MA-203 Combined Quiz-1

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Q1.

Ans: $L\left[e^{3t}\sinh(2t)\sqrt{1+\sin(2t)}\right]$

$$L\left[e^{3t}\underbrace{\sinh(2t)}_{\frac{e^{2t}-e^{-2t}}{2}}\sqrt{1+\sin(2t)}\right]$$

$$\Rightarrow L\left[e^{3t}\left(\frac{e^{2t}-e^{-2t}}{2}\right)\sqrt{1+\sin(2t)}\right]$$

$$\Rightarrow \frac{1}{2}L\left[\left(e^{5t}-e^{t}\right)\sqrt{1+\sin(2t)}\right]$$

$$\left\{ 1+\sin 2t = 1+2\sin t\cdot\cos t = \cos^2 t+\sin^2 t+2\sin t\cdot\cos t \atop = (\cos t+\sin t)^2 \right\}$$

$$\Rightarrow \frac{1}{2}L\left[\left(e^{5t}-e^{t}\right)(\sin t+\cos t)\right]$$

$$\Rightarrow \frac{1}{2}L\left[e^{5t}(\sin t+\cos t)\right]-\frac{1}{2}L\left[e^{t}(\sin t+\cos t)\right]$$

Applying shifting theorem.

$$\Rightarrow \frac{1}{2}\left(\frac{1}{(s-5)^2+1}+\frac{s-5}{(s-5)^2+1}\right)-\frac{1}{2}\left(\frac{1}{(s^2-1)^2+1}+\frac{s-1}{(s-1)^2+1}\right)$$

As $\left\{ L\sin at=\frac{a}{s^2+a^2}\ ;\ L(\cos at=\frac{s}{s^2+a^2}\right\}$

$$\Rightarrow \frac{1}{2}\left(\frac{s-4}{(s-5)^2+1}-\frac{s}{(s-1)^2+1}\right)$$

$$\text{Ans} - \boxed{\left[\frac{1}{2}\left\{\frac{s-4}{(s-5)^2+1}-\frac{s}{(s-1)^2+1}\right.\right]}$$

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Q2 -

Ans - $$\int_0^\infty e^{-\sqrt{2}t} \frac{\sinh(t)\ \sin(t)}{t} \cdot dt$$

$\Rightarrow$ The integral can be written as,

$$\Rightarrow L\left\{\frac{\sinh t \cdot \sin t}{t}\right\} \quad \text{and} \quad s = -\sqrt{2}.$$

$$\Rightarrow \text{As,} \quad \sinh t = \frac{e^{t} - e^{-t}}{2}.$$

$$\Rightarrow L\left\{\frac{e^{t}\sin t}{2t} - \frac{e^{-t}\sin t}{2t}\right\}$$

$$\Rightarrow \frac{1}{2} L\left\{\frac{e^{t}\sin t}{t}\right\} - \frac{1}{2} L\left\{\frac{e^{-t}\sin t}{t}\right\}$$

$$\Rightarrow L\{\sin t\} = 1/s^2+1.$$

In t-division property, $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(y)\,dy$.

So, here $L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{y^2+1}\,dy = \tan^{-1}s\Big|_s^\infty = \frac{\pi}{2} - \tan^{-1}(s)$

Taking actual Integral,

$$\cancel{I = \frac{1}{2}\int_0^\infty e^{-(\sqrt{2}-1)t}\frac{\sin t}{t}\,dt - \frac{1}{2}\int_0^\infty e^{-(\sqrt{2}+1)t}\frac{\sin t}{t}\cdot dt}$$

$\cancel{\text{So, } s = \sqrt{2}-1} \qquad \cancel{s = \sqrt{2}+1}$

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By applying shifting property

$\Rightarrow \frac{1}{2} L\left\{ e^{t} \frac{\sin t}{t} \right\} - \frac{1}{2} L \left\{ e^{-t} \frac{\sin t}{t} \right\}$

$\Rightarrow \frac{1}{2}\left[\cancel{\frac{\pi}{2}} - \tan^{-1}(s-1) - \cancel{\frac{\pi}{2}} + \tan^{-1}(s+1) \right]$

$\Rightarrow \frac{1}{2}\left[\tan^{-1}(s+1) - \tan^{-1}(s-1) \right]$

$\Rightarrow \frac{1}{2}\left[\tan^{-1}(\sqrt{2}+1) - \tan^{-1}(\sqrt{2}-1) \right]$

$\Rightarrow \boxed{\frac{\pi}{8}}$

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Q3.

Ans.

$$L^{-1}\left\{\frac{s^2}{s^4+4a^4}\right\}$$

$$\Rightarrow L^{-1}\left\{\frac{s^2}{(s^2-2as+2a^2)(s^2+2as+2a^2)}\right\}$$

$\Rightarrow$ Rewriting the terms as Partial Fractions.

$$\Rightarrow L^{-1}\left\{\frac{1}{4a}\left[\frac{s}{(s^2-2as+2a^2)}-\frac{s}{(s^2+2as+2a^2)}\right]\right\}$$

$$\Rightarrow \frac{1}{4a}L^{-1}\left\{\frac{s}{(s^2-2as+a^2)+a^2}-\frac{s}{(s^2+2as+a^2)+a^2}\right\}$$

$$\Rightarrow \frac{1}{4a}L^{-1}\left\{\frac{s}{(s-a)^2+a^2}-\frac{s}{(s+a)^2+a^2}\right\}$$

$$\Rightarrow \frac{1}{4a}L^{-1}\left\{\frac{s}{(s-a)^2+a^2}\right\}-\frac{1}{4a}L^{-1}\left\{\frac{s}{(s+a)^2+a^2}\right\}$$

