CS0322 Theory of Computation

Assignment 1

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1.)

(a) To prove: There are countably many finite languages over any alphabet Σ.

Proof :

~~1~~ For ~~any alphabet~~ Σ, any language ~~over Σ~~ is countable

By definition,

a set is countable if and only if is finite or countably infinite

1. Let $\Sigma = \phi$, hence $\Sigma^* = \{e\}$ and we have two languages ϕ and $\{e\}$ over Σ, both finite, so countable

2. Let Σ, ϕ, then Σ^* is countably infinite, so obviously any $L \subseteq \Sigma^*$ is finite or countably infinite, hence countable.

Let the finite alphabet be $\Sigma = \{a_1 \ldots a_\ell\}$ and let σ be some character not in Σ. Let $L = \{w_1, \ldots w_n\}$ be a finite language over Σ. We can consider the strings $\sigma w_1, \sigma w_2, \sigma w_3 \ldots \sigma w_n$ to be a number in base ~~k~~ $|\Sigma| + 1$ by associating the symbols $a_1 \dots a_l$ with the base $-|\Sigma|+1$ digits $0, \dots l-1, l$ respectively.

This gives a ~~m~~ map from the set of finite languages over Σ to a subset of the integers, so that set of languages is countable.

(b) Given:

$M = \{Q, \Sigma, \delta, q_0, F\}$ is a DFA

$q \in Q$ ~~be~~ is a state of M

state q is never visited

K must accept $\{w \in \Sigma^* :$ when M is run on input, the state q is never visited$\}$.

For this, any string that visits q will never be allowed to leave it and all other state would be accepting states (as any string that does not ~~vis~~ visit q must be accepted).

$M' = \{Q, \Sigma, \delta', q_0, F'\}$

where

$F' = Q / \{q\}$

$\delta'(p, a) = \delta(p, a) \; \forall \; p \in Q | \{q\}, a \in \Sigma$

$\delta'(p, a) = q \; \forall \; a \in \Sigma$

(c) $\{a^{n^2} \mid n \geq 0\}$ or $\{a^{2^n} \mid n \geq 0\}$ be the two non-regular languages.

Since, it is non-regular, its words can be arbitrary long.

This ~~me~~ means that its language contains all words $e, a, a^2 - - -$

That is, its prefix language is regular.

~~(2) String $w \in (\Sigma \cup T)^*$,~~

(2.) Given: $A \subseteq (\Sigma \cup \Gamma)^*$ is a regular language.
(a)

~~Tbpr~~ To prove: The language $B = \{\pi(w) : w \in A\}$ is regular

Proof: By ~~K~~ kleen's theorem (Part I), we know that any regular language can be accepted by a finite language.

So, there exists a DFA M, for which language A is accepted.

Since, DFA M will be accepted over alphabet set $(\Sigma \cup T)$, we can define

$M = (Q, \Sigma \cup T, \delta, q_0, F)$ [M accepts A]

$\delta : Q \times (\Sigma \cup T) \rightarrow Q$

To prove that language B is regular, we need to show that there exists a DFA, NFA or ϵ-NFA (finite Automata) which accepts B.

We try to find an ϵ-NFA that accepts B,

So, we define an ϵ-NFA M' over Σ, such that

$M' = (Q, \Sigma, \delta', q_0, F)$, $P(Q)$ = power set of Q

~~$\delta \leq Q \times (\Sigma \cup T)^*$~~ $\delta' = Q \times (\Sigma \cup \{\epsilon\}) \rightarrow P(Q)$ define as:

$\delta'(q, m) = \{\delta(q, m)\}, \forall m \in \Sigma, q \in Q$

$\delta'(q, \epsilon) = \{\delta(q, b), b \in T\}, \forall q \in Q$

Intuitively, we see that we constructed ~~M~~ M' ~~from~~ from M, by replacing all occurances of b (where $b \in \Gamma$) by equivalent ϵ-transition

i.e.

For every $q \in Q$, we required the requirement of a character b ($b \in \Gamma$) to transition from q to $\delta(q, b)$ by providing an epsilon transition as a substitute.

Hence, for example, if our string in A was "ab0a1b", then we constructed M' such that it accepts "abϵaϵb" which is equivalent to "abab" $= \pi(w)$

Hence, ϵ-NFA M', accepts the language

$$B = \{\pi(w) : w \in A\}$$

So, B is a regular language.

(b) Given: $C \subseteq \Sigma^*$ is a regular language.

To prove: $D = \{w \in (\Sigma \cup \Gamma)^* : \pi(w) \in C\}$ is regular

Proof: Let M be a DFA accepting given regular language C (By Kleen's theorem, Part-I) such a finite automaton must exist, and we know that any NFA or ϵ-NFA can be converted ~~int~~ into a DFA. So set M be such a DFA.

~~Sq~~ Suppose, $M = (Q, \Sigma, \delta, q, F)$

Now, we try to define a DFA M', which accepts language D from M.

Since, ~~t~~ strings in D will be of the form

$w_0 x_1 w_1 x_2 w_2$

where $x_1, x_2 \in C$

$w_1, w_2, w_3 \in T^*$

Hence, if we define identity transitions for all $q \in Q$ with input symbol $\in \Gamma$, ~~we~~ has ~~will~~ successfully constructed M' from M.

