$n = 2^k \rightarrow$ +ve integer

$: O(n^2) \Rightarrow$ straightforward

D&C

$O(n^2)$

$X = \boxed{A \mid B} = A \cdot 2^{n/2} + B$

$Y = \boxed{C \mid D} = C \cdot 2^{n/2} + D$

$Z = X \cdot Y = (A \cdot 2^{n/2} + B)(C \cdot 2^{n/2} + D)$

n bit multiplication

$= AC \cdot 2^n + (AD + BC) \cdot 2^{n/2} + BD$

$\frac{n}{2}$ bit multiplication (04 Nos.)

shift/addition: $\Theta(n)$

$$\boxed{T_n = 4\, T_{n/2} + O(n)}$$

$= O(n^2)$

$T_n = 3\, T_{n/2} + O(n)$

$\simeq O(n^{1.59})$

$Z = X \cdot Y = AC \cdot 2^n + (AD + BC) \cdot 2^{n/2} + BD$

03

$U = (A + B) * (C + D) \rightarrow 01$ $\frac{n}{2}$ bit multi—

$V = A * C \rightarrow 01$ $\frac{n}{2}$

$W = B * D \rightarrow 01$

$Z = V \cdot 2^n + (U - V - W) \cdot 2^{n/2} + W$

$$\boxed{Z = 2\, T_{\frac{n}{2}} + O(n)}$$

$T_n = 3\, T_{n/2} + O(n)$

$\simeq O(n^{1.59})$

$n \times n$ n^2 elements

$n = 2^k$

Matrix Multiplication

I/p: A & B

O/p: $C = A \times B$ ↓ n^2 elements

Simplest approach

for i = 1 to n

for j =

for k

3-d loop

$$C[i,j] = \sum_{1 \le k \le n} A[i,k] * B[k,j] \rightarrow O(n)$$

$$n^2 \cdot O(n) \Rightarrow O(n^3)$$

C →

$O(n^3)$ ⟷ $O(n^3)$

D & C soln

A, B $(n \times n)$ → n^2 elements

C $(n \times n)$

$C = A \times B$

n size multiplication

$\frac{n}{2}$

$n = 2^K$

$C = A \times B$

$\frac{n}{2} \times \frac{n}{2} = \frac{n^2}{4}$ elements

$C = A \times B$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

($n/2$)

$$C_{11} = A_{11} B_{11} + A_{12} \cdot B_{21}$$
$$C_{12} = A_{11} B_{12} + A_{12} B_{22}$$
$$C_{21} = A_{21} B_{11} + A_{22} B_{21}$$
$$C_{22} = A_{21} B_{12} + A_{22} B_{22}$$

matrix addition $O(n^2)$

matrix subtract

4

3

$$T_n = 8 T_{n/2} + O(n^2)$$
$$= O(n^3)$$

$$T_n = 7 T_{n/2} + O(n^2)$$
$$= O(n^{2.81})$$

U
V
W

$$2 = x \cdot y$$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Volker Strassen's matrix multiplication

$$P = (A_{11} + A_{22})(B_{11} + B_{22})$$ 01, $\frac{n}{2}$ size mult.

$$Q = (A_{21} + A_{22}) B_{11}$$ 01,

$$R = A_{11}(B_{12} - B_{22})$$ 01

$$S = A_{22}(B_{21} - B_{11})$$

$$T = (A_{11} + A_{12}) B_{22}$$

$$U = (A_{21} - A_{11})(B_{11} + B_{12})$$

$$V = (A_{12} - A_{22})(B_{21} + B_{22})$$

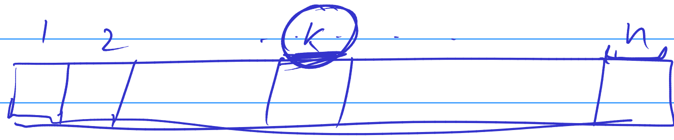
$$C_{11} = P + S - T + V$$
$$C_{12} = R + T$$
$$C_{21} = Q + S$$
$$C_{22} = P + R - Q + U$$

$$T_n = 7T_{n/2} + O(n^2)$$
$$\simeq O(n^{2.81})$$

- Quick Sort
- Merge Sort
- Binary Search

Prob: Finding the k^{th} smallest elements

Sol^n



1) Merge sort

$O(n \log n)$

Heap

2) Build_MaxHeap

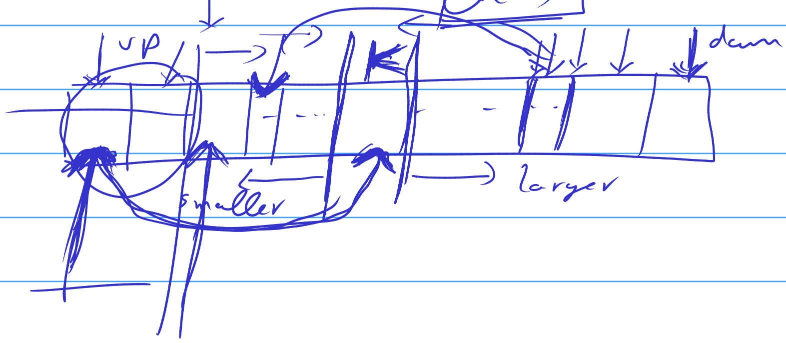
$\boxed{O(n \log n)}$

3) Quicksort X

$\boxed{O(n^2)}$

Partition → $\Theta(n)$

$\Omega(n)$



Next Exp: Volker Strassen Algo. for mm