

Logic.

High (logic 1)
 Low (logic 0)

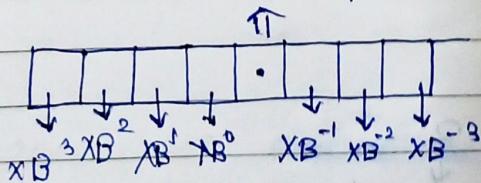
- positive logic : logic 0 (low) = 0 V
 logic 1 (high) = +5 V
- Negative logic : logic 0 = +5 V
 logic 1 = 0 V
- ⇒ largest digit = (Base - 1)

	Number system	Base
①	Decimal	10
②	Binary	2
③	Octal	8
④	Hexadecimal	16

Number system

- ① Positional
- ② Non-positional
 - Grey code
 - Excess-3 code (XS-3)

Radix point.



Conversion related to decimal system

* conversion from any N.S to decimal system

$$\textcircled{1} \quad (1011.01)_2$$

$$\begin{array}{r} \downarrow \quad \downarrow \quad \downarrow \\ 1x2^0 \quad 1x2^{-1} \quad 1x2^{-2} \\ \downarrow \quad \downarrow \quad \downarrow \\ 1x2^3 \quad 1x2^1 \quad 0x2^2 \\ \downarrow \quad \downarrow \\ 0x2^0 \end{array}$$

$$8 + 2 + 4 . 25$$

$$11.25$$

$$\textcircled{2} \quad (314)_8$$

$$\begin{array}{r} \downarrow \quad \downarrow \\ 3x8^0 \quad 1x8^1 \\ \downarrow \quad \downarrow \\ 3x8^2 \end{array}$$

$$192 + 8 + 4$$

$$(204)_{10}$$

$$\textcircled{3} \quad (365.24)_8$$

$$\begin{array}{r} 3 \quad 6 \quad 5 \quad . \quad 2 \quad 4 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 3x8^2 \quad 6x8^1 \quad 5x8^0 \quad 2x8^{-1} \quad 4x8^{-2} \end{array}$$

$$192 + 48 + 5 . 25$$

$$245.3125$$

$$\textcircled{4} \quad (4C8.2)_{16}$$

$$\begin{array}{r} 4 \quad C \quad 8 \quad . \quad 2 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 4x16^2 \quad 12x16^1 \quad 8x16^0 \quad 2x16^{-1} \end{array}$$

$$1024 + 192 + 8 . 125$$

$$= (1224.125)_{10}$$

* conversion of decimal to any other.

① $(105)_{10}$ into binary.

2	105	
2	52	1
2	26	0
2	13	0
2	6	1
2	3	0
1.	1	

↑

$\Rightarrow (1101001)_2$

Steps.

- ① Divide integer part of given decimal system by the base of desired no. system and write remainder.
- ② Keep dividing the quotient by base until nothing is left.
- ③ Write down the remainder in reverse order.
- ④ And it will give you equivalent number.

② $(204)_{10}$

$$\begin{array}{r} 8 | 204 \\ 8 | 25 \quad 4 \\ \quad \quad 3 \quad 1 \\ \quad \quad \quad 3 \end{array} \Rightarrow (314)_8$$

③ $(259)_{10}$

$$\begin{array}{r} 16 | 259 \\ 16 | 16 \quad 3 \\ \quad \quad 1 \quad 0 \\ \quad \quad \quad 1 \end{array} \Rightarrow (103)_16$$

- Successive Multiplication of fractional part.

1. Multiply the given fractional decimal number by the base of desired number system.
2. Record the carry as MSB
3. Multiply only fractional part of result obtained from step 2 by base & record the carry as next bit to MSB
4. Repeat steps 2 + 3 till the end & last carry will correspond to LSB

- Usually the conversion for fractional part is carried out only upto 5 digits

① $(0.42)_{10}$ into binary.

carry

$$0.42 \times 2 = \boxed{0.84}$$

0 MSB

$$0.84 \times 2 = \boxed{1.68}$$

1

$$1.68 \times 2 = \boxed{1.36}$$

1

$$0.36 \times 2 = \boxed{0.72}$$

0

$$0.72 \times 2 = \boxed{1.44}$$

1 LSB

$$\text{Ans} = (0.01101)_2$$

② $(0.8)_{10}$ into binary

$$0.8 \times 2 = \boxed{1.6} \quad 1$$

$$0.6 \times 2 = \boxed{1.2} \quad 1$$

$$\text{Ans: } (0.11001)_2$$

$$0.2 \times 2 = \boxed{0.4} \quad 0$$

$$0.4 \times 2 = \boxed{0.8} \quad 0$$

$$0.8 \times 2 = \boxed{1.6} \quad 1$$

③ $(0.6234)_{10} \rightarrow$ octal

$$0.6234 \times 8 = \boxed{4.9872} \quad 4$$

$$0.9872 \times 8 = \boxed{7.8976} \quad 7$$

$$\text{Ans: } (0.47713)_8$$

$$0.8976 \times 8 = \boxed{7.1808} \quad 7$$

$$0.1808 \times 8 = \boxed{1.4464} \quad 1$$

$$0.4464 \times 8 = \boxed{13.5712} \quad 3$$

4 $(0.122)_{10}$

$$\rightarrow 0.122 \times 16 = 1\boxed{1}.952 \quad 1$$

$$0.952 \times 16 = \boxed{1}5.232 \quad 15$$

$$0.232 \times 16 = \boxed{3}.712 \quad 3$$

$$0.712 \times 16 = \boxed{1}1.392 \quad 11$$

$$0.392 \times 16 = \boxed{6}.272 \quad 6$$

$$\text{Ans} = (0.1\text{E}3\text{B}6)_{16}$$

5 $(85.63)_{10} \rightarrow \text{binary}$.

