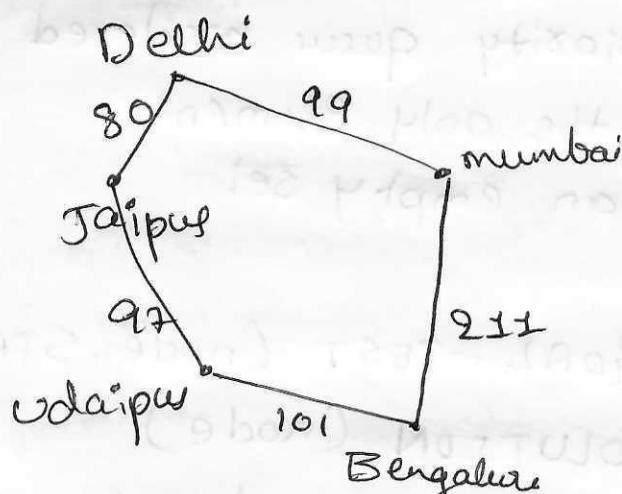


UNIFORM - COST SEARCH

Def Uniform - cost search (UCS) is an search algorithm that finds the least - cost path from start node to goal node.

Work

- Use a priority queue
- Expand the node with the lowest total path cost ($g(n)$) first
- If a newly discovered path to a node is cheaper than the previously found path, update the cost
- The first goal node selected for expansion is guaranteed to be optimal



- (I) Delhi - Jaipur - Udaipur - Bengaluru - 278
 (II) Delhi - Mumbai - Bengaluru - 310.

Ex

- First path from Delhi — Mumbai — Bangalore
Rs. 25,000/-
- Later Delhi — Bhopal — Nagpur — Bangalore
with Rs. 23,000/-
23,000/- cheaper, we update the cost & choose this path.

→ FUNCTION

function UNIFORM-COST-SEARCH (problem)
 returns a solution, or failure

node \leftarrow a node with STATE = problem. INITIAL-STATE, PATH-COST = 0

frontiers \leftarrow a priority queue ordered by PATH-COST
with nodes as the only element

explored \leftarrow an empty set

loop do

 if problem. GOAL-TEST (node. STATE) then
 return SOLUTION (node)

 add node. STATE to explored

 for each action in problem. ACTIONS (node.
 STATE) do



```

child ← CHILD-NODE (problem, node, action)
if child.STATE is not in explored or frontier
then
    frontier ← INSERT (child, frontier)
else if child.STATE is in frontier with higher
    PATH-COST then
    replace that frontier node with child.

```

Expl

1. Initialize the Search

Start with a initial node with cost zero

- Add this node to the priority queue (frontier) which always prioritize the node with the lowest cost.
- Keep an empty explored set to track visited nodes

2. Expand the lowest - cost node

- Remove the node with the smallest path cost from the Queue
- If node is a goal, return the sol?
- otherwise, mark it as explored



3. Explore Child nodes.

for each neighbor (child node) of the current

- if it is not in the explored set or the frontier, add it to the frontier
- If it is in the frontier but with a higher cost, replace it with the cheaper path

4. Repeat until goal is found / no nodes left in the queue.

Depth-First Search

Dep

→ DFS always expands the deepest node in the current frontier of the search tree

→ use LIFO Queue



I. Time Complexity of DFS

In a graph search, DFS explores a maximum of $O(|V| + |E|)$ nodes

↳ number of vertices
 ↳ Number of edges

- In tree search DFS can explore $O(b^m)$
- ↳ branching factor (number of children per node)
- ↳ m → maximum depth of the tree

Use of DFS

- ✓ Space Complexity → DFS stores only $O(bm)$ nodes, whereas BFS requires $O(b^m)$ nodes

↳ dept of shallowest solution
- ✓ Memory Efficiency
- DFS is the foundation for many AI techniques
 - logic programming
 - propositional satisfaction
- ✓ Backtracking Search (help to reduce memory usage)
- ✓ Applications → pathfinding algorithm.
N Queen, Sudoku Solver.



DEPTH - LIMITED SEARCH

Def²

Depth - Limited search (DLS) is a variation of Depth - first search (DFS)

Where the search is restricted to a fixed depth limit l .

Use of Depth - Limited Search

1. Solves the problem of infinite loops in DFS by enforcing a depth limit

2. Uses $O(b^l)$ time complexity

$O(bl)$ → Space "

$b \rightarrow$ branching factor (No. of children / node)

$l \rightarrow$ depth limit

Drawbacks : Incomplete: If the solution is deeper than l , it won't be found

- Non - Optimal → If l is too large, it may find a deeper, suboptimal solution



Function DEPTH - LIMITED - SEARCH (problem, limit)
 returns Solution | failure | cutoff

if problem . is - goal (node . state)
 return node \Rightarrow Solution found

else if limit = 0 then return cutoff

else
 cutoff - occurred \leftarrow false
 for each in problem . actions (node . state)
 for each in problem . result (node . state)
 Child \leftarrow CHILD - NODE (problem, node, action)
 Child \leftarrow RECURSIVE - DLS (Child, problem, limit - 1)
 if result = cutoff then cutoff - occurred?
 \leftarrow true
 else if result \neq failure then return result
 if cutoff - occurred? then return cutoff else
 return failure



ITERATIVE DEEPENING DEPTH-FIRST SEARCH

Def 2

IDDFS is a depth-first search

↳ Combination of Depth first search (DFS) & Breadth-first search

→ It repeatedly runs Depth-limited search (DLS) with increasing depth limit until a solution is found

function ITERATIVE-DEEPENING-SEARCH (problem)
 returns a solution | failure
 for depth = 0 to ∞ do
 result \leftarrow DEPTH-LIMITED-SEARCH
 (problem, depth)
 if result \neq cutoff then return result

Working

Start with depth limit $l = 0$

Run depth-limited search with this limit

If no solution found, increase l & repeat



4. Continue until a solution is found.