$\Theta\left(n^{\log_b a}\right)$ $\quad$ $\boxed{n^{\log_b a}} \longleftrightarrow f(n)$

$\boxed{n^{\log_4 3}}$

Master Method

$T_n = a\,T_{n/b} + f(n)$

$a \,\&\, b > 1$

i) If $f(n) = O\left(n^{\log_b a - \epsilon}\right)$ for some $\epsilon > 0$

$\Rightarrow T(n) = \Theta\left(n^{\log_b a}\right)$

$T_n = a\,T_{n/b} + f(n)$

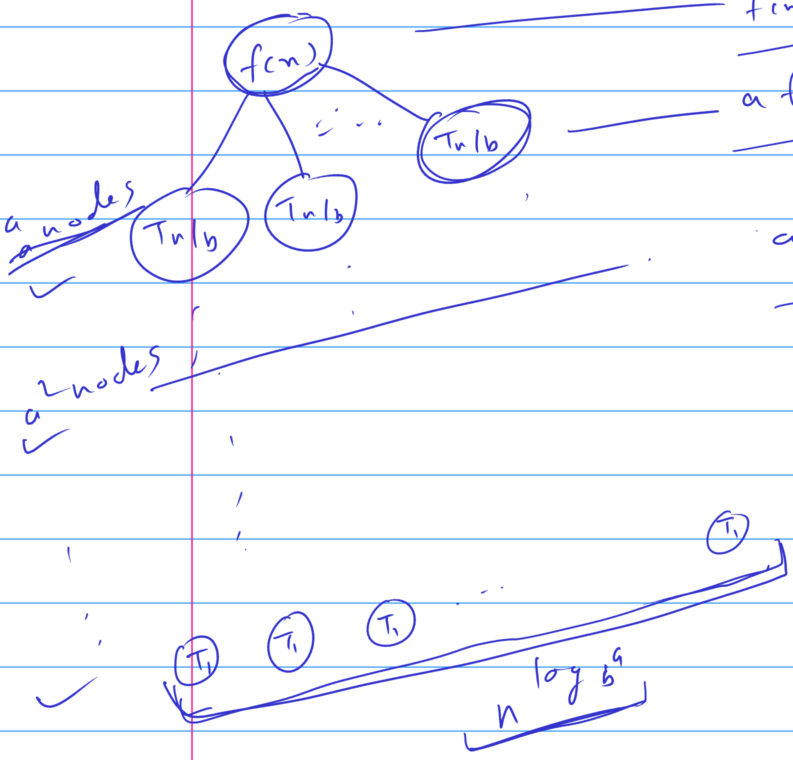
ii) If $f(n) = \Theta\left(n^{\log_b a}\right)$

$\Rightarrow T(n) = \Theta\left(n^{\log_b a} \cdot \log n\right)$

iii) If $f(n) = \Omega\left(n^{\log_b a + \epsilon}\right)$

$\Rightarrow T(n) = \Theta(f(n))$

Proof is left to the students



$f(n)$

$a\,f\left(\frac{n}{b}\right)$

$a^2\,f\left(\frac{n}{b^2}\right)$

$\Theta\left(n^{\log_b a}\right)$

$n^{\log_b a}$

$T_n = a\,T_{n/b} + f(n)$

$n^{\log_b a}$ $\quad$ $f(n)$

Example 1: $\boxed{T_n = 9\,T_{n/3} + n} = \Theta(n^2)$

$a = 9,\ b = 3,\ f(n) = n$

$n^{\log_3 9} = n^2$

Example 2: $\boxed{T_n = T_{\frac{2n}{3}} + 1} = \Theta\left(n^0 \cdot \log n\right) = \Theta(\log n)$

$a = 1,\ b = 3/2,\ f(n) = 1$

$n^{\log_{\frac{3}{2}} 1} = 1$

Example 3: $\boxed{T_n = 3\,T_{n/4} + n \log n}$ $\quad$ $n^{\log_4 3}$

$= \Theta(n \log n)$

Binary multiplication

1) $\Theta(n^2)$] D & C

2) $= \Theta(n^{1.6})$

$\boxed{T_n = 4\,T_{n/2} + cn}$ $\quad n^{\log_2 4} \to n^2$

X: A | B

Y: C | D

$\boxed{T_n = 3\,T_{n/2} + cn}$

$n^{\log_2 3} \simeq n^{1.6}$

Matrix multiplication

$T_n = 8\,T_{n/2} + cn^2$

$n^{\log_2 8} = n^3 \qquad = \Theta(n^3)$

Strassen's algo: $T_n = 7\,T_{n/2} + cn^2$

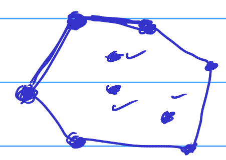
$\cong O(n^{2.81})$

$n^{\log_2 7} \cong n^{2.8}$

Convex Hull Problem

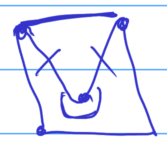
I/P $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