So, defining DFA M':

$M' = (q, \Sigma \cup \Gamma, \delta', q_0, F)$

$\delta' : Q \times (\Sigma \cup \Gamma) \to Q$ as:

$\delta'(q, m) = \delta(q, m)$ for every $q \in Q$, $m \in \Sigma$

and

$\delta'(q, b) = q$ for every $q \in Q$, $b \in \Gamma$ clarly, M' also accepts ~~lan~~ language e. Since, C is a regular language, M' is a DFA.

Since, M' is a DFA, any language accepted by it is a regular language (Kleen's theorem)

But M' accepts language D

Hence, D is a regular language.

Hence proved

(M' is simillar to M, with only difference being that we have accomodated elements of Γ in M', by mapping them to identify transitions.

(3) Given:

$A \subseteq \Sigma^*$ is a regular language

To prove:

$B = \{ uv^R : u, v \in \Sigma^* \text{ and } u, v \in A \}$

Proof:

Define an NFA

$N = \{R, \Sigma, \eta, q_0, G)$

This state set of N is defined as

$R = \cancel{Q} \; Q \cup (Q \times Q)$

and the accepts states of N are

$G = \{(p, p) : p \in Q\}$

It remains to describe the transition function η of N, which will be as follows:

1) $\eta(p, a) = \{ \delta(p, a)\}$ for all $p \in Q$ and $a \in \Sigma$

2) $\eta(p, \varepsilon) = \{(p, q) : q \in F\}$

3) $\eta((p, q), a) = \{(p, r) : \delta(r, a) = q\}$ for all $p, q \in Q$ and $a \in \Sigma$.

4) $\eta((p,q),\varepsilon) = \phi$ for every $p, q \in F$

The idea behind this NFA is that its computation has two phases, which corresponds naturally to the strings u and v, in the definition of B.

The first phase of the computation of N indicated by its state being contained in Q, while the second phase is indicated by its state being contained in $Q \times Q$.

Here, N begins in the state q_0 and therefore in the first phase, simply mimics M directly, which is the point of transitions listed in Item 1 of list above.

The start of the second phase happens when N follows one of the ε-transitions listed in 2 of list.

Such a transition has two effects:

It records the state that N was in immediately prior to the phase change, and it guesses the final state that M would be in if it were given the input uv.

The second phase of N is effectively to sim N backwards, as the rules in 3 allow while all along remembering the state that N was in immediately prior to the phase change.

Acceptance occur when running N backwards in this way reaches this remembered state.

Since, $L(N) = B$

Hence, B is regular.

4. Given:

$A \subseteq \{0\}^*$

$A = \{0^{rs} : r, s \in N, r, s >= 3\}$

To prove: A is ~~a~~ non-regular

Proof:

We can use the pumping lemma to establish that a given language is not regular, by supporting a decomposition of W into xyz and finally show that xyz is not in regular language.

Now, we prove that A is non-regular by using method of contradiction.

Let A be a regular language accepted by finite automaton M.

Then, by pumping lemma, we have -

If A is a regular language, then there exists a constant $n \geq 1$ (which depends on A) ∃ for every string $w \in L$ such that $|w| >= n$, we can break w into 3 substrings, $w = xyz$ such that

(i) $y \neq \epsilon$ - (2)

(ii) $|xy| \leq n$ - (3)

(iii) the string ~~x~~ $xy^k \in A, \forall k \geq 0$ - (4)

Now, from (1)

$r \geq 3, s \geq 3 \Rightarrow rs \geq 9$

Let w be a string of length $n+1$

$w =$ '000 000' $(n+1)$ times

$= xyz$ (say)

Let $|x| = x_1$, $|y| = y_1$, $|z| = z_1$

Using (2) and (3) we can say

$y_1 \neq 0$ – (5) $[\because y \neq \varepsilon]$

$|xy| = |x_1 + y_1| \leq n+1$ – (6)

$x_1 + y_1 + z_1 = n+1$ – (7)

Let us choose

$x = \varepsilon$ $(\Rightarrow x_1 = 0)$

$y = 000 \ldots 0$ (n times $y_1 = n \neq 0$)

$z = 0$ $(z_1 = 1)$ $[n \neq 0, \because n+1 \geq 9 \Rightarrow n \geq 0]$

$\therefore$ (5), (6) are satisfied ($y \neq \varepsilon$ and $|xy| \leq n+1$)

where n is our length of string w.

