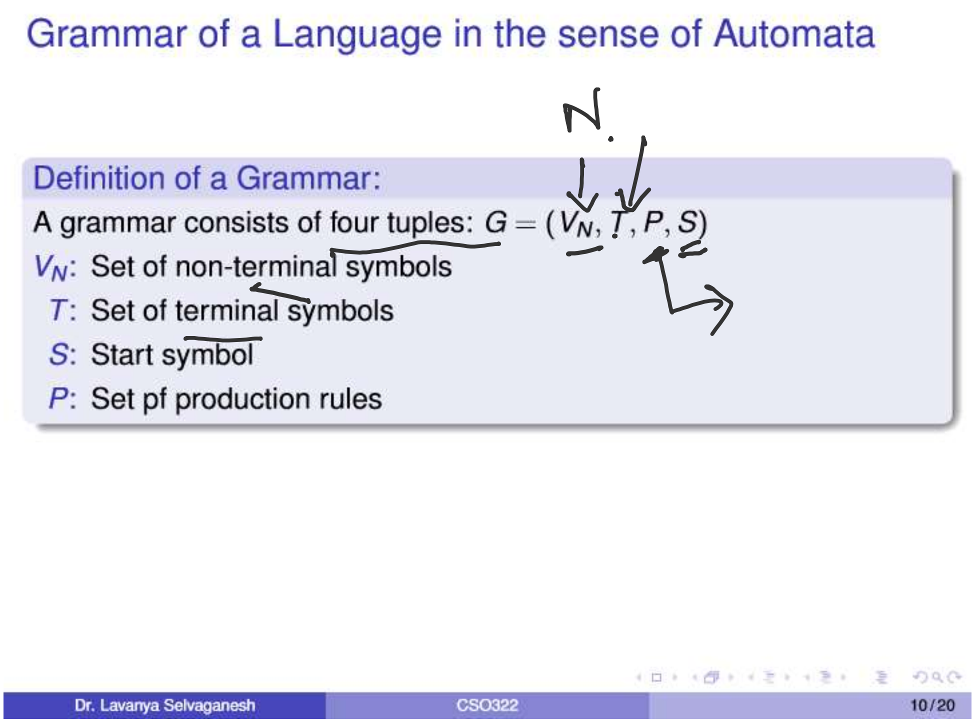
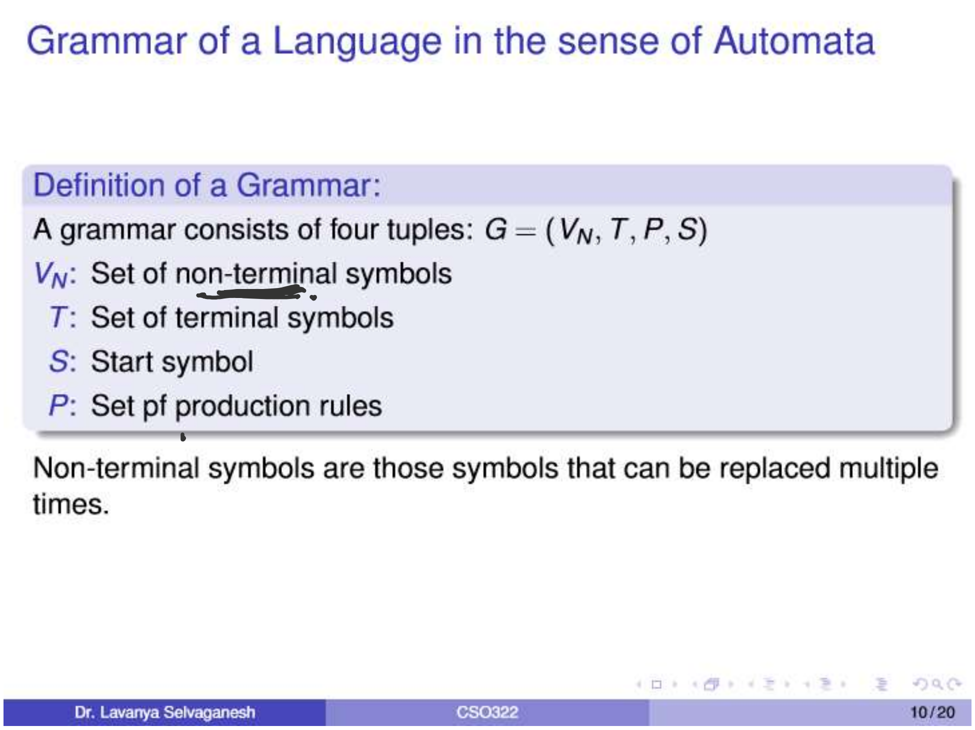
27012022- Grammars

27 January 2022 12:57

Unit-2-Slides





Grammar of a Language in the sense of Automata

Definition of a Grammar:

A grammar consists of four tuples: $G = (V_N, T, P, S)$

V_N: Set of non-terminal symbols

T: Set of terminal symbols

S: Start symbol

P: Set pf production rules

Non-terminal symbols are those symbols that can be replaced multiple times.
Terminal symbols are those symbols that cannot be replaced further.

