NAME- ATHARVA BHATT
ROLL NO- 20095019
BRANCH - ELECTRONICS ENGINEERING

E0203 - MID TERM EXAM

Q.1.
(a) $F(A,B,C,D) = \sum m(0,1,5,6,8,10,12,15)$

$$\therefore F = \underbrace{\bar{A}\bar{B}\bar{C}\bar{D}}_{0} + \underbrace{\bar{A}\bar{B}\bar{C}D}_{1} + \underbrace{\bar{A}B\bar{C}D}_{5}$$
$$+ \underbrace{\bar{A}BC\bar{D}}_{6} + \underbrace{A\bar{B}\bar{C}\bar{D}}_{8} + \underbrace{A\bar{B}C\bar{D}}_{10}$$
$$+ \underbrace{AB\bar{C}\bar{D}}_{12} + \underbrace{ABCD}_{15}$$

$$F = \bar{A}[\bar{B}\bar{C}\bar{D} + \bar{B}\bar{C}D + B\bar{C}D + BC\bar{D}]$$
$$+ A[\bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + B\bar{C}\bar{D} + BCD]$$

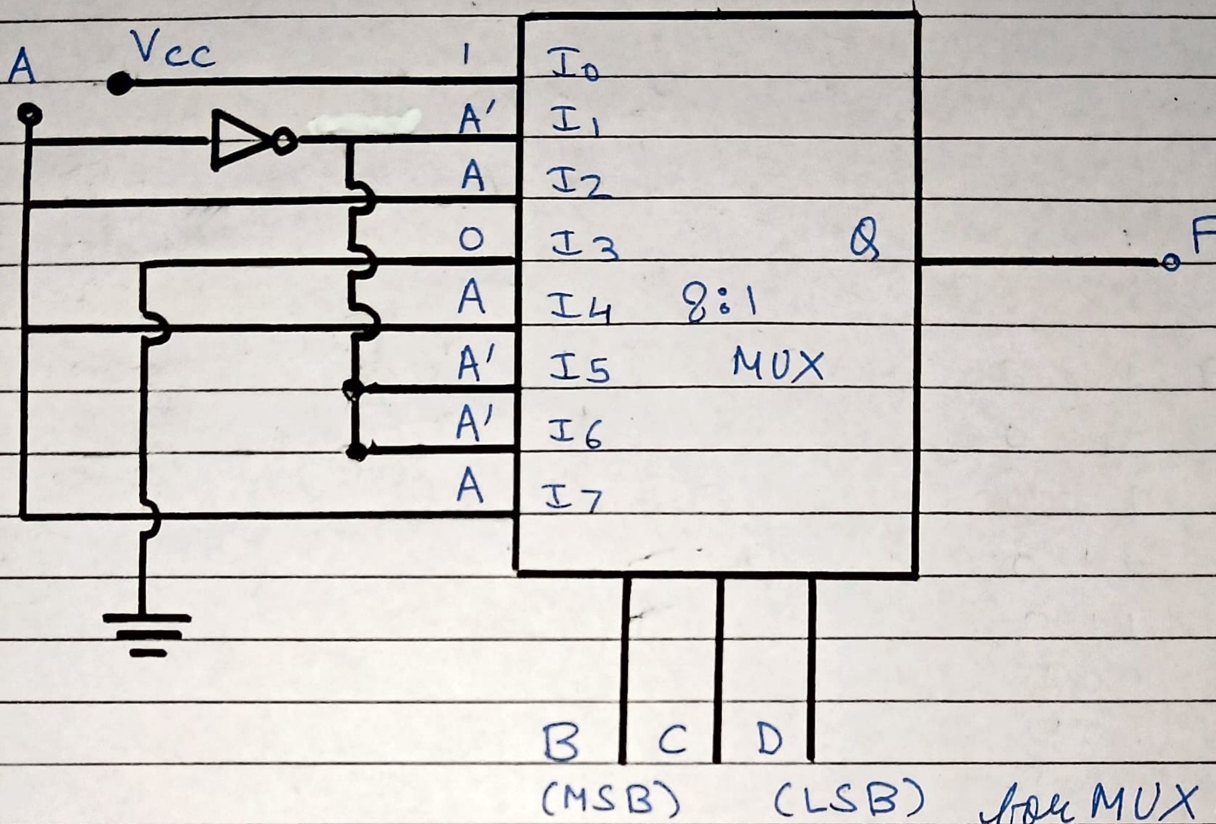
Now, $A + \bar{A} = 1$ {Complement Law}

$$\therefore F = (1)\underbrace{\bar{B}\bar{C}\bar{D}}_{0} + \bar{A}[\underbrace{\bar{B}\bar{C}D}_{1} + \underbrace{B\bar{C}D}_{5} + \underbrace{BC\bar{D}}_{6}]$$
$$+ A[\underbrace{\bar{B}C\bar{D}}_{2} + \underbrace{B\bar{C}\bar{D}}_{4} + \underbrace{BCD}_{7}]$$

- Therefore, the selection lines will be B(MSB), C and D and the input lines will be, $I_0 = 1$ (Vcc), $I_1 = \bar{A}$, $I_2 = A$, $I_3 = 0$ (ground), $I_4 = A$, $I_5 = \bar{A}$, $I_6 = \bar{A}$, $I_7 = A$. for the 8:1 MUX.

P.T.O.