Now, as per our assumption, A is regular

So, (4) should also be satisfied

$\forall k \geq 0$, the string $xy^kz \in A$

$xy^kz = \underset{x}{\underbrace{\varepsilon}}\ \underset{y^k}{\underbrace{000 \ldots 0}}\ \underset{z}{\underbrace{0}} = 00 \ldots 0$

n^k times of '0' $\qquad$ $(nk+1)$ times

To show $xy^kz \in A$

By (1) $A = \{0^{rs} : r, s \in \mathbb{N} ; r, s >= 3\}$

Now, we know that there exist ~~is~~ infinitely many prime members

So, let us choose $n = 11, k = 6$ $(n \geq 0, k \geq 0)$

$nk + 1 = (11)(6) + 1 = 67$ which is a prime number

But if our assumption was true

$\Rightarrow xy^kz \in A$

$0^{nk+1} \in A$

$\cancel{nk+1}\ 0^{nk+1} = 0^{rs}$

$nk + 1 = rs$

but $nk + 1 = 67 = 67 \neq$ product of 2 numbers each ≥ 3

$(\because 1 \leq 3)$

$\therefore xy^kz \notin A$

Our assumption that A is a regular language is wrong

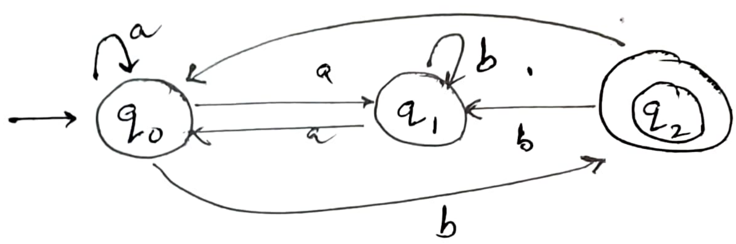
A is non-regular

Hence proved.

5.)

States / Input	a	b
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_2\}$
$\rightarrow q_1$	$\{q_0\}$	$\{q_1\}$
$*q_2$	ϕ	$\{q_0, q_1\}$

$Q = \{q_0, q_1, q_2\}$



We will use subset Construction method:

Subset: $\{\phi, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

Transition Table:

		a	b
A	ϕ	ϕ	ϕ
B	$\{q_0\}$	$\{q_0, q_1\}$	$\{q_2\}$
C	$\{q_1\}$	$\{q_0\}$	$\{q_1\}$
$*$D	$\{q_2\}$	ϕ	$\{q_0, q_1\}$
E	$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$*$F	$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$
$*$G	$\{q_1, q_2\}$	$\{q_0\}$	$\{q_0, q_1\}$
$*$H	$\{q_0, q_1, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$

Renamed State Transition Table

States / Input	a	b
A	A	A
→B	E	D
C	B	C
*D	A	E
E	E	G
*F	E	H
*G	B	E
*H	E	H

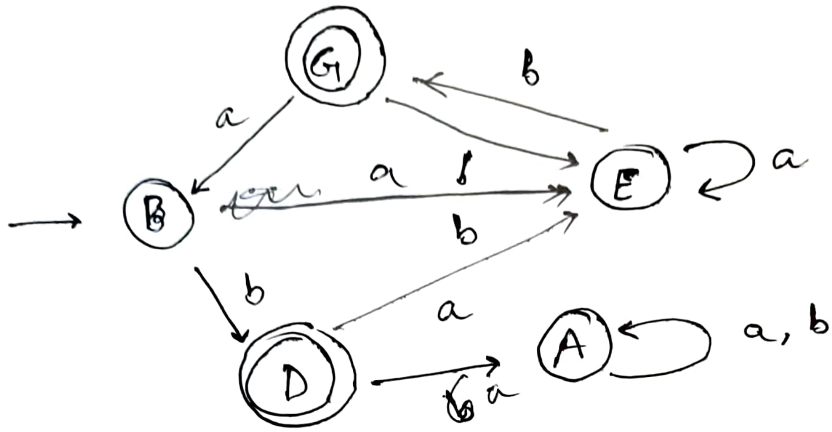
Considering relevant states only

States / Input	a	b
→B	E	D
E	E	G
*G	B	E
*D	A	E
A	A	A

→ denotes initial state

* final state

The DFA can be represented as:



~~G (a~~

$G = a(a+bb+baa)^* b$

$D = B \cdot b = (\varepsilon + G \cdot a)$

$= b + Gab$

$\therefore D = b + a(a+bb+baa)^* bab$

Taking union of both final states

$D \cup G = b + a(a+bb+baa)^* bab + a(a+bb+baa)^* b$

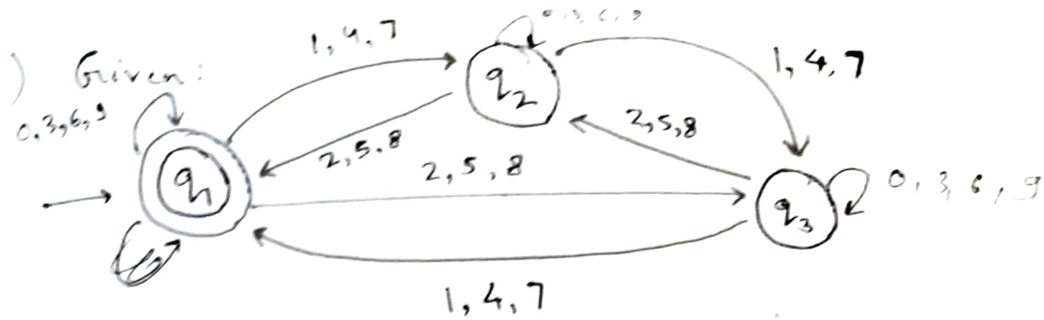
$= \boxed{a(a+bb+baa)^* b(ab+\varepsilon) + b}$

