Total 10 modules.

1. Basics.
2. Mesh & Node Analysis.
3. Linearity, Superposition & Source transformation.
4. Network Th^m.

5) Circuit Analysis.
6) Time & frequency response.
7) Two port N/w.
8) fourier series.
9) fourier Transform

Books :-

A Sudhakan & S P Shyammohan (Circuit & Networks)

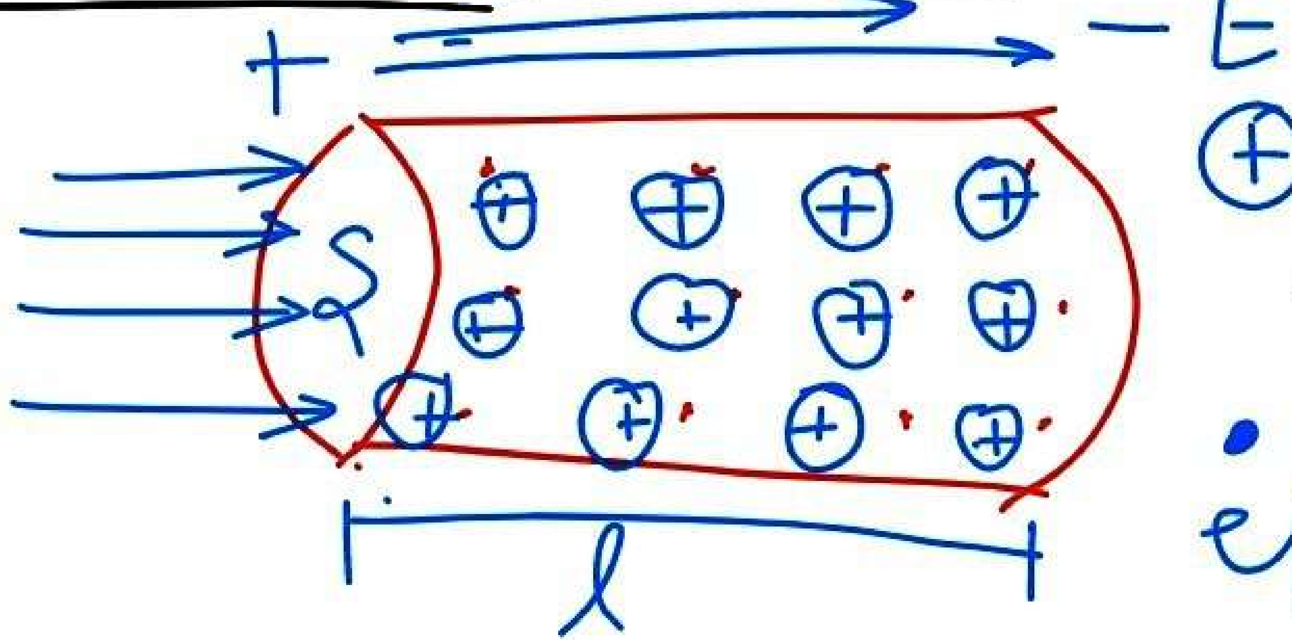
Netework Analysis :- Von Valkenburg

3) Engg ckt Analysis.

William H Hayt.

Jack E Kemmerly.

Mechanism of Electrical Energy flow through the conductor & the Ohm's law



⊕ → immobile, Ag⊕ ion.

• → free electron.

The four best conductors.

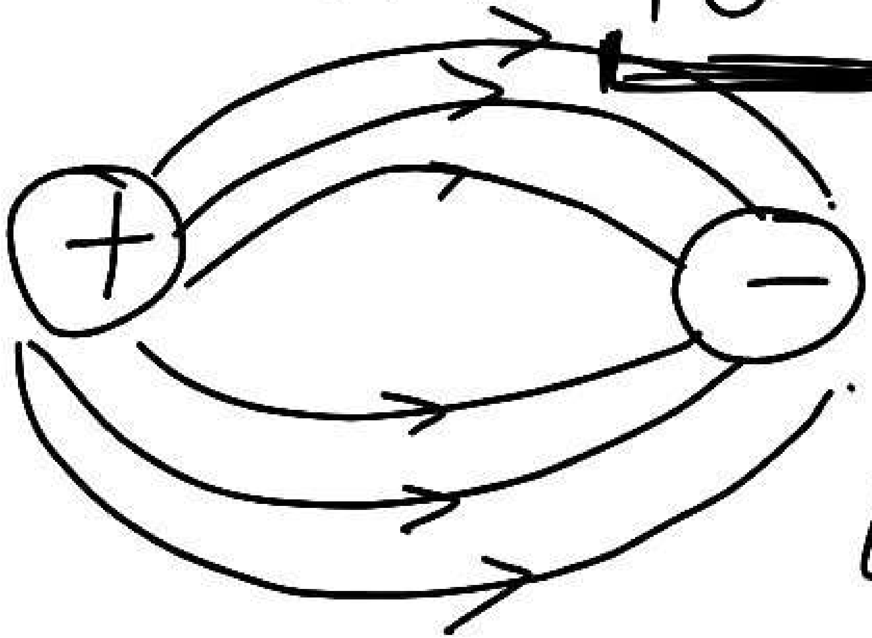
Ag^{+1} ← Silver. ←

Cu^{+2} ← copper.

Au^{+2} ← Gold.

Al^{+3} ← Aluminium.

Generally in any conductor there are 10^{18} to 10^{23} Atoms/ unit volu



Electrical dipole.

When your conductor is subjected to an axial electric field the force exerted by the field on each free electron $\Rightarrow \underline{\underline{F = \vec{E}\boxed{e}\,N.}}$

$e = -1.6 \times 10^{-19}$ coulomb

Due to the opposite direction of F & E there exists a net electron motion. (charge motion)

The amount of charge flowing is given by $\boxed{q = ne}$ Coulomb

ie $q(t) = n(t).e$ coulomb.

$n \Rightarrow$ No. of free electrons crossing cross sectional area

Current :-

$$\boxed{i = \frac{dq}{dt}}$$ Ampere.

The time rate of flow of these electric charges.

Since $\underline{\underline{q}}$ is $-ve$.

The current direction is opposite to that of charge motion. (i.e. free electron motion)

Current density (J)

The current per unit cross section area.

$$\boxed{\underline{\underline{J}} = \frac{i}{s}} \text{ Amp/m}^2$$

Note:- $\underline{\sigma} \Rightarrow$ Scalar quantity.

$\therefore$ The direction of '$\vec{J}$' is in direction of '$\vec{i}$' i.e. in the direction of '$\vec{E}$'

$$F = \vec{E} \cdot e \quad N.$$

$$q = n \cdot e \quad \text{Coulomb.}$$

$$i = \frac{dq}{dt} \quad \text{Amp.}$$

$$J = i/s \quad \text{Amp}/m^2$$

Observation

(1) As $\underline{E}\uparrow \Rightarrow \underline{F}\uparrow \Rightarrow \underline{n}\uparrow \Rightarrow \underline{q}\uparrow \Rightarrow \underline{i}\uparrow \Rightarrow J\uparrow$

linearly

(2) As $E = 0 \Rightarrow F = 0 \Rightarrow n = 0 \Rightarrow q = 0$
$\Rightarrow i = 0 \Rightarrow J = 0$

(All free electrons are at rest)

Ohms law :-

$$J \propto E$$

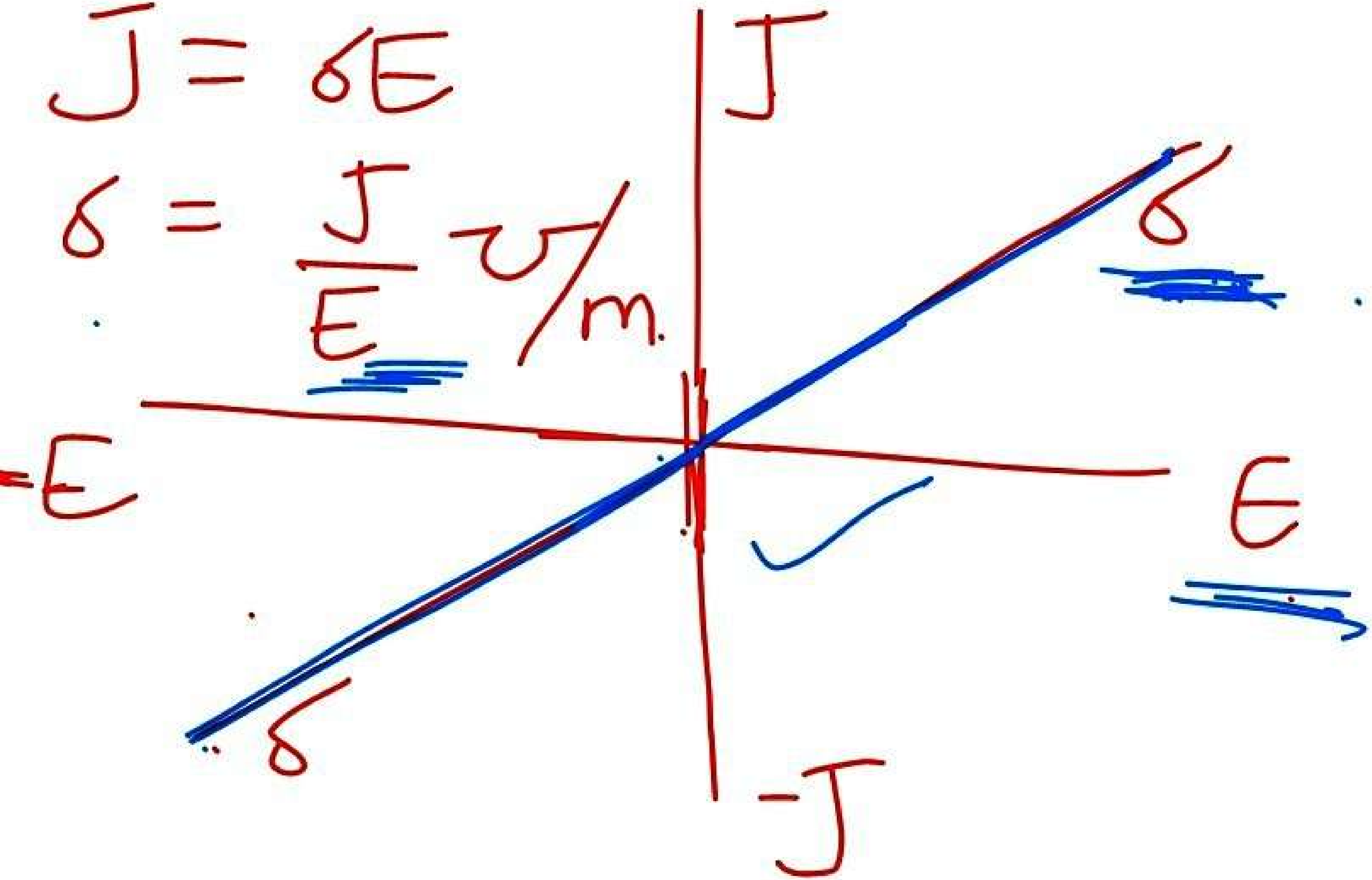
According to Ohms law.
there exist a relation beth.
E & J

$\boxed{J = \sigma E}$ → Basic ohm's law in field theory

σ ⇒ Conductivity of the material

$$J = \sigma E$$

$$\sigma = \frac{J}{E} \ \mho/m.$$

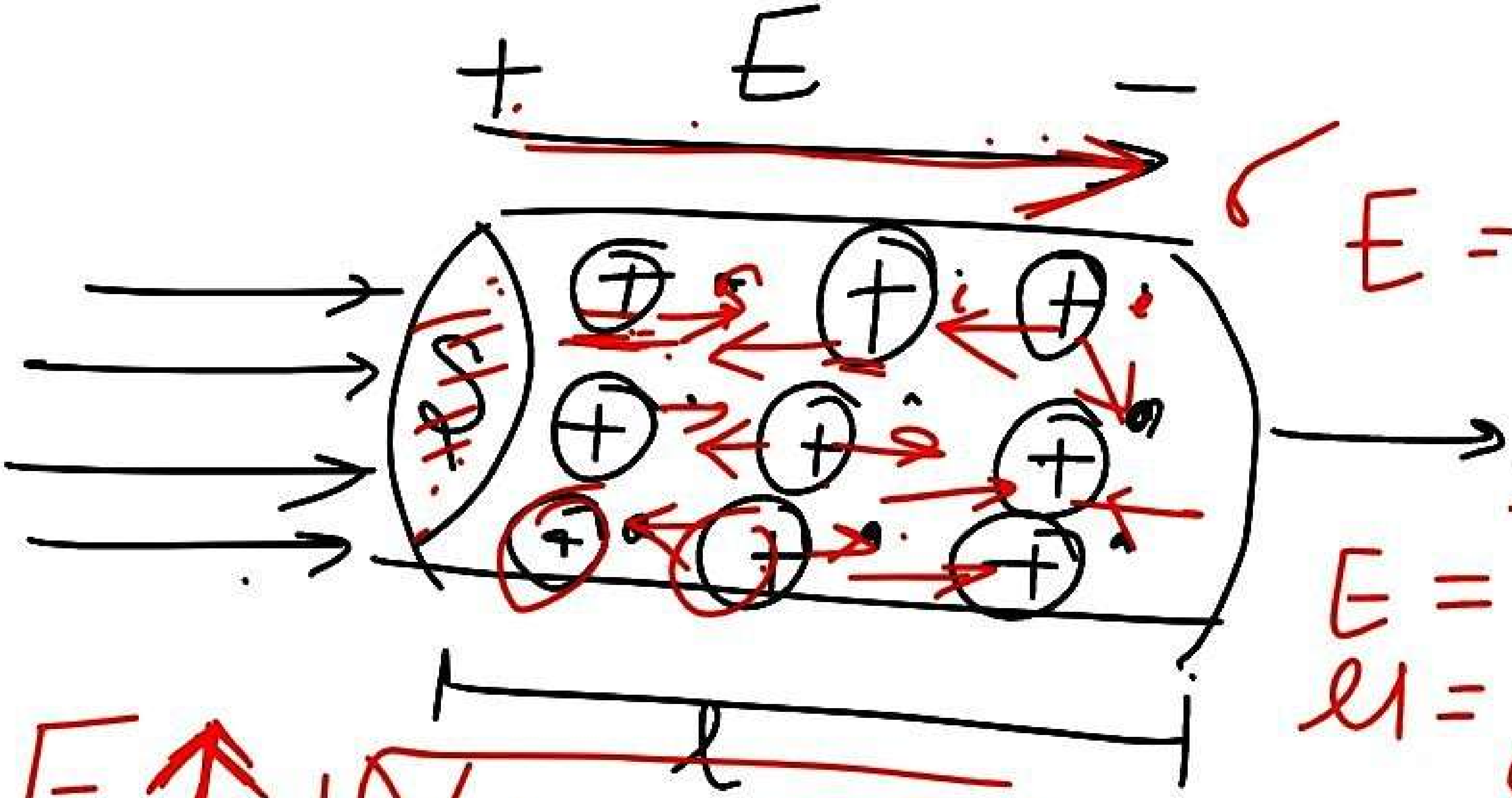


Limitation of Ohm's Law :-

· Ohm's Law, i.e. linear J-E relation is valid only when the proportionality constant σ is kept constant.

i.e. (Temp. is kept constant)

Why voltage drop occurs in any Practical conductor whenever it carries electrical energy?



$E = V/l$

$J = i/S$

$E = V/m$

$\mu = m^2/Vs$

$V_d = m/s$

$E \uparrow \; \downarrow \; \boxed{V_d = \mu E}$

$$K.E = \frac{1}{2} m V_d^2$$

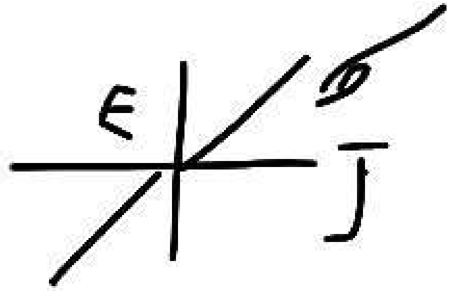
$$m = 9.1 \times 10^{-31} \text{ kg}$$

Here the loss is in the form heat.

$P \Rightarrow$ Power density :-

It is power dissipation per unit volume (unit cube) within the conductor due to the resistance effect of the conductor.

$P = \underline{\underline{J}}E$ $\qquad$ $J = \underline{\underline{\sigma}}\,\underline{\underline{E}}$



$\mathrm{Amp/m^2} \cdot \mathrm{volt/meter}$

$= \sigma E \cdot E \quad \dfrac{\mathrm{Amp \cdot volt}}{\mathrm{m^3}}$

$\boxed{P = \sigma E^2}$ $\mathrm{Watt/m^3}$.

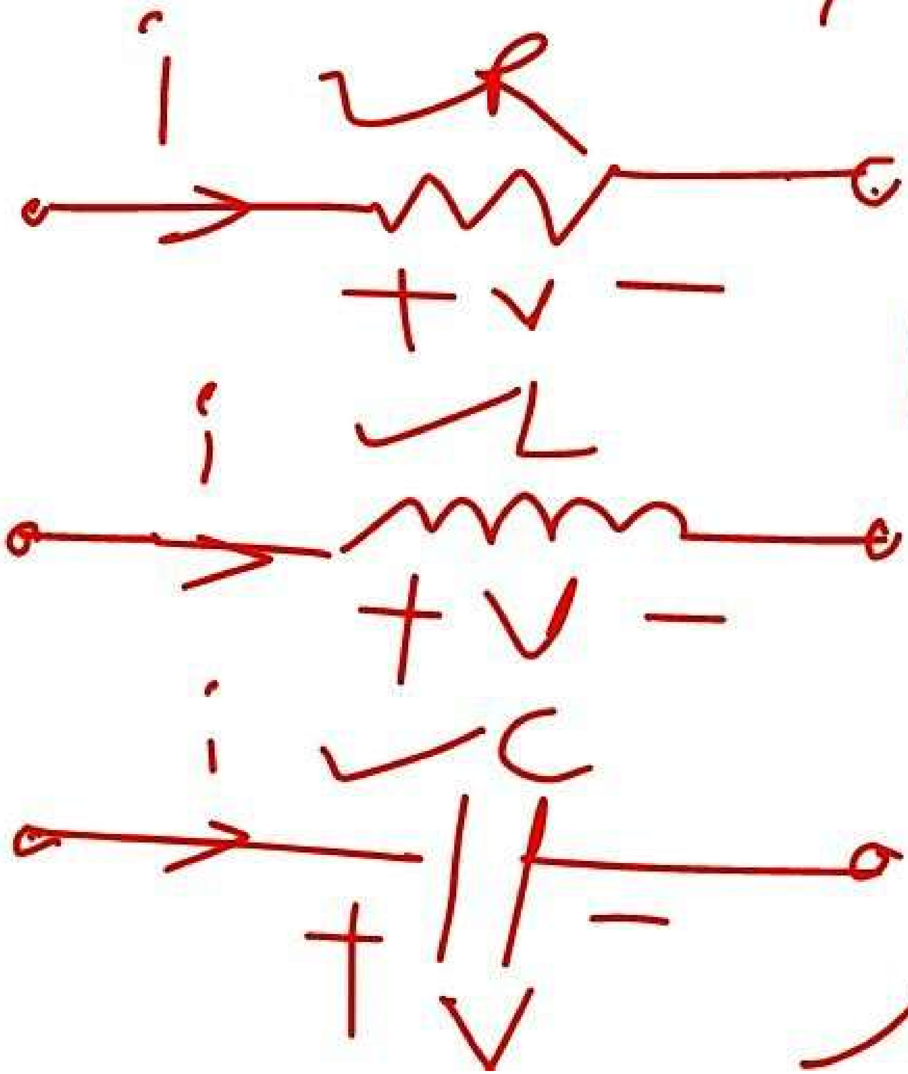
Observation

As $E = 0 \Rightarrow u = 0$ ($\because$ All the electrons are at rest)

$\Rightarrow V_d = 0 \Rightarrow KE = 0 \Rightarrow P = 0$

OR. $P = JE$

$E = 0 \Rightarrow J = 0 \Rightarrow \underline{P = 0}$



$$E = \frac{V}{l}$$

$$J = i/s$$

Conclusion :-

When a conductor is carrying electrical energy, the current through all lumped (R, L, C) will always flow from +ve to −ve terminal.

The opposition by + for the flow of Electrical energy through the conductor is continuous or distributive.

+ → R, L, C } upto 1MHz

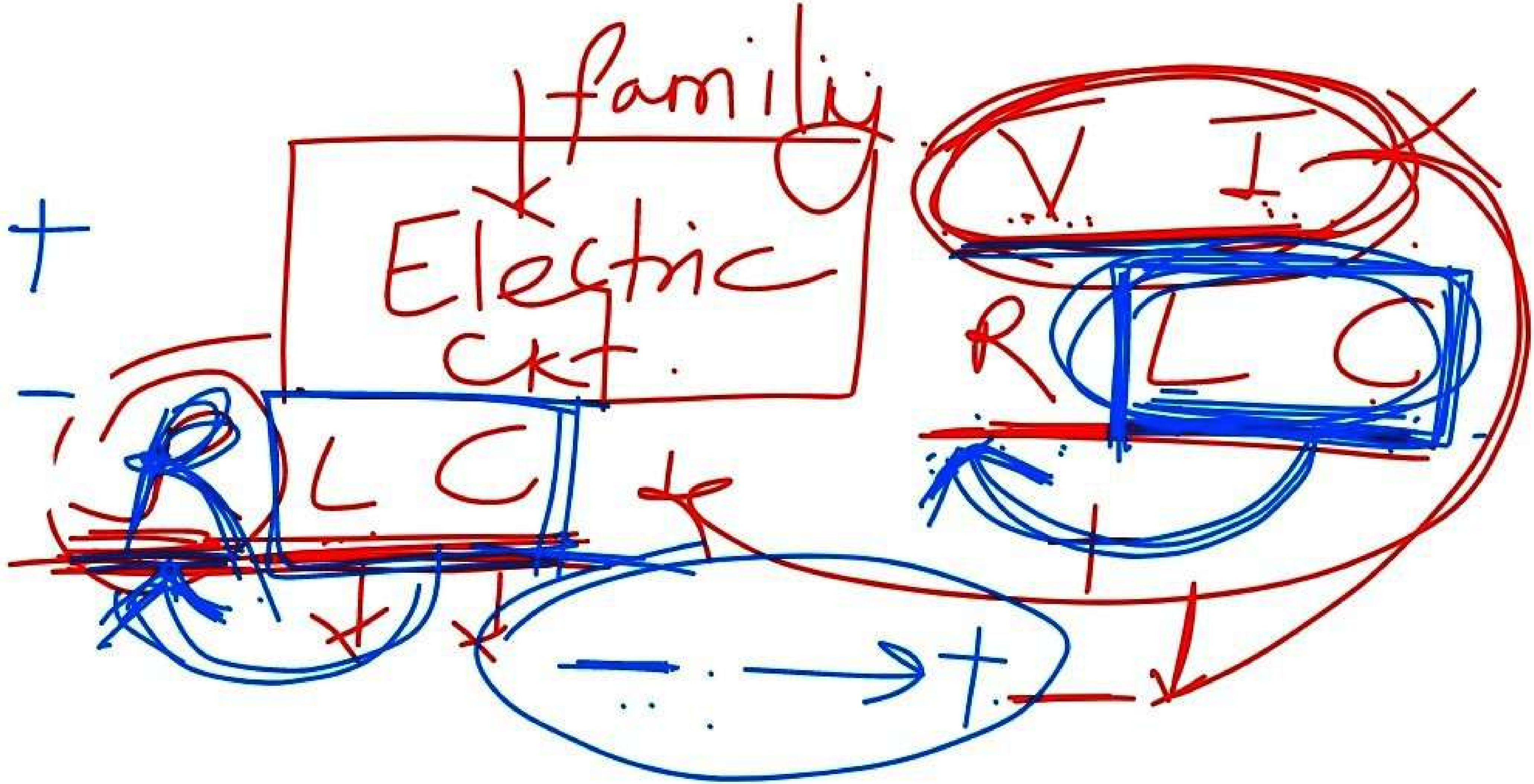
Electrical circuit → Interconnection of R L C.

1 MHz.

>1MHz

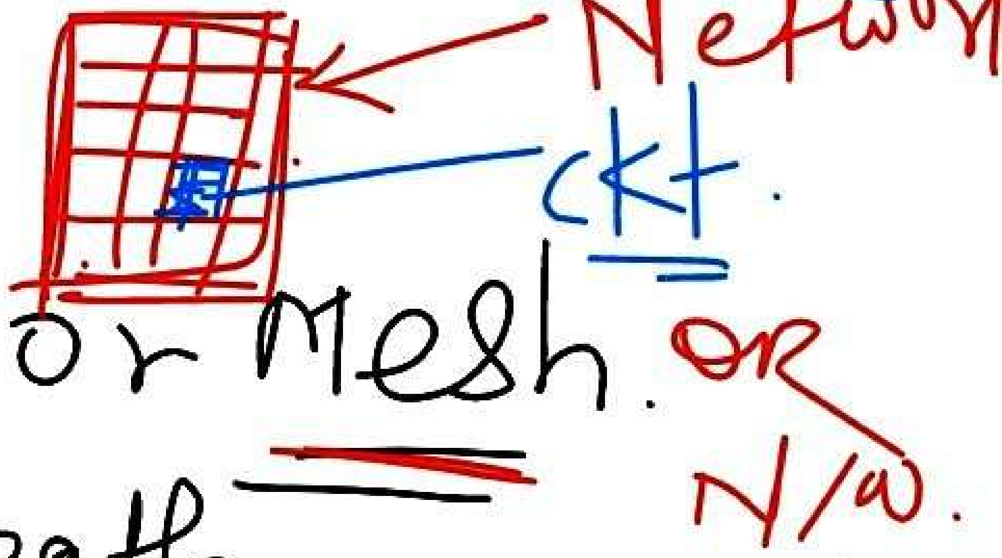
There are two scenarios we will encounter in Network Theory. R L C

1. In Presence of your Active Sources.
2. In absense of Active Sources.



family
Electric
ckt
V
I
R
L
C
R
L
C
+
−

Basic Diff' betn Network & circuit :—



Circuit :— Loop or Mesh.

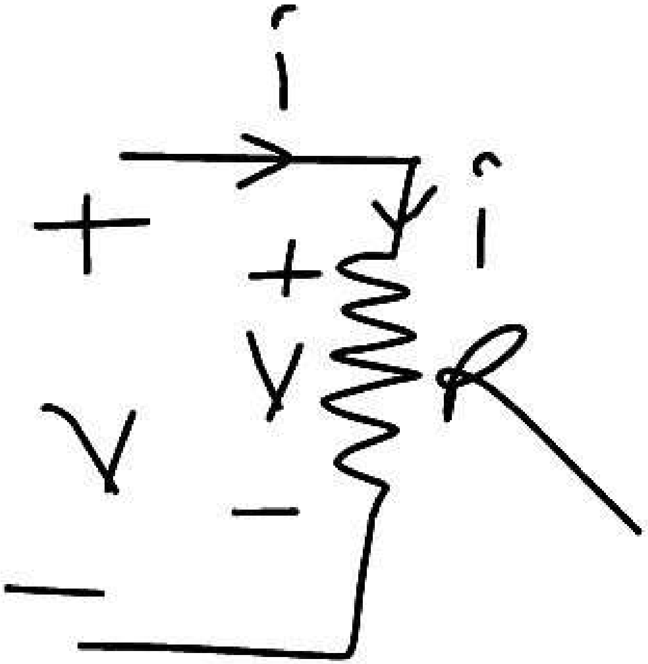
loop → closed path.

mesh → Interconnection of ckt

Resistance:

$$\boxed{J = \sigma E}$$

1st form of ohm's law.



$J = \sigma E$ ← Electrical field theory

$$\frac{i}{s} = \sigma \frac{v}{l} \Rightarrow v = \boxed{\frac{l}{\sigma s}}\, i$$

$$\boxed{v = Ri}$$ → 2nd form of Ohm's law.

Circuit theory form.

$$R = \frac{l}{\sigma S} = \text{ohm} \Rightarrow \Omega \uparrow$$

$$v = R\,i$$

$\sigma \downarrow$

$R \uparrow$

As temp $\uparrow$

$l \uparrow \quad S \uparrow$

l/S = almost constant

$R_t = R_0(1 + \alpha t)$

$\alpha \rightarrow$ Temp coefficient

$t \rightarrow$ temp in $^\circ C$

$t \uparrow \quad R_t \uparrow$

$$R = \frac{l}{\sigma S}$$

$$\sigma = \frac{l}{RS} = \frac{\text{meter}}{\text{ohm} * m^2}$$

$$\sigma = \mho / m.$$

$$V = Ri$$

$$i = \frac{V}{R}$$

$$\boxed{i = GV} \longrightarrow$$ 3rd form of Ohm's law.

$$G \rightarrow \mho \Rightarrow \text{mho.}$$

$$i = \frac{dq}{dt}$$

$$\boxed{V = R\frac{dq}{dt}} \Rightarrow \text{4th form of Ohm's Law.}$$

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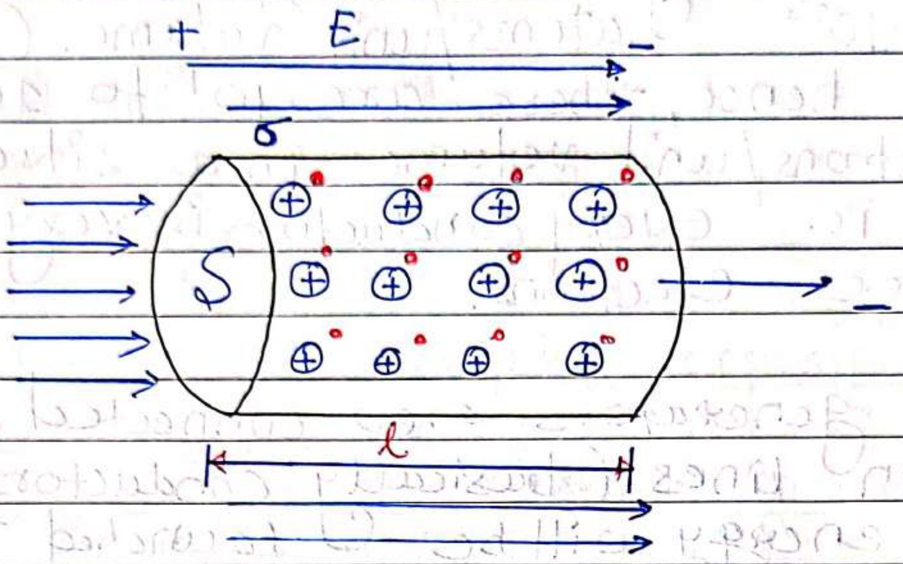
Basic Concepts:→

Mechanism of Electrical Energy flow through the conductor and the Ohm's Law:-



⊕ ⇒ $Ag^{\oplus}$ ion, immobile, larger in size

• ⇒ free electron.

The four best conductors are:

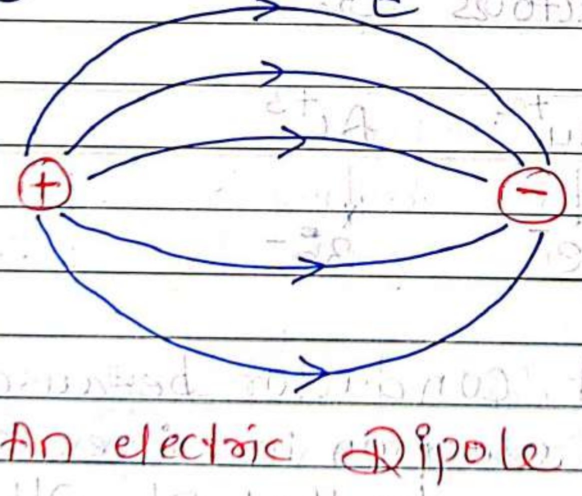
Ag^{+1}	Cu^{+2}	Au^{+2}	Al^{+3}
↓	↓	↓	↓
$1e^-$	$2e^-$	$2e^-$	$3e^-$

→ silver is the best conductor because mobility of free electron in silver conductor is several times to that of other conductors.

so, its conductivity is very high. i.e. the conductivity of any material is mainly a function of free electrons mobility within the material.

→ Generally in any conductor, there are 10^{18} to 10^{23} atoms/unit volume. (i.e. unit cell) and hence, there are 10^{18} to 10^{23} free electrons/unit volume in a silver conductor i.e. every conductor is very rich in free electrons.

→ Whenever generators are connected to the transmission lines (basically conductors), electrical energy will be launched axially, which results in axial electric field (a perpendicular magnetic field) and hence, axial current density along the length of the conductor



An electric Dipole

→ When a conductor is subjected to an axial electric field, the force exerted by the field on each free electron is given by

$$\boxed{\vec{F} = \vec{E} \cdot e} \text{ Newton.}$$

$e = -1.6 \times 10^{-19}$ Coulomb

→ Since 'e' is negative, the direction of force is in the direction opposite to that of 'E' and hence, there is exists a net electron motion (ie. charge motion) in the direction opposite to that of 'E'.

→ The amount of charge flowing is given by

$$\boxed{q = ne} \text{ Coulomb.}$$

ie. $q(t) = n(t) \cdot e$ Coulomb.

n = The number of free electrons crossing a reference cross section area 'S', a variable quantity due to large number of free electrons within the conductor (so called random variable)

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→ The time rate of flow of these electric charges (i.e. free electrons) is nothing but the electric current.

$$\boxed{i = \frac{dq}{dt} \text{ Ampere}}$$

→ Since 'q' is negative, the current direction is opposite to that of charge motion (i.e free electrons motion) i.e. in the direction of 'E'.

→ The current per unit cross section area is nothing but the current density resulted within the conductor. i.e.

$$\boxed{J = \frac{i}{S} \text{ Ampere/meter}^2}$$

→ Since 'S' is scalar, the direction of 'J' is in the direction of 'i' i.e. in the direction of 'E'.

* $\vec{F} = \vec{E} \cdot e$ Newton.
* $q = ne$ coulomb.
* $i = dq/dt$ Ampere
* $J = i/S$ Ampere/meter2

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Observation :-

$\Rightarrow$ As $E\uparrow \Rightarrow F\uparrow \Rightarrow n\uparrow \Rightarrow q\uparrow \Rightarrow i\uparrow \Rightarrow J\uparrow$ linearly.

$\rightarrow$ As $E=0 \Rightarrow F=0 \Rightarrow n=0 \Rightarrow q=0 \Rightarrow i=0 \Rightarrow J=0$

ie. All the free electrons are at rest.

Definition of Ohm's Law :-

According to Ohm, there exists a linear relation between the applied electric field (E) and the resulted current density (J) by

$$J \propto E$$

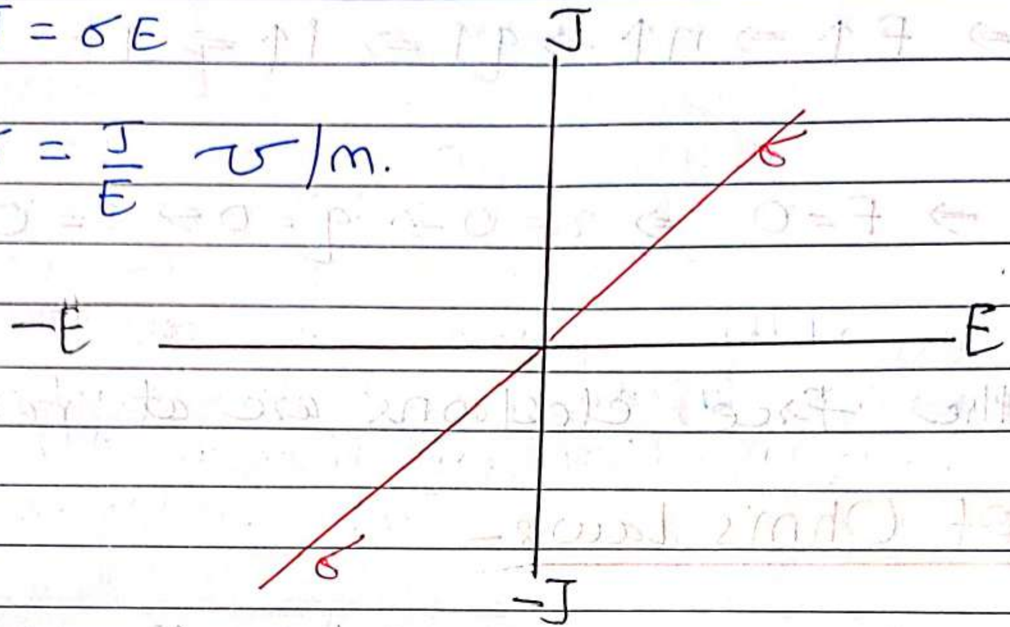
$$\boxed{\vec{J} = \sigma \vec{E}}$$ $\leftarrow$ Basic Ohm's law in field theory form.

σ = Conductivity of the material ie conductor.

J-E Characteristics :-

$J = \sigma E$

$\sigma = \frac{J}{E}$ ℧/m.



Limitations :-

Ohm's law i.e. a linear J-E relation is valid only when the proportionality constant σ is kept constant. i.e. temprature is kept constant.

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* What happens to the conductivity σ of all the conductors, if the temprature increases beyond room temprature?

→ When the temprature increases from room temprature (ie. 27°C or 300°K), free electrons will acquire extra thermal energy at higher temperatures, which results increase in collisions among the free electrons and hence, net free electron mobility (ie carrier mobility) falls, so conductivity decreases.

* Why voltage drop occurs in any practical conductor whenever it carries electrical energy?

→ Voltage Drop :- Decrease in electrical potential along the path of current flowing.

As 'E' increases, there exists an increase in collisions between the free electrons and immobile positive ions (larger in size), which results fall in drift velocity ($v_d = \mu E$) m/s.

μ ⇒ Average mobility of free electrons in m^2/Vs. (meter²/ volt second)

E ⇒ External electric field in volt/meter.

∴ Loss in kinetic energy

$$KE = \frac{1}{2} m v_d^2 \text{ Joule}$$

$(m = 9.11 \times 10^{-31} \text{ kg})$,

This loss in energy will be dissipated in the form of heat energy, which results in voltage drop across the conductor.

Power density (P)

Amount of power dissipated within the conductor per unit volume (unit cube) within the conductor due to ~~this effect~~ resistance effect of the conductor.

Due to this effect, the conductor gets heated up.

$P = J \cdot E$ Ampere/meter^2 . volt/meter

$P = \sigma E \cdot E$ $\frac{\text{Amper} \cdot \text{volt}}{\text{meter}^3}$

$P = \sigma E^2$ watt/meter^3.

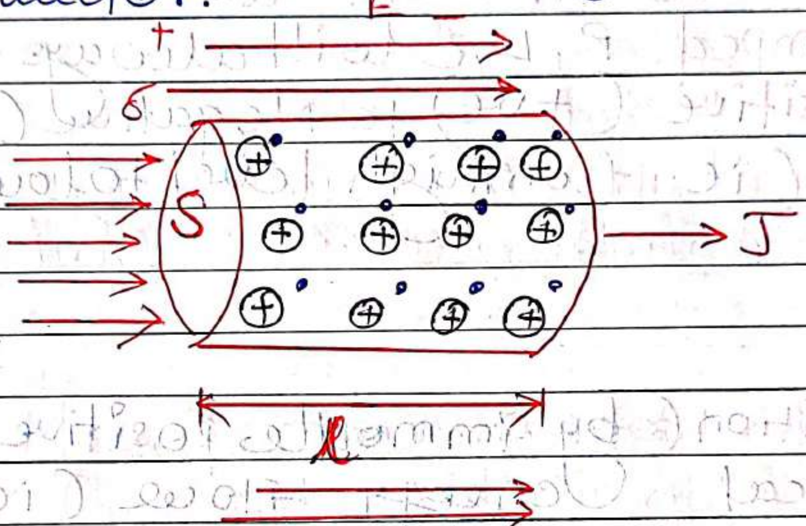
Observation :-

→ As $E=0 \Rightarrow u=0$ (since all the electrons are at rest)

$\Rightarrow v_d = 0 \Rightarrow KE = 0 \Rightarrow P = 0$ Watt/m^3.

→ As $E=0 \Rightarrow J=0 \Rightarrow P = J \cdot E = 0$ Watt/m^3.

So when no energy flow through the conductor ⇒ no voltage drop across the conductor.



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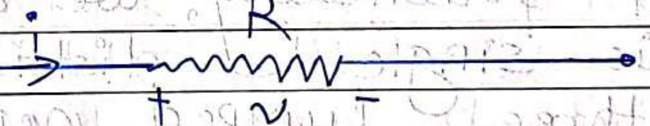
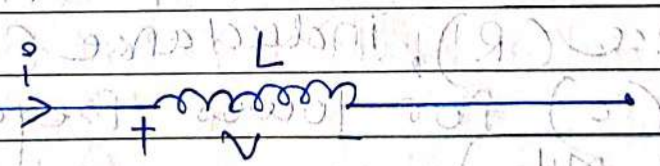
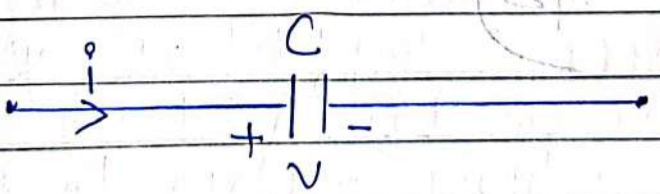






→ Since $E = \frac{v}{l}$ & $J = \frac{i}{S}$ as 'l' & 'S' are scalars, the drop polarities are same as electric field 'E' polarities and the current direction is same as current density 'J' direction.

Conclusion :-

Whenever a conductor is carrying electrical energy, currents through all the three passive lumped R, L, C will always flow from positive (+ve) to Negative (−ve) terminals (ie. from high level to low level)

See

The opposition (by immobile positive ions) for electrical energy flow (ie free electron motion) through the conductor is continuous or distributive in nature, but practically, we can approximate this single distributive opposition into three lumped oppositions called resistance (R), inductance (L) & capacitance (C) for lower frequencies (ie upto 1 MHz) magnitude of these lumped R, L, C are function of length (l), cross section area (S) & conductivity (σ) of the conductor.

→ Interconnection of these lumped (R, L, C) elements is nothing but a lumped electric circuit. Network theory is used to analyse only these lumped electric circuits at lower frequencies (ie. upto 1MHz)

→ At higher frequencies (ie above 1MHz) we cannot approximate distributive elements into lumped elements & hence, no lumped electric circuit, so no Network theory. Here Field theory approach is used directly on distributive conductors.

→ field theory approach of solving distributive electric networks is valid for all the frequencies starting from zero (ie. 0 Hz ⇒ D.C)

There are two scenarios we will encounter in Network theory.

Scenario 1 :- In the presence of Active Sources :-

In the presence of sources, all the three passive lumped R, L, C will always absorbs energy. Whenever they are absorbing energy, current through them will always flow from positive (+ve) to Negative (-ve) terminals (ie from high level to low level)

Scenario 2 :- In the absence of Active sources :

In the absence of sources, stored energy in the memory elements (L & C) will be delivered on memoryless ~~rest~~ resistance Whenever L & C elements are delivering energy, currents through them will always flow from low level (-ve) to high level (+ve) terminals.
As the resistor always absorbs energy. current through it will always flow from Positive (+ve) to Negative (-ve) terminals.

* What is the difference between Network & Circuit ?

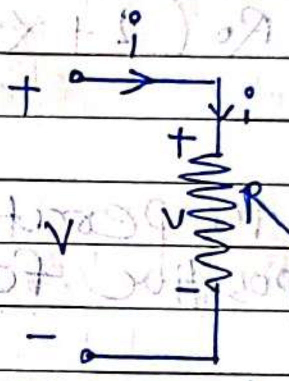
→ Circuit → Loop or Mesh
i.e. A closed path.

Network ⇒ Interconnection of circuits.

Eg. Network is like a building, where as the Circuits are like rooms in a building.

Passive Lumped RLC Elements & Ohm's Law :-

1 Resistance (R)



Since $J = \sigma E$

$$\Rightarrow \frac{i}{S} = \sigma \frac{v}{l}$$

$$v = \frac{l}{\sigma S} i$$

(2nd form of Ohm's law)

$$\boxed{v = Ri}$$ ← Ohm's law in circuit theory form.

Where $R = \frac{l}{\sigma S}$ = Ohm (Ω)

R → Resistance parameter of the conductor.

Limitations:-

→ Ohm's law i.e. a linear voltage and current relation is valid only when the proportionality constant 'R' is kept constant. i.e. the temperature is kept constant.

→ As temperature increases ⇒ $l\uparrow$, $S\uparrow$

$\frac{l}{S}$ = Almost constant, $\sigma\downarrow$ so $R\uparrow$.

→ $R_t = R_0(1+\alpha t)$

Where
α = temperature coefficient in /°C, positive for all the conductors

t = temperature in °C.

So $t\uparrow \Rightarrow R_t\uparrow$ for all the conductors

→ $R = \frac{l}{\sigma S} \Rightarrow \sigma = \frac{l}{RS} = \frac{\text{meter}}{\text{ohm} - \text{meter}^2}$

$= (\Omega\text{-m})^{-1} = \Omega^{-1}m^{-1} = \mho/m$ or S/m.

→ Resistivity or specific resistance = ρ

$$= \frac{1}{\sigma} = \frac{RS}{l} = \frac{\Omega m^2}{m} = (\Omega\,\text{meter})$$

since $V = Ri$

$$i = \frac{V}{R}$$

$$\boxed{i = GV}$$ ← 3^{rd} form of Ohm's law.

where $G = \frac{1}{R} = \text{mho}\ (\mho)$

$G \Rightarrow$ conductance parameter of the conductor

Since $i = \frac{dq}{dt}$

$$\boxed{V = R\frac{dq}{dt}}$$ ← 4th form of ohm's law.

Electric Power (P)

$$P = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt}$$

$$\boxed{P = VI \text{ watt}}$$ ← Instantaneous power.

i.e. $P(t) = V(t) \cdot i(t)$ watt.

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Electrical Energy (Watt)

Since $\frac{dW}{dt} = P \Rightarrow dW = P\,dt$

$$\boxed{W = \int P\,dt} \quad \text{Joule}$$

For R :-

$P = V \cdot i = R \cdot i \cdot i$

$$\boxed{P = i^2 R} \quad \text{Watt}$$

i.e. $P(t) = i^2(t) \cdot R$ Watt

$P = VI = V \cdot \frac{V}{R}$

$$\boxed{P = \frac{V^2}{R}}$$

i.e. $P(t) = \frac{V^2(t)}{R}$ Watt

$W = \int P\,dt$ Joule

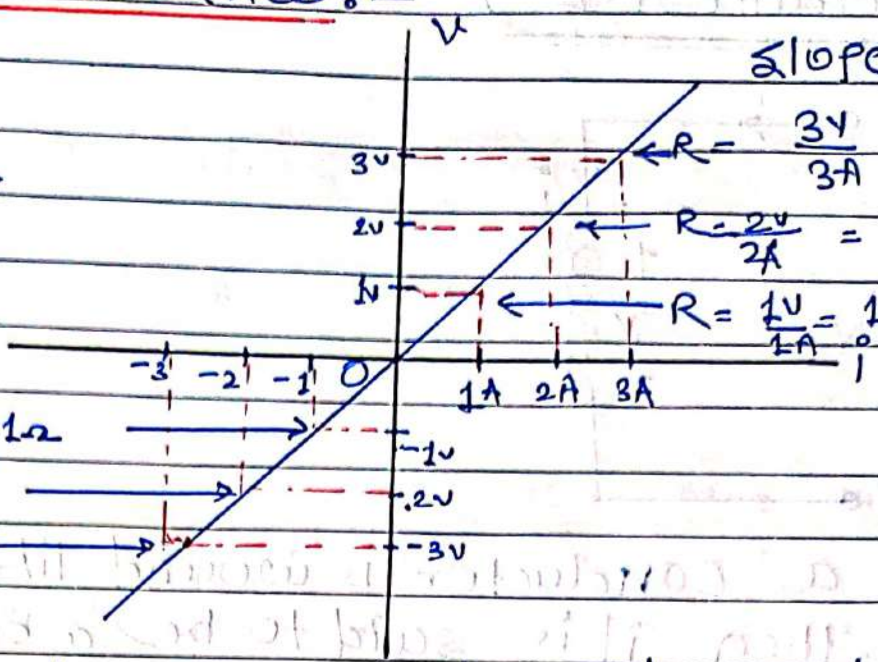
$W = \int i^2 R\,dt$ Joule &

$$\boxed{W = \int \frac{V^2}{R}\,dt \quad \text{Joule}}$$

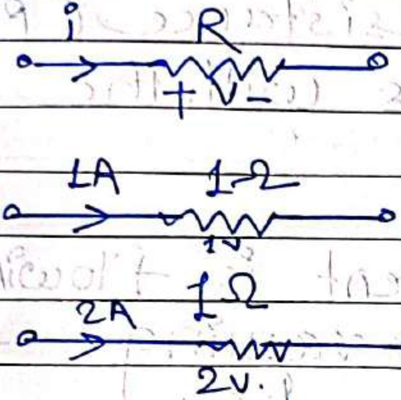
V-i Characteristics :-

$V = Ri$

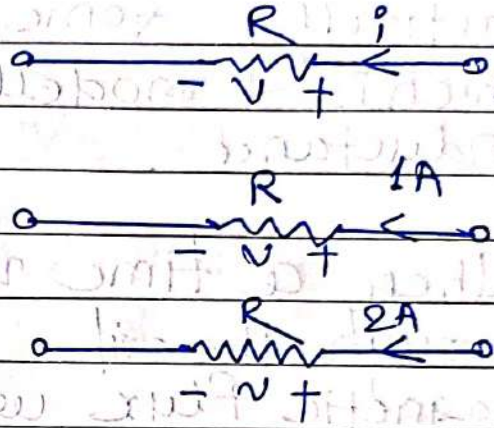
$R = \frac{V}{i}\ \Omega$



1st quadrant



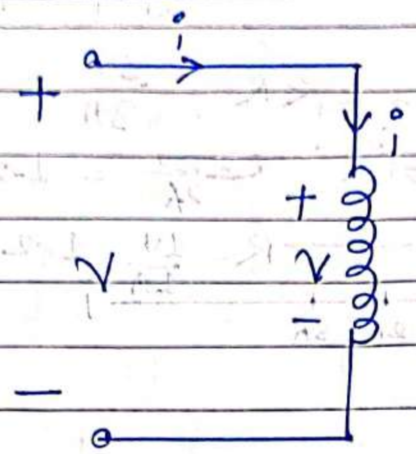
3rd quadrant



Observation :-

So, from the characteristics, resistor is a linear, passive, bilateral and time invariant in V-i plane (these are possible only at a constant temperature, where length (l) cross section area (S) and conductivity (σ) are kept constant. ie 'R' is kept constant.

② Inductance (L)



→ When a conductor is wound like a spring, then it is said to be a coil. The coil will exhibit ideally inductance (L) & practically some leakage resistance (R) which is modelled in series with the inductance.

→ When a time varying current is flowing through the coil, a time varying magnetic flux will be produced (by Faraday's law). The total flux produced is

$\psi = N\phi$ Weber

where N = The number of turns

ϕ = flux per turn.

ie. $\psi(t) = N \cdot \phi(t)$ Weber.

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→ This total flux produced is proportional to the current flowing through the coil

i.e. $\phi \propto i \Rightarrow \phi = Li$

Where L = inductance parameter of the coil.

→ The voltage drop across the coil is

$$v = \frac{d\phi}{dt} \quad (\text{By faraday's law})$$

$$v = \frac{d}{dt}(Li)$$

$$v = L\frac{di}{dt} \text{ volt.}$$

$$di = \frac{1}{L} v\,dt$$

$$\boxed{i = \frac{1}{L}\int_{-\infty}^{t} \text{ Ampere}}$$

$$p = v.i = L\frac{di}{dt}.i$$

$$P = Li\frac{di}{dt} \text{ watt (Instantaneous power)}$$

$$W = \int P\,dt \text{ Joule}$$

$$W = \int Li\left(\frac{di}{dt}\right).dt \quad \text{Joule}$$

$$P = Li\frac{di}{dt} = \frac{d}{dt}\left(\frac{1}{2}Li^2\right)$$

Since $W = \int P\,dt$ Joule

$$W = \int \frac{d}{dt}\left(\frac{1}{2}Li^2\right)dt$$

$$W = \frac{1}{2}Li^2 \text{ Joule}$$

= Instantaneous energy
= Total energy

i.e. the energy stored in an inductor at any instant will depend on the current flowing through the inductor at that instant.

Since $W = \int P\,dt$ Joule

$$W = \int \frac{d}{dt}\left(\frac{1}{2}Li^2\right)dt$$

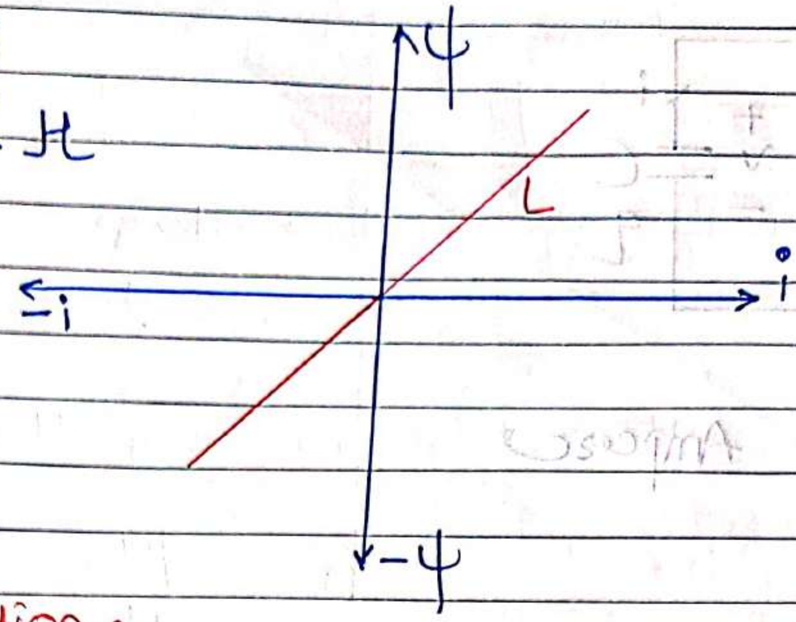
$$W = \frac{1}{2}Li^2 \text{ Joule}$$

This is the total energy stored by the inductor from past $(-\infty)$ to the present time t.

ψ - i Characteristics :-

$\psi = Li$

$L = \frac{\psi}{i}$ H



Observation :-

So from the characteristics, inductor is a linear, passive, bilateral and time-invariant in ψ - i plane (these are possible only at a constant temperature), where length (l), cross section area (s) and conductivity (σ) are kept constant. i.e. 'L' is kept constant.

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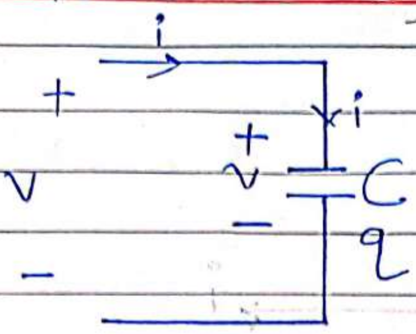
(3) Capacitance (C)



$i = \frac{dq}{dt}$ Ampere

$q \propto v$

$q = Cv$

where

C = Capacitance parameter of the conductor.

$i = \frac{d}{dt}(Cv)$

$i = C\frac{dv}{dt}$ Ampere

$dv = \frac{1}{C} i\,dt \Rightarrow \boxed{v = \frac{1}{C}\int_{-\infty}^{t} i\,dt \text{ volt}}$

$$P = v \cdot i$$
$$= v \cdot C \frac{dv}{dt}$$

$$P = Cv \frac{dv}{dt} \text{ Watt.} =$$ Instantaneous power.

$$W = \int P\, dt \text{ Joule}$$

$$W = \int Cv \left(\frac{dv}{dt}\right) dt \quad \text{Joule}$$

$$\boxed{W = \int Cv \left(\frac{dv}{dt}\right) dt \quad \text{Joule}}$$

$$P = Cv \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{2} Cv^2\right)$$

Since $W = \int p.dt$ Joule

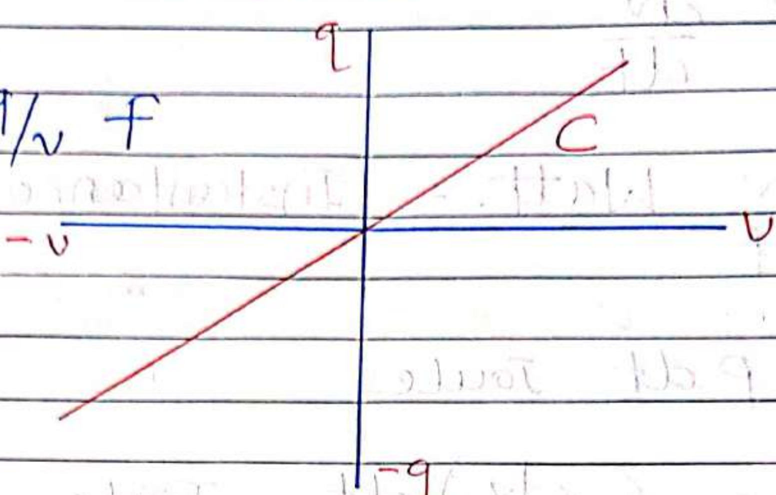
$$W = \frac{1}{2} Cv^2 \text{ Joule}$$

$W \Rightarrow$ Instantaneous energy = Total energy

q - v Characteristic

$q = cv$

$c = q/v \; F$



Observation :-

So from the characteristic, capacitor is a linear, passive, bilateral and time invariant in q-v plane (these are possible only at constant temperature where length (l), cross section area (s), and conductivity (σ) are kept constant. i.e. 'C' is kept constant.

Relation between voltage & current in L & C elements :-

(1) L :-

$$v = L\frac{di}{dt}$$

$$v_1 = L\frac{di_1}{dt} \quad \text{—— (1)} \qquad v_1 \leftarrow i_1$$

$$v_2 = L\frac{di_2}{dt} \quad \text{—— (2)} \qquad v_2 \leftarrow i_2$$

$$v = L\frac{di_1}{dt} + L\frac{di_2}{dt}$$

$$v = L\frac{d}{dt}(i_1 + i_2) \qquad v_1 + v_2 \leftarrow i_1 + i_2$$

$$v = v_1 + v_2$$

relation between voltage and current in an inductor is linear & hence

$$\boxed{v = L\frac{di}{dt}} \rightarrow 5^{th} \text{ form of Ohm's law.}$$

Date
Page

② C :→

$$i = C\frac{dv}{dt}$$

$$i_1 = \frac{C\,dv_1}{dt} \qquad \leftarrow i_1 \leftarrow v_1$$

$$i_2 = C\frac{dv_2}{dt} \qquad i_2 \leftarrow v_2$$

$$i = C\frac{d}{dt}(v_1 + v_2) \qquad i_1 + i_2 \leftarrow v_1 + v_2$$

The relation between voltage & current in a capacitor is linear & hence

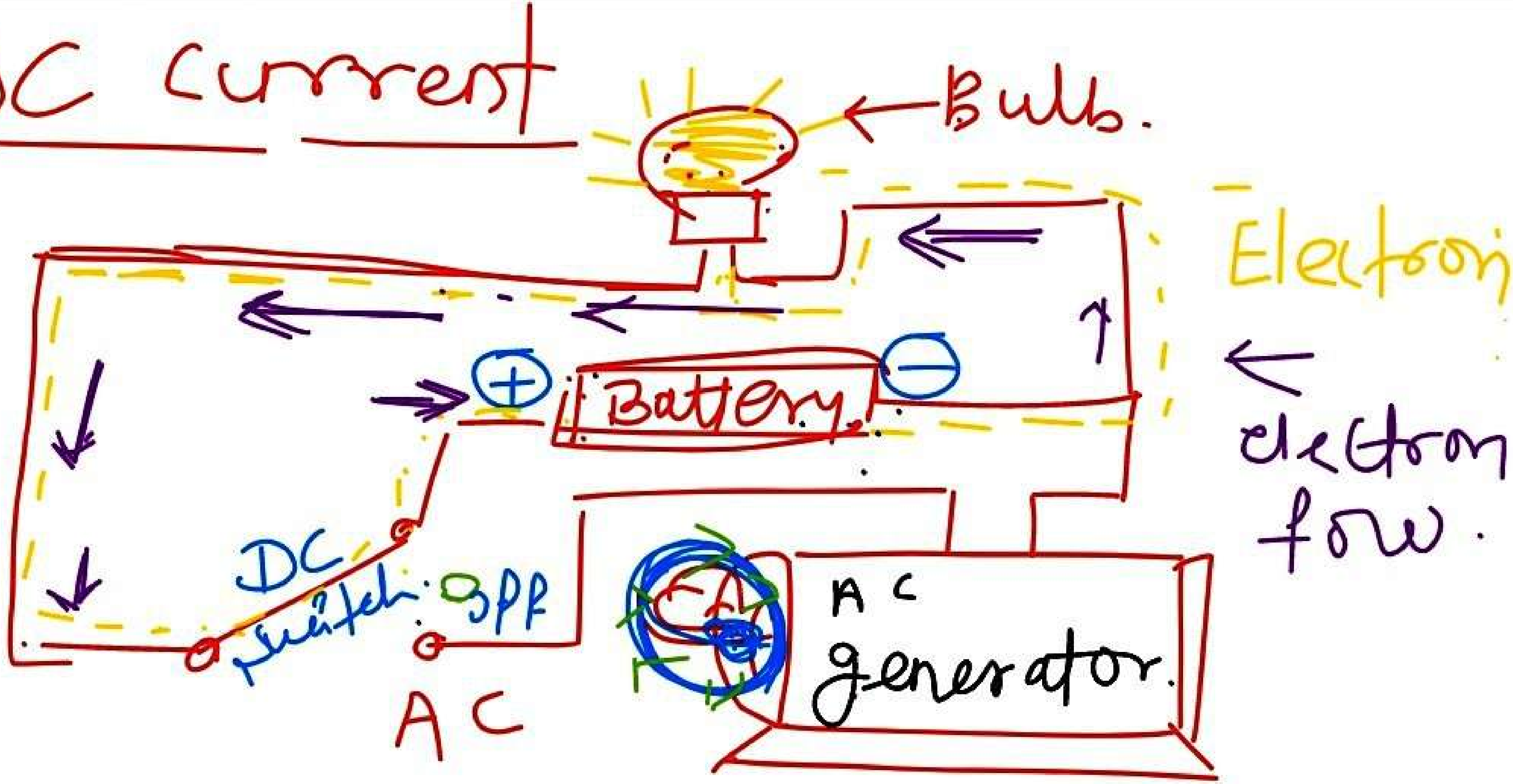
$$\boxed{i = C\frac{dv}{dt}} \leftarrow 7^{th} \text{ form of Ohm's law.}$$

Similarly

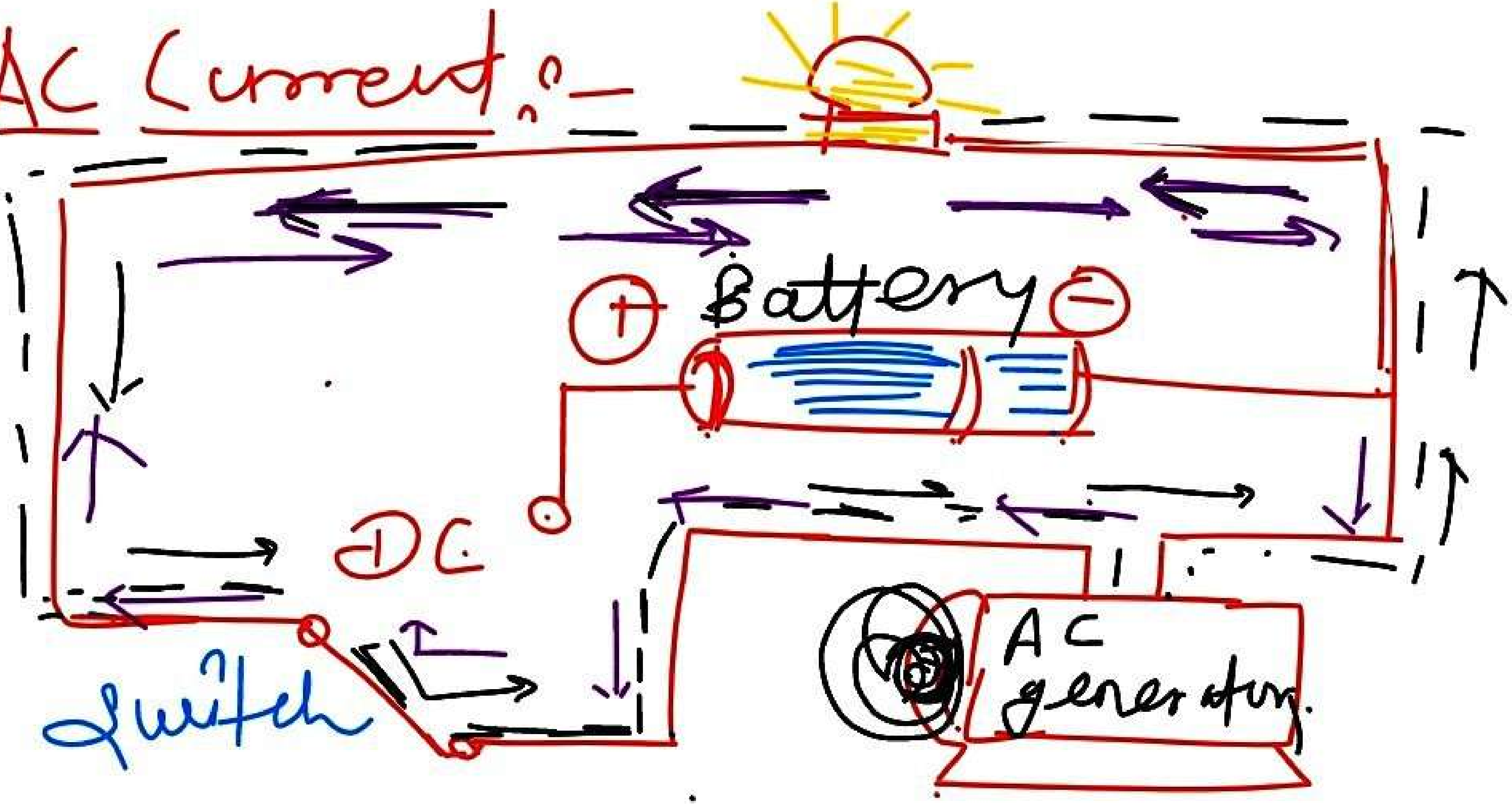
$$\boxed{v = \frac{1}{C}\int_{-\infty}^{t} i\,dt} \leftarrow 8^{th} \text{ form of Ohm's law.}$$

Due to resistance, inductance & capacitance effects of the conductor, energy loss will occur in the conductor.

DC current


Bulb.
Electron.
electron
flow.
Battery
DC
switch
Off
AC
AC
generator.

AC Current :-



DC → use for shorter distance.

AC → Is safe for longer distance withouge any power loss.

(less power loss)

→ Houses, offices, buildings use the AC supply.

→ freq. of AC supply is 50Hz or 60Hz.

(Depending upon country)

freq of DC is zero.

→ Power factor.

for AC → 0 to 1

for DC → Always 1

Types of AC signal.

Sinusoidal, Trapezoidal, Triangular, Square.

Types of DC :-

Pure & pulsating.

Passive Parameter

AC → Impedance → (Generators)

DC → Resistance only → (cell or battery)

Frequency of AC in India is 50Hz.

AC supply 50Hz/220volt.

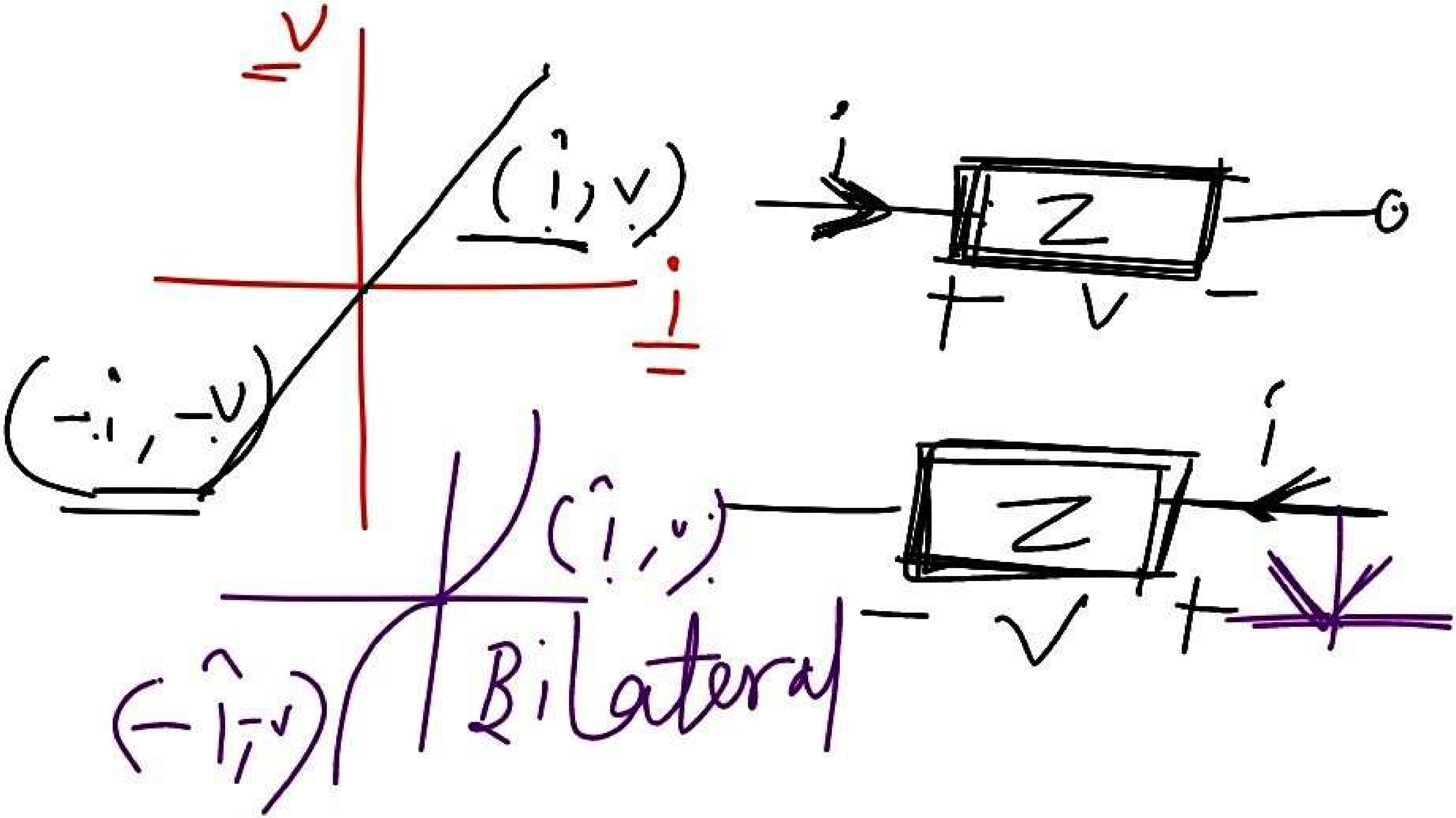
In USA

60Hz/110volt.

Types of Element :-

(5 pairs. total elements 10)

1. Linear & Non linear.
2. Passive & Active.
3. Bilateral & Unilateral
4. Distributed & lumped.
5. Time invariant & Time varying.


v
(i, v)
i
(-i, -v)
i
Z
+ v −
(i, v)
Z
i
− v +
(-i, -v)
Bilateral

$\underline{\underline{Z}} \rightarrow$ Leakage Resistance

$$Z = \sqrt{R^2 + \underline{\underline{(X_L - X_C)}}^2}$$

$Z \downarrow$ Impedence

$R \downarrow$ Resistor.

$(X_L) \downarrow$ Inductive Reactance

$X_C \rightarrow$ Capacitive Reactance.

(4) Distributed & Lumped

(R, L, C) ← lumped

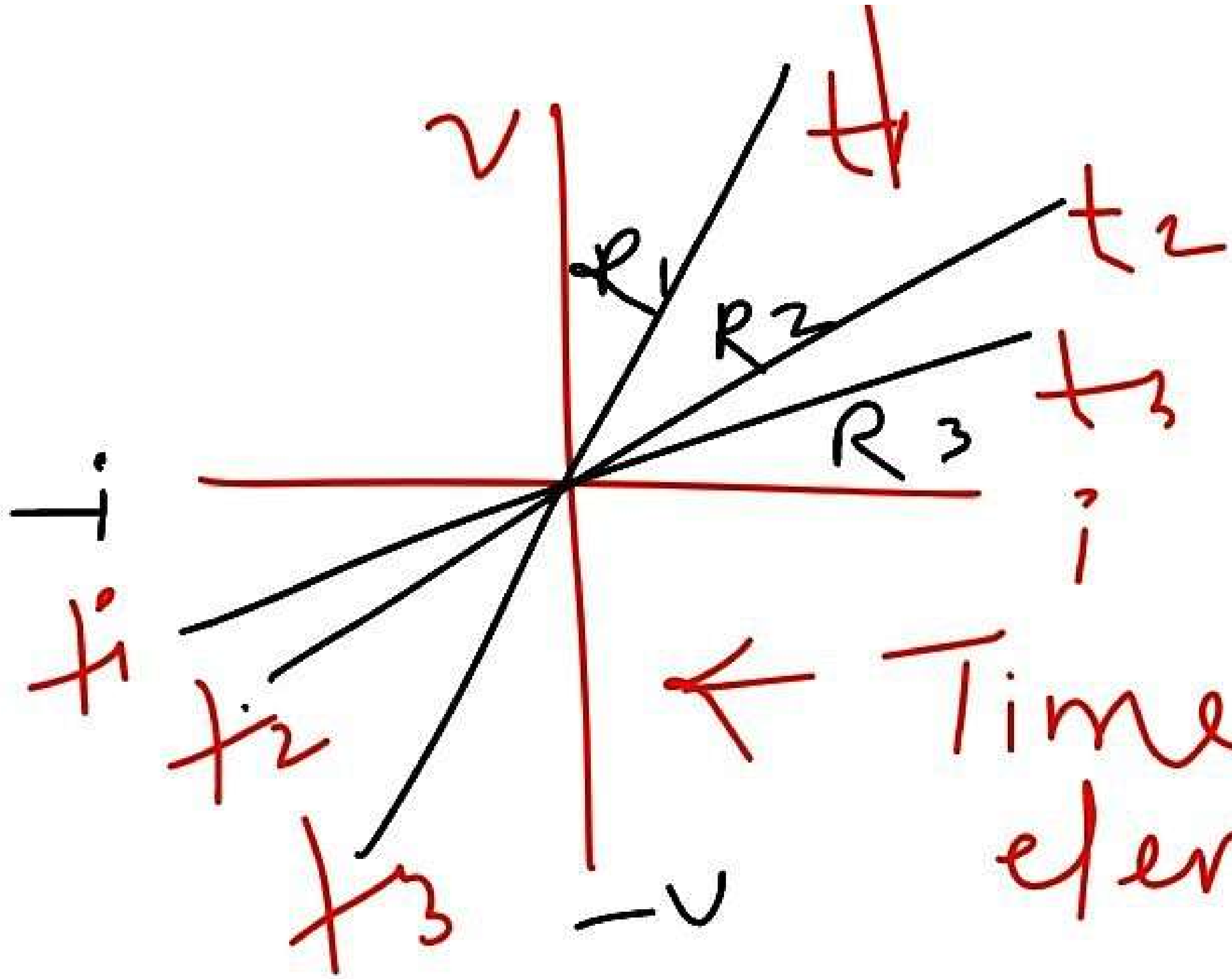
1 MHz

R L C → Distributed across entire ckt. > 1 MHz

They are related to size of the system.

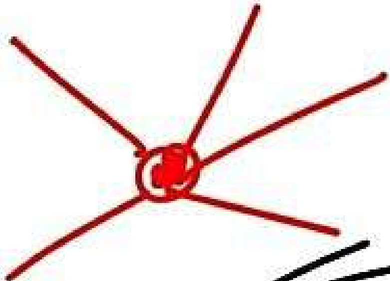
5) Time invariant & Time varying

An element is said to time invariant if, for all time t its characteristics do not change with time



← Time varying element.

Kirchhoff's Law

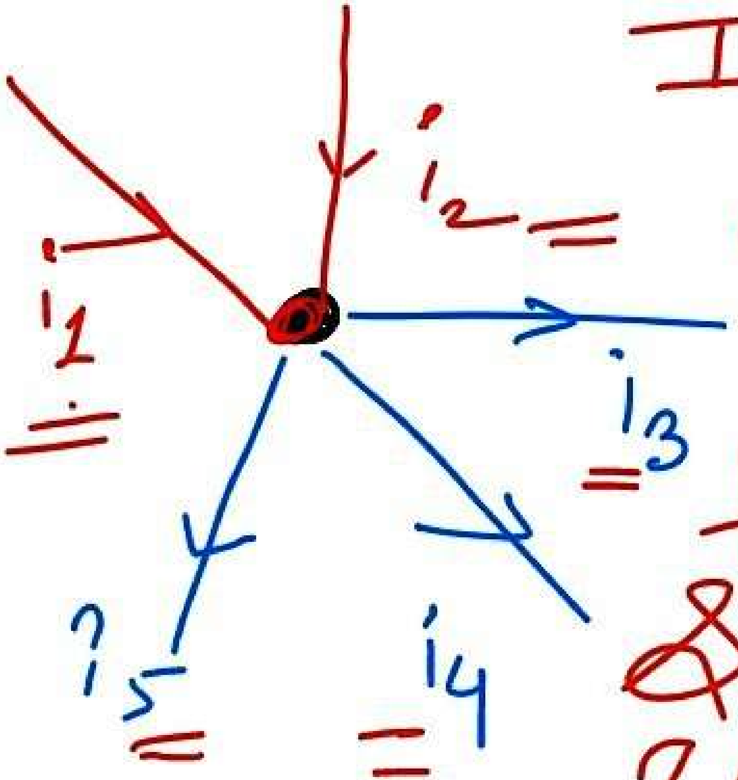


KCL (Node)

1) Principle
2) Simpler

KVL (Mesh)

KCL (It is defined at node)



In a lumped electric circuit for any of its nodes and at any time 't' the algebric sum of brach currents is zero.