⇒ Thus, the implementation using 8:1 MUX is,



for MUX.

(b) The 2's complement representation for decimal number (−28) is 2's complement of decimal number (28).

Now, 16-bit binary representation of decimal number (28) is

0000 0000 0001 1100

∴ 1's complement : 1111 1111 1110 0011

∴ 2's complement : 1111 1111 1110 0011 + 1

= 1111 1111 1110 0100 (−28)

⇒ The 16-bit 2's complement representation for decimal number (−28) is,

1111 1111 1110 0100.

(C) To convert Binary to Gray Code:,

- In the Gray code, the MSB will be the same as the MSB of the given binary number.

- Now, to get the 2nd bit of the Gray code, we perform exclusive-or (XOR) of the 1st and 2nd bit of the binary number.

- Similarly, the n^{th} bit of Gray code $(n>1)$ is the XOR of $(n-1)^{th}$ and n^{th} bit of the binary number.

Now, the given binary number is 1011,

$\therefore$ MSB of Gray Code = 1,

$\therefore$ 2^{nd} bit of Gray Code $= \underbrace{1 \oplus 0}_{1^{st} \oplus 2^{nd} \text{ bit (binary)}} = 1$

$\therefore$ 3^{rd} bit of Gray Code $= \underbrace{0 \oplus 1}_{2^{nd} \oplus 3^{rd} \text{ bit (binary)}} = 1$

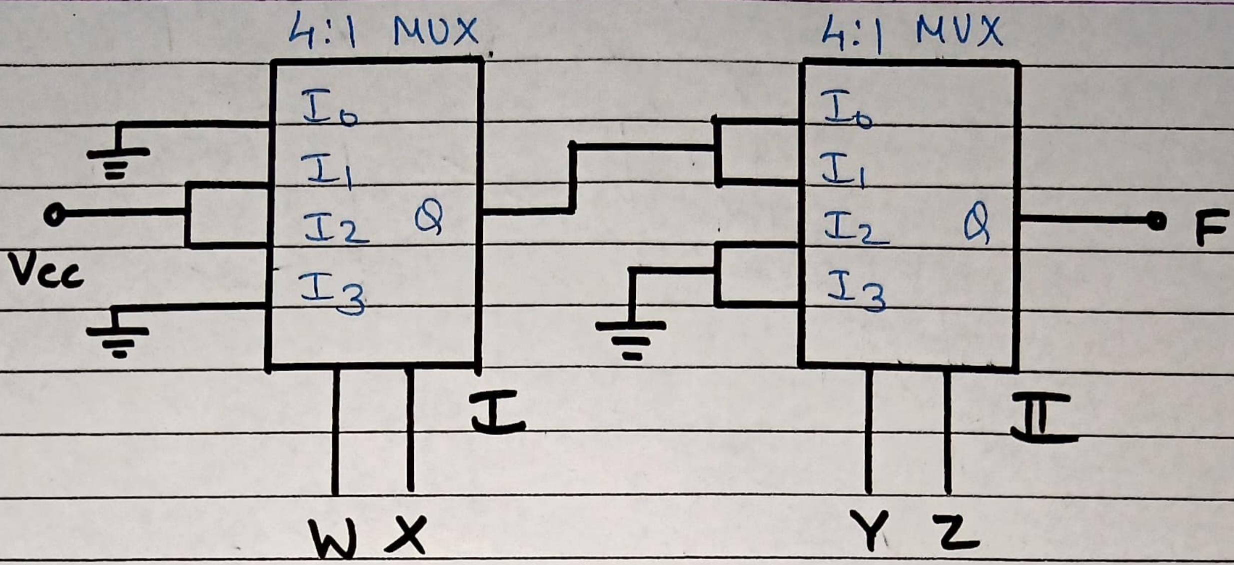
$\therefore$ 4^{th} bit of Gray Code $= \underbrace{1 \oplus 1}_{3^{rd} \oplus 4^{th} \text{ bit (binary)}} = 0$