$$0.63 \times 2 = 1.26$$

$\rightarrow 1$

$$0.26 \times 2 = 0.52$$

$\rightarrow 0$

$$0.52 \times 2 = 1.04$$

$\rightarrow 1$

$$0.04 \times 2 = 0.08$$

$\rightarrow 0$

$$0.08 \times 2 = 0.16$$

$\rightarrow 0$

2	85	
2	42	1
2	21	0
2	10	1
2	5	0
2	2	1
1	0	

$$(1010101.10100)_2$$

② $(3000.45)_{10}$ to Octal.

$$\begin{array}{rcl}
 0.45 \times 8 & \rightarrow & 3.6 \\
 0.6 \times 8 & \rightarrow & 1.8 \\
 0.8 \times 8 & \rightarrow & 6.4 \\
 0.4 \times 8 & \rightarrow & 3.2 \\
 0.2 \times 8 & \rightarrow & 1.6
 \end{array}$$

3 LSD

1

6

3

1 LSD

8	3000	
8	375	0
8	46	7 ↑
8	5	6

$$\text{Ans} = (5670.34631)_8$$

③ $(2003.31)_{10}$ to hexa

$$\begin{array}{rcl}
 0.31 \times 16 & \rightarrow & 4.96 \rightarrow 4 \quad \text{MSD} \\
 0.96 \times 16 & \rightarrow & 15.36 \rightarrow 15 \\
 0.36 \times 16 & \rightarrow & 5.76 \rightarrow 5 \\
 0.76 \times 16 & \rightarrow & 12.16 \rightarrow 12 \\
 0.16 \times 16 & \rightarrow & 2.56 \rightarrow 2 \quad \text{LSD}
 \end{array}$$

16	2003	
16	125	3 ↑
16	7	13 7

$$(7D3.4F5B2)_{16}$$

Conversion from binary to other systems.

Binary to octal.

$$\textcircled{1} \quad (11010010)_2$$

0	1	1	0	1	0	1	0
↓	↓	↓					
3	2	2					

0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	1
1	0	0	1	0	1	0	1
1	0	0	1	0	1	0	1
1	0	0	1	0	1	0	1
1	0	0	1	0	1	0	1
1	0	0	1	0	1	0	1

\rightarrow Ans = $(322)_8$

- * Group the bits into group of 3 LSB.
- * Always add extra zero's on MSB side and not on LSB side.

Binary to hex

$$\textcircled{2} \quad (11010010)_2$$

1	1	0	1	0	0	1	0
↓	↓						
1	3	2					

$\Rightarrow (D2)_{16}$

1	0	1	1	1	0	1	0	1	0
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
A	F	B	2						

$(AFB2)_{16}$

conversion from any N.S to binary.

① octal to binary

$$(364)_8$$

$$(011110100)_2$$

② Hex to Binary

$$(AFB2)_{16}$$

$$(101011110110010)_2$$

+ octal + hex

$$(436)_8$$

$$00(10001110)_2$$

$$(11E)_{16} \quad \text{Ans}$$

0	0	0	0	-	0
0	0	0	1	-	1
0	0	1	0	-	2
0	0	1	1	-	3
0	1	0	0	-	4
0	1	0	1	-	5
0	1	1	0	-	6
0	1	1	1	-	7
1	0	0	0	-	8
1	0	0	1	-	9
1	0	1	0	-	A
1	0	1	1	-	B
1	1	0	0	-	C
1	1	0	1	-	D
1	1	1	0	-	E
1	1	1	1	-	F

→ Hex +> Octal

① $(F6A)_{16}$

$\begin{array}{ccccccccc} 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ \boxed{2} & \boxed{1} & \boxed{5} & \quad & \end{array}$

$(2152)_8$

$\begin{array}{ccccccccc} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ \boxed{2} & \boxed{3} & \boxed{1} & \quad & \end{array}$

$(2312)_8$

② $(0.12E)_{16}$

$0.\underbrace{0001}_{0} \underbrace{001}_{4} \underbrace{0}_{5} \underbrace{111}_{5} 0$

0 4 . 5 5

$(0.0456)_8$

③ $(68.4B)_{16}$

0 will be added
to MSB side
(integer part)

$\begin{array}{ccccccccc} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & . & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ \boxed{1} & \boxed{5} & \quad & \end{array}$

1 5 0 . 2 2 6

0 added after LSB
in frac. part

$(150.226)_8$

* Convert following decimal no to binary, octal, hexadecimal.

$$\textcircled{1} \quad (25.55)_{10}$$

$$\textcircled{2} \quad (456)_{10}$$

* Convert hex \rightarrow decimal, binary & octal

$$\textcircled{1} \quad 48 \quad \textcircled{2} \quad 2E$$

* Convert following number into decimal

$$\textcircled{1} \quad (10110.0101)_2 \quad \textcircled{2} \quad (16.5)_{16}$$

* Represent decimal numbers in binary, octal and hexadecimal.

$$\textcircled{1} \quad (3.96)_{10} \quad \textcircled{2} \quad (4096)_{10}$$

* Convert following octal no to hexa, binary, decimal.

$$\textcircled{1} \quad (76)_8 \quad \textcircled{2} \quad (0.7034)_8 \quad \textcircled{3} \quad (1567)_8 \quad \textcircled{4} \quad (65.04)_8$$

(Q.1)

$$\textcircled{1} \quad (25.55)_{10}$$

2	2	5	
2	1	2	1
2	6	0	$0.55 \times 2 = 1.1 = 1$
2	3	0	$0.1 \times 2 = 0.2 = 0$
	1	1	$0.2 \times 2 = 0.4 = 0$
		1	$0.4 \times 2 = 0.8 = 0$
			$0.8 \times 2 = 1.6 = 1$

$$\begin{array}{r} 611001.10010 \\ \hline 31.42 \end{array}_2$$

$$(31.42)_8$$

00011001.10001000

1 9 . 88

(19.88)₁₆

②

(456)₁₀

2	456
2	228
2	114
2	57
2	28
2	14
2	7
2	3
1	1
	1

(111001000)₂

(7 10)₈

(1C8)₁₆

①

4 A

411

2	4	1	1
2	2	0	5
2	1	0	2
2	5	1	0
2	2	5	1
2	1	2	1
2	6	0	
2	3	0	
	1	1	
		1	

 $(\underbrace{1 \ 1 \ 0 \ 0}_{6} \ \underbrace{1 \ 1}_{3} \ \underbrace{0 \ 1 \ 1}_{3})_2$

