MA-203 Quiz-1

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Q1.

Ans - $f(s) = \log\left(1 - \frac{a^2}{s^2}\right)$

$L\left\{\log\left(1-\frac{a^2}{s^2}\right)\right\}$

$L\{t f(t)\} = -F'(s) = \frac{-d}{ds}\log\left(1-\frac{a^2}{s^2}\right)$

$= \frac{-d}{ds}\{\log(s-a) + \log(s+a) - 2\log(s)\}$

$\Rightarrow F'(s) = \frac{1}{s-a} + \frac{1}{s+a} - \frac{2}{s} = L\{t f(t)\}$.

Taking laplace inverse on both sides,

$L^{-1}(F'(s)) = L^{-1}\left\{\frac{1}{s-a}\right\} + L^{-1}\left\{\frac{1}{s+a}\right\} - L^{-1}\left\{\frac{2}{s}\right\}$

$L^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

$L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$

$L^{-1}\left\{\frac{2}{s}\right\} = 2L^{-1}\left\{\frac{1}{s}\right\} = 2.$

Known values.

$\therefore F'(s) = e^{at} + e^{-at} - 2.$

Now,

$L^{-1}\{L\{t f(t)\}\} = e^{at} + e^{-at} - 2.$

$$\boxed{\therefore L^{-1}\{f(s)\} = \frac{-1}{t}\left(e^{at} + e^{-at} - 2\right)}$$

Camlin

Q2.

Ans - $y'' + 9y = \cos 2t \quad y\left(\frac{\pi}{2}\right) = -1 \quad y(0) = 1.$

Applying L.T. and its linearity property, we get above

$L(y'') + 9L(y) = L(\cos 2t)$

$\Rightarrow s^2\bar{y} - sy(0) - y'(0) + \bar{y} = \frac{s}{s^2+4}.$

$\Rightarrow$

on solving we have,

$\frac{1}{5}\cos 2t - \frac{1}{5}\cos 3t + \cos 3t + y'(0)\frac{\sin 3t}{3}.$

$y(t) = \frac{\cos 2t}{5} + \frac{9}{5}\cos 3t + y'(0) \cdot \frac{\sin 3t}{3}.$

for $t = \frac{\pi}{2} \quad y\left(\frac{\pi}{2}\right) = -1$

$-1 = \frac{-1}{5} + 0 + y'(0)\left(\frac{-1}{3}\right)$

$\left(\frac{1}{5} - 1\right) = \frac{-y'(0)}{3} = -\frac{4}{5}$

$y'(0) = \frac{12}{5}$

$\therefore \boxed{y(t) = \frac{\cos 2t}{5} + \frac{4}{5}\cos 3t + \frac{4}{5}\sin(3t)}$