Regular Expression

$\boxed{\{a\}, \{a, bb, baa\}^* \{b\}, \{ab, \varepsilon\} \cup \{b\}}$

Language

6.) Given:



We have an automaton $A(Q, \Sigma, \delta, q_0, F)$

where $q_0 = \{q_1\}$

$Q = \{q_1, q_2, q_3\}$

$\Sigma = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$F = \{q_1\}$

for different cases:

1.) when a is divisible by 3 i.e $a \in \{0, 3, 6, 9\}$

$\delta(q_1, a) = q_1$, $\delta(q_2, a) = q_2$, $\delta(q_3, a) = q_3$

2.) when a gives remainder 1 when divided by 3

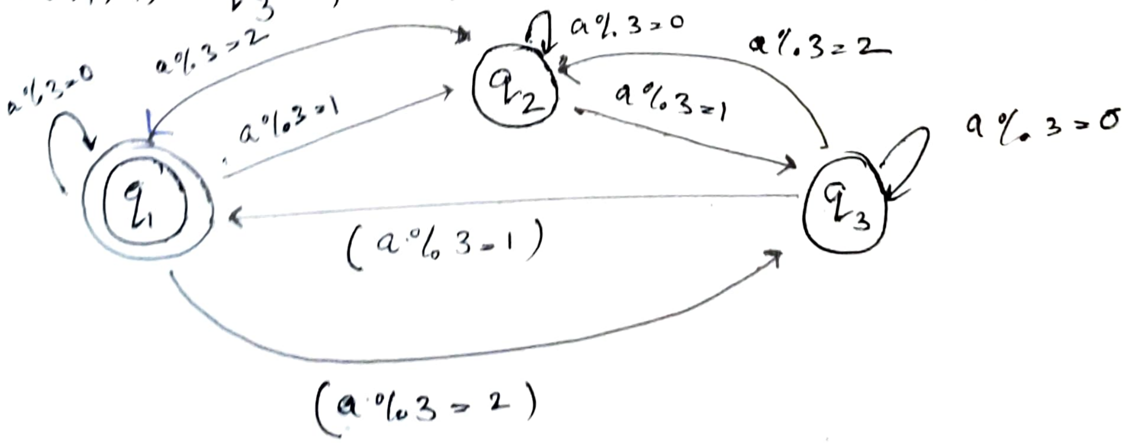
$a \in \{1, 4, 7\}$

$\delta(q_1, a) = q_2$, $\delta(q_2, a) = q_3$, $\delta(q_3, a) = q_1$

3.) when a gives remainder 2 when divided by 3

. $a \in \{2, 5, 8\}$

$\delta(q_1, a) = q_3$, $\delta(q_2, a) = q_1$, $\delta(q_3, a) = q_2$



Let there be a string w, then S_1, S_2, S_3 ~~rep~~represent set of possible values for:
(sum of all digits) mod 3, where w is such that:
$\delta(q, w) = q_1$ (for S_1), q_2 (for S_2) q_3 (for S_3)

1. If all characters of string w are divisible by 3 i.e. $a \bmod 3 = 0$, then only possible state in such condition is q_1. So, we can say $0 \in S_1$.

2. Let us say $w = xa$
Here, x is a string which follows condition (1) i.e. all characters of x are divisible by 3.
$\Rightarrow$ ~~to~~ Let $a =$ (value of mod $3 = 1$)
then $\delta(q_1, w) = \delta(\delta(q, x), a)$
$= \delta(\delta(q, x), 1)$
$= \delta(q_1, 1)$
$= q_2$

$\therefore$ we can say $w \in q_0$
then $S_2 =$ (sum of digits of x) % 3
+ (sum of digits of a) % 3
$= 0 + 1 = 1$
$1 \in S_2$

$\Rightarrow$ Let $a = 2$ (value of mod $3 = 2$)
then $\delta(q_1, w) = \delta(\delta(q_1, x), a)$
$= \delta(\delta(q_1, x), 2)$
$= \delta(q_1, 2)$
$= q_3$

In this case $w \in$ ~~q_3~~ q_3

the S_3 = (Sum of digits of x)%3 + (Sum of digits of q)%3

$= 0 + 2 = 2$

$2 \in S_3$

Finally we get:

$S_1 = \{0\}$, $S_2 = \{1\}$, $S_3 = \{2\}$

$\delta(q_1, w) = q_1$ (Sum of digits)%3 = 0

$\delta(q_1, w) = q_2$ (Sum of digits)%3 = 1

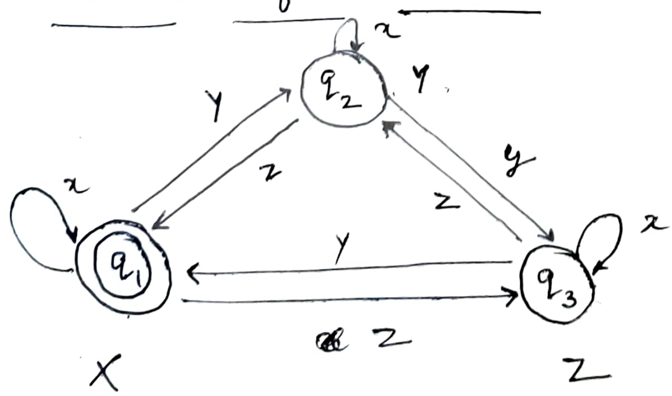
$\delta(q_1, w) = q_3$ (Sum of digits)%3 = 2

$\therefore$ when a ~~sitting~~ string w has sum of all digits divisible by 3 (non-empty) are accepted by automata, while if sum is not divisible by 3 then string is not accepted.

