End Semester Examination - April 2022
CSO322 - Theory of Computation
Department of Mathematical Sciences
Indian Institute of Technology (BHU) Varanasi

Time: 24 **Hours** **Marks: 35**

Instructions to students:

- All the questions are compulsory.
- Write your name, roll number and signature on every page of the assignment.
- Write each and every step clearly and show all your workings for proper evaluation.
- Each question should begin on a new page.
- Upload a s a single .pdf file in the TEAMS portal before the deadline. No excuses shall be entertained.

PART A ($1 \times 5 = 5$)

1. Construct the string $aaabbabbba$ from the grammar $S \to aB|bA$, $A \to a|aS|bAA$, $B \to b|bS|aBB$ by using (i) leftmost derivation and (ii) rightmost derivation.
2. Check whether the grammar with production rules $\{S \to a|Sa|bSS|SbS|SSb\}$ is ambiguous or not. Justify your answer.
3. State True or False and Justify your answer: Suppose L_1 and L_2 are Context-free languages, then the intersection of L_1 and L_2 is also Context-free.
4. Describe the conditions to declare a string to be accepted by PDA.
5. An ordinary Turing machine can also be simulated by one in which the transition function takes values in $(Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\} \cup \{L, R\})$, so that writing a symbol and moving the tape head are not both allowed on the same move. Explain how the move $\delta(p, a) = (q, b, R)$ of an ordinary Turing machine could be simulated by the above described restricted Turing machine.

PART - B

1. Describe the Chomsky classification of grammar. Mention the languages produced from each type of grammar along with the corresponding machine format for recognizing the same. (2)
2. (a) Construct a CFG to generate the set of strings of 0 and 1 where consecutive 0 can occur but no consecutive 1 can occur. (2)

   (OR)

   (b) Convert the following grammar into CNF:
   $S \to aA|B|C|a$
   $A \to aB|E$
   $B \to aA$
   $C \to cCD$
   $D \to abd$

3. Prove that a language is accepted by a PDA by empty stack if and only if the language is accepted by a PDA by final state. (3)

4. Construct a PDA to accept the language $L = \{a^n b^m c^m d^n : m, n \geq 1\}$ (2)

5. Define a Deterministic Push Down Automata. Test whether the following PDA is deterministic or not? Justify your answer. (2)

   $P = (\{q_0, q_1\}, \{0, 1\}, \{X, Y, Z\}, q_0, Z_0, \{q_1\}, \delta)$ where the transition is defined as:
   $\delta(q_0, 0, Z) = \{(q_1, Z)\}$, $\delta(q_0, 1, Z) = \{(q_0, XZ)\}$, $\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$,
   $\delta(q_0, 1, X) = \{(q_0, XX)\}$, $\delta(q_0, 0, Z) = \{(q_1, YZ)\}$, $\delta(q_1, 1, Z) = \{(q_1, Z)\}$,
   $\delta(q_1, 0, Y) = \{(q_1, YY)\}$, $\delta(q_1, 1, Y) = \{(q_1, \epsilon)\}$

6. Prove or Disprove: $L = \{ss : s \in \{a, b\}^*\}$ is Context-free language. (3)

7. (a) Describe the language accepted by the Turing Machine shown in Figure 1. (2)

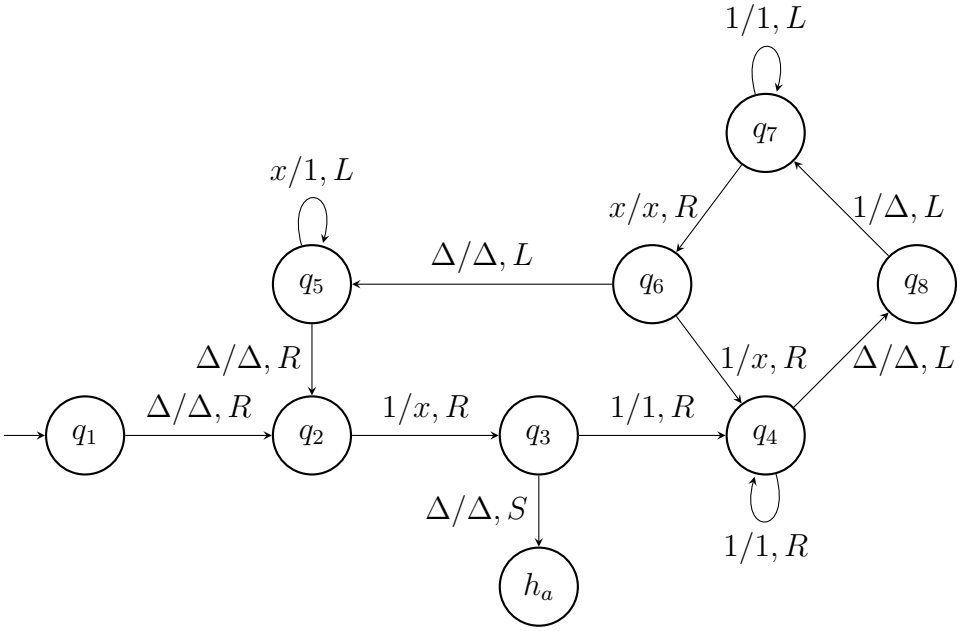

Figure 1: Figure for Question 7a

   (b) Describe the language generated by the unrestricted grammar with the given productions. The symbol a is a terminal and all other symbols are variables. (2)

$$\{S \to LaR, L \to LD|\epsilon, Da \to aaD, DR \to R, R \to \epsilon\}$$

8. Design and Explain: A Turing machine that takes as input a string of 0's and 1's and replaces '0' by '1' and '1' by '0'. (3)

9. Show that the language $\{scs|s \in \{a, b\}^*\}$ on $\Sigma = \{a, b, c\}$ is recursively enumerable. Modify the TM to show that this language is also recursive. (3)

10. (a) Show that the problem ACCEPTS is unsolvable. (3)

    (b) State and Prove Rice's Theorem. (3)

*****ALL THE BEST*****