$\therefore$ Gray Code form is 1110.

$\Rightarrow$ The binary number 1011 in the form of Gray Code is:

1110.

Q.2.



Function Table.

W	X	QI
0	0	$I_0(I)$
0	1	$I_1(I)$
1	0	$I_2(I)$
1	1	$I_3(I)$

Y	Z	QII
0	0	$I_0(II)$
0	1	$I_1(II)$
1	0	$I_2(II)$
1	1	$I_3(II)$

∴ The boolean expression for QI is :

$$Q_I = \bar{W}\bar{X}(I_0(I)) + \bar{W}X(I_1(I)) + W\bar{X}(I_2(I)) + WX(I_4(II))$$

$$\left\{ I_0 = GND \equiv 0,\ I_1 = Vcc \equiv 1,\ I_2 = Vcc \equiv 1,\ I_3 = GND \equiv 0 \right\}.$$

$$\therefore Q_I = \bar{W}X + W\bar{X} = W \oplus X \quad \text{—— (1)}$$

Now, the boolean expression for QII is :

$$Q_{II} = \bar{Y}\bar{Z}(I_0(II)) + \bar{Y}Z(I_1(II)) + Y\bar{Z}(I_2(II)) + YZ(I_3(II)).$$

$$\{ I_0 = I_1 = Q_I ,\ I_2 = I_3 = GND \equiv 0 \}$$

$$\therefore\ Q_{II} = \overline{Y}\,\overline{Z}\,Q_I + \overline{Y} Z Q_I \quad \text{, from (1),}$$

$$\therefore\ Q_{II} = \overline{Y}\,\overline{Z}(\overline{W}X + W\overline{X}) + \overline{Y}Z(\overline{W}X + W\overline{X})$$

$$\therefore\ Q_{II} = \overline{W}X\overline{Y}\,\overline{Z} + W\overline{X}\,\overline{Y}\,\overline{Z} + \overline{W}X\overline{Y}Z + W\overline{X}\,\overline{Y}Z .$$

$$\therefore\ Q_{II}(W,X,Y,Z) = \Sigma m(4,5,8,9) = F$$

WX \ YZ	00	01	11	10
00	0_0	0_1	0_3	0_2
01	1_4	1_5	0_7	0_6
11	0_{12}	0_{13}	0_{15}	0_{14}
10	1_8	1_9	0_{11}	0_{10}

K-Map.