By KCL

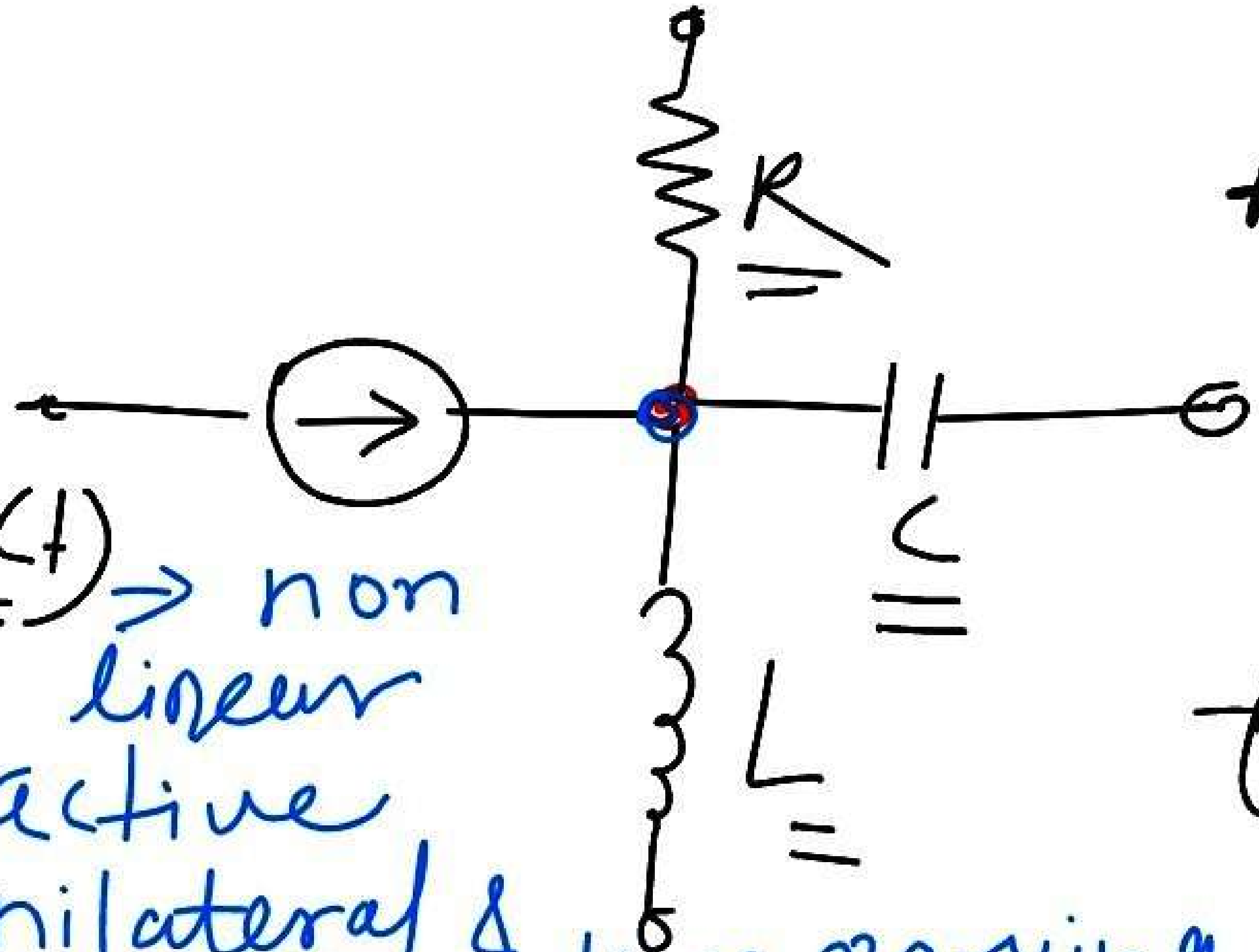
$$i_1 + i_2 = i_3 + i_4 + i_5$$

$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

$$i = \frac{dq}{dt} \Rightarrow \frac{dq_1}{dt} + \frac{dq_2}{dt} = \frac{dq_3}{dt} + \frac{dq_4}{dt} + \frac{dq_5}{dt}$$

$$\Rightarrow q_1 + q_2 = q_3 + q_4 + q_5$$

i.e. Sum of entering charges is equal to sum of leaving charges.



$i(t)$ → non linear
active
unilateral & time varying

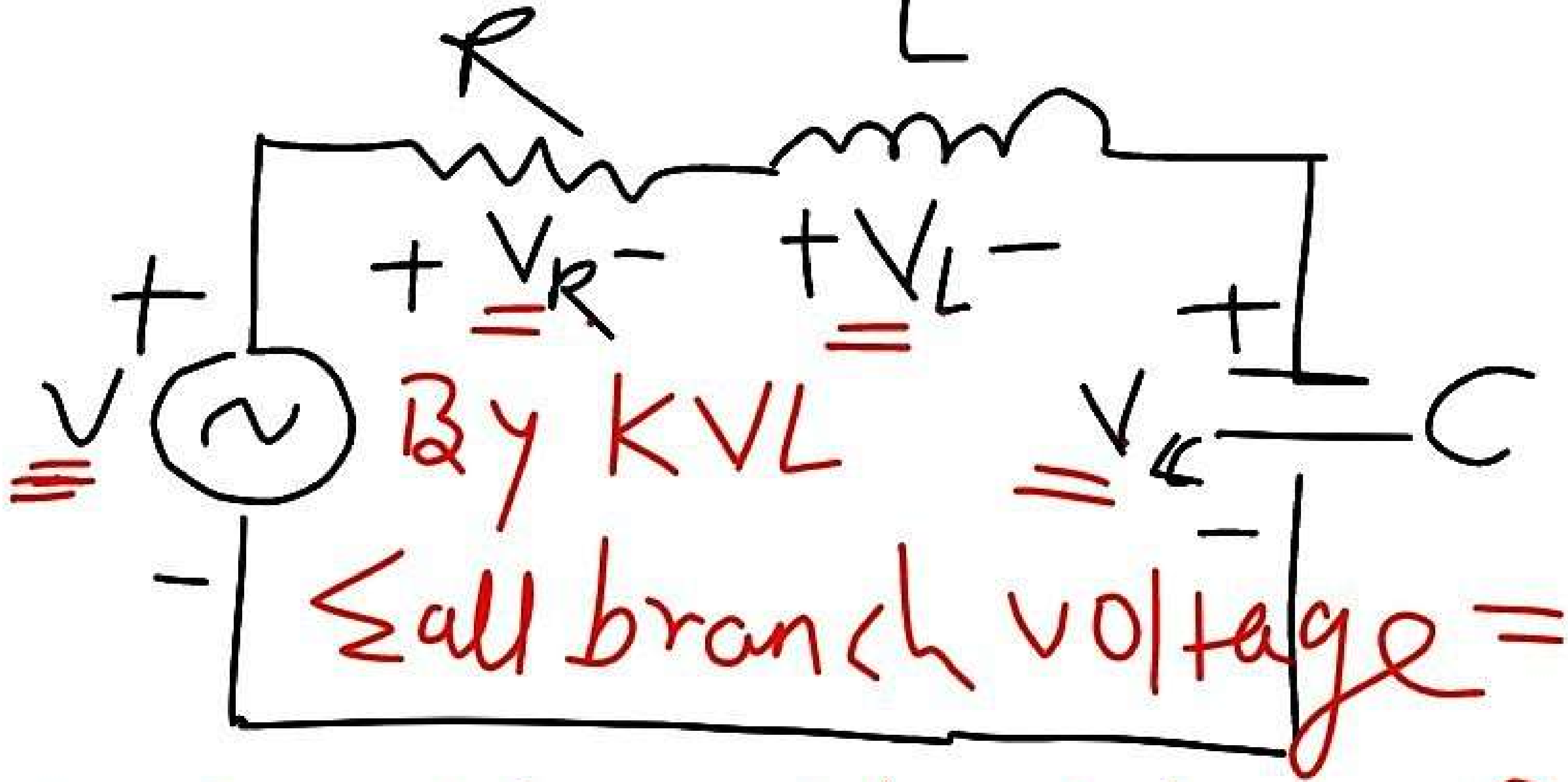
RLC → linear
→ passive
bilateral
time-invariant

→ KCL is independent of the nature of the element

→ It expresses conservation of charge at each & every node in lumped Electric ckt.

KVL $\Rightarrow$ It is defined in a loop or mesh. (i.e. closed path)

KVL : In a lumped electric circuit for any of its loops and at any time 't' the algebric sum of branch voltages around the loop is zero.



By KVL

Σ all branch voltage $= 0$

$$= V - V_R - V_L - V_C = 0$$

Total voltage

$$V = V_R + V_L + V_C$$

Algebric Sum = 0

$$V = \frac{W}{q} = \frac{W_R}{q} + \frac{W_L}{q} + \frac{W_c}{q}$$

$W = W_R + W_L + W_C$

1. KVL is independent of nature of the element
2. KVL expresses conservation of energy in every loop of lumped electric ckt.

Nodal & Mesh Analysis Technique.

Nodal Analysis $\Rightarrow$ KCL + Ohm's law.

Mesh Analysis $\Rightarrow$ KVL + Ohm's law.

Ohm's law → function of nature of element (8 diff forms of Ohm's law)

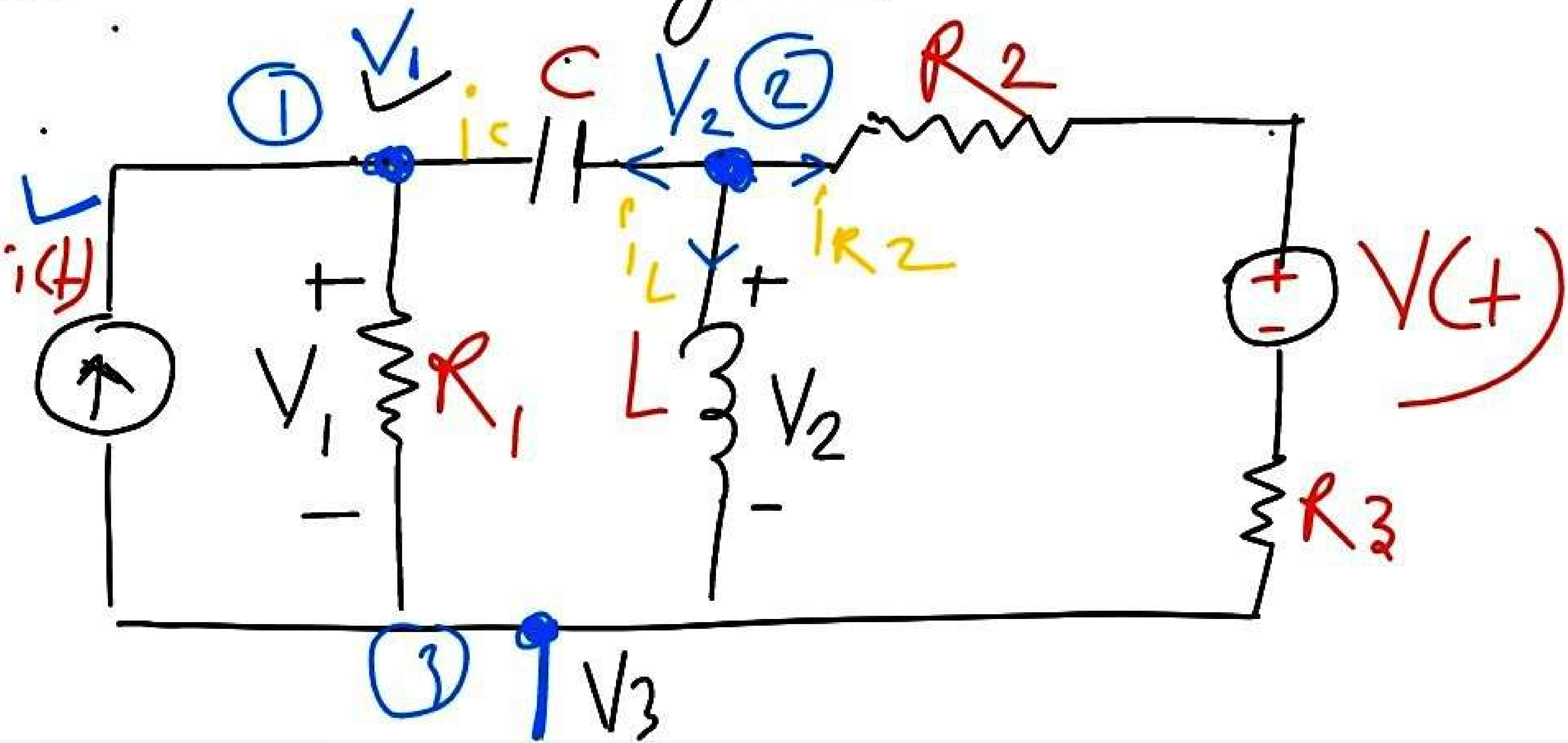
(R, L C) ✓

$J = \sigma E$

Not Applicable for active elements

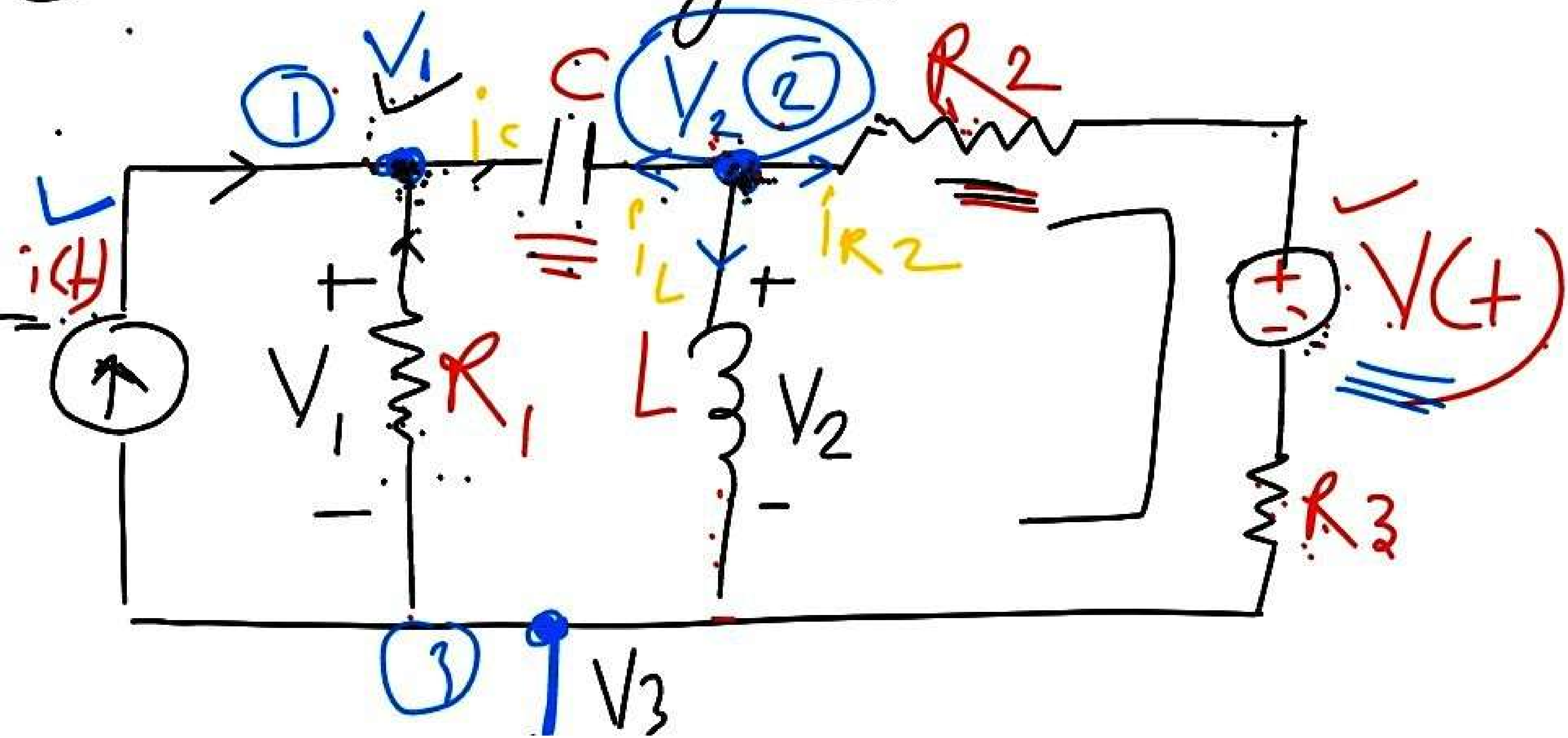
Ex. Source (Generators)

① Nodal Analysis



1. Identify the nodes
2. Assign node voltages w.r.to ground node (whose voltage is always zero)
3. KCL first ⟸ 1st
   Ohms law ⟸ 2nd

1 Nodal Analysis



(1) Identify the nodes

(2) Assign node voltages w.r.to ground node (whose voltage is always zero)

(3) KCL first ⟸ 1st ✓

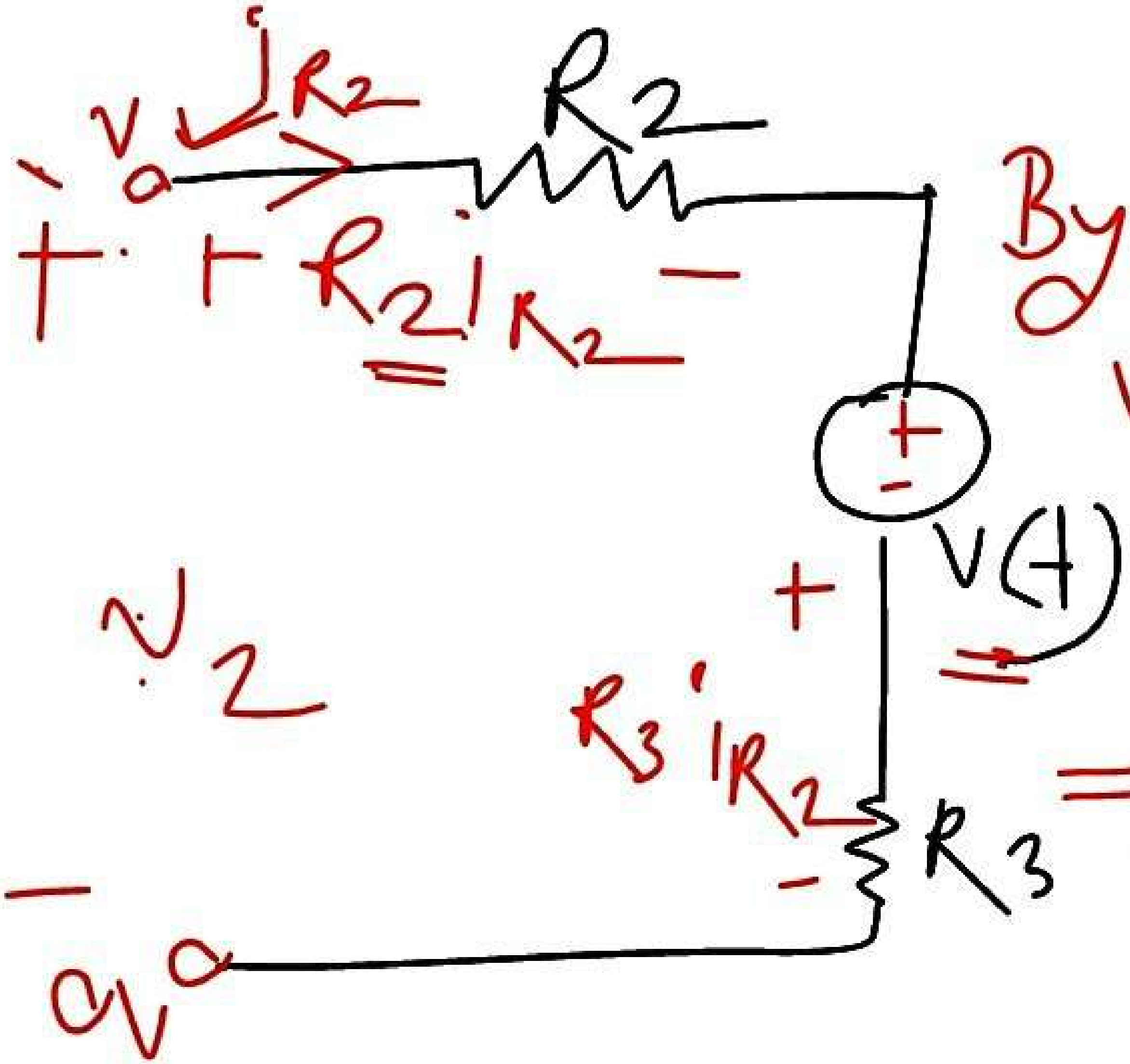
Ohms law ⟸ 2nd ✓

Node 2 $\left(\begin{array}{l} V_2 > V_1 \\ V_2 > 0 \\ V_2 > V(t) \end{array} \right)$

By KCL $\Rightarrow$ $I_C + I_L + I_{R_2} = 0$

$$C \frac{d}{dt}(V_2 - V_1) + \frac{1}{L}\int_{-\infty}^{t} V_2 \, dt + \frac{V_2 - V(t)}{R_2 + R_3} = 0 \quad \text{--- (1)}$$

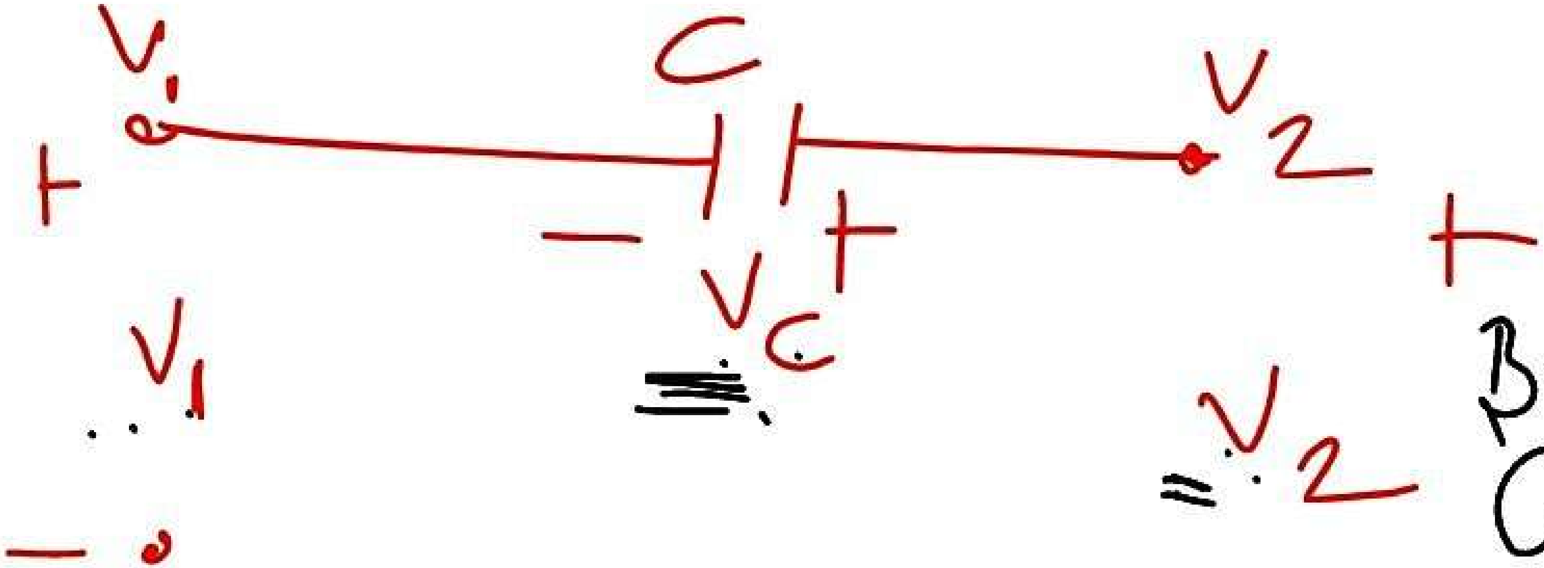
(by Ohm's Law)



By KVL

$$v_2 - R_2 i_{R_2} - v(t) - R_3 i_{R_2} = 0$$

$$\Rightarrow i_{R_2} = \frac{v_2 - v(t)}{R_2 + R_3} \text{ A.}$$



By KVL

$$-v_1 + v_c - v_2 = 0$$

$$v_c = v_2 - v_1$$

By ohm's law

$$i_c = C\frac{dv_c}{dt}$$

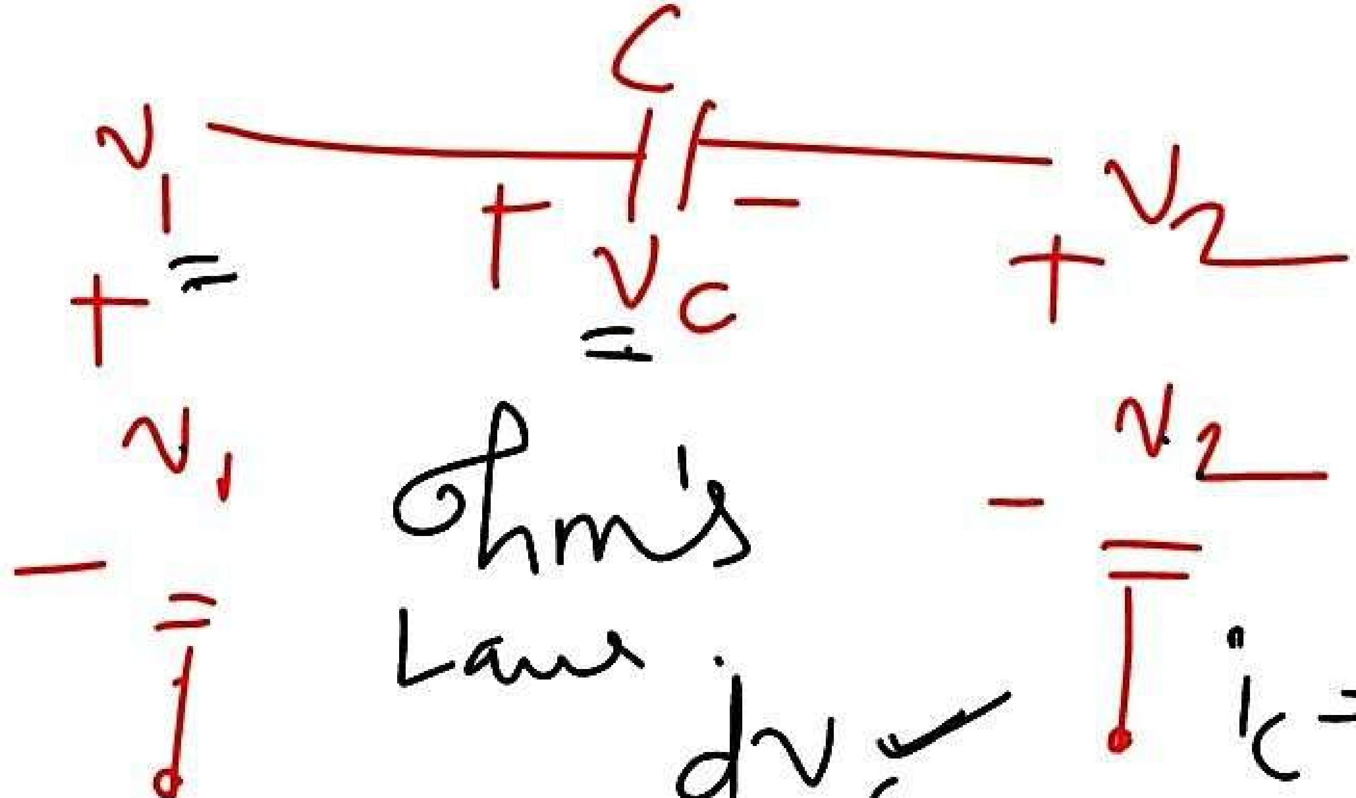
Current through C.

$$\Rightarrow i_C = C \frac{d}{dt}(v_2 - v_1)$$

Node 1. $(v_1 > 0,\ v_1 > v_2.)$

$$-i(t) + \frac{v_1}{R_1} + C\frac{d}{dt}(v_1 - v_2) = 0 \quad — (2)$$

(By Nodal Analysis)



By KVL

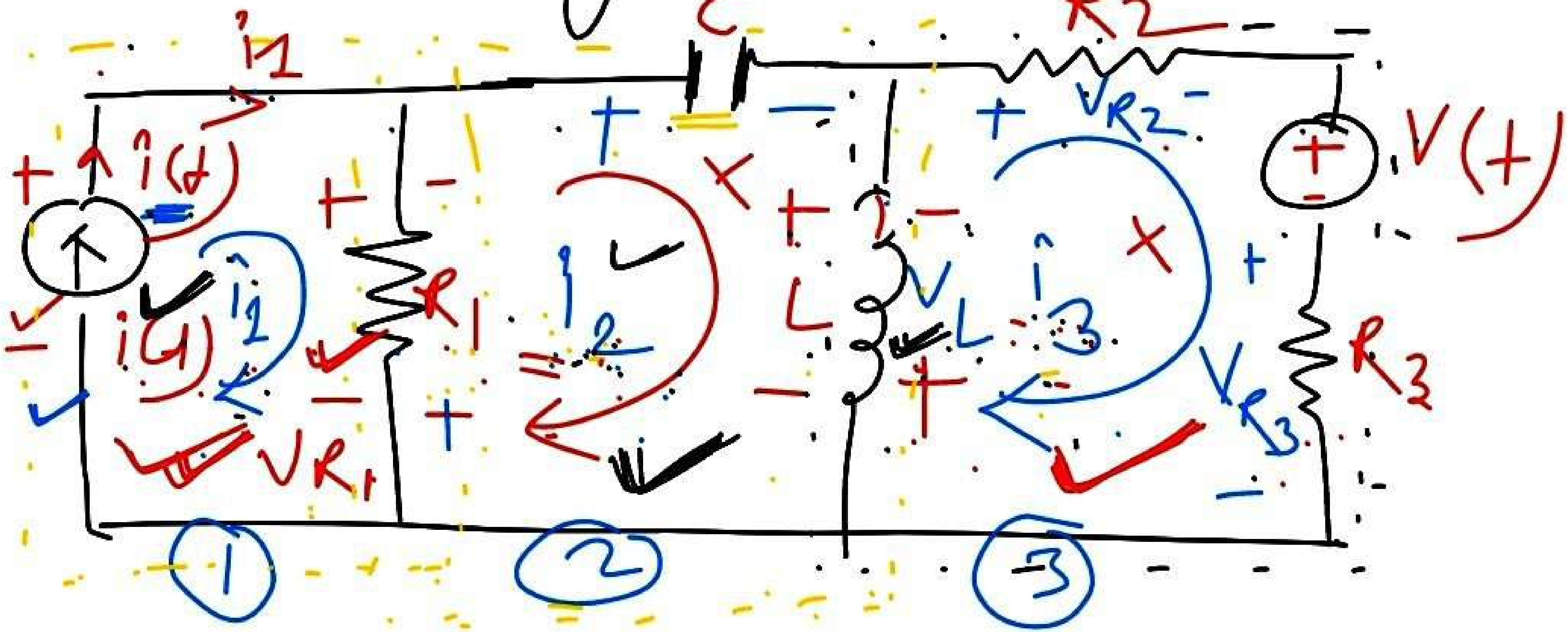
$$v_C = v_1 - v_2$$

Ohm's Law

$$i_c = C\frac{dv_c}{dt}$$

$$i_c = C\frac{d}{dt}(v_1 - v_2)$$

Mesh Analysis :-

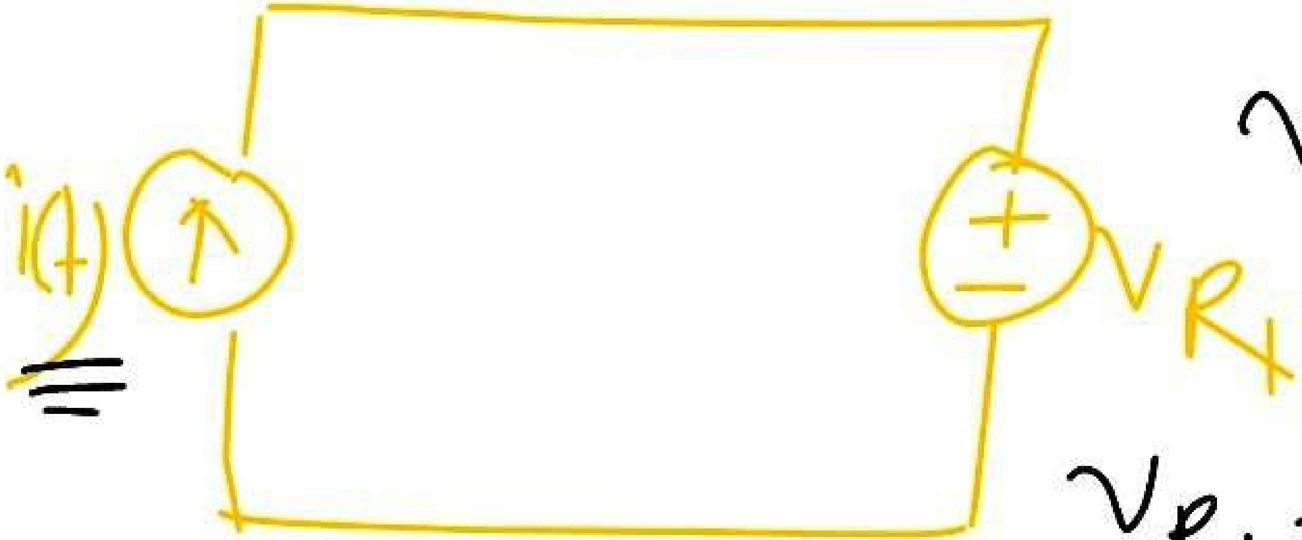


(1) Identify the mesh.

(2) Assign mesh current (clockwise direction)

(3) By using KVL & Ohm's law next write the mesh eqn.

Mesh ① $(i_1 > i_2$ & $i_1 > i_3)$



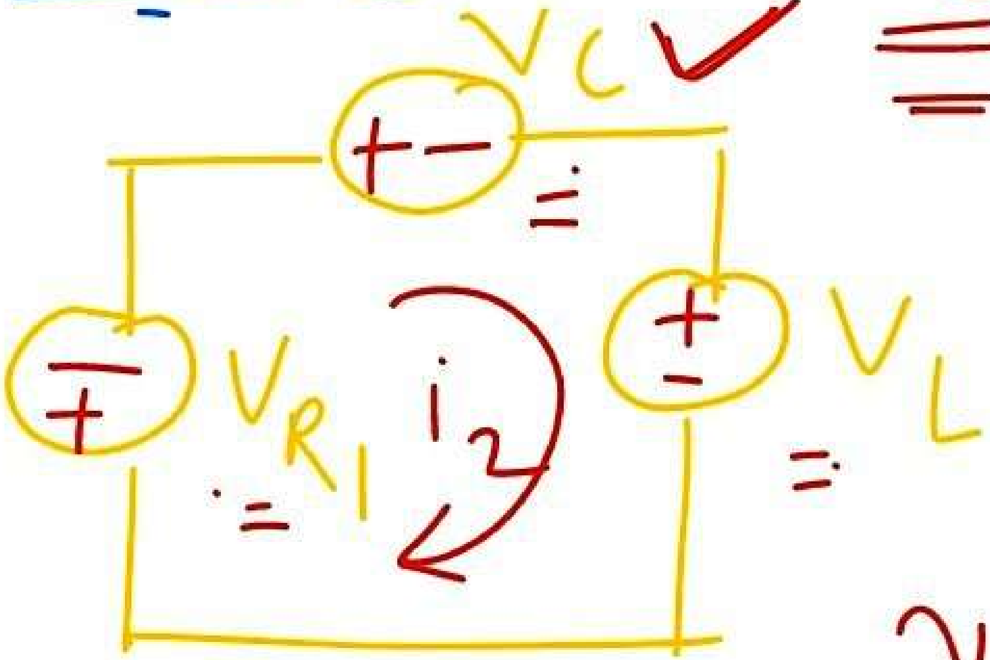
$$v_{R_1} = R_1(i_1 - i_2)$$

$$v_{R_1} = v_{R_1}$$

$$v_{R_1} - v_{R_1} = 0$$

So $i_1 = i(t)$ (Both currents are in same direction)

Mesh (2)



$$V_C = \frac{1}{C}\int_{-\infty}^{t} i_2 \, dt$$

$$V_L = L\frac{d}{dt}(i_2 - i_3)$$

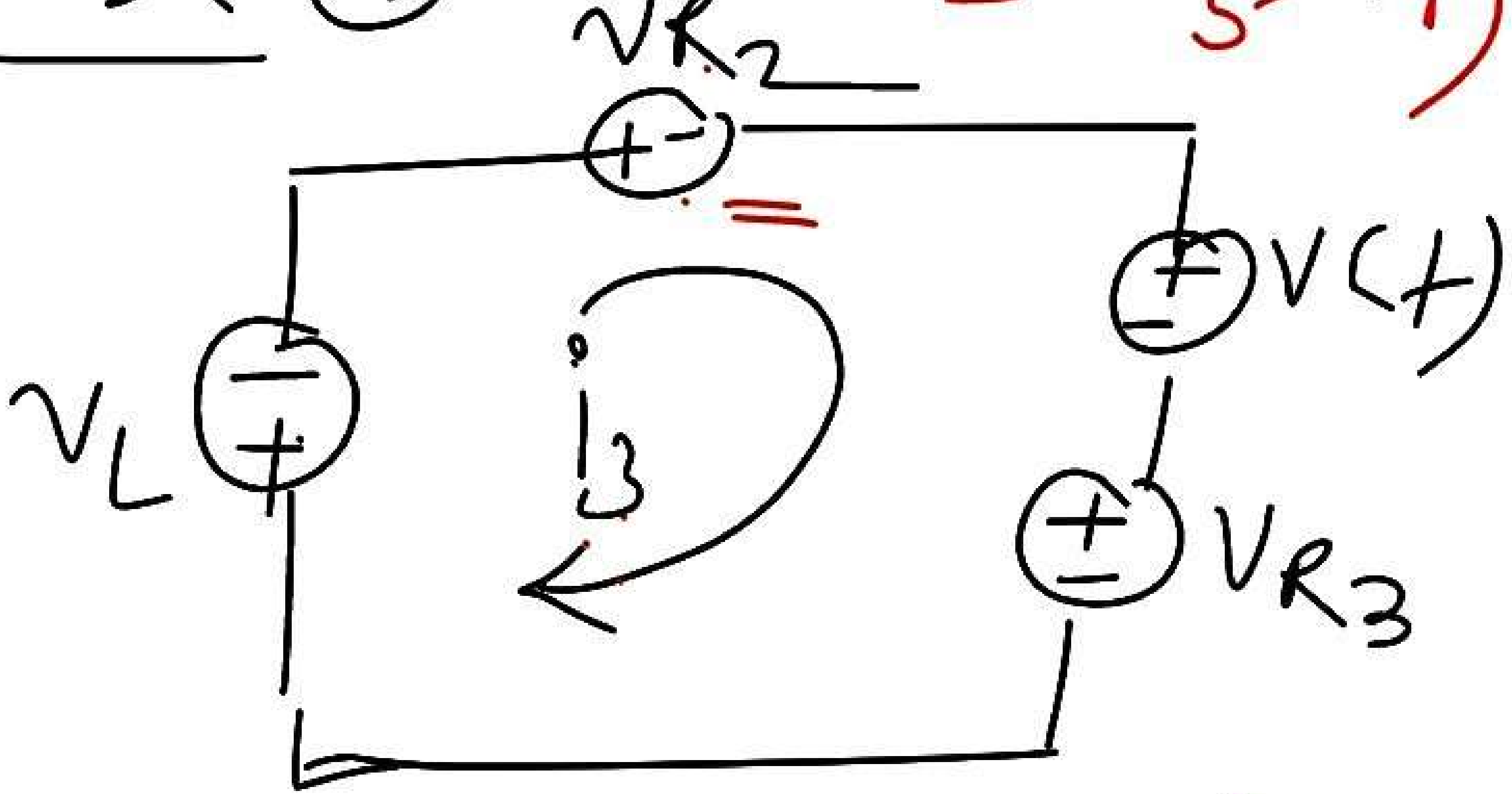
$$V_{R_1} = R_1(i_2 - i_2)$$

By KVL $v_{R_1} + v_C + v_L = 0$

$\Rightarrow$ By Ohm's Law.

$$-R_1(i_2 - i_1) - \frac{1}{C}\int_{-\infty}^{t} i_2\,dt - L\frac{d}{dt}(i_2 - i_3) = 0 \quad \text{—} \quad (2)$$

Mesh 3 $(i_3 > i_2 \quad i_3 > i_1)$



$$v_{R_2} = R_2 i_3$$

$$v_{R_3} = R_3 i_3$$

$$v_L = L \frac{d}{dt}(i_3 - i_2)$$

$$v(t) =$$

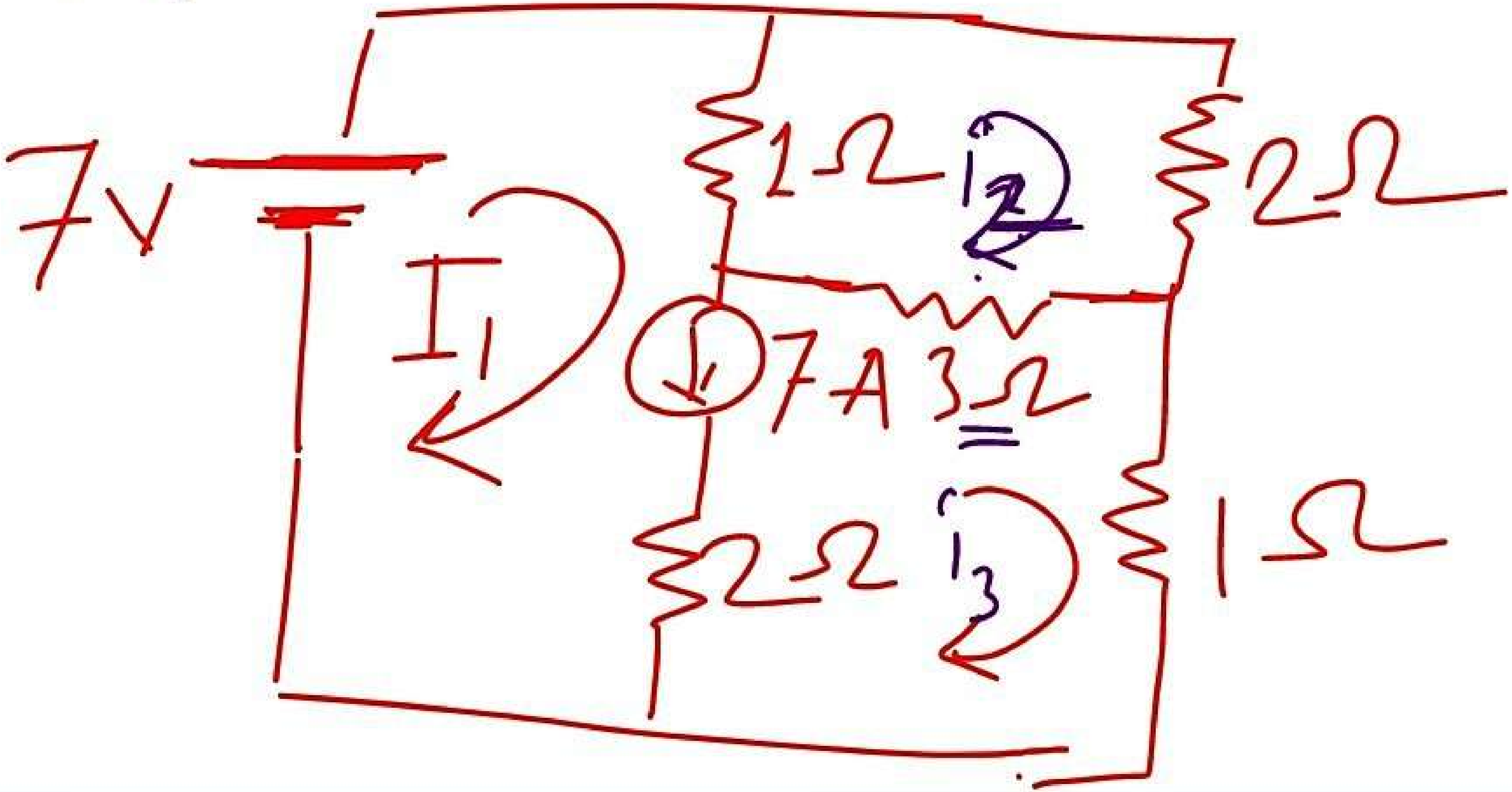
$$-v_L - v_{R_2} - v_{R_3} - v(t) = 0$$

(KVL)

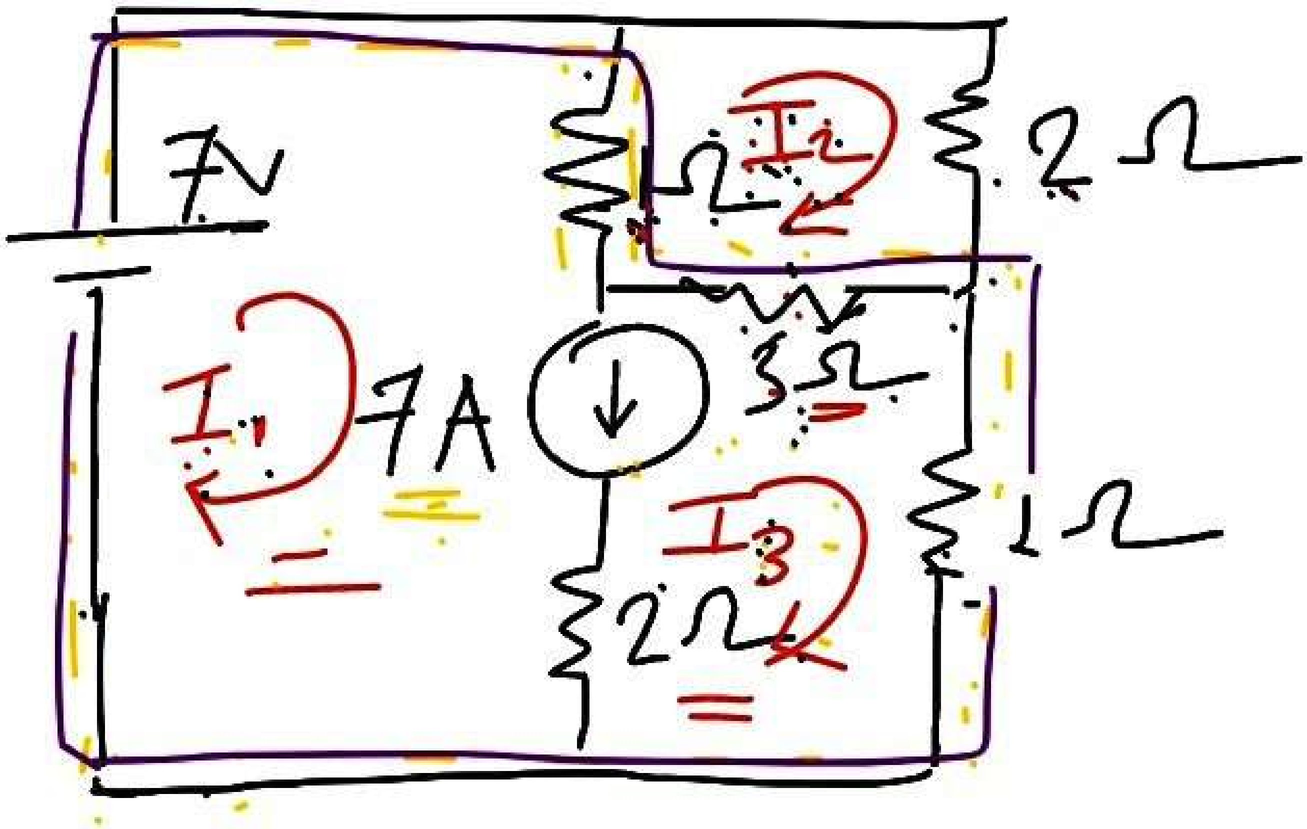
$$= -L\frac{d}{dt}(i_2 - i_3) - R_2 i_3 - R_3 i_3 - v(t) = 0$$

(ohm's law)

find current in 3 Ω



find the current in 3-Ω resistor



$I_2 - I_3$

$I_1 - I_3 = 7 \quad \text{—} \quad (1)$

By Applying KVL to Supermesh.

$7 - 1(I_1 - I_2) - 3(I_3 - I_2) - I_3 = 0$

$7 - I_1 + 4I_2 - 4I_3 = 0$

$= -I_1 + 4I_2 - 4I_3 = -7 \quad \text{—} \quad (2)$

Applying KVL to mesh 2

$$\Rightarrow -1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$= I_1 - 6I_2 + 3I_3 = 0 \quad \text{--- } (3)$$

$I_1 = 9$ Amp.

$I_2 = 2.5$ Amp.

$I_3 = 2$ Amp.

Current through $3\,\Omega = I_2 - I_3$

$= 2.5 - 2 = 0.5$ Amp.

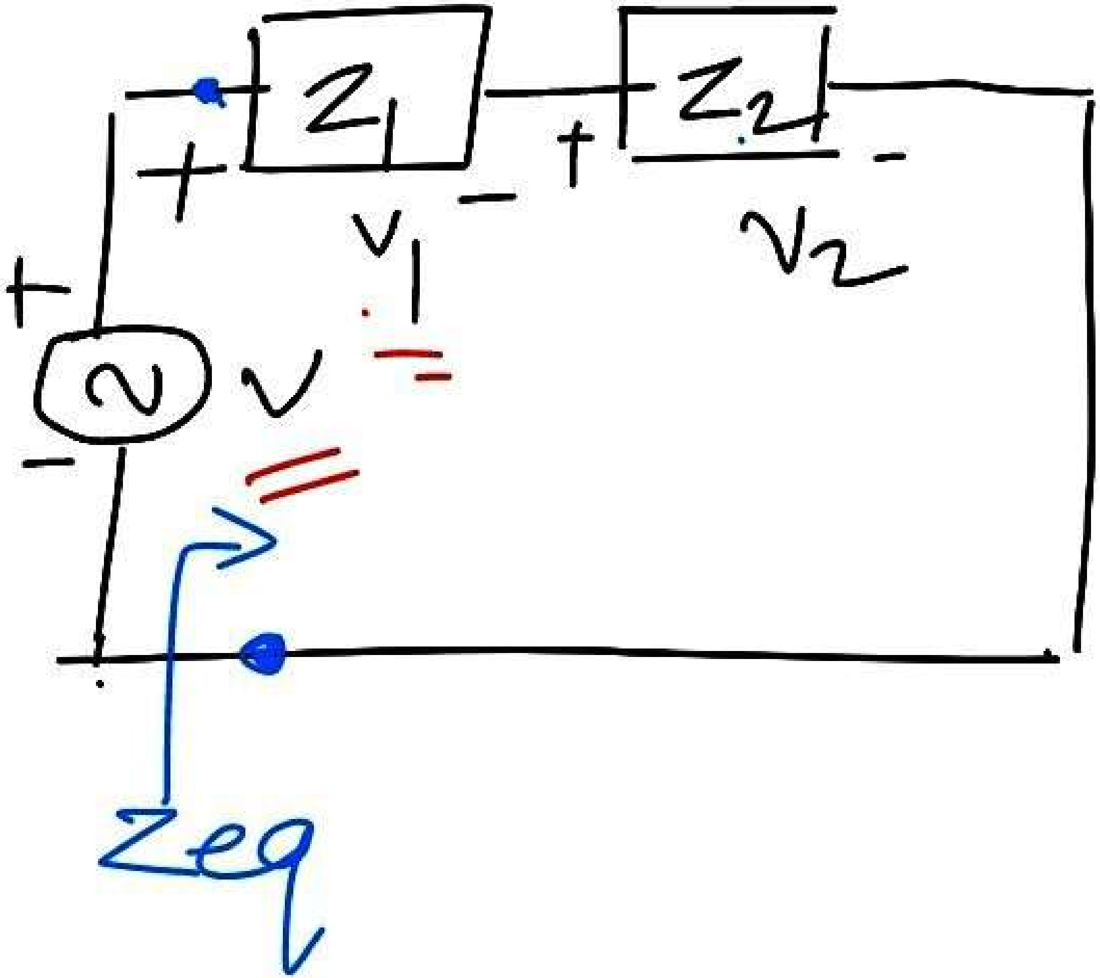
Equivalent ckt w.r. to passive RLC

Two elements are in series

→ Same current through the element

→ Two elements are in ||el.

→ Same voltage across the eleme



$$Z = Z_R = R\,\Omega$$

$$Z_L = j\omega L\,\Omega$$

$$Z_C = \frac{1}{j\omega C}\,\Omega$$

$$Z_{eq} = Z_1 + Z_2$$

$$R_{eq} = R_1 + R_2$$

$$L_{eq} = L_1 + L_2$$

$$C = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

If C_1 & $C_2 = C$

$$C_{eq} = \frac{C}{2}$$

Voltage division principle.

$V = Z_{eq} I$ (By Ohm's law)

$$I = \frac{V}{Z_{eq}} = \frac{V}{Z_1 + Z_2}$$

$\underline{V}_1 = Z_1 \underline{I}$ (Ohms law)

$$V_1 = \frac{V Z_1}{Z_1 + Z_2}$$

$$V_2 = Z_2 I \text{ (ohm's law)}$$

$$V_2 = \frac{Z_2 V}{Z_1 + Z_2}$$

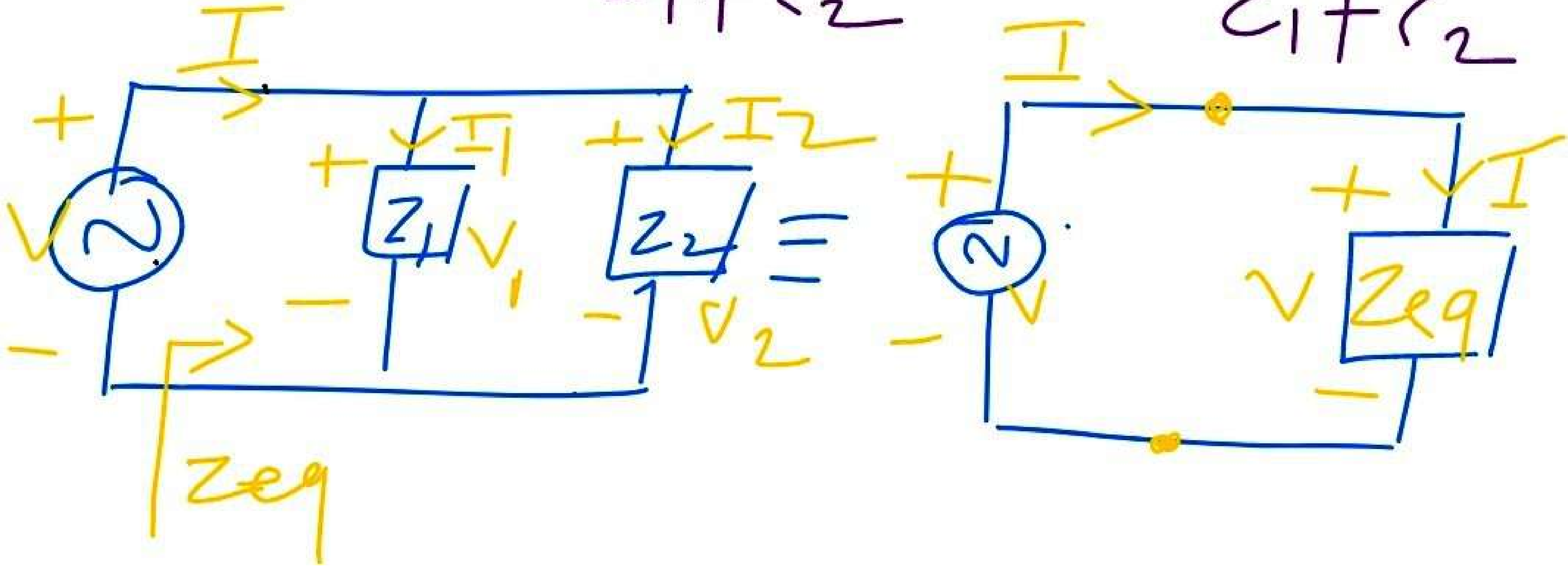
By KVL $v - v_1 - v_2 = 0$

$$\boxed{v_1 + v_2 = v}$$

R : $v_1 = \dfrac{v R_1}{R_1 + R_2}$ $v_2 = \dfrac{v R_2}{R_1 + R_2}$

L : $v_1 = \dfrac{v L_1}{L_1 + L_2}$ $v_2 = \dfrac{v L_2}{L_1 + L_2}$

$\underline{\underline{C}}$: $v_1 = \dfrac{v C_1}{C_1 + C_2}$ $v_2 = \dfrac{v C_2}{C_1 + C_2}$



$$Y_{eq} = Y_1 + Y_2$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$R = \frac{1}{R_{eq}} = \frac{1}{R_1 + R_2} \text{ if } R_1 = R_2 = R$$

$$R_{eq} = R/2$$

$$L = \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$C = C_{eq} = C_1 + C_2$$

Current division Principle

$$V = Z_{eq} \cdot I \quad \text{(By ohm's law)}$$

$$V = \frac{Z_1 Z_2}{Z_1 + Z_2} \cdot I$$

$$I_1 = \frac{V}{Z_1}$$

$$I_1 = \frac{I \cdot Z_2}{Z_1 + Z_2}$$

$$I_2 = \frac{V}{Z_2} \Rightarrow \frac{I Z_1}{Z_1 + Z_2}$$

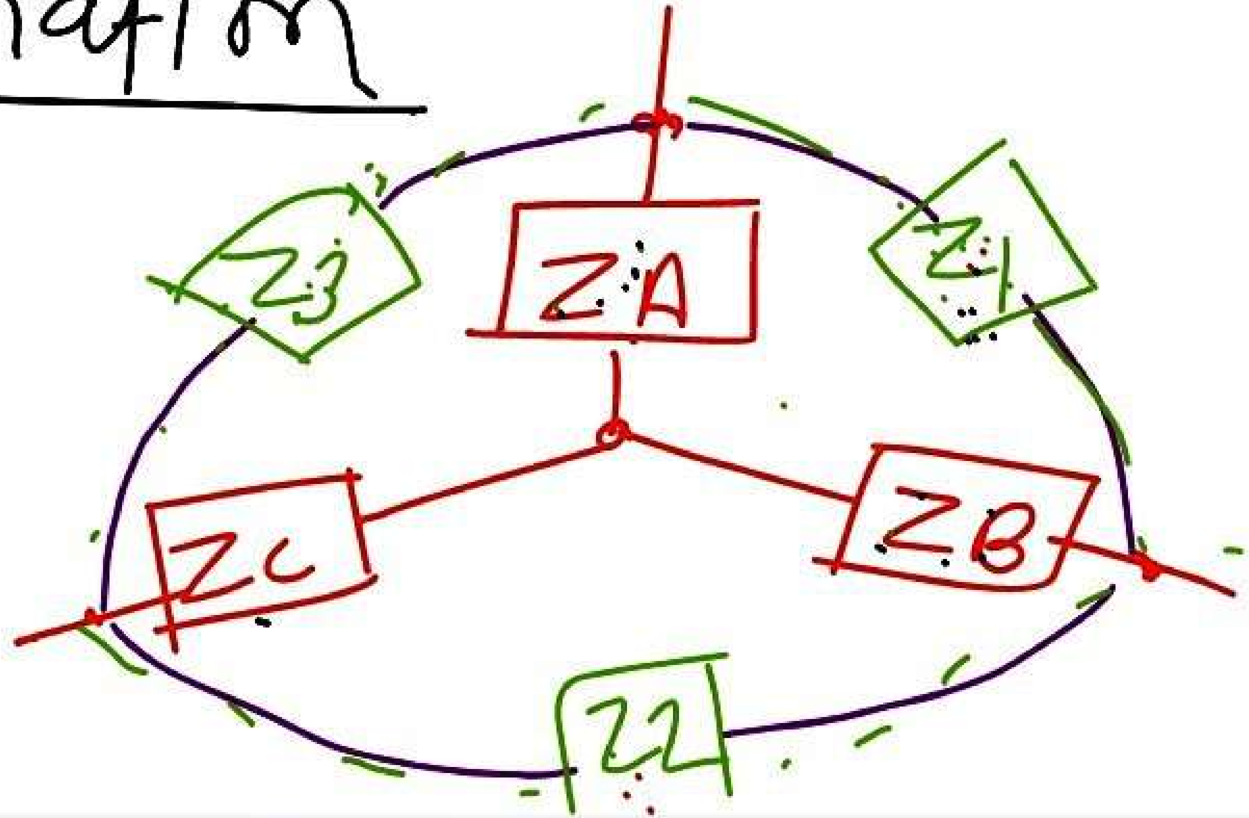
By KCL $\boxed{I = I_1 + I_2}$

$R \Rightarrow I_1 = \dfrac{IR_2}{R_1 + R_2} \quad I_2 = \dfrac{IR_1}{R_1 + R_2}$

$L \Rightarrow I_1 = \dfrac{IL_2}{L_1 + L_2} \quad I_2 = \dfrac{IL_1}{L_1 + L_2}$

$$C \Rightarrow I_1 = \frac{I C_1}{C_1 + C_2} \quad I_2 = \frac{I C_2}{C_1 + C_2}$$

Star(Y) - Delta(Δ) Transformation



Case 1 $\Delta \rightarrow Y$.

Δ is given (Z_1, Z_2, Z_3)

1 $Z_A = \dfrac{Z_1 \cdot Z_3}{Z_1 + Z_2 + Z_3} \Rightarrow \left(\dfrac{\text{Product}}{\text{Total}}\right)$

(2) $Z_B \Rightarrow \dfrac{Z_1 \cdot Z_2}{Z_1 + Z_2 + Z_3}$

(3) $Z_C \Rightarrow \dfrac{Z_2 \cdot Z_3}{Z_1 + Z_2 + Z_3}$

If

$Z_1 = Z_2 = Z_3$

then

$Z_A = Z_B = Z_C$

$\Rightarrow Z/3$ ✓

Note :- Δ to Y Transformation Impedance reduces by 3 times.

$\underline{\underline{Y - \Delta}}$ $\quad (Z_A, Z_B, Z_C)$

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A \cdot Z_B}{Z_C}\right)$$

$$= \text{Sum} + \left(\frac{\text{Their Product}}{\text{Remaining}}\right)$$

$$Z_2 = Z_B + Z_C + \left(\frac{Z_B \cdot Z_C}{Z_A} \right)$$

$$Z_3 = Z_C + Z_A + \left(\frac{Z_C \cdot Z_A}{Z_B} \right)$$

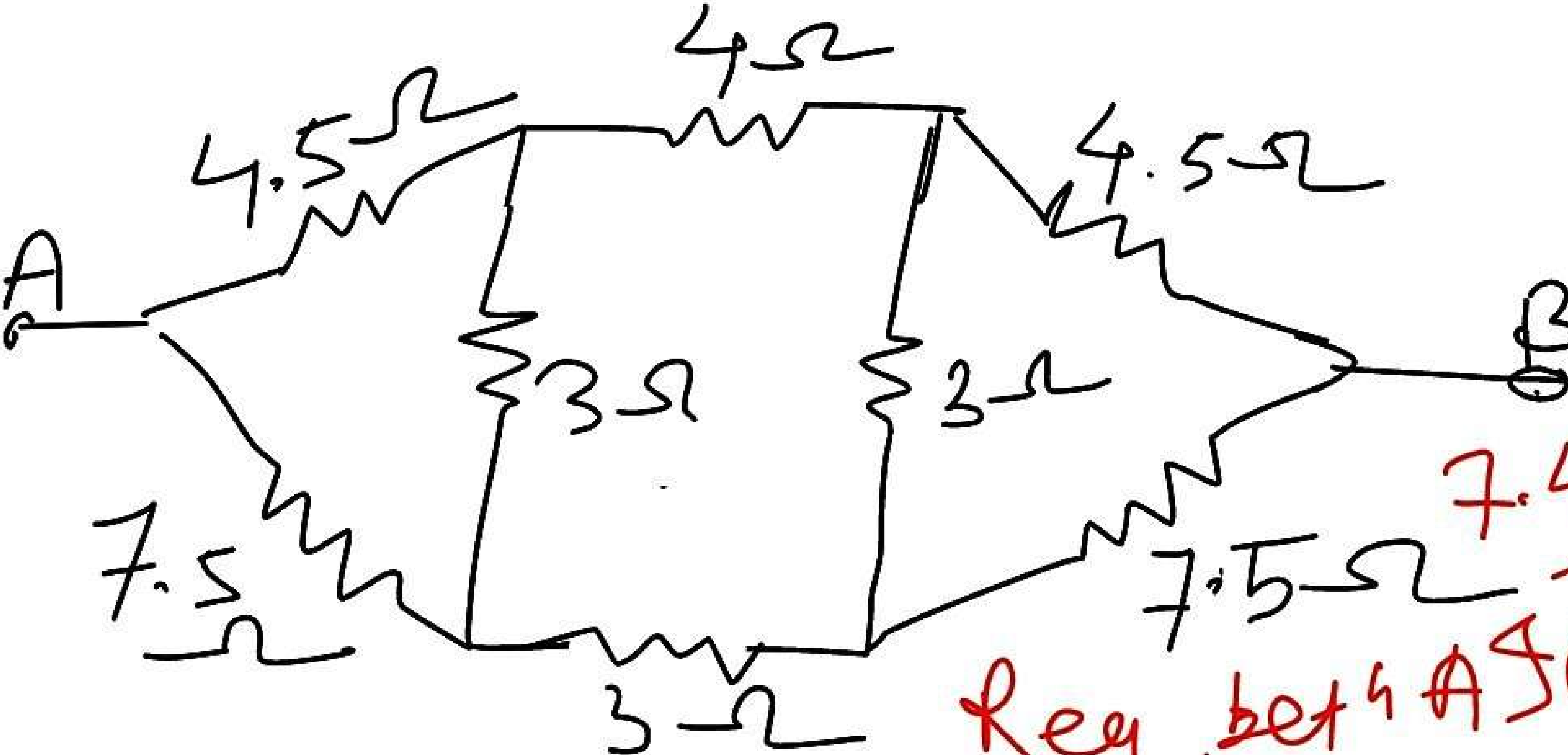
If $Z_A = Z_B = Z_C = Z$

$Z_1 = Z_2 = Z_3 = 3Z$ ✓

If $Z_A = Z_B = Z_C = Z$

then $Z_1 = Z_2 = Z_3 = 3Z$

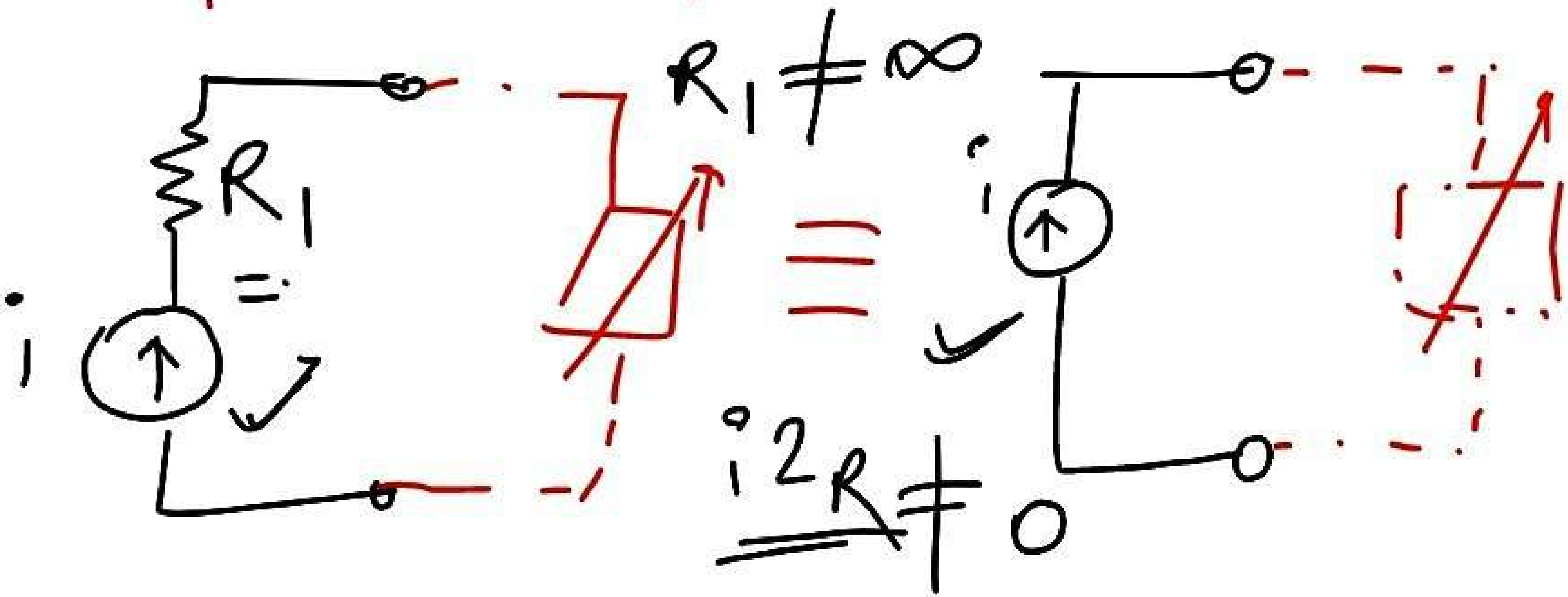
(Y to $\Delta \Rightarrow$ Impedance increase by 3 times.

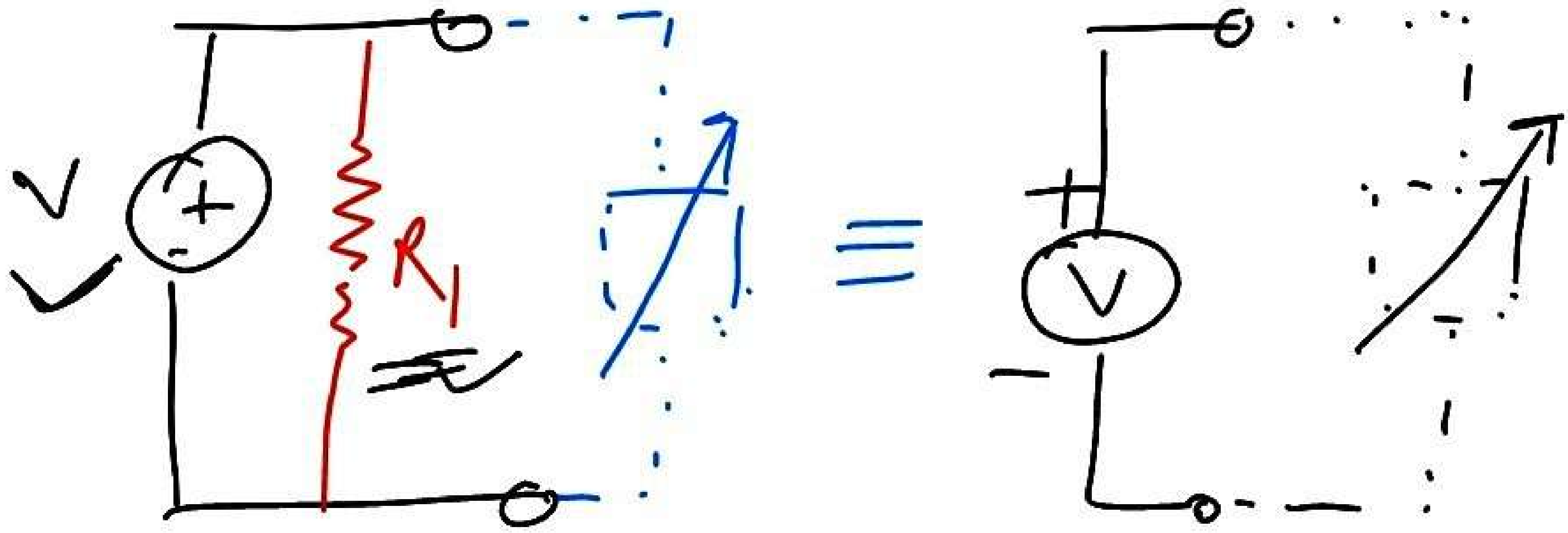


7.45 Ω

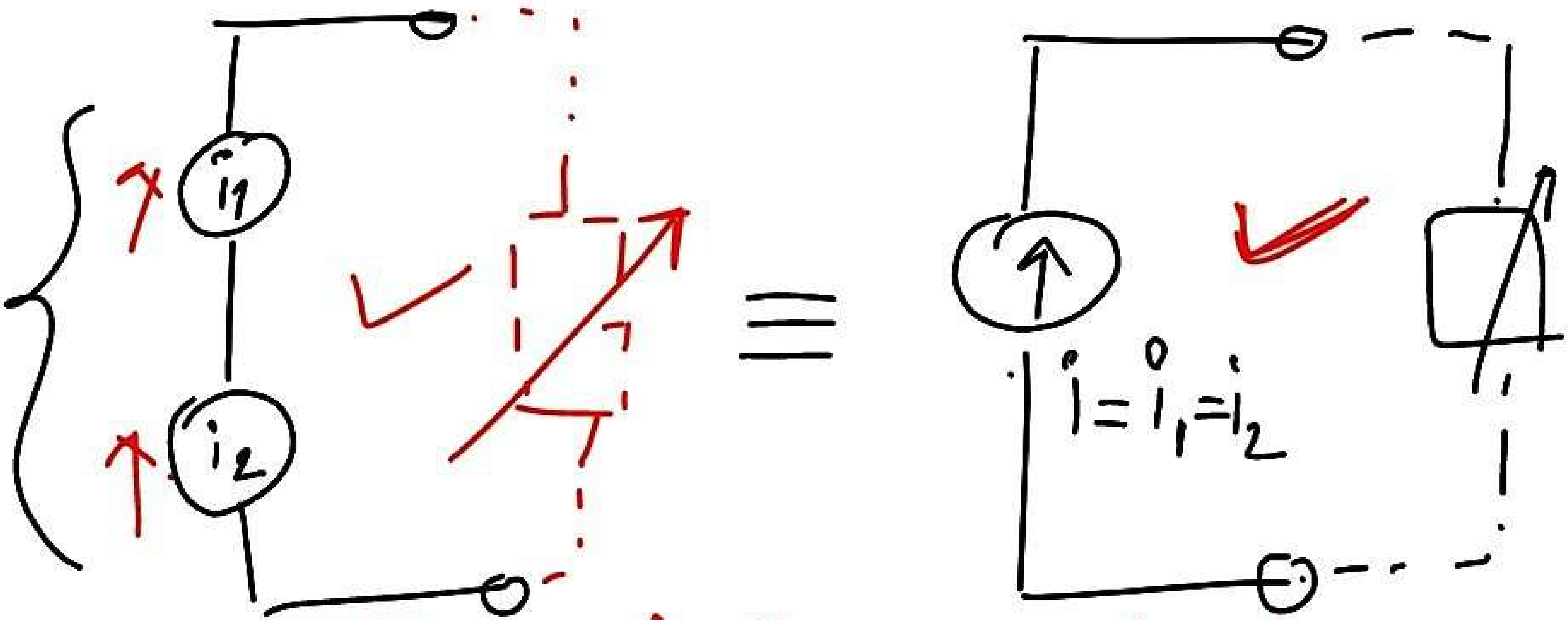
Req. betⁿ A & B.