6 3 3

 $(633)_8$

$$\begin{array}{r} 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \\ 2^8 \ 2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \cancel{2} \ \cancel{2} \end{array}$$

 $(639)_{10}$

②

2 E

2	2	1	4
2	1	0	7
2	5	3	1
2	2	6	1
2	1	3	0
2	6	1	
2	3	0	
	1	1	

 $(\underbrace{0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0}_{3 \ 2 \ 6})_2$
 $(326)_8$ decimal
 $2 \times 16^4 + 14 \times 16^0$ $(46)_{10}$

① $(101100101)_2$

$$\begin{array}{ccccccccc} 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

$$256 + 32 + 64 + 4 + 1 \\ (357)_{10}$$

② $(16.5)_{16}$

$$\begin{array}{r} 1 \quad 6 \quad . \quad 5 \\ \times 16 \quad 1 \quad 6 \times 16^0 \quad 5 \times 16^{-1} \end{array}$$

$$\begin{array}{r} 16 + 6.3125 \\ 22.3125 \\ \hline \end{array}$$

Q. ① $(3.96)_{10}$

$$\begin{array}{r} 2 \quad | \quad 3 \\ \quad \quad | \quad 1 \\ \quad \quad \quad | \quad 1 \\ \quad \quad \quad \quad | \quad 1 \end{array}$$

Binary (-1)

$$(11.11110)_2$$

Octal

$$(3.74)_8$$

hexa

$$(3.FD)_{16}$$

$$0.96 \times 2 = 1.92 + 1$$

$$0.92 \times 2 = 1.84 + 1$$

$$0.84 \times 2 = 1.68 + 1$$

$$0.68 \times 2 = 1.36 + 1$$

$$0.36 \times 2 = 0.72 + 0$$

②

$(4096)_{10}$

2	4096	
2	2048	0
2	1024	0
2	512	0
2	256	0
2	128	0
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	0
		1

① $(76)_8$

$(111110)_2$

$(3E)_{16}$

$$\begin{array}{r} 7 \\ 6 \times 8^1 \quad 6 \times 8^0 \\ \hline (54)_{10} \end{array}$$

$(62)_{10}$

$$\textcircled{2} \quad (1567)_8$$

$$(0\underline{0}\underline{1}\underline{1}\underline{0}\underline{1}\underline{1}\underline{0}\underline{1}\underline{1})_2$$

$$\begin{array}{r} (1567)_8 \\ \times 8^3 \\ \hline 512 & + 320 & + 48 & + 7 = (887)_{10} \end{array}$$

$$(377)_{16}$$

$$\textcircled{3} \quad (65.04)_8$$

$$(110\underline{101.000100})_2$$

$$(35.10)_{16}$$

$$\begin{array}{r} 6 \quad 5 \quad . \quad 0 \quad 4 \\ | \quad \downarrow \quad \quad \uparrow \quad 4 \times 8^{-2} \\ 6 \times 8^1 \quad 5 \times 8^0 \end{array}$$

$$48 + 5.0625$$

$$(53.0625)_{10}$$

$$\textcircled{2} \quad (0.7634)_8$$

$$(0.\underline{1111}\underline{0011}\underline{100})_2$$

$$(0.F9C)_{16}$$

$$(0.7634)_8$$

1	1	1	1
8^{-1}	8^{-2}	8^{-3}	8^{-4}

$$= 0.875 + 0.09375 + 0.00585 + 0.000976$$

$$(0.0975)_{10}$$

$$(0.0975)_{10}$$

→ Binary Addition.

carry sum

$$\begin{array}{r}
 0 + 0 & 0 & 0 \\
 0 + 1 & 0 & 1 \\
 1 + 0 & 0 & 1 \\
 1 + 1 & 1 & 0
 \end{array}$$

$$A = (10111)_2$$

$$B = (11001)_2$$

$$\begin{array}{r}
 & 1 & 1 & 1 & 1 \\
 & 1 & 0 & 1 & 1 \\
 + & 1 & 1 & 0 & 0 \\
 \hline
 (110000)_2
 \end{array}$$

Binary Multiplication

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

$$(9)_{10} \times (8)_{10}$$

$$(1001) \times (1000)$$

$$\begin{array}{r} 1001 \\ \times 1000 \\ \hline \end{array}$$

$$\begin{array}{r} 1001 \\ \times 1000 \\ \hline 0000 \\ 0000 \times \\ 1001 \times \\ \hline 1001000 \end{array}$$

Q. Binary subtraction:

subtract borrow

1	0 - 0	0	0
	0 - 1	1	1
	1 - 0	1	0
	1 - 1	0	0

$$A = (11011)_2 \text{ and } B = (10110)_2$$

$$\begin{array}{r}
 1101100000 \\
 - 10110 \\
 \hline
 00101
 \end{array}$$

$$Q. 38 - 29$$

$$38 = 100110$$

$$29 = 11101$$

$$38 - 29$$

$$\begin{array}{r}
 \boxed{1} \quad \boxed{0} \quad 11 \quad 0 \\
 - \quad \boxed{1} \quad \boxed{1} \quad 10 \quad 1 \\
 \hline
 0 \quad \boxed{0} \quad \boxed{1} \quad 0 \quad 1
 \end{array}$$

4 3 2 1 2⁰

$$\begin{array}{r}
 2 \quad 38 \\
 2 \quad 19 \quad 0 \\
 2 \quad 9 \quad 1 \\
 2 \quad 4 \quad 1 \\
 2 \quad 2 \quad 0 \\
 \hline
 1 \quad 0
 \end{array}$$

$$\begin{array}{r}
 2 \quad 29 \\
 2 \quad 14 \quad 1 \\
 2 \quad 7 \quad 0 \\
 2 \quad 3 \quad 1 \\
 \hline
 1 \quad 1
 \end{array}$$