$\therefore$ we define language L as:

$L(A)$ = {w : sum of all characters of w is divisible by 3}

→ Grammar of Automata:



where $x : x \% 3 = 0$

$y : y \% 3 = 1$

$z : z \% 3 = 2$

Let Grammar $G(L) = (V_n, T, P, S)$

$V_n = \{X, Y, Z\}$

$T = \{x, y, z\}$

$S \rightarrow \epsilon$ (epsilon condition)

for P:

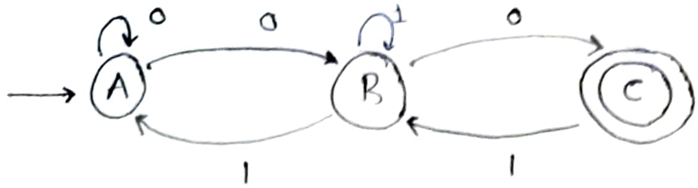
$S \rightarrow X$

$X \rightarrow \epsilon \mid xX \mid yY \mid zZ$

$Y \rightarrow xY \mid yZ \mid zX$

$Z \rightarrow xZ \mid yX \mid zY$

7.



<u>Arden's</u> <u>Theorem:</u> If $R = Q + RP$ then $R = QP^*$

$$A = \varepsilon + A\cdot 0 + B\cdot 1 \quad -(1)$$

$$B = A\cdot 0 + B\cdot 1 + C\cdot 1 \quad -(2)$$

$$C = B\cdot 0 \quad -(3)$$

Substituting (3) in (1)

$$B = A\cdot 0 + B\cdot 1 + B\cdot 0 \cdot 1$$

$$= \underbrace{A\cdot 0}_{Q} + \underbrace{B}_{R}\,[\underbrace{1+01}_{P}]$$

$$B = A\cdot 0(1+01)^* \quad - (4)$$

$$A = \varepsilon + A\cdot 0 + A\cdot 0\cdot(1+01)^*\cdot 1$$

$$A = \varepsilon \cancel{+A}\,(0 + 0(1+01)^*\cdot 1)^*$$

$$= (0 + 0(1+01)^* 1)^*$$ [ε is identity of concatenation]

$$B = A\cdot 0(1+01)^*$$

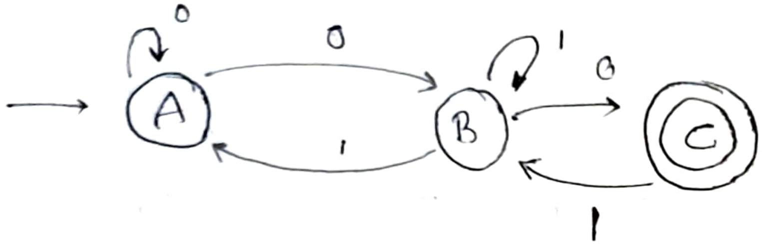
$$= (0 + 0(1+01)^* 1)^* 0(1+01)^*$$

$$C = B\cdot 0 = (0+0(1+01)^* 1)^* 0(1+01)^* 0$$

Since, C is the final state, this is the regular expression for the automata.

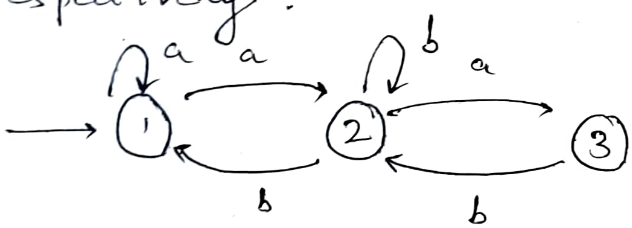
$$C = (0 + 0\cdot(1+01)^* 1)^* 0 (1+01)^* 0$$

Using Kleen's Theorem :



~~For simplicity, I will~~

For simplicity, we will denote A, B, C as 1, 2, 3 respectively.