Equivalent Circuits w.r. to Active Sources

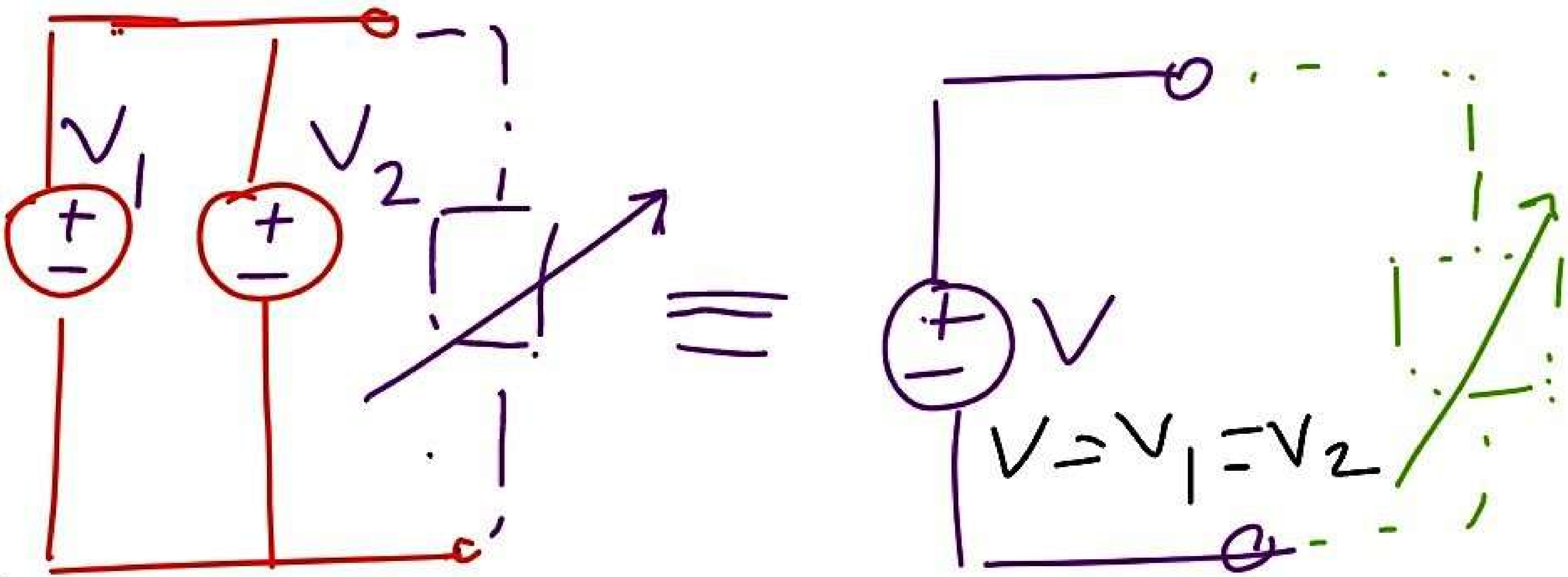




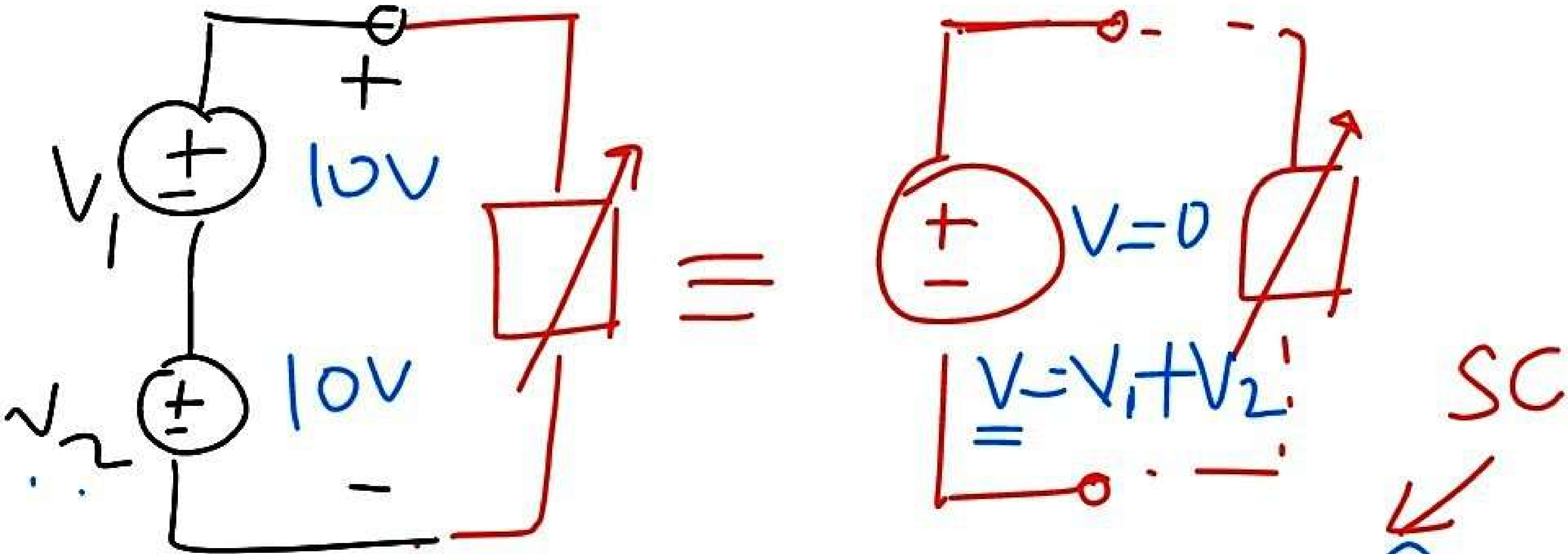
$$\frac{V^2}{R} \neq 0$$



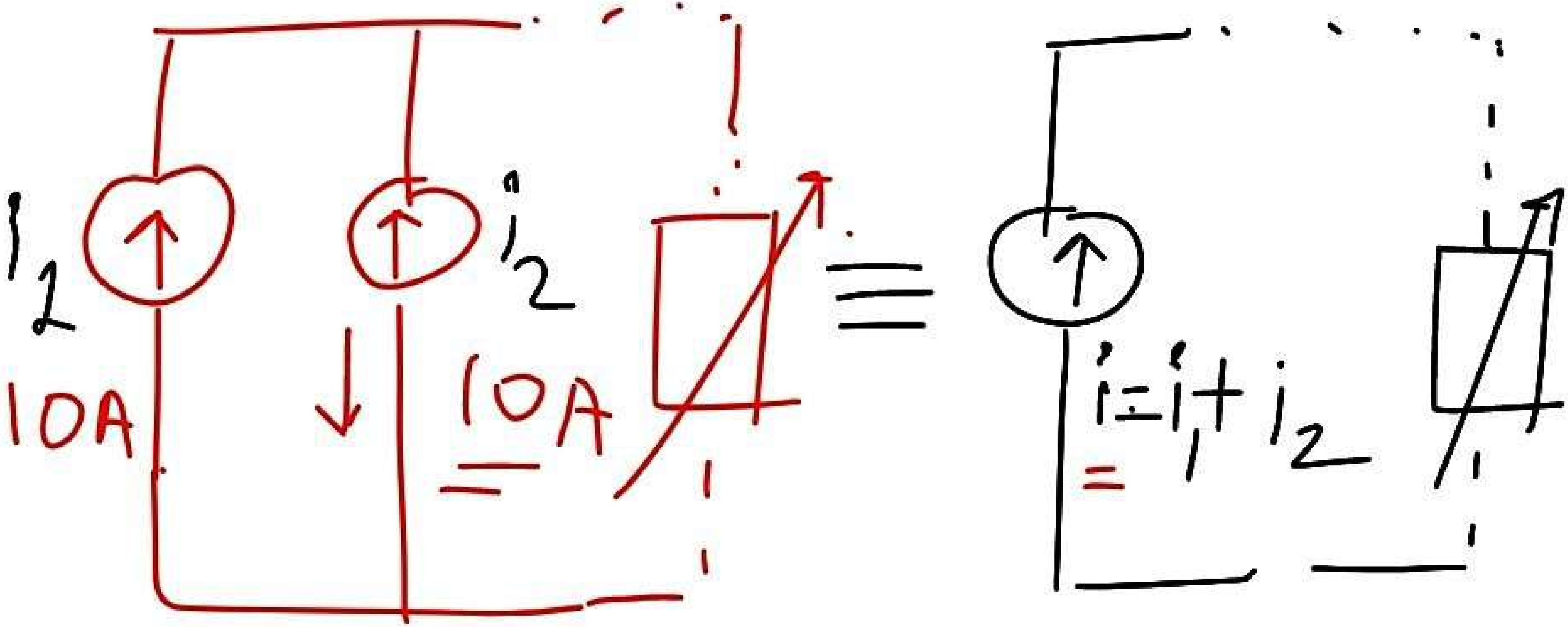
By KCL $\Rightarrow i_2 + i_1 = 0$ $\quad i_1 = i_2$



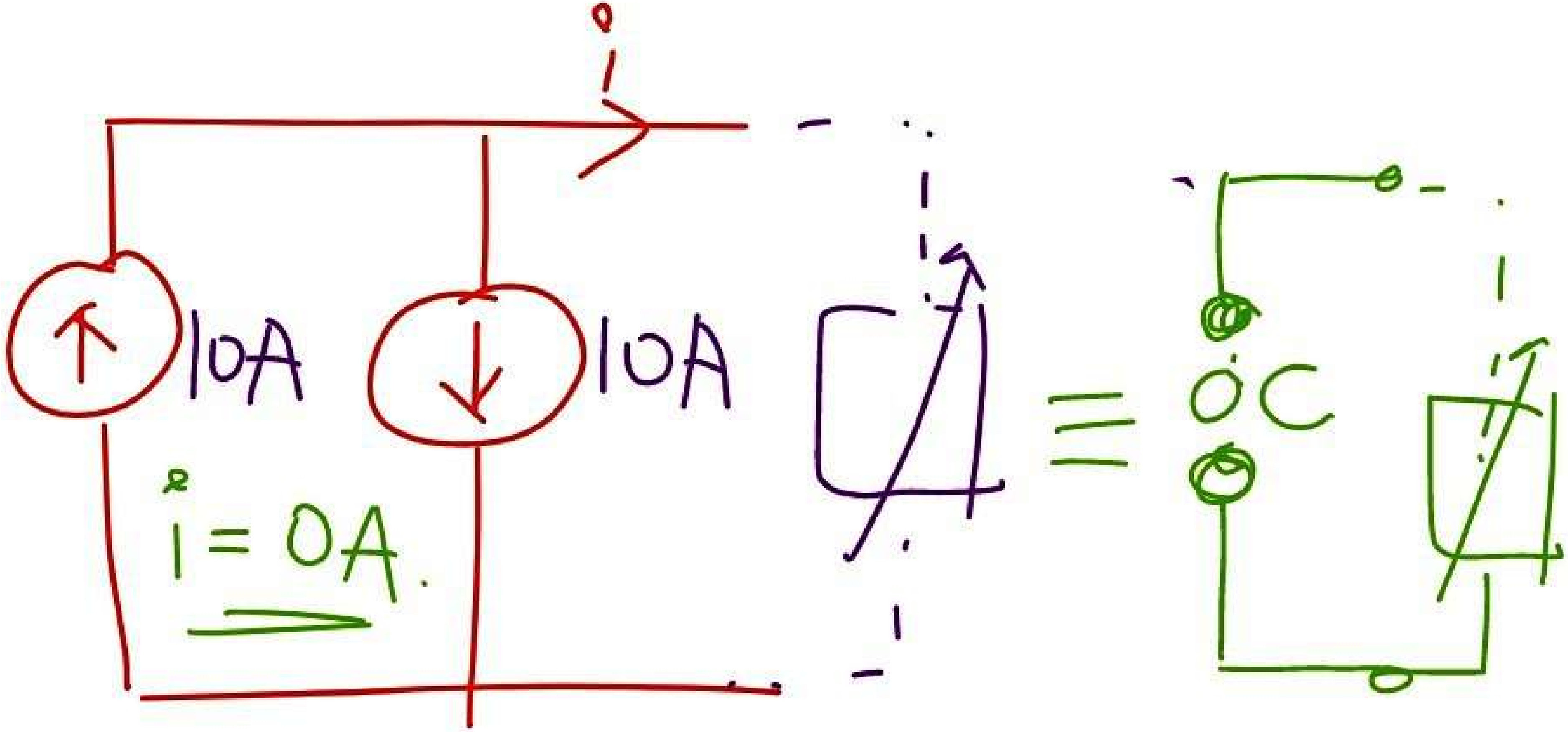
By KVL $V_1 - V_2 = 0 \quad \therefore \; \underline{\underline{V_1 = V_2}}$



$10 - 10 + V \Rightarrow \underline{\underline{V = 0}}$

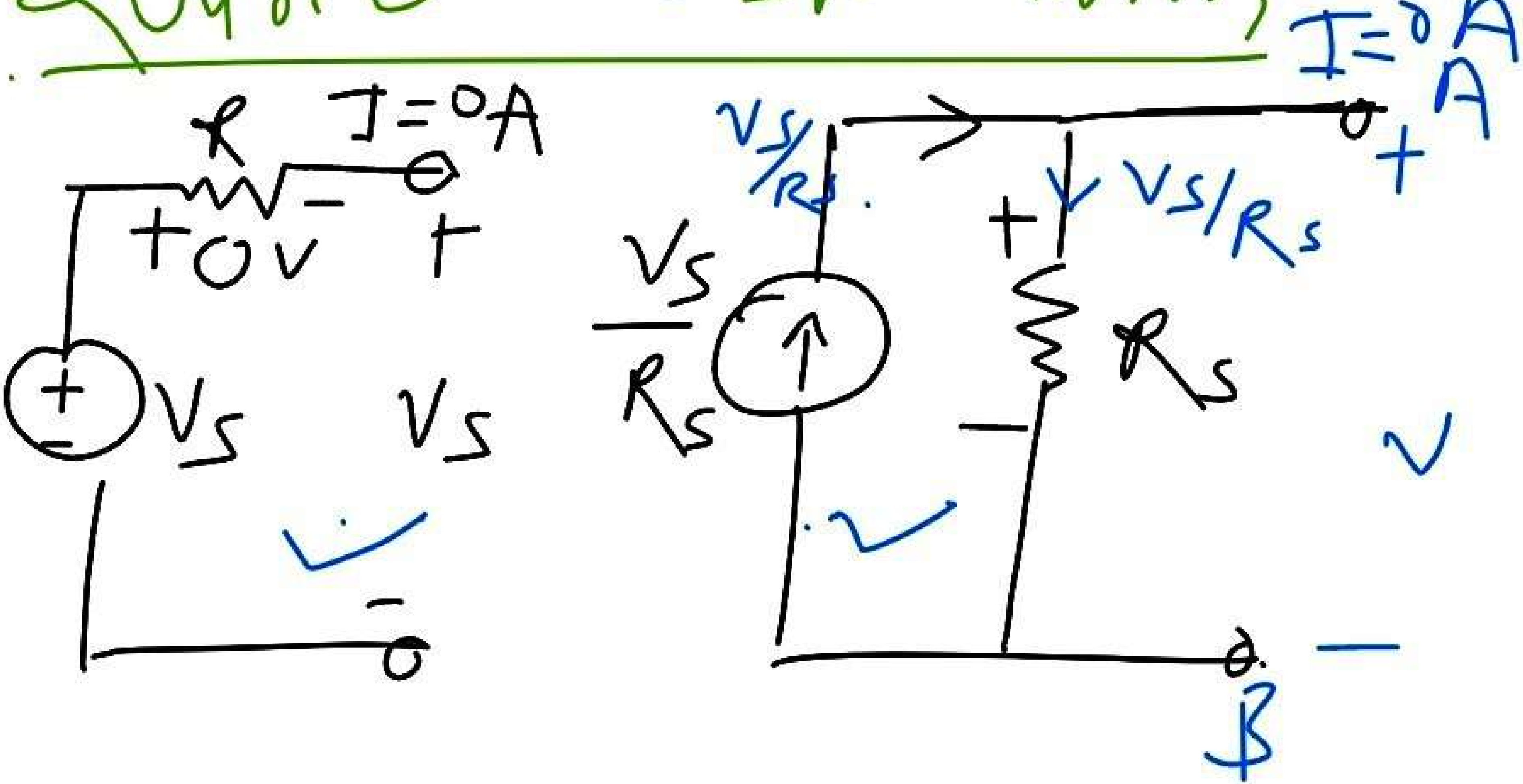


i_1
10A
i_2
10A
$i = i_1 + i_2$



i
10A
10A
i = 0A.
≡ OC

Source Transformation



Power calculations by Tellegen's thm.

Defn → In an arbitrary n/w, the algebric sum of powers at any given instant is zero i.e. power delivered by some element is equal to power absorbed by remaining element present in N/W.

$$\sum_{k=1}^{b} v_k i_k = 0$$

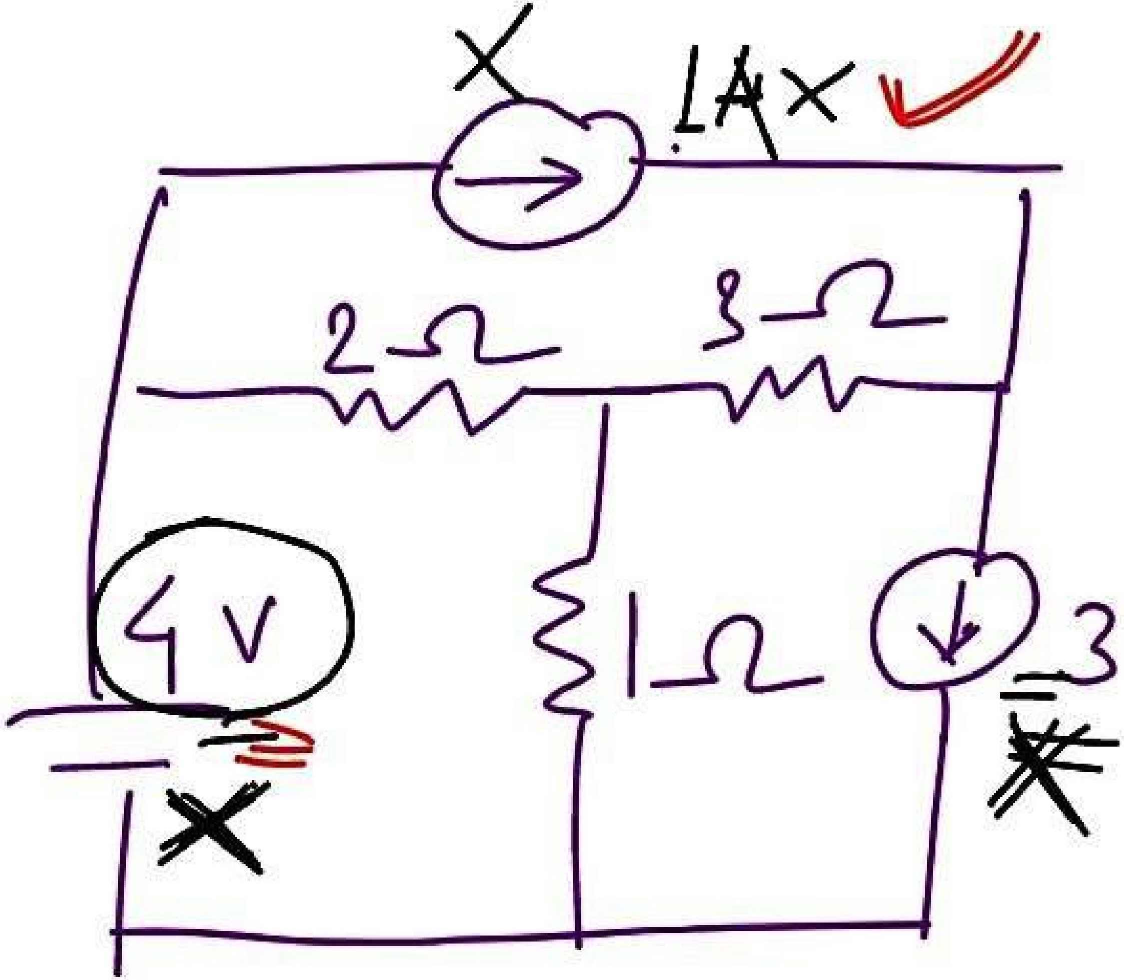
b = Total No. of branches.

→ When current enters at -ve terminal of an element then that element will deliver power, otherwise it will absorb the power.

Superposition Thm :- (Module 3)
(SPT)

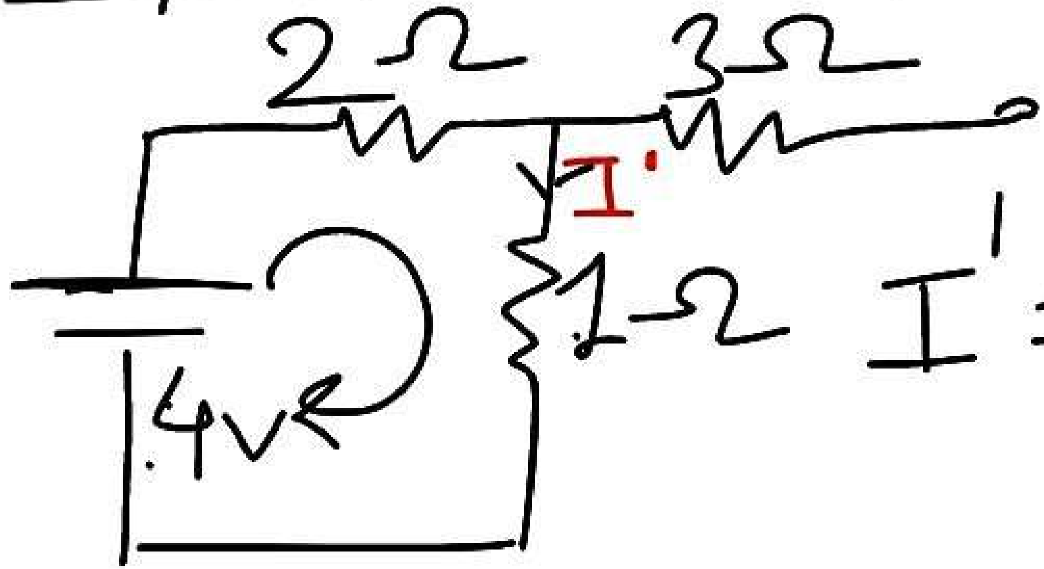
Defn :- In a linear network with several independant sources the response in particular branch when all the sources are acting simultaneously is equal to linear sum of individual responses calculated by taking one independent source at a time.

is equal to linear sum of individual responses calculated by taking one independent source at a time.



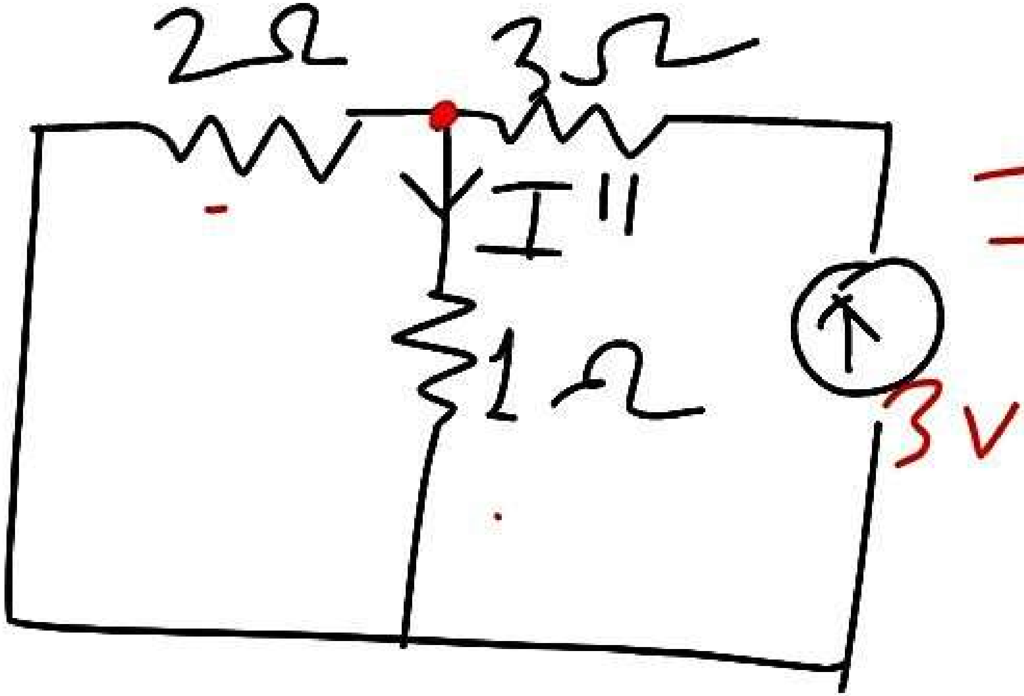
find current in 2Ω resistor using SPT

Step 1 When 4V source is acting.



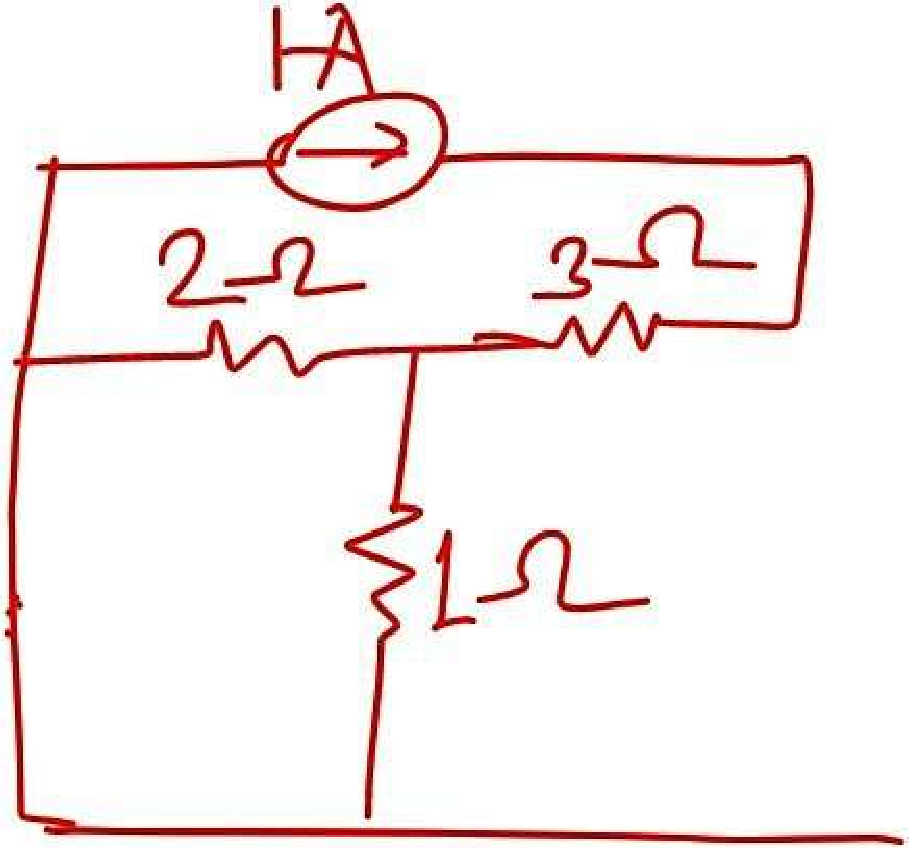
$$I' = \frac{4}{2+1}$$

$$= 1.33 \text{ Amp.}$$

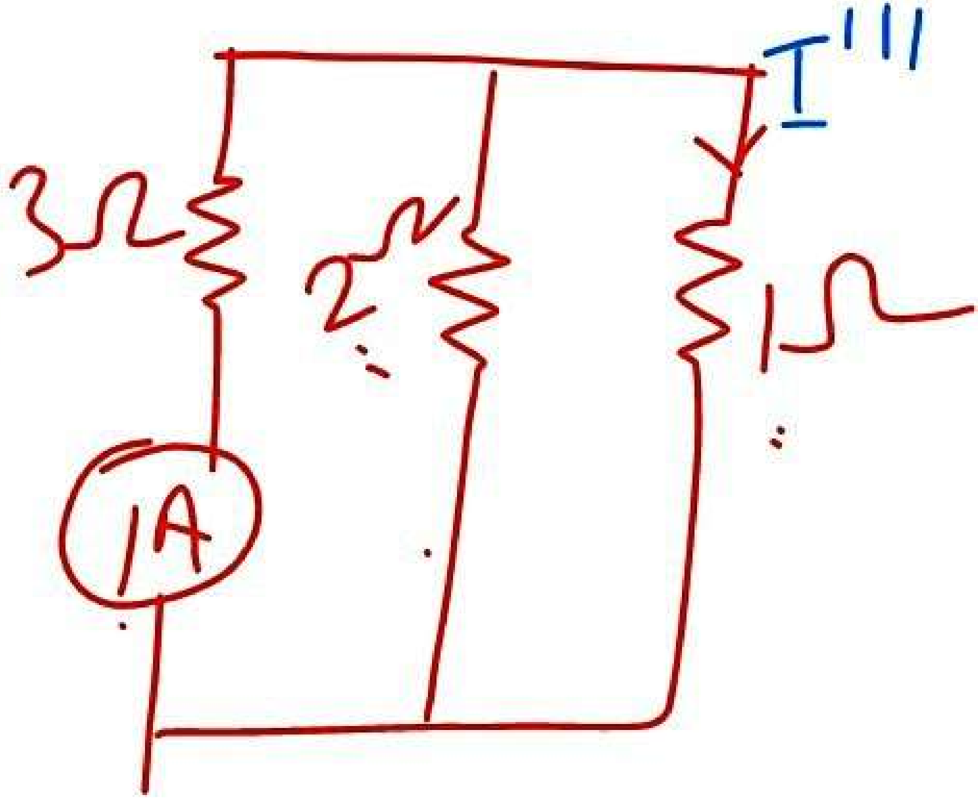


$$I'' = 3 \times \frac{2}{1+2}$$

$$= 2 \text{ Amp.}$$



$\equiv$



By current division formula.

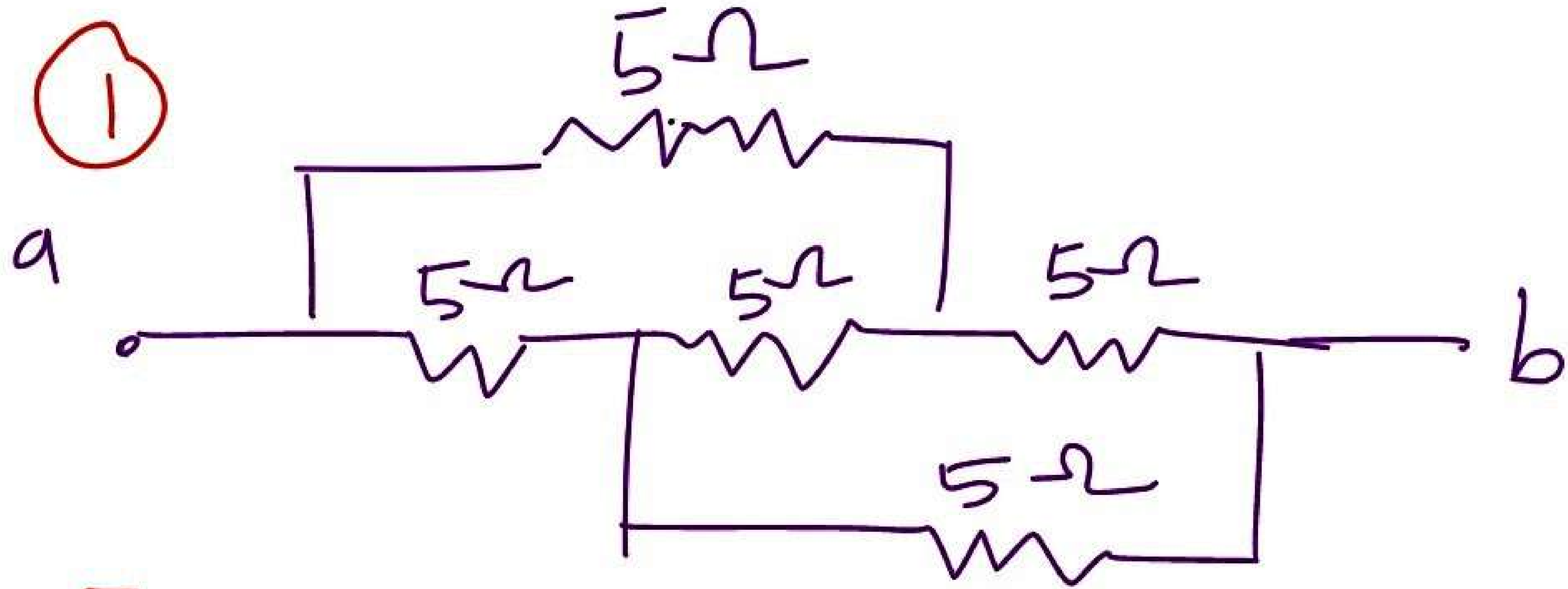
$$I''' = 1 \times \frac{2}{2+1} = 1 \times 2/3 = 0.66 \text{ Amp}$$

By superposition Thm.

$$I = \underline{I' + I'' + I'''} = 1.33 + 2 + 0.66$$

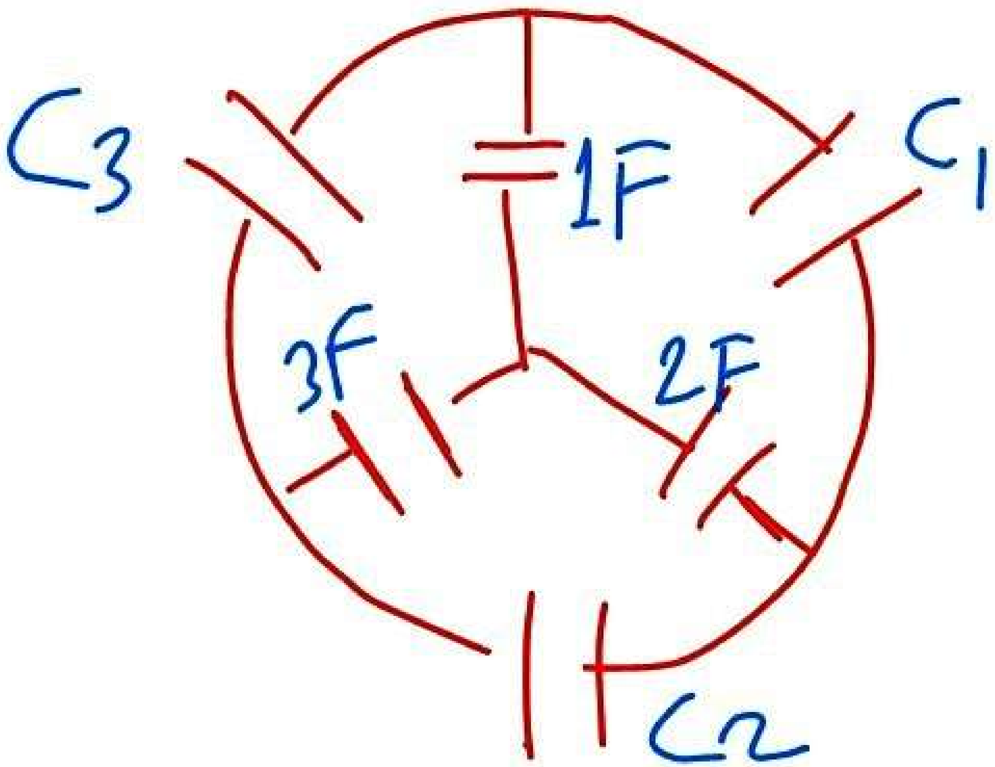
$$= 4 \text{ Amp.}$$

1



Equivalent Resistance across a & b.

② Determine C_1, C_2, C_3



Types of Elements :-

There are ten types of elements (ie five pairs) as follows.

1) Linear & Non linear

2) Active and passive

3) Bilateral and unilateral

4) Distributed and lumped

5) Time invariant and Time varying.

Definitions :-

→ A two terminal element is said to be linear if for all time 't', its characteristics is a straight line through the origin, otherwise it is said to be nonlinear.

→ An element is said to be active if it delivers a net amount of energy to the outside world, otherwise it is said to be passive.

Active elements have the ability to supply energy while passive elements only receive the energy.

→ An element is said to be bilateral if it offers same impedance for the different directions of same current flow. Otherwise it is said to be unilateral.

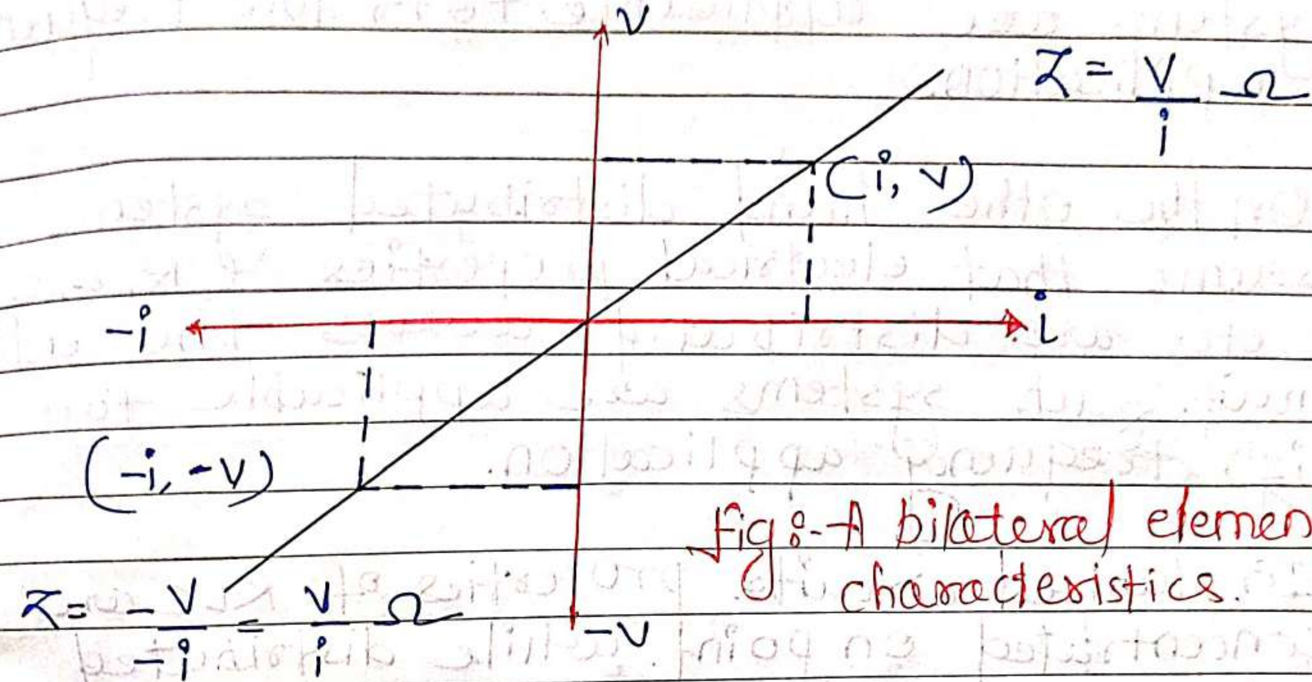
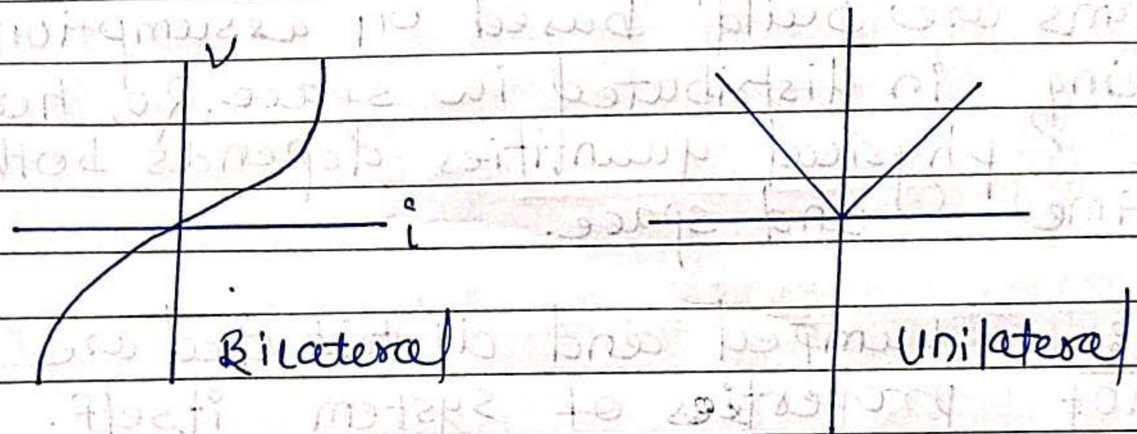


fig:- A bilateral element characteristics.



In other words, for a bilateral elements, if (i, v) is on the characteristics then (-i, -v) must also be on the characteristic then only same voltage for the different directions of same current flow and hence same impedance i.e. for a bilateral element, symmetry w.r.to origin exists.

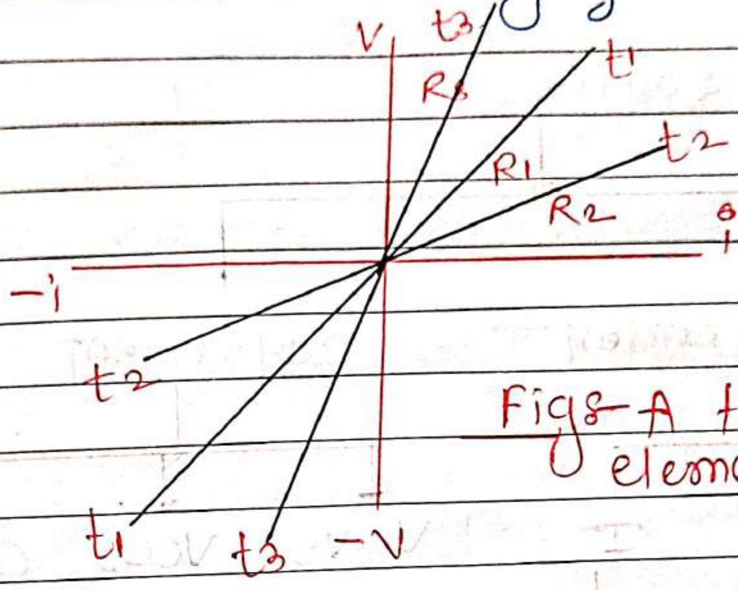
→ Lumped systems are those systems in which electrical properties like R, L, C etc are assumed to be located on a small space of the circuit. These systems are applicable for low frequency application.

→ On the other hand distributed systems assume that electrical properties (R, L, C) etc. are distributed across the entire circuit. such systems are applicable for high frequency application.

→ In lumped circuits. properties of RLC are concentrated on point. while distributed systems are build based on assumption of being in distributed in space. So, here the physical quantities depends both on time and space.

Note:- Lumped and distributed are not properties of system itself. These are related to the size of the system compared to the wavelength of applied voltage & current.

→ An element is said to be time invariant if for all time 't', its characteristics do not change with time, otherwise it is said to be time varying.



Figs- A time varying element characteristics.

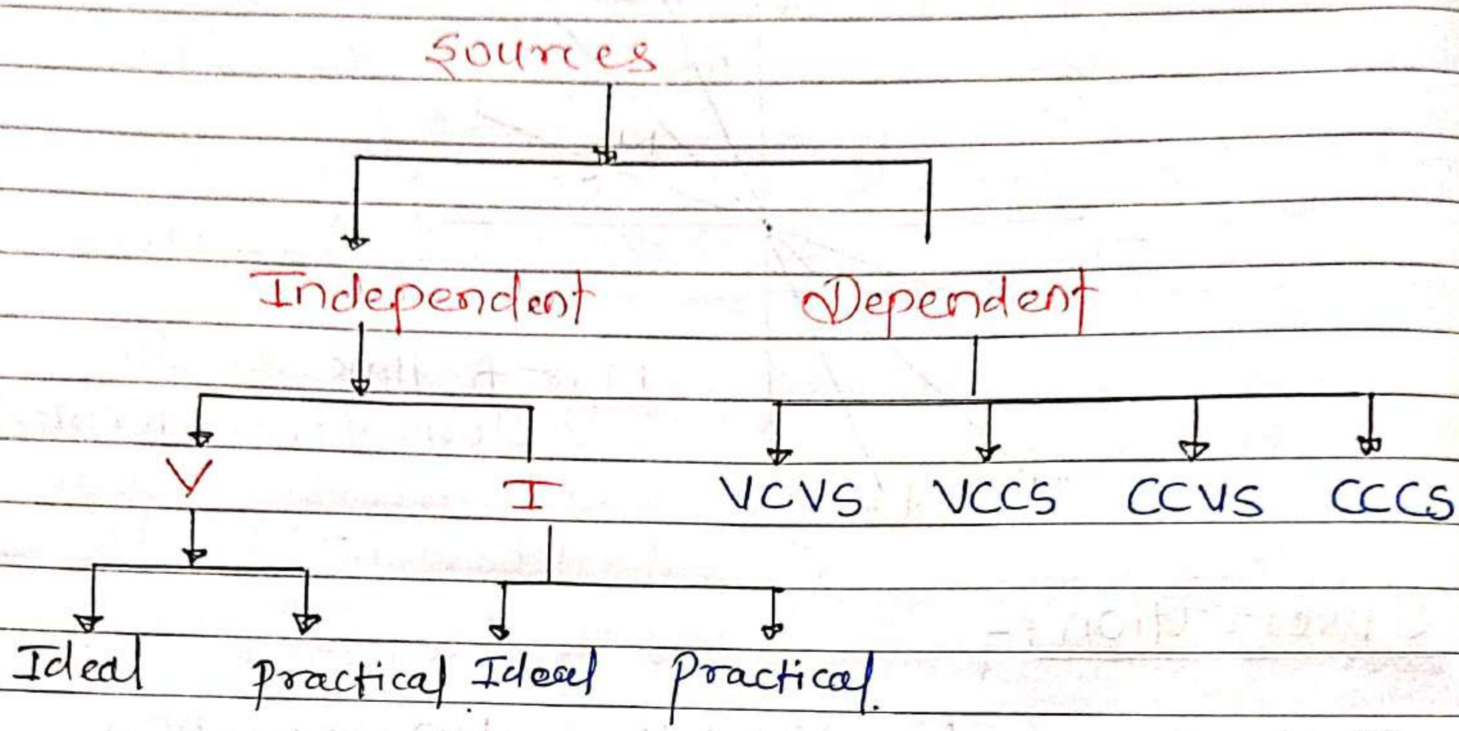
Observation :-

The above characteristics also present a linear, passive and bilateral element.

Note :- The resistors, inductors and capacitors are said to be passive if and only if $R \geq 0$, $L \geq 0$ and $C \geq 0$ (i.e. Nonnegative slop of the $v-i$ characteristics), otherwise they are said to be active i.e. $R < 0$, $L < 0$ & $C < 0$ (ie. Negative slop of the $v-i$ characteristic.

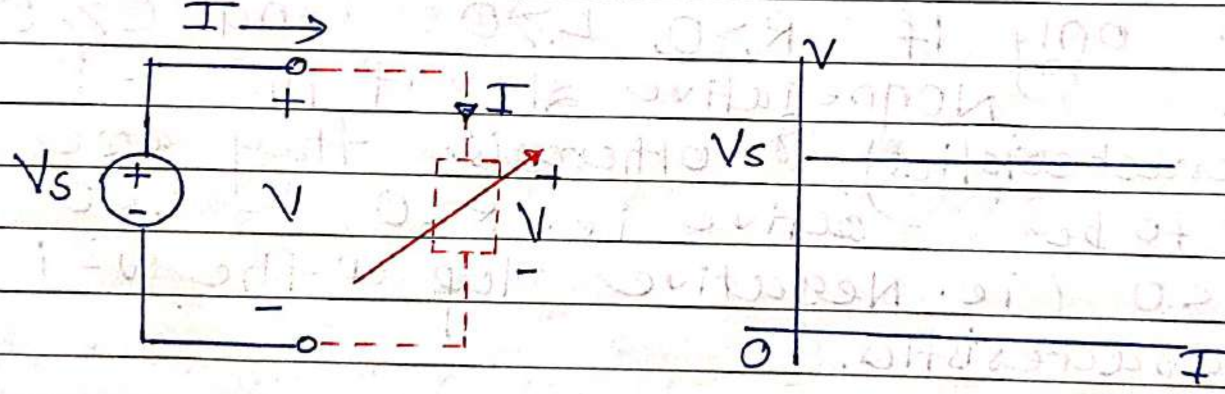
Sources :-

Sources are classified as shown below.



(1) Independent Sources :-

(1) Ideal Voltage Source



Notes:-

→ Energy delivery i.e. current delivery in any source is always from the positive terminal i.e. current flows from negative to positive terminals.

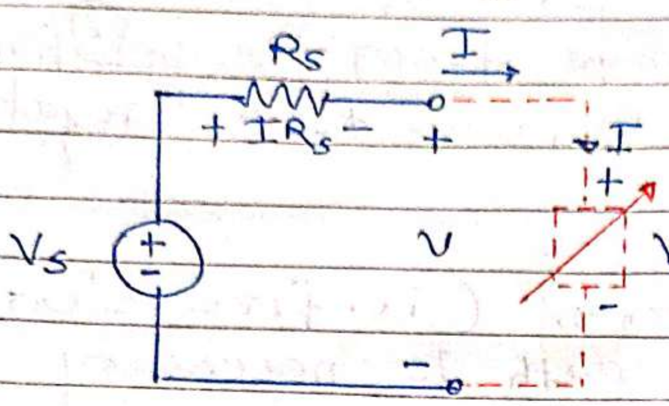
→ for the flow of current (i.e. free electron motion) always closed path is necessary i.e. $V = V_s$ for all I.

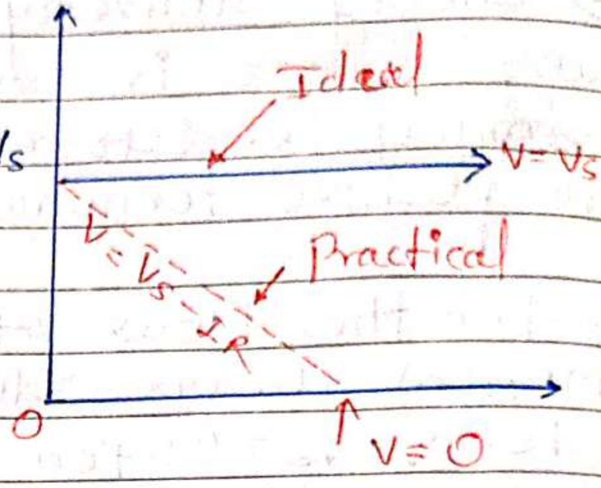
Property :-

→ In an ideal voltage source, load voltage is independent of the load current drawn.

Note:- Inherently all the sources are nonlinear in nature, since their voltage and current relation is non-linear. They are basically active and unilateral elements. DC means time invariant and AC means time varying elements.

② Practical Voltage Source





By KVL $\Rightarrow$ $V_s - IR_s - V = 0$

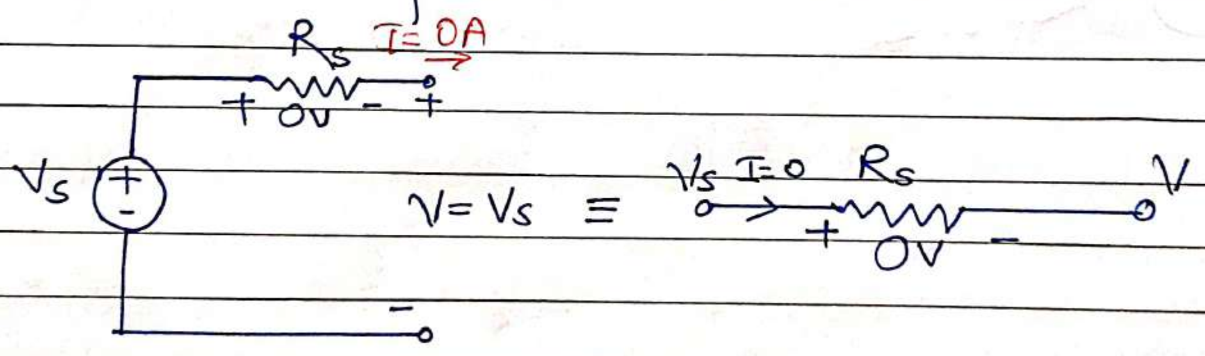
$\Rightarrow V = V_s - IR_s$.

Property :-

In a practical voltage source, load voltage is a function of load current drawn.

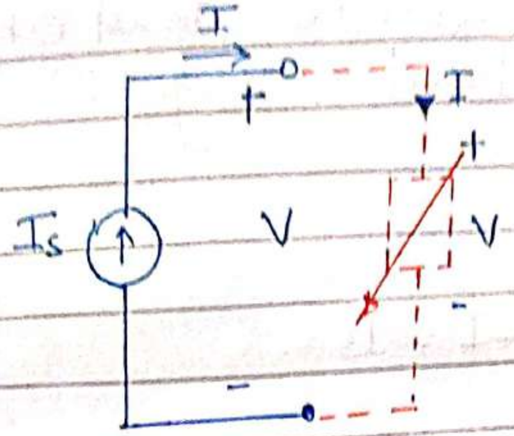
Observation :-

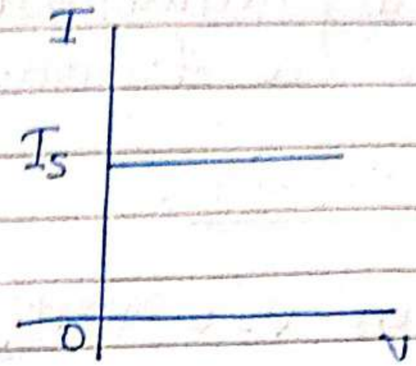
When $I = 0$ Ampere, then $V = V_s$.



Conclusion :- When current through any passive element is zero, then the two end voltages are equal and vice versa.

Ideal Current Source :-

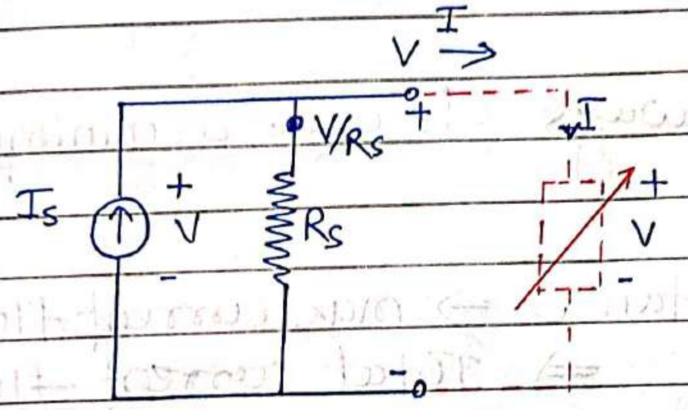


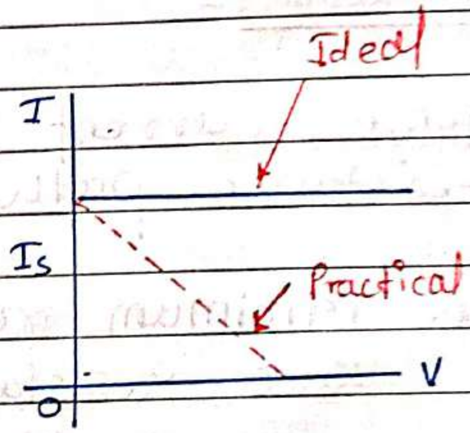


i.e. $I = I_s$ for all V.

Property :- In an ideal current source, load current is independent of load voltage.

Practical Current Sources :-





$$\text{By KCL} \Rightarrow -I_s + \frac{V}{R_s} + I = 0$$

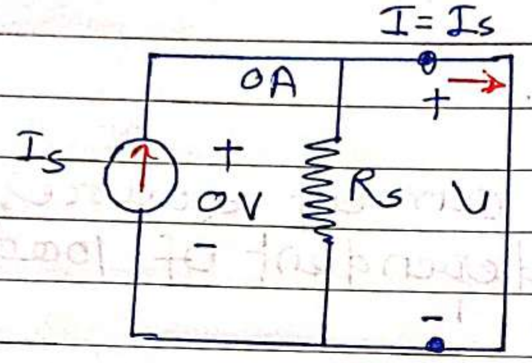
$$\Rightarrow I = I_s - \frac{V}{R_s}$$

Property :-

In a practical current source, load current is a function of load voltage.

Observation :

When $V = 0$ volt, then $I = I_s$



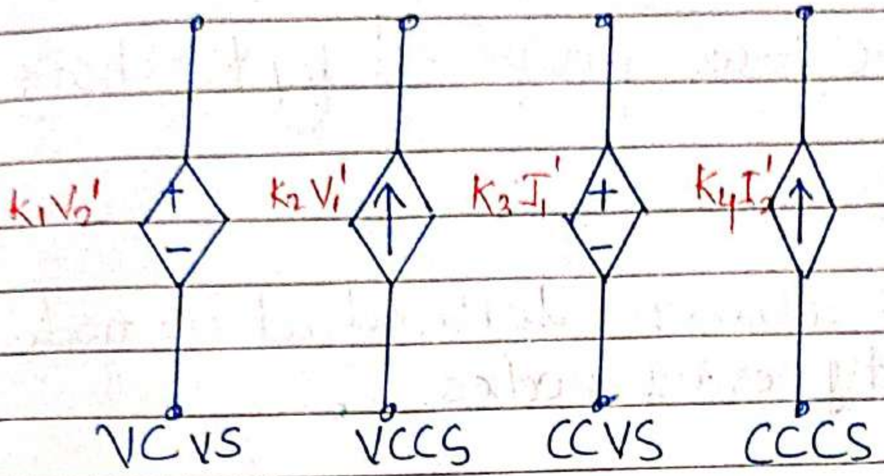
$\Rightarrow V = 0$ volt.

Conclusion :-

Hence, current always chooses a minimum resistance path.

i.e. Minimum resistance $\Rightarrow$ max current flows
Zero Resistance $\Rightarrow$ Total current flows.

Dependent OR controlled Sources :-



Linear controlled sources (with respect to the controlled variable, not w. r. to the voltage & current relation.

→ With respect to controlled variable only, the dependent source are said to be linear, active & bilateral. The presence of these elements makes the network a linear, an active and a bilateral.

→ Controlled sources are said to be sources i.e. active elements in the presence of least one independent source, then only controlled variables are non zero and hence their magnitudes are non zero.

Kirchoff's Laws :-

There are two laws proposed by Kirchoff's

KCL and KVL.

① KCL :- It is always defined at a node.
There are two types of nodes.

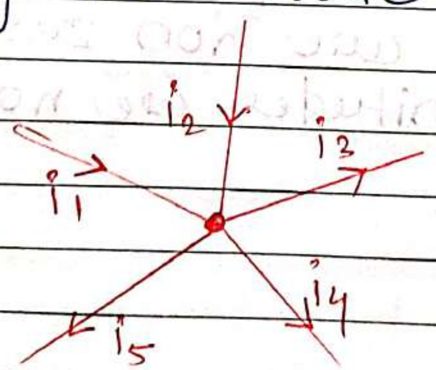
(a) Simple Node :-
It is an inter connection of only two branches.

(b) Principle Node :-

It is an interconnection of atleast three branches.

Definition :-

"In a lumped electric circuit for any of its nodes and at any time 't', the algebric sum of branch currents leaving the node is zero."



By KCL $\Rightarrow$ Σ currents $= 0$

$$\Rightarrow \boxed{-i_1 - i_2 + i_3 + i_4 + i_5 = 0}$$

$$i_1 + i_2 = i_3 + i_4 + i_5$$

i.e. Sum of entering currents is equal to sum of leaving currents.

Since

$$i = \frac{dq}{dt} = \frac{dq_1}{dt} + \frac{dq_2}{dt} = \frac{dq_3}{dt} + \frac{dq_4}{dt} + \frac{dq_5}{dt}$$

At a given time 't', $\frac{d}{dt}$ is same for all branches.

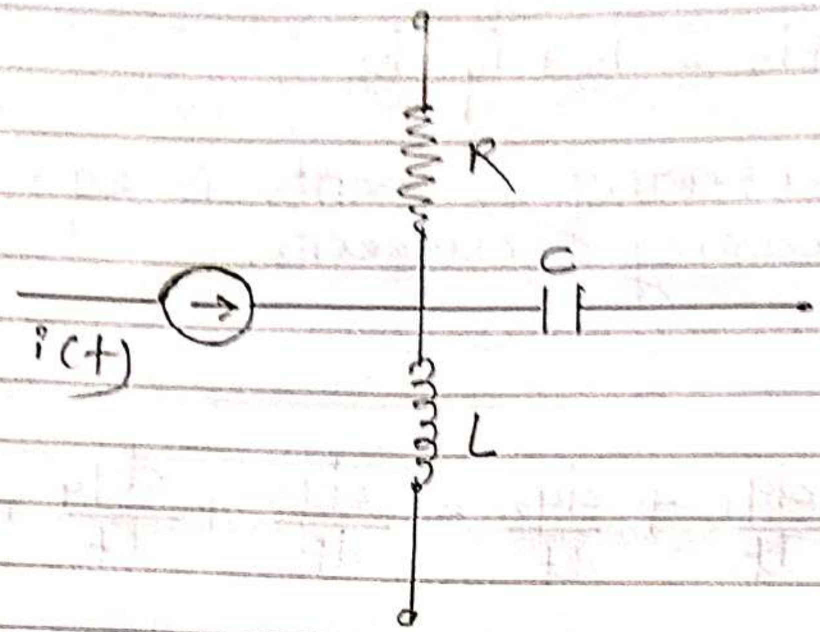
$$\Rightarrow q_1 + q_2 = q_3 + q_4 + q_5$$

i.e. sum of entering charges is equal to sum of leaving charges.

Properties :-

→ KCL applies to any lumped electric circuit, it does not matter whether the elements are linear, non-linear, active, passive, time varying, time invariant etc. ie. KCL is independent of the nature of the elements connected to the node.

Ex.

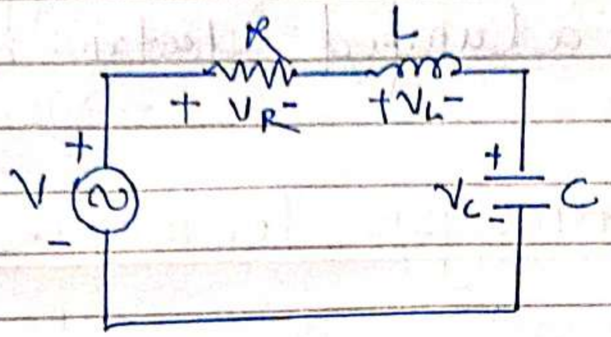


→ Here R, L, C are linear, passive, bilateral time invariant elements and i(t) is a non linear, active, unilateral, time varying element.

→ Since there is no accumulation of charge at any node, KCL expresses conservation of charge at each and every node in a lumped electric circuit.

KVL :-

"In a lumped electric circuit, for any of its loops and at any time 't' the algebraic sum of branch voltages around the loop is zero.



$Z_R = R\ \Omega$

$Z_L = j\omega L\ \Omega$

$Z_C = \frac{1}{j\omega C}\ \Omega$

By KVL $\Rightarrow$ Σ Branch voltages $= 0$

$$V - V_R - V_L - V_C = 0$$

$$\boxed{V = V_R + V_L + V_C}$$

Since $V = \frac{W}{q}$ and $i = \frac{dq}{dt}$ in series, current is same and hence charge flow is same through all the elements.

$$\frac{W}{q} = \frac{W_R}{q} + \frac{W_L}{q} + \frac{W_C}{q}$$

$$\boxed{W = W_R + W_L + W_q}$$

Properties :-

→ KVL is independent of nature of the elements (linear, non-linear, active, passive etc) present in a loop.

→ KVL expresses conservation of energy in every loop of a lumped electric circuit

Nodal and Mesh Analysis Techniques :-

→ KCL + Ohm's law = Nodal Analysis

→ KVL + Ohm's law = Mesh Analysis.

→ The above techniques are network solving procedures, valid only for lumped electric circuits (where KCL & KVL are valid) and that too at a constant tempratures (where Ohm's law is valid)

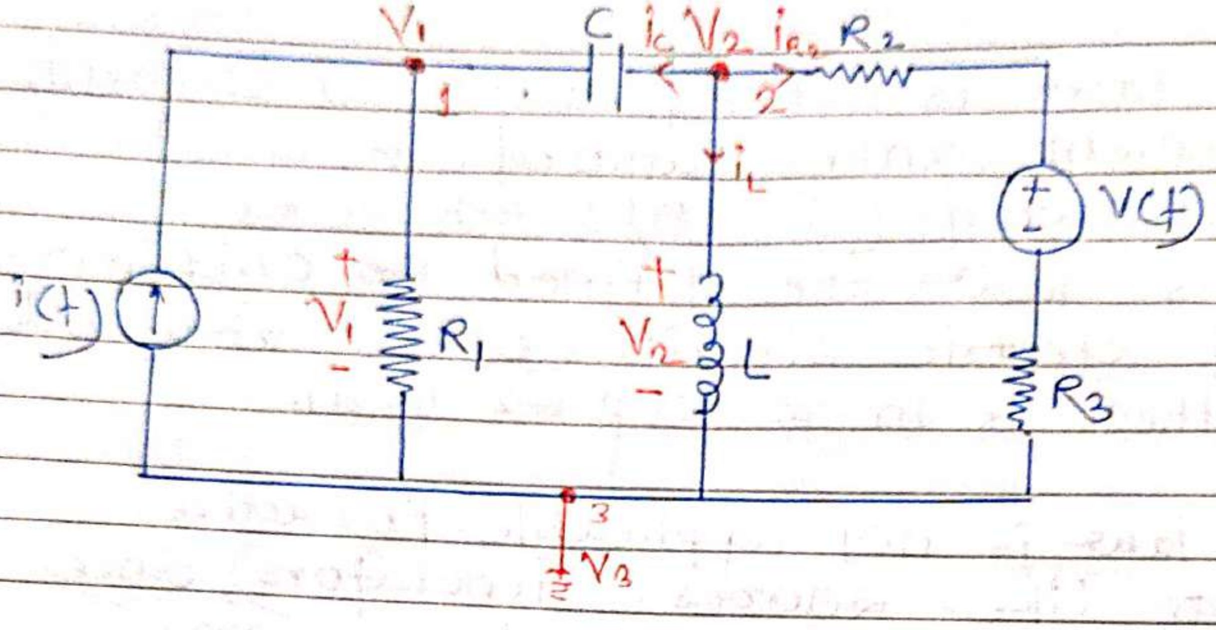
→ Nodal & Mesh procedures are independent of each other, since KCL & KVL are independent of each other.

→ Ohm's law is a function of nature of the elements (eight different forms of Ohm's law), where as Kirchoff's laws are independent of nature of the elements.

→ Ohm's law is defined across an element, that element canbe lumped (R, L, C) or distributed ($J = \sigma E$), whereas Kirchoff's laws are defined to only for lumped electric circuits (one is at a node and another is in a loop or mesh.

→ Ohm's law is not applicable for active elements like sources (Generators), since the voltage & current relation is non linear. It is applicable only for linear passive elements like R, L, C where the voltage and current relation is linear.

Nodal Analysis :-



Steps :-

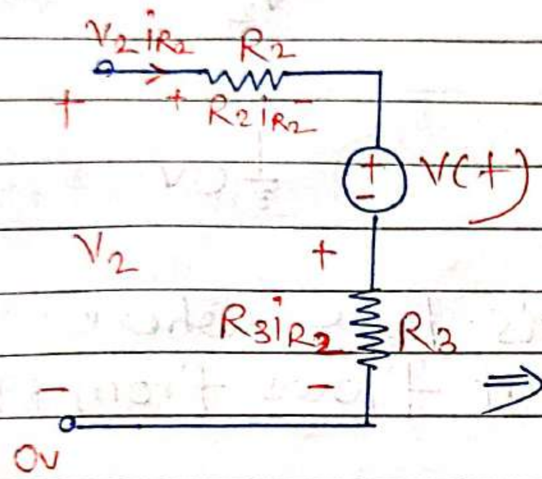
1. Identify number of nodes.
2. Assign node voltages with respect to the ground node, whose voltage is always equal to zero.
3. By using KCL first and ohm's law next write the nodal equations.

Node 2 :- $\begin{bmatrix} V_2 > V_1 \\ V_2 > 0 \\ V_2 > V(t) \end{bmatrix}$

$i_C + i_L + i_R = 0$ (By KCL)

$$C\frac{d}{dt}(V_2 - V_1) + \frac{1}{L}\int_{-\infty}^{t} V_2\,dt + \frac{V_2 - V(t)}{R_2 + R_3} = 0$$

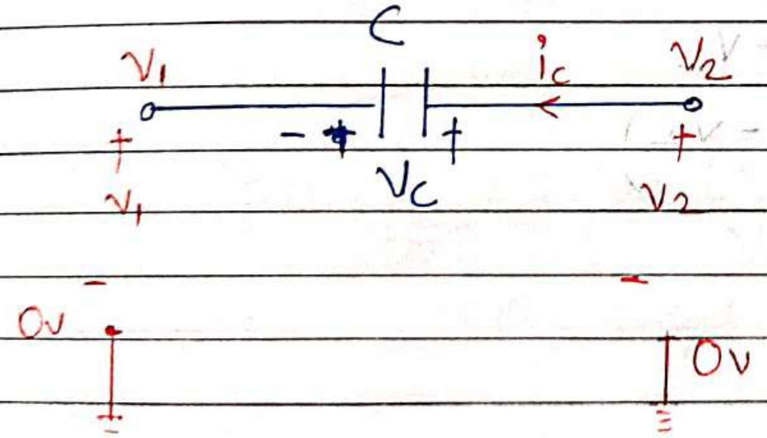
(By ohm's law)



By KVL

$$V_2 - R_2 i_{R_2} - V(t) - R_3 i_{R_2} = 0$$

$$\Rightarrow i_{R_2} = \frac{V_2 - V(t)}{R_2 + R_3} \text{ Ampere}$$



$i_C = C\frac{dV_C}{dt}$ (By Ohm's law, where current flows from + to −)

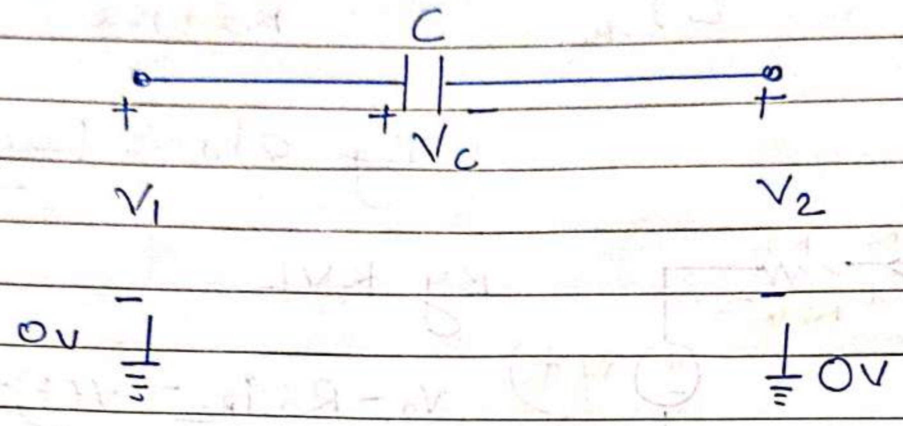
By KVL $\Rightarrow$ $V_1 + V_C - V_2 = 0$

$$V_C = V_2 - V_1 \Rightarrow i_C = C\frac{d}{dt}(V_2 - V_1)$$

Node 1 :- $\begin{bmatrix} V_1 > 0 \\ V_1 > V_2 \end{bmatrix}$

Nodal $\Rightarrow -i(t) + \frac{V_1}{R_1} + C\frac{d}{dt}(V_1 - V_2) = 0$

(By KCL and Ohm's laws simultaneously)



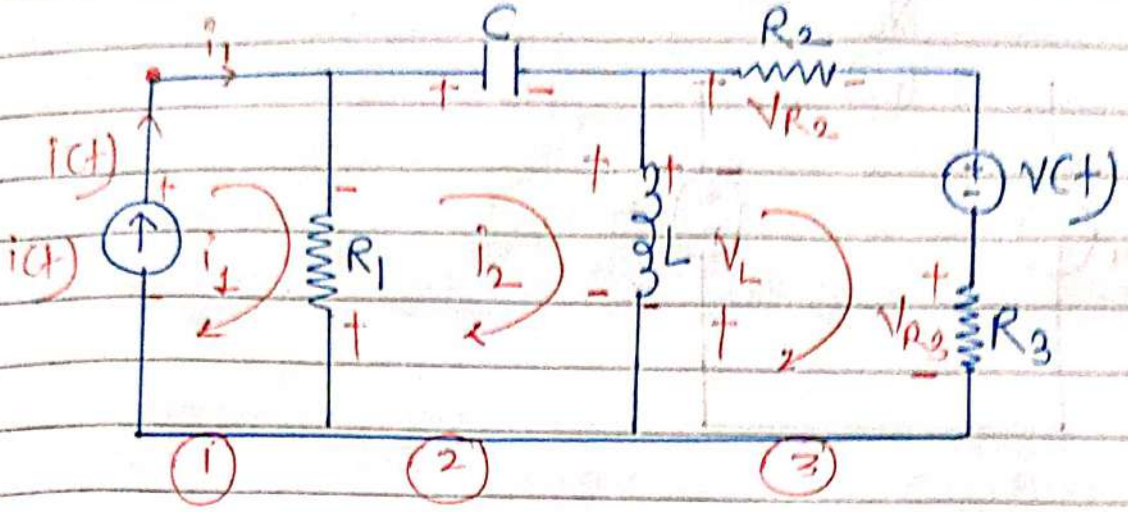
$i_c = C\frac{dV_c}{dt}$ (By ohm's law, where current flows from + to -)

By KVL $\Rightarrow V_1 - V_c - V_2 = 0$

$V_c = V_1 - V_2$

so $i_c = C\frac{d}{dt}(V_1 - V_2)$

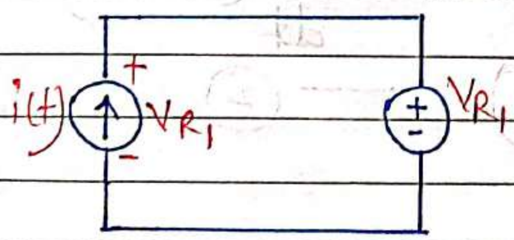
Mesh Analysis:-



Steps:-

1. Identify number of Meshes.
2. Assign mesh currents in the clockwise direction.
3. By using KVL first and Ohm's Law next write the mesh equations.

Mesh 1 :- $\begin{bmatrix} i_1 > i_2 \\ i_1 > i_3 \end{bmatrix}$



$V_{R_1} = R_1(i_1 - i_2)$

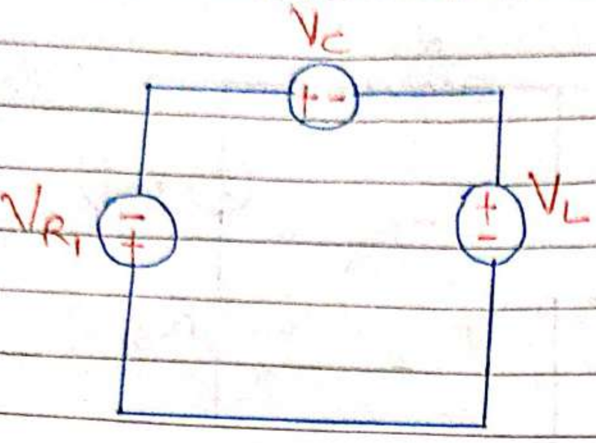
By KVL $\Rightarrow V_{R_1} - V_{R_1} = 0$

$0 = 0$

So, $i_1 = i(t) \rightarrow (1)$ (As both currents are in the same direction)

By KCL $\Rightarrow -i(t) + i_1 = 0 \Rightarrow i_1 = i(t)$ — (1)

Mesh (2) :- $\begin{bmatrix} i_2 > i_3 \\ i_2 > i_1 \end{bmatrix}$



$$V_C = \frac{1}{C}\int_{-\infty}^{t} i_2 \, dt$$

$$V_L = L\frac{d}{dt}(i_2 - i_3)$$

$$V_{R_1} = R_1(i_2 - i_1)$$

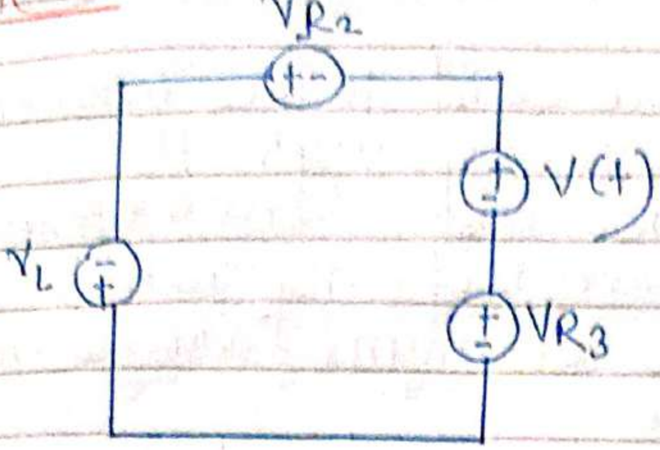
$$-V_{R_1} - V_C - V_L = 0$$

(By KVL)

$$-R_1(i_2 - i_1) - \frac{1}{C}\int_{-\infty}^{t} i_2 \, dt - L\frac{d}{dt}(i_2 - i_3) = 0$$

(By ohm's law) —— (2)

Mesh (3) [$i_3 > i_2$ & $i_3 > i_1$



Note :- In a single branch, the branch current is equal to mesh current and in a common branch, the branch current is equal to the difference of mesh currents.

$$V_{R2} = R_2 i_3$$

$$V_{R3} = R_3 i_3$$

$$V_L = L \frac{d}{dt}(i_3 - i_2)$$

$$-V_L - V_{R2} - V(t) - V_{R3} = 0 \quad \text{(By KVL)}$$

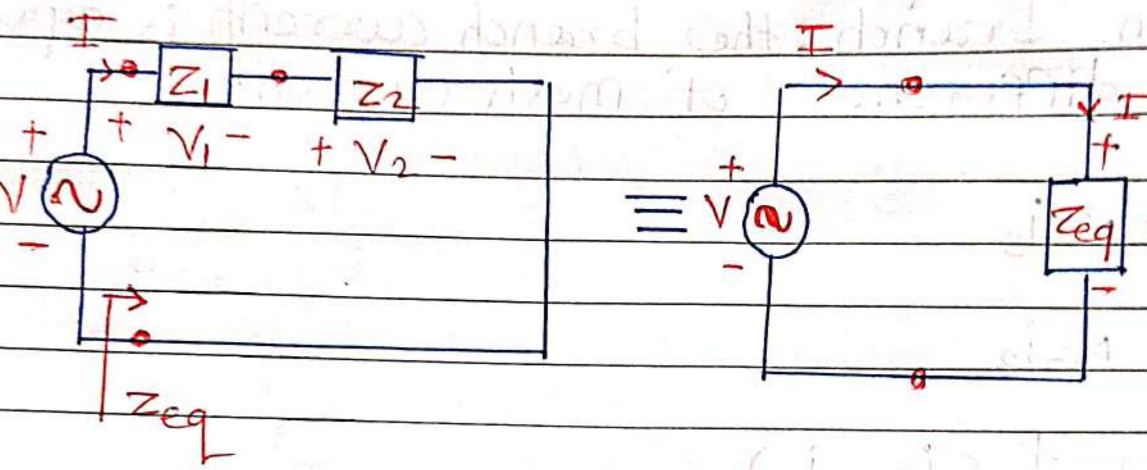
$$-L\frac{d}{dt}(i_3 - i_2) - R_2 i_3 - V(t) - R_3 i_3 = 0$$

(By ohm's law) —— (3)

Equivalent Circuits w.r.to Passive R, L, C

→ Two elements are said to be in series only when currents through the elements are same and two elements are said to be in parallel only when voltages across the elements are same.

→ Impedances in series and admittances in parallel we can add.



$Z = Z_R = R\,\Omega$

$Z_L = j\omega L\,\Omega$

$Z_C = \dfrac{1}{j\omega C}\,\Omega$

$Z_{eq} = Z_1 + Z_2$

R : $R_{eq} = R_1 + R_2$

L : $L_{eq} = L_1 + L_2$

C : $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

If $C_1 = C_2 = C$

$C_{eq} = \dfrac{C}{2}$

Voltage Division Principle:-

$V = Z_{eq} \cdot I$ (By Ohm's law)

$$I = \frac{V}{Z_{eq}} = \frac{V}{Z_1 + Z_2}$$

$V_1 = Z_1 I$ (By Ohm's law)

$$\boxed{V_1 = \frac{V \cdot Z_1}{Z_1 + Z_2}}$$

$V_2 = Z_2 \cdot I$ (By Ohm's law)

$$\boxed{V_2 = \frac{V \cdot Z_2}{Z_1 + Z_2}}$$

Check :-

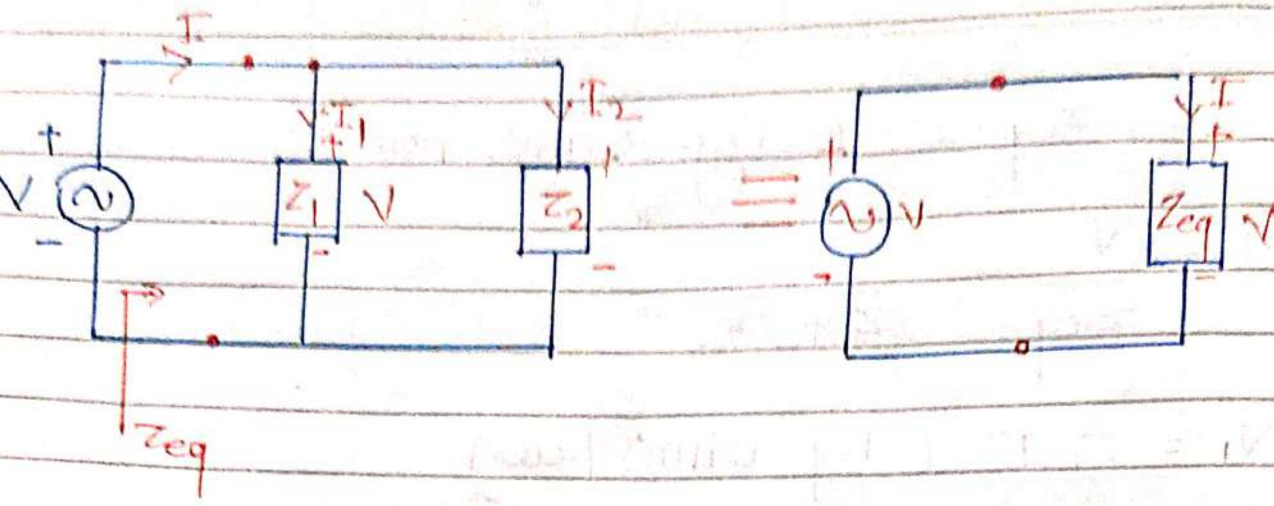
By KVL $\Rightarrow$ $V - V_1 - V_2 = 0$

$V = V_1 + V_2$

R : $V_1 = \dfrac{V \cdot R_1}{R_1 + R_2}$; $V_2 = \dfrac{V \cdot R_2}{R_1 + R_2}$

L : $V_1 = \dfrac{V L_1}{L_1 + L_2}$; $V_2 = \dfrac{V L_2}{L_1 + L_2}$

C : $V_1 = \dfrac{V C_1}{C_1 + C_2}$; $V_2 = \dfrac{V C_1}{C_1 + C_2}$



$$Y_{eq} = Y_1 + Y_2$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} \Rightarrow Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

R: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$; $R_1 = R_2 = R$

$$R_{eq} = \frac{R}{2}$$

L: $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

C: $C_{eq} = C_1 + C_2$

Current Division Principle :-

$V = Z_{eq} \cdot I$ (By Ohm's law)

$$\Rightarrow V = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \cdot I$$

$$I_1 = \frac{V}{Z_1} \text{ (By Ohm's law)}$$

$$I_1 = \frac{I \cdot Z_2}{Z_1 + Z_2}$$

$$I_2 = \frac{V}{Z_2} \text{ (By Ohm's law)}$$

$$I_2 = \frac{I \cdot Z_1}{Z_1 + Z_2}$$

Check :- By KCL $\Rightarrow -I + I_1 + I_2 = 0$

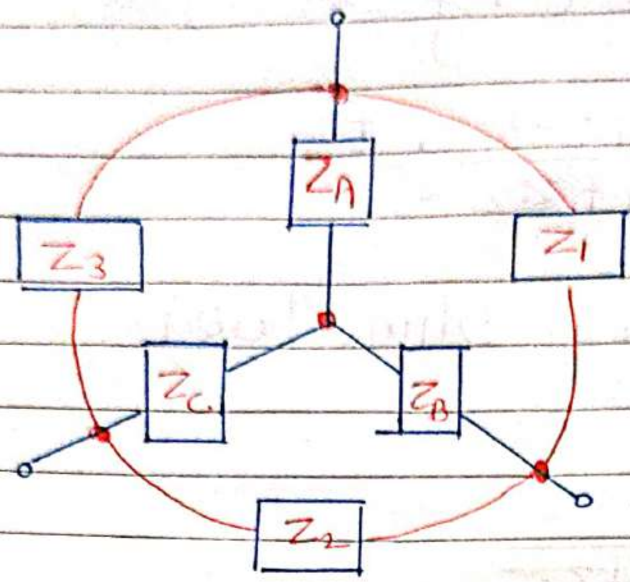
$\Rightarrow I_1 + I_2 = I$

R : $I_1 = \frac{IR_2}{R_1 + R_2}$; $I_2 = \frac{IR_1}{R_1 + R_2}$

L : $I_1 = \frac{I \cdot L_2}{L_1 + L_2}$; $I_2 = \frac{IL_1}{L_1 + L_2}$

C : $I_1 = \frac{I \cdot C_1}{C_1 + C_2}$; $I_2 = \frac{I \cdot C_2}{C_1 + C_2}$

Star (Y) - Delta (Δ) Transformation



Let Δ- Network be given i.e. Z_1, Z_2 & Z_3 are known,

$$Z_A = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} = \left(\frac{\text{Product}}{\text{Total}}\right)$$

$$Z_B = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z_C = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

Observation

IF $Z_1 = Z_2 = Z_3 = Z$, then $Z_A = Z_B = Z_C = Z/3$

So, for the balanced case, Δ to Y transformation will reduce the impedance by 3 times.

