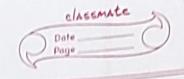


Mathematical Methods MA-203 Assignment -1 Name: Vanshika Kejsimal Roll No: 20075095 Branch: computer Science and Engineering f(t) = et d [tre-t] Laplace of t'et, Using the property L $e^{at} f(t) = f(s-a)$ $\frac{L\left[+^{n}e^{-t}\right] = \frac{\left[n+1\right)}{\left(s+1\right)^{n+1}}}{\left(s+1\right)^{n+1}}$ Next using the derivative property laplace; $L\left(\frac{d^{2}+d^{2}+d^{2}}{d^{2}+d^{2}}\right) = S^{2}L\left(\frac{d^{2}+d^{2}}{d^{2}+d^{2}}\right) - S^{2}H\left(\frac{d^{2}+d^{2}+d^{2}}{d^{2}+d^{2}}\right) - S^{2}H\left(\frac{d^{2}+d^{2}+d^{2}}{d^$ We can charly see that all norder desirative of f(t) become 0 at t=0 From Now to calculate L(d' g(t) we put the Values in eq (1) to get: $L\left[\frac{d^{n}}{dt^{n}}t^{n}e^{-t}\right] - \frac{s^{n}(n+1)}{(s+1)^{n+1}}$



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Again we use the property L[eat f(t)]=f(s-a) So [Let do (to e-) = (5-1) (0+1)

Oriner equation is 2 4 - AS(x-1)

Taking laplace transform both the sides:

 $L\left[\frac{d^{4}u}{dx^{4}}\right] = AL\left[\delta\left(x-L\right)\right]$

RHS: we know that Laplace transform of 8 (dirac -delta) is equal to 1

using the second shifting theorem of laplace transform which states that; $L[G(t)] = e^{-\alpha s}f(s) \text{ were } G(t) = \{f(t-\alpha) \mid t\}, \alpha$

 $L\left[S(x-1)\right] = e^{\frac{1}{4}S}$ [f(s) = 1 for dirac delta]

·· RHS = Ae年 Using the Laplace of desinatives for 4th order;

LHS becomes: SAL[u(x)] - SAI u(0) - S4-2 u'(0) - S4-3 u'(0) - \$ u''(0)

Putting the given values: u(0) = 0 and u''(0) = 0Also let u'(0) = B and u'''(0) = CLHS = $S''L(u(x)) - BS^2 - C$

Vanshika

The orierall equation becomes;

 $S^{4}L[u(x)] - BS^{2} - C = Ae^{\frac{Lx}{4}}$ $= 2 L[u(x)] = Ae^{\frac{Lx}{4}} + BS^{2} + C$ $= 3^{4}$ $= 2 L[u(x)] = Ae^{\frac{Lx}{4}} + B + C$ $= 3^{4}$

 $L[u(x)] = A e^{\frac{13}{4}} \frac{3!}{5^2} + B \frac{5}{5^2} + \frac{3!}{6 5^4}$

Taking laplace inverse on both sides: $u(x) = A(x+1)^3 + Bx + Cx^3$

Putting u(l) = 0 and u'(l) = 0 in the above equation: $0 = A (51)^3 + Bl + Cl^3 - - 0$

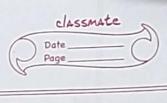
 $u'(x) = A(x+1)^2 + B + Cx^2$

 $= \frac{1}{2} \left(\frac{51}{4} \right)^2 + 3 + \frac{1}{2} \left(\frac{2}{3} - \frac{1}{3} \right)$

Derviding (1) by e on noth sides:

Subtracting (1) from (1): $0 = A \cdot 25 \cdot 1^2 - A \cdot 125 \cdot 1^2 + C \cdot 1^2 - C \cdot 1^2$ $2 \cdot 16 \cdot 6 \cdot 64 \cdot 2 \cdot 6$

