11/01/2022 Yash Verma 19075083 CSE

Theorem 1:

If $D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ is the DFA constructed from NFA $N = (Q_N, \delta_N, q_0, F_N)$ by subset construction, then $L(D) = L(N)$

Proof: By induction on $|w|$, we prove $-\ \delta_D^*(\{q_0\}, w) = \delta_N^*(q_0, w)$

Basis: Let $|w| = 0$ i.e. $w = \epsilon$

By definition of DFA and NFA,

$$\delta_D^*(\{q_0\}, \epsilon) = \{q_0\}$$
$$\delta_N^*(q_0, \epsilon) = \{q_0\}$$

Induction: Let w be of length $n+1$ and assume the statement for length n. Let $w = xa$ where a is the final symbol of w.

By Inductive hypothesis,

$$\delta_D^*(\{q_0\}, x) = \delta_N^*(\{q_0\}, x) \quad -(1)$$
$$= \{p_1, p_2 \ldots p_k\}$$

If both these sets of N's states be $\{p_1, p_2 \ldots p_k\}$,
By inductive defn of δ^* for NFA's

$$\delta_N^*(q_0, w) = \bigcup_{i=1}^{k} \delta_N(p_i, a) \quad - (2)$$

The subset Construction,

$$\delta_D(\{p_1, p_2 \ldots p_k\}, a) = \bigcup_{i=1}^{k} \delta_N(p_i, a) \quad - (3)$$

$$\delta_D^*(\{q_0\}, x) = \{p_1, p_2 \ldots p_k\} \quad \text{(By (3))}$$

Theorem 2:

A language L is accepted by some DFA, iff L is accepted by some NFA:

Proof: (If): The 'if' part is the subset construction method and Theorem 1.

Only If: We have only to convert a DFA into an identical NFA.

Let $D = (Q, \Sigma, \delta_Q, q_0, F)$

Define $N = (Q, \Sigma, \delta_N, q_0, F)$ to be an equivalent NFA by:

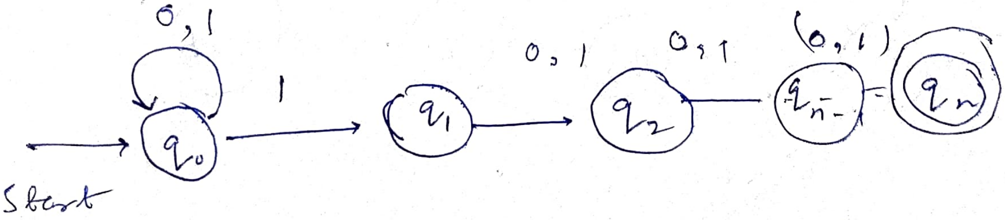
If $\delta_D(q, a) = p$, then $\delta_N(q, a) = \{p\}$

Then by induction ~~of~~ on $|w|$, we have

If $\delta^*_D(q_0, w) = p$, then $\delta^*_N(q_0, w) = \{p\}$.

Therefore, w is accepted by ~~some DFA~~ D iff w is accepted by N i.e. $L(D) = L(N)$

Bad Case for Subset Construction:



$L(N)$ is a set of all strings of 0's and 1's, s.t. the nth symbol from the end is 1.

- DFA D accepts this language must remember the last n symbols it has read.
- Since any 2^n subsets of last n symbols could have been 1, if D have fewer than 2^n states, there would be some state q s.t. D can be in a state q

- Since, the seq. are different, they must differ in some position, say $a_i = b_i$.
- Suppose that, $a_i = 1$ and $b_i = 0$. ~~a_i~~ If $i = 1$, then q must be both an accepting state and a non-accepting state since $a_1, a_2 \ldots a_n$ is accepted and $b_i, b_{i+1} \ldots b_n, 0, \ldots 0$ is not.