Let Y - Network be given i.e. Z_A, Z_B and Z_C are known

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A \cdot Z_B}{Z_C}\right)$$

$$= \text{Sum} + \left(\frac{\text{Their Product}}{\text{Remaining}}\right)$$

$$Z_2 = Z_B + Z_C + \left(\frac{Z_B \cdot Z_C}{Z_A}\right)$$

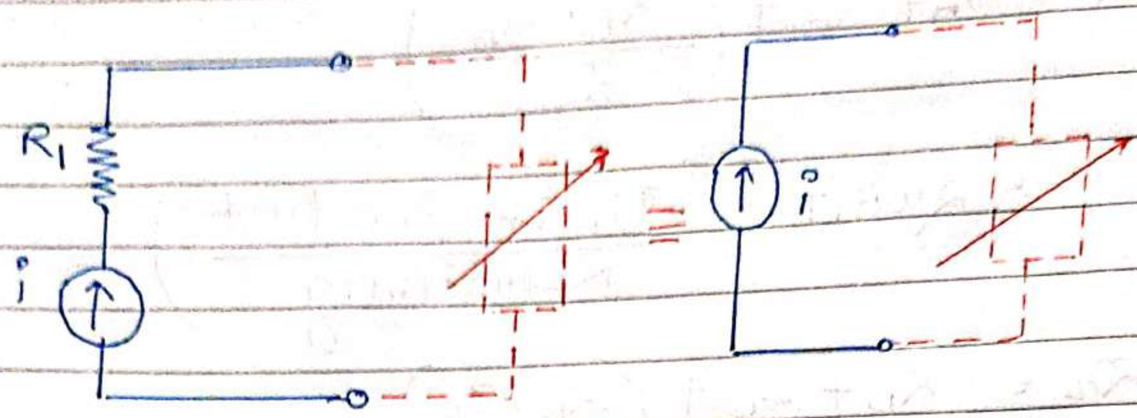
$$Z_3 = Z_C + Z_A + \left(\frac{Z_C \cdot Z_A}{Z_B}\right)$$

Observation :-

If $Z_A = Z_B = Z_C = Z$ then $Z_1 = Z_2 = Z_3 = 3Z$

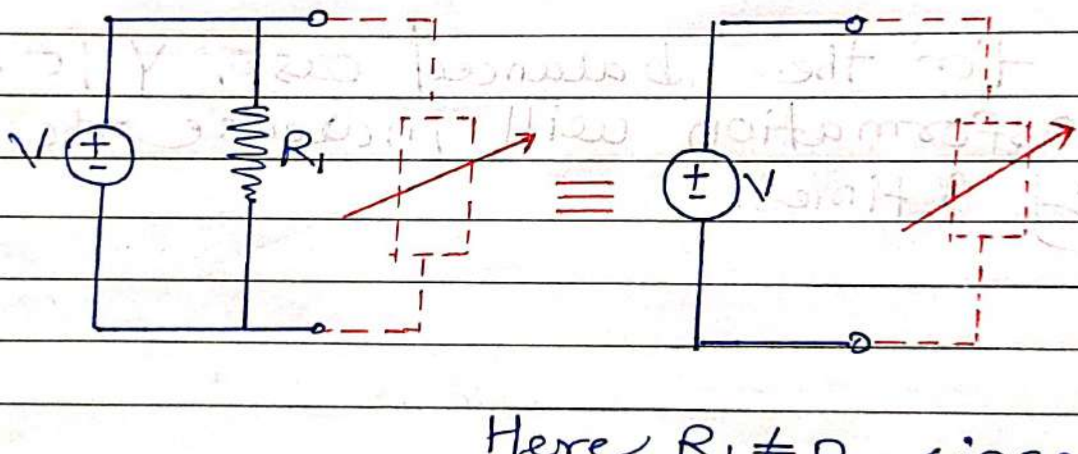
So for the balanced case, Y to Δ transformation will increase the impedance by 3 times.

Equivalent Circuits with respect to Active Sources :-



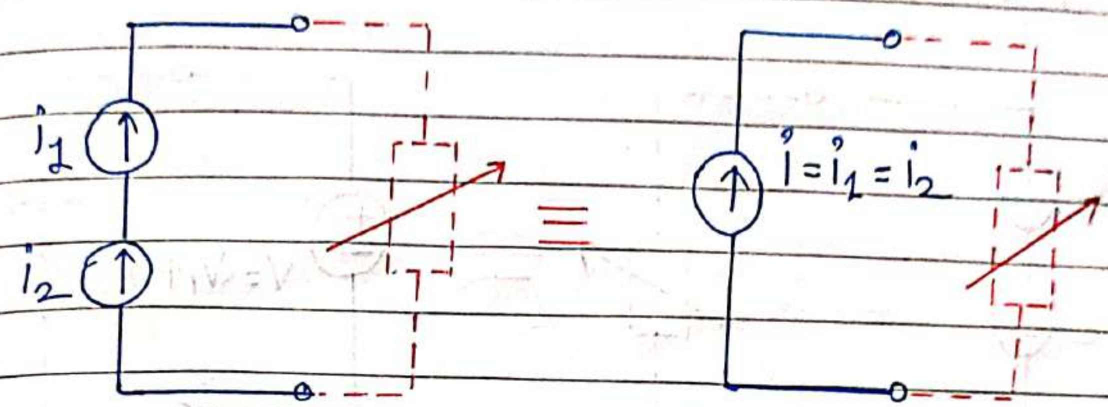
Here $R_1 \neq \infty$
since violation of KCL

So, a resistor in series with an ideal current source canbe neglected in the analysis ie. the load current is independent of R_1 value. We cannot omit this resistor in power calculations since $i^2 R_1 \neq 0$

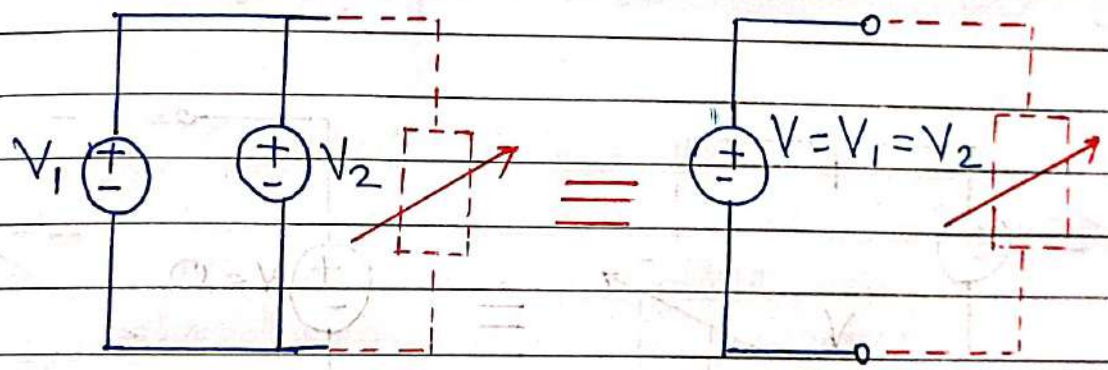


Here $R_1 \neq 0$, since violation of KVL so, a resistor in parallel with an ideal voltage source can be neglected in the analysis i.e. the load voltage is independent of R_1 value. We cannot omit this resistor in power calculations

Since $\frac{V^2}{R_1} \neq 0$



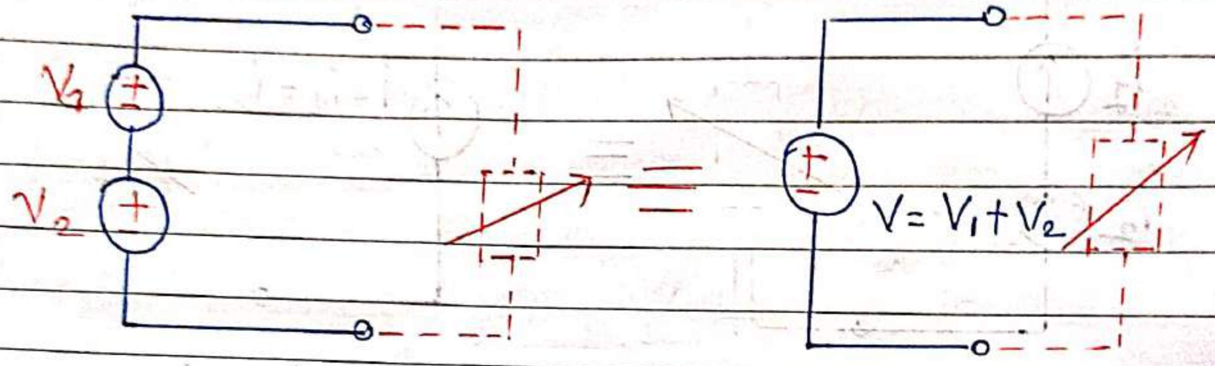
By KCL $\Rightarrow -i_2 + i_1 = 0 \Rightarrow i_1 = i_2$



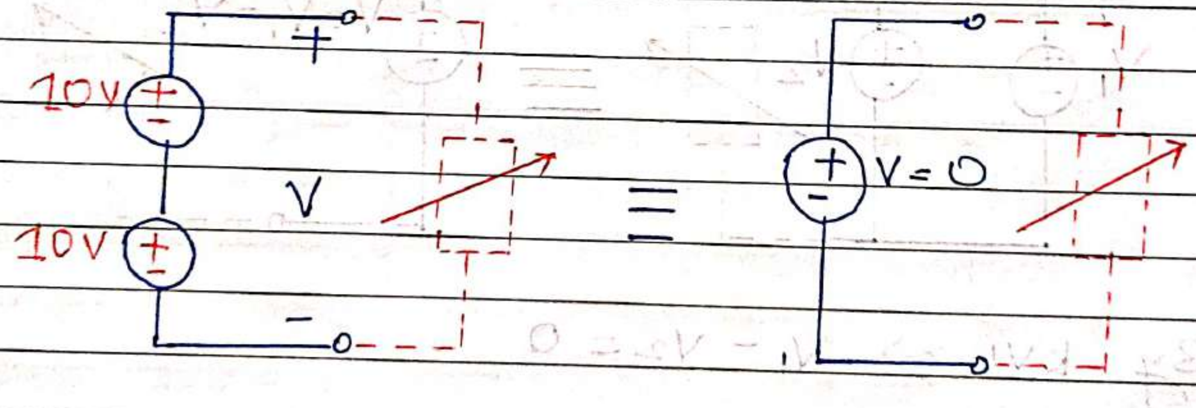
By KVL $\Rightarrow V_1 - V_2 = 0$

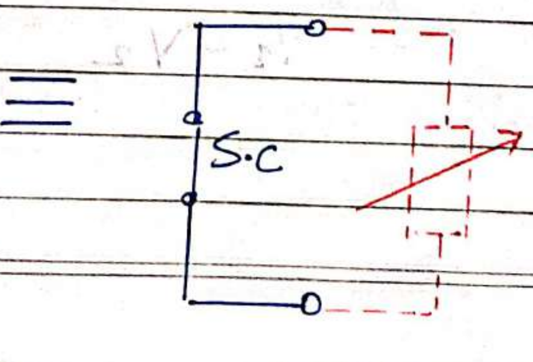
$V_1 = V_2$

So, two ideal current sources are connected in series only when their magnitudes are equal, otherwise violation of KCL, which results in the unstability of the system and hence, physical connections is not possible ie. the circuits does not exist. Similarly, for parallel connection of two ideal voltage sources, violation of KVL.



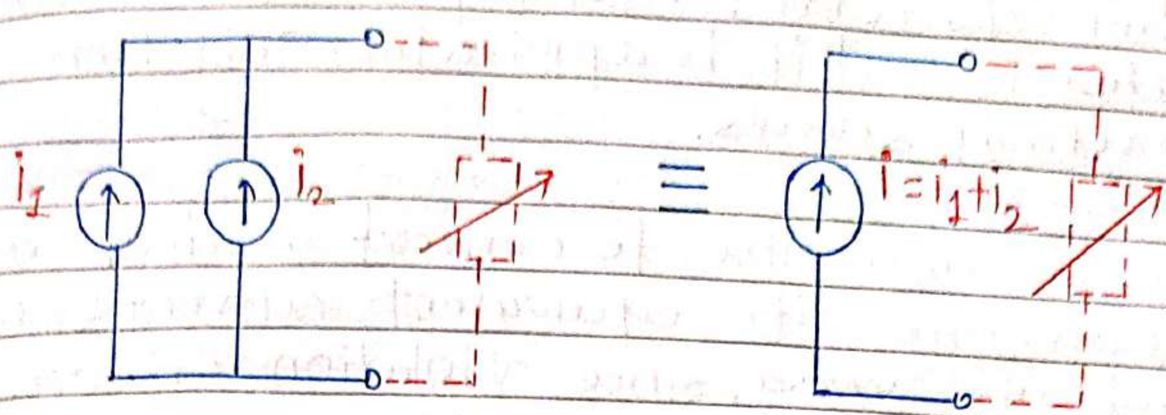
By KVL $\Rightarrow V_2 + V_1 - V = 0 \Rightarrow V = V_1 + V_2$



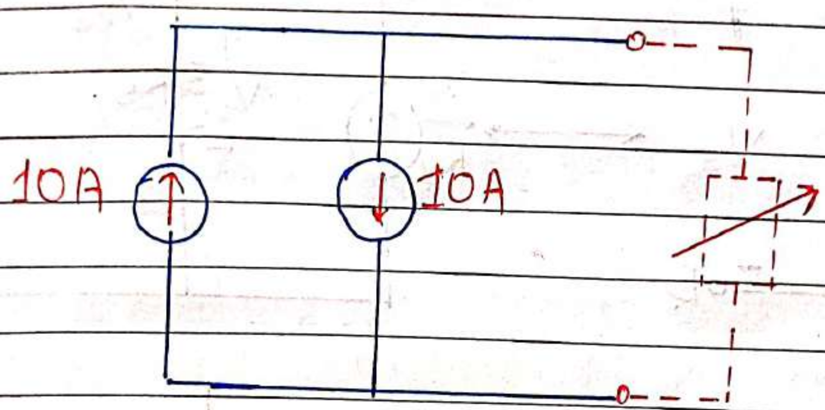


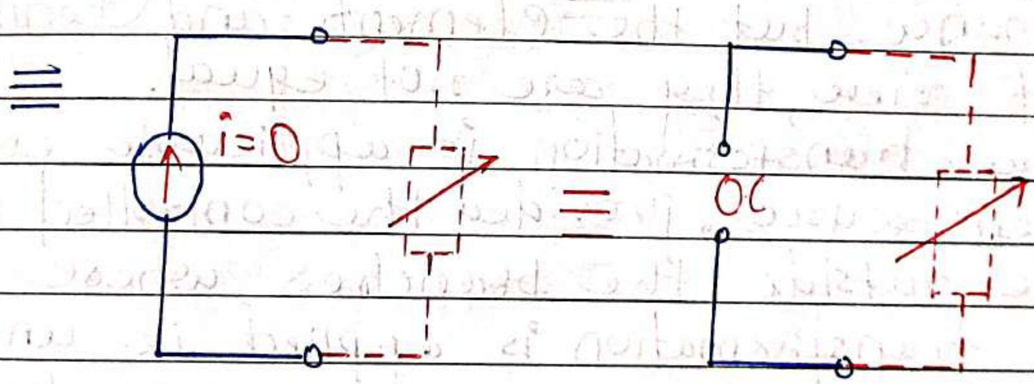
By KVL $\Rightarrow 10-10-V=0$

$\Rightarrow V=0 \Rightarrow$ S.C.



By KCL $\Rightarrow -i_1-i_2+i=0 \Rightarrow i=i_1+i_2$



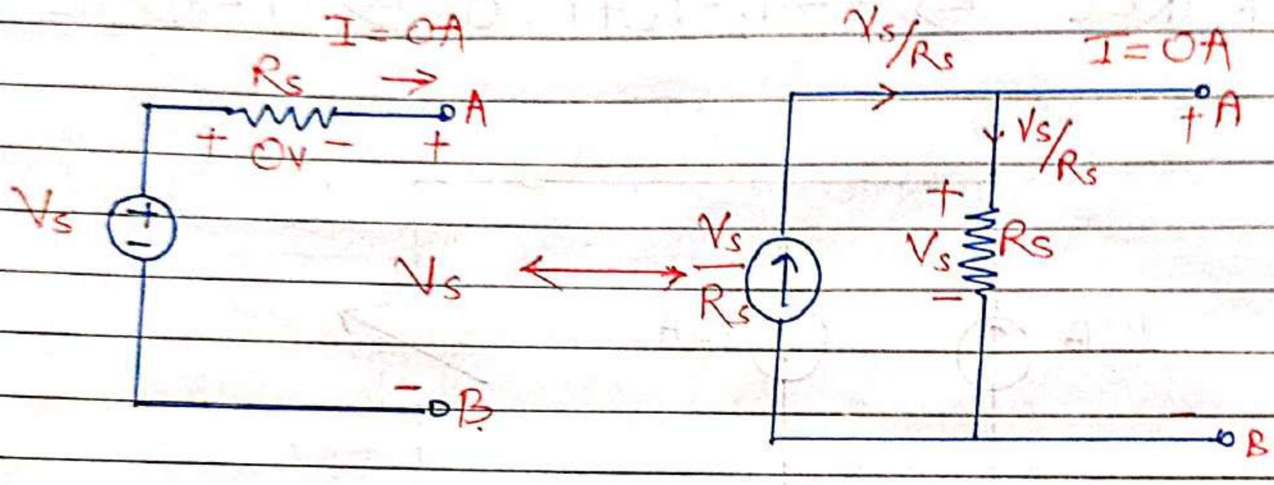


By KCL $\Rightarrow -10+10+i=0 \Rightarrow i=0 \Rightarrow$ OC.

Source Transformation Techniques:-

→ It is a simplification technique, which eliminates extra nodes present in the network and it is applicable only for practical sources.

→ It is impossible to convert an ideal voltage source into its equivalent current source and vice versa, since violation of KVL and KCL (as there is no resistance in the circuit)



→ The above circuit are equal only w.r.to performance, but the elements and connections point of view, they are not equal.

→ Source transformation is applicable even for dependent sources, provided the controlled variable must be outside the branches, where the source transformation is applied i.e. under any circumstances, controlled variable branch should not be disturbed, otherwise the controlled source magnitude will effect.

Power Calculations By Tellegen's Theorem:-

Definition:-

→ In an arbitrary network, algebraic sum of powers at any given instant is zero. ie. the power delivered by some elements is equal to the power absorbed by remaining elements present in the network.

Mathematically $\sum_{k=1}^{b} V_k i_k = 0$

Where b = total number of branches.

→ When current enters at the −ve terminal of an element, then that element will deliver the power, otherwise it will absorb the power.

→ Sources can deliver power or they can absorb power. (In order to protect KCL & KVL in the network ie. to ensure the stability of the network). Whereas the passive elements will always absorb power, since current will enter at the positive terminal in the respective R, L, C (as the sources are present there.)

Properties:-

→ This theorem depends on the voltage & current product in an element, but not on the type of element (Linear, non linear, active, passive etc) i.e. Independent of the nature of the element (Like KCL & KVL)

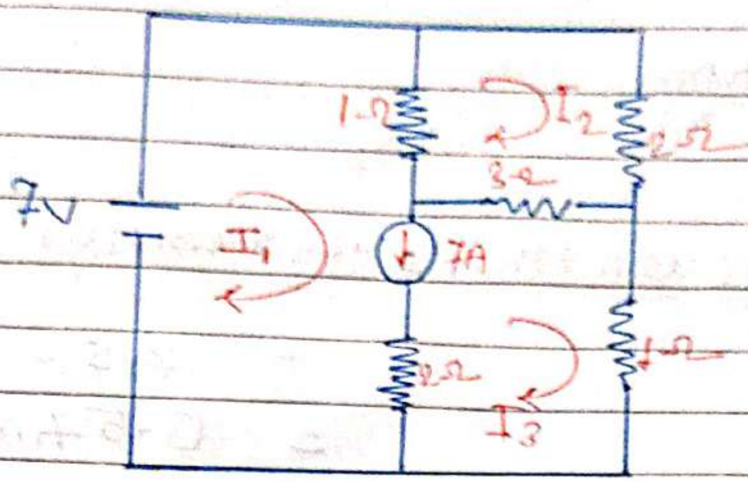
→ Tellegen's theorem expresses conservation of power (energy) in every lumped electric circuit.

Note:-

While verifying Tellegen's theorem do not disturb the original network given for evaluating voltages and currents in each and every elements of the network

(i.e. Source transformation technique is not applied on the network)

Que (1) Find the current in 3-Ω Resistor.



→ Mesh (1) & (3) will form a supermesh writing current equation for supermesh

$$I_1 - I_3 = 7 \quad \text{——— (1)}$$

Applying KVL to the Other path of the supermesh

$$7 - 1(I_1 - I_2) - 3(I_3 - I_2) - I_3 = 0$$

$$-I_1 + 4I_2 - 4I_3 = 7 \quad \text{——— (2)}$$

Applying KVL to Mesh (2)

$$-1(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

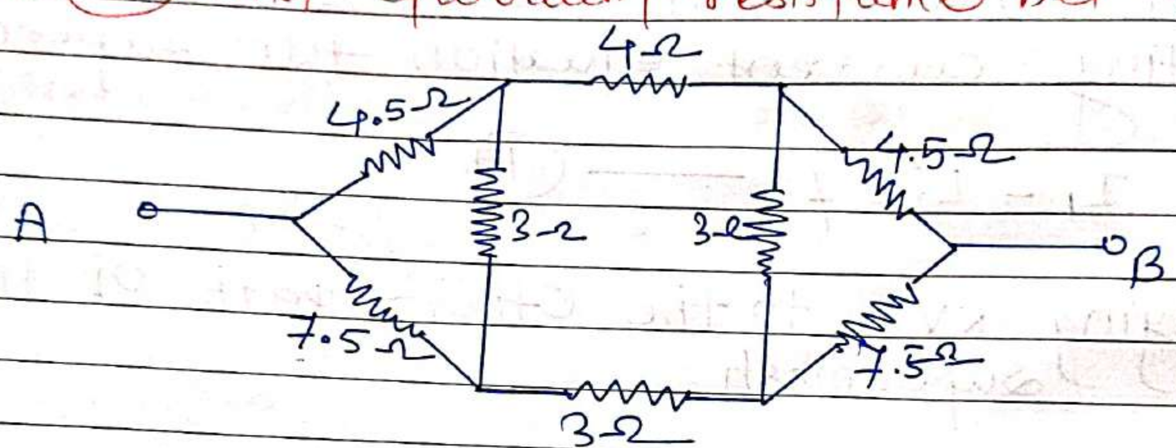
$$I_1 - 6I_2 - 3I_3 = 0 \quad \text{——— (3)}$$

Solving eqⁿ ①, ② & ③

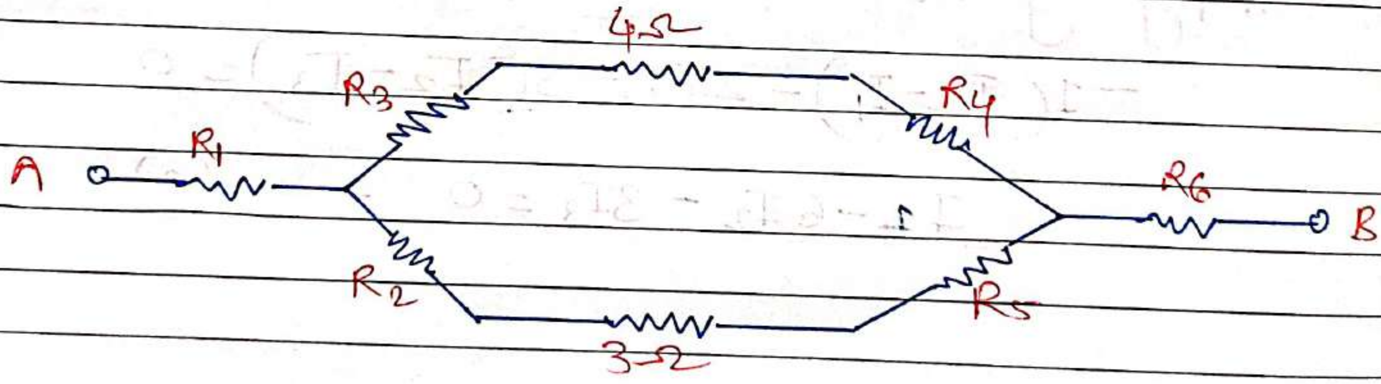
$I_1 = 9$ Amp
$I_2 = 2.5$ Amp
$I_3 = 2$ Amp.

Current through the 3Ω resistor $= I_2 - I_3$
$= 2.5 - 2$
$= 0.5$ Amp.

Que ② Find equivalent resistance betⁿ A & B.



→ Converting the two delta networks formed by resistors 4.5Ω, 3Ω & 7.5Ω into equivalent star networks, we have

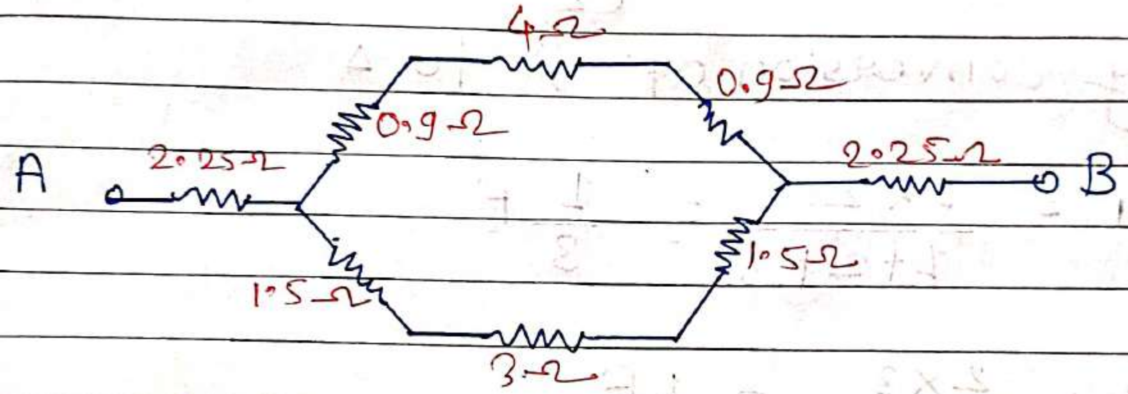


$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5+7.5+3} = 2.25\,\Omega$$

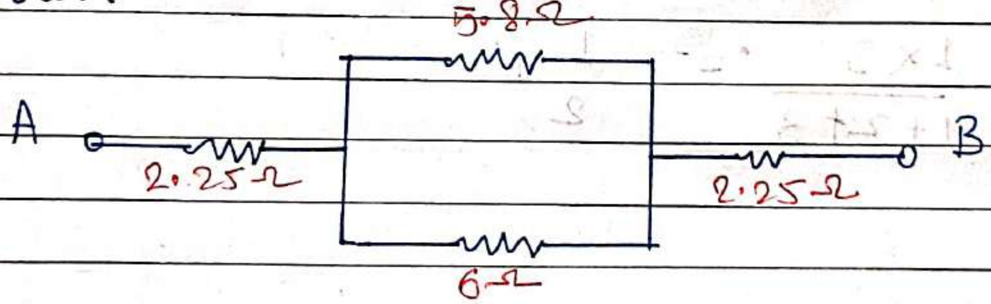
$$R_2 = R_5 = \frac{7.5 \times 3}{4.5+7.5+3} = 1.5\,\Omega$$

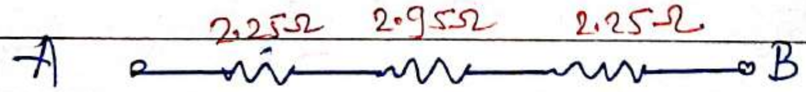
$$R_3 = R_4 = \frac{4.5 \times 3}{4.5+7.5+3} = 0.9\,\Omega$$

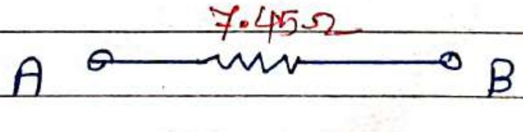
The simplified network is shown in fig.



The network can be further simplified as follows.

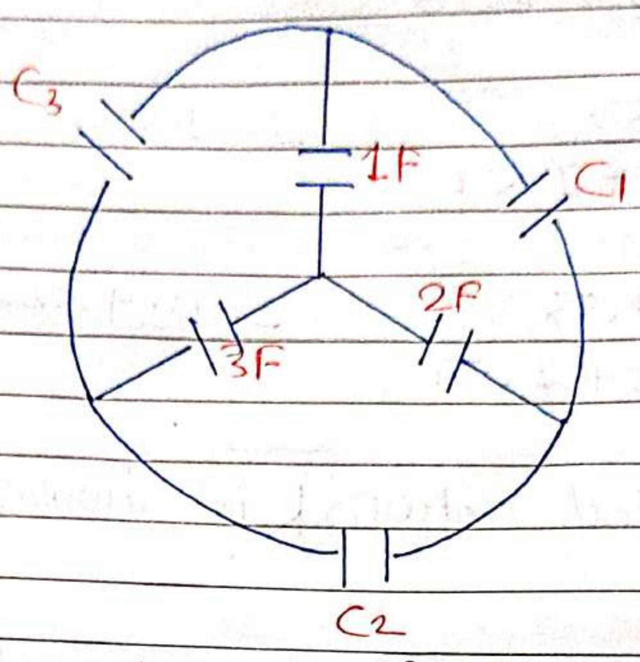






$$\boxed{R_{AB} = 7.45\,\Omega}$$

Que.(3) Determine C_1, C_2, C_3 ?



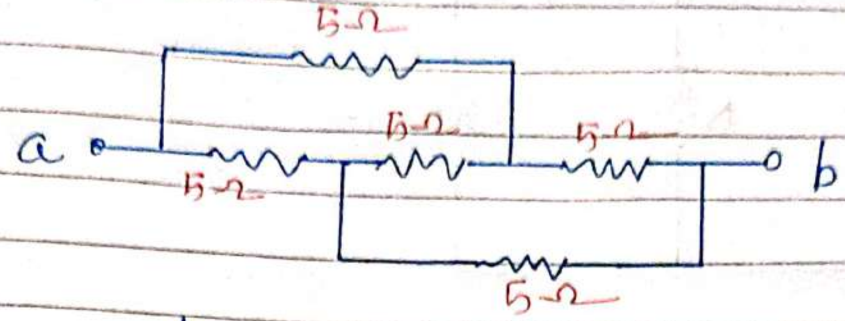
$\Rightarrow$ By conversion of λ to Δ

$$C_1 = \frac{1\times 2}{1+2+3} = \frac{1}{3} F$$

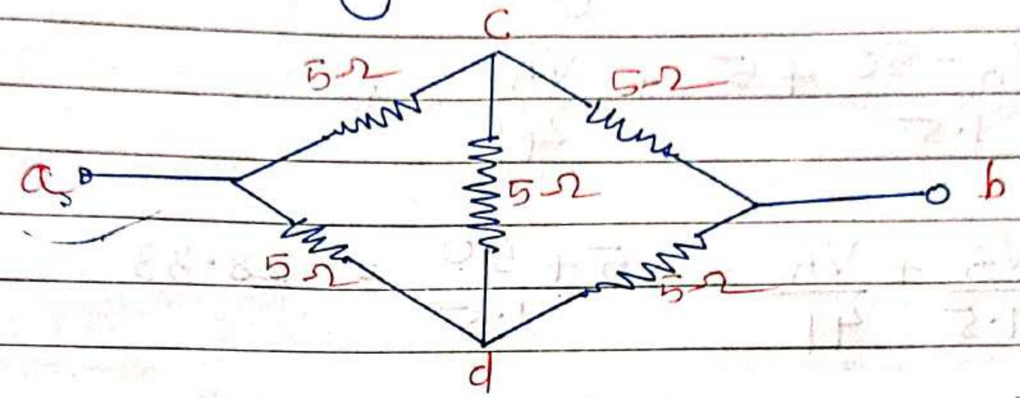
$$C_2 = \frac{2\times 3}{1+2+3} = 1 F$$

$$C_3 = \frac{1\times 3}{1+2+3} = \frac{1}{2} F$$

Que(4) Find the equivalent resistance across a & b.



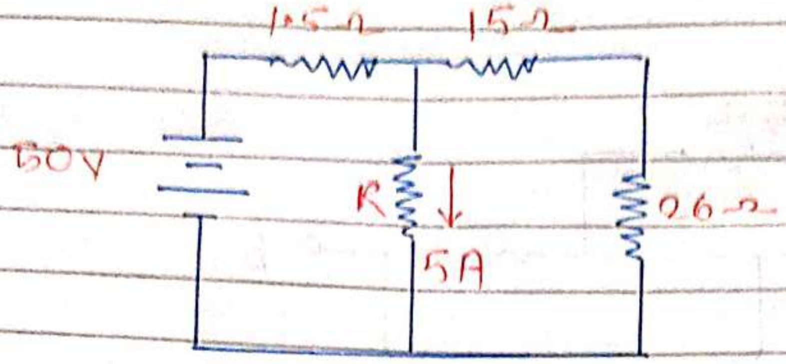
→ By redrawing



Here bridge is balanced, so no current will flow in branch cd.

$$R_{ab} = 10 \parallel 10 = \frac{10 \times 10}{10+10} = \frac{100}{20} = 5\Omega$$

Que (5) Find the value of R



→ By nodal analysis

$$\frac{V_A - 50}{1.5} + 5 + \frac{V_A}{41} = 0$$

$$\frac{V_A}{1.5} + \frac{V_A}{41} = -5 + \frac{50}{1.5} = 28.33$$

By solving

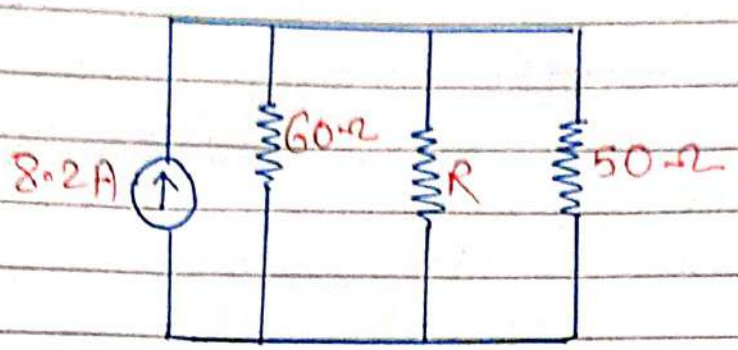
$$V_A = 40.99 \approx 41$$

$$\text{so } R = \frac{41}{5} = 8.199$$

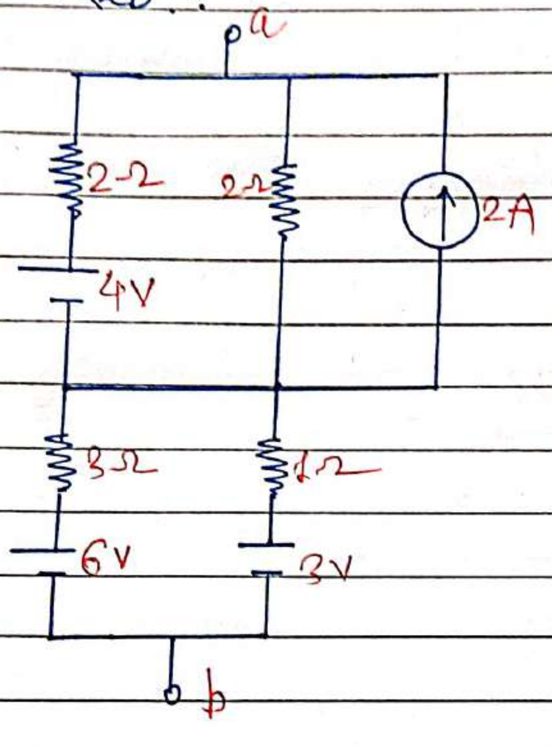
$$\underline{R = 8.2\,\Omega}$$

Practice questions :-

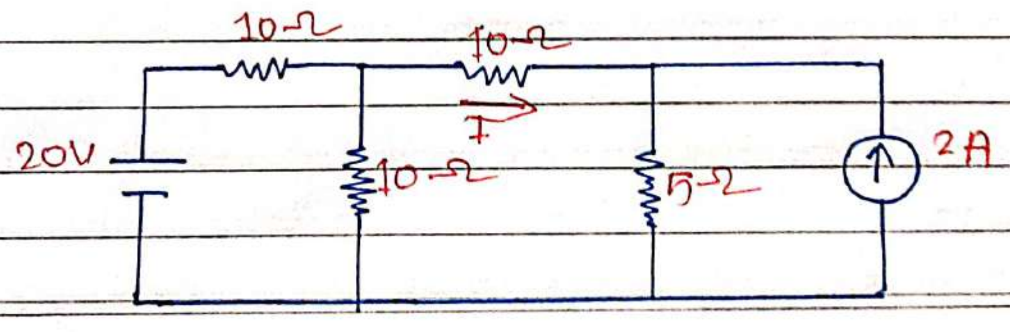
① What value of R ensures that the current through the 60-Ω resistor of this circuit is 1A ?



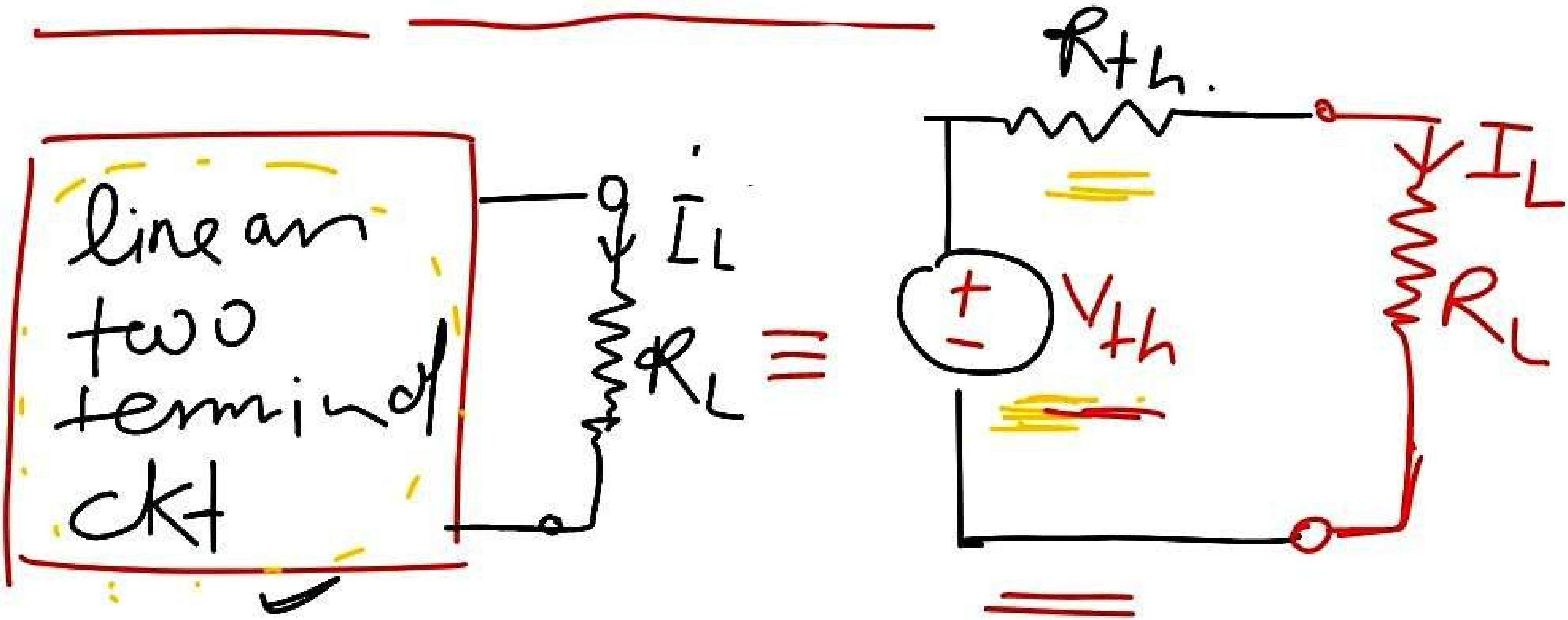
② Find Rab ?



③ find current indicates in below figure.



Thevenin's Theorem

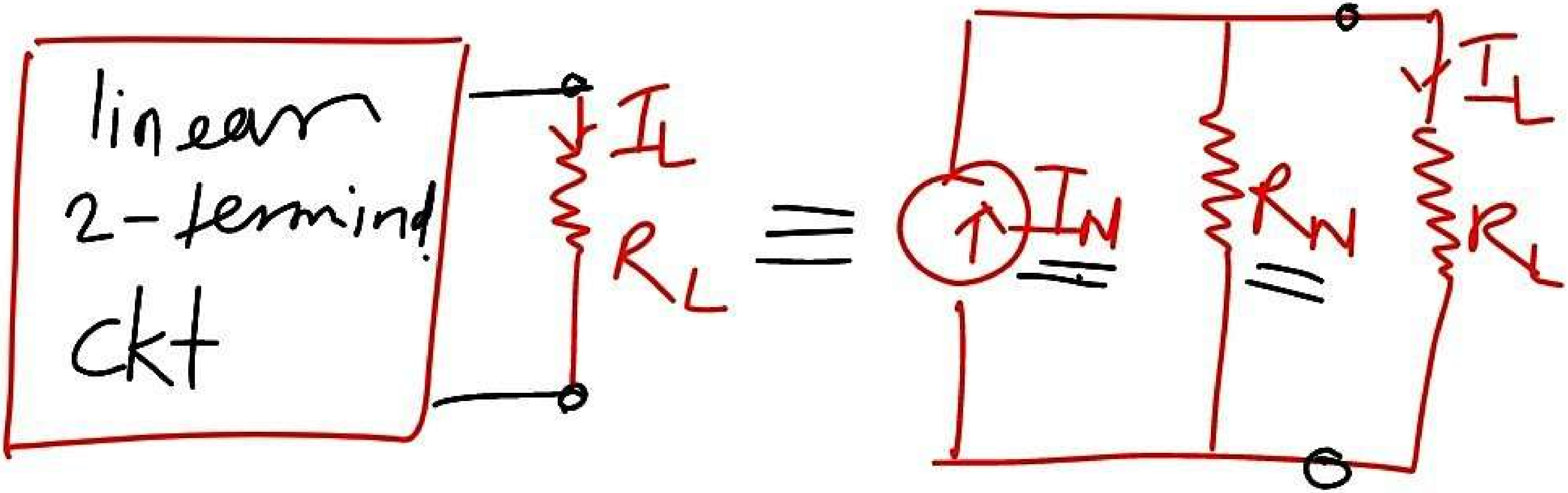


Thevenin's th^n States that a linear two terminal ckt can be replaced by equivalent ckt consisting of a voltage source V_{th} in series with a resistor R_{th} where

$V_{th} \Rightarrow$ open ckt voltage at load terminal

$R_{th} \Rightarrow$ I/p or equivalent resistance at load terminal when the independent sources are turned off i.e. killed.

Norton's Theorem :-



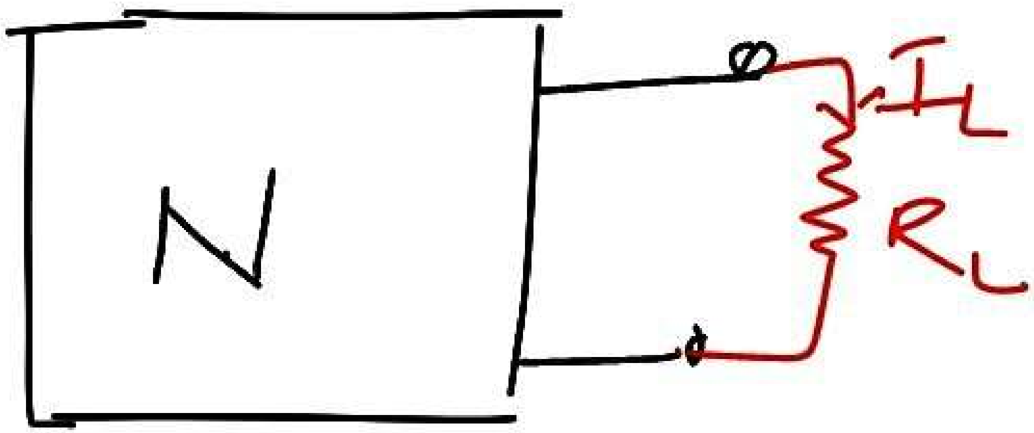
Statement

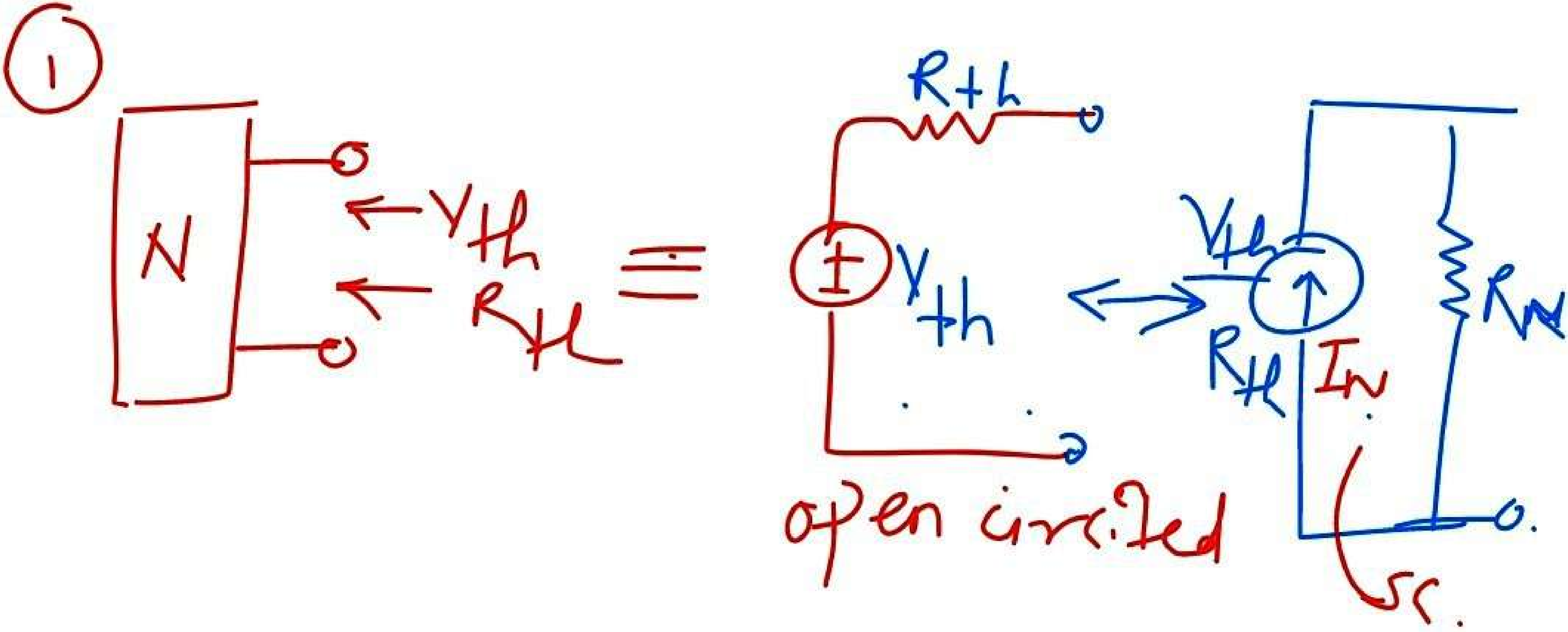
Norton's Th^m states that a linear two terminal circuit can be replaced by an equivalent ckt consisting of current source I_N in ||el with resistor R_N

$I_N \Rightarrow$ Short circuited current through the load terminal

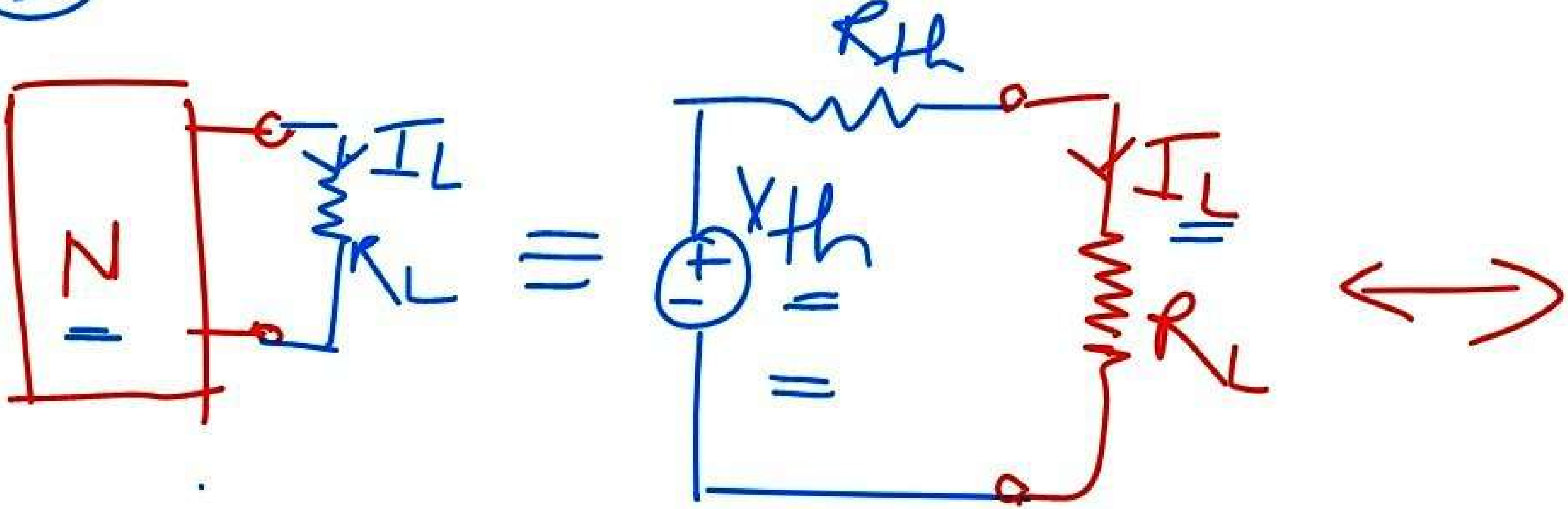
$R_N \Rightarrow$ i/p or equivalent resistance at load terminal when the independent sources are turned off ie killed. $R_N = R_{th}$.

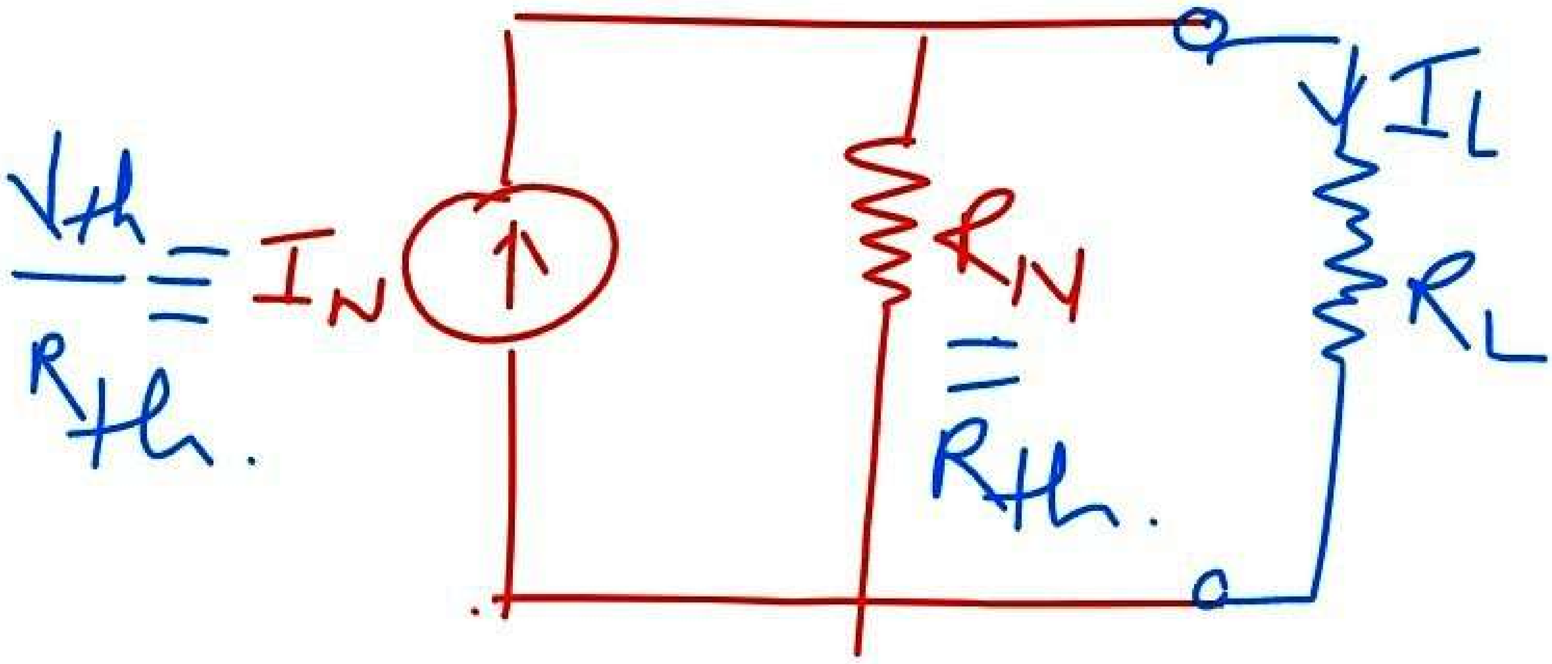
Procedure




1
N
Vth
Rth
Rth
Vth
Vth
Rth
IN
RN
open circuited
sc.

2





$\frac{V_{th}}{R_{th}} = I_N$
$R_N = R_{th}$
I_L
R_L

By ohm's Law.

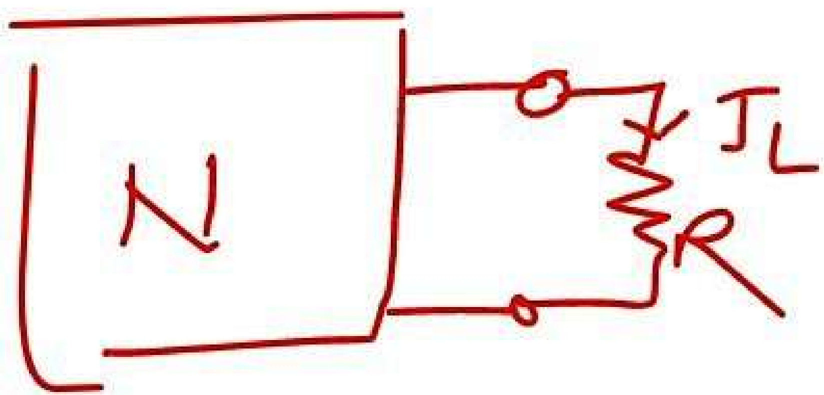
$$I_L = \frac{V_{th}}{R_{th} + R_L} \text{ Amp}.$$

$$I_L = \frac{\frac{V_{th}}{R_{th}} \times R_{th}}{R_{th} + R_L} =$$

3 cases In thevenin's thm
& Nortons. Thm based on
sources.

(1) All are Independent
source: V_{th} R_{th}

Q.

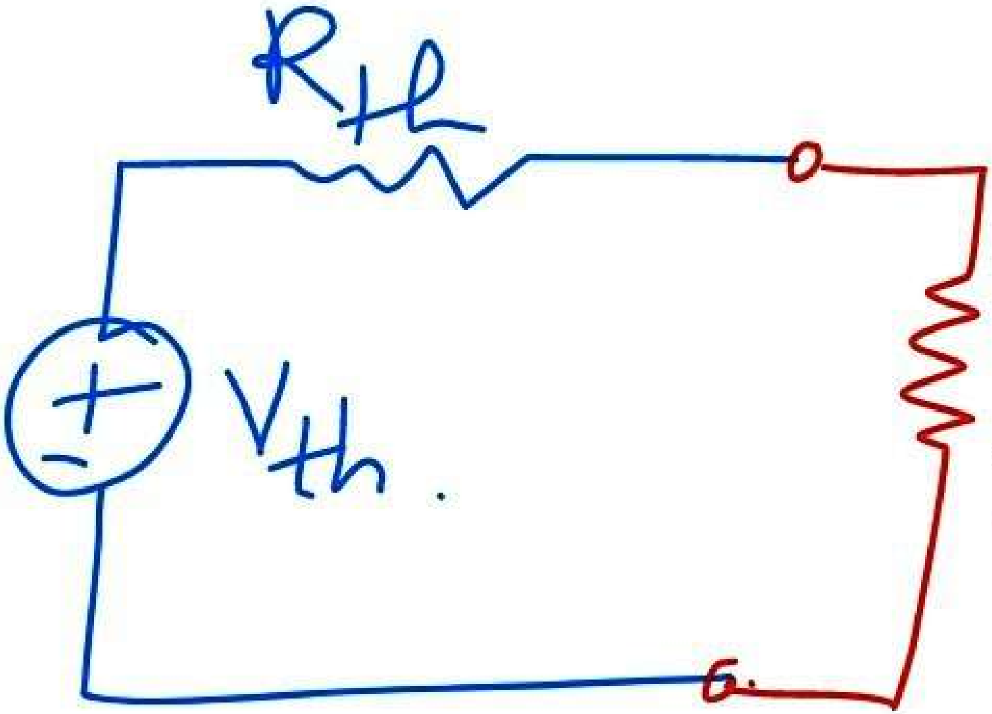


Several resistance & DC sources

The current $I = 3$ Amp & 1.5 Amp when $R = 0\,\Omega$ & $2\,\Omega$ resp. Determine I when $R = 1\,\Omega$

Method ① by Thevenin's

Note :- Any n/w. with any no. of elements w.r.to load element canbe written as a 3 element n/w.



$$\Rightarrow \quad I = \frac{V_{th}}{R_{th} + R}$$

$$3 = \frac{V_{th}}{R_{th}} \qquad (1)$$

$$1.5 = \frac{V_{th}}{R_{th}+2} \quad \text{---} \quad (2)$$

$$\frac{3}{1.5} = \frac{R_{th}+2}{R_{th}}.$$

$$\boxed{R_{th} = 2\Omega}$$

$$2 = 1 + 2/R_{th}.$$

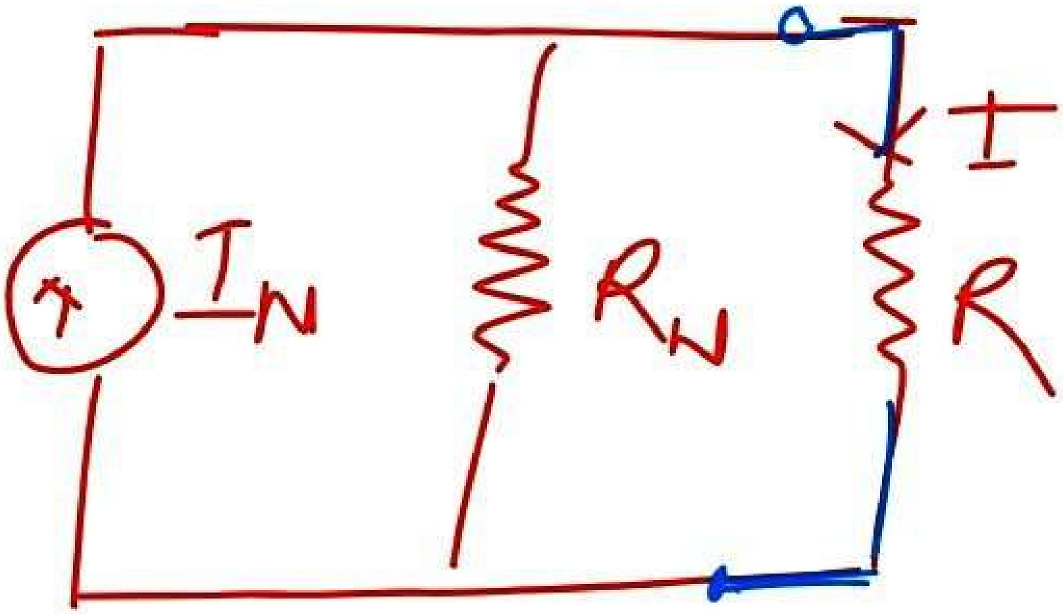
$$3 = \frac{V_{th}}{2}$$

$$\boxed{V_{th} = 6V}$$

V_{th}, R_{th} are independent of Load value

$$I = \frac{6}{2+1} = 2\,\text{Amp.}$$

Nortons Thm



$$I = \frac{I_N R_N}{R + R_N}$$

$$3 = \frac{I_N R_N}{R_N} \quad \text{--- (1)}$$

$$1.5 = \frac{I_N R_N}{2 + R_N} \text{ —— } (2)$$

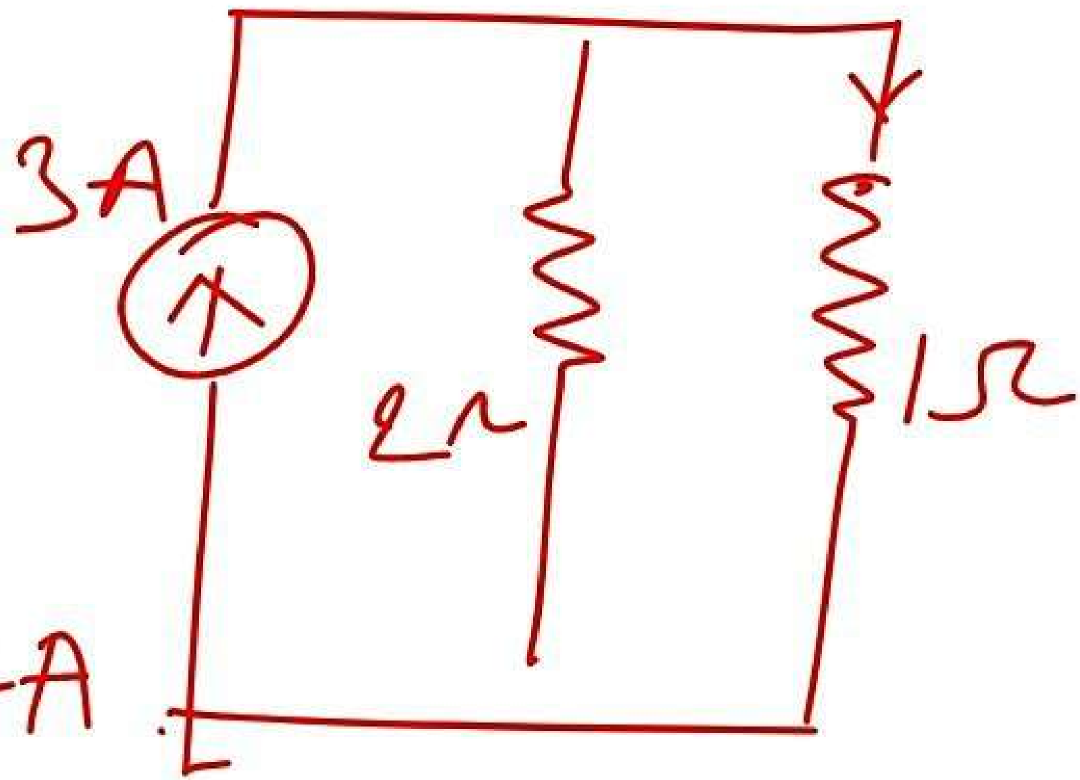
$$\frac{3}{1.5} = \frac{2 + R_N}{R_N}$$

$$2 = 2/R_N + 1 \Rightarrow \boxed{R_N = 2\,\Omega}$$

$$3 = \frac{I_N R_N}{R_N}$$

$$I_N = 3 \text{ Amp.}$$

$$I = \frac{3 \times 2}{1+2} = 2A.$$



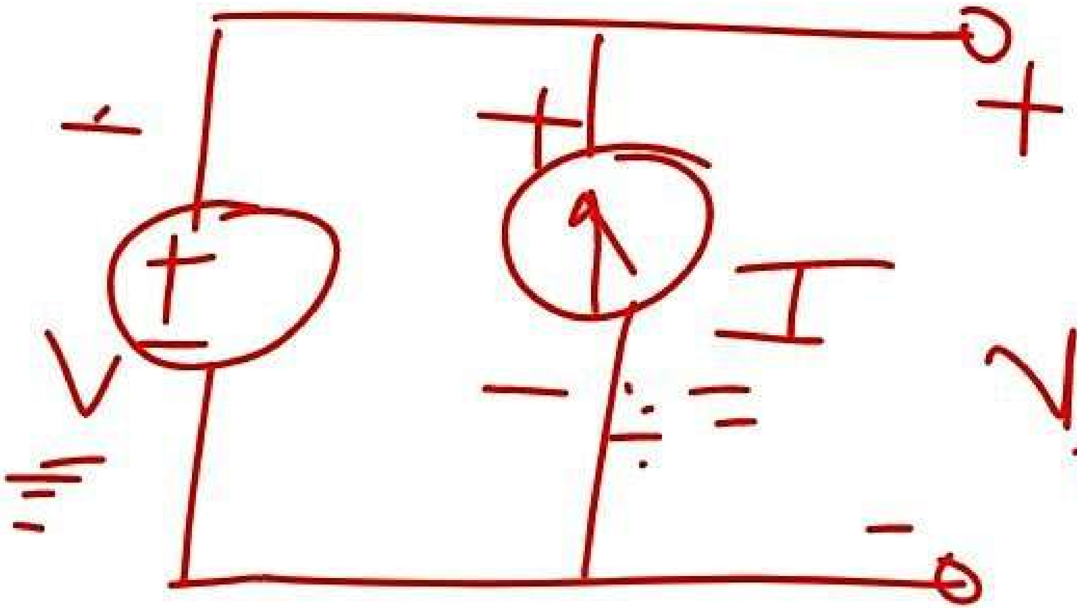
(2) An Ideal voltage & current sources are connected in ||el. Then combination will have

(1) Thevenins & Nortons.

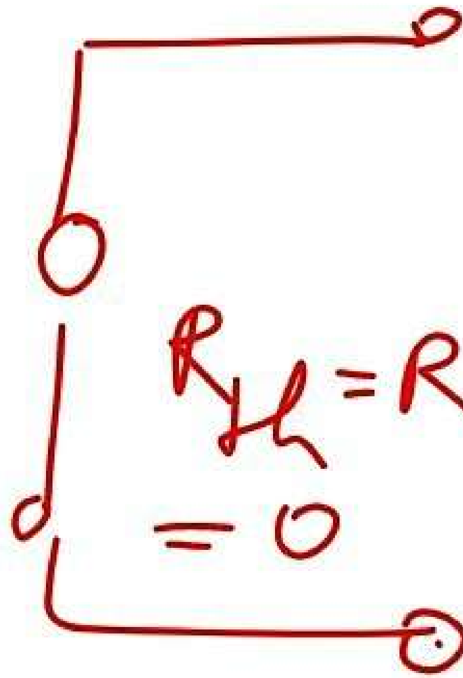
(2) Thevens.

(3) Norton

(4) Neither Thevenin nor Norton.



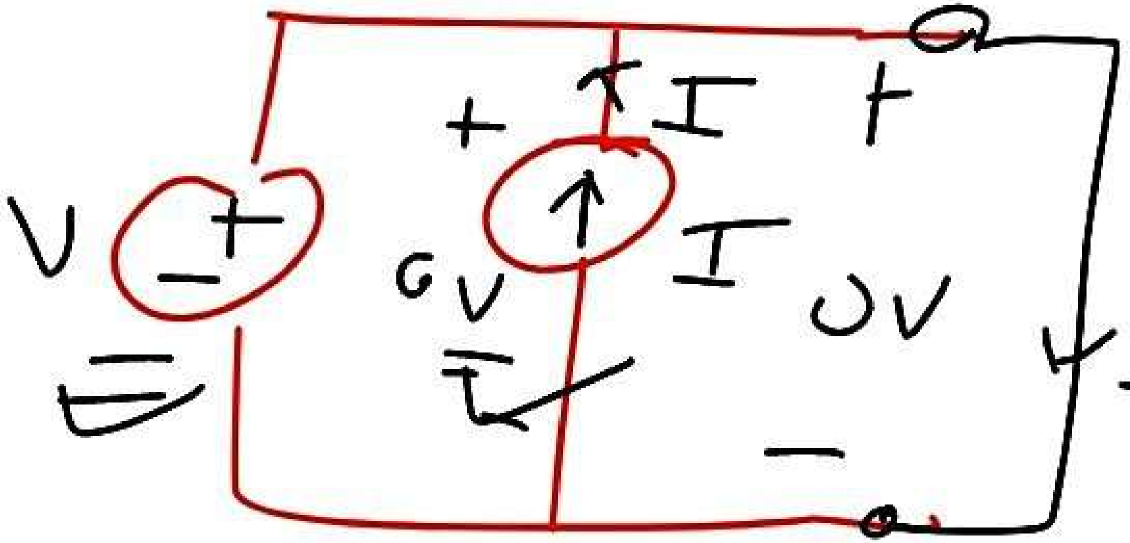
$V_{th} = V.$



$R_{th} = R$

$= 0$

Reason



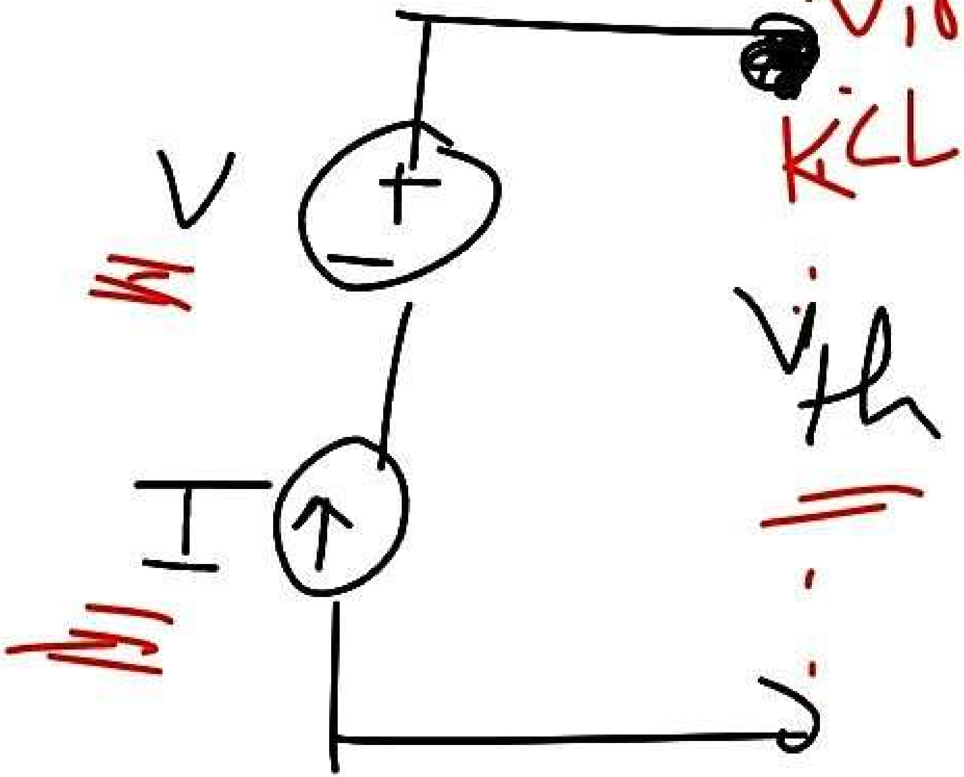
By KCL

$I_N = I + \text{any value}$

$I_{sc} = I_N$

$V - 0 = 0$

$V = 0$ ✗



violation of KCL at this Node.

KVL

$V_{th} = V + \text{any value}.$

(2) At least one Independent and one dependent source.

V_{th}, I_{sc}

$$R_{th} = \frac{V_{th}}{\underline{\underline{I_{sc}}}} \Omega$$

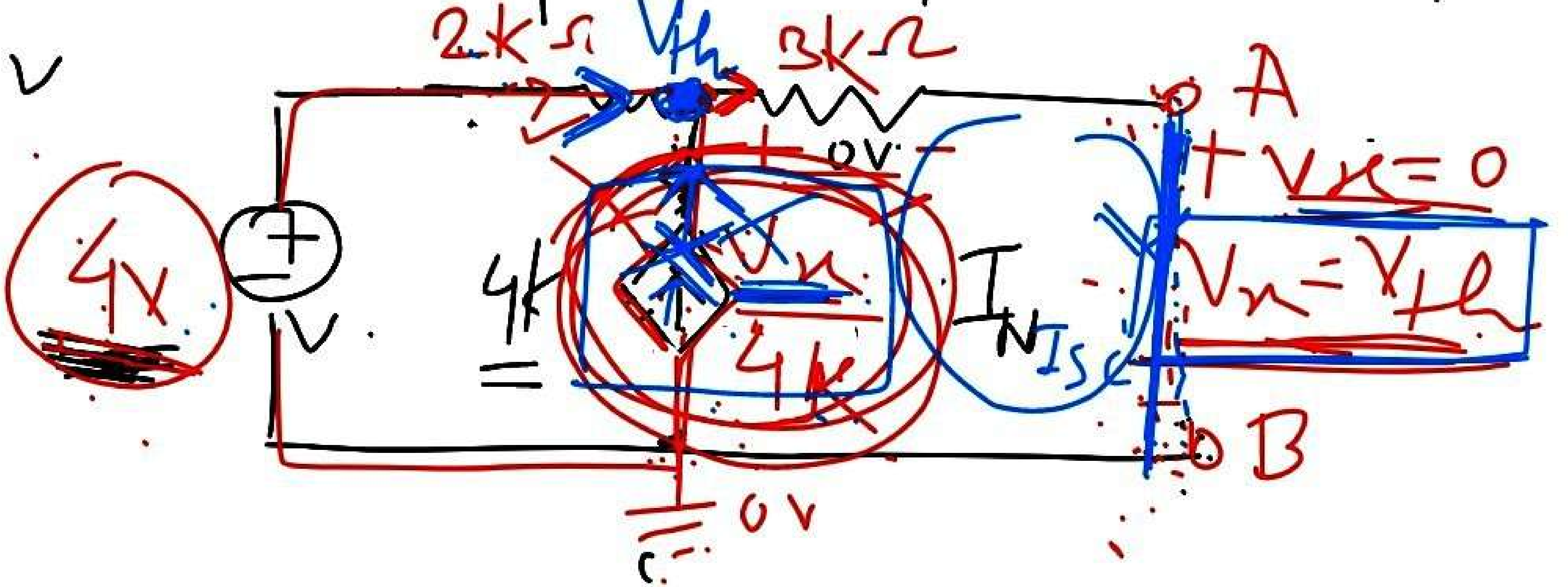
$$I_{sc} = \frac{V_{th}}{R_{th}} = \underline{\underline{I_N}}$$

Case 2 :- At least one independent and one dependent source (Inside) (outside)

$$\Rightarrow R_{th} = \frac{V_{th}}{I_{sc}}$$

$$I_{sc} = I_N$$

Q Determine the Thevenin's & Norton's equivalent across AB.



$\rightarrow V_x = V_{th}$.

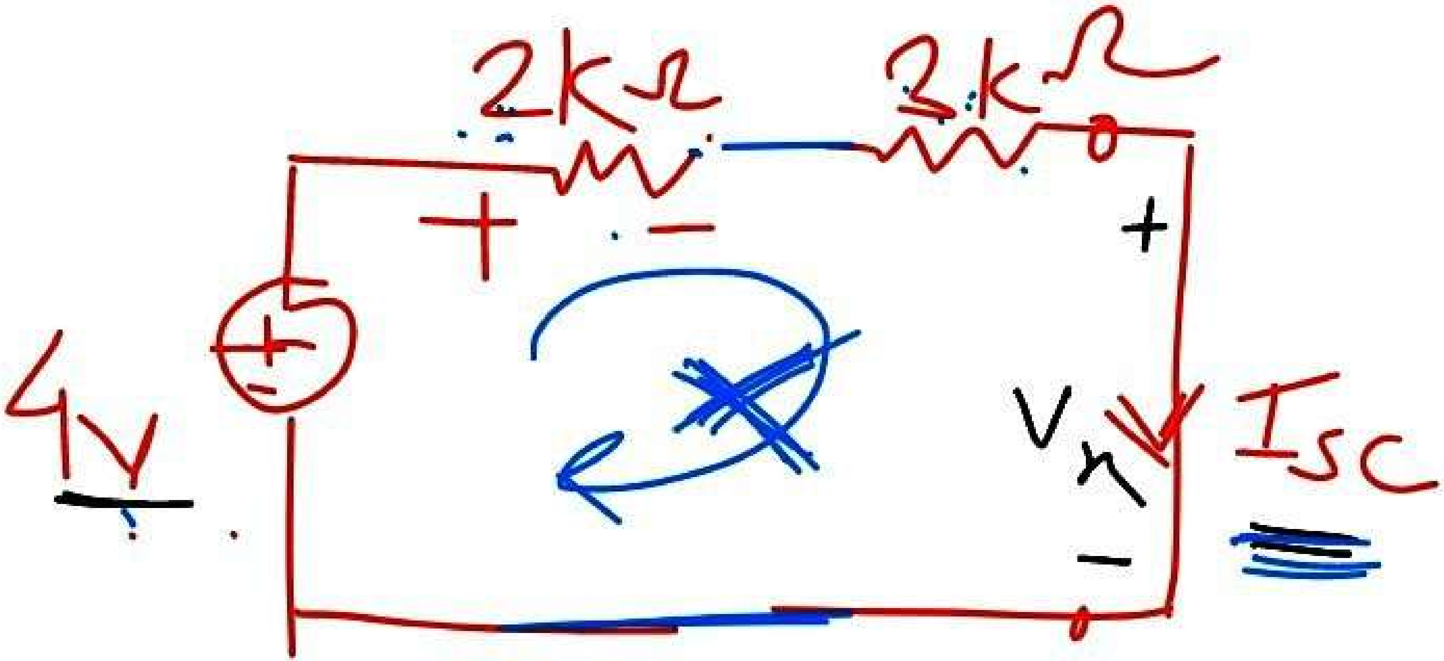
<u>Nodal</u> $\Rightarrow \dfrac{V_{th} - 4V}{2K} - \dfrac{V_x}{4K} + 0 = 0$

$\Rightarrow \dfrac{V_{th} - 4}{2K} - \dfrac{V_{th}}{4K} = 0$

$\Rightarrow 2V_{th} - 8 - V_{th} = 0$

$\Rightarrow V_{th} - 8 = 0$

$\boxed{V_{th} = 8V}$



V_{th}

I_{sh}

R_{th}

By KVL $\Rightarrow 4 - 5\times10^3 I_{sc}$

$I_{sc} = 4/5\,mA$ — Im.