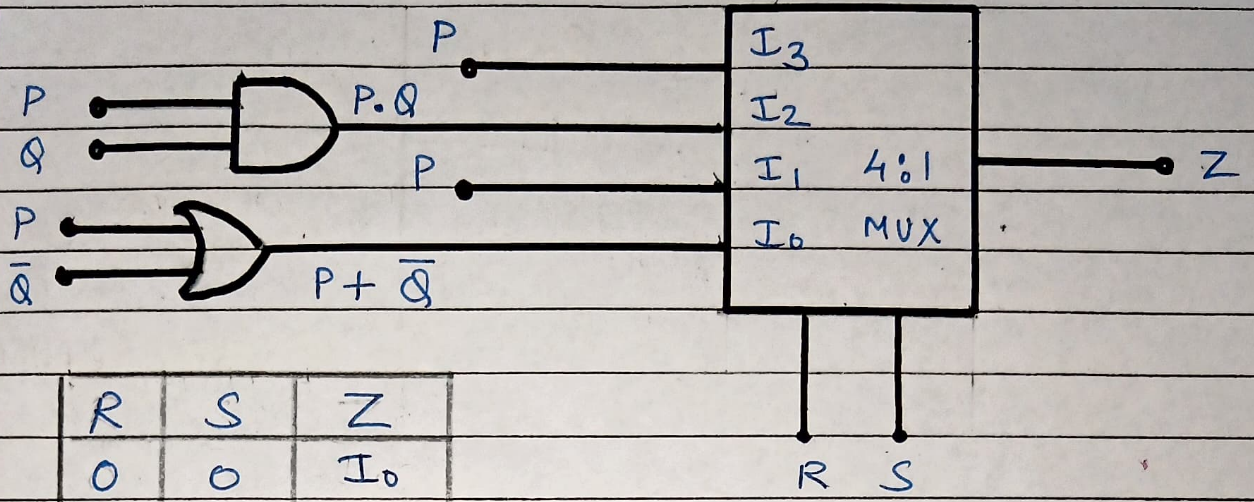
$$\therefore\ F = W\overline{X}\,\overline{Y} + \overline{W}X\overline{Y}$$

⇒ The minimized Boolean expression corresponding to output F is,

$$F = W\overline{X}\,\overline{Y} + \overline{W}X\overline{Y} = (W \oplus X)\overline{Y}$$

⇒ $F = (W \oplus X)\overline{Y}$ $\quad \{ W\overline{X} + \overline{W}X = W \oplus X \}$

Q.3.



R	S	Z
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

Function Table

∴ the boolean expression is given by :

$$Z = \overline{R}\,\overline{S}(I_0) + \overline{R}S(I_1) + R\overline{S}(I_2) + RS(I_3)$$

$$\therefore Z = RS(P) + R\overline{S}(P.Q) + \overline{R}S(P) + \overline{R}\,\overline{S}(P+\overline{Q})$$

$$= PRS + PQR\overline{S} + P\overline{R}S + P\overline{R}\,\overline{S} + \overline{Q}\,\overline{R}\,\overline{S}$$

$$= PRS(Q+\overline{Q}) + PQR\overline{S} + P\overline{R}S(Q+\overline{Q})$$
$$+ P\overline{R}\,\overline{S}(Q+\overline{Q}) + \overline{Q}\,\overline{R}\,\overline{S}(P+\overline{P})$$

{∵ $A+\overline{A}=1$}
{Complement Law}

$$\therefore Z = PQRS + P\overline{Q}RS + PQR\overline{S} + P\overline{R}SQ + P\overline{R}S\overline{Q}$$
$$+ P\overline{R}\,\overline{S}Q + \underbrace{P\overline{R}\,\overline{S}\,\overline{Q} + P\overline{Q}\,\overline{R}\,\overline{S}}_{(P\overline{Q}\,\overline{R}\,\overline{S})} + \overline{P}\,\overline{Q}\,\overline{R}\,\overline{S}$$

$$\therefore Z_{(P,Q,R,S)} = \Sigma m(0,8,9,11,12,13,14,15)$$

{∵ $A+A=A$}

Thus, the K-Map is given by,

PQ \ RS	00	01	11	10
00	1_{0}	0_{1}	0_{3}	0_{2}
01	0_{4}	0_{5}	0_{7}	0_{6}
11	1_{12}	1_{13}	1_{15}	1_{14}
10	1_{8}	1_{9}	1_{11}	0_{10}

K-Map.

$$\therefore\ Z = \overline{Q}\,\overline{R}\,\overline{S} + PQ + PS$$

=) The minimized expression corresponding to the output Z is,

$$Z = P(Q+S) + \overline{Q}\,\overline{R}\,\overline{S}.$$

Q.4.