 $= \frac{1}{3} - \frac{125}{3} - \frac{125}{32} = \frac{125}{6x64}$

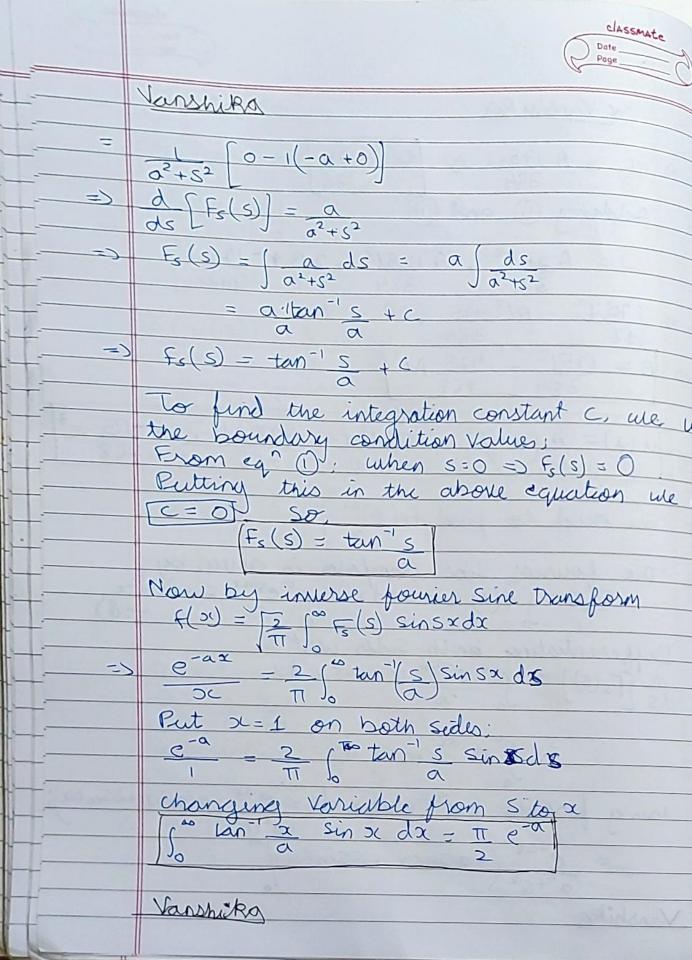


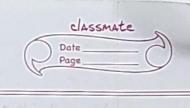
3 3 Varshika $\frac{-3}{3} - \frac{-175}{384} = \frac{-175}{128}$ Adding (1) and (11) $0 = A 251^{2} + A 1251^{2} + 28 + -1751^{2}$ 32 384 182 $u(x) = A \left(x+1\right)^3 + 1751^2 x - 425A1^2 x + -175 x^3$ $6 \left(4\right)^3 + 384 + 768 + 768$ 3) we need to find Fs[e-ax] The fourier Sine Transform is given by:

F_S(S) = S F(X) Sin SX dX = Setax sin Sx dx

L(Differentiating both sides with s;

de [Fs(s)] = 5 \int \int \cossx dsi = for cos sx dx Using formula: Jeax cosbx dx = eax (acosbx +bsinbx) $= \frac{e^{-\alpha z}}{\alpha^2 + s^2} \left(-\alpha \cos s \propto + s \sin s \propto \right)^{\alpha}$





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Girlen: f(x) = e-ax

To fried journer cosine transform: Fe(s) = \frac{7}{17} \left(x) cos sx dx

= 1 100 c-ax cossx dx

Drawing an analogy with laplace transform and wring the second shifting theorem we get

 $F_{c}(f(x)) = \begin{cases} 2 & \alpha \\ \sqrt{11} & \alpha^{2} + S^{2} \end{cases} \qquad [-1] (\cos \alpha x) = \frac{S}{\alpha^{2} + S^{2}}$

Let $g(x) = \begin{cases} 1 & 0 < x < b \\ 0 & x > b \end{cases}$ $\Rightarrow F_{\epsilon}(g(x)) = \int_{\pi}^{2} \int_{0}^{\infty} g(x) \cos x \, dx$ [Definition]