First shifting theorem applied.

$$\Rightarrow \frac{1}{4a}\left\{L^{-1}\left(\frac{s-a}{(s-a)^2+a^2}\right)+L^{-1}\left(\frac{a}{(s-a)^2+a^2}\right)-L^{-1}\left(\frac{s-(-a)}{(s-(-a))^2+a^2}\right)+L^{-1}\left(\frac{a}{(s-(-a))^2+a^2}\right)\right.$$

$$\Rightarrow \frac{1}{4a}\left\{e^{at}\cos(at)+e^{at}\sin at-e^{-at}\cos(at)+e^{-at}\sin(at)\right\}$$

$$\Rightarrow \frac{1}{4a}\cdot\cos(at)\left(e^{at}-e^{-at}\right)+\frac{\sin(at)}{4a}\left(e^{at}+e^{-at}\right)$$

$$\Rightarrow \frac{1}{2a}\left[\cos(at)\left(\frac{e^{at}-e^{-at}}{2}\right)+\frac{\sin(at)}{\cancel{4a}}\left(\frac{e^{at}+e^{-at}}{2}\right)\right]$$

$$= \boxed{\frac{1}{2a}\left\{\cosh(at)\sin(at)+\sinh(at)\cos(at)\right\}}$$

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Q4.

Ans-

$$\frac{dy}{dt} + 4y = \delta(t-2) \qquad y(0) = 6.$$

Taking LT on both sides.

$$L\left[\frac{dy}{dt}\right] + 4L(y) = L(\delta(t-2))$$

$$\left\{ \because L(\delta(t)) = 1 \Rightarrow L(\delta(t-2)) = e^{-2s} \right]$$

~~$L(\delta(t))$~~

$$sL(y) - y(0) + 4L(y) = e^{-2s}.$$

$$L(y) = \frac{e^{-2s} + y(0)}{4+s} = \frac{e^{-2s} + 6}{4+s} \text{ (substituting).}$$

Taking Inverse.

$$y = L^{-1}\left(\frac{e^{-2s}}{4+s}\right) + L^{-1}\left(\frac{6}{4+s}\right)$$

$$\because \left\{ L^{-1}\left(\frac{1}{4+s}\right) = e^{-4t} \right\}$$

Using second shifting,

$$\therefore y = \begin{cases} e^{-4(t-2)} + 6e^{-4t} & ; t > 2 \\ 0 + 6e^{-4t} & ; t < 2. \end{cases}$$

$$= \begin{cases} e^{-4t+8} + 6e^{-4t} & , t > 2 \\ 6e^{-4t} & , t < 2. \end{cases}$$

for $t = 2$.

$$\lim_{t \to 2^-} y(t) = 6e^{-8}$$

$$\lim_{t \to 2^+} y(t) = e^{0} + 6e^{-8} = 1 + 6e^{-8}.$$

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As, Left hand limit $\neq$ right hand limit

Hence, the function is not continuous at $t=2$.

i.e. $\boxed{\text{function } y(t) : [0,3] \rightarrow (-\infty, \infty) \text{ is not continuous}}$

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Q5.

Ans- $y'' + 2y' + 5y = e^{-t}\sin(t) \qquad y(0) = 0 \quad \& \; y'(0) = 1.$

Applying the Laplace Transformation.

$$L\{f(t)\} = \int_0^\infty f(t)\, e^{(-st)}\, dt$$ to both the sides.

$$L\left\{ \frac{d^2 Y(t)}{dt^2} + 2\frac{dY(t)}{dt} + 5Y(t) \right\}(s) = L\left\{ e^{(-t)} \sin(t) \right\}$$

$$\Rightarrow \left[s^2\bar{y} - sy(0) - \bar{y}(0)\right] + 2\left[s\bar{y} - y(0)\right] \cancel{+} 5\bar{y} = L\left(e^{-x}\sin x\right)$$

$$\left[s^2\bar{y} - sy(0) - y'(0)\right] + 2\left[s\bar{y} - y(0)\right] + 5\bar{y} = \frac{1}{(s+1)^2 + 1}.$$

Substituting values of $y(0)$ and $y'(0)$;

$$(s^2\bar{y} - 1) + 2(s\bar{y}) + 5\bar{y} = \frac{1}{s^2 + 2s + 2}$$

$$\Rightarrow (s^2 + 2s + 5)\bar{y} = 1 + \frac{1}{s^2 + 2s + 2} = \frac{s^2 + 2s + 3}{s^2 + 2s + 2}.$$

$$\Rightarrow \bar{y} = \frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$

Rewriting into Partial fractions,

$$\bar{y} = \frac{2}{3} \frac{1}{s^2 + 2s + 5} + \frac{1}{3} \frac{1}{s^2 + 2s + 2}$$

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Taking Inverse.

$$y = \frac{2}{3} L^{-1}\left\{\frac{1}{s^2+2s+5}\right\} + \frac{1}{3} L^{-1}\left\{\frac{1}{s^2+2s+2}\right\}$$

$$y = \frac{1}{3} L^{-1}\left\{\frac{2}{(s+1)^2+(2)^2}\right\} + \frac{1}{3} L^{-1}\left\{\frac{1}{(s+1)^2+(1)^2}\right\}$$

$$\Rightarrow \quad y = \frac{1}{3} e^{-t} \sin 2t + \frac{1}{3} e^{-t} \sin t$$

$$\Rightarrow \quad \boxed{y = \frac{1}{3} e^{-t} (\sin t + \sin 2t)}$$