* Binary division

$$(110)_2 \div (10)_2 = (11)_2$$

$$6 \div 2 = 3$$

- Unsigned binary number:

smallest 8 bit unsigned no -

$$0000\ 0000 \quad (00)_H \quad (00)_{10}$$

largest 8 bit unsigned no

$$1111\ 1111 \quad (FF)_H \quad (255)_{10}$$

unsigned no

for 16 bit

$$(0000\ 0000\ 0000\ 0000)_{10} \quad (65535)_{10}$$

- Signed binary number:

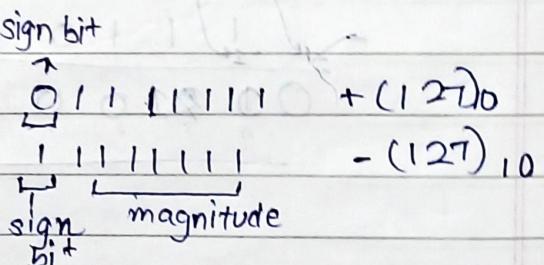
for 8 bit

+ve signed no = 0

-ve signed no = 1

for 16 bit

$$-(32767)_{10} \text{ to } (32767)_{10}$$



Complements.

1. Radix complemented form. (r 's complement form)
2. Diminished radix complemented form ($r-1$)'s complemented form.

- Binary 1's 2's
- Hexadecimal 15's 16's
- Octal 8's 9's
- Decimal 10's 9's

1's complement.

- One's complement of +ve no have same representation as signed no +5 \Rightarrow 0101
- One's complement of -ve no by inverting +ve no i.e. -5 \Rightarrow 1010

Range.

$$+ (2^m - 1) \leq 0 \leq (2^m - 1) \quad \text{for } 4\text{-bit} \quad + (2^{m-1} - 1) \Rightarrow - (2^{m-1} - 1)$$

+ve	-ve
Dec. No	Binary eq.

+ 7	0111	1000	-7
+ 6	0110	1001	-6
+ 5	0101	1010	-5
+ 4	0100	1011	-4
+ 3	0011	1100	-3
+ 2	0010	1101	-2
+ 1	0001	1110	-1
+ 0	0000	1111	0

drawback of 1's c it have +0 & -0

BOSS
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For two no signed magnitude form, one's complement & complement are all the same.

$$2^{\text{sc}} = 1^{\text{sc}} + (\text{LSB})$$

ex- -6

$$+6 = 0110$$

$$1^{\text{sc}} = 1001 \ (-6)$$

$$+ \boxed{1}$$

$$\overline{1010} \quad -2^{\text{sc}} [-6]$$

1's c

signed magnitude

2^sc signed magni.

$$+7 \quad 0111 \quad 0111 \quad 0111 \quad 0111$$

$$+6 \quad 0110 \quad 1010 \quad 0110 \quad 0110$$

$$+5 \quad 0101 \quad 1101 \quad 0101 \quad 0101$$

$$+4 \quad 0100 \quad 1110 \quad 0100 \quad 0100$$

$$+3 \quad 0011 \quad 1111 \quad 0011 \quad 0011$$

$$+2 \quad 0010 \quad 1111 \quad 0010 \quad 0010$$

$$+1 \quad 0001 \quad 1111 \quad 0001 \quad 0001$$

$$+0 \quad 0000 \quad 1111 \quad 0000 \quad 0000$$

$$-1 \quad 1110 \quad 0000 \quad 1111 \quad 1001$$

$$-2 \quad 1101 \quad 0001 \quad 1110 \quad 1010$$

$$-3 \quad 1100 \quad 0010 \quad 1101 \quad 1011$$

$$-4 \quad 1011 \quad 0011 \quad 1100 \quad 1011$$

$$-5 \quad 1010 \quad 0100 \quad 1011 \quad 1100$$

$$-6 \quad 1001 \quad 0101 \quad 1010 \quad 1101$$

$$-7 \quad 1000 \quad 0110 \quad 1001 \quad 1110$$

$$-8 \quad 0111 \quad 1111 \quad 0000 \quad 1111$$

$$\begin{array}{r}
 +5 \\
 -5 \\
 \hline
 (-5) \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}
 \end{array}$$

0 111
+
1000

100

- Binary addition.

Case-I same smaller no is -ve

$$A = -7$$

$$B = 12 \rightarrow 0.0001100$$

$$+7 = 00000111$$

$$-7 = 1111000$$

+ 1

$$\overline{11111001} + 2'sc \rightarrow -7$$

+ 00001.100

$$\boxed{X}00000101 = 5$$

- Binary subtraction by 2's complement

$$A - B \Rightarrow A + (-B)$$

① Add A to 2's complement of B

② If carry is generated then result is positive in true form. If not generated then result is negative & in 2's complement form

$$\begin{array}{r} 110 \\ + 101 \\ \hline 1001 \end{array}$$

$$0101 + 1 = 1010$$

2	1
2	1
2	0
2	0
1	0
	1

$$\begin{array}{r} 1001 \\ + 1010 \\ \hline \boxed{10011} \\ + \quad \curvearrowleft 1 \\ \hline 0100 \end{array} = +4 = (0100)_2$$

2	5
2	2
2	1
1	0
	1

② $(-48)_{10}$ and $(-23)_{10}$ using 8 bit representation

$$48 = 00110000 \quad 11001111$$

$$1's \text{ of } 48 = 1000111 \rightarrow 2^3 =$$

$$23 = 00010111$$

$$1's \text{ of } 23 = 101000$$

2	18
2	21
2	12
2	6
2	3
2	1
	0

$$\begin{array}{r} 000111 \\ + 001000 \\ \hline 001000 \end{array} \quad \begin{array}{r} 01000 \\ + 01001 \\ \hline 01001 \end{array}$$

2	23
2	11
2	5
2	2
1	0
	1

$$\begin{array}{r} 001000 \\ + 001001 \\ \hline 010001 \end{array}$$

$$\begin{array}{r} 1100111 \\ + 1110100 \\ \hline 1100000 + 11101001 \end{array}$$

$$\begin{array}{r}
 11010000 \\
 +11101001 \\
 \hline
 10110001
 \end{array}$$

$$\begin{array}{r}
 10111000 \\
 -1 \\
 \hline
 10111000
 \end{array}$$

\rightarrow 1's c. + 1's c
 \downarrow inverting.