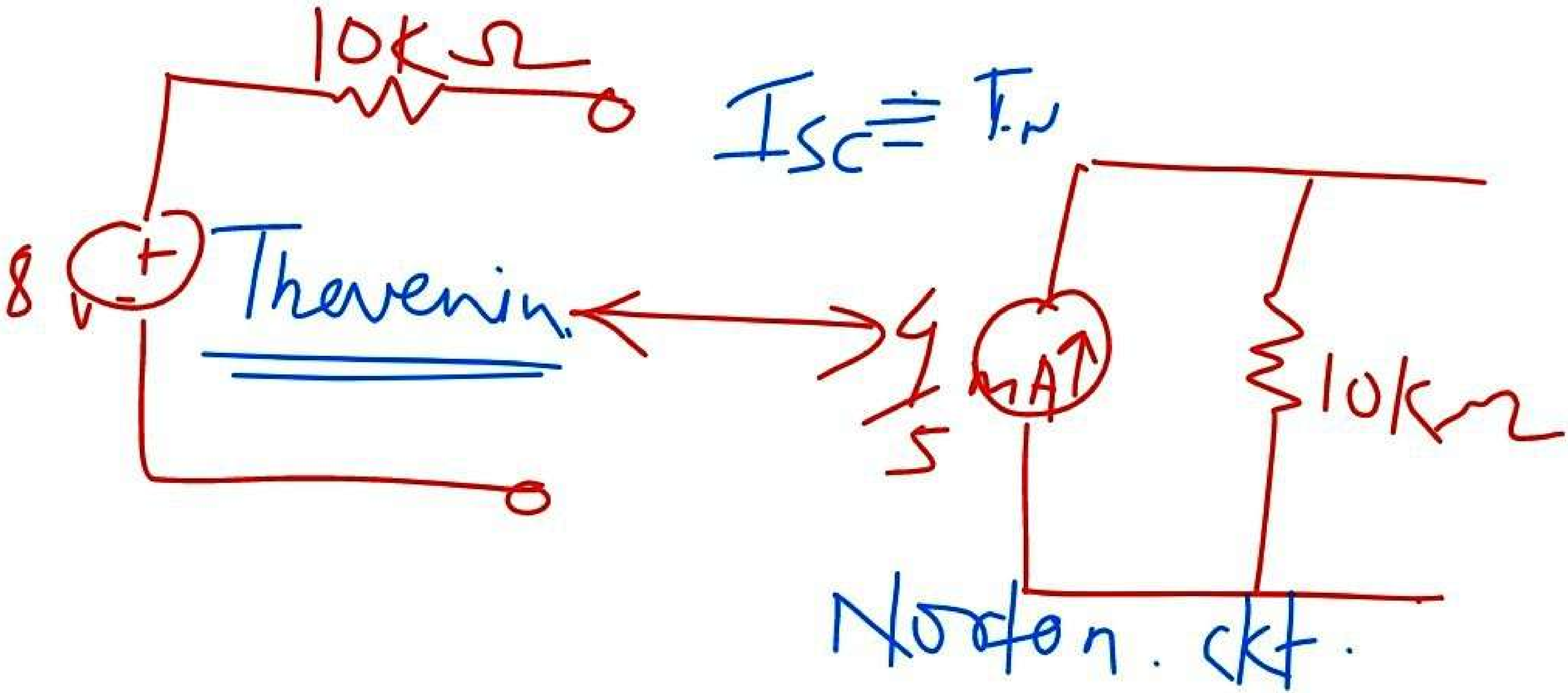
$$R_{Th} = \frac{V_{th}}{I_{sc}}$$

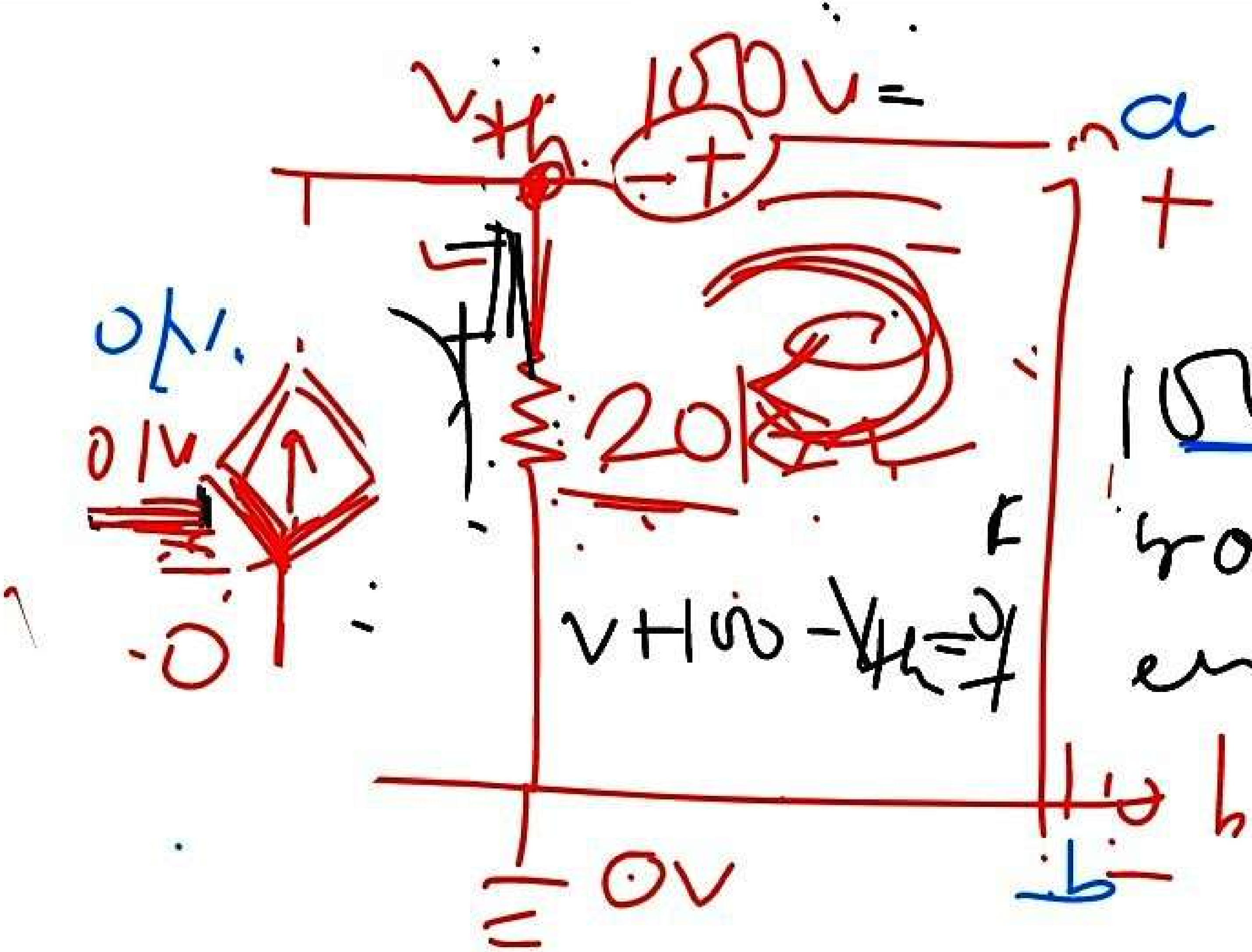
$$= \frac{8\,v}{4/5 \times 10^{-3}}$$

$$R_{th} = 10K\,\Omega$$

$$R_{th} = \frac{V_{th}}{I_{sc}}$$

$$I_{sc} = \frac{V_{th}}{R_{th}} = I_N$$

10kΩ
8 V
Thevenin
Isc = IN
5 mA
10kΩ
Norton ckt.



+ a $V_1 = V_{th}$

100 Ω Res

ross AB

ew I

V =

① $\underline{\underline{V_{th}}} \Rightarrow \dfrac{V_{th}-100}{}$

$\underline{\underline{\text{Nodal}}} \Rightarrow -0.01V_1 + \dfrac{V}{20k} + 0 = 0$

$= -200V_1 + \underline{\underline{V}} = 0$

$-200V_1 + V_{th} - 100 = 0$

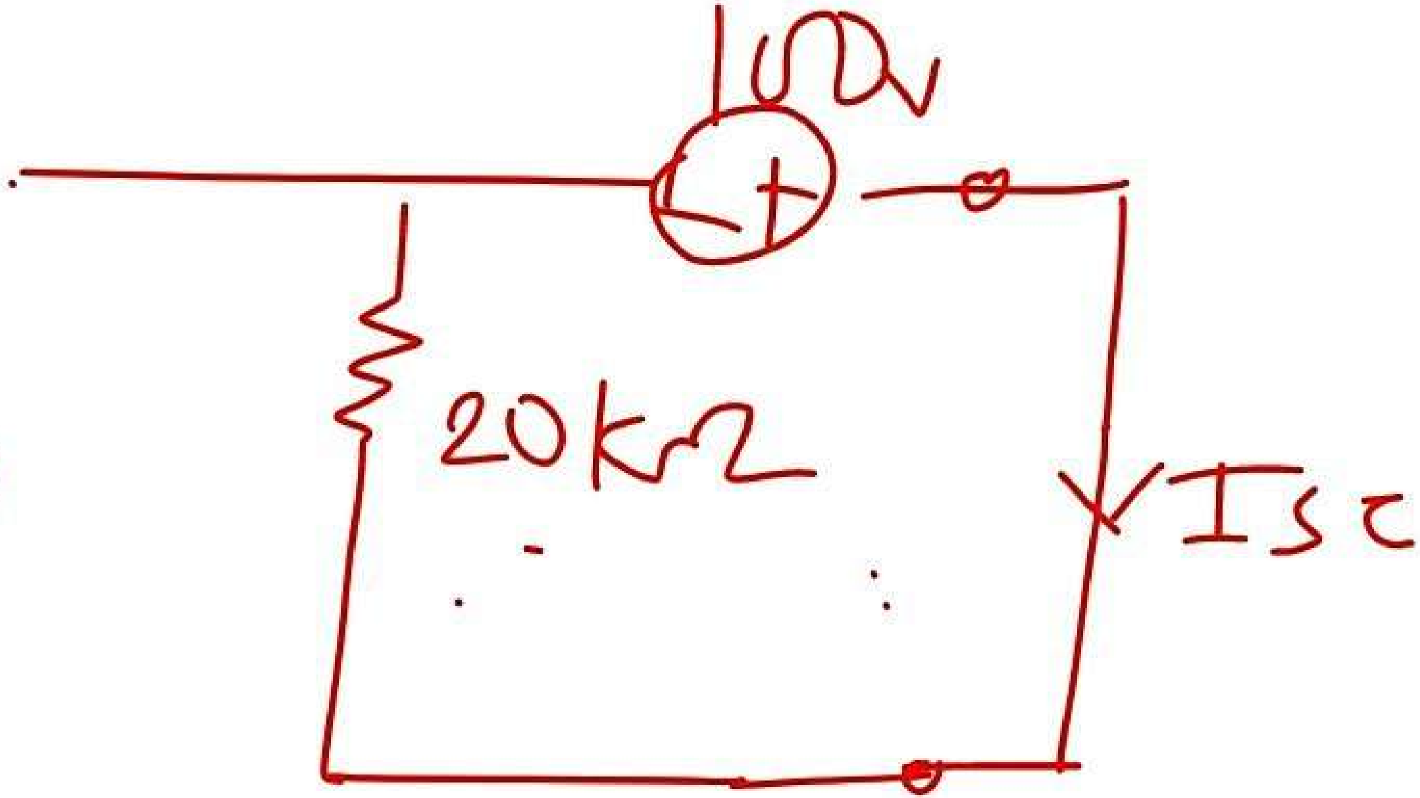
$= -2\underline{\underline{00}}V_1 + V_1 - 100 = 0$

$$-199v_1 = 100$$

$$\underline{\underline{v_1}} = \frac{100}{-199}$$

$$\underline{v_1 = V_{th}} = -502.5\,mv$$

*



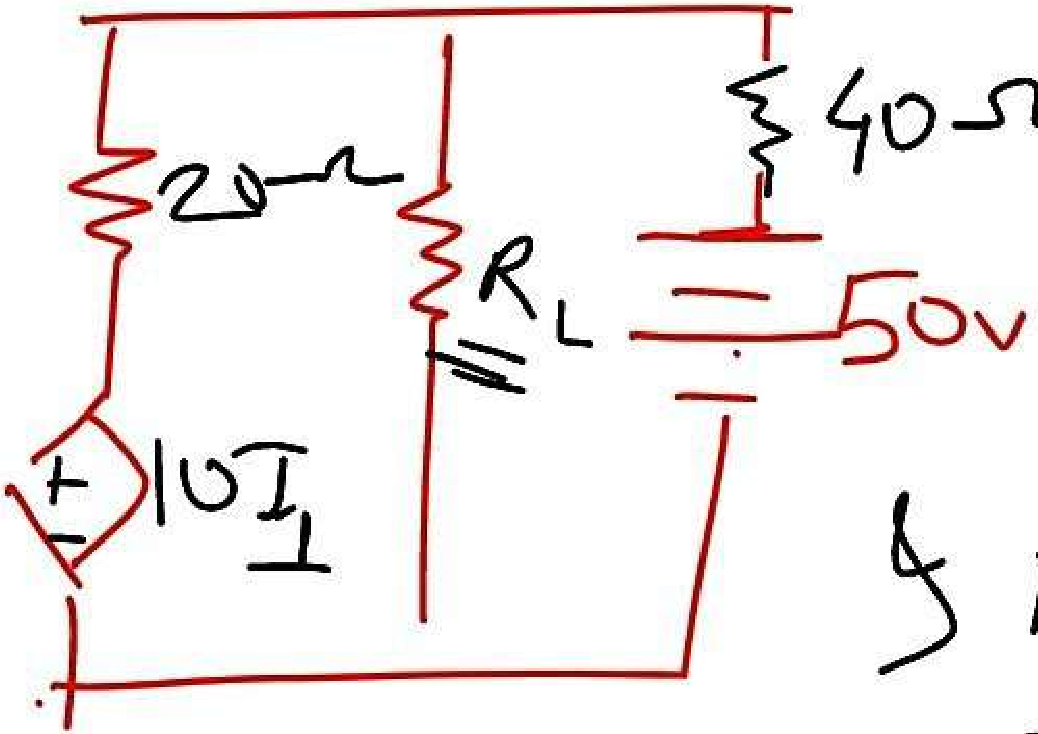
$$100 - 20k I_{sc} = 0$$

$$100 - 20\times10^{3} I_{sc} = 0 \quad \text{KVL}$$

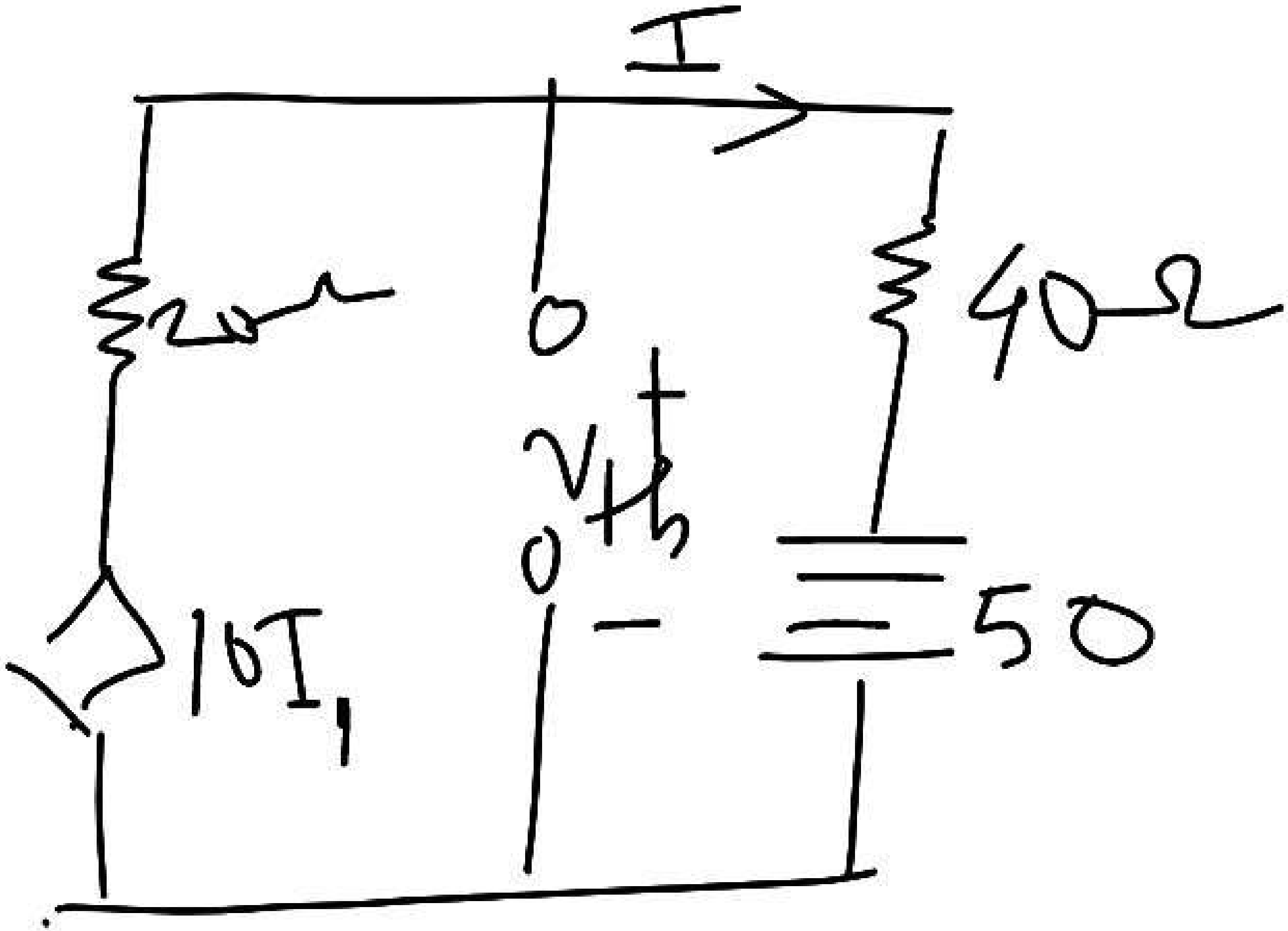
$$I_{sc} = 5\,\text{mA}$$

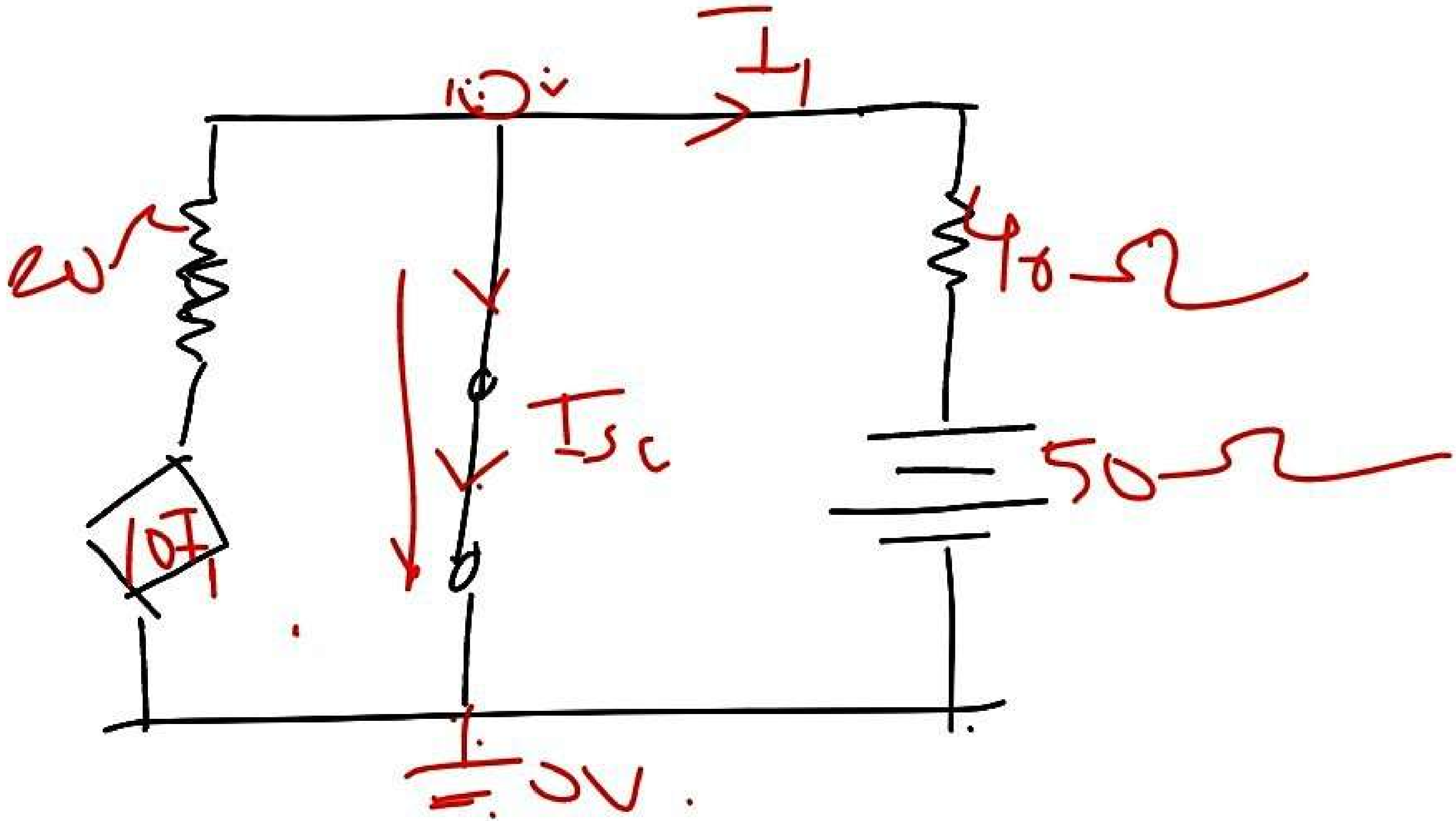
$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{V_{th}}{5\times10^{-3}} = R_{th}.$$

$$= 100.5\,\Omega$$

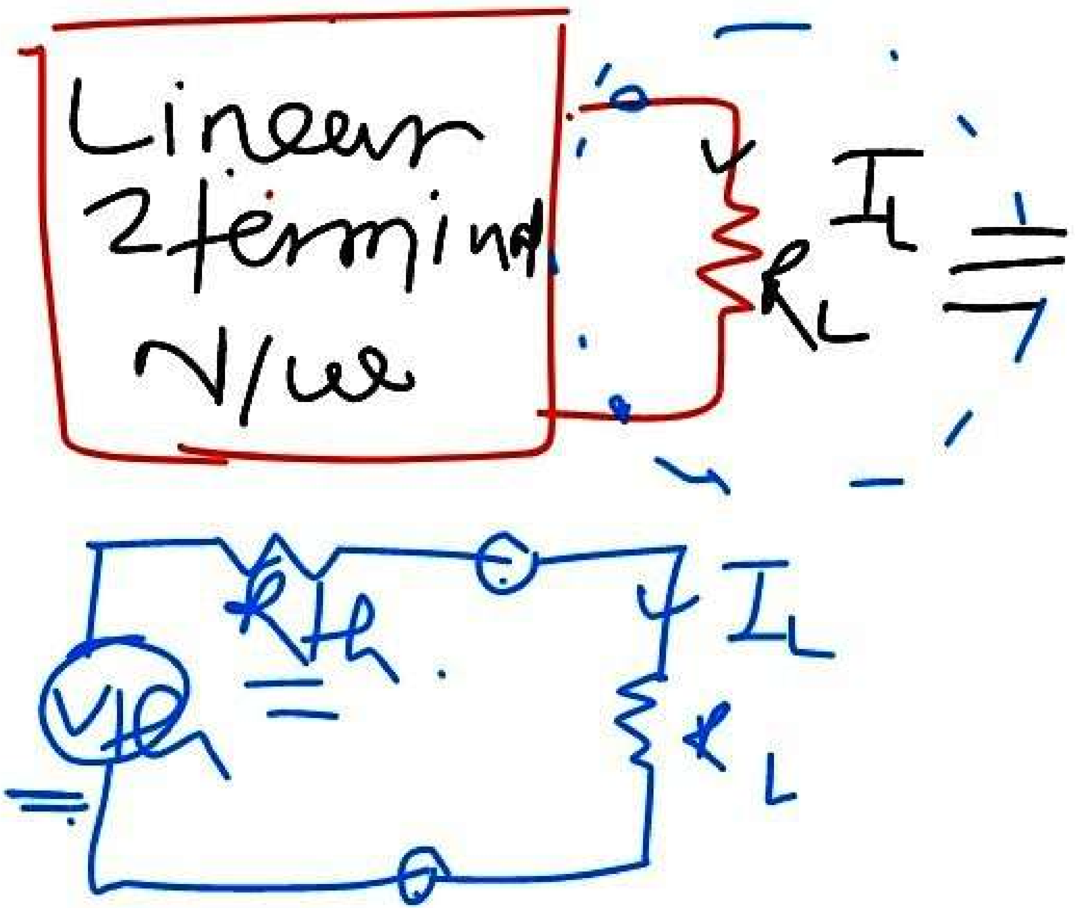


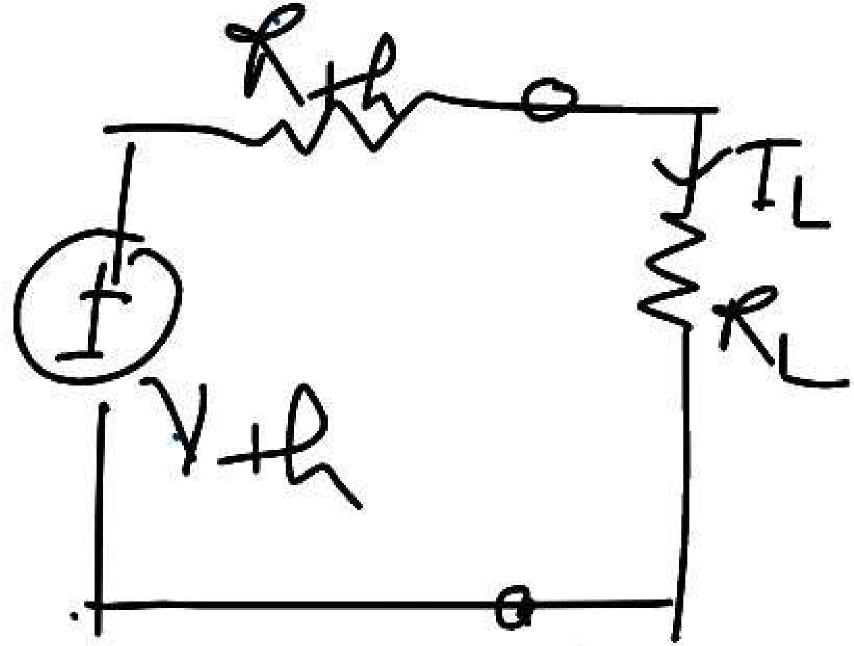
Determine Thevenins & Norton's equivalent across R_L


I
20Ω
40Ω
+
v_{th}
−
$10I_1$
50

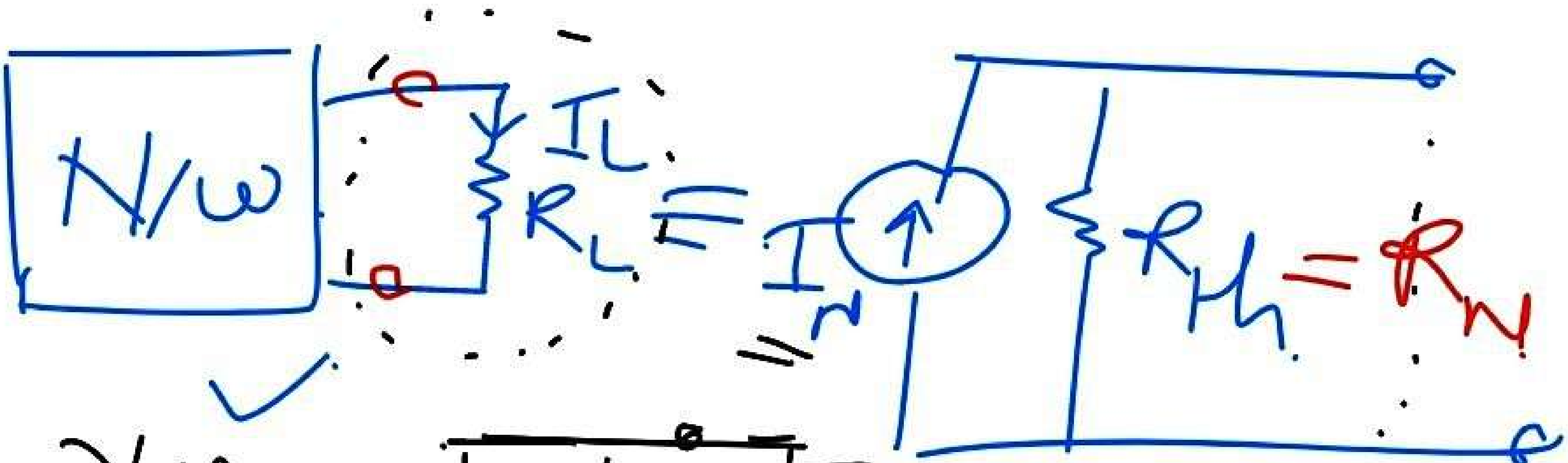


20Ω
10I1
10V
I1
Isc
40Ω
50Ω
0V



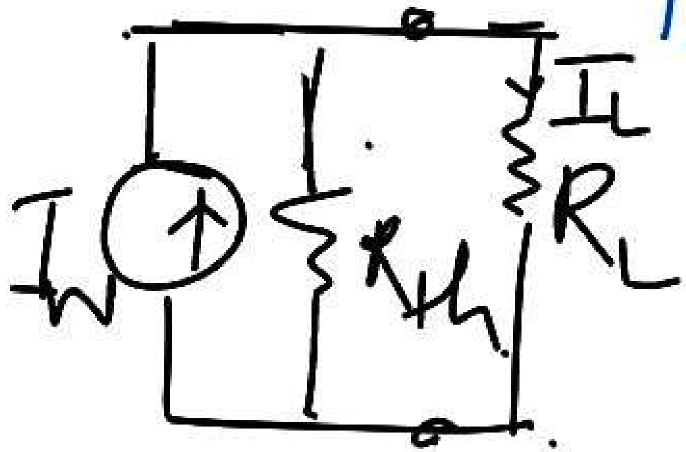


Thevenin's Thm.



Norton Thm

$$\frac{V_{Th}}{R_{Th}}$$

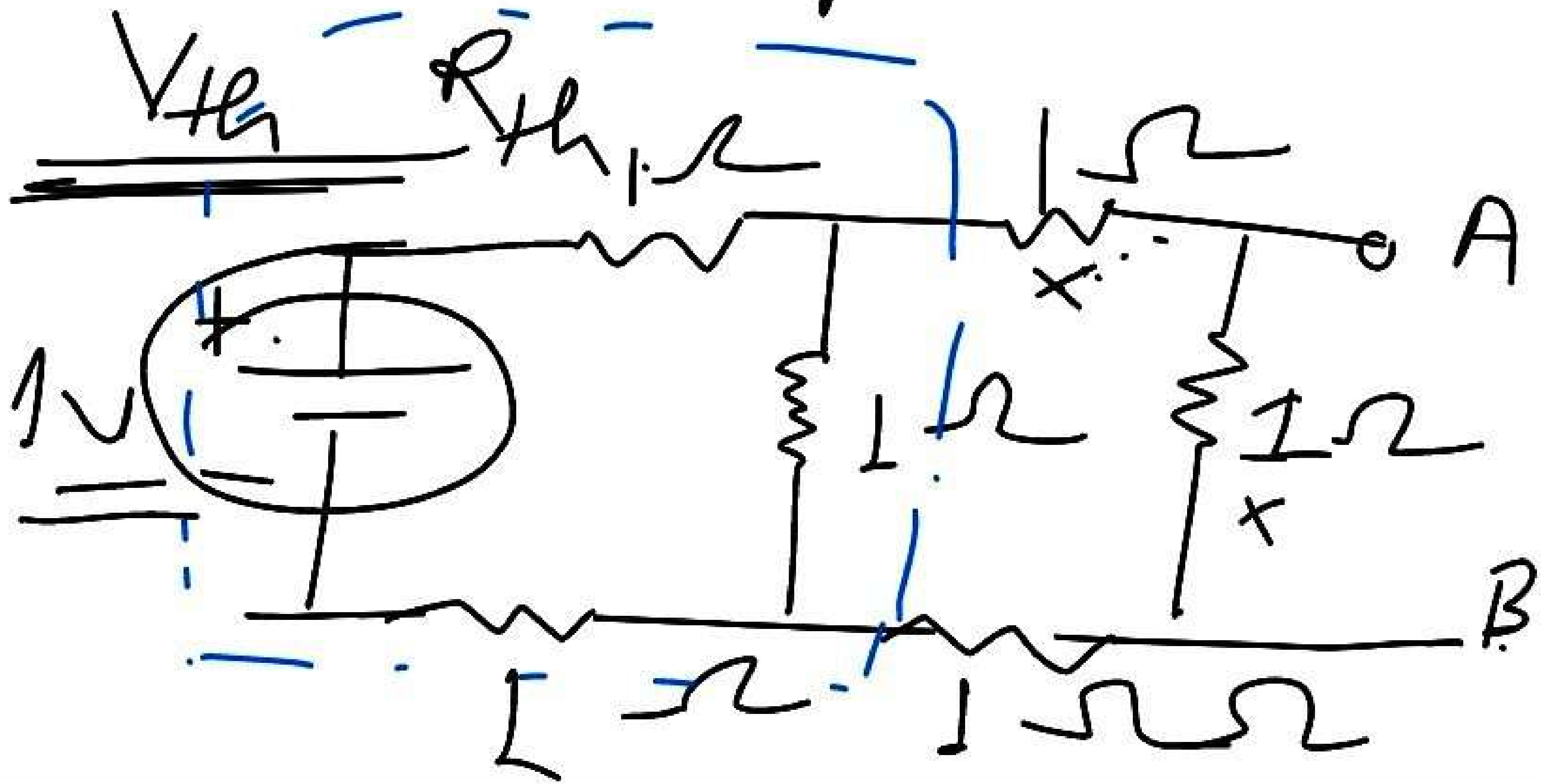


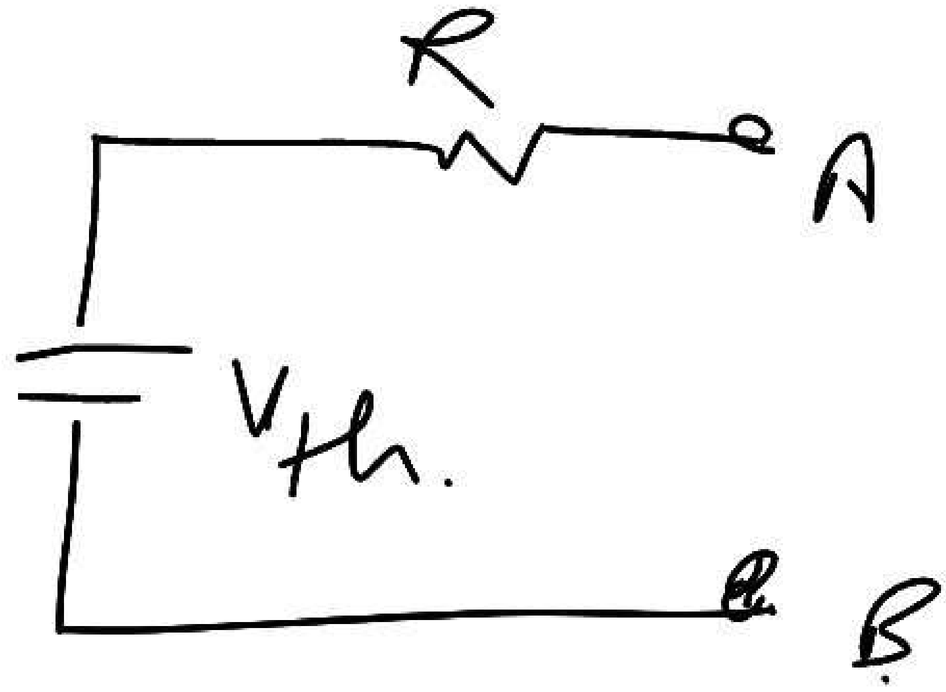
$R_{Th} = R_N$

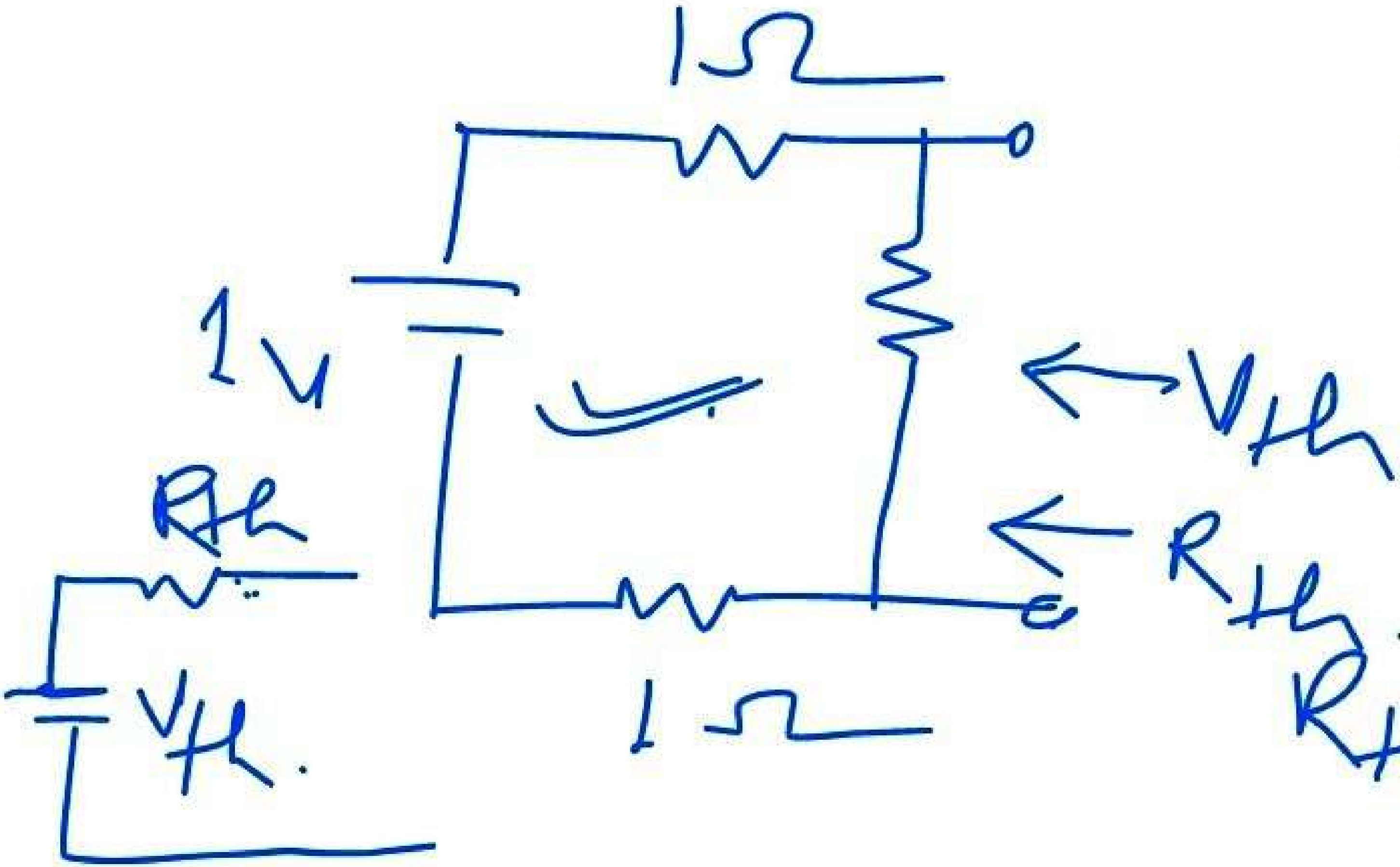
$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

V_{th} R_{th} .

All are Independent source




R
A
Vth.
B.

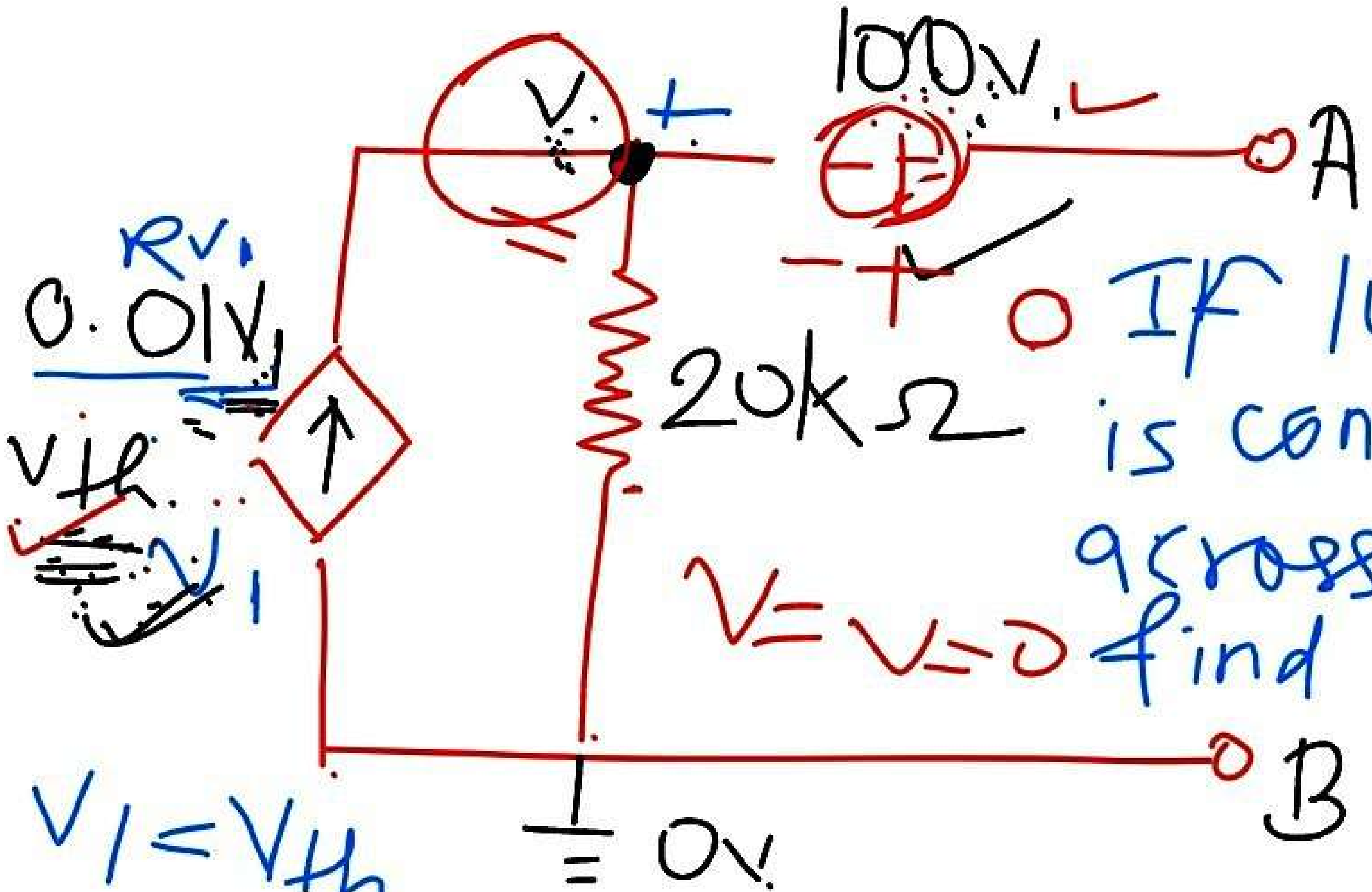


$V_{th} = 1/3\,v$

$R_{th} = 2/3$

(2) At least one independent & one dependent source.

(1) V_{th}
(2) I_{sc}
(3) R_{th}



If 100Ω Res is connected across AB. then find current.

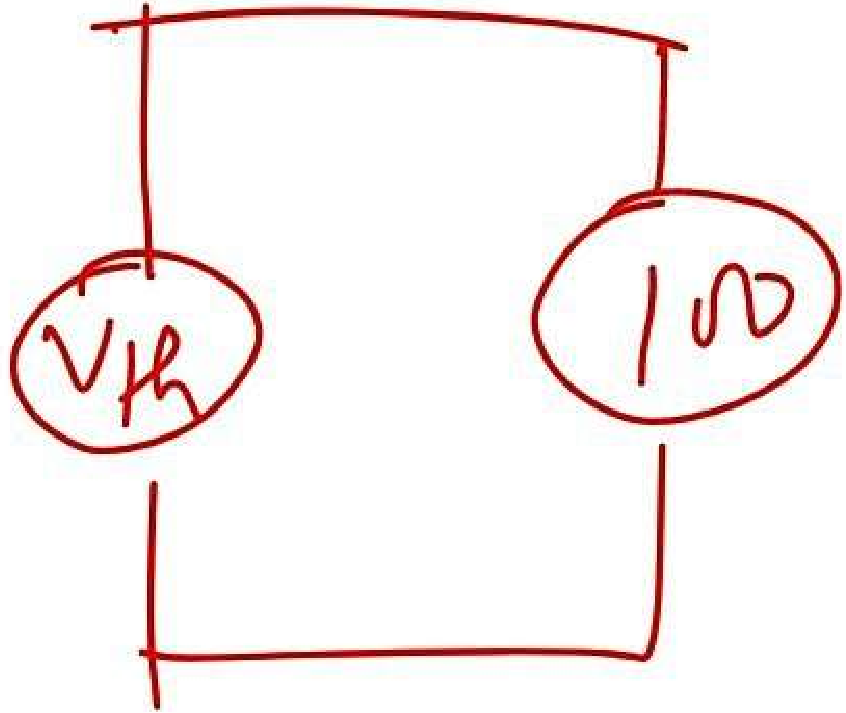
$V_1 = V_{th}$.

Assume $V_1 = V_{th}$. — (1)

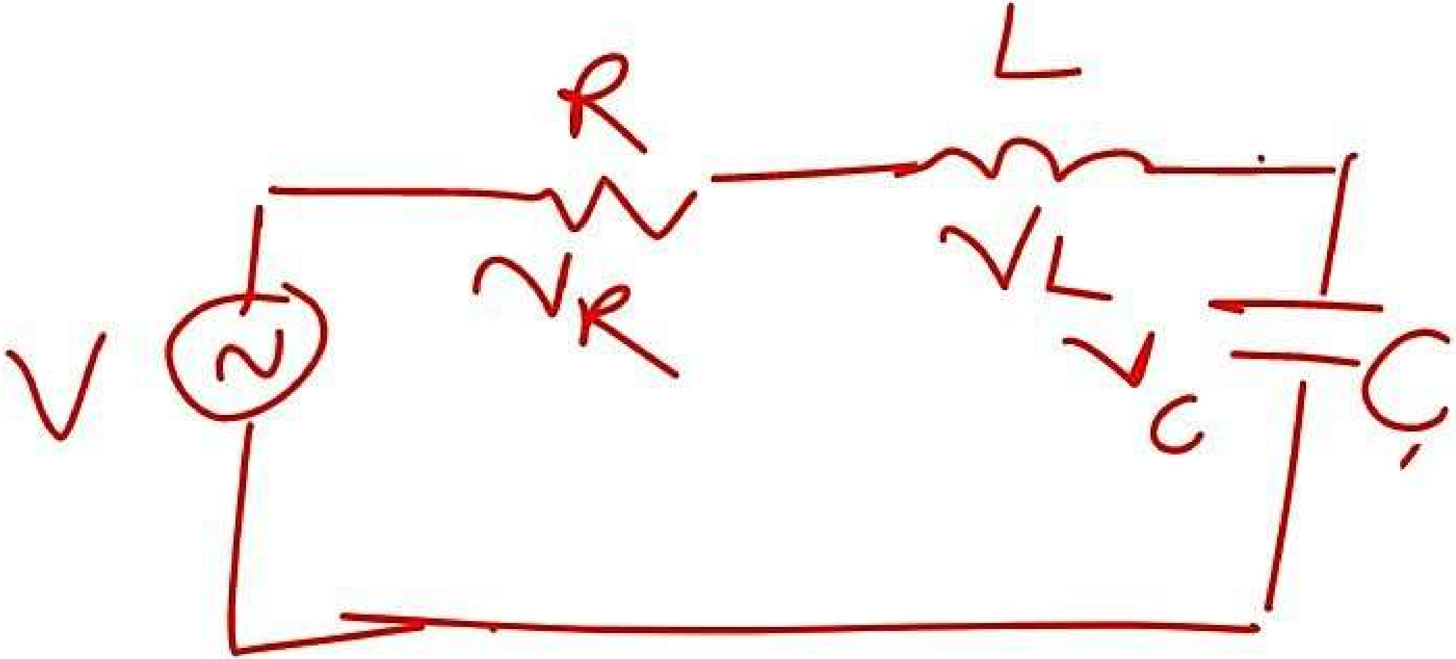
By KVL $\Rightarrow V + 100 - V_1 = 0$

$V + 100 - V_{th} = 0$

$$\boxed{V = V_{th} - 100}$$



V .



$$V = V_R + V_L + V_C$$

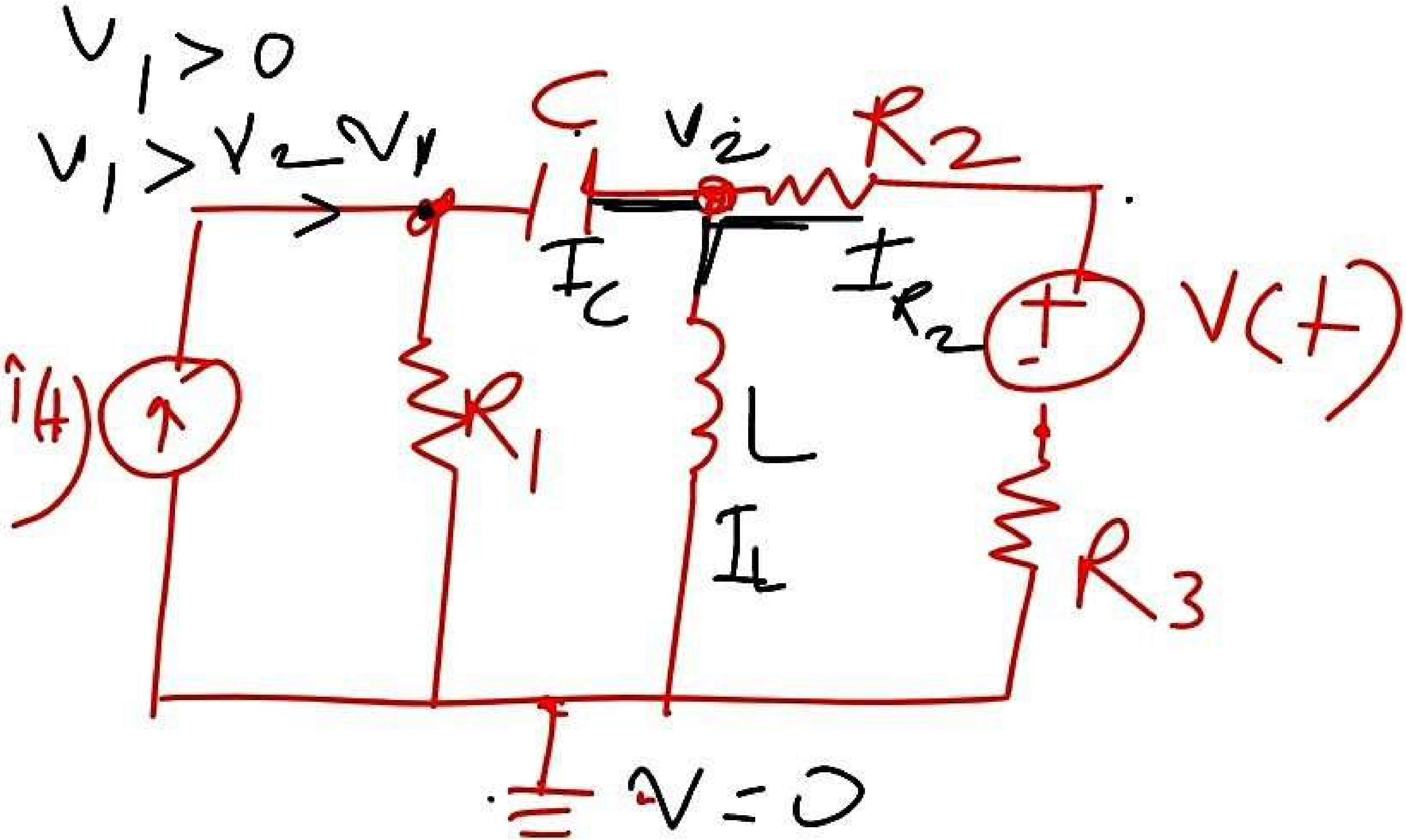
Nodal

$$-0.01v_1 + \frac{v}{20k} + 0 = 0$$

$$-200v_1 + v = 0$$

$$-200v_1 + v_{th} - 100 = 0$$

$$v_1 - 100 = 0$$



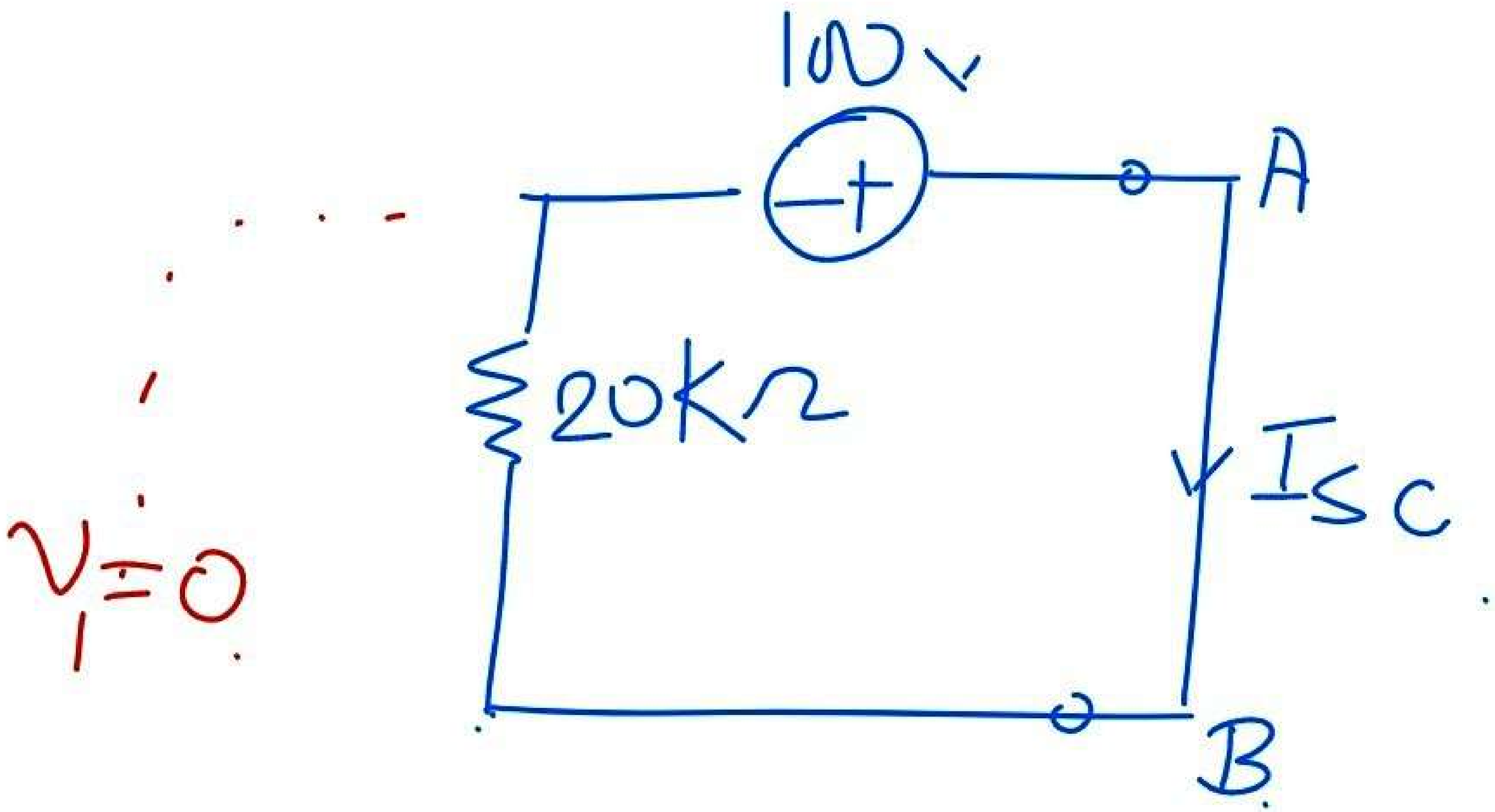
v1 > 0
v1 > v2 ~ v1
C
v2
R2
IC
IR2
V(t)
i(t)
R1
L
IL
R3
v = 0

$$-200v_1 + v_1 - 100 = 0$$

$$-199v_1 = 100$$

$$v_1 = \frac{100}{-199} = v_{th}$$

$$-502.5\,mV$$



100v
A
20kΩ
Isc
B
V=0

By KVL = $100 - 20\times10^{3} I_{sc}$

$$\boxed{I_{sc} = 5mA}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{-502.5mV}{5mA}$$

$R_{th} = -100.5\,\Omega$

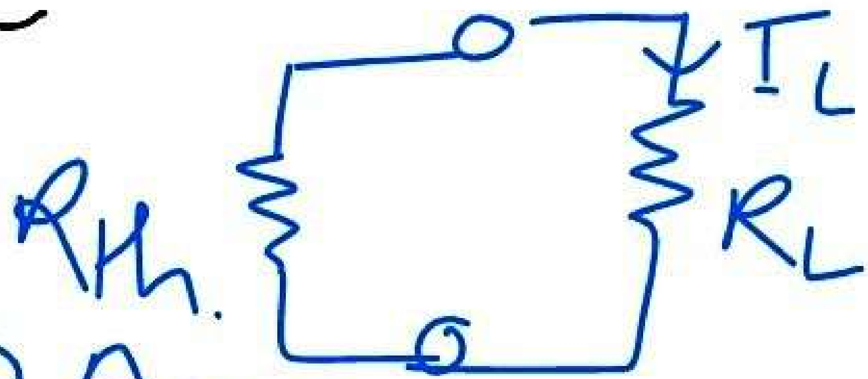
The presence of dependent sources in the network makes R_{th} sometime to have a negative value.

Note

→ All are dependent sources. (Dead N/w)

$V_{th} = 0$ $I_{sc} = 0$

$R_{th} \neq 0$

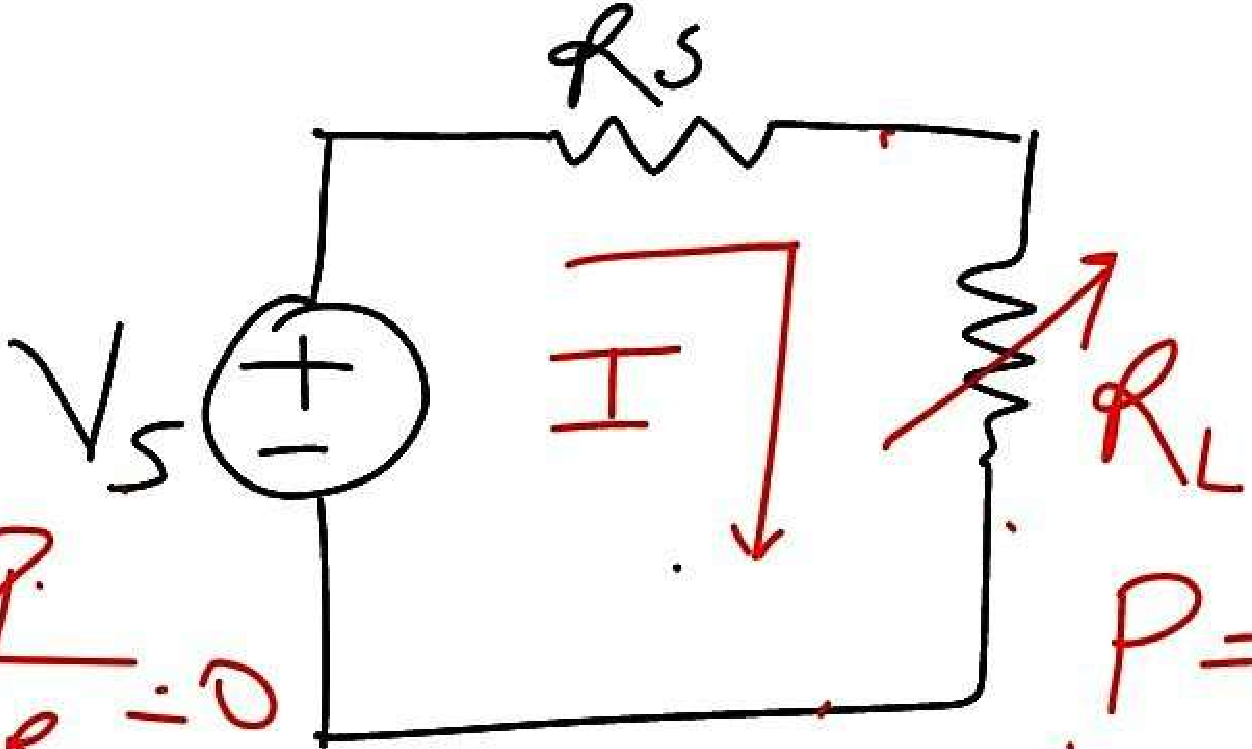


$I_L = 0$ Amp.

Maximum power Transfer Theorem

(1) Under Variable Load

$\Rightarrow R_S$ and R_L



$$I = \frac{V_S}{R_S + R_L}$$

$$P = I^2 R_L \ W$$

$$\frac{\partial P}{dR_L} = 0$$

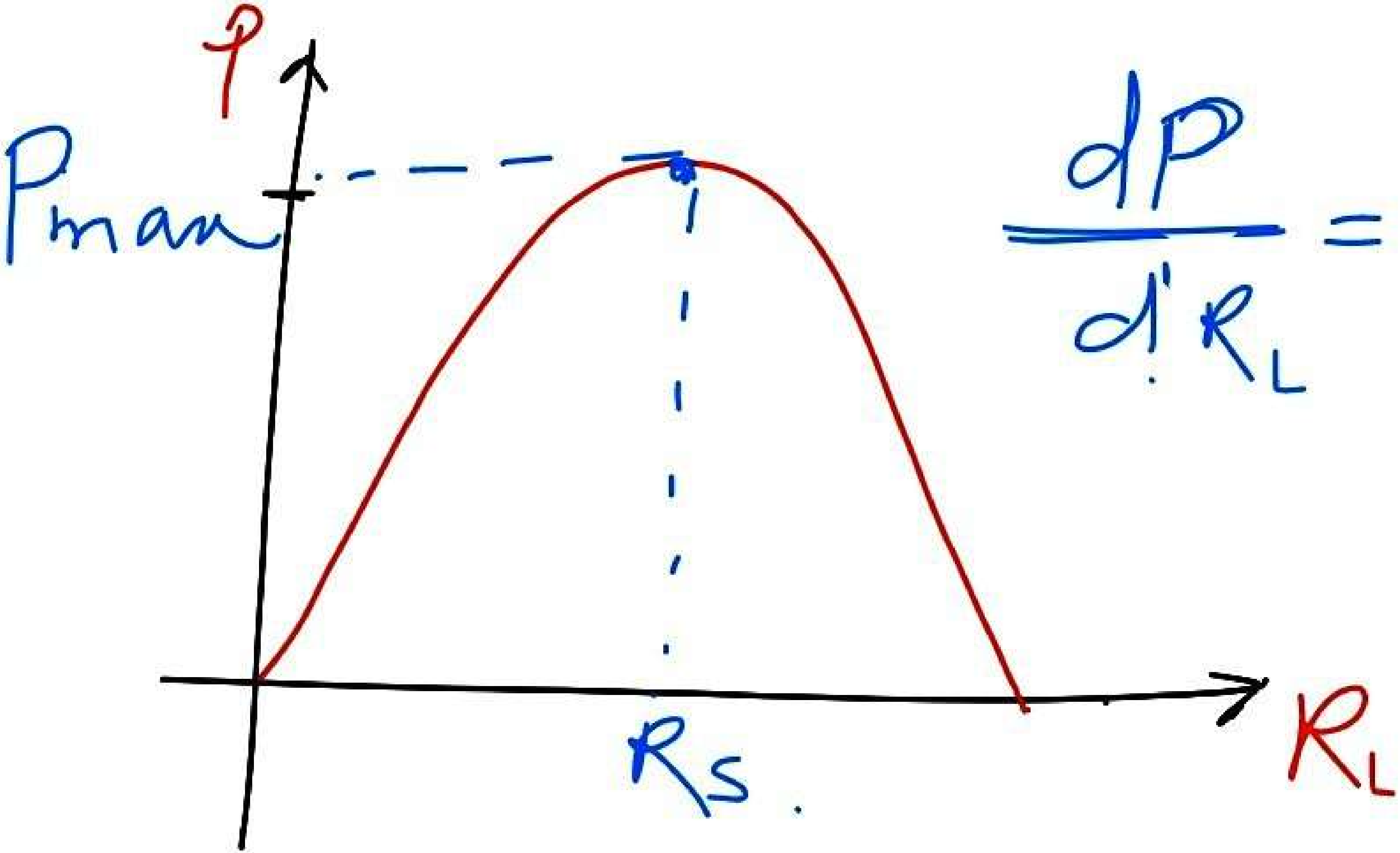
$$P = I^2 R_L \Rightarrow I = \frac{V_s}{R_s + R_L}$$

$$\boxed{P = \frac{V_s^2 \cdot R_L}{(R_s + R_L)^2}}$$

$R_L = 0$

$R_L = \infty$

P_{min}



$$\frac{dP}{dR_L} = 0$$

$$\frac{dp}{dR_L} = \frac{V_S^2 (R_S+R_L)^2 \cdot 1 - R_L \, 2(R_S+R_L) \cdot V_S^2}{(R_S+R_L)^2}$$

$$\Rightarrow \frac{V_S^2 \left[(R_S+R_L)^2 \cdot 1 - R_L \cdot 2 (R_S+R_L) \right]}{(R_S+R_L)^2}$$

$$\frac{Vs^2\left[(R_s+R_L)^2\cdot 1-R_L\cdot 2(R_s+R_L)\right]}{(R_s+R_L)^2}=0$$

$$\Rightarrow Vs^2\left[(R_s+R_L)^2\cdot 1-R_L 2(R_s+R_L)\right)=0$$

$$(R_S + R_L)^2 - R_L \cdot 2(R_S + R_L)$$

$$(R_S + R_L)^2 = R_L \cdot 2(R_S + R_L)$$

$$R_S + R_L = 2R_L$$

$$R_S = 2R_L - R_L \Longrightarrow \boxed{R_S = R_L \, \Omega}$$

$$P_{max} = P \Big|_{R_S = R_L} \qquad P = \frac{V_S^2 R_L}{(R_S + R_L)^2}$$

$$= \frac{V_S^2 R_L}{(R_S + R_L)^2} \Bigg|_{R_S = R_L}$$

$$\Rightarrow$$

$$\frac{V_s^2 R_s}{(R_s + R_s)^2}$$

$$= \frac{V_s^2 R_s}{4R_s^2} = \frac{V_s^2}{4R_s} \text{ Watt.}$$

$$P_{max} = \frac{V_s^2}{4R_L} \text{ Watt}$$

Useful power

$$P_{del} = P_{abs} \Leftarrow \text{Tellegen's Thm.}$$

$$P_T = I^2 R_S + I^2 R_L \Big|_{R_L = R_S}$$

Total power

$$= \frac{V_s^2}{4R_S} + \frac{V_s^2}{4R_L}$$

$$= \frac{2V_s^2}{4R_S} = \frac{1}{2}\frac{V_s^2}{R_s} \text{ or.}$$

$$\frac{V_s'^2}{2R_L} \text{ Watt} = \frac{V_s^2}{2R_s}.$$

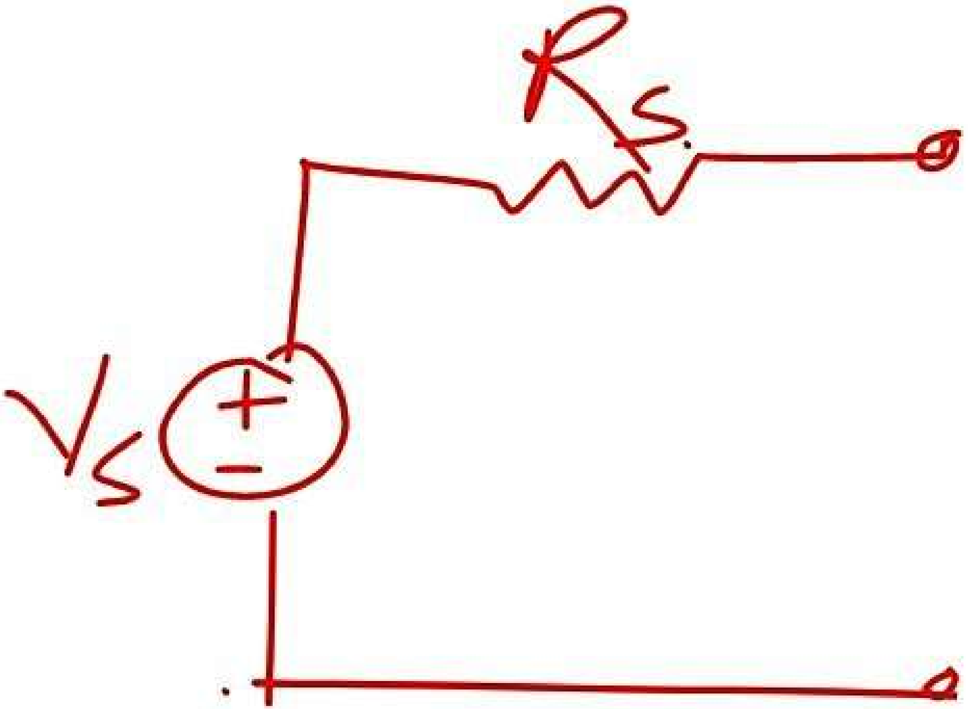
$$\eta = \frac{\text{Used forward power}}{\text{Total power}}.$$

$$\frac{\dfrac{V_s^2}{4R_s}}{\dfrac{V_s^2}{2R_s}} = \frac{1}{2} = 0.5$$

$$\% \eta = 50\%$$

The efficiency of maximum power ~~ter~~transfer is almost 50%.

Ex ⇒ A practical DC source when short circuited supplies a current of 2.5 Amp & it can provide a power of 80 Watt to a 20 Ω load. Determine the open ckt voltage & R_L for MPT.