By Kleen's Theorem, $L = \bigcup_{q \in A} (q_0, q) = L(1, 3)$

Now, $L(P, q) = L(p, q, n)$

Here, $n = 3$ $\therefore$ required expression is $L(1, 3, 3)$

$$L(P, q, 0) = \begin{cases} \{a \in \Sigma \mid \delta(p, a) = q\}, & P \neq q \\ \{a \in \Sigma \mid \delta(p, a) = p\} \cup \{\epsilon\}; & p = q \end{cases}$$

$L(1, 1, 0) = \{a \in \Sigma \mid \delta(1; a) = 1\} \cup \{\epsilon\}$ i.e. self loop $+ \epsilon$

$\therefore r(1, 1, 0) = a + \epsilon$

$L(1, 2, 0) = \{a \in \Sigma \mid \delta(1, a) = 2\}$

$r(1, 2, 0) = a$

$r(1, 3, 0) = \phi$

$\therefore$ The tabulation we get is

P	$r(P,1,0)$	$r(P,2,0)$	$r(P,3,0)$
1	$a+\varepsilon$	a	ϕ

Moving further,

$r(2,1,0) = b$

$r(2,2,0) = b+\varepsilon$

$r(2,3,0) = a$

$r(3,1,0) = \phi$

$r(3,2,0) = b$

$r(3,3,0) = \varepsilon$

$\therefore$ Tabulation after first round

P	$r(P,1,0)$	$r(P,2,0)$	$r(P,3,0)$
1	$a+\varepsilon$	a	ϕ
2	b	$b+\varepsilon$	a
3	ϕ	b	ε

$$L(p,q,k+1) = L(p,q,k) \cup L(p,k+1,k) \cdot L(k+1,k+1,k)^* \\ L(k+1,q,k)$$

$\therefore r(1,1,1) = r(1,1,0) + r(1,1,0) . r(1,1,0)^* r(1,1,0)$

Here, $k=0$ and $k+1=1$

$\therefore r(1,1,1) = (a+\varepsilon) + (a+\varepsilon) . (a+\varepsilon)^* (a+\varepsilon)$
$= a^*$

$\therefore r(1,2,1) = r(1,2,0) + r(1,1,0)\, r(1,1,0)^*\, r(1,2,0)$
$= a + (a+\varepsilon)(a+\varepsilon)^* a \quad = a^* a$

$$r(1,3,1) = r(1,3,0) + r(1,1,0) \cdot r(1,1,0)^* \, r(1,3,0)$$
$$= \phi + (a+\varepsilon)(a+\varepsilon)^* \phi$$
$$= \phi$$

$$r(2,1,1) = r(2,1,0) + r(2,1,0) \cdot r(1,1,0)^* \, r(1,1,0)$$
$$= b + b(a+\varepsilon)^* a$$
$$= b a^*$$

$$r(2,2,1) = r(2,2,0) + r(2,1,0)\, r(1,1,0)^* \, r(1,2,0)$$
$$= (b+\varepsilon) + b(a+\varepsilon)^* a$$
$$= b a^* + \varepsilon$$

$$r(2,3,1) = r(2,3,0) + r(2,1,0)\, r(1,1,0)^* \, r(1,3,0)$$
$$= a + b(a+\varepsilon)^* \phi = a$$

$$r(3,1,1) = r(3,1,0) + r(3,1,0)\, r(1,1,0)^* \, r(1,1,0)$$
$$= \phi + \phi (a+\varepsilon)^* (a+\varepsilon)$$

$$r(3,2,1) = r(3,2,0) + r(3,1,0)\, r(1,1,0)^* \, r(1,2,0)$$
$$= b + \phi (a+\varepsilon)^* \phi$$
$$= b$$

$$r(3,3,1) = r(3,3,0) + r(3,1,0)\, r(1,1,0)^* \, r(1,3,0)$$
$$= \varepsilon + \phi (a+\varepsilon)^* \phi$$
$$= \varepsilon$$

$\therefore$ Tabulation after 2^{nd} round:

P	$r(P,1,1)$	$r(P,2,1)$	$r(P,3,1)$
1	a^*	a^*a	ϕ
2	ba^*	$ba^*+\varepsilon$	a
3	ϕ	b	ε

$$r(1,1,2) = r(1,1,1) + r(1,2,1)\, r(2,2,1)^*\, r(2,1,1)$$
$$= a^* + a^*a\,(ba^*+\varepsilon)^*\, ba^*$$
$$= a^* + a^*a\,(ba^*)^*(ba^*)$$

$$r(1,2,2) = r(1,2,1) + r(1,2,1)\, r(2,2,1)^*\, r(2,2,1)$$
$$= a^*a + a^*a(ba^*+\varepsilon)^*(ba^*+\varepsilon)$$
$$= a^*a\,[(ba^*+\varepsilon)^*(ba^*+\varepsilon)+\varepsilon]$$
$$= a^*a\,[(ba^*+\varepsilon)^*] = a^*a\,[(ba^*)^*]$$

$$r(1,3,2) = r(1,3,1) + r(1,2,1)\, r(2,2,1)^*\, r(2,3,1)$$
$$= \phi + a^*a(ba^*+\varepsilon)^*a$$
$$= \phi + a^*a\,(ba^*)^*a = a^*a\,(ba^*)^*a$$

$$r(2,1,2) = r(2,1,1) + r(2,2,1)\, r(2,2,1)^*\, r(2,1,1)$$
$$= ba^* + (ba^*+\varepsilon)(ba^*+\varepsilon)^*\, ba^*$$
$$= ba^*(ba^*)^*$$

$$r(2,2,2) = r(2,2,1) + r(2,2,1)\, r(2,2,1)^*\, r(2,2,1)$$
$$= ba^*+\varepsilon + (ba^*+\varepsilon)(ba^*+\varepsilon)^*(ba^*+\varepsilon)$$

$$r(3,1,2) = r(3,1,1) + r(3,2,1)\, r(2,2,1)^*\, r(2,1,1)$$
$$= \phi + b(ba^* + \varepsilon)^* ba^*$$
$$= b(ba^*)^* ba^*$$

$$r(3,2,2) = r(3,2,1) + r(3,2,1)\, r(2,2,1)^*\, r(2,2,1)$$
$$= b + b(ba^* + \varepsilon)^* (ba^* + \varepsilon)$$
$$= b(ba^*)^*$$