$$\begin{array}{r}
 10100011 \\
 \downarrow \\
 10100011
 \end{array}$$

+ Final ans. $(-71)_{10}$

I To perform octal addition add +

-
- ① Add Octal numbers considering them as decimal numbers.
 - ② If the sum is greater than or equal to 8, subtract 8 from it and generate carry of 1

ex- $(2)_8 + (4)_8$

$$(2)_{10} + (4)_{10} = (6)_{10}$$

$$\begin{array}{r}
 (7)_8 + (4)_8 = (13)_8 \\
 \underline{+} \\
 \hline
 \end{array}$$

in decimal

$$\begin{array}{r}
 (11)_8 \rightarrow \text{greater than } 8 \\
 - 8 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1 \boxed{1} 3 \\
 \hline
 4
 \end{array}
 \Rightarrow (7)_8 + (4)_8 = (13)_{10}$$

carry

Q.

$$(765)_8 + (365)_8 = (1352)_8$$

$$\begin{array}{r}
 (7)_{10} (6)_{10} (5)_{10} \\
 + (3)_{10} (6)_{10} (5)_{10} \\
 \hline
 \begin{array}{r}
 \begin{array}{c} 1 \\ - 8 \\ \hline 1 \end{array} & \begin{array}{c} 1 \\ - 8 \\ \hline 5 \end{array} & \begin{array}{c} 1 \\ - 8 \\ \hline 2 \end{array}
 \end{array}
 \end{array}$$

$(1352)_8$

Q. $(354)_8 + (266)_8 + (123)_8$

$$\begin{array}{r}
 3 5 4 \\
 2 6 6 \\
 \hline
 \begin{array}{r}
 \begin{array}{c} 1 \\ - 8 \\ \hline 1 \end{array} & \begin{array}{c} 2 \\ 14 \\ - 8 \\ \hline 6 \end{array} & \begin{array}{c} 3 \\ 13 \\ - 8 \\ \hline 5 \end{array}
 \end{array}
 \end{array}$$

$(765)_8$

- Octal subtraction using 7's complement.

- ① Obtain 7's complement of number to be subtracted.
- ② It will be added to first number
- ③ If carry is generated add it to the sum in step 2
- ④ If carry is not generated take 7's complement of the result

$$\textcircled{1} \quad (536)_8 - (345)_8 = (171)_8$$

is complement of 345 is

$$\begin{array}{r} 777 \\ - 345 \\ \hline 1352 \end{array}$$

$$\begin{array}{r} 435 \\ + 536 \\ \hline \begin{array}{c} 7 \\ 11 \\ \boxed{13} \end{array} \end{array}$$

$$\begin{array}{r} 432 \\ + 536 \\ \hline \begin{array}{c} 9 \\ 6 \\ \boxed{1} \end{array} \end{array}$$

$$\begin{array}{r} 170 \\ + 1 \\ \hline 171 \end{array}$$

$$\textcircled{2} \quad (161)_8 - (243)_8 = -(62)_8$$

$$\begin{array}{r} 777 \\ - 243 \\ \hline 534 \\ + 161 \\ \hline \begin{array}{c} 95 \\ - 8 \\ \hline \boxed{1}5 \end{array} \end{array}$$

no carry

715

so take 7's complement

$$\begin{array}{r} 777 \\ - 715 \\ \hline - 62 \end{array}$$

Octal subtraction using 8's complement.

$$\textcircled{1} \quad (556)_8 - (345)_8$$

$$\begin{array}{r}
 7\ 7\ 7 \\
 - 3\ 4\ 5 \\
 \hline
 4\ 8\ 2 \\
 + \quad 1 \\
 \hline
 4\ 3\ 3 \rightarrow 8's \ c.
 \end{array}$$

$$\begin{array}{r}
 + 5\ 3\ 6 \\
 \hline
 \overset{-9}{\cancel{7}} \quad \overset{-8}{\cancel{7}} \quad \text{carry so discard it} \\
 \hline
 \boxed{\boxed{1}} \rightarrow \boxed{\boxed{1}} 71
 \end{array}$$

$$(556)_8 - (345)_8 = (171)_8$$

\textcircled{1} steps-

- \textcircled{1} obtain 8's complement of no. to be subtracted
- \textcircled{2} Add it to 1'st no
- \textcircled{3} if carry generated discard carry to get correct ans
- \textcircled{4} if carry is not generated take 8's c of the result.

$$\textcircled{2} \quad (161)_8 - (243)_8 = -(62)_8$$

$$\begin{array}{r}
 \overset{-2}{\cancel{7}} \quad \overset{-2}{\cancel{7}} \quad \overset{-2}{\cancel{3}} \\
 \hline
 5\ 8\ 4 \\
 + \quad 1 \\
 \hline
 5\ 3\ 5
 \end{array}
 \quad
 \begin{array}{r}
 + 5\ 3\ 5 \\
 \hline
 \overset{-9}{\cancel{1}} \quad \overset{-8}{\cancel{6}} \\
 \hline
 \boxed{\boxed{1}} 1
 \end{array}$$

$\therefore 716 + \text{no carry so take 8's c}$

$$\begin{array}{r}
 777 \\
 -716 \\
 \hline
 61 \\
 +1 \\
 \hline
 62
 \end{array}
 \quad \text{ans}$$

- Hexadecimal addition.

$$(C_2)_H + (SE)_H = (100)_H$$

$$(12^2 + (-314))$$

$$\begin{array}{r}
 & 2 \\
 & 2 \\
 + & 3 & 1 \\
 \hline
 & 4 & 3 & 6
 \end{array}
 \quad
 \begin{array}{r}
 3 \\
 1 \\
 \hline
 16 \\
 -16 \\
 \hline
 0 & 0
 \end{array}
 \quad
 = 100$$

$$\textcircled{2} \quad (\text{DDCC})_H + (\text{BBAA})_H$$

$$\begin{array}{cccc}
 D & D & C & C \\
 B & B & A & A \\
 \hline
 \begin{array}{r} 26 \\ -16 \end{array} & \begin{array}{r} 25 \\ -16 \end{array} & \begin{array}{r} 23 \\ -16 \end{array} & \begin{array}{r} 22 \\ -16 \end{array} \\
 \hline
 \begin{array}{r} 19 \\ 19 \end{array} & \boxed{7} & \boxed{7} & \boxed{6}
 \end{array}$$

Hex subtraction using 15's complement.

