Mid-Semester Examination - Even Semester 2021-2022
CSO 322 - Theory of Computation
Department of Mathematical Sciences
Indian Institute of Technology (BHU) Varanasi

Time: 1 **Day** **Marks: 25**

Answer all questions

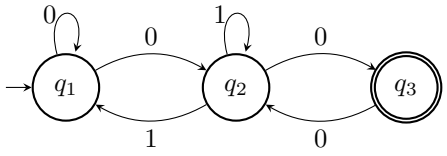
1. Let us say that a string x is obtained from a string w by ***deleting symbols*** if it is possible to remove zero or more symbols from w so that just the string x remains. For example, the following strings can all be obtained from 0110 by deleting symbols: $\epsilon, 0, 1, 00, 01, 10, 11, 010, 011, 110$, and 0110.

   Let $\Sigma = \{0, 1\}$, let $A \subseteq \Sigma^*$ be a regular language, and define

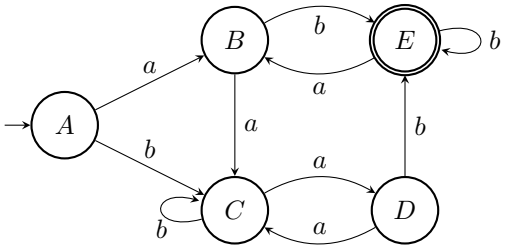
   $$B = \{x \in \Sigma^* : \text{ there exists a string } w \in A \text{ such that } x \text{ is obtained from } w \text{ by deleting symbols}\}.$$

   Prove or Disprove: B is regular. **(2 marks)**

2. Solve by Arden's method to find a regular expression for the FA: **(2 marks)**

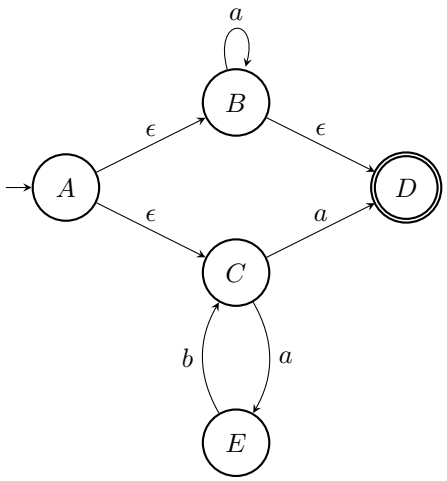


3. Minimize using Myhill Nerode theorem. Also, find the language recognized by the FA. **(3 marks)**



4. Construct a a regular grammar for the miniminum-state finite automata recognizing the regular expression $R = (010)^*1 + (1^*0)^*$ over $\Sigma = \{0, 1\}$. **(3 marks)**

5. For the given $\epsilon - NFA$, find an equivalent finite automata accepting the same language. **(3 marks)**



6. Let $M = (Q, \Sigma, q_0, \delta, A)$ be an $\epsilon - NFA$. Use the recursive definition of ϵ-closure ECLOSE, show that $ECLOSE(S)$ for a set S satisfies the following. **(3 marks)**

   (a) If S and T are subsets of Q, such that $S \subseteq T$, then $ECLOSE(S) \subseteq ECLOSE(T)$.

   (b) If $S, T \subseteq Q$, then $ECLOSE(S \cup T) = ECLOSE(S) \cup ECLOSE(T)$.

   (c) Draw a transition diagram to illustrate the fact that $ECLOSE(S \cap T)$ not always equals $ECLOSE(S) \cap ECLOSE(T)$. What is the relation between the two sets?

7. Let L be the language $\{0^n 1^n | n \geq 0\}$. **(3 marks)**

   (a) Find two distinct strings x and y that are indistinguishable with respect to L.

   (b) Show that L is not regular, by showing that there is an infinite set of strings, any two of which are distinguishable with respect to L.

8. Let $\Sigma = \{0, 1\}$, and define a language **(3 marks)**

$$MIDDLE = \{u0v : u, v \in \Sigma^* \text{ and } |u| = |v|\}.$$

   In words, *MIDDLE* is the language of all binary strings of odd length whose middle symbol is 0. Prove that *MIDDLE* is not regular.

9. Construct a CFG to generate set of strings of 0's and 1's, where consecutive 0 can occur but no consecutive 1 can occur. **(3 marks)**