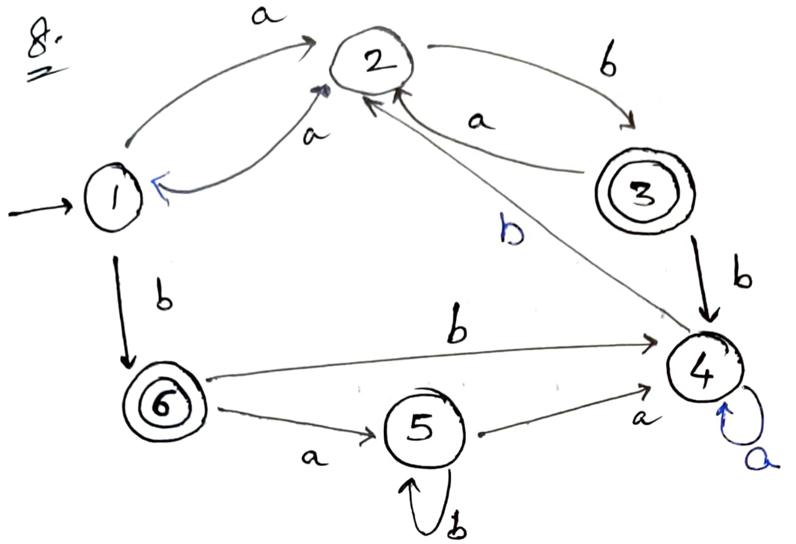
$$r(3,3,2) = r(3,3,1) + r(3,2,1)\, r(2,2,1)^*\, r(2,3,1)$$
$$= \varepsilon + b(ba^* + \varepsilon)^* a$$
$$= \varepsilon + b(ba^*)^* a$$

Tabulation after 3rd round:

P	$r(P,1,2)$	$r(P,2,2)$	$r(P,3,2)$
1	$a^*(\varepsilon + a(ba^*)^*(ba^*))$	$a^* a(ba^*)^*$	$a^* a(ba^*)^* a$
2	$ba^*(ba^*)^*$	$ba^*(ba^* + \varepsilon)$	$a(ba^*)^*$
3	$b(ba^*)^* ba^*$	$b(ba^*)^*$	$\varepsilon + b(ba^*)^* a$

$$r(1,3,3) = r(1,3,2) + r(1,3,2)\, r(3,3,2)^*\, r(3,3,2)$$
$$= a^* a(ba^*)^* a + a^* a(ba^*)^* a\, (\varepsilon + b(ba^*)^* a)^* (\varepsilon + b(ba^*)^* a)$$
$$= a^* a(ba^*)^* a [\varepsilon + b(ba^*)^* a]^*$$
$$= a^* a(ba^*)^* a\, (b(ba^*)^* a)^*$$

8.



	1	2	3	4	5	6
1						
2	×					
3	✓	✓				
4	×	×	✓			
5	×	×	✓	×		
6	✓	✓	×	✓	✓	

Unmarked pairs : (1, 2), (1, 4), (1, 5), (2, 4), (2, 5),
(3, 6), (4, 5)

For (1, 2) ⇒ $\delta(1, a) = 2$ $\delta(1, b) = 6$

$\delta(2, a) = 1$ $\delta(2, b = 3$

Neither (1, 2) is marked nor (3, 6).

∴ (1, 2) remaining unmarked.

For (1, 4) ⇒ $\delta(1, a) = 2$ $\delta(1, b) = 6$

$\delta(5, a) = 4$ $\delta(5, b) = 5$

The pair (2, 6) is marked ∴ mark (1, 4)

For (1, 5) ⇒ $\delta(1, a) = 2$ $\delta(1, b) = 6$

$\delta(5, a) = 4$ $\delta(5, b) = 5$

The pair (5, 6) is marked ∴ mark (1, 5)

For (2, 4) ⇒ $\delta(2, a) = 1$ $\delta(2, b) = \cancel{4}\,3$

$\delta(4, a) = 4$ $\delta(4, b) = 2$

The pair (2, 3) is marked ∴ mark (2, 4)

For $(2,5) \Rightarrow \delta(2,a) = 1$ $\delta(2,b) = 3$

$\delta(5,a) = 4$ $\delta(5,b) = 5$

The pairs $(1,4)$ and $(3,5)$ are marked $\therefore$ mark $(2,5)$

For $(3,6) \Rightarrow \delta(3,a) = 2$ $\delta(3,b) = 4$

$\delta(6,a) = 5$ $\delta(6,b) = 4$

The pair $(2,5)$ is marked $\therefore$ mark $(3,6)$

For $(4,5) \Rightarrow \delta(4,a) = 4$ $\delta(4,b) = 2$

$\delta(5,a) = 4$ $\delta(5,b) = 5$

The pair $(2,5)$ is marked $\therefore$ mark $(4,5)$

Preparing the table again

	1	2	3	4	5	6
1						
2	✓					
3	✓	✓				
4	✓	✓	✓			
5	✓	✓	✓	✓		
6	✓	✓	✓	✓	✓	

For $(1,2)$

$\delta(1,a) = 2$ $\delta(1,b) = 6$

$\delta(2,a) = 1$ $\delta(2,b) = 3$

Since, $(3,6)$ is marked, mark $(1,2)$ also.