$f(A,B,C,D) = \sum m(2,3,8,10,11,12,14,15)$

$\therefore f = \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$
$+ A\bar{B}CD + ABC\bar{D} + ABC\bar{D} + ABCD$

$\therefore$ The K-Map is given by.

K-Map.

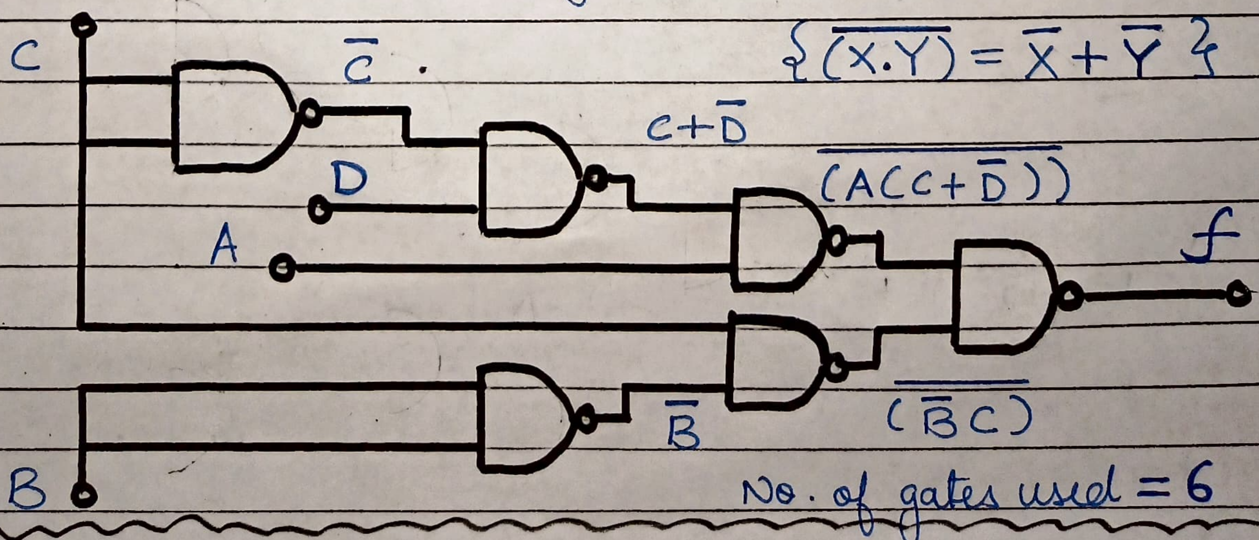
AB \ CD	00	01	11	10
00	0 (0)	0 (1)	1 (3)	1 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	1 (12)	0 (13)	1 (15)	1 (14)
10	1 (8)	0 (9)	1 (11)	1 (10)

$\therefore f = A\bar{D} + AC + \bar{B}C$
(Sum of products)

$\Rightarrow$ The minimised expression in the form SOP:

$$f = A(C+\bar{D}) + \bar{B}C$$

$\Rightarrow$ The realization using two input NAND gates,



No. of gates used = 6

Q.5.

$$f(w,x,y,z) = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}.$$

$$= wy(x+\bar{x})(z+\bar{z}) + xy(w+\bar{w})(z+\bar{z}) + \bar{w}xyz + \bar{w}\bar{x}y(z+\bar{z}) + \bar{x}\bar{y}\bar{z}(w+\bar{w}) + xz(y+\bar{y})(w+\bar{w}).$$

$\{A + \bar{A} = 1\}$

$$= wxyz + wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + wxyz + wxy\bar{z} + \bar{w}xyz + \bar{w}xy\bar{z} + \bar{w}xyz + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z} + wxyz + wx\bar{y}z + \bar{w}xyz + \bar{w}x\bar{y}z.$$

Now, $A + A = A$, therefore,

$$= wxyz + wxy\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}xy\bar{z} + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z} + wx\bar{y}z + \bar{w}x\bar{y}z$$

Thus, the K-map is given by,

wx \ yz	00	01	11	10
00	1 (0)	0 (1)	1 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	1 (8)	0 (9)	1 (11)	1 (10)

K-Map

$$f = \sum m(0,2,3,5,6,7,8,10,11,13,14,15)$$

P.T.O.