Same as 7's c just find 15's c.

$$\textcircled{1} \quad (\text{B D C E})_H - (\text{B B A A})_H$$

B	B	A	A
11	11	10	10

$$\begin{array}{r}
 15 & 15 & 15 & 15 \\
 - 11 & 11 & 10 & 10 \\
 \hline
 4 & 4 & 5 & 5
 \end{array}
 \quad + 15\text{'s c}$$

$$\textcircled{1} \quad (\text{D 8 A}) - (\text{4 2 6})$$

$$\begin{array}{r}
 15 & 15 & 15 \\
 - 4 & 2 & 6 \\
 \hline
 11 & 13 & 9
 \end{array}$$

$$\begin{array}{r}
 + 13 & 8 & 10 \\
 \hline
 25 & 22 & 19 \\
 - 16 & - 16 & - 16 \\
 \hline
 \boxed{9} & \boxed{6} & \boxed{3}
 \end{array}$$

$$\begin{array}{r}
 + \quad \quad \quad 1 \\
 \hline
 (9 6 4)_H
 \end{array}$$

16's c

$$\textcircled{2} \quad (426)_{16} - (D8A)_{16} = (964)_{16}$$

$$\begin{array}{r}
 15 & 15 & 15 \\
 13 & 8 & 10 \\
 \hline
 2 & 7 & 5
 \end{array}
 \quad
 \begin{array}{r}
 2 & 7 & 5 \\
 + & & 1 \\
 \hline
 2 & 7 & 6
 \end{array}$$

$$\begin{array}{r}
 + 4 & 2 & 6 \\
 \hline
 6 & 9 & 14
 \end{array}
 \quad
 \begin{array}{r}
 \boxed{6} & 9 & 12 \\
 \hline
 \text{no carry}
 \end{array}$$

$$\begin{array}{r}
 15 & 15 & 15 \\
 - & 6 & 9 & 12 \\
 \hline
 9 & 6 & 3
 \end{array}
 \quad
 \begin{array}{r}
 + 1 \\
 \hline
 10
 \end{array}
 \quad
 \begin{array}{l}
 \rightarrow \text{add 1 to make} \\
 16's \text{ c.}
 \end{array}$$

$$\textcircled{3} \quad (587)_H - (4EB)_H = (9C)_H$$

$$\begin{array}{r}
 15 & 15 & 15 \\
 - 4 & 14 & 14 \\
 \hline
 11 & 1 & 4
 \end{array}
 \quad
 (0912)_H$$

$$\begin{array}{r}
 + \\
 \hline
 1 & 1 & 1 & 5
 \end{array}$$

$$\begin{array}{r}
 + 5 & 8 & 7 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 + 16 & 9 & 12 \\
 - 16 \\
 \hline
 0 & 0 & 0
 \end{array}$$

Codes.

1. Weighted codes
2. Non-weighted codes
3. Error-detecting codes

1. Weighted codes
 - ① Binary code
 - ② BCD [Binary coded decimal]
only valid for 0-9
10-15 → invalid.

BCD

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

ex - BCD 10

0001 0000 + 8 bit

- while in binary only 4 bit i.e 1010
- ^{Disadv} ① So BCD is less efficient than binary
- ^{Adv} ② In BCD we only require to remember equivalent of 0-9 & represent them which is not in case of binary

BCD addition

1. sum ≤ 9 , final carry = 0 \Rightarrow correct ans
2. sum ≤ 9 , final carry = 1 \Rightarrow Incorrect [add 6 to it to correct it]
3. sum > 9 , final carry = 0 \Rightarrow Incorrect [add 6 to it -11-]

① ex- $(2)_{10} + (6)_{10}$

$$\begin{array}{r} 0 \ 0 \ 1 \ 0 \\ + 0 \ 1 \ 1 \ 0 \\ \hline 1 \ 0 \ 0 \ 0 \end{array} \rightarrow \text{correct}$$

$$a_1(2) + a_1(6)$$

$$\begin{array}{r} 1 \ 1 \ 1 \ 0 \ 1 \ 0 \\ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \\ \hline 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \\ 0 \ 1 \ 1 \ 0 \end{array}$$

② $(3)_{10} + (7)_{10}$

$$\begin{array}{r} 0 \ 0 \ 1 \ 1 \\ + 0 \ 1 \ 1 \ 1 \\ \hline 1 \ 0 \ 1 \ 0 \end{array} \rightarrow \text{invalid as } [10]$$

so case ③ so add 6 to it

$$\begin{array}{r} 1 \ 0 \ 1 \ 0 \\ + 0 \ 1 \ 1 \ 0 \\ \hline \boxed{0} \ 0 \ 0 \ 0 \ 0 \\ \hline 1 \ 0 \end{array} = (10)_{10}$$

$$a_1(3) + a_1(7)$$

$$1 \ 1 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 0$$

$$1 \ 1 \ 1 \ 0$$

$$a_1(3) + a_1(7)$$

③ $(8)_{10} + (9)_{10}$

$$\begin{array}{r} 1 \ 0 \ 0 \ 0 \\ + 1 \ 0 \ 0 \ 1 \\ \hline \boxed{1} \ 0 \ 0 \ 0 \ 1 \\ \hline \boxed{1} \ 0 \ 0 \ 0 \ 1 \\ + 0 \ 1 \ 1 \ 0 \\ \hline \boxed{0} \ 0 \ 0 \ 1 \ 0 \ 1 \ 1 \\ \hline 1 \ 7 \end{array} = (17)_{10}$$