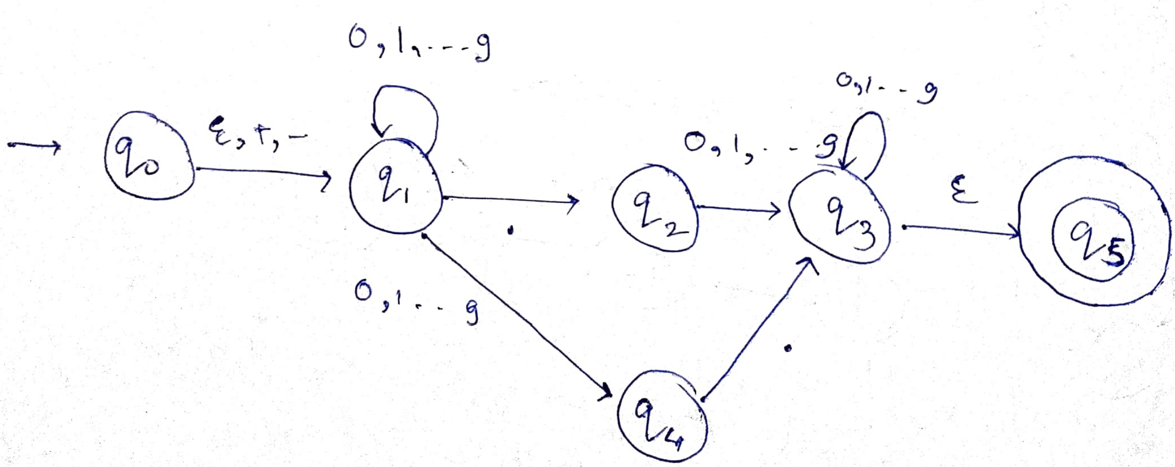
Finite Automata with Epsilon Transitions:

Example:

ϵ-NFA that accepts decimal ~~of~~ numbers consisting of L.

1. Optional sign $(+, -)$
2. String of digits - $\{0, 1, \ldots 9\}$
3. Decimal point
4. Another string of digits

(2) and (4) can be empty but atleast one must be ~~ø~~ non-empty.



ϵ-NFA.

Formally, $E = (\{q_0, q_1 \ldots q_5\}, \{., +, -, 0, 1 \ldots 9\}, \delta, q_0, \{q_5\})$.

$\delta(q_0, \epsilon) = q_1$

$\delta(q_1, 4) = \{q_1, q_4\}$

$\delta(q_1, .) = \{q_2\}$

$\delta(q_4, .) = \{q_3\}$

$\delta(q_2, 8) = \{q_3\}$

$\delta(q_3, 8) = \{q_5\}$

$E\,Close(q) = \{q\}$

$P \in EClose(q)$

$\delta(p, \epsilon) = r \Rightarrow r \in EClose(q)$ $[EClose(q) \cup r]$

Extended Transitions for ϵ-NFA:

Basis: $\delta^*(q, \epsilon) = ECLOSE(q)$

Induction: Suppose w is of the form xa,

Compute $\delta^*(q, w)$

- Let $\{p_1, p_2 \ldots p_k\}$ be $\delta^*(q, x)$ i.e. the p_i's are all and only the states that can be reached q following a path labelled x.
- Let $\bigcup_{i=1}^{k} \delta(p_i, a)$ be $\{r_1, r_2 \ldots r_m\}$
- $\delta^*(q, w) = \bigcup_{j=1}^{m} ECLOSE(r_j)$

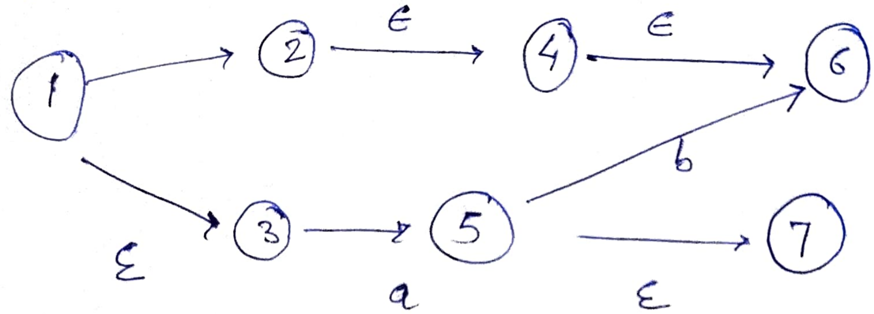
$\cancel{\delta^*(q_0, \varepsilon) = ECLOSE}$

$$\delta^*(q_0, 4.8) = \delta(q_2, 8) \cup (q_3, 8) \; \delta(q_4; 8)$$

$$= \{q_3\} \cup \{q_3, q_5\}$$

$$= \cancel{\phi}. \{q_3, q_5\}$$

Question:



ECLOSE(1) = {1, 2, 3, 4, 6}

ECLOSE(2) = {2, 4, 6}

ECLOSE(3) = {3}

ECLOSE(4) = {4, 6}

ECLOSE(5) = {5, 7}

ECLOSE(6) = {6}

ECLOSE(7) = {7}