Divide & Conquer

- compute n inputs}

k subsets

$1 \le k \le n$

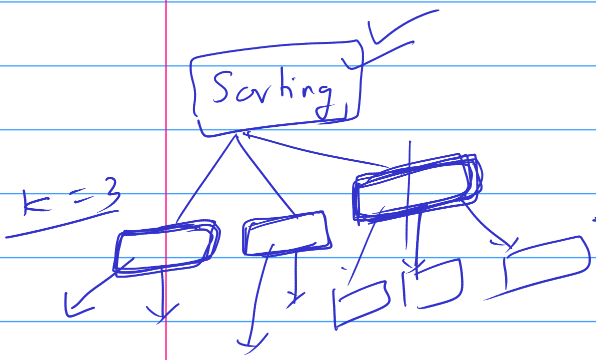
$\boxed{k}$ subproblems

↓ solving

Combine partial results

↓

Overall sol^n



→ Subproblems are of same type

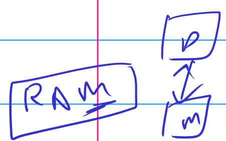
→ Size of a subproblem is smaller than the size of the original problem

→ original problem nature with subproblems

→ subproblems are solved seperately

⇒ DANDC can be expressed as recursive procedure

terminating step



Procedure DANDC(p, q)

global n, A[1, n], int p, q

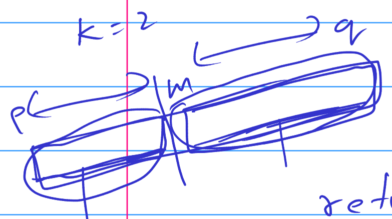
→ if small(p, q)

then return G(p, q) ← straight forward sol^n

else

→ m ← DIVIDE(p, q) $p \le m \le q$

return COMBINE(DANDC(p, m), DANDC(m, q))



Finding the maximum and minimum

A

1 2 n

max = min = A[1]

max

min

for i = 2 to n

Compare max/min, A[i]

A[i] > max

max ← A[i]

A[i] < min

min ← A[i]

2(n−1) comparisons

Best case n−1

(A) ⟹ $O(n)$

⇓ Rewritten

(B) ⟶ $O(n)$

1) Split
2) Solving subproblems
3) Combine

1 2 ... mid ... n

max_1, min_1 max_2, min_2

→ $mid = \lfloor \frac{lb+ub}{2} \rfloor$

$O(1)$

$k=2$

$\frac{n}{2}$ max ← MAX (max_1, max_2)

min ← MIN (min_1, min_2)

$n = 2^k$ ← +ve integer

$n = 3^k$ $n = b^k$ $k=2$

$n = 8 = 2^3$

| 1 | 4 | 5 | 6 | 3 | 2 | 10 | 9 |

1st subproblem $n = 4$

2nd subproblem $n = 4$

straight form

$n = 1$ → max = min = A[i]

$n = 2$

$min_1 = 1$ | $min_2 = 5$

$max_1 = 4$ | $max_2 = 6$

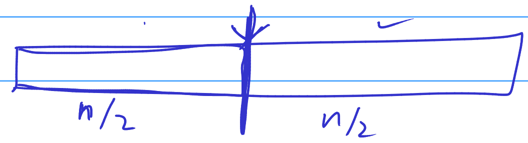
$max_1 = 6$
$min_1 = 1$

$max_2 = 10$
$min_2 = 2$

max = 10
min = 1

Time complexity of D&C method for MINMAX

Recurrence relation



$n = 2^k$

$T_n = c_2 + T_{n/2} + T_{n/2} + c_1$

$\Downarrow$ $\boxed{T_n = 2T_{n/2} + c}$ $\boxed{T_1 = 1c}$

$T_n = 2T_{n/2} + c$

$= 2(2T_{n/4} + c) + c$ (substitution method)

$=$

$T_{n/2^3}$

$T_{n/2^k} \rightarrow T_1$

$n = 2^k$

$=$

$= \boxed{nT_1 + (n-1)c} = \boxed{O(n)}$

$= n + nc - c$

16
↓
8
↓
4
↓
2
↓
1

Next lab experiment: D&C implementation

Multiplication of 2 n-bit numbers

0111

$n = 2^k$

$$\begin{array}{r} 1101 \\ \times\ 0101 \\ \hline 1101 \\ 0000 \\ 1101 \\ 0000 \\ \hline \end{array}$$

addition

X, Y $*$ $\quad Z = X * Y$

$$\begin{array}{rcccc} & x_n & x_{n-1} & \cdots & x_1 \\ * & y_n & y_{n-1} & \cdots & y_1 \\ \hline \end{array}$$

$O(n^2)$ multiplications

$O(n^2)$ addition

$O(n^2)$

$M * 2^n$

$M\,000\cdots0$

X	A	B
Y	C	D

$A\,\underbrace{00\cdots0}_{n/2\ 0\text{'s}} + B$

$$X = A * 2^{n/2} + B$$

$$Y = C * 2^{n/2} + D$$

$$Z = X * Y = (A \cdot 2^{n/2} + B)(C \cdot 2^{n/2} + D)$$

$Z = X * Y$

$$Z = \underset{1}{AC} \cdot 2^n + (\underset{2}{AD} + \underset{3}{BC}) \cdot 2^{n/2} + \underset{4}{BD}$$

I. $n/2$ bit multiplications (4 Nos.)

II. Shift opn $(O(n))$

III. Binary addition $O(n)$

$$T_n = 4T_{n/2} + O(n)$$

$$= O(n^2)$$