$$1 \ 1 \ 1 \ 0$$

$$0 \ 0 \ 0 \ 1$$

$$1 \ 1 \ 1 \ 1$$

$$0 \ 1 \ 1 \ 0$$

$$1 \ 0 \ 1 \ 0$$

$$0 \ 0 \ 0 \ 0$$

$$a_1(8) + a_1(9)$$

$$1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1$$

$$0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0$$

$$P = 1 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \rightarrow P_1$$

$$0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0$$

④ $(57)_{10} + (26)_{10}$

$$\begin{array}{r}
 01010111 \\
 + 00100110 \\
 \hline
 01111101 \rightarrow \text{case ③}
 \end{array}$$

$$\begin{array}{r}
 + 0110 \\
 \hline
 \underbrace{10000001}_{\begin{matrix} 8 \\ 3 \end{matrix}} = (83)_{10}
 \end{array}$$

② $(3)_{10} + (4)_{10}$

$$\begin{array}{r}
 0011 \\
 + 0100 \\
 \hline
 0111 = (7)_{10}
 \end{array}$$

③ $(7)_{10} + (9)_{10}$

$$\begin{array}{r}
 0111 \\
 + 1000 \\
 \hline
 1111 \rightarrow \text{case ③}
 \end{array}$$

$$\begin{array}{r}
 + 0110 \\
 \hline
 \underbrace{000110101}_{\begin{matrix} 16 \end{matrix}} = (16)_{10}
 \end{array}$$

⑤ $(83)_{10} + (34)_{10} = (117)_{10}$

$$\begin{array}{r}
 10000011 \\
 + 00110100 \\
 \hline
 \underbrace{1011}_{+ 9} \quad \underbrace{100}_{+ 8} = 117
 \end{array}$$

$$\begin{array}{r}
 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
 + & 0 & 1 & 1 & 0 \\
 \hline
 0 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & 1 \\
 \hline
 & 1 & 1 & 7
 \end{array}$$

- Non-weighted code
Excess-3 code [XS-3]

Decimal \rightarrow BCD, $\frac{+3}{10}$ × 5-3 code

Decimal	BCD	XS-3 (Self Complementing)
0	0 0 0 0	0 0 1 1 →
1	0 0 0 1	0 1 0 0 →
2	0 0 1 0	0 1 0 1 →
3	0 0 1 1	0 1 1 0 →
4	0 1 0 0	0 1 1 1 →
5	0 1 0 1	1 0 0 0 ←
6	0 1 1 0	1 0 0 1 ←
7	0 1 1 1	1 0 1 0 ←
8	1 0 0 0	1 0 1 1 ←
9	1 0 0 1	1 1 0 0 ←

XS-3 code is sequential codes

as we add 1 to it. it will result next no

ex 0000 + 0

$$\begin{array}{r}
 + \\
 \hline
 0 & 0 & 0 & 1 \\
 + & 1 & 1 & 0 \\
 \hline
 0 & 0 & 1 & 0
 \end{array}
 \rightarrow 2$$

① $(+28)_10$

$$\begin{array}{r}
 0100 \quad 0010 \quad 1000 \\
 + 0011 \quad 0011 \quad 0011 \\
 \hline
 0111 \quad 0101 \quad 1011
 \end{array}
 \begin{array}{l}
 \text{BCD} \\
 \downarrow + (3)_{10} \\
 \rightarrow \text{XS-3 code}
 \end{array}$$

XS-3 addition

case-

- ① When final carry = 0 , subtract 3
- ② When final carry = 1 , add 3

ex- ① $(2)_{10} + (5)_{10} = (7)_{10} \xrightarrow{+ (3)_{10}} (10)_{10}$

$$\begin{array}{r}
 0010 \\
 + 0101 \\
 \hline
 0111
 \end{array}
 \begin{array}{r}
 010100 \\
 - 0011 \\
 \hline
 1010
 \end{array}
 \begin{array}{l}
 \text{carry not generate} \\
 \text{so subtract 3}
 \end{array}$$

② $(27)_{10} + (39)_{10} = (66)_{10}$

$$\begin{array}{r}
 010010 \\
 + 010100 \\
 \hline
 100010
 \end{array}
 \begin{array}{r}
 010110 \\
 - 0011 \\
 \hline
 1000
 \end{array}
 \begin{array}{l}
 \text{+ 0011} \\
 \hline
 1000
 \end{array}$$

ii] Gray code.

Frank Grey

Reflected binary code (RBC)

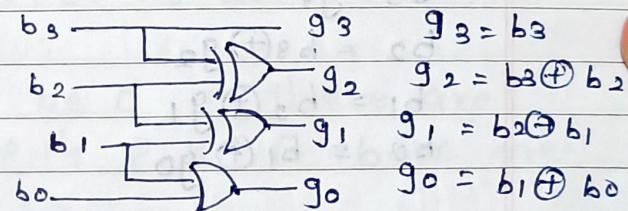
Unit distance code

- 2 successive values differ in only 1 bit
- Binary no is converted to grey code to reduce switching operation.

Conversion from binary to grey.

1. Record MSB as it is (XOR operation)
2. Add MSB with next bit, compute sum & ignore carry
3. Repeat the process.

$$\begin{array}{cccc}
 b_3 & b_2 & b_1 & b_0 \\
 \text{---} & \text{---} & \text{---} & \text{---} \\
 0 & 0 & 1 & 1 \\
 | \oplus & | \oplus & | \oplus & | \oplus \\
 g_3 & 0 & 0 & 1 \\
 & g_2 & g_1 & g_0
 \end{array}$$



Decimal	Binary	Gray	Decimal	Binary	Gray
0	0000	0000	13	1101	1011
1	0001	0001	14	1110	1001
2	0010	0011	15	1111	1000
3	0011	0010			
4	0100	0110			
5	0101	0111			
6	0110	0101			
7	0111	0100			
8	1000	1100			
9	1001	1101			
10	1010	1111			
11	1011	1110			
12	1100	1010	7-8, 6-9, 5-10		

unit distance code

bcoz when we changing from 1 binary to other in gray code only 1 bit is changing (i.e. 1 → 2 → 3 → 4)

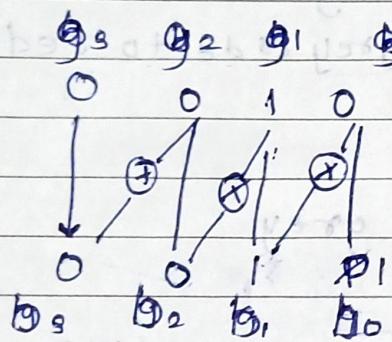
from 8 they are mirror images like 7-8, 6-9, 5-10

Gray to binary.

