DEPARTMENT OF MATHEMATICAL SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY (BHU), VARANASI-221005
ODD SEMESTER 2020 - 2021
MA- 203 : MATHEMATICAL METHODS
ASSIGNMENT – II

[9 February 2021] [M.M. 5]

Instructions

- Write your name, roll number and department clearly at the top of the first page . Start each question on a new page and write your roll number at the top of each page .
- Answers all questions with proper justification on A4 size paper (or Mathematical word-processor) and assemble them into a single pdf file.
- Each question carrying 1 marks. If any question is incorrect or there is any printing mistake, then kindly inform us by a comment in Google-classroom.
- Due date : **13 February 2021 , time : 11:59 PM**. Upload your work to Google-classroom for your tutorial section with your official (@itbhu.ac.in) email account.

Notations

- Fourier transform of φ is notated as $\widehat{\varphi}$ or $\mathcal{F}(\varphi) = f(\xi)$ and its inverse is denoted by $\mathcal{F}^{-1}(f) = \varphi(x)$.

1. Let $\varphi(x) = \begin{cases} x, & -1 \le x \le 1 \\ 0, & otherwise, \end{cases}$ then find the following :

$(i)\ \widehat{\varphi}(0), \quad (ii) \int_{-\infty}^{\infty} \left(\frac{\xi\, cos\xi - sin\xi}{\xi^2}\right)^2 d\xi.$

2. Does Fourier transform of constant function exist? If not, write the suitable existence of it.

3. Exploiting the Fourier transform technique solve the following diffusion equation :

$$\begin{cases} u_t = \alpha\, u_{xx} \\ u(x,0) = \varphi(x), \end{cases} \text{ for } \alpha > 0 \text{ on } (x,t) \in (-\infty,\infty) \times (0,\infty)$$

and find $K(x-\xi,t)$ which satisfies $u(x,t) = \int_{-\infty}^{\infty} K(x-\xi,t)\, \varphi(\xi) d\xi$.

4. Find Fourier sine transform of the following functions :

$(i)\ \varphi(x) = \frac{2}{\pi}\, tan^{-1}\left(\frac{x}{a}\right), a > 0, \quad (ii)\ \sigma(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$.

5. Find $\varphi(x)$ if its Fourier cosine transform is $\sum_{n=0}^{\infty} (-1)^n\, s^{2n}$.

** End of the sheet **