$R_L = 20\,\Omega$

$\frac{V_S}{R_S} = 2.5\,\text{Amp} \quad \text{(i)}$

$P_{20\Omega} = 80\ \text{Watt.}$

$I^2_{20\Omega} R = 80$

$$I^2_{R=20}(20) = 80\,\text{Watt}$$

$$I^2_{R_{20}} = \frac{80}{20} = 4$$

$$\frac{V_s}{R_s + R_L}$$

$$I_{R_{20}} = 2A = \frac{V_s}{R_s + 20} \rightarrow (2)$$

$$\frac{V_s}{R_s} = 2.5 \quad \& \quad \frac{V_s}{R_s + 20} = 2A$$

$$\begin{cases} V_s = 2.5 R_s \\ V_s = 2(R_s + 20) \end{cases}$$

$$2.5 R_s = 2R_s + 40$$

$$2.5R_s = 2R_s + 40$$

$$2.5R_s - 2R_s = 40$$

$$0.5R_s = 40$$

$$R_s = \frac{40}{0.5}$$

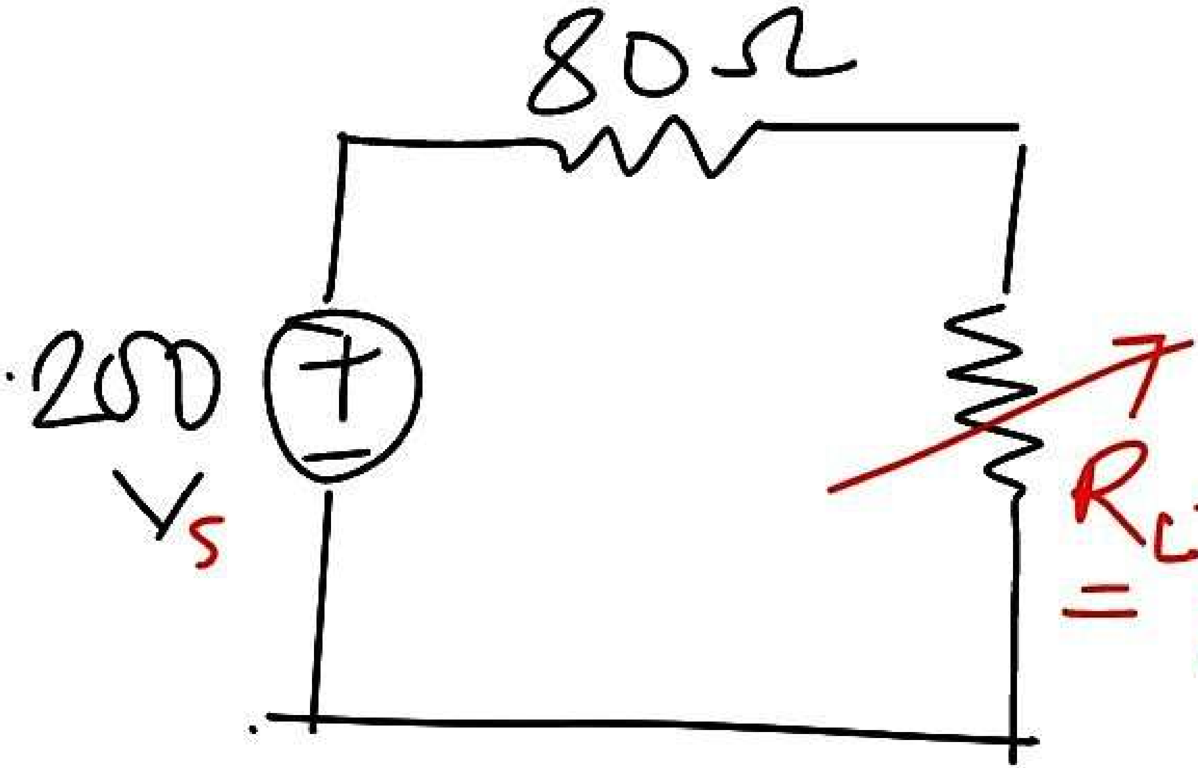
$$\boxed{R_s = 80\,\Omega}$$

$$\frac{V_s}{R_s} = 2.5$$

$$\frac{V_s}{80} = 2.5$$

$$V_s = 2.5 \times 80$$

$$\boxed{V_s = 200\,V}$$



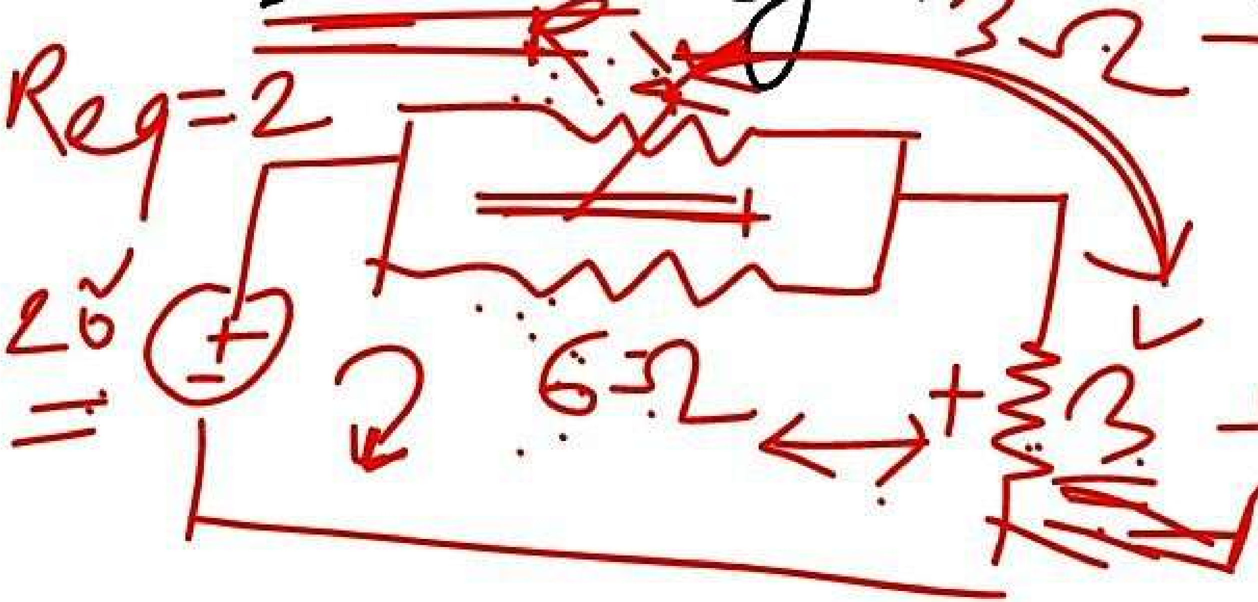
$R_L = R_S$
$= 80\Omega$ for MPT

$$P_{max} = \frac{V_S^2}{4R_S}$$

$$= \frac{(200)^2}{4 \times 80}$$

$$P_{max} = \frac{40000}{320} = \underline{\underline{125\,W}}$$

(2) Determine the value of R at which power transferred to $3\,\Omega$ Reg is maximum. P_{max}



$$I = \frac{20}{(6 \| R) + 3} \text{ A}$$

$R_L = R$

$$P_{3\Omega} = I^2_{3\Omega} R_{3\Omega}$$

$$P_{3\Omega} = I^2_{3\Omega} \cdot 3$$

$$= \left(\frac{20}{R_{eq} + 3} \right) A.$$

$$=$$

$$R_{eq} = 6 \,||\, R$$

$$6 \,||\, 3$$

$$= \frac{6 \times 3}{6 + 3} = \frac{18}{9}$$

$$= 2\Omega$$

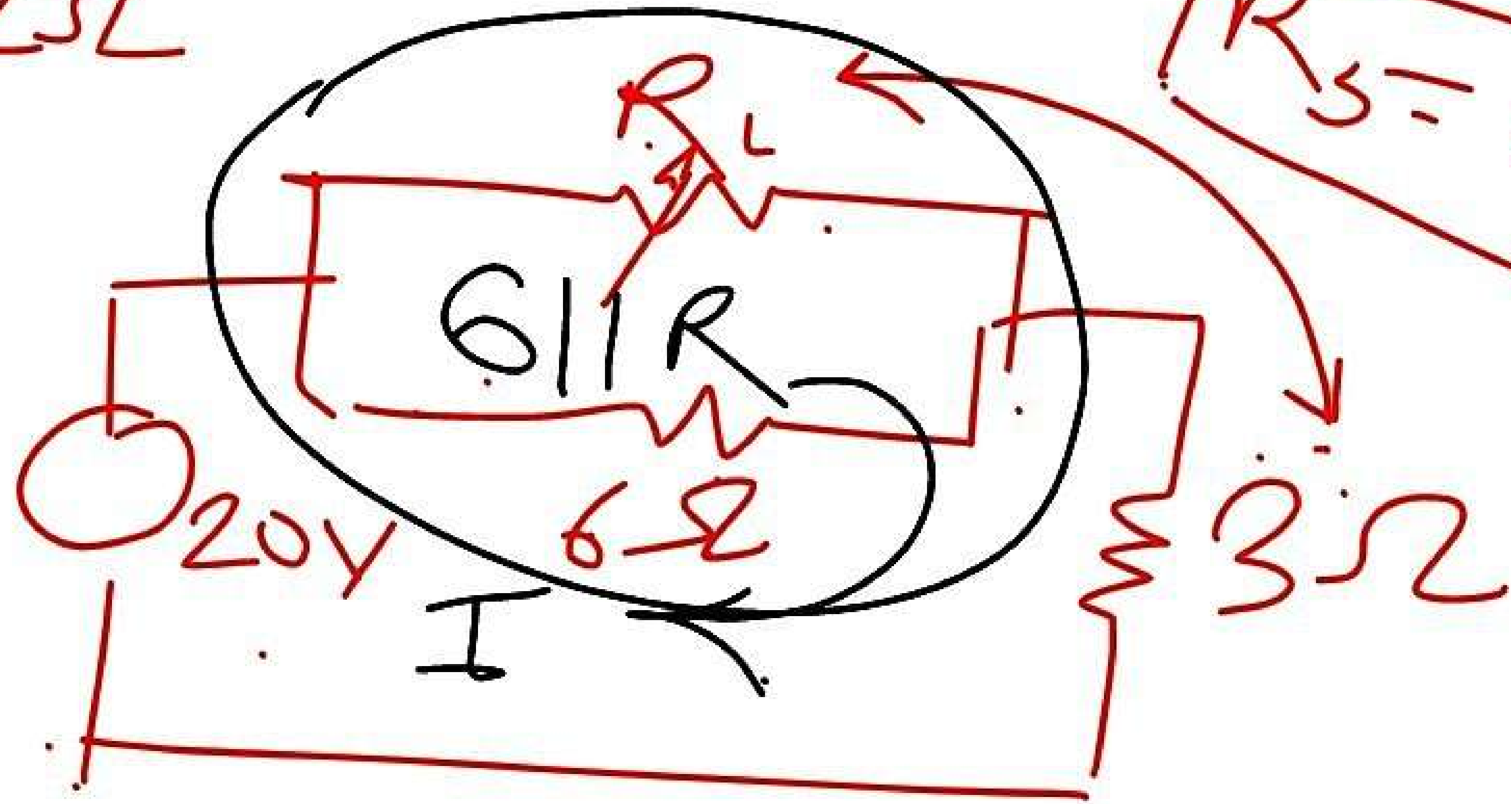
$R = 3\Omega$

$R_{eq} = 2\Omega$

I

$6 \parallel 3$

$\boxed{R_S = R_L}$



Re

$I = \frac{V}{R}$

$$I = \frac{V}{R_s + R} \qquad I^2 R \qquad R = R_{eq}$$

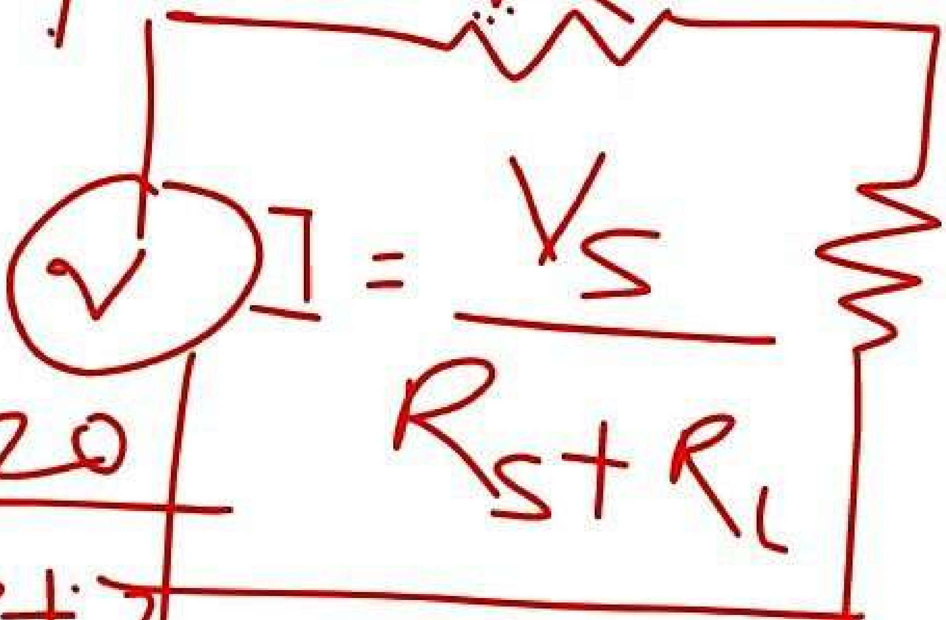
$$= \frac{V}{3 + (6 \| R)} \qquad I_{max} = 4\,Amp$$

$$= I^2 R$$

$$= \frac{v}{2+3} = \frac{20}{5} = 4^2 \times 3$$

$$= 48\,W$$

R_{eq} 3Ω R



$$I = \frac{V_S}{R_S + R_L}$$

$R_L = R$

$R = 3$

3Ω 6Ω

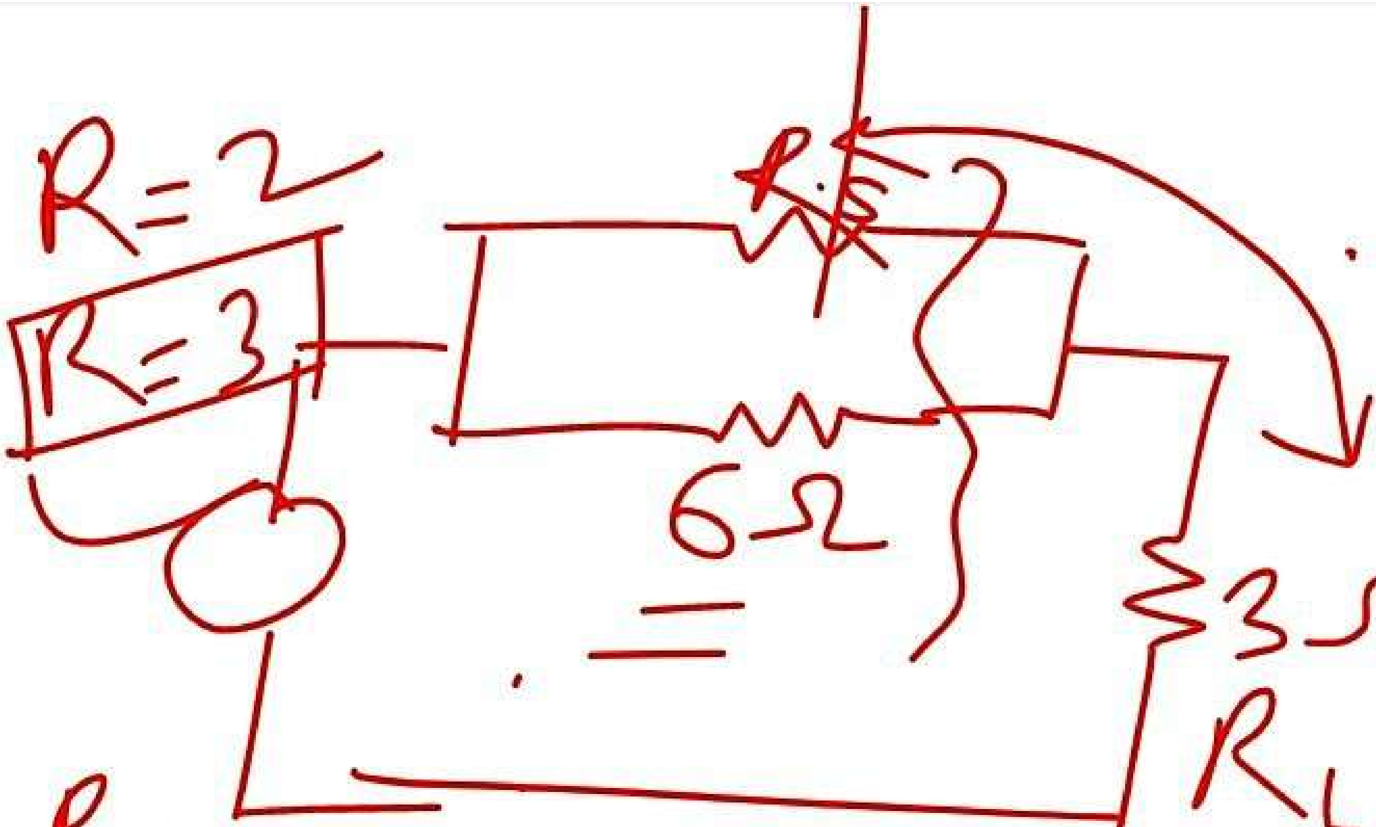
$R \parallel 6\Omega$

$R \parallel 6$

$= 3 \parallel 6$

$$= \frac{20}{3+3} = 20/6$$

$$\frac{3\times 6}{3+6} = 18/9 = 2\Omega$$



$R_s \parallel 6 = 3$

$$\frac{R_s \times 6}{R_s + 6} = 3$$

$R_s \times 6 = 3R_s + 18 \Rightarrow 3R_s = 18$

$= 6R_s - 3R_s = 18 \Rightarrow R_s = 6$

$$\frac{dP}{dX_L}=0 \quad \frac{V_S^2\left[\left[(R_S+R_L)^2+(X_S+X_L)^2\right]\cdot 0-R_L\cdot 2(X_L+X_S)\right]}{\left[(R_S+R_L)^2+(X_S+X_L)^2\right]^2}=0$$

$$R_L i_0^2 (X_S + X_L) = 0$$

$$\Rightarrow X_S + X_L = 0$$

$$P_{max} = P\big|_{X_S + X_L = 0}$$

$$\text{①}\ R_L = \sqrt{R_S^2 + (X_S + X_L)^2}\ \Omega$$

$$\text{②}\ X_S \mp X_L = 0$$

(3) Both R_L & X_L are varied simultaneously

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$X_s + X_L = 0$$

are valid at a time.

$$\boxed{R_L = R_S}$$

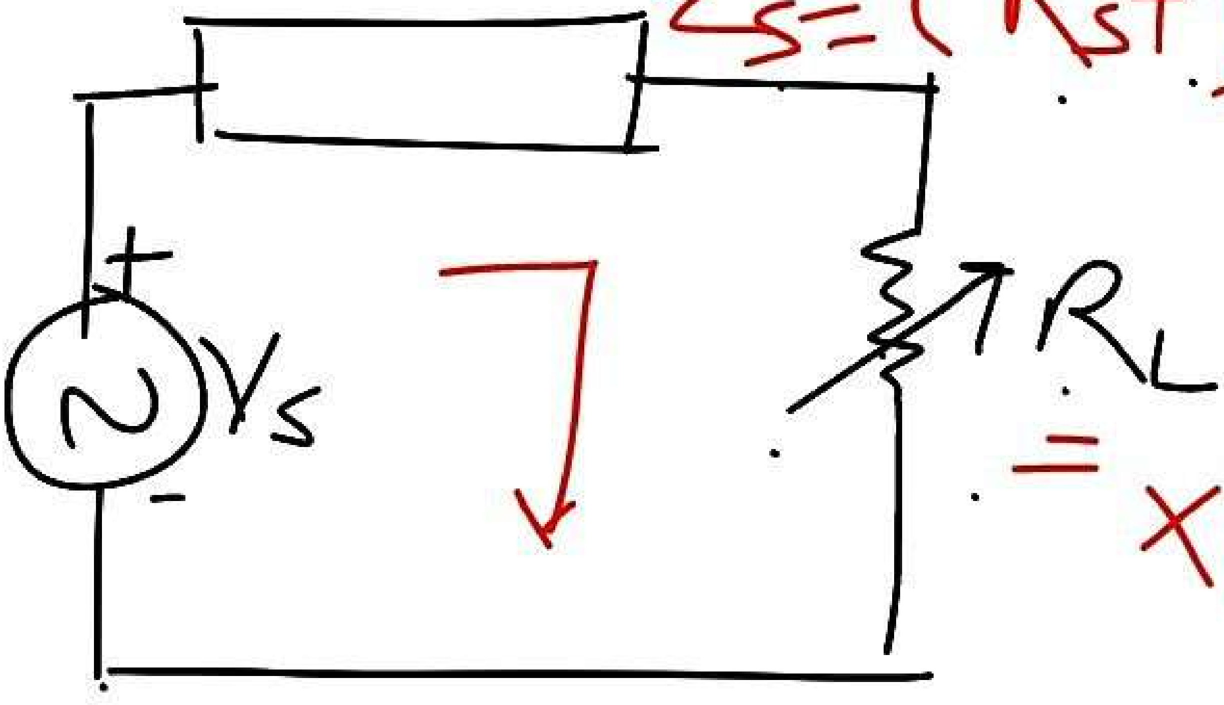
$$\boxed{X_L = -X_S}$$

$$Z_L = Z_S^*$$

$$P_{max}\Big|_{\substack{Z_S \,\&\, Z_L \\ R_L = R_S \\ X_L = -X_S}} = \frac{V_S^2}{4R_L} = \frac{V_S^2}{4R_S}$$

Z_s & R_L

$Z_s = (R_s + jX_s)$



$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$

$= 0$

$= X_L = 0$

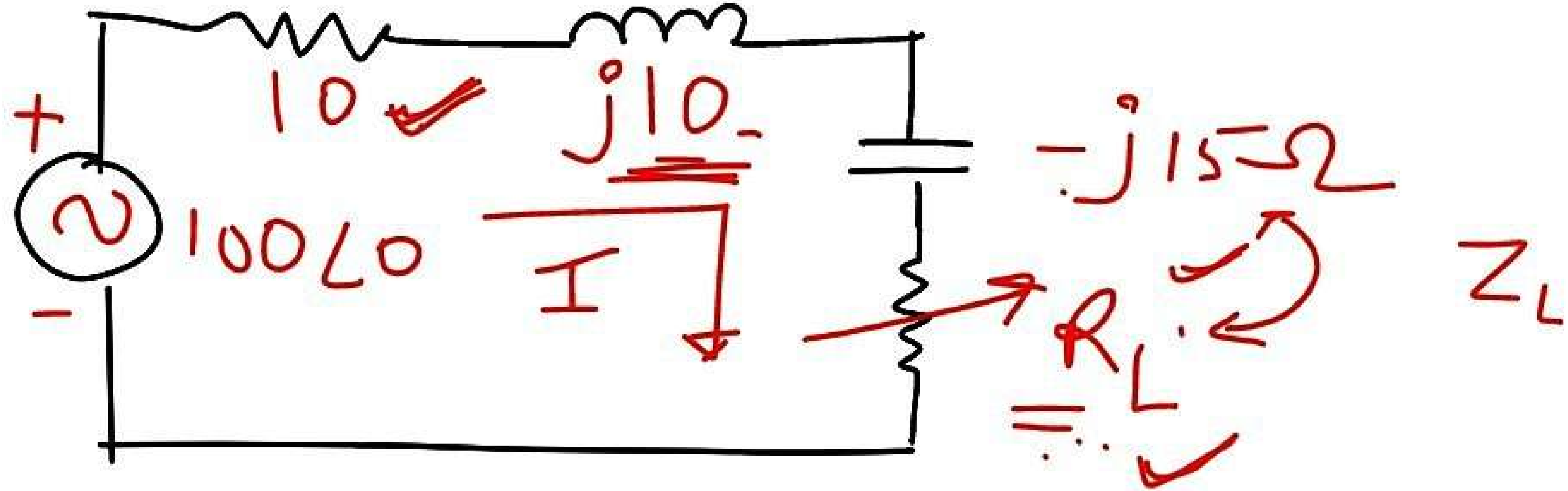
$$R = \sqrt{R_s^2 + X_s^2} \quad \Omega$$

$$R_L = |R_s + jX_s|$$

$$\boxed{R_L = |Z_s|}$$

$$P_{max}\Big|_{R_L = |Z_s|}$$

1. Determine the max power transfer to R_L



$$Z_L = (R_L - j15)$$

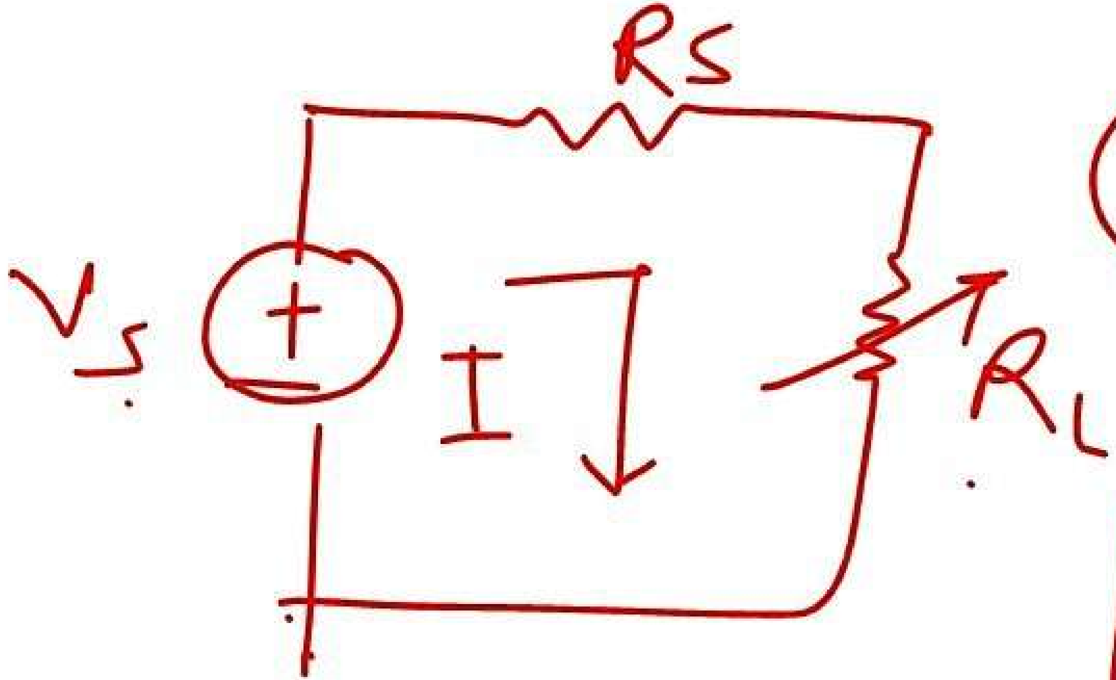
$$= R_L + j(-15)$$

$$= R_L + jX_L \Big|_{X_L = -15}$$

$$R_L = \sqrt{R_S^2 + (X_L + X_S)^2}$$

① Khan Z_s & Z_L.

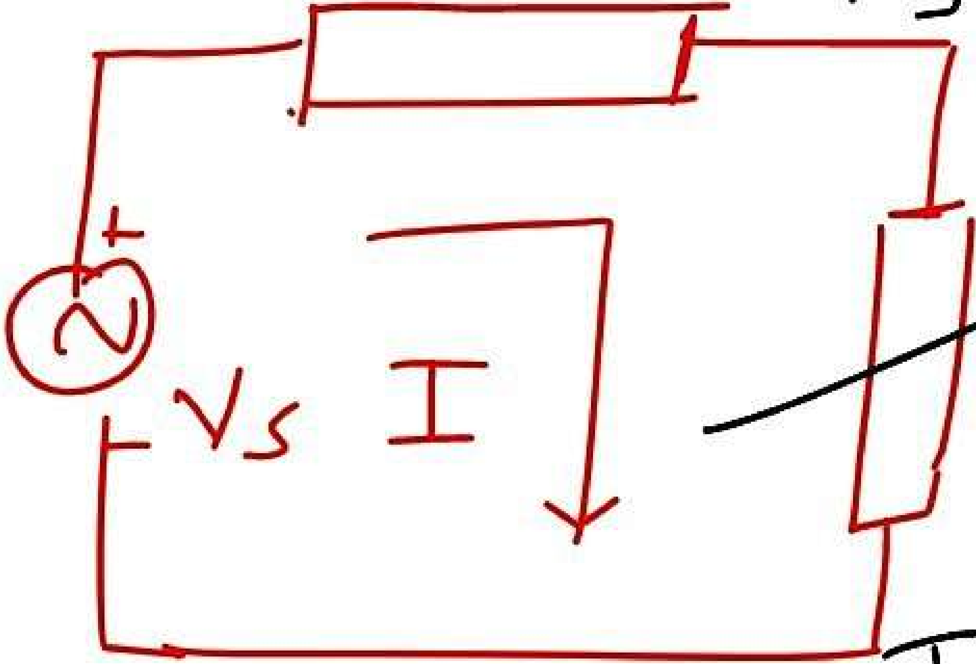
$$P = \frac{V_s^2 R_L}{(R_s+R_L)^2+(X_s+X_L)^2} \text{ W.}$$



1 under variable load

$$\boxed{R_S = R_L}$$

$$Z_S = (R_S + jX_S)$$



$$Z_L = (R_L + jX_L)\,\Omega$$

$$I = \frac{V_S}{R_S + R_L + j(X_S + X_L)}$$

$$P = I^2 R_L$$

$$P = \frac{V_s^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2} \text{ Watt}$$

$$\boxed{\frac{dP}{dR_L} = 0} \Leftarrow \text{for MPT}$$

'R_L' is variable

R_L is variable.

$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}\ \Omega$$

$$P_{max} = P\Big|_{R_L = \sqrt{R_S^2 + (X_S + X_L)^2}}$$

$X_L \Rightarrow$ is variable.

$$\frac{dP}{dX_L} = 0$$

$$\boxed{X_S + X_L = 0}$$

$P_{max} \mid P_{at\ X_S + X_L = 0}$

R_L & X_L are varying.

$$\boxed{Z_L = Z_S^*}$$

$R_L = R_S$ & $X_L = -X_S$.

$P_{max} \Big|_{P \text{ ket}}$ $R_L = R_S$.

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

$$X_s + X_L = 0$$

$$\boxed{Z_L = Z_s^*}$$

$$\boxed{R_L = R_s} \qquad \boxed{X_s = -X_L}$$

$$Z_L = R_L + jX_L = R_s - jX_s$$

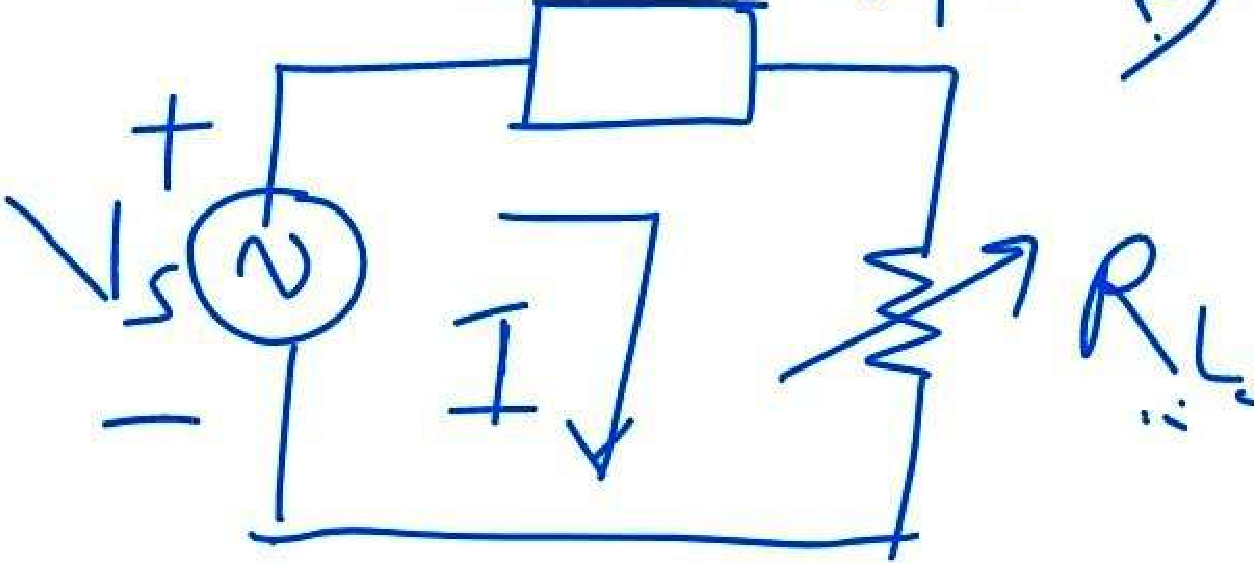
$$X_L = -X_S.$$
$$R_L = R_S$$

$$\frac{V_s^2}{4R_L} = \frac{V_s^2}{4R_s}.$$

Watt.

④ Z_s & R_L

$Z_s = (R_s + jX_s)$



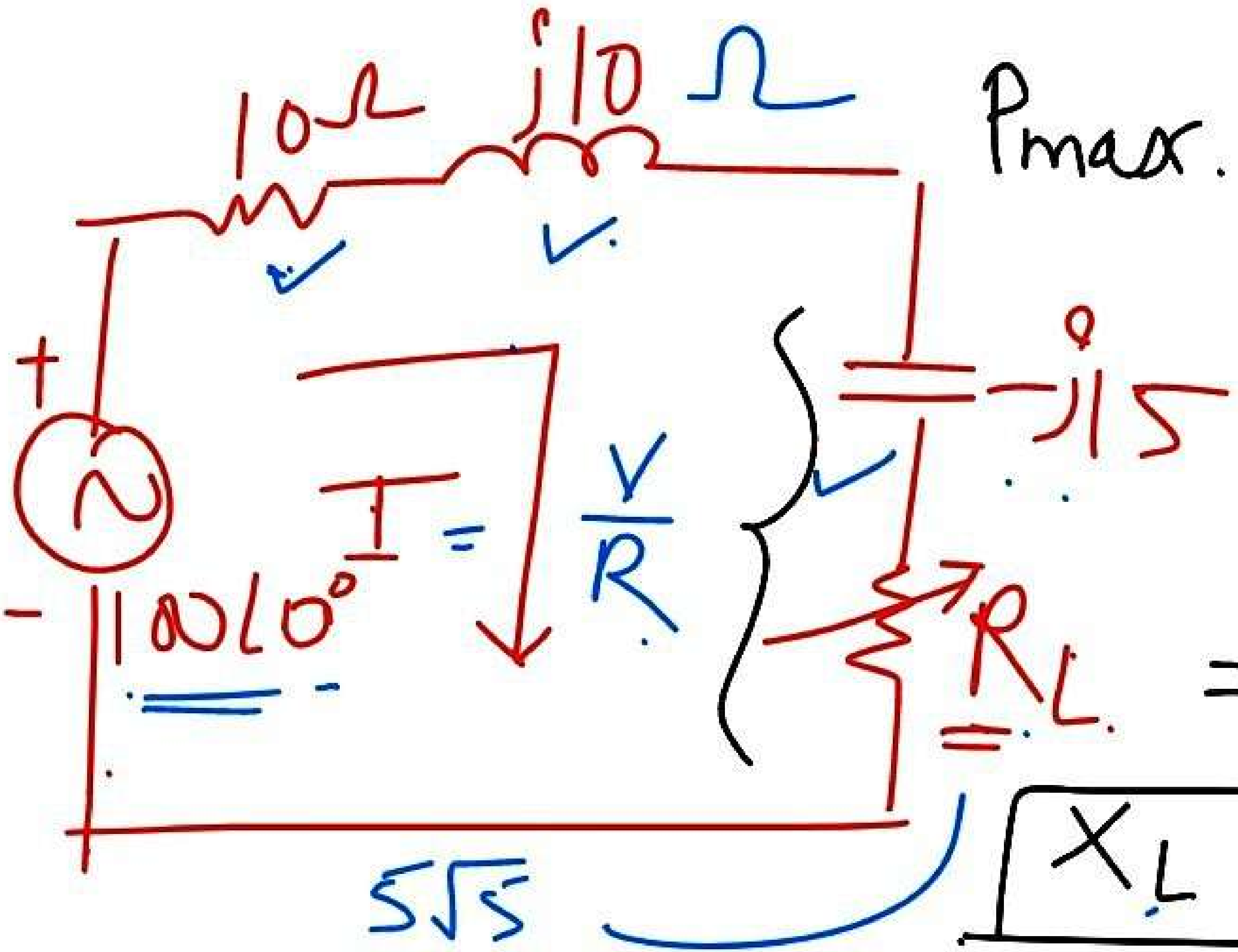
$\boxed{Z_L = R_L}$

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

~~At~~ $X_L = 0$

$$\boxed{R_L = \sqrt{R_s^2 + X_s^2}}\ \Omega$$

$$R_L = |R_s + jX_s| \Rightarrow R_L = |Z_s|$$



P_{max}. at R_L

$$Z_L = R_L - j15$$
$$= R_L + j(-15)$$
$$= R_L + jX_L$$

$$\boxed{X_L = -15}$$

for MPT

$$R_L = \sqrt{R_S^2 + (X_L + X_S)^2}$$

$$= \sqrt{10^2 + (-15 + 10)^2}$$

$$= \sqrt{10^2 + (-5)^2} = 5\sqrt{5}\ \Omega$$

$$\underline{I} = \frac{100\angle 0}{10 + j10 - j15 + 5\sqrt{5}} \Rightarrow \frac{100\angle 0}{10 + 5\sqrt{5} - j5}$$

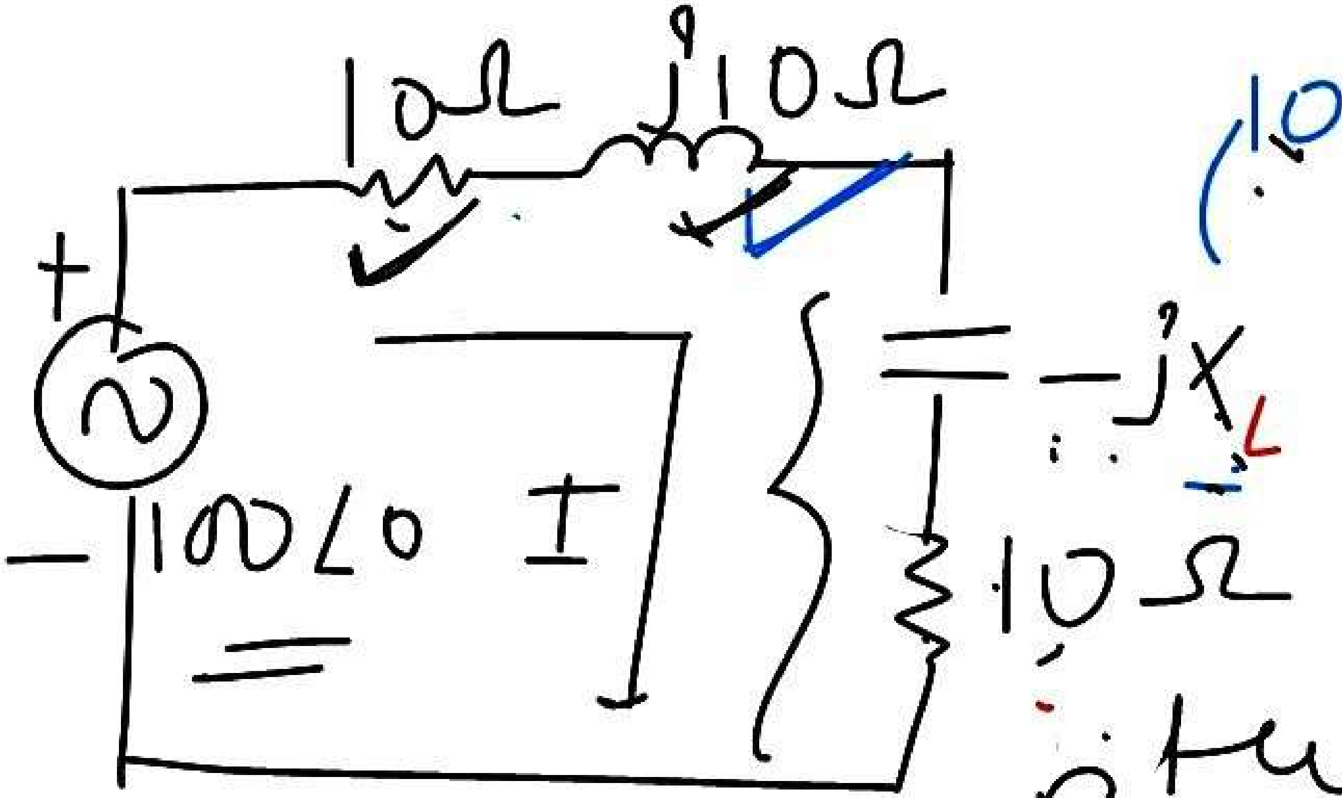
$$P_{max} = I^2 R_L$$

$$P_{max} = \left| \frac{100\angle 0}{21.18 - j5} \right|^2 \cdot 5\sqrt{5}$$

$$P_{max} = \underline{\underline{236 \text{ Watt}}}$$

$$=$$

$$4.59 \angle 13.28$$



Determine the value of X_L at which power transfer to $\underline{10\ \Omega}$ (R_L) load resister is max.

$$Z_L = 10 - jX_L$$
$$= 10 + j(-X_L)$$
$$= R_L + jX_L \quad \Omega$$

for MPT $\quad X_S + X_L = 0$

$$= 10 - X_L = 0$$

$$\boxed{X_L = 10}$$

$$I = \frac{100\angle 0^0}{10 + j10 - j10 + 10}$$

$$= \frac{100\angle 0}{20}$$

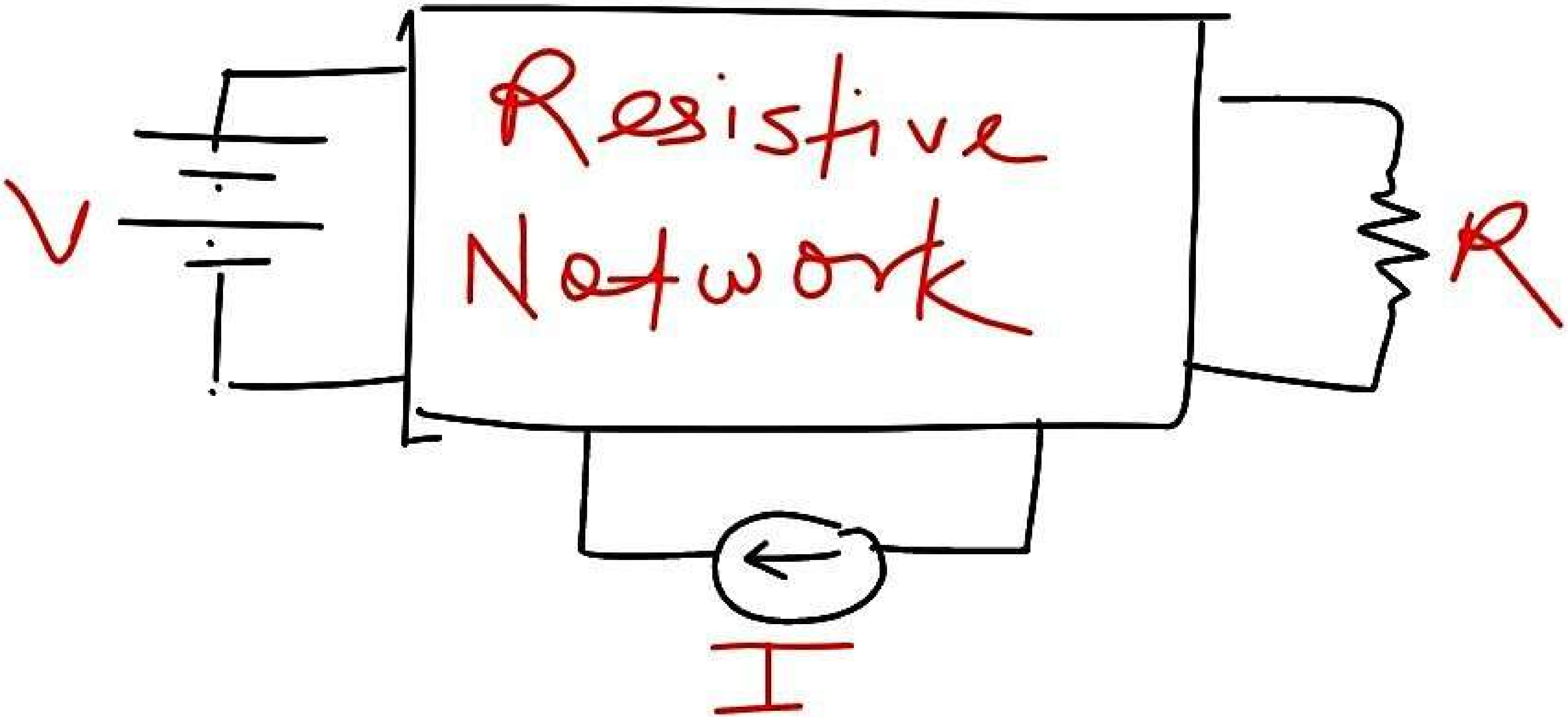
$$I = 5\angle 0^\circ$$

$$P_{max} = |I|^2 \cdot R_L$$

$$= (5)^2 \times 10$$

$$\boxed{P_{max} = 250 \text{ Watt}}$$

A particular resister R dissipates a power of 4W when V alone is active. The same Resister R dissipates a power of 9 W. when I alone is active The max & min power dissipated by R when both sources are active


Resistive
Network
V
R
I

$P = I_1^2 R$ (when ν is active)

$4 = I_1^2 R$

$$I_1 = \frac{2}{\sqrt{R}}$$

When I is active.

$$I_2^2 R = 9$$

$$I_2 = \frac{3}{\sqrt{R}}$$

when both active. $\left\{ \begin{array}{l} I_1 + I_2 \quad \text{OR} \\ I_1 - I_2 \end{array} \right.$

$$P = (I_1 + I_2)^2 R$$

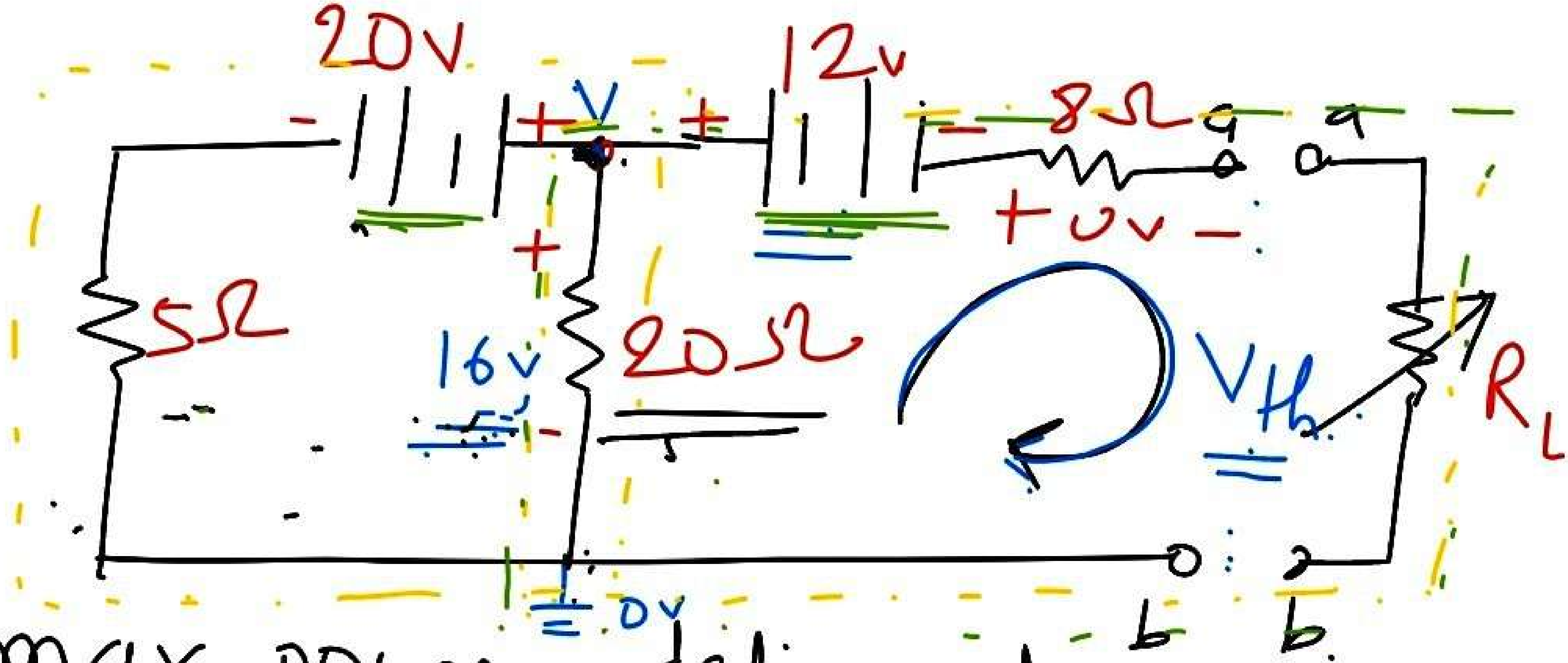
$$= \left(\frac{2}{\sqrt{R}} + \frac{3}{\sqrt{R}}\right)^2 R$$

$$= \left(\frac{5}{(\sqrt{R})}\right)^2 R \Rightarrow 25 \text{ Watt}$$

$$P = (I_1 - I_2)^2 R$$

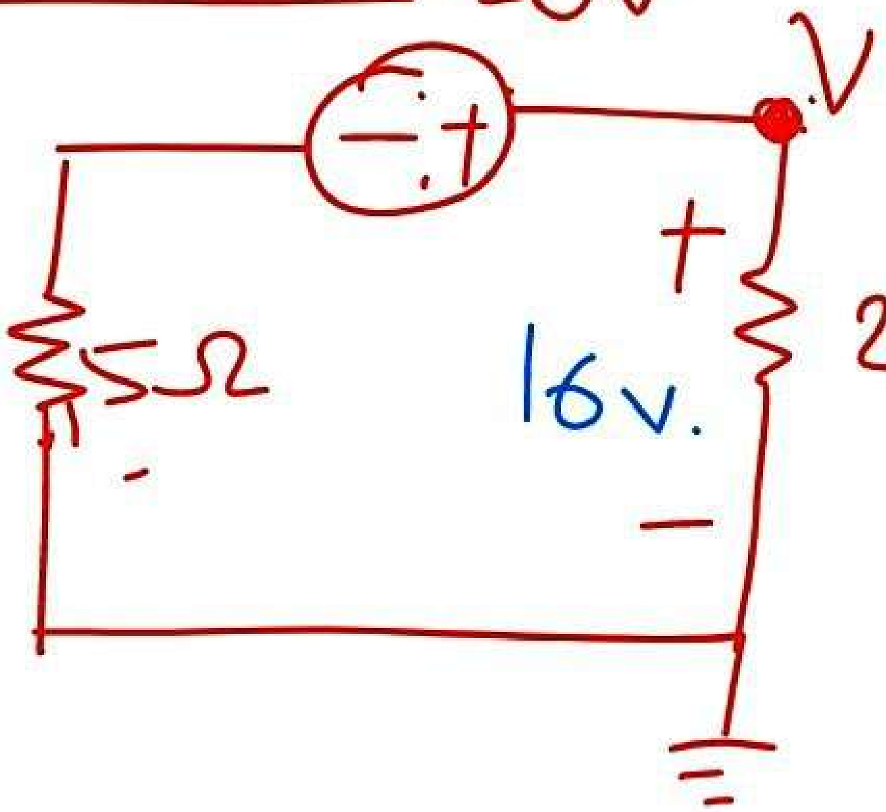
$$= \left(\frac{2}{\sqrt{R}} - \frac{3}{\sqrt{R}}\right)^2 R.$$

$$= \left(\frac{-1}{\sqrt{R}}\right)^2 \cdot R = 1 \text{ Watt}.$$



max power delivered to R_L

By Nodal



$$\frac{V}{5} - \frac{20}{5} + \frac{V}{20} = 0$$

$$\frac{4V - 80 + V}{20} = 0$$

$$5v - 80 = 0$$

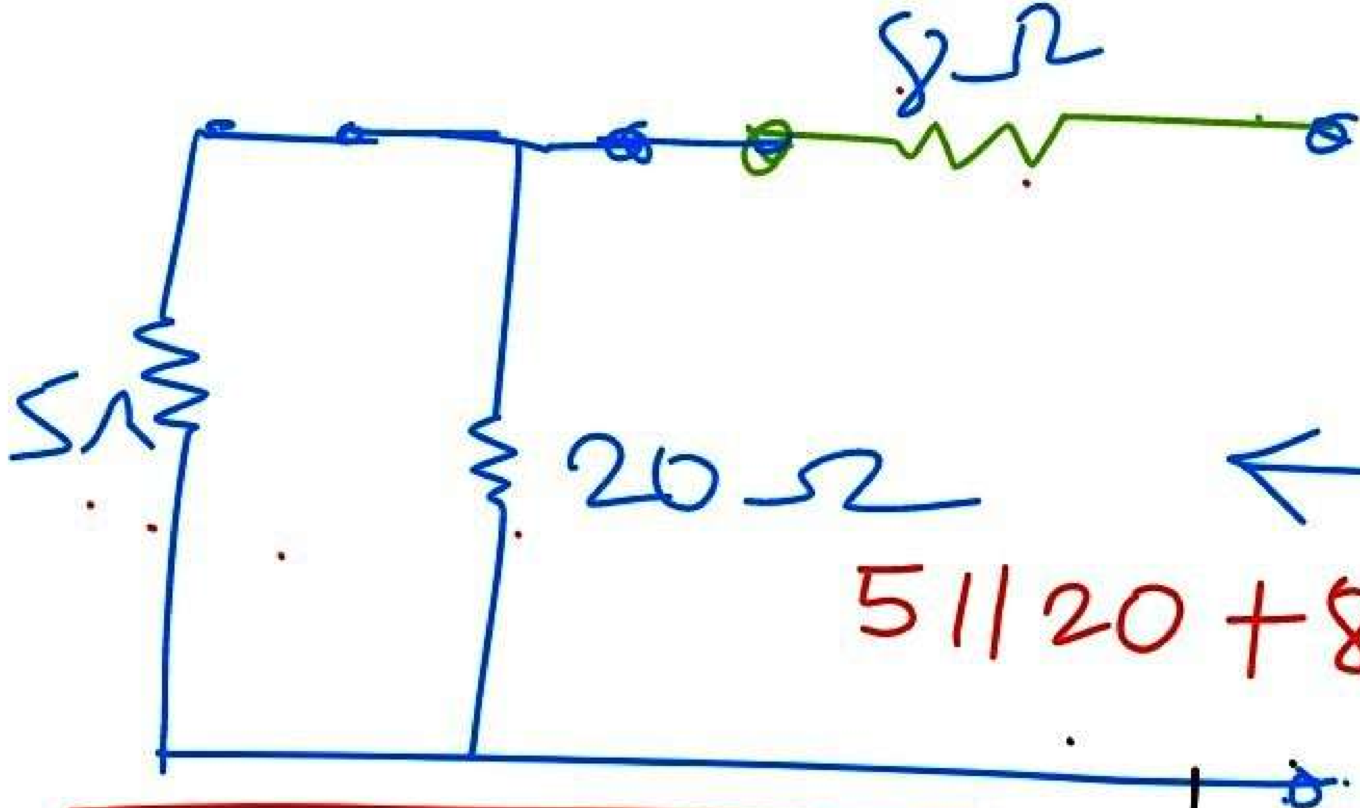
$$5v = 80$$

$$\boxed{v = 16\,V}$$

By KVL.

$$16v - 12 - V_{th} = 0$$

$$\boxed{V_{th} = 4V}$$

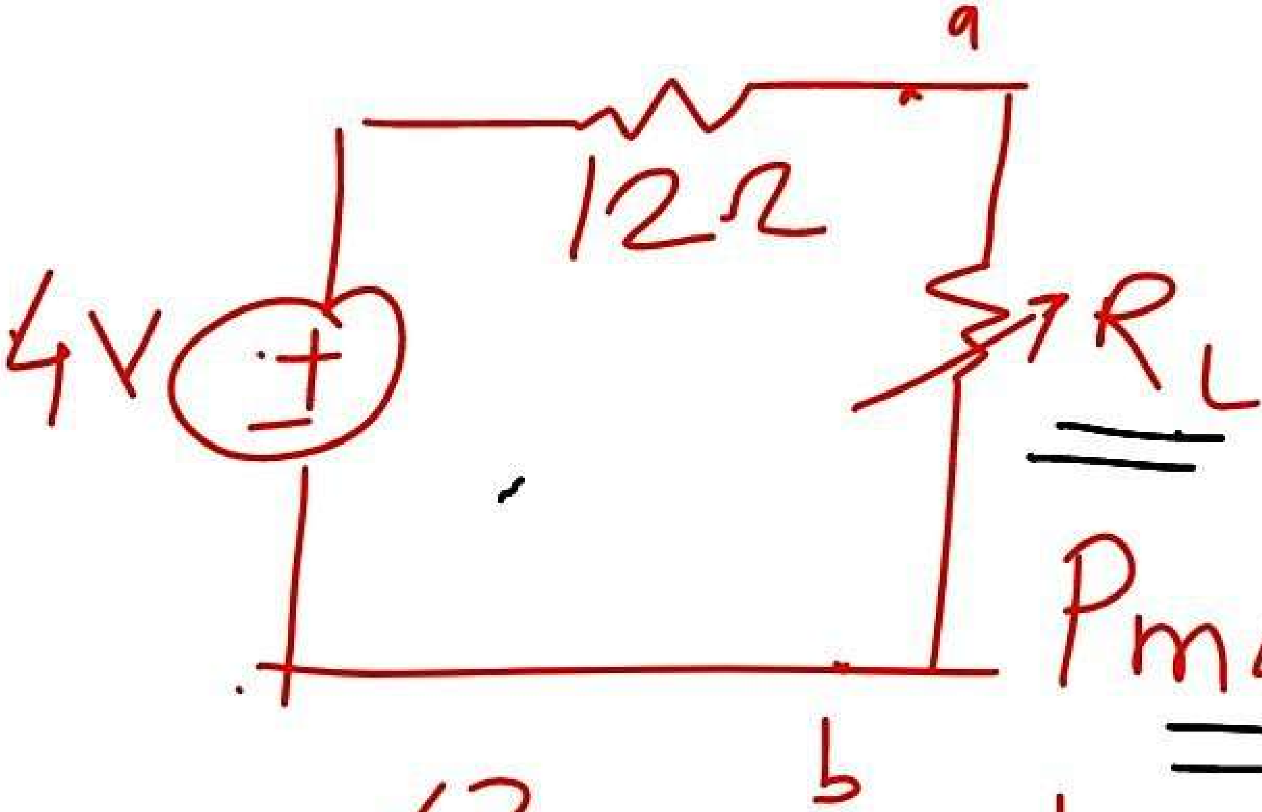


$\leftarrow R_{Th}$

$$5 \parallel 20 + 8 = \frac{5 \times 20}{5+20} + 8$$

$$= \frac{100}{25} + 8$$

$$\boxed{R_{Th} = 12\,\Omega}$$



$$R_L = R_s$$

$$= 12\,\Omega$$

$$P_{max} = \frac{V_s^2}{4R_L}$$

$$= \frac{4^2}{4 \times (12)} = \frac{1}{3}$$

$$\boxed{P_{max} = \tfrac{1}{3}\ \text{Watt}}$$

Circuit Theorems

Superposition Theorem :-

Definition :-

→ In a linear network with several independent sources, the response in a particular branch when all the sources are acting simultaneously is equal to the linear sum of individual responses calculated by taking one independent source at a time.

→ All the ideal voltage sources are eliminated from the network by shorting the sources, all the ideal current sources are eliminated from the network by opening the sources and do not disturb the dependent sources present in the network.

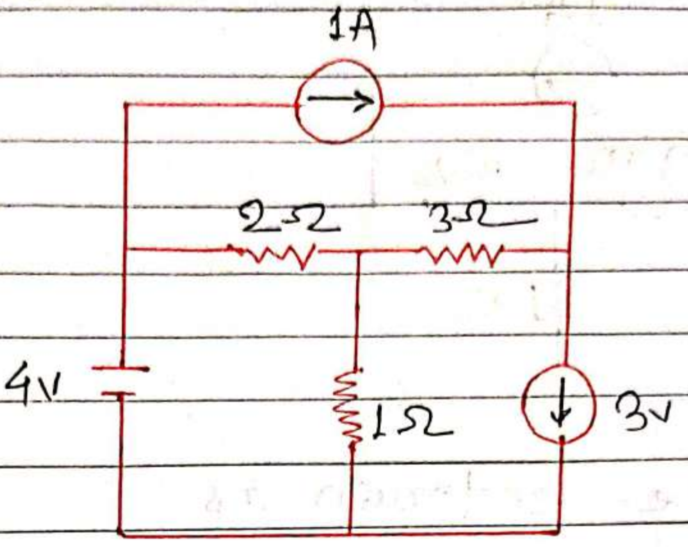
Properties :-

→ This theorem is applicable only to linear networks i.e. the networks with R, L, C transformer and linear controlled sources as elements.

→ The presence of dependent sources makes the network an active and hence SPT is used for both active as well as passive networks.

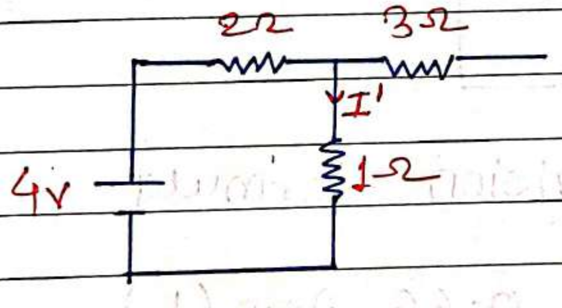
Example :-

1) find the current in the 1 Ω resistor using super position theorem.



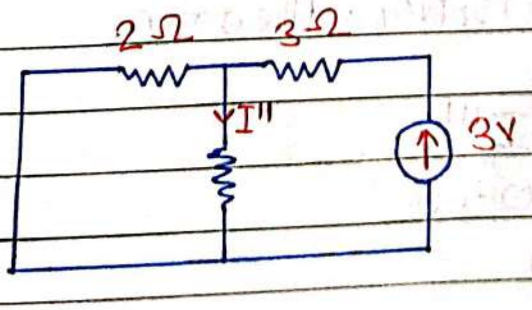
step 1 :-

when the 4 V source is acting alone.



$$I' = \frac{4}{2+1} = 1.33 \text{ Ampere } (\downarrow)$$

step II :-

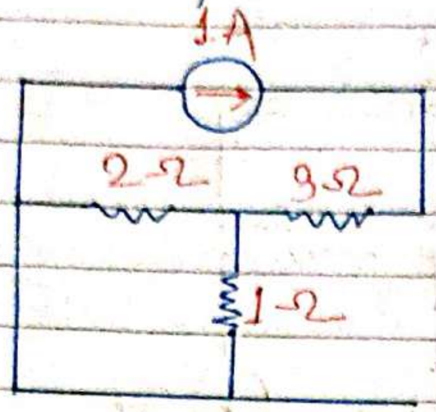


By current division formula

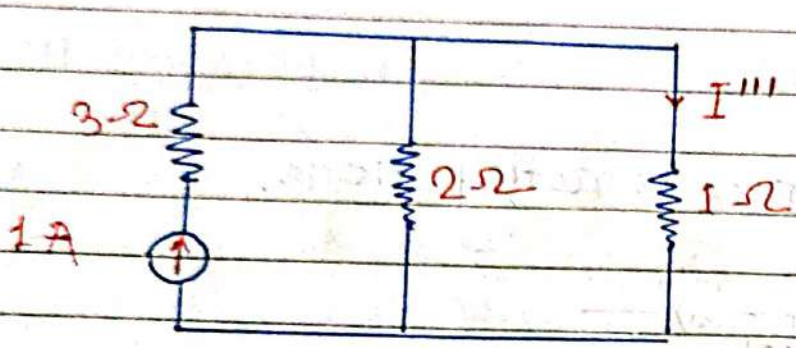
$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ Amp} (\downarrow)$$

Step III

When the 1-ampere source is acting alone



Circuit can be redrawn as



By current - division formula

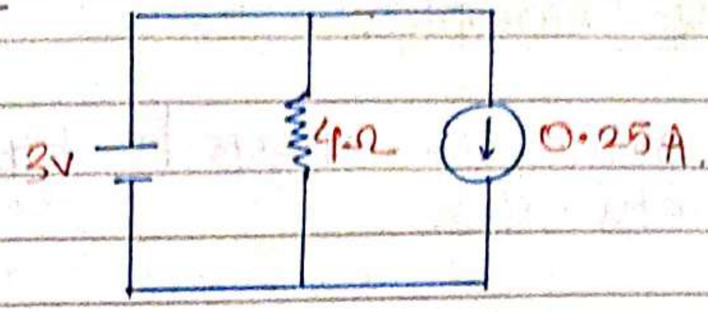
$$I''' = \frac{1 \times 2}{2+1} = 0.66 \text{ Amp} (\downarrow)$$

Step IV

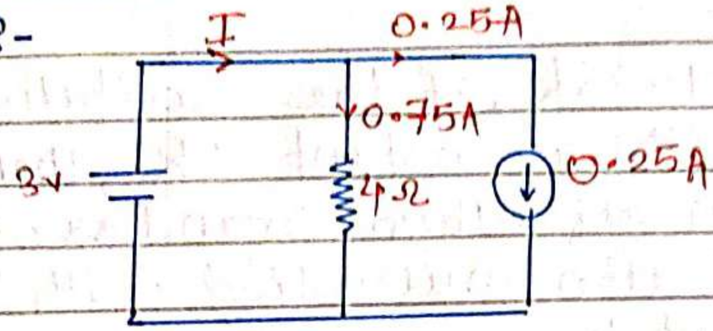
By superposition theorem

$$I = I' + I'' + I'''$$
$$= 1.33 + 2 + 0.66$$
$$= 4 \text{Amp} (\downarrow)$$

(2) Determine I



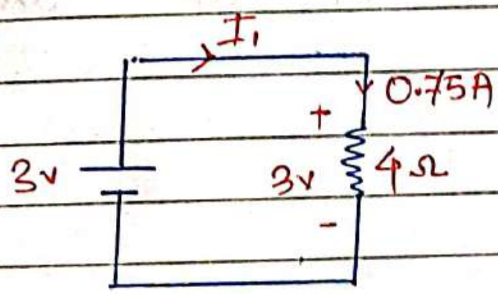
By KCL :-



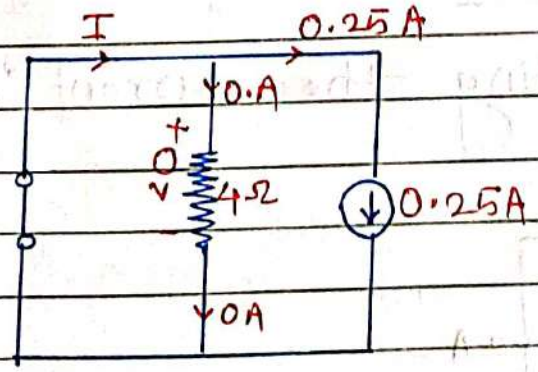
By KCL ⇒ $-I + 0.75 + 0.25 = 0$

$\therefore$ $\underline{I = 1.0 \text{ Amp.}}$

By Superposition Theorem :-



$I_1 = \frac{3}{4} = 0.75 \Omega$



$I_2 = 0.25 A$

By SPT

$I = I_1 + I_2$

$= 0.75 + 0.25$

$= 1.0 \text{ Amp.}$

$\underline{I = 1 \text{ Amp.}}$

Homogeneity Principle:-

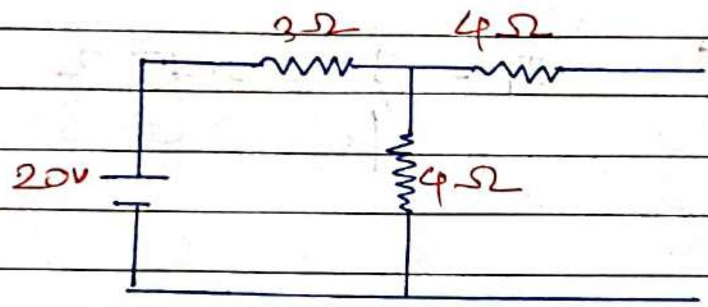
It is the principle obeyed by all the linear networks.

Definition:-

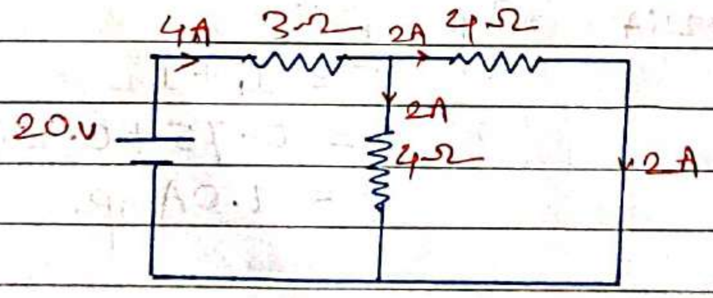
In a linear network, if the excitation is multiplied with a constant 'K', then the responses in all other branches of the network are also multiplied with the same constant K.

Example:-

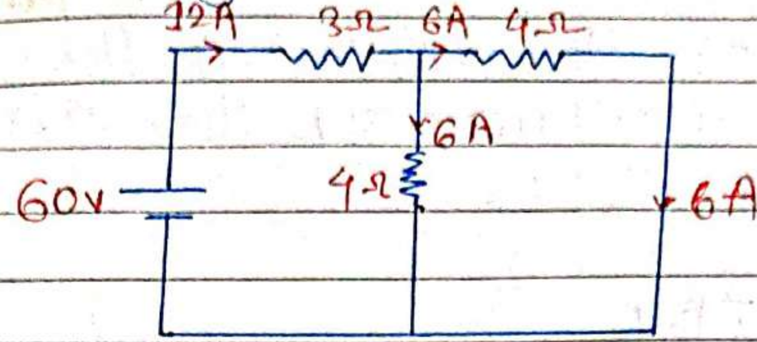
1) Find all branch currents if source voltage is increased to 60 volts in the circuit below.



Solution :- first calculating the current in each branch.



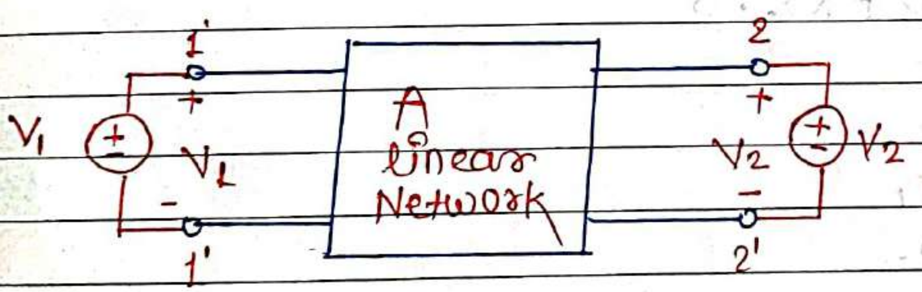
So here, the excitation is multiplied by '3' and hence the responses are also proportionally multiplied by '3'.



Note:-

When multiple sources are present superposition theorem to be applied first and later Homogeneity principle. i.e. Whenever Homogeneity principle is to be applied always ensure only one source in the network.

2) Determine I when $V_1 = 10V$ & $V_2 = -20V$.



V_1	V_2	I
2V	0V	0.5A
0V	5V	-1A

⇒ Method 1

→ When $V_1 = 2V$ alone then the current is 0.5A (Given)

→ When $V_1 = 10V$ alone then current $I_1 = 5 \times 0.5$

$I_1 = 2.5$ Amp

(Required)

When $V_2 = 5V$ alone then the current is $-1A$ (Given)
When $V_2 = -20V$ alone the current $I_2 = -4 \times (-1)$

$I_2 = 4 Amp$
(Required)

By superposition theorem, when the both sources acting simultaneously the overall current $I = I_1 + I_2$

$= 2.5 + 4$
$= 6.5$ Amp.

Method (2)

By superposition theorem,

$$I = I_1 + I_2$$

$$I = K_1 V_1 + K_2 V_2 \quad \text{(By homogeneity)}$$

$$0.5 = K_1 \times 2 + 0$$
$$-1 = 0 + K_2 \cdot 5$$

$$0.5 = 2K_1$$
& $$-1 = 5K_2$$

$$I = \left(\frac{V_1}{4} - \frac{V_2}{5}\right) \text{ Amp.}$$

$$= \left[\frac{10}{4} - \left(\frac{-20}{5}\right)\right]$$

$$= \left(\frac{10}{4} + \frac{20}{5}\right)$$

$$= \frac{50+80}{20}$$

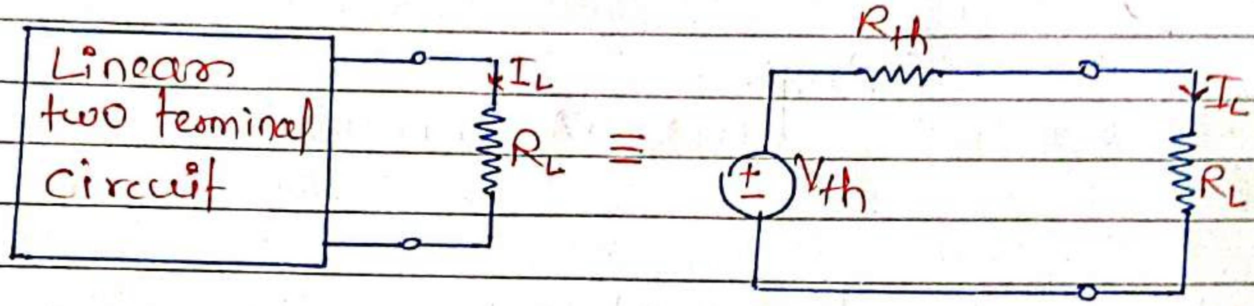
$$= \frac{130}{20}$$

$$= 6.5 \text{ Amp.}$$

Thevenins & Nortons Theorem :-

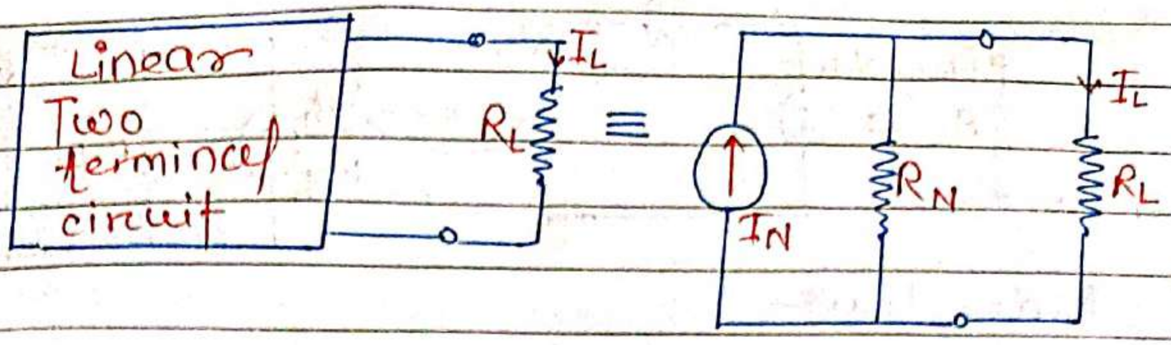
(1) Thevenin's Theorem :-

Thevenin's theorem states that a linear two terminal circuit canbe replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, where V_{th} is the open circuit voltage at the load terminals and R_{th} is the input or equivalent resistance at the load terminals when the independent sources are turned off. i.e. killed.



(2) Norton's Theorem :-

Norton's theorem states that a linear two terminal circuit canbe replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short circuit current through the load terminals and R_N is the input or equivalent resistance at the load terminals when the independent sources are ~~tern~~ turned off. i.e. killed.

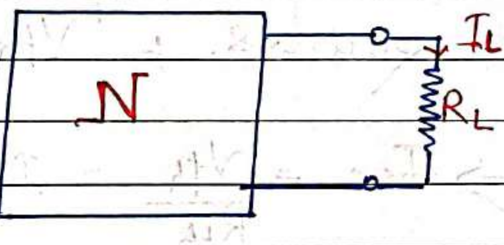


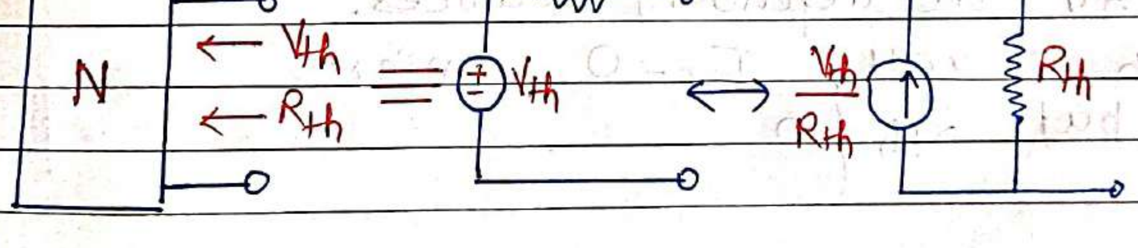
Properties :-

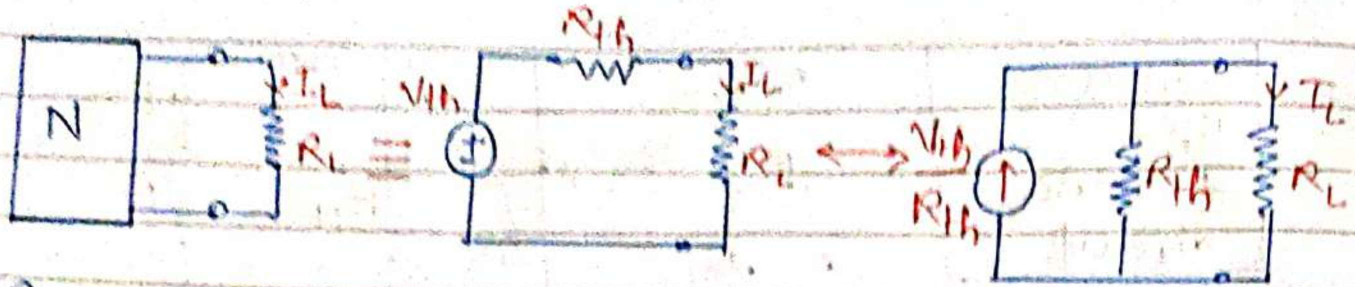
→ These theorems are applicable only for linear networks i.e. networks with R, L, C transformer and linear controlled sources as elements.

→ Presence of dependent sources makes the network active and hence these theorems are used for both active as well as passive networks.

Procedure :-







By ohm's law

$$I_L = \left(\frac{V_{th}}{R_{th}+R_L}\right) \text{Amp.}$$

$$I_L = \frac{\frac{V_{th}}{R_{th}} \cdot R_{th}}{R_{th}+R_L} = \left(\frac{V_{th}}{R_{th}+R_L}\right) \text{Amp.}$$

Cases in Thevenin Theorem (Norton's theorem) based on sources.

Case 1 :- (All are Independent sources) : V_{th}, R_{th}

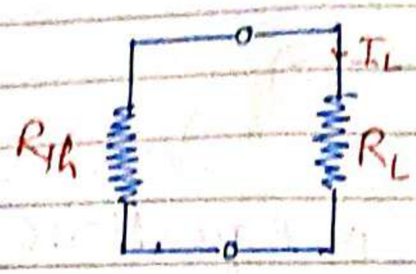
Case 2 :- (Atleast one independent and one dependent sources. : V_{th}, I_{sc}

$$R_{th} = \frac{V_{th}}{I_{sc}} \,\Omega \Rightarrow I_{sc} = \frac{V_{th}}{R_{th}} = I_N$$

Case 3 :- All are dependent sources.

$V_{th} = 0$ volt, $I_{sc} = 0$ Ampere

but $R_{th} \neq 0$



$\Rightarrow$ $I_L = 0$ Amp.

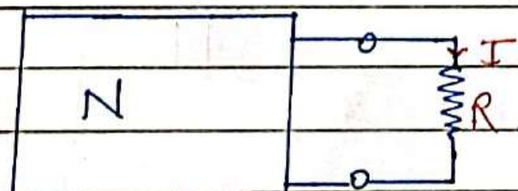
i.e. Thevenin's equivalent = Norton's Equivalent.

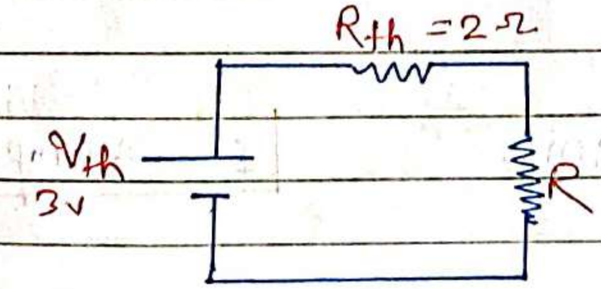
Q. (1) Network N contains several resistances & DC sources. The current I = 3 Amp. & 1.5 Amp. when $R = 0\,\Omega$ and $2\,\Omega$ respectively. Determine I when $R = 1\,\Omega$.

Sol^n:- (case 1 :- All are independent sources, so called passive Networks)

By Thevenin :-

Any network with any number of elements with respect to load element canbe written as a 3 element network.





$$I = \frac{V_{th}}{R_{th} + R}$$

$$3 = \frac{V_{th}}{R_{th}} \quad \text{—— (1)}$$

$$1.5 = \frac{V_{th}}{R_{th}+2} \quad \text{——— (2)}$$

Equation (1) Divided by Equation (2)

$$\frac{3}{1.5} = \frac{R_{th}+2}{R_{th}}$$

$$2 = 1 + \frac{2}{R_{th}}$$

$$\therefore \quad 2-1 = \frac{2}{R_{th}}$$

$$1 = \frac{2}{R_{th}}$$

$$\therefore \quad \boxed{R_{th} = 2\,\Omega}$$

from eqn (1)

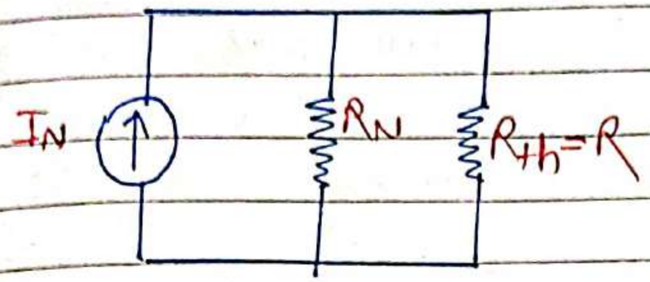
$$3 = \frac{V_{th}}{2}$$

$$\therefore \quad \boxed{V_{th} = 6V.}$$

& $I = \frac{6}{2+1} = 2\,\text{Amp.}$ $\quad \boxed{I = 2\,\text{Amp.}}$

Method (2) By Norton. Theorem :-

Any network with any number of elements with respect to a load element can be written as a 3 element network by.



$$I = \frac{I_N R_N}{R + R_N}$$

$$3 = \frac{I_N R_N}{0 + R_N} \quad \text{--- (1)}$$

$$1.5 = \frac{I_N \cdot R_N}{2 + R_N} \quad \text{--- (2)}$$

eq^n (1) divided by equation (2)

$$\frac{3}{1.5} = \frac{\dfrac{I_N \cdot R_N}{0 + R_N}}{\dfrac{I_N R_N}{2 + R_N}}$$

$$2 = \frac{2 + R_N}{R_N}$$

$$2 = \frac{2}{R_N} + 1$$

$$2 - 1 = \frac{2}{R_N}$$

$$R_N = \frac{2}{1}$$

$$\boxed{R_N = \cancel{1}\ 2\,\Omega}$$

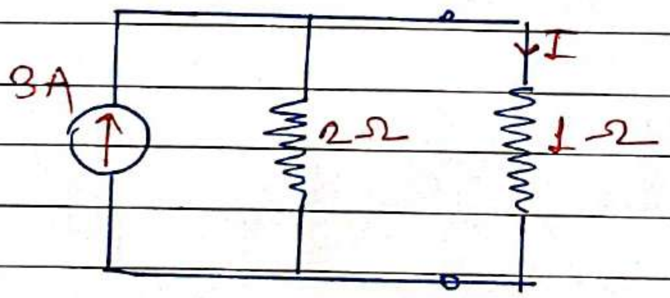
Now

$$3 = \frac{I_N \cdot R_N}{R_N}$$

$$I_N = \frac{3 R_N}{R_N}$$

$$I_N = \frac{3 \times 2}{2}$$

$$\boxed{I_N = 3\ \text{Amp}}$$



$$I = \frac{3 \times 2}{1+2} = \frac{6}{3}$$

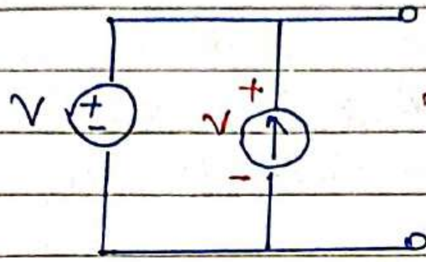
$$\boxed{I = 2\ \text{Amp.}}$$

Observation :- Both I_N & R_N are independent of the load magnitude (value)

Q(2) An ideal voltage and current sources are connected in parallel. This combination will have.

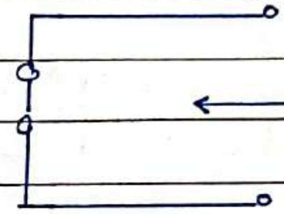
(a) Both Thevenin and Norton's equivalents
(b) Thevenin's but not Nortons (option b)
(c) Norton's but Not Thevenin
(d) Neither Thevenin Nor Norton's

→



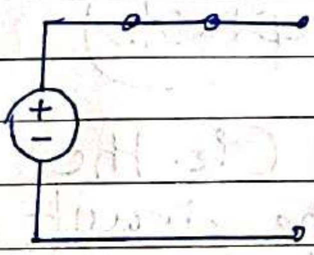
$V_{th} = V$

(i) Kill the sources



$\leftarrow R_{th} = R = 0\ \Omega$

Entire network is in series with R_{th}.

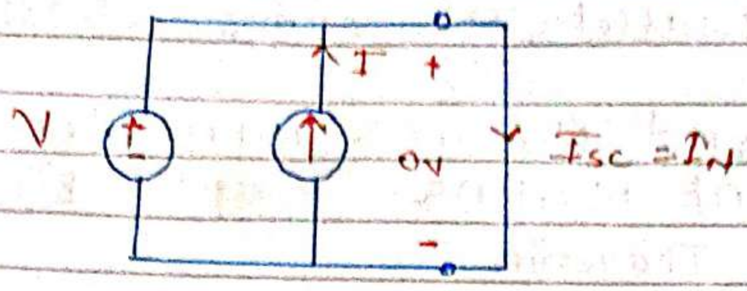


An ideal voltage source (w. r. to load)

Note:- As discussed in source transformation it is impossible to convert an ideal voltage source into its equivalent current source. Hence Norton's equivalent does not exist.

Reason :-

By direct Norton's Approach



By KCL

$$I_{sc} = I + \text{Any value}$$
$$= \text{Any value}$$

(Any value for metal or conductor is not possible)

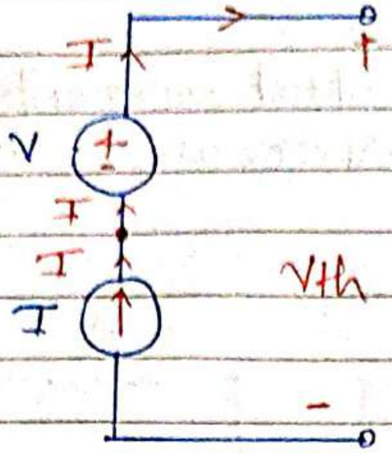
(OR)

By KVL,

$$V - 0 = 0$$
$$V = 0 \quad \text{(Not acceptable)}$$

Since the violation of KVL (i.e. the conversation of energy) in the circuit, the physical connection is not possible. i.e. the circuit does not exist & Hence it is impossible to evaluate I_{sc} so, Norton's equivalent does not exist.

When both the sources are in series.