The given automata is already in minimised form because ~~none of~~ all the pairs are marked which are required to minimise the automata. (set ~~of~~ pairs required to minimise the automata is empty / no ~~unma~~ unmarked pair)

→ The DFA is quite complex, hence finding regular expression by hand is practically impossible

9. Equivalence classes for I_L where

$L = \{ww \mid w \in \{0,1\}^*\}$

(i) {0, which is the only string for which the continuations that lead to acceptance are all strings that would be in L except that they are missing a leading 0}

(ii) {1, which is the only string for which the continuations that lead to acceptance of all strings that would be in L, except that they are missing a leading 1}

(iii) {01, which is the only string for which the continuations that lead to acceptance of all strings that would be in L, except that they are missing a leading 01}

(iv) {00, which is the only string for which the continuations that lead to acceptance of all strings that would be in L, except that they are missing a ~~le~~ ~~lea~~ leading 00}.

10. $L \subseteq \{a, b\}^*$

$\{\epsilon\} \notin L$, $b \in L$, $ba \notin L$, $baba \in L$

$\exists I_L a$ and $a I_L bb$.

Let an automata $A = (Q, \Sigma, \delta, q, F)$

where $q = q_0$

$Q = \{q_0, q_1, q_2\}$ [3 state given]

$\Sigma = \{a, b\}$

$\delta(q_0, \epsilon) = q_0$

$\therefore \delta(q_0, a) = q_0$ [as $\exists I_L a$)

$\{\epsilon\} \notin L$ $\therefore q_0$ cannot be our final stage.

as $b \in L$ so $\delta(q_0, b) \neq q_0$

Let $\delta(q_0, b) = q'$

where q' come to be either q_1 or q_2 ~~after~~ for now,
we take q' as q_1.

$\delta(q_0, b) = q_1$ $\therefore q_1$ is our final stage

$F = q_1$

$\delta(q_0, bb) = q_0$ (as $a I_L bb$)

$\delta(\delta(q_0, b), b) = q_0$

$\delta(q_1, b) = q_0$

as $ba \notin L$ $\delta(q_0, ba) \neq q_1$

$\delta(\delta(q_0, b), a) \neq q_1$

$\delta(q_1, a) \neq q_1$

So, Let $\delta(q_1, q) = q''$

where q'' can be written either q_0 or q_2.

$\rightarrow$ baba $\in L$

$\delta(q_0, baba) = q_1$

$\delta(\delta(q_0, b)\ aba) = q_1$

$\delta(q_1, aba) = q_1$

$\delta(\delta(q_1, a)\ ba) = q_1$

as $ba \notin L$ q'' cannot be q_0

$\therefore q'' = q_2$

So, $\delta(q_1, a) = q_2$

$\delta(q_2, ba) = q_1$

$\delta(\delta(q_2, b), a) = q_1$

$\delta(q_2, b) \neq q_1$ as $\delta(q_1, a) \neq q_1$

also $\delta(q_2, b) \neq q_0$ as $\delta(q_0, q) \neq q_1$

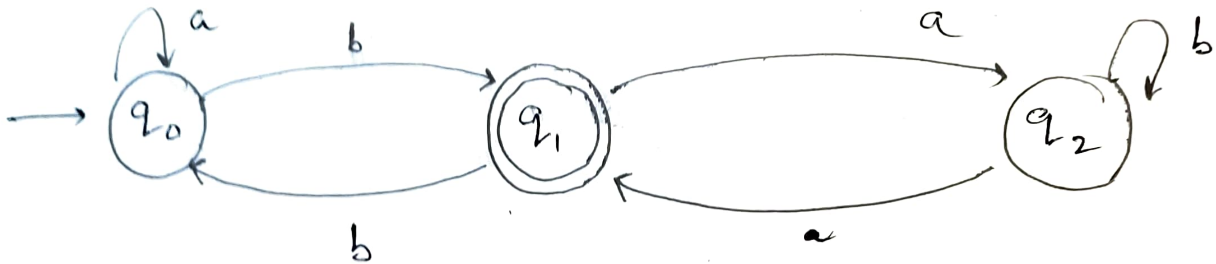
$\therefore \delta(q_2, b)$ can only be q_2.

$\delta(q_2, b) = q_2$

and $\delta(q_2, q) = q_1$

$\therefore$ Finally we have :-

	a	b
$\rightarrow q_0$	q_0	q_1
$* q_1$	q_2	q_0
q_2	q_1	q_2



a
b
a
b
q0
q1
q2
b
a