① Record MSB as it is

② Add MSB bit with next bit

③ Repeat the process.



$$\begin{aligned} g_3 &= b_3 \\ g_2 &= g_3 \oplus b_2 \\ g_1 &= g_2 \oplus b_1 \\ g_0 &= b_0 \end{aligned}$$

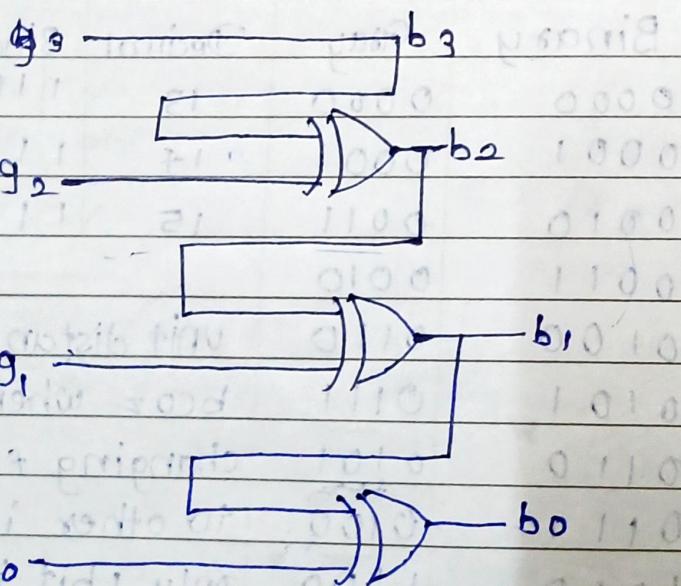
$$\begin{aligned} g_3 &= b_3 \\ g_3 \oplus b_2 &= g_2 \\ g_2 \oplus b_1 &= g_1 \\ g_1 \oplus b_0 &= g_0 \end{aligned}$$

$$b_3 = g_3$$

$$b_2 = b_3 \oplus g_2$$

$$b_1 = b_2 \oplus g_1$$

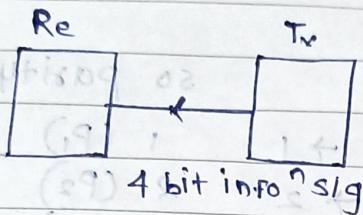
$$b_0 = b_1 \oplus g_0$$



Error detecting code and error correcting code

Parity

- To detect error
- Single error will be detected



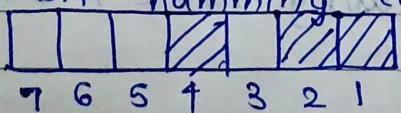
Types

1. Odd parity
 2. Even parity - Total no. of 1's are even.
 - Even parity \Rightarrow Parity will be 0 if there are even no. of 1's & 1 if there are odd no. of 1's
 - Odd parity \Rightarrow If total no. of 1's are odd.
- [Parity '0' if odd no. of 1's and Parity is '1' if even no. of 1's]

ex- $\begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{matrix}$

- Hamming code [R-W hamming code]

- 7 bit hamming code



D₇ D₆ D₅ P₁ D₃ P₂ P₁

task which has position 1

D_1	1 0 0 0 0 1 1	$P_1 \rightarrow D_8 D_5 D_7$
D_2	2 0 1 0	$P_2 \rightarrow D_8 D_6 D_7$
D_3	3 0 0 1 1	$P_3 \rightarrow D_5 D_6 D_7$
D_4	4 1 0 0	
D_5	5 1 0 1 1	
D_6	6 1 1 0 0	
D_7	7 1 0 1 1	

so parity position

$$\text{parity } 2^n \text{ i.e } \begin{matrix} 2^0 & \rightarrow 1 \\ 2^1 & \rightarrow 2 \\ 2^2 & \rightarrow 4 \end{matrix} \begin{matrix} 1 & (P_1) \\ 2 & (P_2) \\ 4 & (P_4) \end{matrix}$$

Q.1) perform binary subtraction.

- ① $(26)_{10} - (15)_{10}$
- ② $(-31)_{10} - (6)_{10}$
- ③ $(144)_{10} - (156)_{10}$

Q.2) Perform Octal addition.

$$1. (777)_8 + (77)_8$$

Q.3) perform octal subtraction using 7's c and 8's c

$$(161)_8 - (243)_8$$

$$\begin{array}{r} 0 \quad 0 \quad 0 \quad 1 \quad 0 \\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \end{array}$$

Q.4) Perform Hex addition

$$1. (658)_{16} + (975)_{16}$$

$$2. (888)_{16} + (999)_{16}$$

Q.5) perform hex subtraction using 15's.c + 16's.c

$$(426)_{16} - (D8A)_{16}$$

$$16's.c : (578)_{16} - (7EB)_{16}$$

8

If 7 bit hamming code received by receiver is 1011011 assuming even parity state whether received code word is correct or incorrect if incorrect, locate the bit having error.

D_7	D_6	D_5	D_4	D_3	P_2	P_1
1	0	1	1	0	1	1

$P_1 \quad D_5 \quad D_6 \quad D_7$

1 0 1 1 \Rightarrow odd 1

$P_2 \quad D_3 \quad D_6 \quad D_7$

1 0 0 1 \Rightarrow even 0

$P_4 \quad D_5 \quad D_6 \quad D_7 \quad \Rightarrow$ odd 1

1 1 0 1