By KVL,

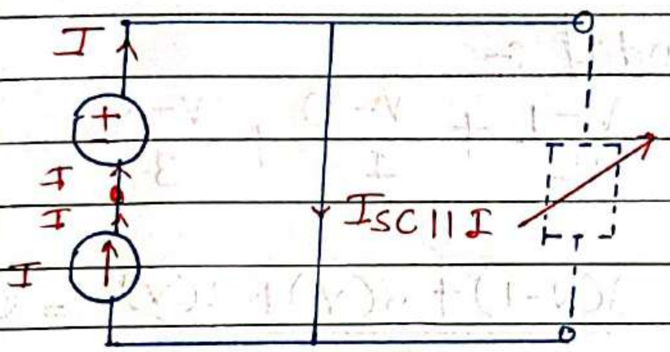
$V_{th} = V + \text{Any value}$

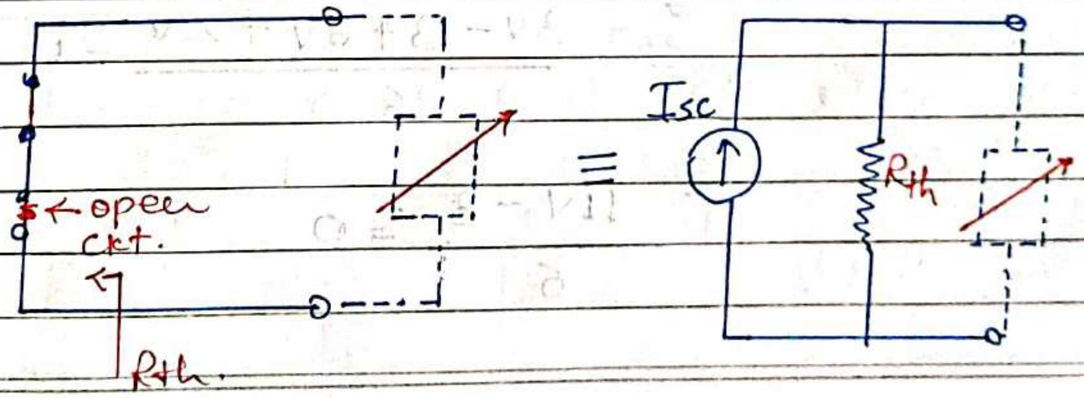
Any value for metal
(Not acceptable)

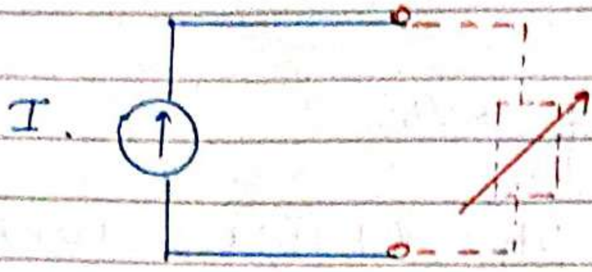
(OR)

Here we cannot open the load for V_{th} since the violation of KCL at the node in the circuit & hence it is impossible to evaluate V_{th} so, Thevenin's equivalent does not exist.

By Direct Norton's Approach:-

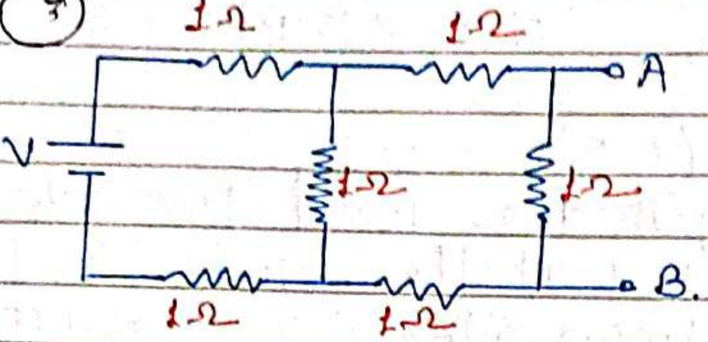


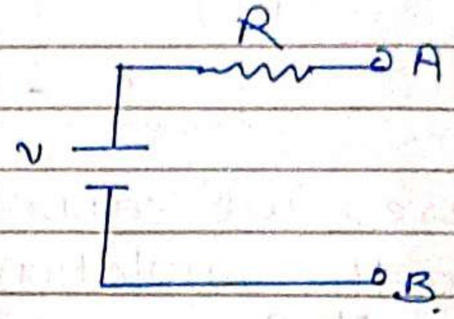




An ideal current source w. r. to load.

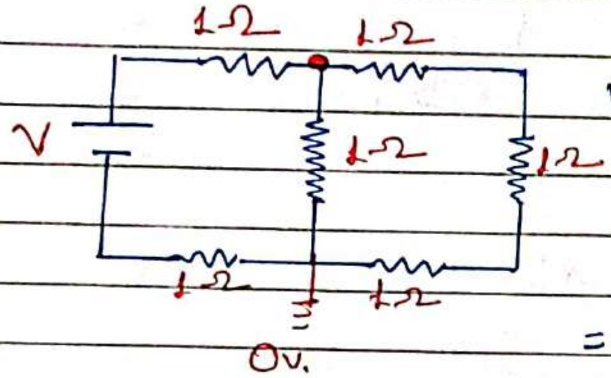
Q. (7)





find V_{th} & R_{th}.

By Nodal Analysis ⇒



Nodal :-

$$\frac{V-1}{2}+\frac{V-0}{1}+\frac{V-0}{3}=0$$

$$=\frac{3(V-1)+6(V)+2(V)}{6}=0$$

$$=\frac{3V-3+6V+2V}{6}=0$$

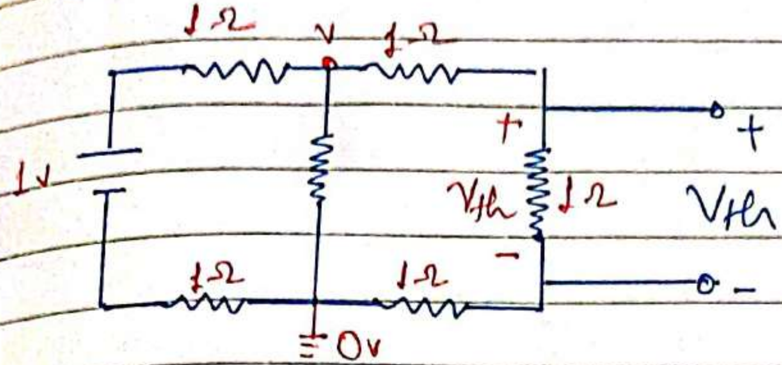
$$=\frac{11V-3}{6}=0$$

$$11V - 3 = 0$$

$\therefore$

$$11V = 3$$

$$V = \frac{3}{11} V$$



$$V_{th} = V_{1\Omega}$$

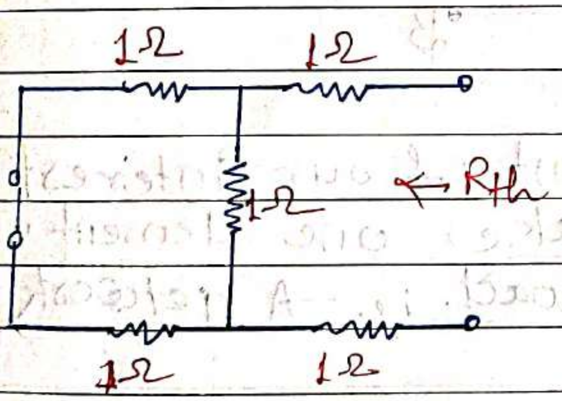
$$= I_1 \cdot 1$$

$$= \left(\frac{V - 0}{3}\right) \cdot 1$$

$$= \frac{V}{3}$$

$$\left(\because V = \frac{3}{11} V\right) \qquad = \frac{\frac{3}{11}}{3}$$

$$V_{th} = \frac{1}{11} V$$



$$R_{th} = 1 \| \left(1 + \frac{2 \times 1}{2+1} + 1\right)$$

$$= 1 \| \left(1 + \frac{2}{3} + 1\right)$$

$$= 1 \| \left(\frac{8}{3}\right)$$

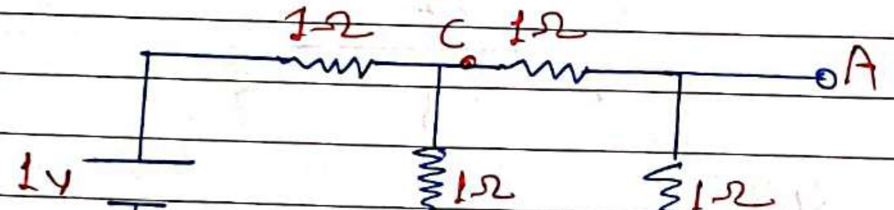
$$\frac{1 \times \frac{8}{3}}{1 + \frac{8}{3}}$$

$$\frac{\frac{8}{3}}{\frac{11}{3}}$$

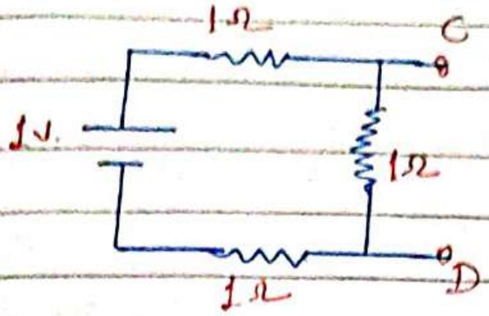
$$= \frac{8}{3} \times \frac{3}{11}$$

$$R_{th} = \frac{8}{11}\,\Omega$$

Method (2) By Thevenin Theorem



<u>Note:-</u> Generally the element of our interest is a load. We can take one element or group of elements as a load. i.e. A network canbe a load.



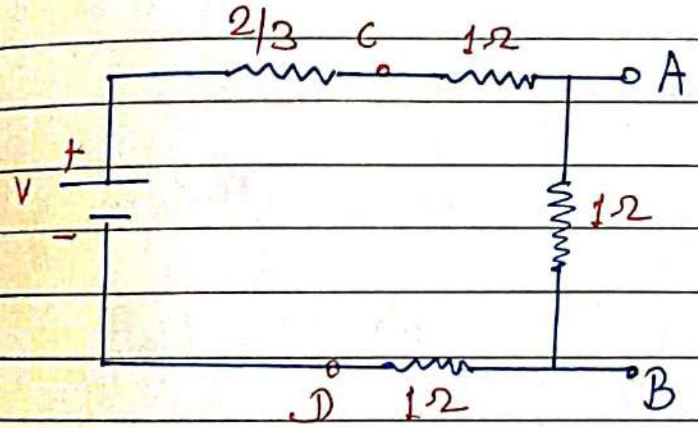
$$V_{th} = \frac{1}{3} \times 1$$

$$= \frac{1}{3} V.$$

$$R_{th} = 1 \parallel 2$$

$$= \frac{1 \times 2}{1+2}$$

$$= \frac{2}{3} \Omega$$



$$V_{th} = \left(\frac{1/3}{2/3 + 1 + 1 + 1} \right) \cdot 1$$

$$= \frac{\frac{1}{3}}{2/3 + 3}$$

$$= 1/3 \times 3/11 = 1/11 \ V$$

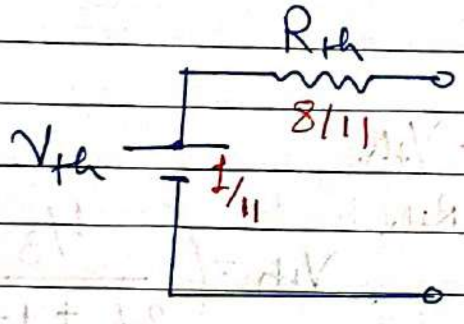
$$R_{th} = 1 \parallel \left(1 + \frac{2}{3} + 1\right)$$

$$= 1 \parallel \frac{8}{3}$$

$$= \frac{1 \times 8/3}{1 + 8/3}$$

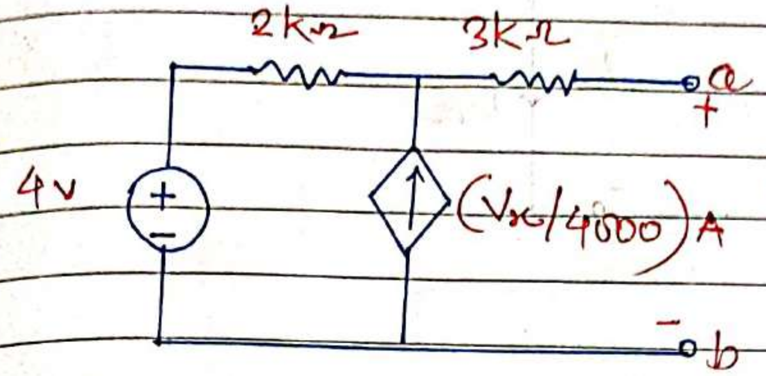
$$= \frac{8/3}{11/3}$$

$$R_{th} = \frac{8}{11}\ \Omega$$

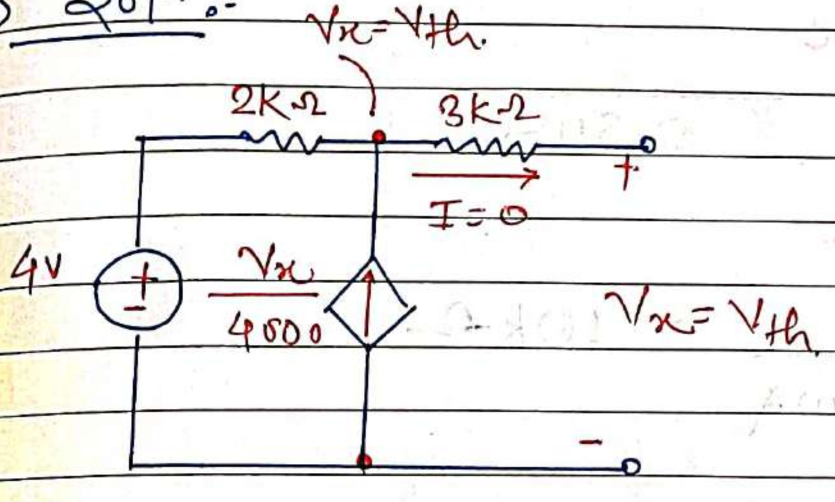


Case 2 :- (Atleast one dependent (Inside) and Independent (outside) sources so called Active Network.

Q(1) Determine the Thevenin's & Norton's equivalent across.



⇒ Soln :-



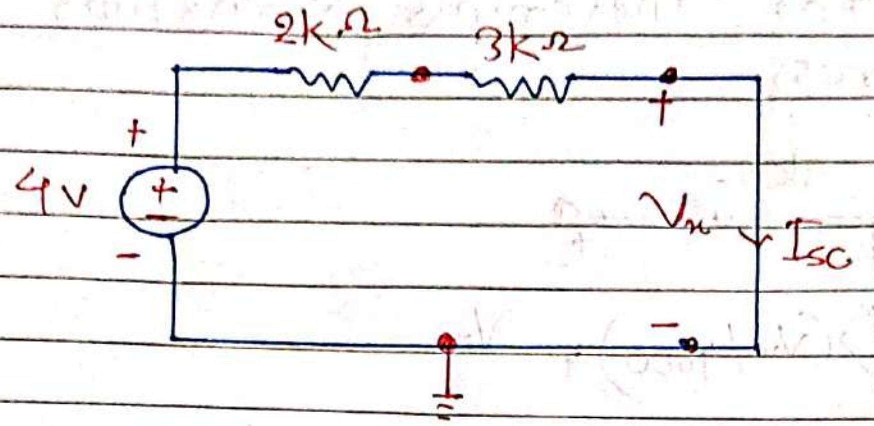
Assume $V_x = V_{th}$

By Nodal ⇒

$$\frac{V_x - 4}{2k} - \frac{V_x}{4000} = 0$$

$$\frac{V_x}{2K} - \frac{2}{1K} = \frac{V_x}{4000}$$

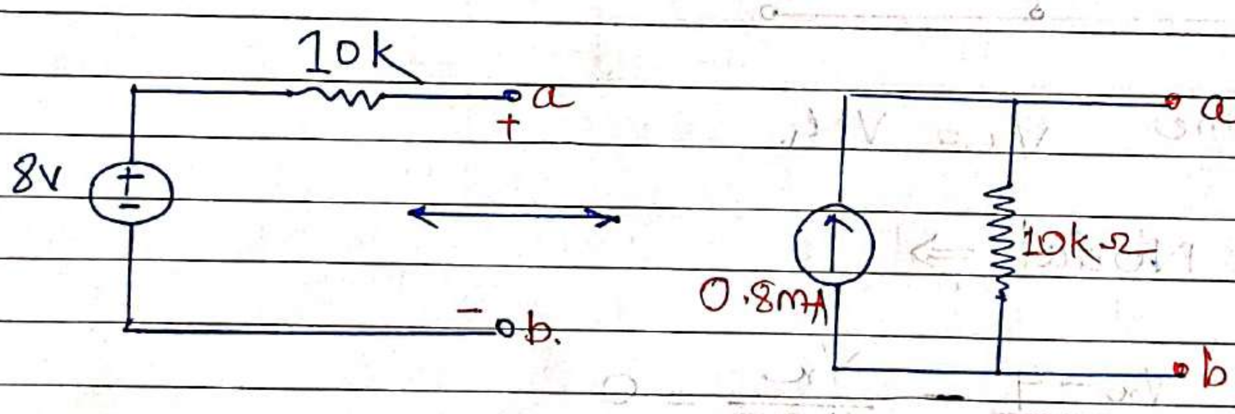
$$V_x = 8V = V_{th}$$



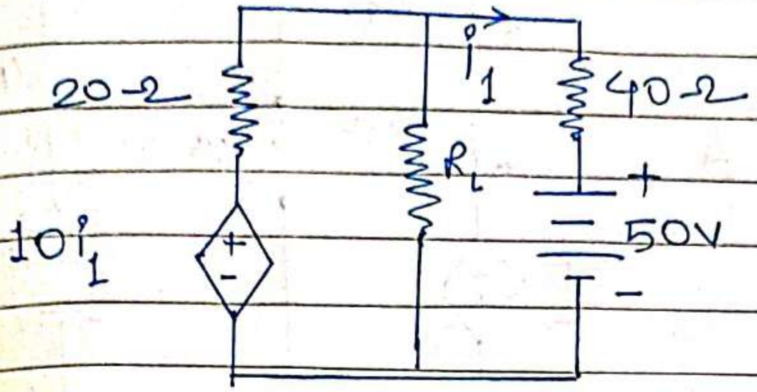
$$R_{th} = \frac{V_{th}}{I_{sc}}$$

$$I_{sc} = \frac{4}{5K} = 0.8\,mA$$

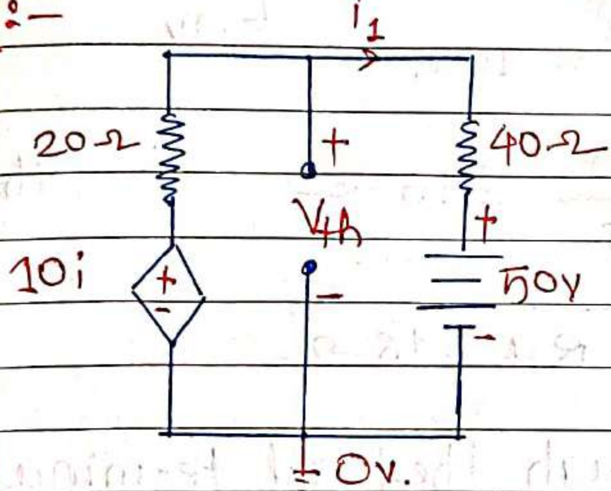
$$R_{th} = \frac{8}{0.8\,mA} = 10k\Omega$$



Q. Consider the following circuit and determine Thevenin's and Norton's equivalent across R_L.



Sol^n :-



By Nodal ⇒

$$\frac{V_{th}}{20} - \frac{10i_1}{20} + \frac{V_{th}}{40} - \frac{50}{40} = 0$$

$$\frac{2V_{th} - 20i_1 + V_{th} - 50}{40} = 0$$

$$3V_{th} - 20i_1 - 50 = 0$$

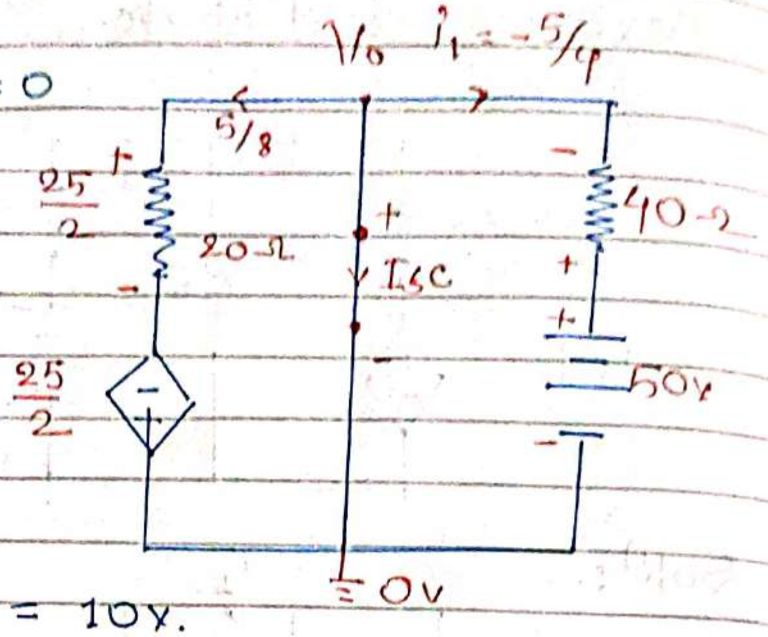
$$3V_{th} = 20i_1 + 50$$

By KVL.

$10i_1 - 20i_1 - 40i_1 - 50 = 0$

$i_1 = -1A.$

$i_1 = \frac{-50}{40} = \frac{-5}{4} A.$



$I_{sc} = \frac{5}{8}$ Amp

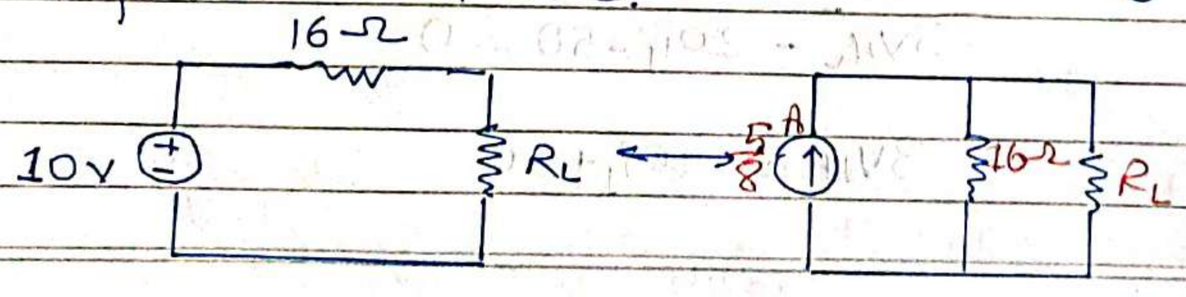
and $V_{th} = \frac{50-20}{3} = 10v.$

$\therefore R_{th} = \frac{10}{\frac{5}{8}} = \frac{80}{5} = 16\Omega$ $\qquad \therefore R_{th} = \frac{V_{th}}{I_{sc}}$

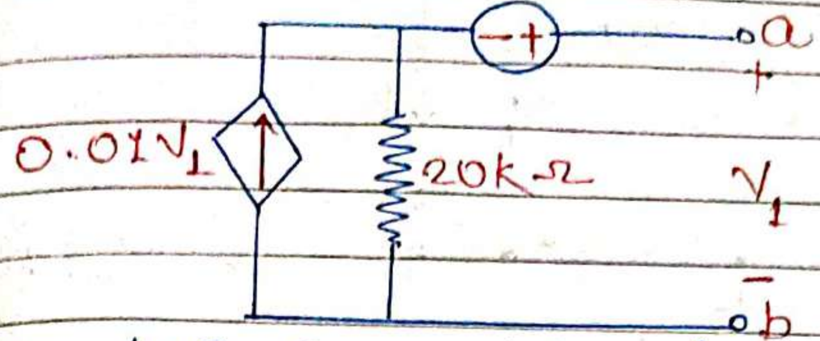
$\therefore V_{th} = 10v$ and $R_{th} = 16\Omega$

Note:- Here, even though the load terminals are shorted, The controlled variable is non zero hence dependent source magnitude is non zero.

To flow a current ~~through~~ through the short circuit (ie zero impedance path) As well as ideal current source, There is no need of Potential difference (ie. Height difference) across the terminals. It is required only ~~for~~ for the passive R, L, C.

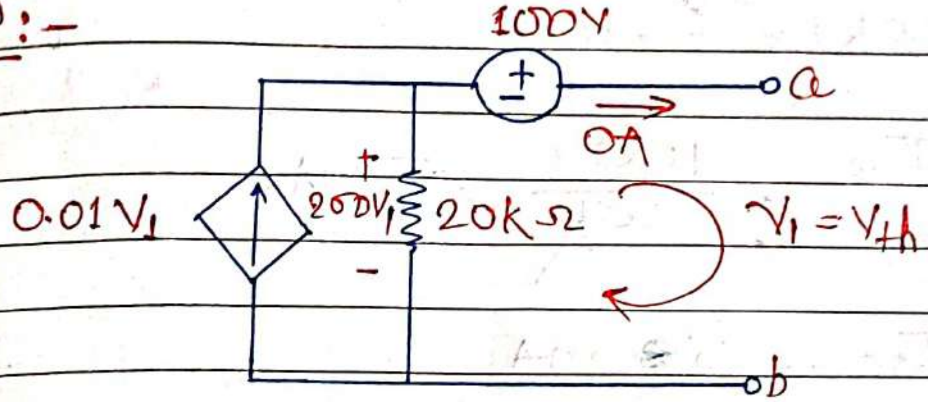


Q. Consider the following circuit.



If a 100 Ω resister is connected across a, b, then determine current through it by using Thevenin's Theorem.

Soln :-



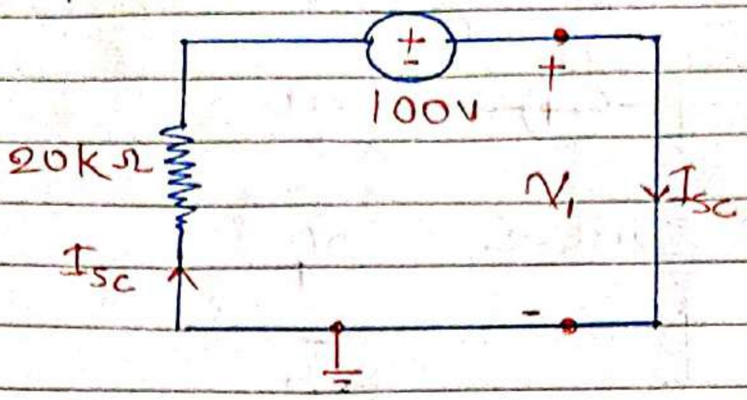
Assume $V_1 = V_{th}$

By KVL

$$200V_1 + 100 - V_1 = 0$$

$$199V_1 + 100 = 0$$

$$\therefore V_1 = \frac{-100}{199} = -502.5\ mV$$



By KVL

$$-20k I_{sc} + 100 = 0$$

$$-20k I_{sc} = -100$$

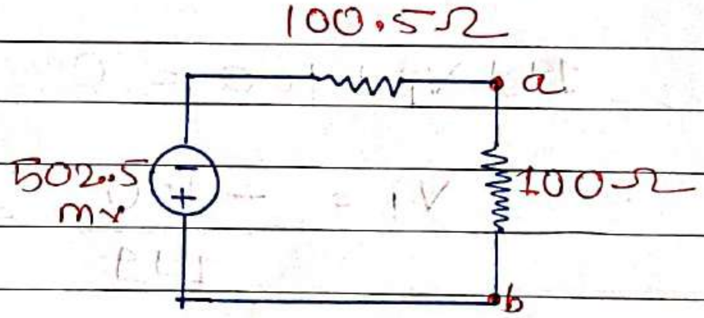
$$I_{sc} = \frac{100}{20k} = 5\,mA$$

$$I_{sc} = 5\,mA$$

$$R_{th} = \left|\frac{V_{th}}{I_{sc}}\right| = \frac{-502.5\,mV}{5\,mA} = \frac{-502.5\,mV}{5\times10^{-3}}$$

$$= |-100.5\,\Omega| = 100.5\,\Omega$$

$$R_{th} = 100.5\,\Omega$$



So,

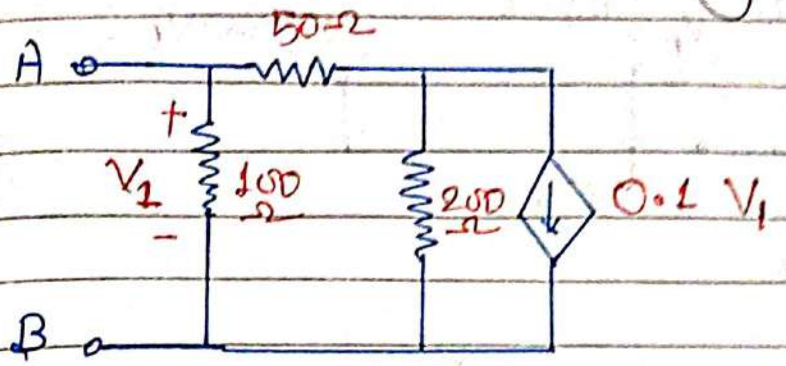
$$I_{100} = \frac{502.5\,mV}{200.5}$$

$$= 2.5\,mA$$

Note:- Presence of dependent sources in the network makes R_{th} sometimes, to have a negative value.

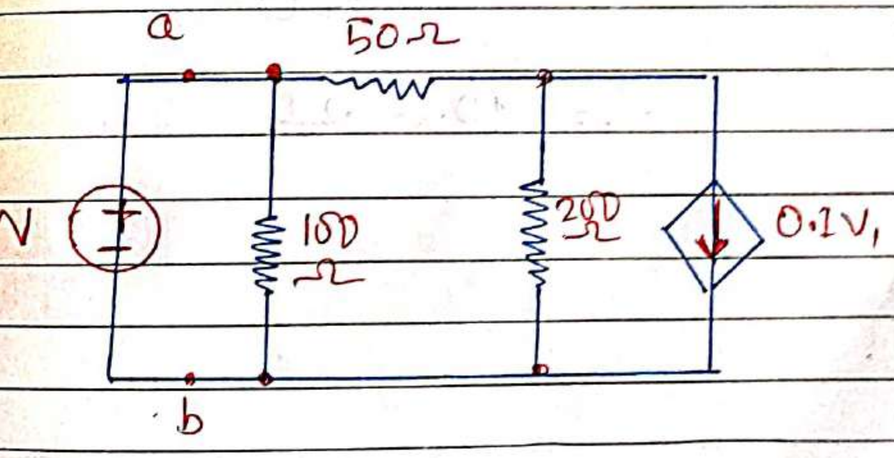
Case 3 :- All are dependent sources, so called Dead Networks.

Q. Determine Thevenin's and Norton's equivalent across AB of the following Networks.



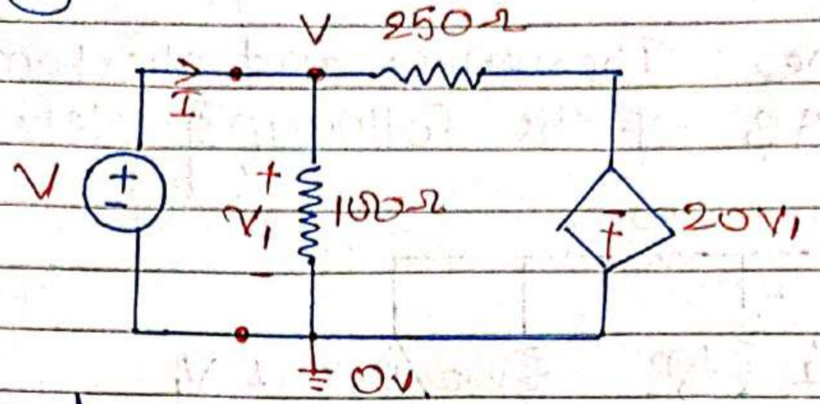
The above ~~A~~ network is a dead network, so we need to energize it with "V".
The dead network behavior is a load resistor behavior.

Note:- Here we can apply source transformation for dependent source. As controlled variable is outside.



$V = V_1$

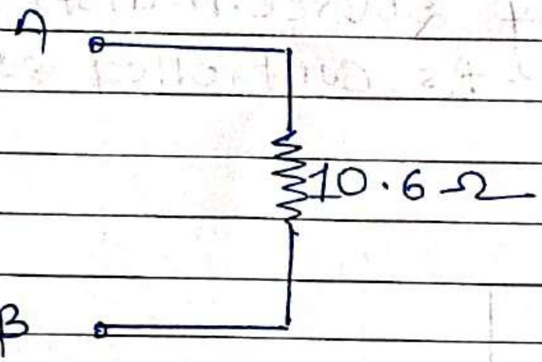
On doing source transformation



By Nodal ⇒

$$-I + \frac{V}{100} + \left(\frac{V + 20V}{250}\right) = 0$$

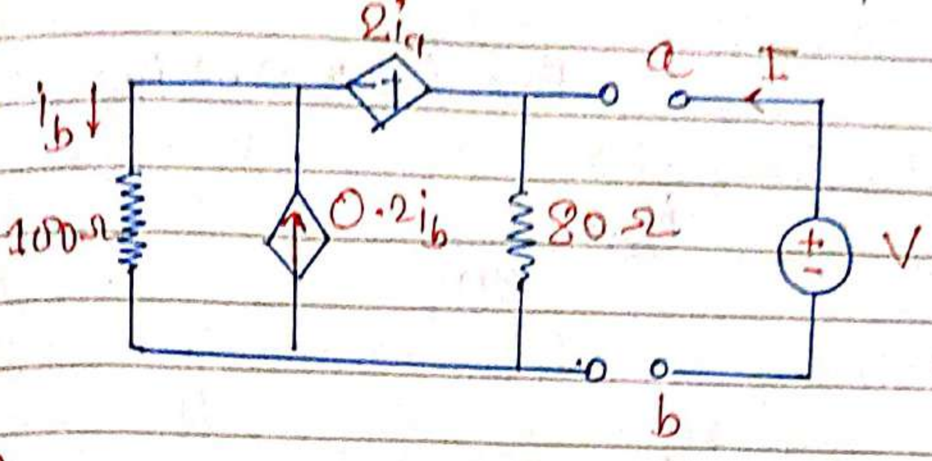
$$\frac{V}{I} = R_{th} = R_N = \frac{1}{\frac{1}{100} + \frac{21}{250}}$$



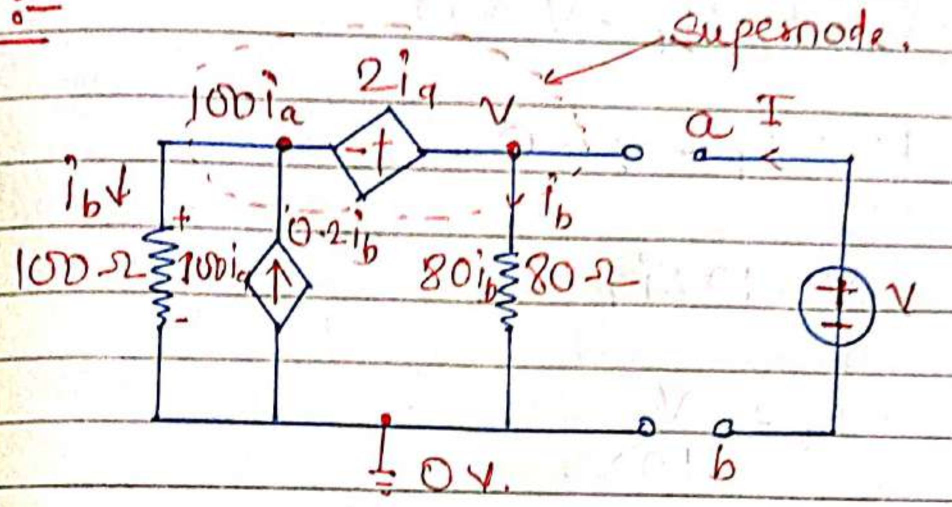
$$= \frac{100 \times 250}{2350}$$

$$= 10.6\,\Omega$$

Q. For the circuit shown in figure, determine the Thevenin and Norton's equivalent.



Soln :-



Note :- Here we cannot apply source transformation for dependent source as the controlled variable is inside. Hence supernode procedure is followed here for the dependent source. It is also to note that dependent source also behaves like independent source in the presence of atleast one external independent source.

Supernode Equation $= i_a - 0.2 i_b + i_b - I = 0$

$$I = i_a + 0.8\, i_b$$

$$i_b = \frac{V}{80}$$

By KVL

$$100 i_a + 2 i_a - V = 0$$

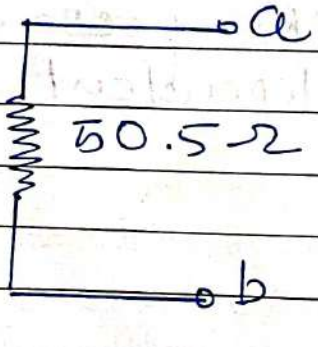
$$102 i_a - V = 0$$

$$V = 102 i_a$$

$$i_a = \frac{V}{102}$$

$$I = \frac{V}{102} + 0.8 \times \frac{V}{80}$$

$$\frac{V}{I} = R_{th} = R_N = \frac{1}{\frac{1}{102} + \frac{1}{100}}$$



$$= \frac{100 \times 102}{202}$$

$$= 50.5\,\Omega$$

Maximum Power Transfer Theorem (MPTT)

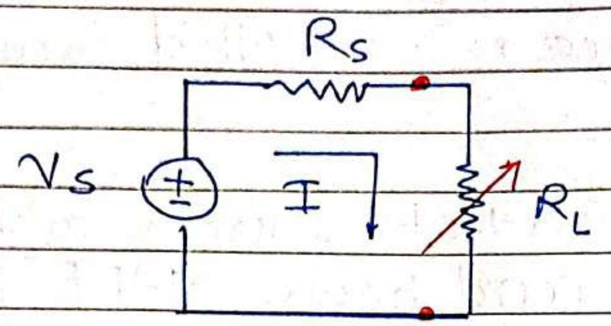
→ This theorem is applicable only for linear networks i.e. the networks with R, L, C, transformer and linear controlled sources as elements.

→ The presence of dependent sources makes the network active and hence, MPTT is used for both active as well as passive networks.

→ This theorem is applicable only when load is variable, otherwise (i.e. load is fixed) choose the minimum internal impedance of the source, which results in maximum current through the fixed load and hence, a maximum power dissipation across the load.

Under variable Load conditions :-

① R_S & R_L



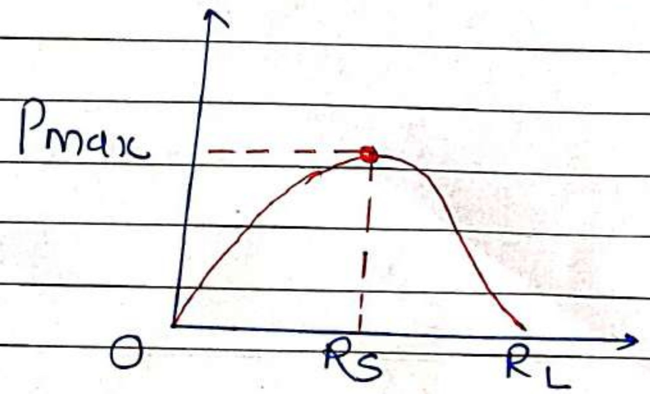
$$I = \frac{V_S}{R_S + R_L}$$

$$P = I^2 R_L \text{ watt}$$

$$= V \cdot I$$

$$= R_L I \cdot I$$

$$P = \frac{V_S^2 R_L}{(R_S + R_L)^2} \text{ watt}$$



$$\frac{dP}{dR_L} = \frac{V_S^2[(R_S+R_L)^2 \cdot 1 - R_L \cdot 2(R_S+R_L)]}{[(R_S+R_L)^2]^2}$$

for MPT $\Rightarrow \frac{dP}{dR_L} = 0$

$\Rightarrow$

$$\frac{V_S^2[(R_S+R_L)^2 \cdot 1 - R_L \cdot 2(R_S+R_L)]}{[(R_S+R_L)^2]^2} = 0$$

$$V_S^2[(R_S+R_L)^2 \cdot 1 - R_L \cdot 2(R_S+R_L)] = 0$$

$$\therefore \quad (R_S+R_L)^2 - R_L\, 2(R_S+R_L) = 0$$

$$(R_S+R_L)^2 = R_L \cdot 2(R_S+R_L)$$

$$R_S+R_L = 2R_L$$

$$R_S = 2R_L - R_L$$

$$\boxed{R_S = R_L\ \Omega}$$

$$P_{max} = P_{R_L=R_S} = \frac{V_S^2}{4R_L}\ \text{Watt} = \frac{V_S^2}{4R_S}\ \text{Watt}$$

$P_{delivered} = P_{absorbed}$ (By Tellegen's Theorem)

$$= I^2 R_S + I^2 R_L \Big|_{R_L = R_S}$$

$$= \frac{V_s^2}{2R_L} \text{ Watt} = \frac{V_s^2}{2R_S} \text{ Watt.}$$

$$\eta = \frac{\text{Useful Power}}{\text{Total Power}}$$

$$= \frac{\frac{V_s^2}{4R_L}}{\frac{V_s^2}{2R_L}}$$

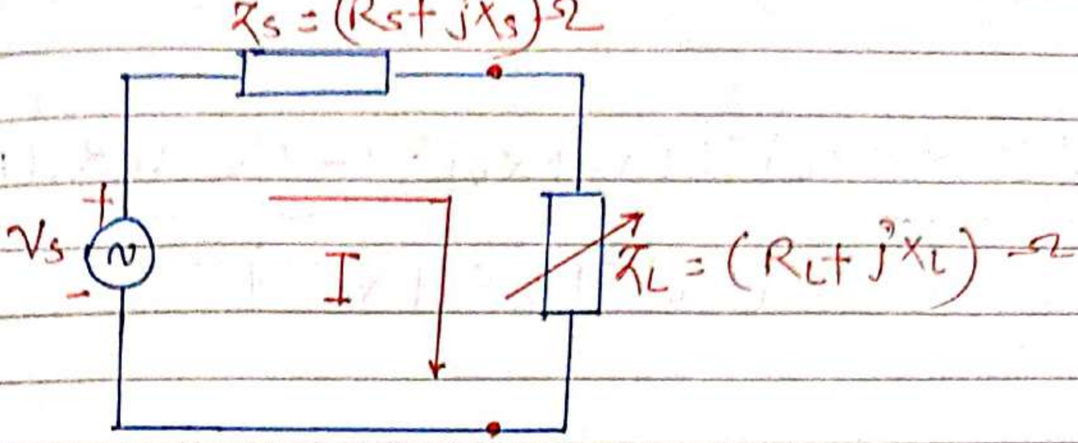
$$= \frac{1}{2}$$

$$= 0.5$$

$$\% \eta = 50\%$$

So the efficiency of maximum power Transfer theorem is atmost 50%

(2) Z_S and Z_L



$$I = \frac{V_S}{R_S + R_L + j(X_S + X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S+R_L)^2 + (X_S+X_L)^2}}$$

$$P = |I|^2 \cdot R_L \text{ Watt}$$

$$P = \frac{V_S^2 R_L}{(R_S+R_L)^2 + (X_S+X_L)^2} \text{ Watt}$$

(a) Only 'R_L' is variable :-

$$\frac{dP}{dR_L} = \frac{V_S^2\left[\left((R_S+R_L)^2 + (X_S+X_L)^2\right)\cdot 1 - R_L \cdot 2(R_S+R_L)\right]}{\left[(R_S+R_L)^2 + (X_S+X_L)^2\right]^2}$$

For MPT $\frac{dP}{dR_L} = 0$

$\therefore$

$$\frac{v_s^2[(R_s+R_L)^2+(X_s+X_L)^2 \cdot 1 - R_L \cdot 2(R_s+R_L)]}{[(R_s+R_L)^2+(X_s+X_L)^2]^2} = 0$$

$$v_s^2[(R_s+R_L)^2 + (X_s+X_L)^2 \cdot 1 - R_L\, 2(R_s+R_L)] = 0$$

$$\Rightarrow (R_s+R_L)^2 + (X_s+X_L)^2 - R_L\, 2(R_s+R_L)] = 0$$

$$\Rightarrow R_s^2 + R_L^2 + 2R_s \cdot R_L + (X_s+X_L)^2 = R_L \cdot 2R_s + 2R_L^2$$

$$\Rightarrow R_s^2 + R_L^2 + (X_s+X_L)^2 = 2R_L^2$$

$$R_s^2 + (X_s+X_L)^2 = 2R_L^2 - R_L^2$$

$$R_s^2 + (X_s+X_L)^2 = R_L^2$$

$$\therefore \boxed{R_L = \sqrt{R_s^2+(X_s+X_L)^2}\ \Omega}$$

$$P_{max} = P\Big|_{R_L = \sqrt{R_s^2+(X_s+X_L)^2}}$$

(b) Only 'X_L' is variable

$$\frac{dP}{dX_L} = \frac{V_s^2\left[(R_s+R_L)^2+(X_s+X_L)^2 \cdot 0 - R_L\, 2(X_s+X_L)\right]}{\left[(R_s+R_L)^2+(X_s+X_L)^2\right]^2}$$

for MPT $\frac{dP}{dX_L} = 0$

$$\frac{V_s^2\left[(R_s+R_L)^2+(X_s+X_L)^2 \cdot 0 - R_L^2(X_s+X_L)\right]}{\left[(R_s+R_L)^2+(X_s+X_L)^2\right]^2} = 0$$

$\Rightarrow$

$$V_s^2\left[(R_s+R_L)^2+(X_s+X_L)^2 \cdot 0 - R_L^2(X_s+X_L)\right] = 0$$

$$\left\{(R_s+R_L)^2+(X_s+X_L)^2\right\} \cdot 0 - R_L^2(X_s+X_L)] = 0$$

$$-R_L^2(X_s+X_L) = 0$$

$$\therefore \quad \boxed{X_s+X_L = 0}$$

$$P_{max} = P\Big|_{X_s+X_L=0}$$

(c) Both R_L & X_L are varied simultaneously

In this case, both the conditions

$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$$

& $\quad X_S + X_L = 0$

valid at time.

$\Rightarrow \quad R_L = R_S$

$-X_S = X_L \quad \Rightarrow \quad X_L = -X_S.$

so, $\quad Z_L = R_L + jX_L$

$\quad = R_L - jX_S.$

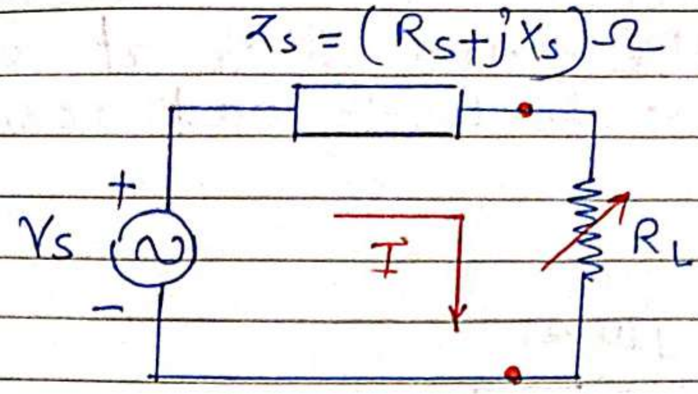
$$Z_L = Z_S^* \ \Omega$$

$$P_{max} = P\Big|_{R_L = R_S}$$

$$X_L = -X_S$$

$$= \frac{V_S^2}{4R_L} \text{ Watt} = \frac{V_S^2}{4R_S} \text{ Watt}$$

(8) Z_s and R_L



It is special case of (9) $X_L = 0$

i.e. $Z_L = R_L$ only when $X_L = 0$
so for MPT

$$R_L = \left.\sqrt{R_s^2 + (X_s + X_L)^2}\right|_{X_L = 0}$$

$$\boxed{R_L = \sqrt{R_s^2 + X_s^2}\ \Omega}$$

$$R_L = |R_s + jX_s|$$

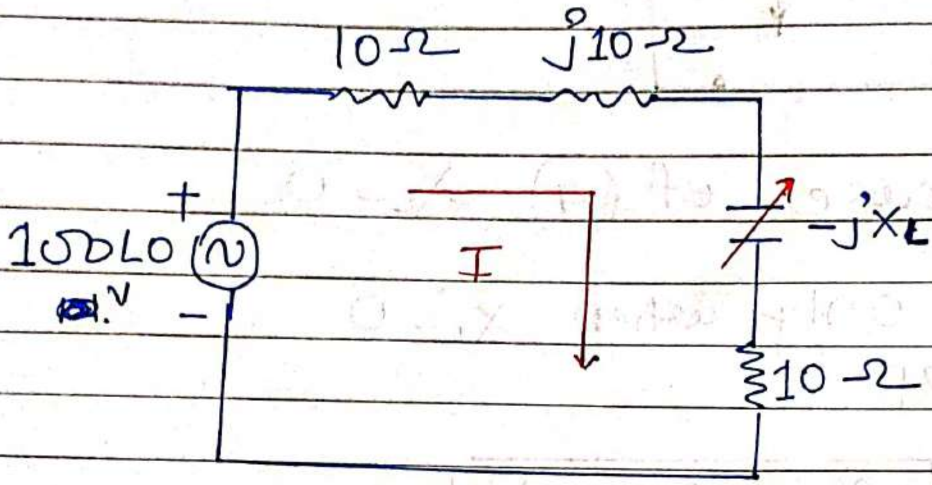
$$\boxed{R_L = |Z_s|\ \Omega}$$

$$P_{max} = P\Big|_{R_L = \sqrt{R_s^2 + X_s^2}}$$

$\%\eta < 50\%$ (Nearer to 50%)

Problems based on MPTT :-

Q. Determine the value of X_L at which power transfer to 10 Ω load resister is maximum



$\Rightarrow \quad Z_L = 10 - jX_L.$

$= 10 + j(-X_L)$

$= R_L + j(X_L) \ \Omega$

For MPTT

$X_s + X_L = 0$

$10 - X_L = 0$

$\boxed{X_L = 10}$

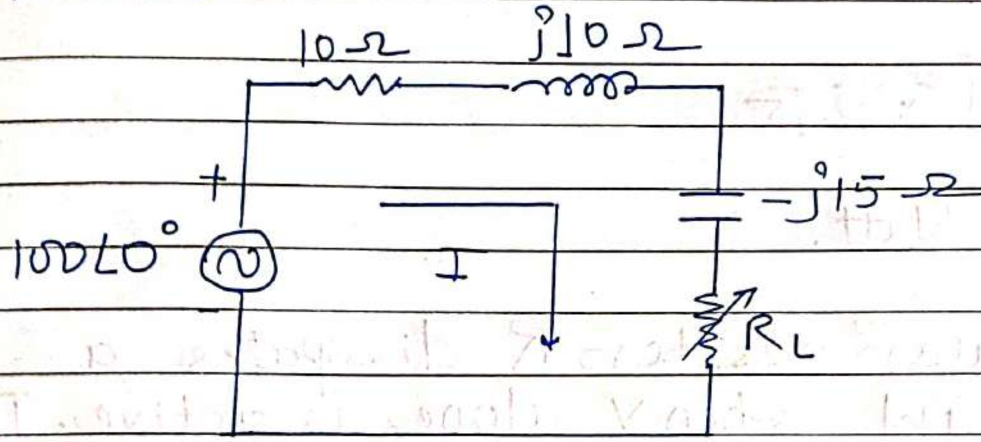
$$I = \frac{100\angle 0^\circ}{10 + j10 - j10 + 10} = 5\angle 0^\circ \text{ Amp.}$$

$$P_{max} = |I|^2 R_L$$

$$= 5^2 \times 10$$

$$= 250 \text{ watt}$$

Q. Determine the maximum power transfer to R_L



⇒

$$Z_L = (R_L - j15)\ \Omega$$

$$= R_L + j(-15)$$

$R_L + jX_L$ | where $X_L = -15$

for MPTT

$$R_L = \sqrt{R_S^2 + (X_L + X_S)^2}$$

$$= \sqrt{10^2 + (10-15)^2}$$

$$= 5\sqrt{5}\ \Omega$$

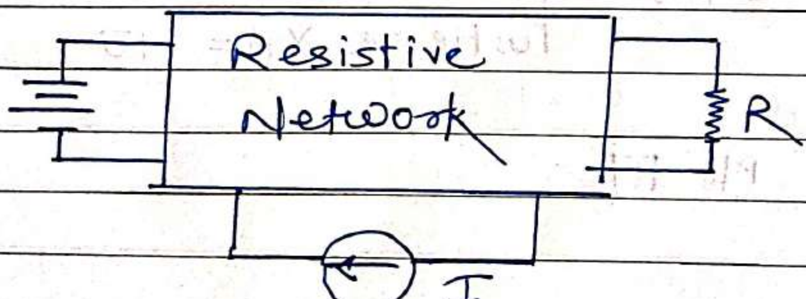
$$I = \frac{100\angle 0^\circ}{10 + j10 - j15 + 5\sqrt{5}}$$

$$P_{max} = |I|^2 R_L$$

$$= |I|^2 \times 5\sqrt{5}$$

$$= 236 \text{ Watt.}$$

Q. A particular resister R dissipates a power of 4W when V alone is active. The same resistor R dissipates a power of 9 Watts when I alone is active. The maximum and minimum powers dissipated by R when both sources are active simultaneously will be



→ When only V is active

$$I_1^2 R = 4$$

$$I_1 = \frac{2}{\sqrt{R}}$$

When only I is active

$$I_2^2 R = 9$$

$$I_2 = \frac{3}{\sqrt{9}}$$

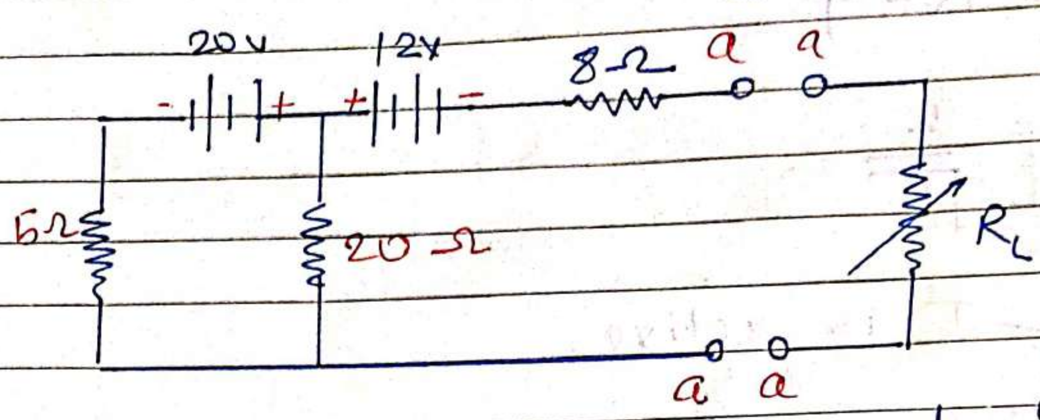
When both are active

$$I_{total} = I_1 + I_2 \quad OR \quad I_1 - I_2$$

$$P = (I_1 + I_2)^2 R = \left(\frac{3}{\sqrt{R}} + \frac{2}{\sqrt{R}}\right)^2 R = 25 \text{ Watt}$$

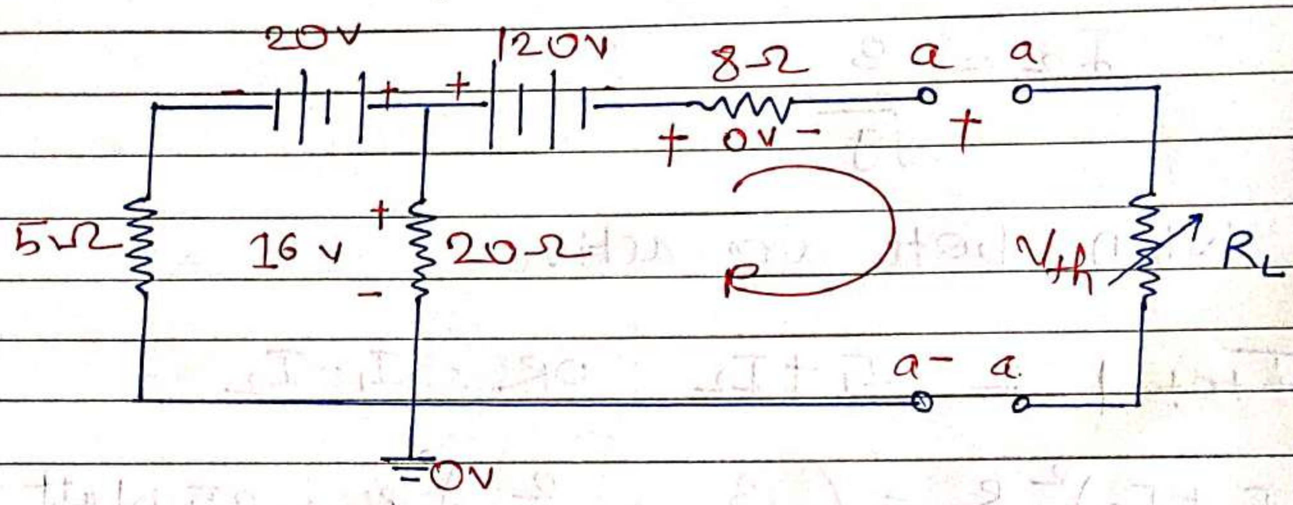
$$P = (I_1 - I_2)^2 R = \left(\frac{2}{\sqrt{R}} - \frac{3}{\sqrt{R}}\right)^2 R = 1 \text{ Watt.}$$

Q. consider the following circuit



The maximum power delivered to 'R_L' is

⇒



By Nodal

$$\frac{V}{5} - \frac{20}{5} + \frac{V}{20} = 0$$

$$\frac{4V - 80 + V}{20} = 0$$

$$5V - 80 = 0$$

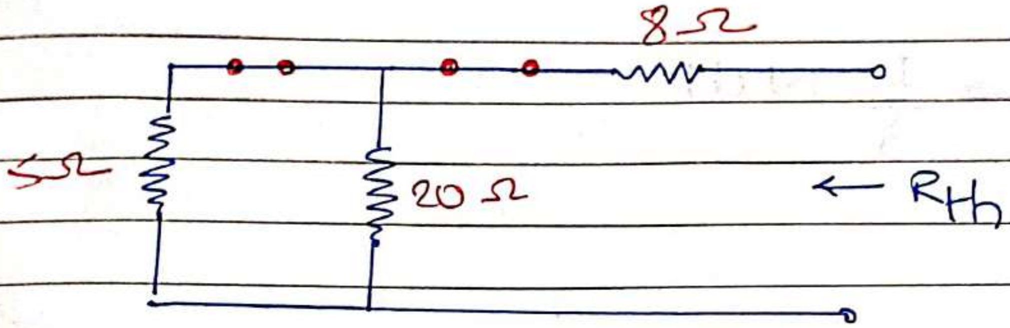
$$V = \frac{80}{5} \Rightarrow 16V.$$

By KVL $\Rightarrow$

$$16 - 12 - V_{th} = 0$$

$$V_{th} = 4V.$$

R_{th} :-

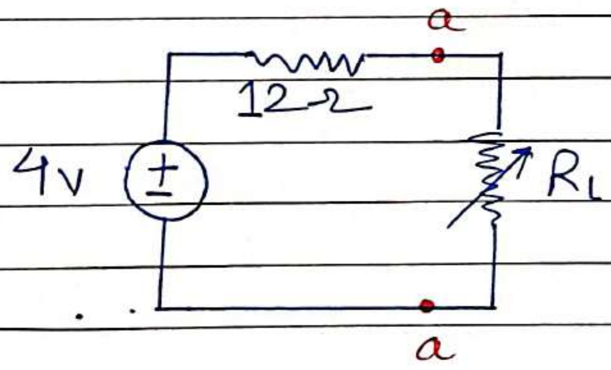


$$R_{th} = (20 \parallel 5) + 8$$

$$= \frac{20 \times 5}{20 + 5} + 8$$

$$= 4 + 8$$

$$R_{th} = 12\,\Omega$$



For maximum power transfer

$$R_L = R_s = 12\,\Omega$$

$$P_{max} = \frac{V_s^2}{4R_L} = \frac{4^2}{4(12)} = \frac{16}{48} = \frac{1}{3}$$

$$P_{max} = \frac{1}{3}\text{ Watt}$$

Graph Theory :-

William Thomas Tutte.

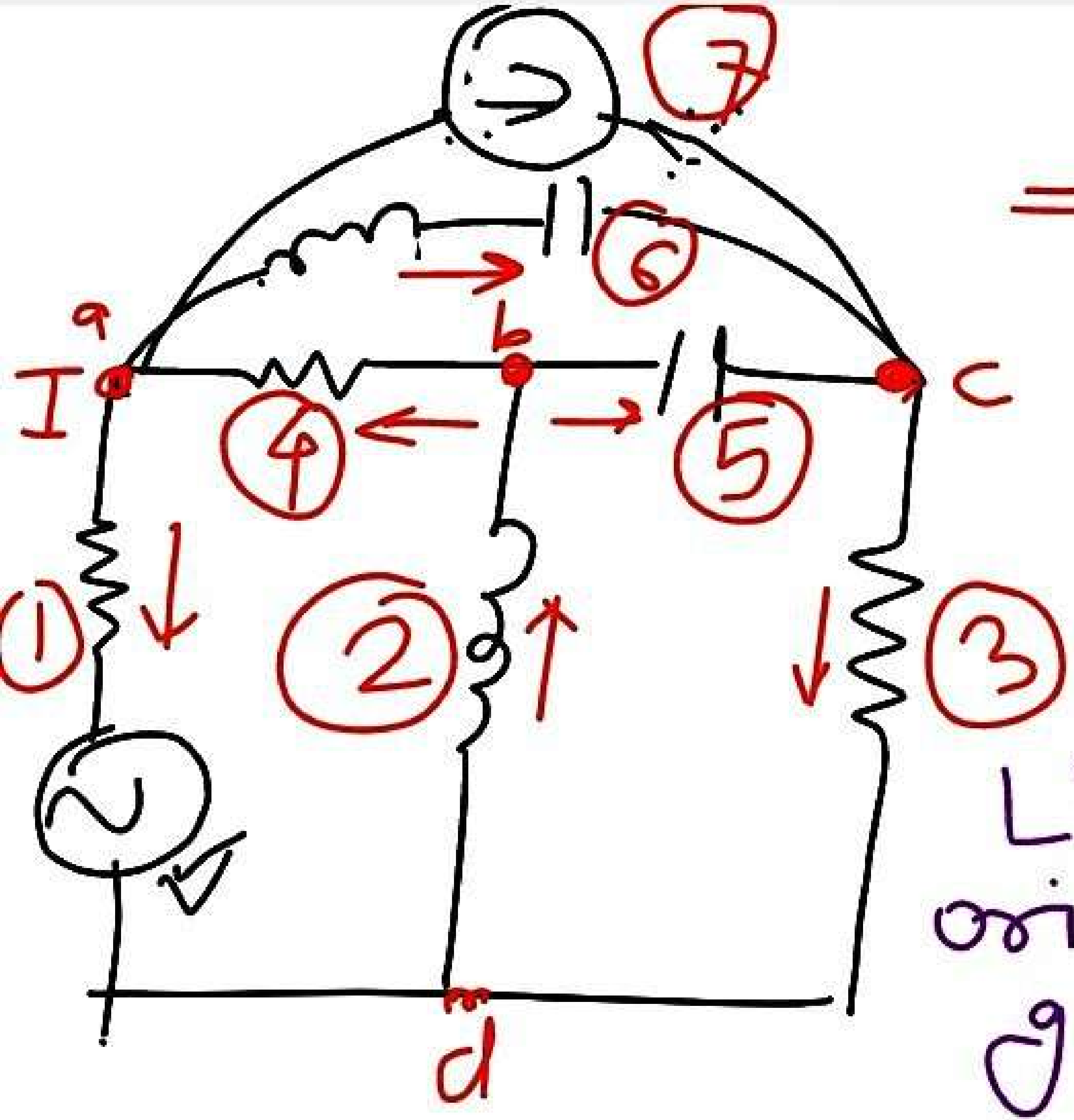
(1917 - 2002)

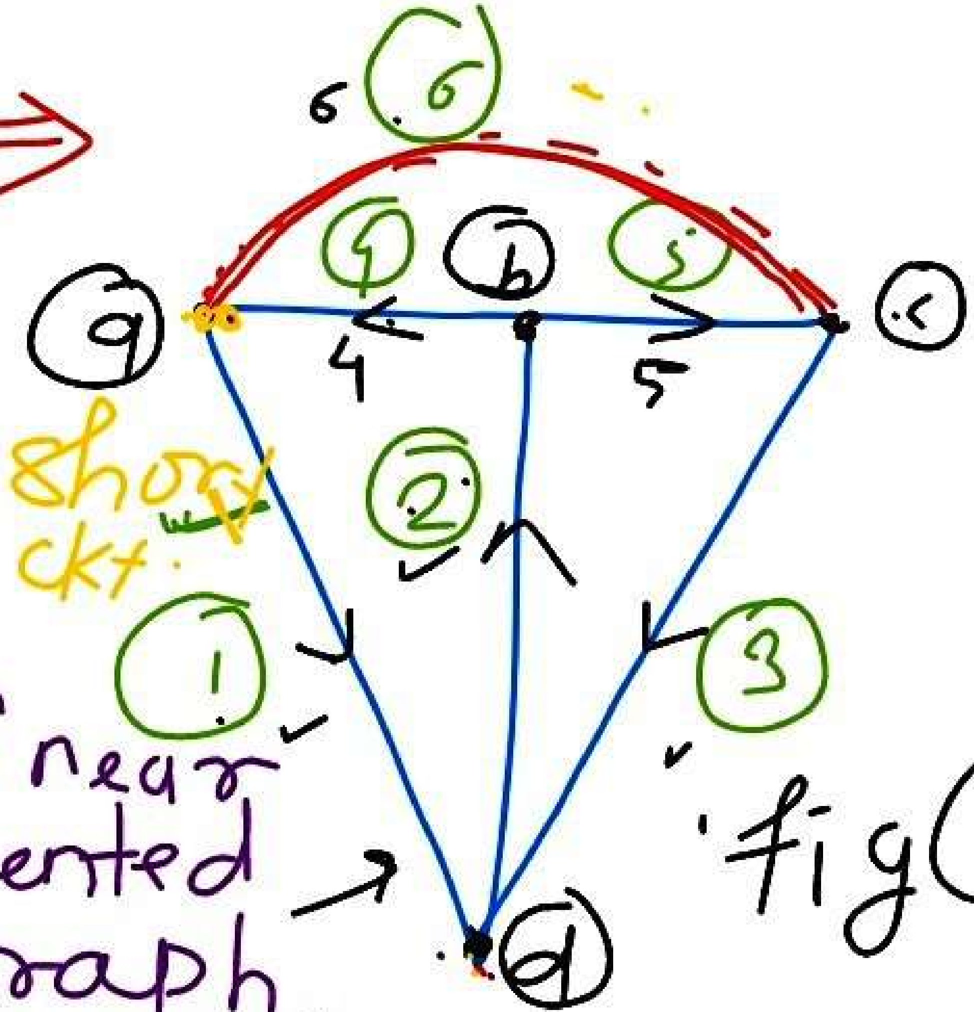
Introduction

Graph theory is geometrical representation of given electrical Network.

⇒ We are replacing all passive elements by line segments
(Branches or edges)

⇒ Ideal voltage sources are replace by short circuits & Ideal current sources are replace by open circuit.





Linear oriented graph.

fig 9

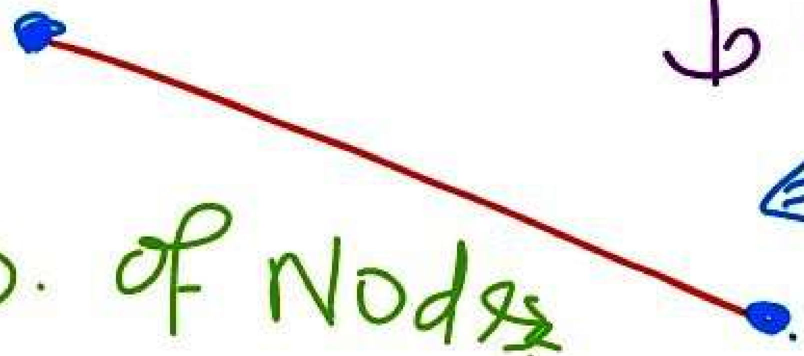
Observation

1. Graph branches are less than or equal to the network branch.
2. No. of nodes or vertices = 4
3. No. of edges or branches = 6

Complete Graph or Standard graph

Def^n:- In a complete graph between any pair of nodes, only one branch is connected for all the combinations.

standard graph



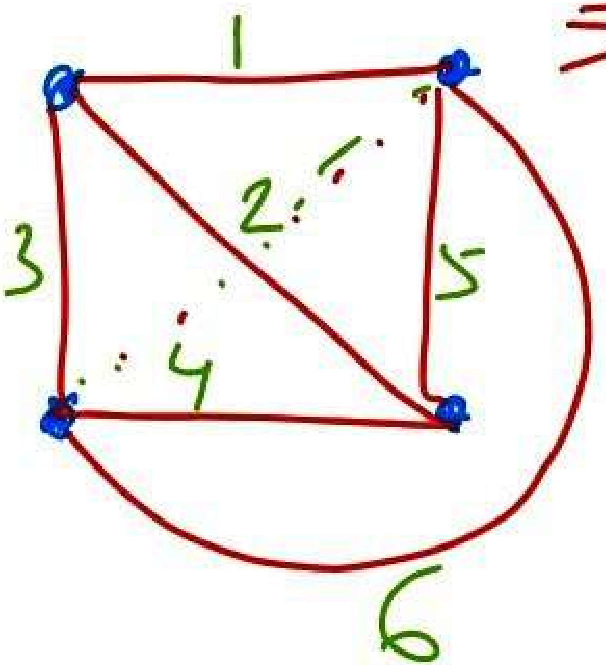
$$b = {}^{n}C_2 = \frac{n(n-1)}{2}$$

$$= \frac{2(2-1)}{2}$$

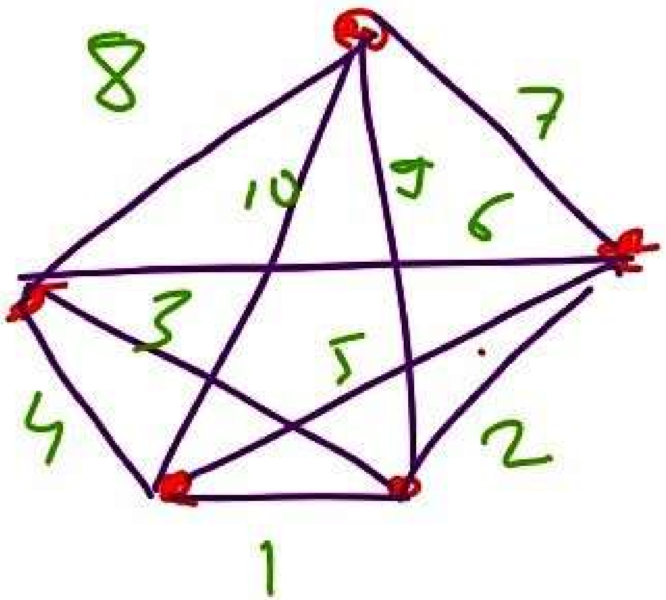
$$= 1$$

n = no. of Nodes

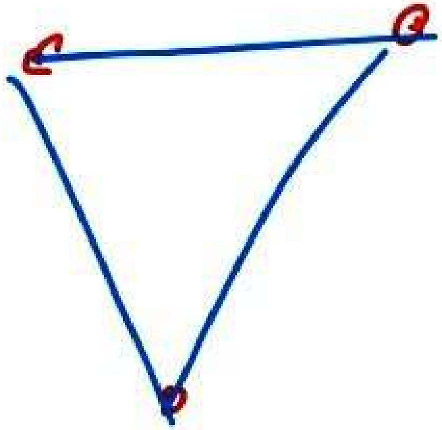
The minimum no. of edges to make a graph complete with n nodes is ${}^{n}C_2$



$$\Rightarrow \frac{4(4-1)}{2} = 6$$



$$\Rightarrow \frac{5(5-1)}{2} = 10$$



$$\frac{3(3-1)}{2} = 3.$$

for the given graph (fig 9)

$n = 4$

$$\frac{4(4-1)}{2} = 6$$

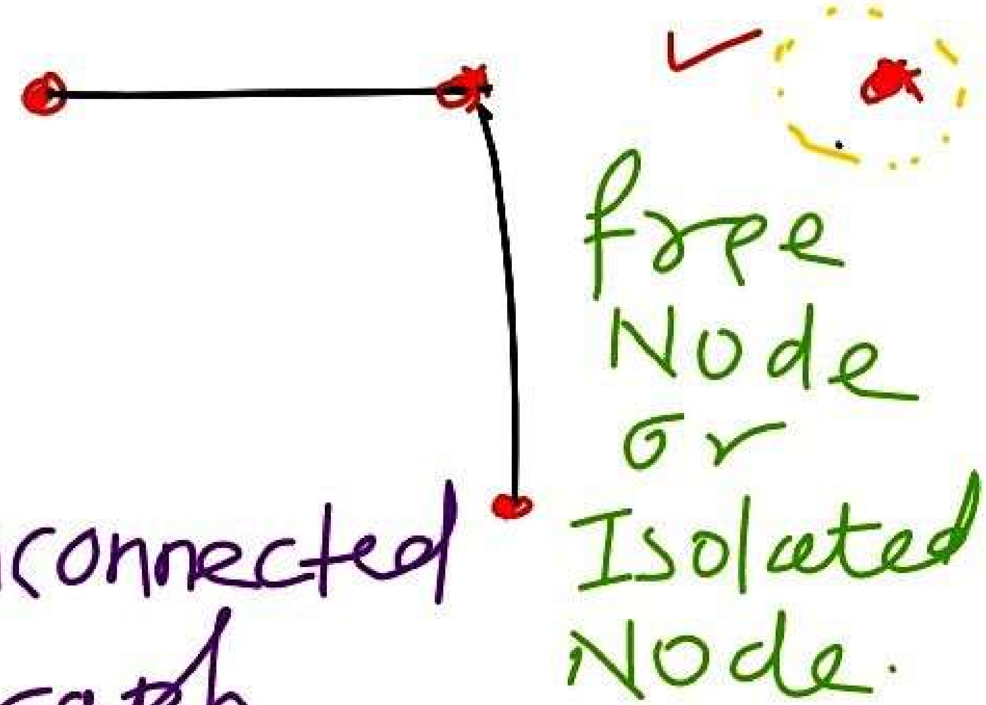
Connected Graph

In connected graph, all the nodes are connected by atleast one branch. Otherwise it is said to be unconnected.

fig. b

Connected graph.

fig c



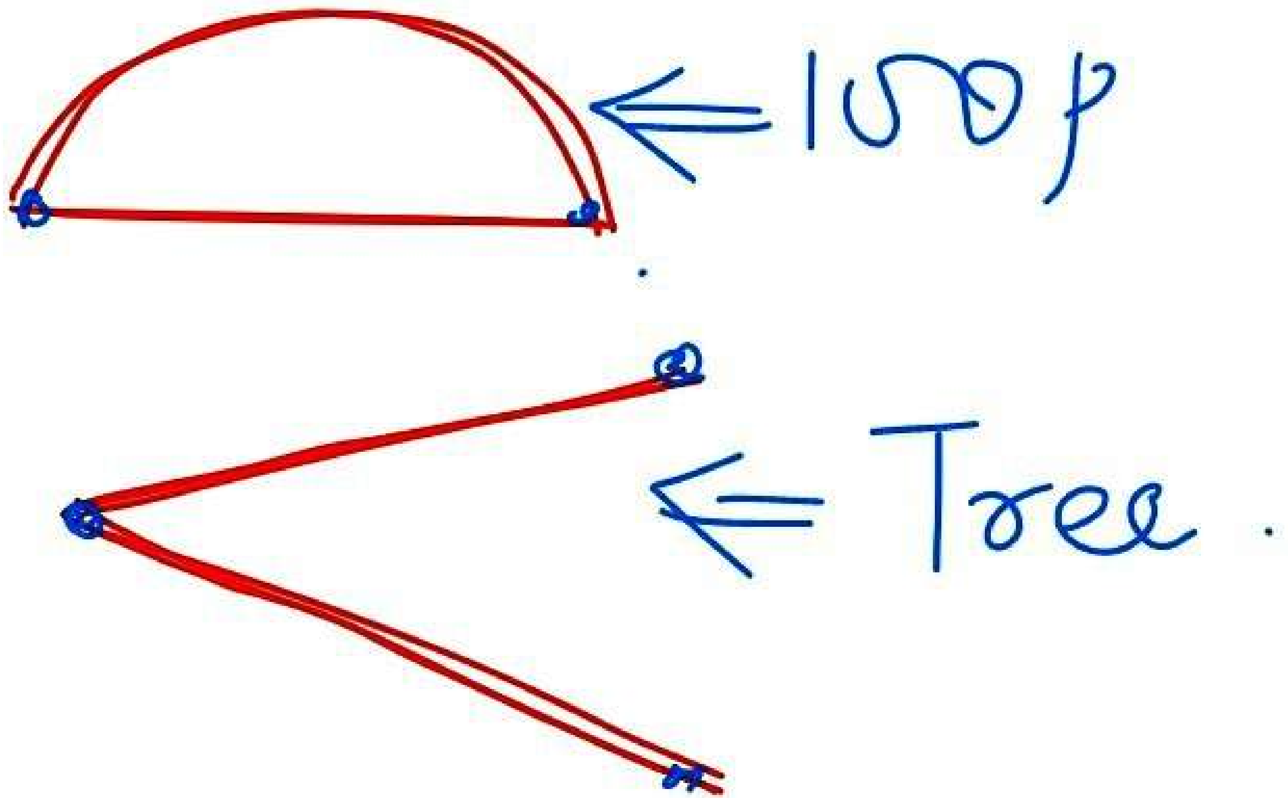
Unconnected
Graph.

Sub graph

It is the graph with less number of branches (least one branch) as compared to original Graph)

Tree of Graph

Defn:- Tree is a connected subgraph, which connects all the nodes without any closed loop.

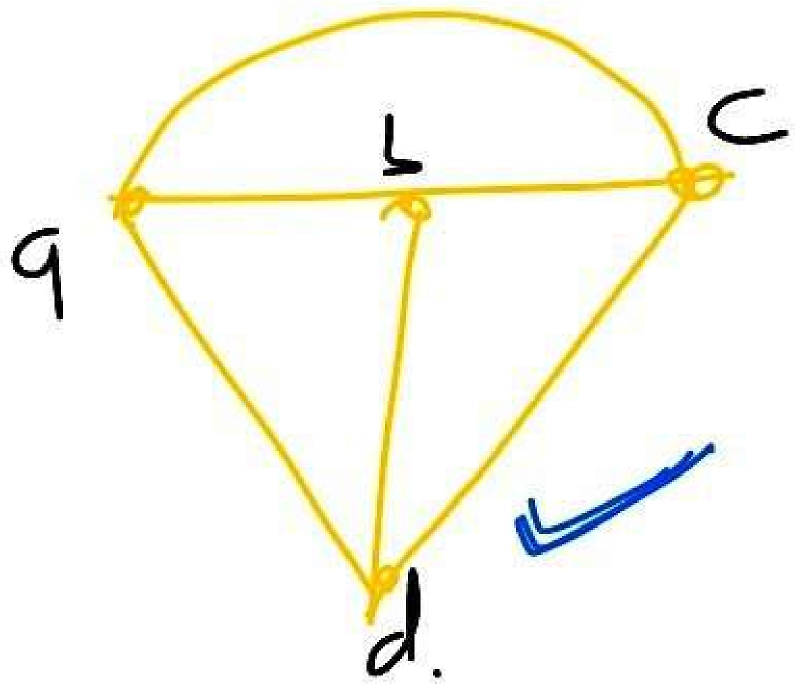

loop
Tree.