Example

$$V_N = \{S, A\}, S - \text{ starting symbol}$$
$$T = \{a, b\},$$
$$P = \{S \to aA, S \to b, A \to aa\}$$

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Language of this grammar

$S \to aA \to aaa$ (obtained by replacing $A \to aa$)
or $S \to b$.
So, $L(G) = \{b, aaa\}$.

Chomsky Classification of Grammar

Chomsky classified the grammar into four types depending on the production rules.

1. Type-0 Grammar
2. Type-1 Grammar
3. Type-2 Grammar
4. Type-3 Grammar

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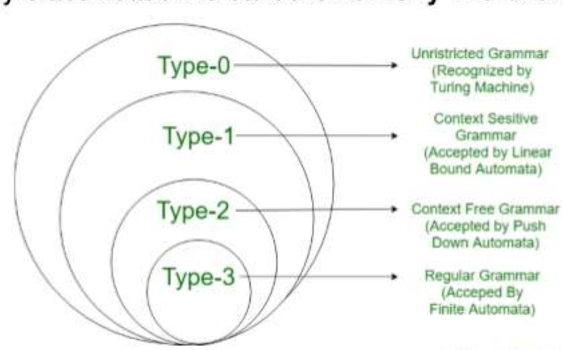
The Chomsky classification is called **Chomsky Hierarchy**.



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where L_c: left context, R_c: right context, $NT \in V_N$: non-terminal symbol and $\alpha \in (V_N \cup T)^*$ is a string of non-terminals or terminals or both.

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- Type-1 Grammar
  - Context-sensitive grammar
  - All production rules are in the format of context-sensitive if all rules

in P are of the form -

$$\alpha A\beta \to \alpha\gamma\beta,$$

where $A \in V_N$ i.e. A is single non-terminal symbol, $\alpha, \beta \in (V_N \cup T)^* = \Sigma^*$ are strings of non-terminals and terminals and $\gamma \in (V_N \cup T)^*$ is non empty string of non-terminals and terminals.

Chomsky Hierarchy

- Type-2 Grammar
  - Context-free grammar
  - In left hand side of production, there will be no left or right context.
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- Type-3 Grammar
  - Regular Grammar
  - Production rule format -

$$A \to \alpha \text{ or } A \to \alpha B$$

where $A, B \in V_N$ and $\alpha \in T$.

The Languages and Machine Format for different Grammars

Grammar	Language	Machine Format
Type-0	Unrestricted Language/ Recursively Enumerable	Turing Machine
Type-1	Context-Sensitive Language	Linear Bounded Automata
Type-2	Context-Free Language	(Non-deterministic) Push-down Automata
Type-3	Regular Language	Finite Automata

Examples- Grammar to Language

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  $V_N = \{S\}, T = \{a, b\}, S = \{S\}, P = \{S \rightarrow aSb|\epsilon\}$

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  $V_N = \{S\}, T = \{a, b\}, S = \{S\}, P = \{S \rightarrow aSb|\epsilon\}$
  $S \rightarrow aSb \rightarrow aaSbb \rightarrow$ ... or $S \rightarrow \epsilon$
  So, $L = \{\epsilon, ab, aabb, aaabbb, ...\}$

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  $L(G) = a^n b^n$ for $n \geq 0$.

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- **Example 2:**
  $P : \{S \to aCa, C \to aCa|b\}$

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- **Example 3:**
  $S \to AbB, A \to aA|a, B \to aB|a$
  $L(G) = a^n ba^m, n, m > 0$.

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- **Example 4:**
  $S \rightarrow aS|bS|\epsilon$
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- **Example 5:**
  $S \rightarrow aSa|bSb|c$ (c is also a terminal symbol)

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  $L(G) = \{a, b\}^*$.
- **Example 5:**
  $S \rightarrow aSa|bSb|c$ (c is also a terminal symbol)
  $L(G) = WcW^R$, W is any string from $\{a, b\}^*$ and W^R is reverse of string W.

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  $L(G) = a^n c^i b^n, n, i \geq 0$

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- **Example 4:**
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  Grammar: $G = \{V_N = \{S, A\}, T = \{a, b, c\}, S, P\}$, where
  $P = \{S \rightarrow aSb|A, A \rightarrow Ac|\epsilon\}$.

Examples- Context-Sensitive Grammar

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Examples- Context-Sensitive Grammar

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- **Example 2:**
  $L(G) = xx, x \in \{a, b\}^*$

Examples- Context-Sensitive Grammar

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- **Example 2:**
  $L(G) = xx, x \in \{a, b\}^*$
  Grammar: $G = \{V_N = \{S, A, B, Z\}, T = \{a, b\}, S, P = \{S \rightarrow aAS|bBS|aAZ|bBZ, Aa \rightarrow aA, Bb \rightarrow bB, AZ \rightarrow Za, BZ \rightarrow Zb, Z \rightarrow \epsilon\}\}$.

DIY Exercise

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- Construct a grammar for palindrome of binary numbers.

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Questions!!!