O/P:

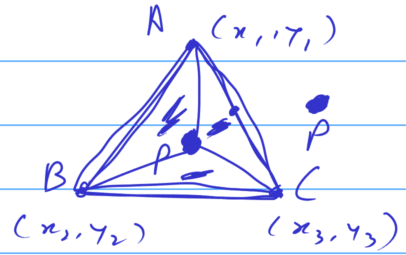


interior points

exterior points



Problem:



$P(x, y)$

Sol^n

Exte $\rightarrow$

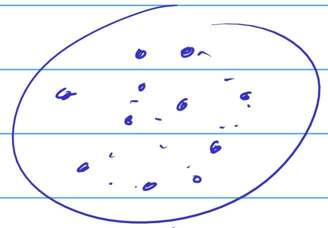
Inter $\rightarrow$ Area($\triangle ABC$) =

Area($\triangle ABP$) + Area($\triangle BPC$) + Area($\triangle APC$)

$O(1)$

Convex Hull

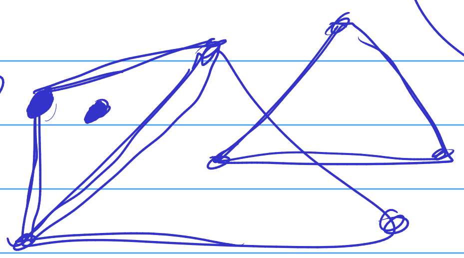
n points



$f(n) = {}^nC_3 = \Theta(n^3)$

${}^nC_3$ triangles $= O(n^3)$ triangles

P



$P(x,y)$ n points

If P is not there in any of the triangles → then P is an exterior point

Q^n: Time complexity

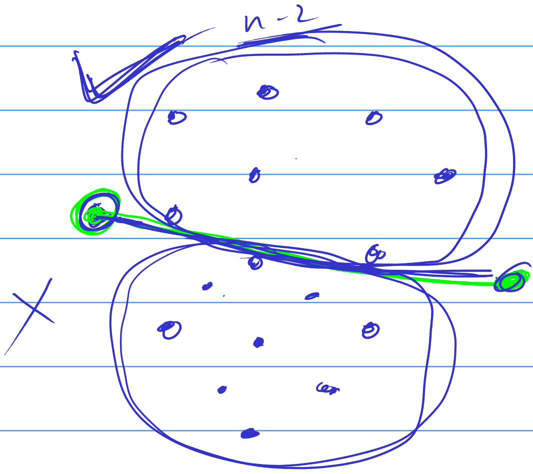
$n^3 \cdot n$

$O(n^4)$

Convex Hull Lab Exp (Next)

D & C Sol^n

$n-2$



X_{min} X_{max}

Step 1:

Step 2

worst case

$T_n = T_{n-2} + O(n)$

$= O(n^2)$

Greedy Method

Binary Heap

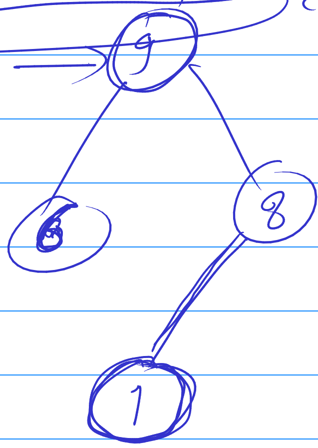
Binary Heap

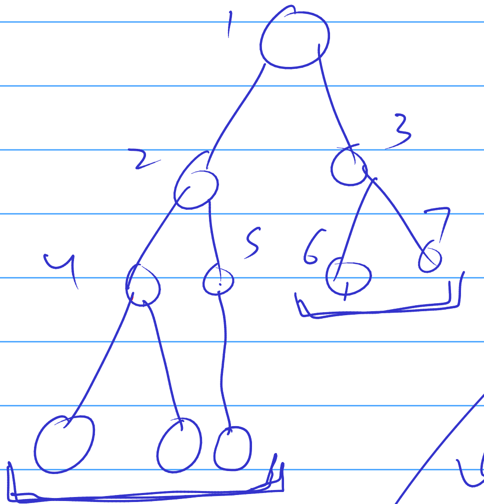
1) —

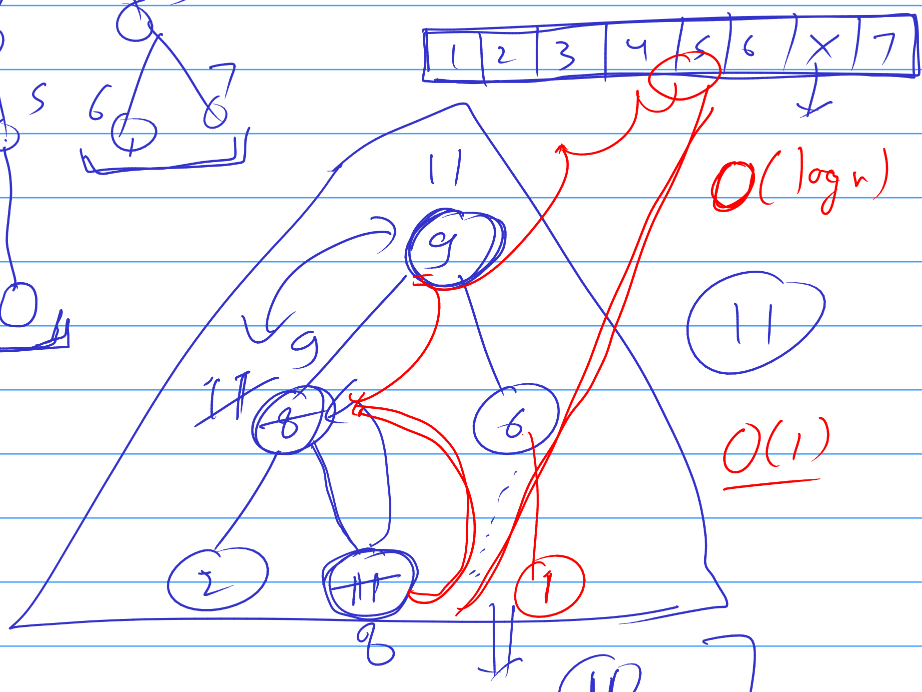
2) — structure
- complete BT
- Almost complete BT

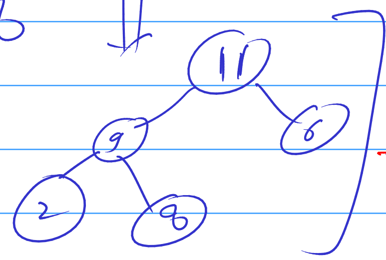
Max Heap — father ≥ children

Min Heap — father ≤ children





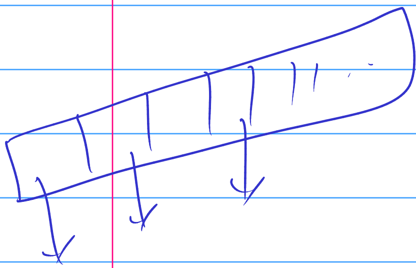


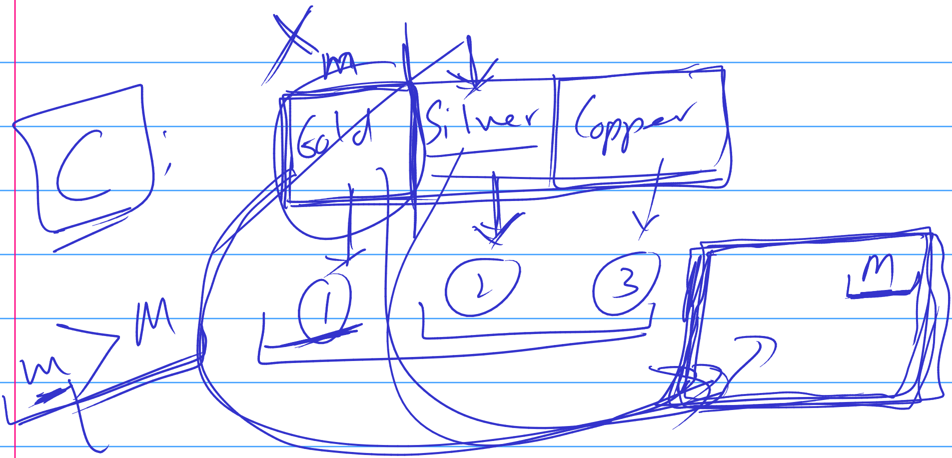


MaxHeap

Greedy method

- optimization problem
- we look for a set of candidates that minimize or maximize the objective
- step by step
- We start with an empty sol^n set
- We have to add the candidates





$m < M$

n $\quad M - m$

$m + n$

$S = \{ \phi \}$