⇒ From the K-map,

$$f = \bar{x}\bar{z} + xz + y$$

⇒ Therefore, the number of essential prime implicants corresponding to this function is 3 and their complete list is,

$$y,\ xz,\ \bar{x}\bar{z}$$

$$f = y + xz + \bar{x}\bar{z}$$

Q-6.

⇒ The BCD representation of $(2)_{10}$ is 0010.

∴ Excess-3 representation is 0010 + 0011 $(3)_{10}$.

∴ Excess-3 representation of $(2)_{10}$ is 0101. —— 1

⇒ The BCD representation of $(5)_{10}$ is 0101.

∴ Excess-3 representation is 0101 + 0011 $(3)_{10}$.

∴ Excess-3 representation is 1000. —— 2.

Excess-3 addition $(2)_{10} + (5)_{10}$.

$$
\begin{array}{r}
0101 \\
+\ 1000 \\
\hline
1101 \\
-\ 0011 \\
\hline
1010 \\
\hline
\end{array}
$$

from 1 and 2.

$(3)_{10}$ (∵ no carry)

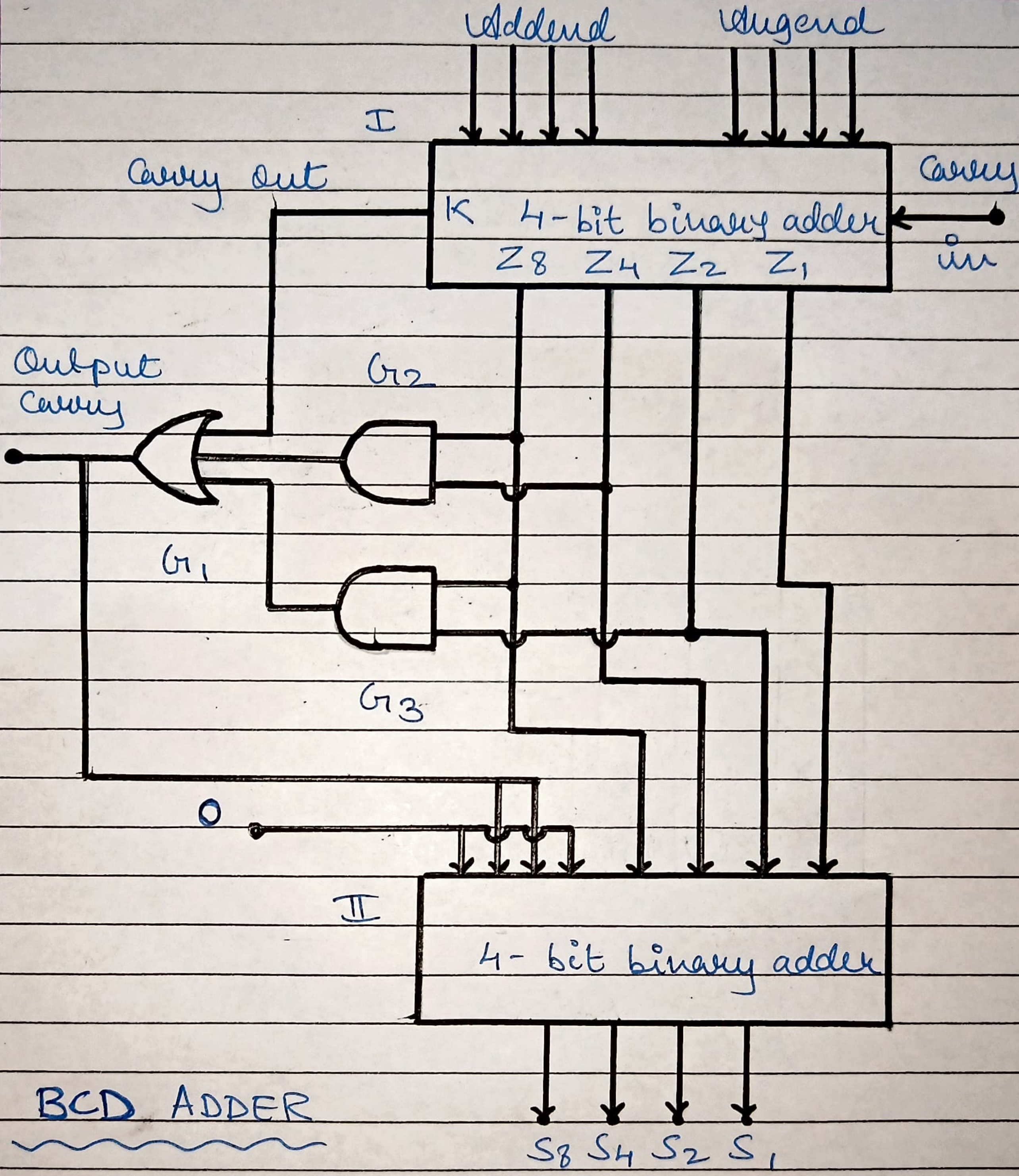
1010 ⟶ Excess-3

⇒ The answer for $(2)_{10} + (5)_{10}$ for Excess-3 addition is :

1010

Decimal form $= (10)_{10}$

Q.7.



BCD ADDER

Consider the arithmetic addition of two decimal digits in BCD, together with an input carry from a previous stage. Since each input digit does not exceed 9, the output sum cannot be greater than 9+9+1 = 19, the 1 in the sum being input carry.

When we apply two BCD digits to a four-bit binary adder, the adder forms the sum in binary and produce a result that ranges from 0 through 19. These binary numbers are labelled by symbols K, Z_8, Z_4, Z_2, Z_1. K is the carry, and the subscripts under the letter Z represent the weights 8, 4, 2 and 1 that can be assigned to the four bits in the BCD code.