= $\int \frac{2}{\pi} \int_{0}^{\pi} \cos 3x \, dx$

 $= \int_{\overline{\Pi}}^{2} \left[\sin sx \right]^{\delta}$

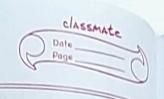
= $\frac{2}{\sqrt{11}} \frac{\sin bx}{\sin bx}$

Now consider $I = \int_0^\infty (t-3) \frac{\sin(t+3) + (t+3) \sin(t-3)}{(t^2+4)(t^2-9)} dt$

 $= \int_{0}^{\infty} \frac{1}{t^{2}+4} \left(\frac{\sin(t+3)}{t+3} + \frac{\sin(t-3)}{t-3} \right) dt$

On repracing t with s in above integral

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where
$$f(x) = e^{-\alpha x}$$
, $\alpha = 2$

$$I = \pi \int_{2}^{\infty} F_{c}(f(x)) \int_{2}^{\infty} \left[\frac{\pi}{2} \frac{\sin(s+3)}{\sin(s+3)} + \frac{\pi}{2} \frac{\sin(s-3)}{s-3} \right] ds$$
We know that:
$$f_{c}(g(x)(\cos \alpha x)) = \int_{2}^{\infty} f_{c}(s-\alpha) + F_{c}(s+\alpha)$$

$$I = \pi \int_{2}^{\infty} F_{c}(s+\alpha) \int_{2}^{$$

$$= \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} F[f(x)] \cdot F[h(x)] ds$$

where
$$h(x) = g(x) \cos 3x$$
, $b=1$
Using the Preserval's identity for the fourier
Cosine transform we have:
 $I = \pi \int_{-\infty}^{\infty} f(x) \cdot h(x) dx$

$$\Gamma = \pi \int_{\infty}^{\infty} e^{-2x} q(x) (x) 3x$$

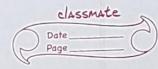
$$T = \frac{\pi}{2} \int_0^\infty e^{-2x} g(x) \cos 3x dx$$

$$= \frac{\pi}{2} \int_{0}^{1} e^{-2x} \cos 3x \, dx \qquad \left[\begin{array}{c} \text{definition of } g(x) \\ \text{is applied} \end{array}\right].$$
Using formula:
$$\int_{0}^{2x} e^{ax} \cos bx = \frac{e^{ax}}{a^{2} + b^{2}} \left[a \cos bx + b \sin bx\right]_{+}^{+}$$

$$T = \frac{\pi}{2} e^{-2x} \left(\frac{1}{2} - \frac{1}{2} - \frac{1}$$

$$I = \frac{1}{2} \left[-\frac{2}{13} \left[-2\cos 3\pi + 3\sin 3x \right]^{1} \right]$$

$$\Gamma = \frac{\pi}{26} \left[-2e^2 \cos 3 + 3e^2 \sin 3 + 2 \right] e \quad \text{Ans}$$



Vanshika 5) Crimen equation: $\frac{\partial u}{\partial t} = \frac{\sqrt{\partial^2 u}}{\partial x^2}$ Since the values U(0,t) or $U_{\infty}(0,t)$ are not given and $SCE(-\infty,\infty)$ we will use the complex fourier transform to Solve the problem. Taking fourier transporm on both sides of equation: $\frac{du}{dt} = -\alpha s^2 u$ Here to represents the fourier transform of u 12 = - (xs2 d+ =) $\log \pi = -\alpha s^2t$ [A is integration constant]

=) $\pi = Ae^{-\alpha s^2t}$ or $\pi(s,t) = Ae^{-\alpha s^2t}$ ---(1)

Putting t = 0 on both sides; $\pi(s,0) = A$ Mso it is given that $u(z,0) = e^{-\alpha s^2t}$ Taking fourier transform on both sides; $\frac{U(SD) = \int_{-\infty}^{\infty} e^{-x^2} e^{iSx} dx}{\sqrt{2\pi} \int_{-\infty}^{\infty} e^{-(x^2 - iSx)} dx}$ = 1 00 - 2c - 15 - 4x Putting (x-is):t da = dt