Now output of AND gate G_2 is $g_2 = Z_4 \cdot Z_8$ and of AND gate G_3 is $g_3 = Z_2 \cdot Z_8$.

Output of OR gate G_1 is $g_1 = g_2 + g_3 + k$.

$g_1 = C = Z_4 \cdot Z_8 + Z_2 \cdot Z_8 + k$.

Thus, G_1 gives output 1 whenever Z_4 and Z_8 bit are set or, Z_2 and Z_8 bit are set, or the carry K of first adder is 1.

Together this combination gives 1 as output when the sum of the two (BCD) decimal digits and input carry is greater than 9.

When the sum is greater than 9, gate G_1's output causes $(0110)_2 = (6)_{10}$ to be added to the sum from the first adder using a new second adder. When $g_1 = C = 1$, $(0110)_2$ is added to sum otherwise $(0000)_2$ is added, i.e., sum is unchanged.

The required output is $S_8 S_4 S_2 S_1$. The output

carry generated by the 4-bit adder II can be ignored, since it supplies information already available at output carry terminal (C).
A decimal parallel adder that adds n decimal digits needs n BCD adder stages. The output carry (C) from one stage must be connected to the input carry of the next higher ~~stage~~ order stage.

$$C = Z_4 \cdot Z_8 + Z_2 \cdot Z_8 + K$$

⇒ Thus, this is how the circuit of BCD adder works.

K is 1 for sum from 4-bit adder I, $Z_4 \cdot Z_8$ is 1 for sum greater than or equal to 12, $Z_2 \cdot Z_8$ is 1 for sum equal to 11 and 10 (also for 14 and 15). Thus C is 1 for sum from 4-bit adder I greater than 9 (≥10).