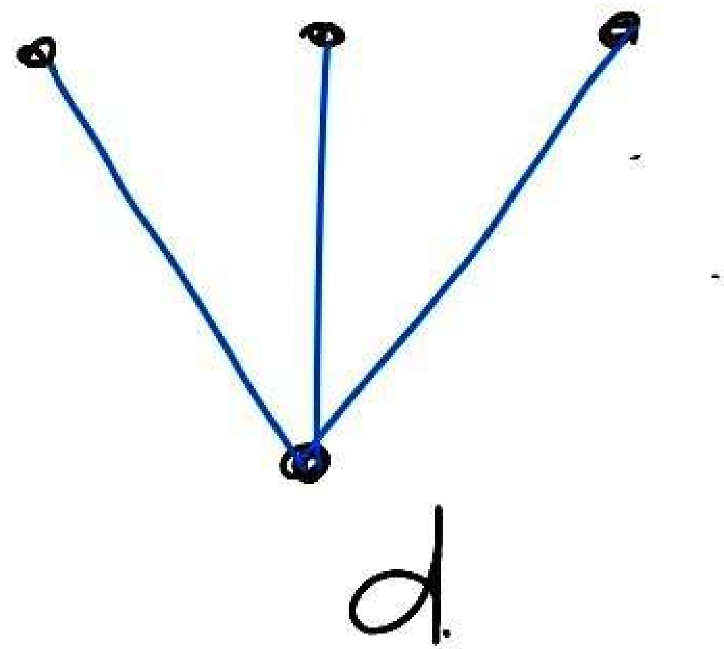
⇒ No. of tree branches of any graph = $(n-1)$

⇒ Tree branches are called twigs.

ref fig 9

⇒ for present graph $(4-1=3)$ Tree branches.


a
b
c
d.


d.

⇒ The no. of possible trees of a complete graph with n nodes is $n^{(n-2)}$

Present graph

$4^{(4-2)} = 16$ possible Trees

Co-tree or the complimented Tree

Defn :- It is a tree formed with all the removed branches from the original graph in order to construct a tree.

⇒ co-tree branches are called links or chords.

⇒ No. of links

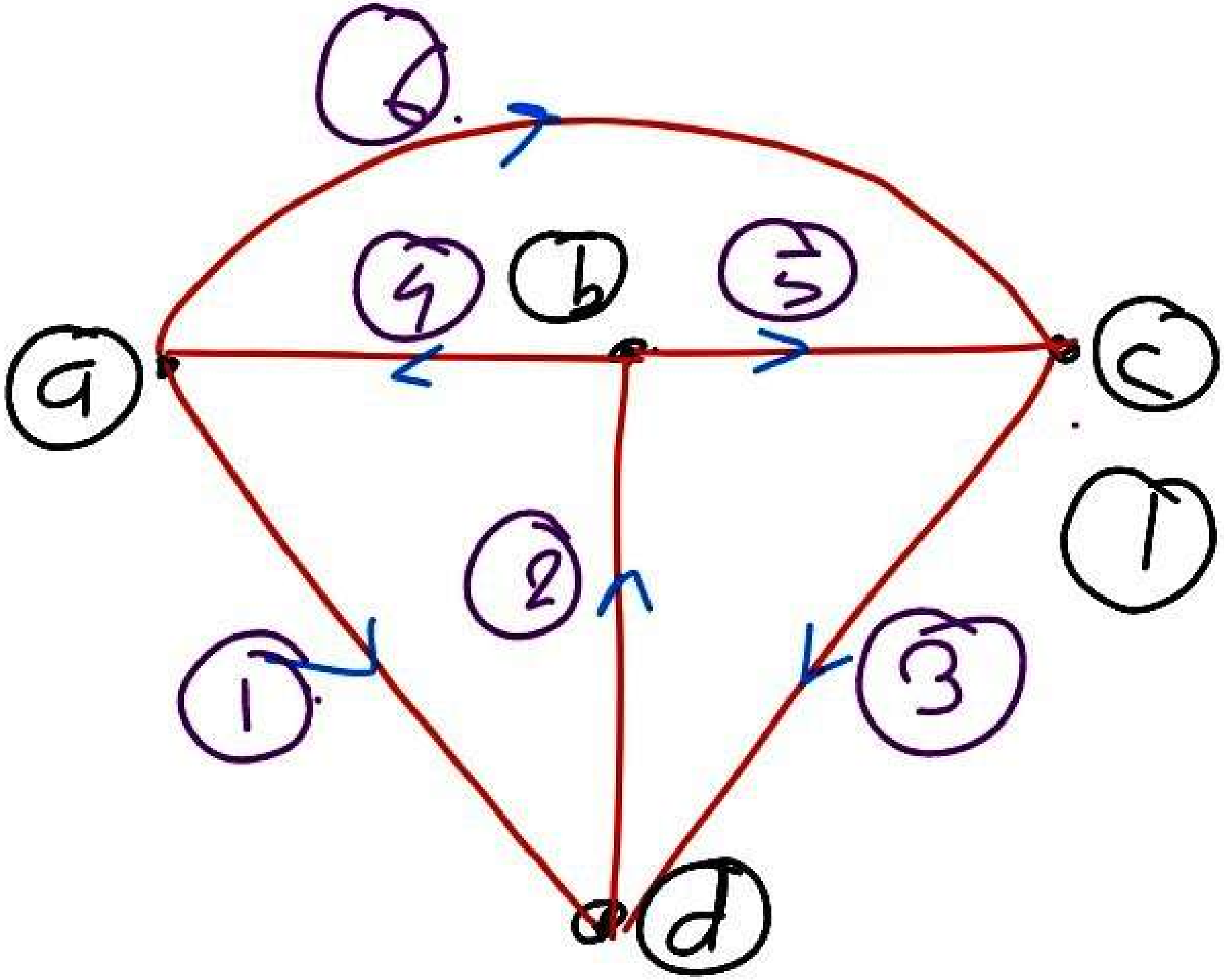
$L = b - (n-1) \Rightarrow b - n + 1$

$b \Rightarrow$ Branches.

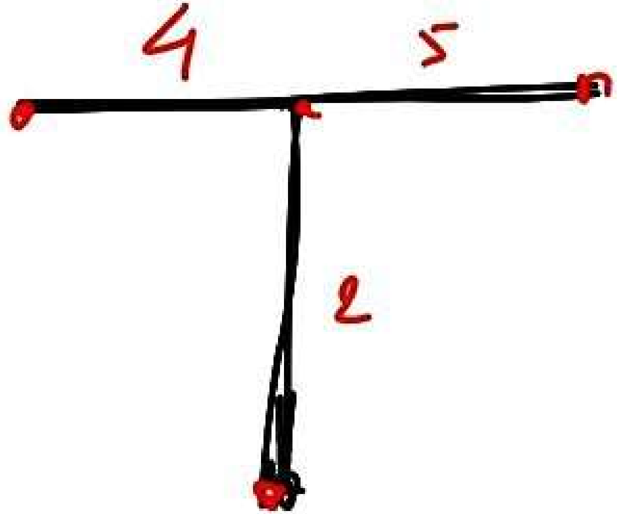
$n \Rightarrow$ Nodes.

$L = 6 - 4 + 1$

$= 3$ links

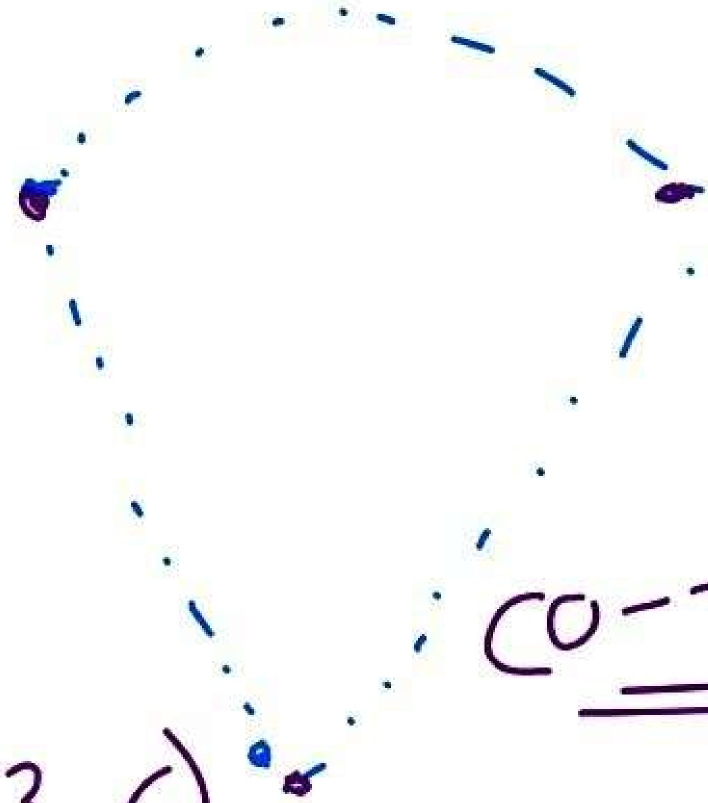


T(2, 4, 5).



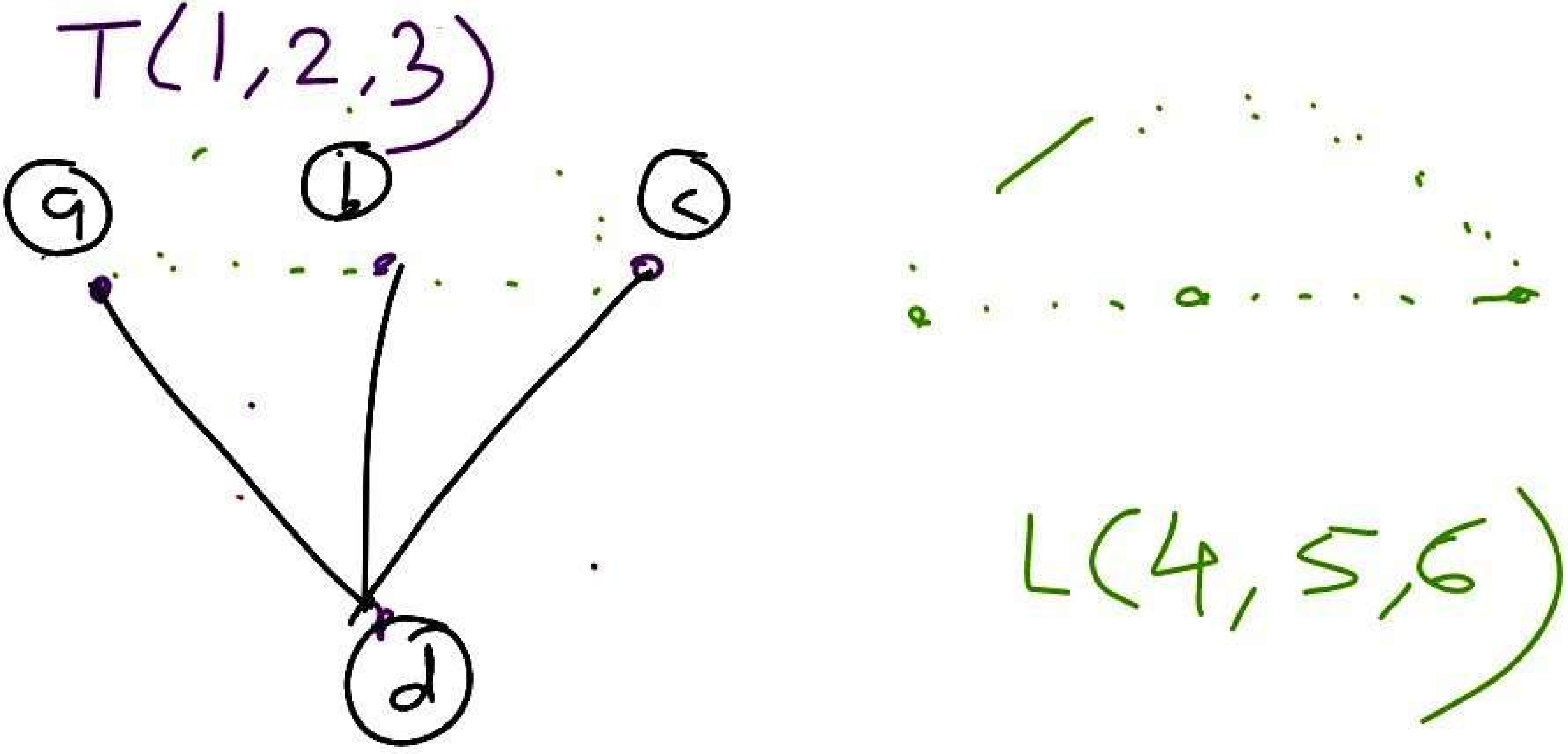
$T(2,4,5)$

Tree of graph.



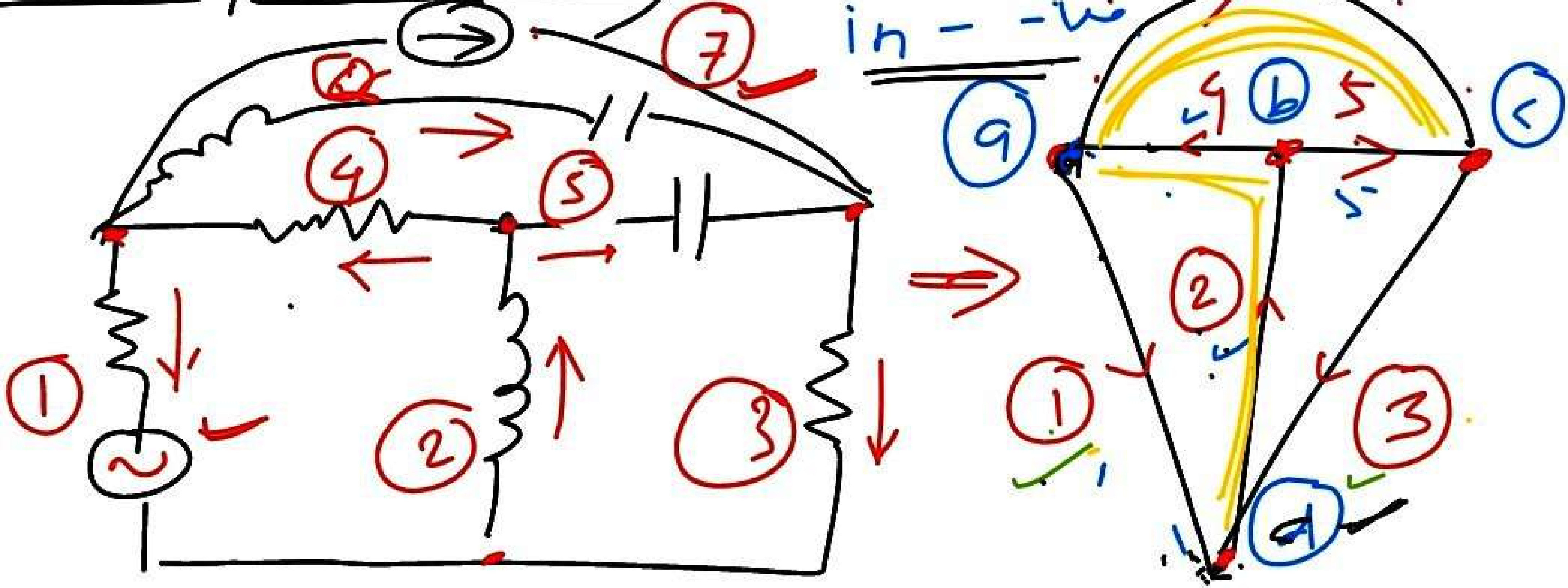
co-tree

$L(1,3,6)$

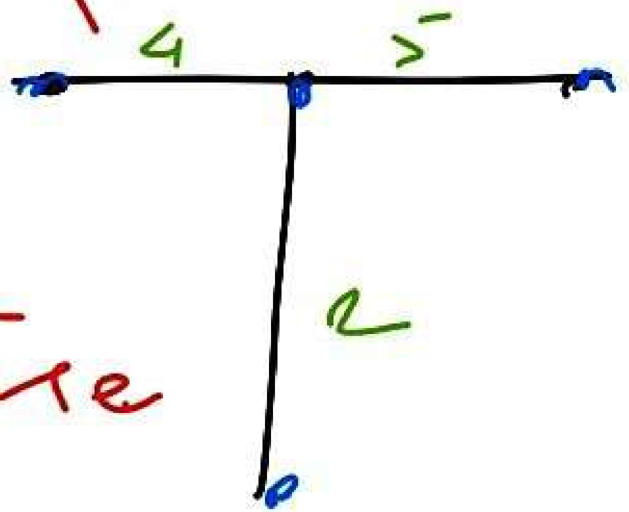

T(1,2,3)
a
b
c
d
L(4,5,6)

Graph theory:-

out → +ve
in - -ve

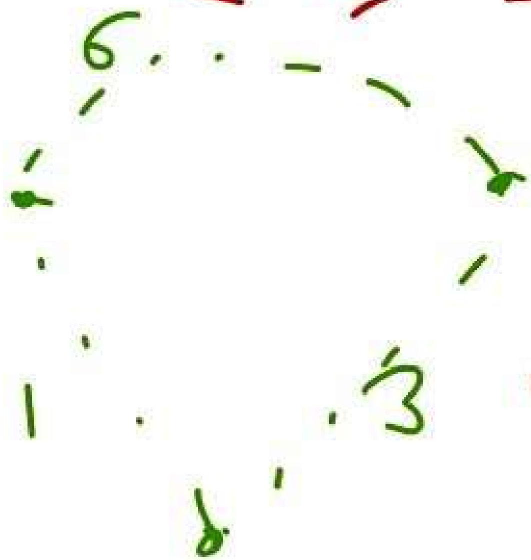


(2, 4, 5)



Tree

(1, 3, 6)



Co-tree

<u>Obs.</u> :- Tree + co-tree $\Rightarrow$ original graph.

Incidence Matrix (A_r)

Let us take the branch current $i_1, i_2, i_3 \ldots i_6$ for the branches $1, 2, 3 \ldots 6$ respectively.

Writing the KCL eqn at each Node.

(a) I $\Rightarrow$ $i_1 - i_4 + i_6 = 0$

(b) II $\Rightarrow$ $-i_2 + i_4 + i_5 = 0$

(c) III $\Rightarrow$ $i_3 - i_5 - i_6 = 0$

(d) IV $\Rightarrow$ $-i_1 + i_2 - i_3 = 0$

$$
\begin{array}{c}
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\mathrm{I} & & & & & & \\
\mathrm{II} & & & & & & \\
\mathrm{III} & & & & & & \\
\mathrm{IV} & & & & & &
\end{array}
\end{array}
\begin{array}{cc}
\mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV}
\end{array}
\underbrace{\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 1 \\
0 & -1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & -1 & -1 \\
-1 & 1 & -1 & 0 & 0 & 0
\end{bmatrix}}_{\substack{4\times 6 \\ n \times b}}
\underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}}_{\substack{6\times 1 \\ b\times 1}}
=
\underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{4\times 1}
$$

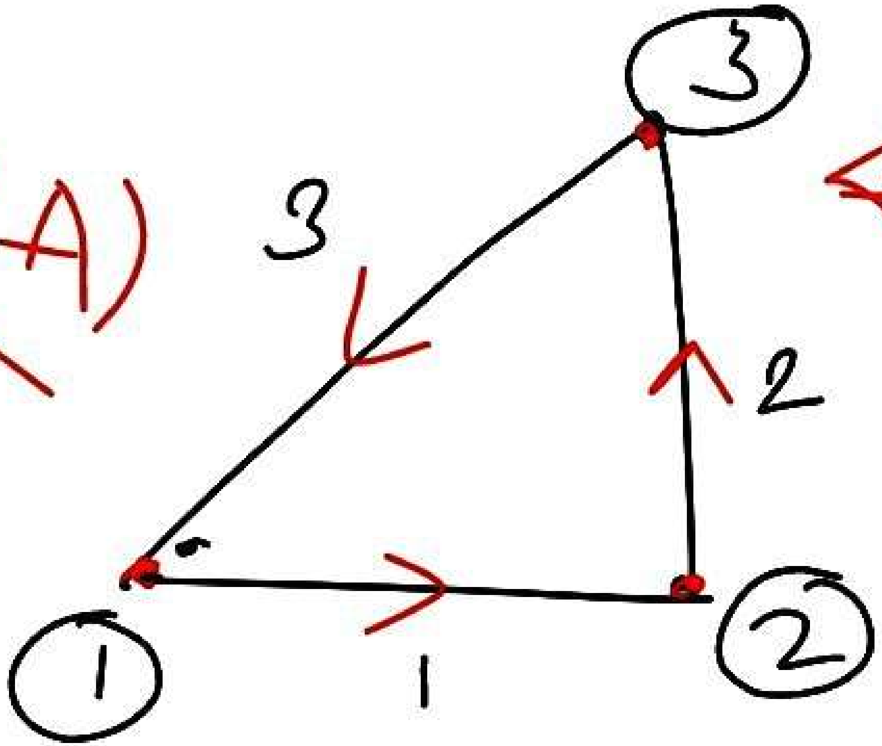
$\Downarrow$ A_r $\quad n \times b \Leftarrow$

1) The order of incidence matrix
$(n \times b) \Rightarrow (4 \times 6)$

2) The rank of incidence matrix is equal to the rank of grap.
$= (n-1) \Rightarrow (4-1) = 3.$

(3) The determinant of the complete incidence matrix of closed loop is always equal zero.

(A)



Closed loop.

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline \mathrm{I} & 1 & 0 & -1 \\ \mathrm{II} & -1 & 1 & 0 \\ \mathrm{III} & 0 & -1 & 1 \end{array}$$

$\begin{matrix} n \times b \\ 3 \times 3 \end{matrix}$

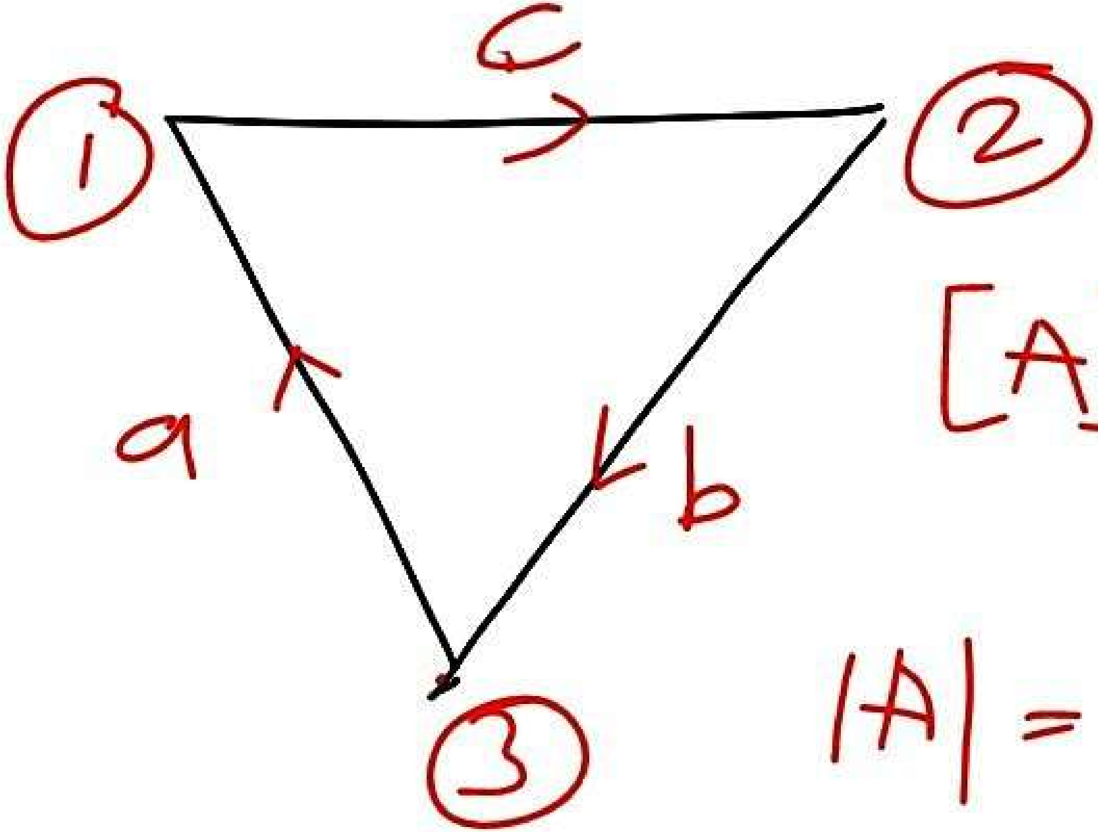
$|A| = 0$

$\text{Rank} = r \neq n \neq 3$

Rank of this matrix $= r = n-1$

$= 3-1 = 2$

$\Rightarrow$ Incidence matrix of a closed loop graph is of $n \times n$



$$[A] = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$|A| = 0$$

Tie-set matrix :-

(fundamental loop matrix or fundamental circuit matrix)

Defⁿ :- It is a closed path which contains only one link together with any number of twings (tree branches)

Let us take the branch voltages $v_1, v_2, v_3 \ldots \ldots v_6$ for the branches 1, 2 . . . 6 resp.

f. loops or f. cuts or tie set.

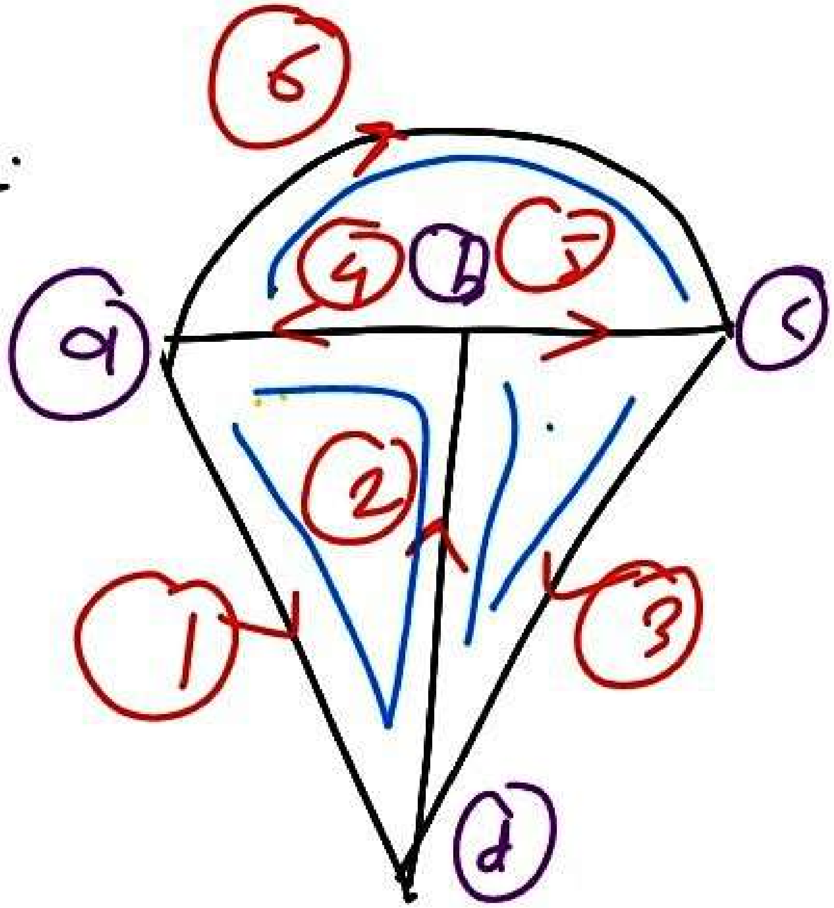
Minimum no. of loop or mesh eqn required to solve the n/w.

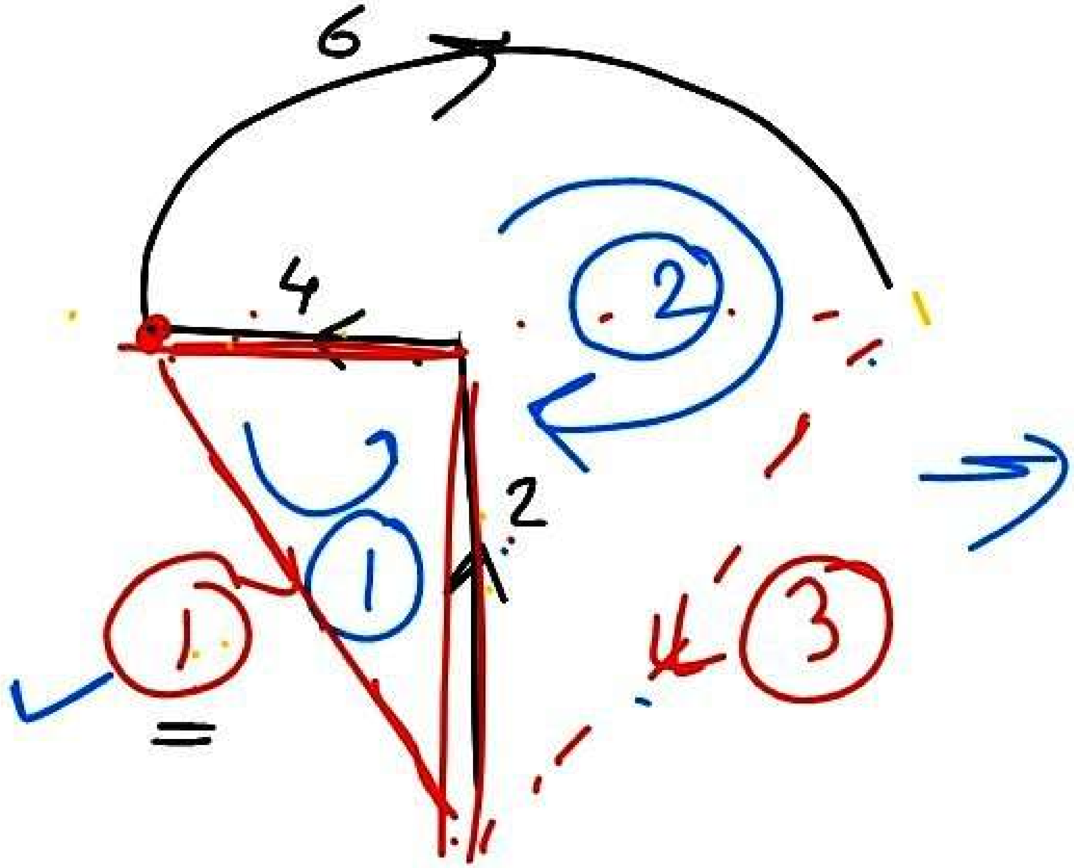
Steps.

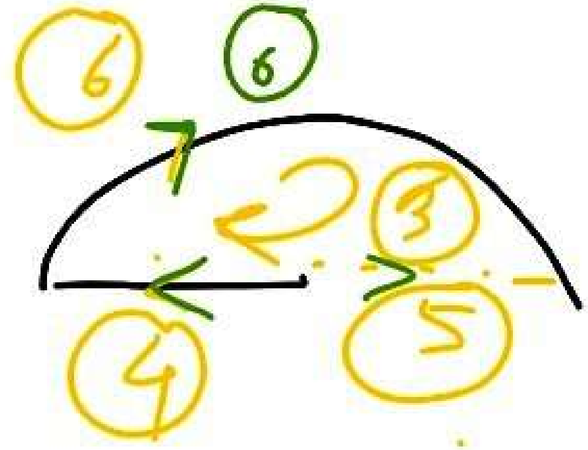
1. Select a tree.

T (2, 4, 6) ✓

L (1, 3, 5)



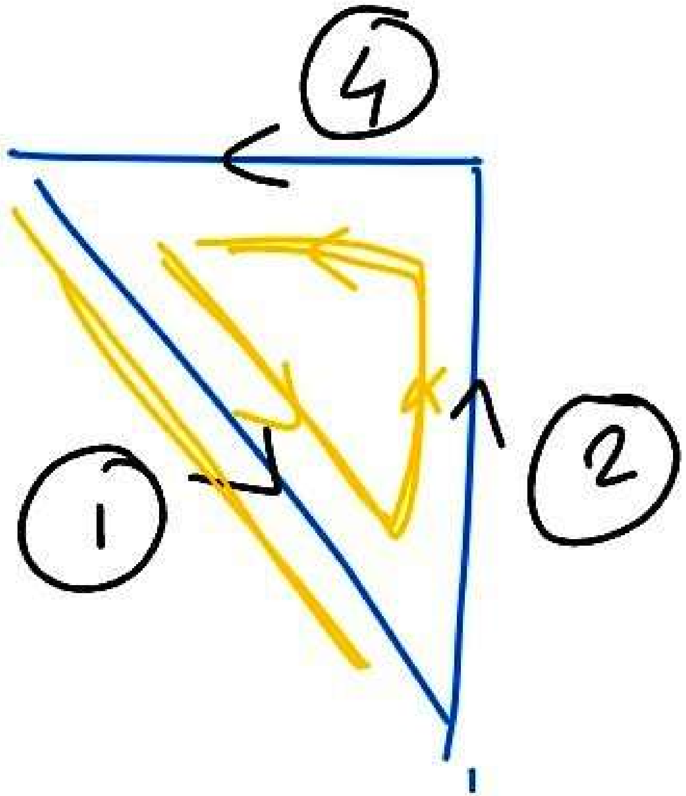

6
4
2
2
1
1
3
=

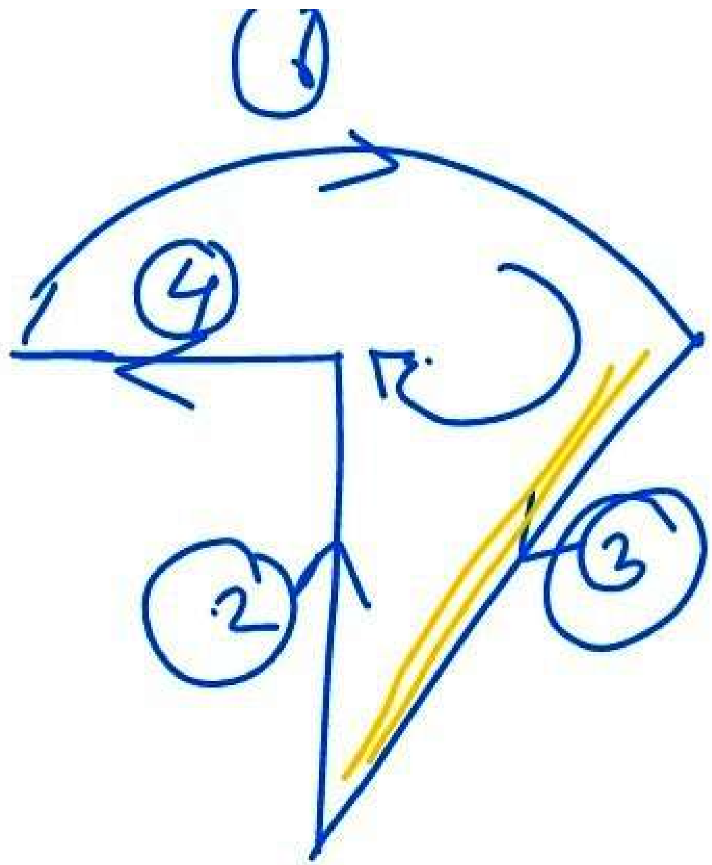

6
6
3
4
5

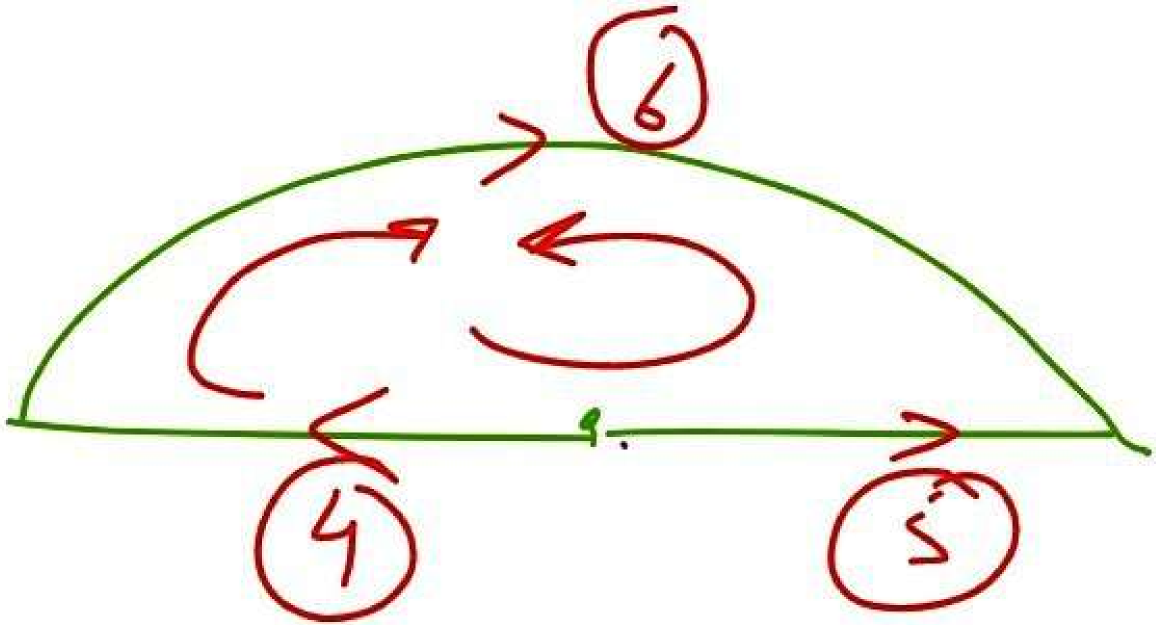
$$f_1 \Rightarrow v_1 + v_2 + v_4 = 0$$

$$f_2 \Rightarrow v_2 + v_3 + v_4 + v_6 = 0$$

$$f_3 \Rightarrow$$


4
1
2


1
4
2
3



$$V_5 - V_4 - V_6 = 0$$

Tie set matrix (Br)

$$
\begin{array}{c}
 \\ f_1 \\ f_2 \\ f_3
\end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & -1 & 1 & -1
\end{bmatrix}
\end{array}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}_{6\times 1}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{(3\times 1)}
$$

$(b-n+1)\times b \Rightarrow$

$= 3\times 6$

$(b - n + 1) \Rightarrow$ Rank of Tie set matrix.

$6 - 4 + 1 = 3$.

(2) No. of f-loop $= b - n + 1$

$= 3$.

(3) For complete graph there are $n^{(n-2)}$ f loop matrices.

Construct the correct graph of Network shown below.

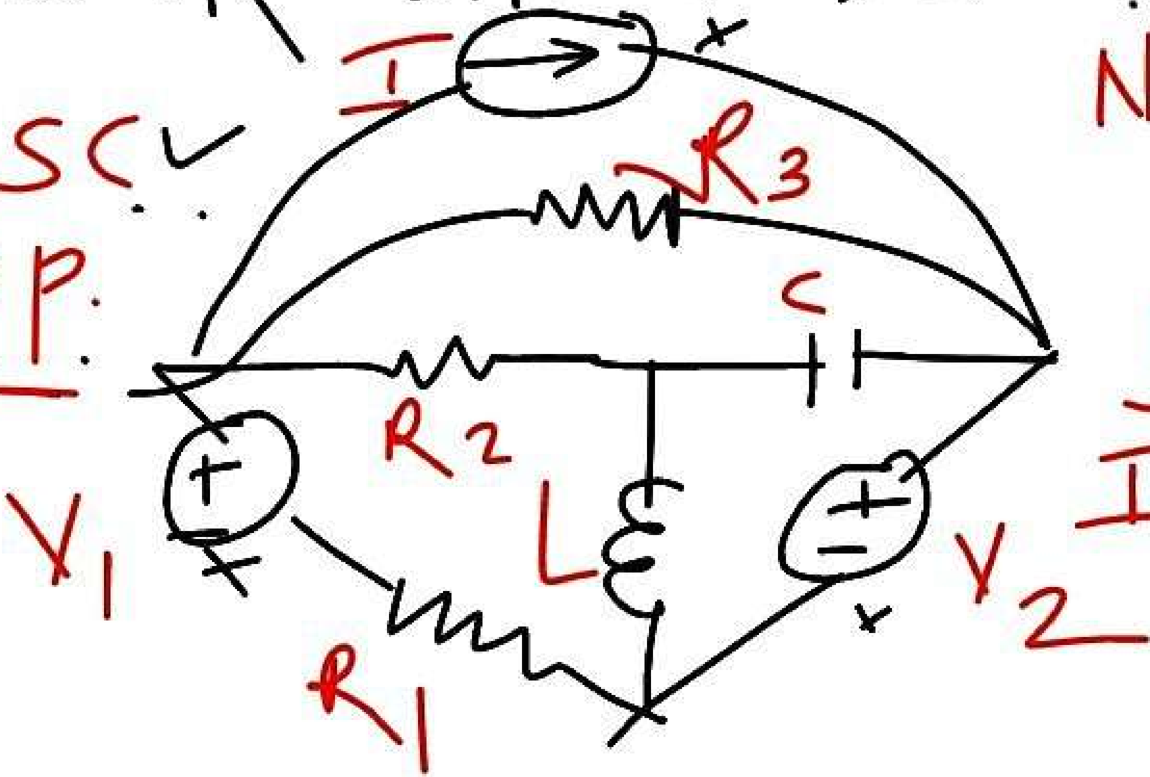
VS ⇒ SC ✓

CS ⇒ OP.

4 ✓

6 ✓

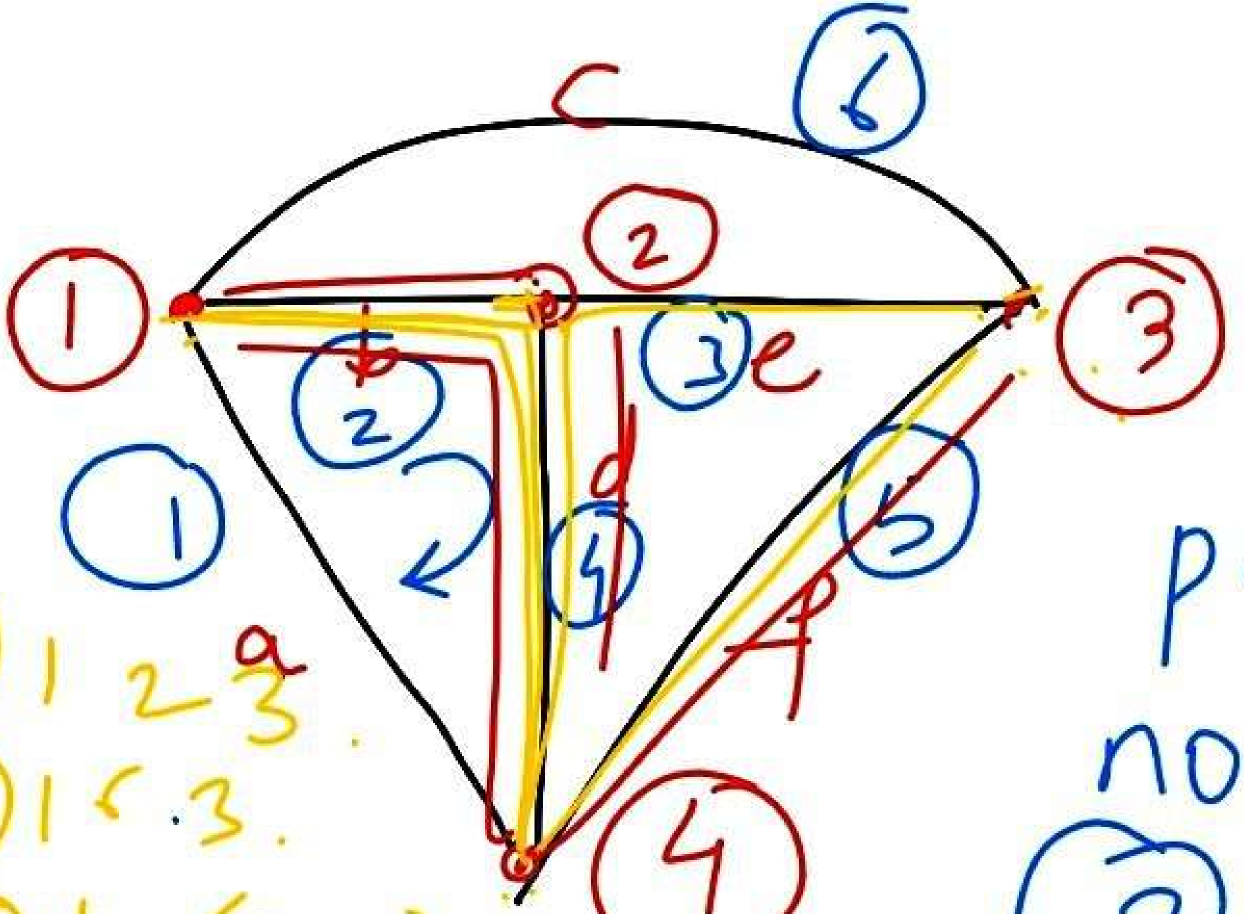
3 ✓



No. of Nodes = ?

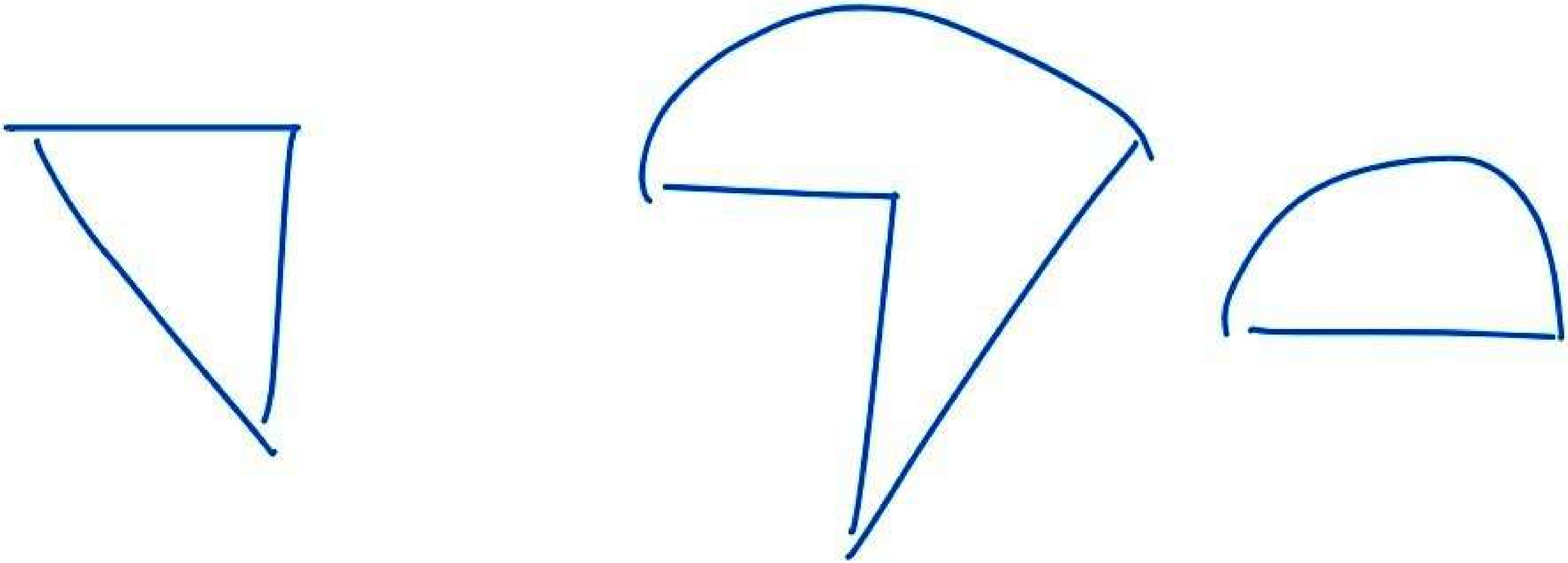
Branches = ?

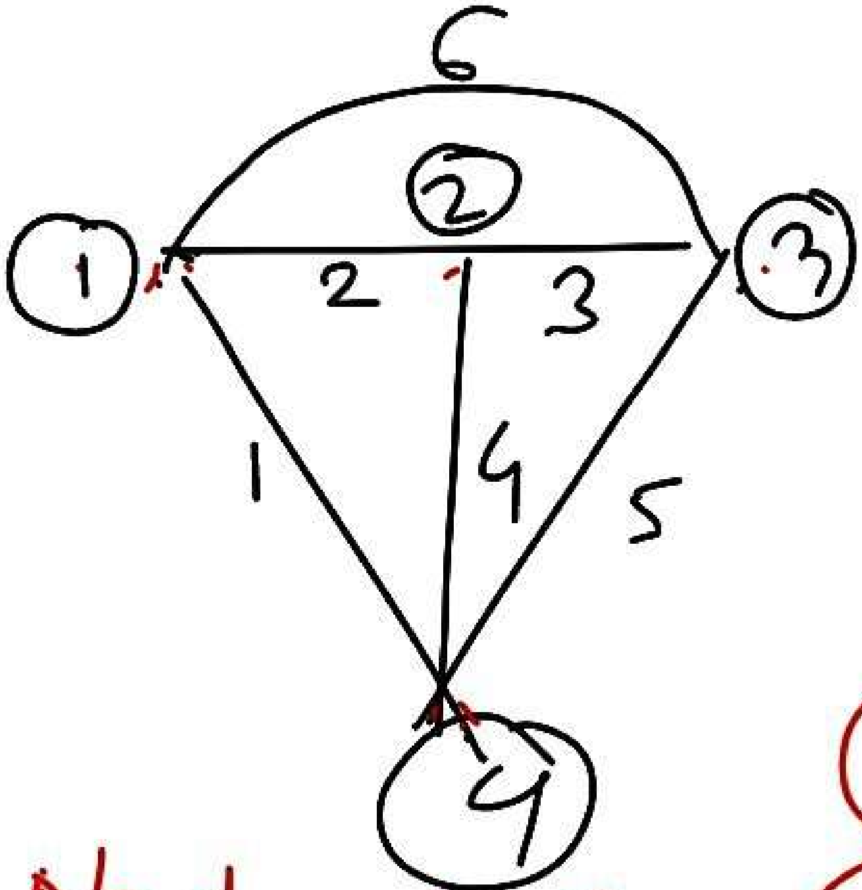
Independent loops.



No. of possible path from node 1 to 3 4

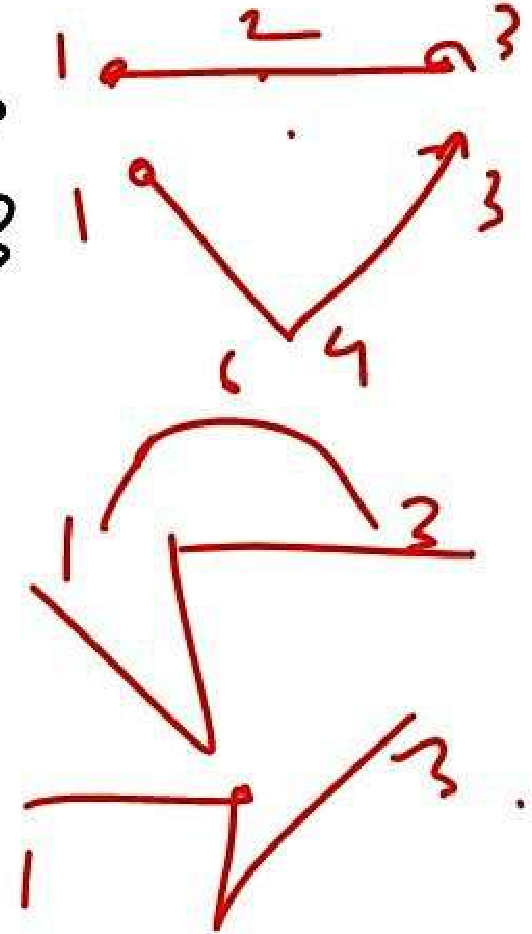
1 1 2 3.
2 1 5 3.
3 1 4 3

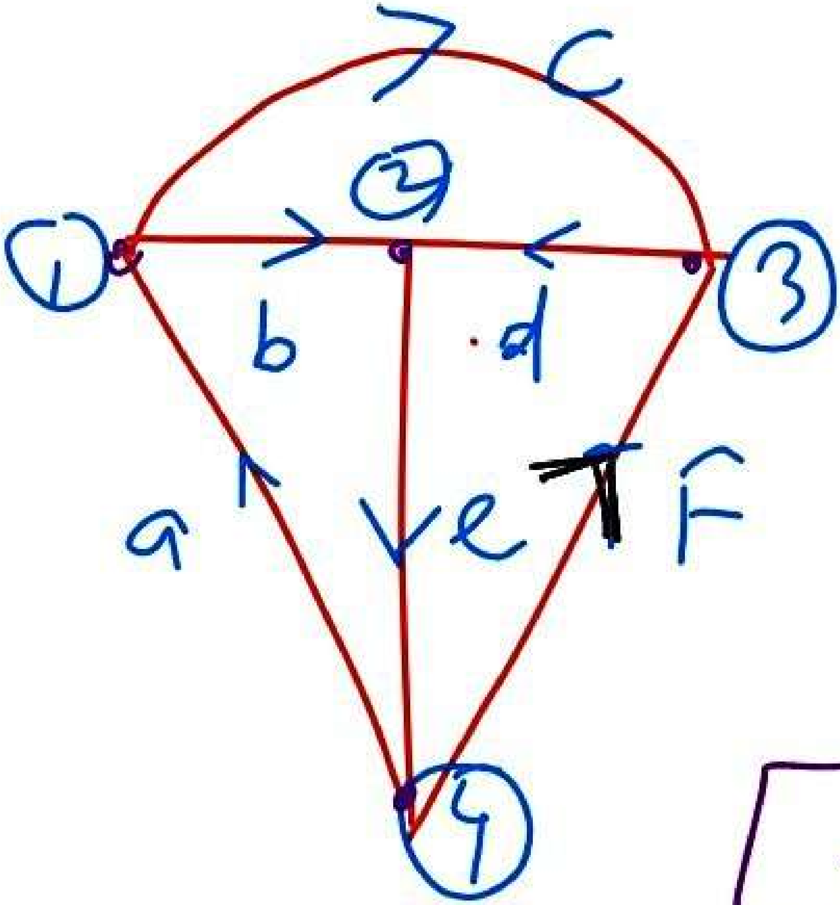




Node 1 to 3

1	1	2	3	
2	1	4	3	
3	1	6	3	
4	1	4	2	3
5	1	2	4	3





Construct the complete incidence matrix for the oriented graph.

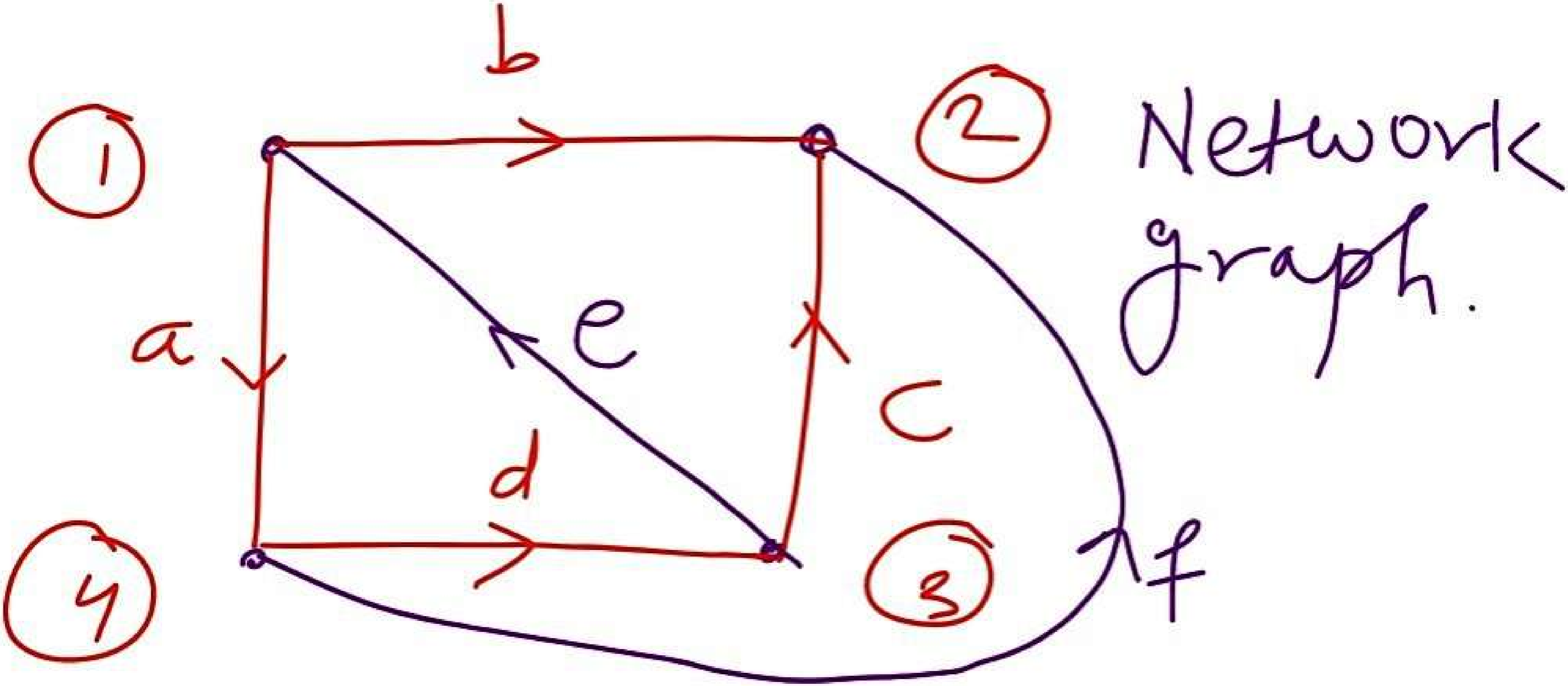
[A]

$$[A] = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline (1) & -1 & 1 & 1 & 0 & 0 & 0 \\ (2) & 0 & -1 & 0 & -1 & 1 & 0 \\ (3) & 0 & 0 & -1 & 1 & 0 & -1 \\ (4) & 1 & 0 & 0 & 0 & -1 & 1 \\ \hline & 0 & 0 & 0 & 0 & 0 & 0 \end{array}_{4\times 6}$$

Construct a oriented graph of a Network where incidence matrix is given as

$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline 1 & 1 & 1 & 0 & 0 & -1 & 0 \\ 2 & 0 & -1 & -1 & 0 & 0 & -1 \\ 3 & 0 & 0 & 1 & -1 & 1 & 0 \\ 4 & -1 & 0 & 0 & 1 & 0 & 1 \end{array}$$

No. of Nodes - 4
No. of branches = 6


1
2
3
4
a
b
c
d
e
f
Network graph.

A graph of an electrical N/W has 4 Nodes & 7 branches. The no. of links l, with respect to choosen tree would be.

$$\Rightarrow \boxed{4 \Rightarrow b-(n-1)=4}$$

A N/ω. has 8 Nods &
5 independent loops.
No. of branches = ?

$\Rightarrow \underline{12}$

$5 = b - 7$

$l = \underline{\underline{b}} - (n-1) \Rightarrow \boxed{b = 12}$

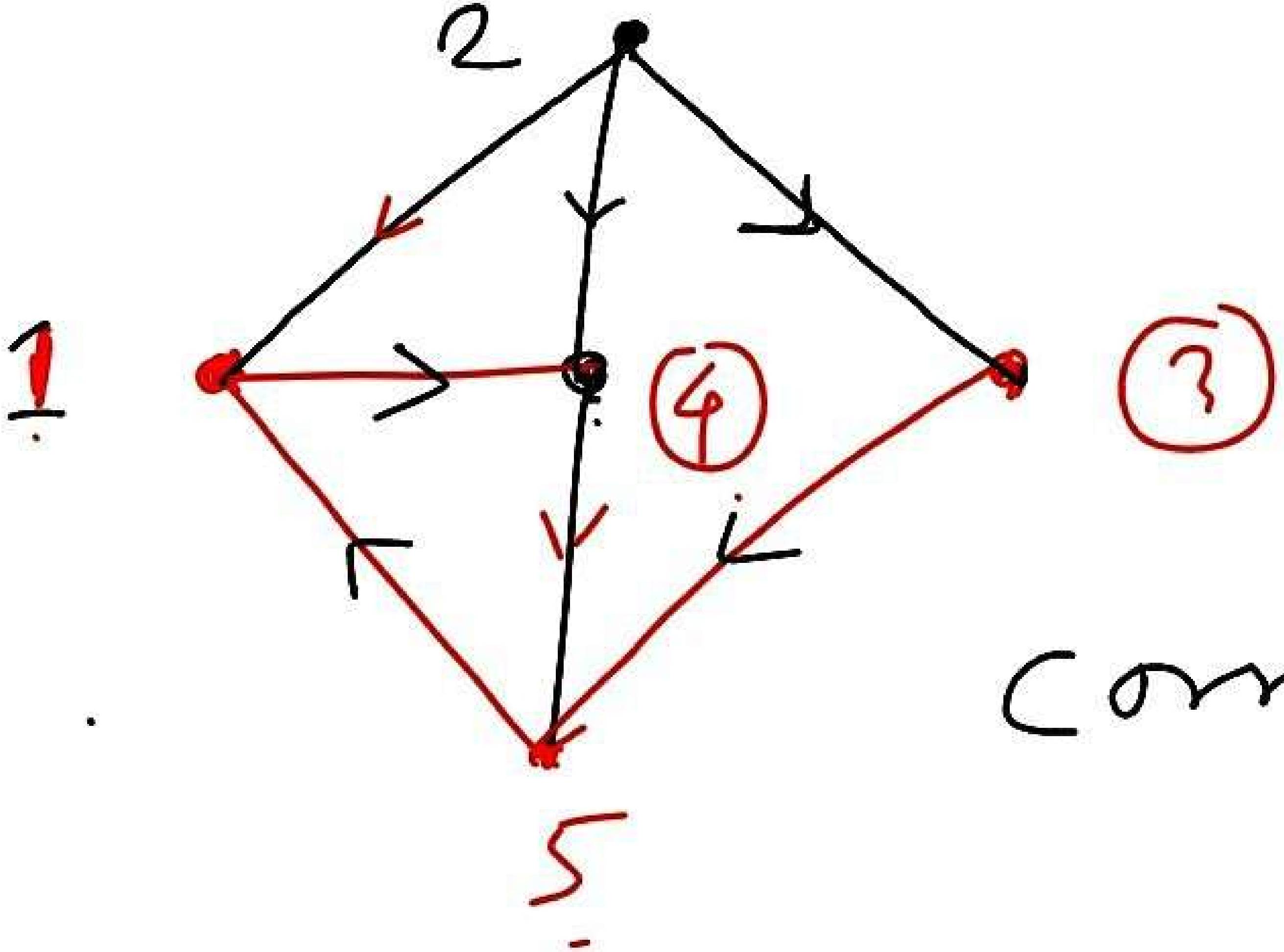
→ 6 Nodes & 9 Branch

Independent loops = l

$l = b - (n-1) =$

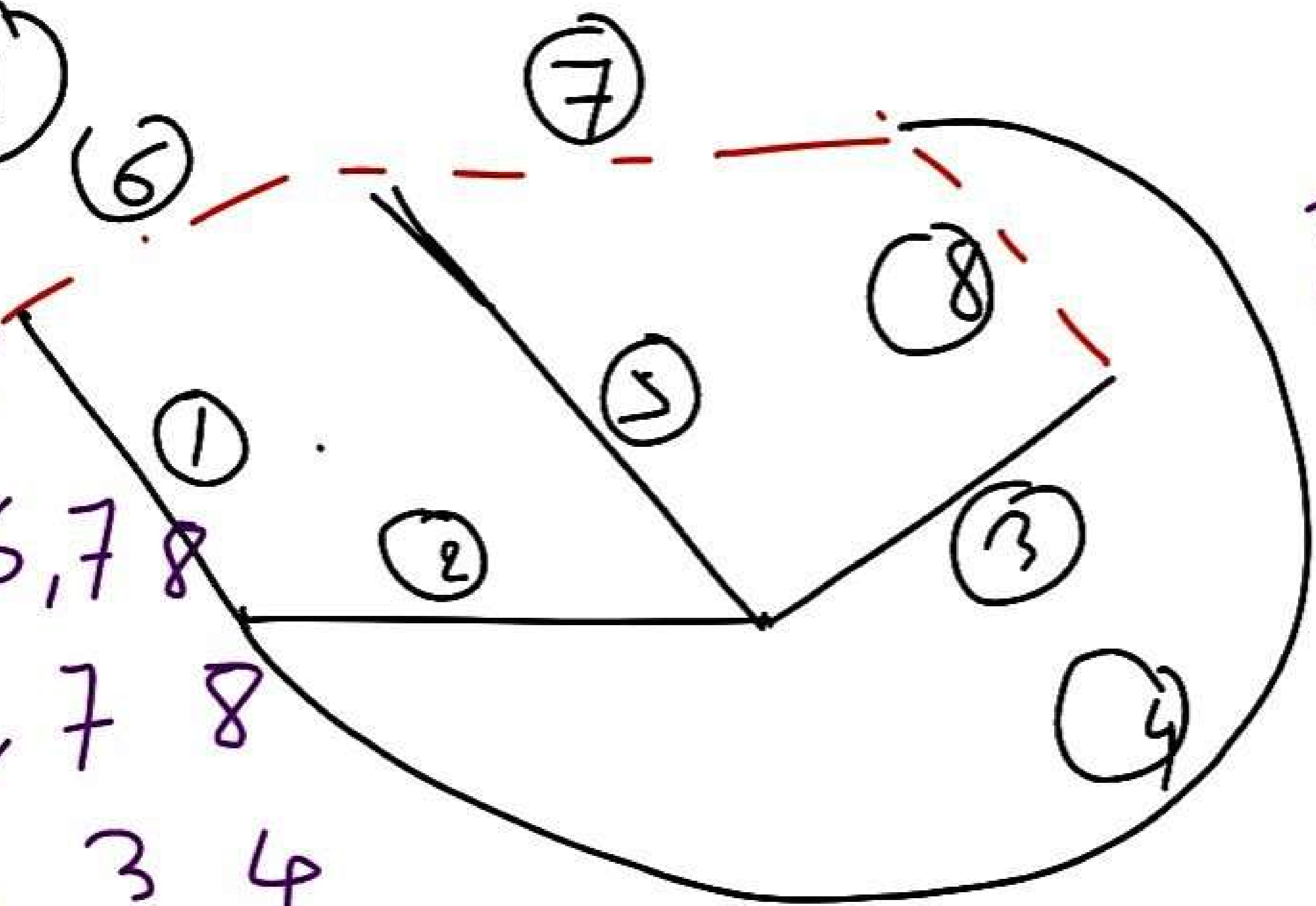
$9 - (6-1) \Rightarrow \underline{4}$

$$
A = \begin{array}{c}
\begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\
\begin{bmatrix}
-1 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -1 & -1 & 0 \\
1 & -1 & -1 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$



correct ✓

Q. 1



find Invalid cut set.

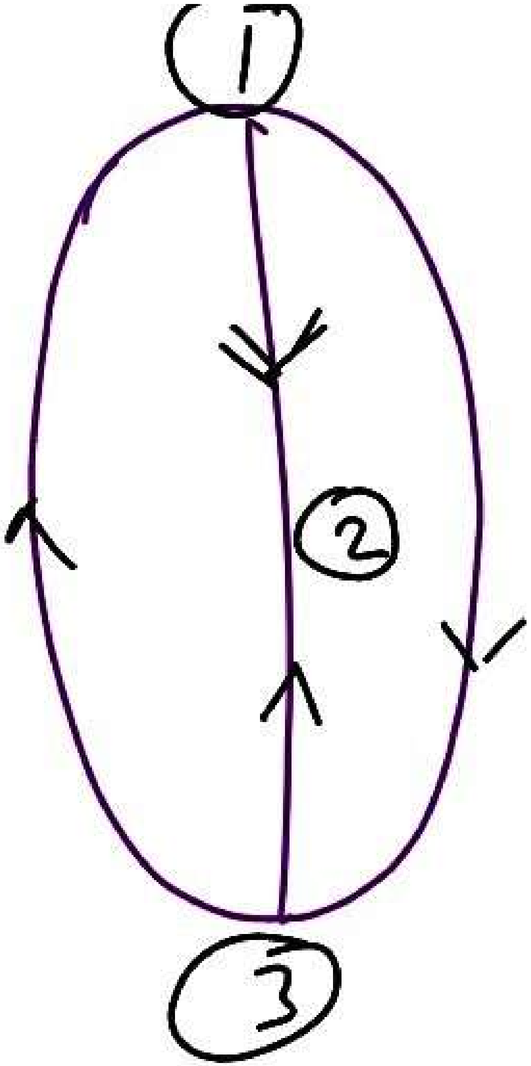
(a) 1, 6

(b) 2, 6, 7 8

(c) 4, 7 8

(d) 2 3 4

for the graph shown. find the no of possible tree.



(a) 6

(b) 5

(c) 4

(d) 3

Q. 3) A network graph consist of 17 branches & 10 nodes. Determine the no. of eqn required to solve the corresponding Network

Gauss elimination Method

Consider A system eqn.

$$I_1 - 2I_2 = 1$$

$$3I_1 + 2I_2 = 11$$

1st step ⇒ Write eqn in matrix form.

$$\begin{bmatrix} 1 & -2 \\ \underline{3} & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

$$AX = B.$$

4×4

$C = A : B$. $\quad R_2 \to R_2 - 3R_1$

$$\begin{array}{c} R_1 \\ R_2 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & 2 & 11 \end{array}\right] \Rightarrow \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$

$\hookrightarrow$ convert this into upper triangular matrix.

Elementry Row transformation for upper triangular matrix.

$$R_2 \longrightarrow R_2 - 3R_1$$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 8 & 8 \end{array}\right]$$

$$I_1 - 2I_2 = 1$$

$$8I_2 = 8$$

$$\boxed{I_2 = 1}$$

& $I_1 - 2(1) = 1 \Rightarrow \boxed{I_1 = 3}$

$$x - 2y + 3z = 9$$

$$-x + 3y = -4$$

$$2x - 5y + 5z = 17$$

$$C = A : B.$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

A

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

$$R_2 \longrightarrow R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{array}\right] \Rightarrow \quad R_3 - 2R_1$$

$R_3 \longrightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array}\right]$$

$R_3 \longrightarrow R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$$x - 2y + 3z = 9$$

$$y + 3z = 5 \Rightarrow$$

$$2z = 4$$

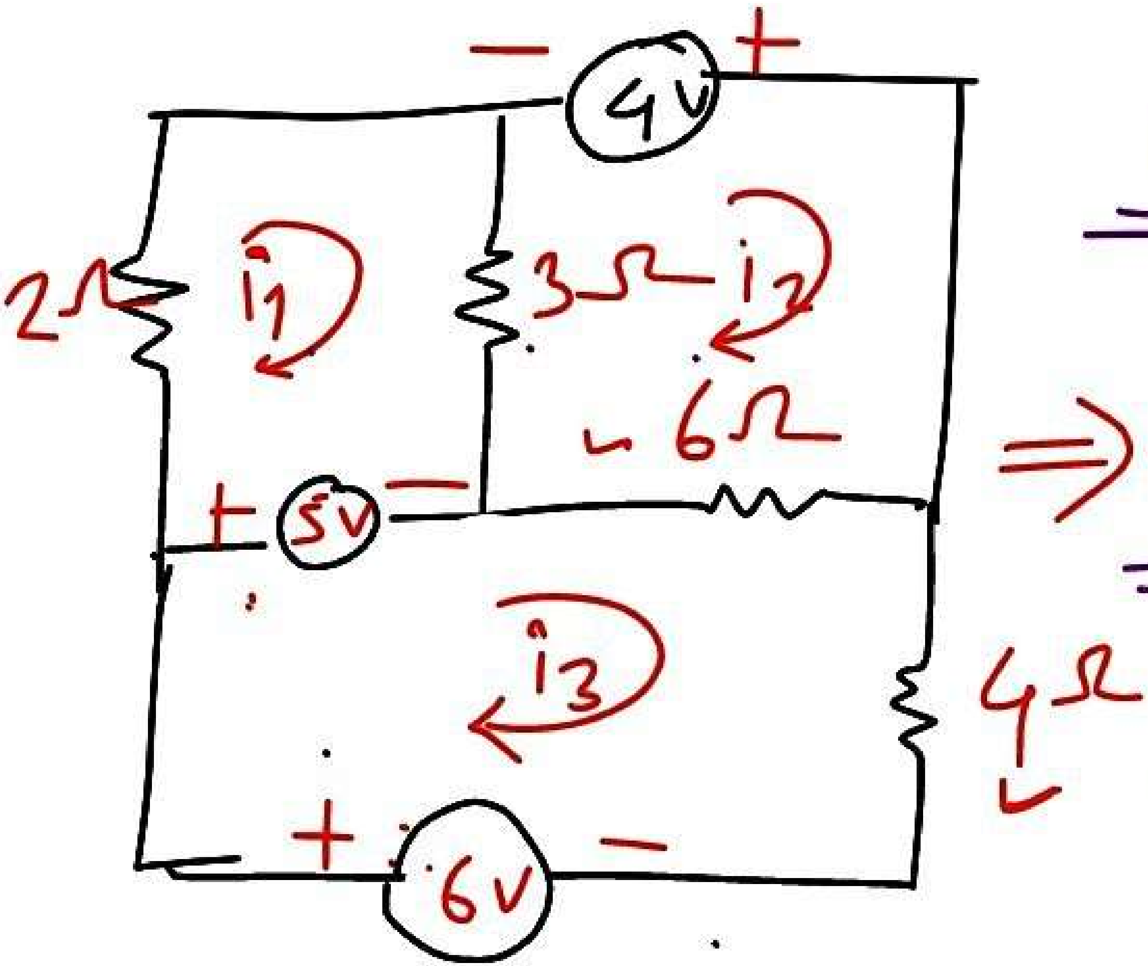
$$\boxed{z = 2}$$

$$y + 3(2) = 5$$

$$\boxed{y = -1}$$

$$x - 2(-1) + 3 \times 2 = 9$$

$$= \boxed{x = 1}$$



loop 1

$$2i_1 + 3(i_1 - i_2) = 5$$

$$= 2i_1 + 3i_1 - 3i_2 = 5$$

$$= 5i_1 - 3i_2 = 5 \quad \text{--- (1)}$$

Loop. 2

$$6(i_2 - i_3) + 3(i_2 - i_1) = 4$$

$$6i_2 - 6i_3 + 3i_2 - 3i_1 = 4$$

$$-3i_1 + 9i_2 - 6i_3 = 4 \quad \text{--- (2)}$$

Loop 3

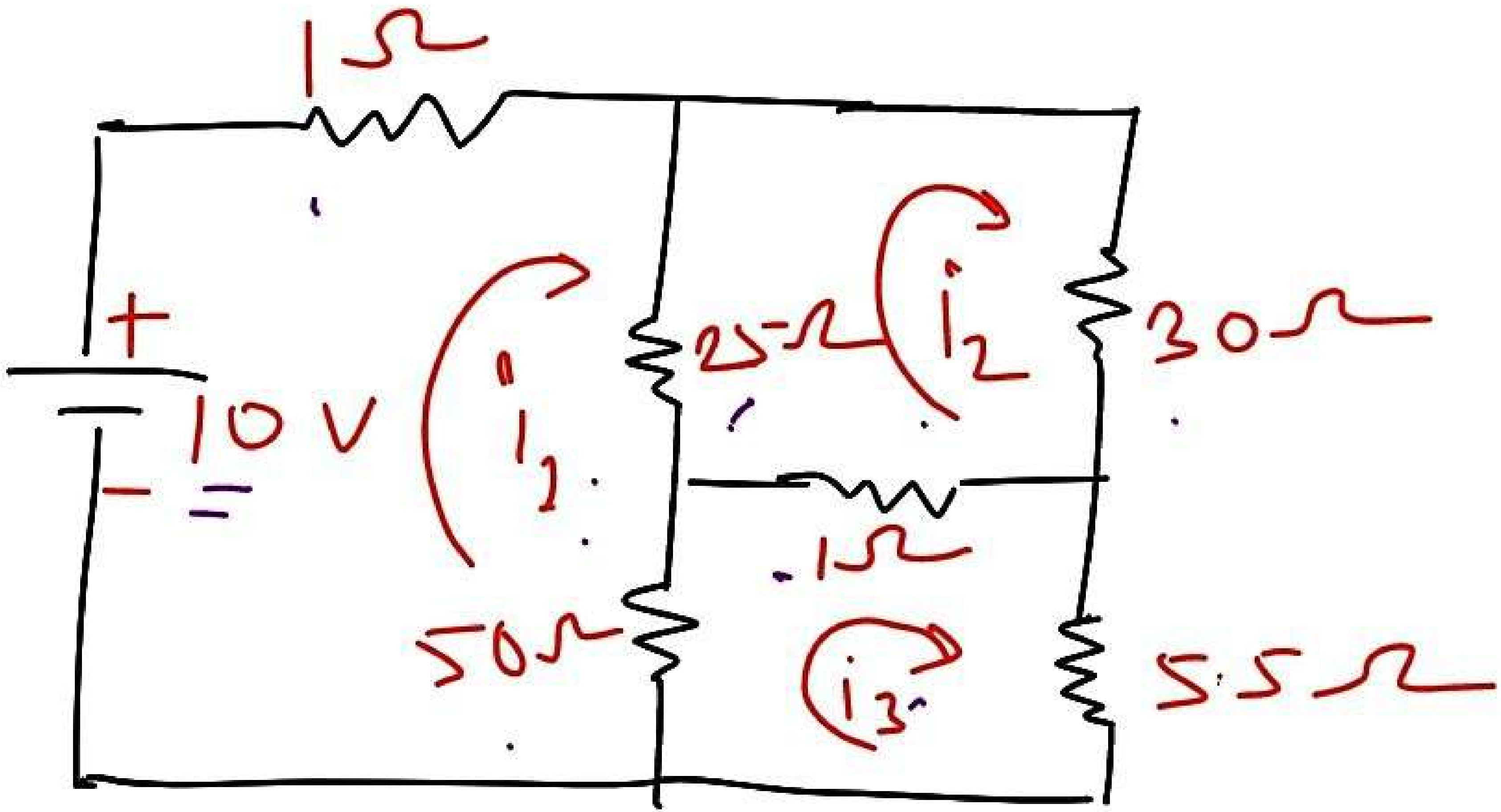
$$\Rightarrow 6(i_3 - i_2) + 4i_3 = 6 - 5$$

$$6i_3 - 6i_2 + 4i_3 = 1$$

$$= \quad -6i_2 + 10i_3 = 1 \quad \text{—— } (3)$$

$$\begin{bmatrix} 5 & -3 & 0 \\ -3 & 9 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 5 & -3 & 0 & 5 \\ -3 & 9 & -6 & 4 \\ 0 & -6 & 10 & 1 \end{array}\right]$$



1Ω
+
10 V
−
i_1
25Ω
i_2
30Ω
1Ω
50Ω
i_3
5.5Ω

loop. 1

$$2i_1 + 25(i_1 - i_2) + 50(i_1 - i_3) = 10$$

$$i_1 + 25i_1 - 25i_2 + 50i_1 - 50i_3 = 10$$

$$\Rightarrow \quad 76i_1 - 25i_2 - 50i_3 = 10 \qquad \text{——— } (1)$$

Loop 2

$\rightarrow 25(i_2 - i_1) + 30i_2 + 1(i_2 - i_3) = 0$

$\rightarrow 25i_2 - 25i_1 + 30i_2 + i_2 - i_3 = 0$

$\rightarrow -25i_1 + 56i_2 - 1i_3 = 0$ ——— (2)

Loop 3

$$50(i_3-i_1)+1(i_3-i_2)+55i_3=0$$

$$\rightarrow 50i_3-50i_1+i_3-i_2+55i_3=0$$

$$\rightarrow -50i_1-i_2+106i_3=0 \quad \text{——— (3)}$$

$$\begin{bmatrix} 76 & -25 & -50 \\ -25 & 56 & -1 \\ -50 & -1 & 106 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 76 & -25 & -50 & 10 \\ -25 & 56 & -1 & 0 \\ -50 & -1 & 106 & 0 \end{array}\right]$$

Q.1 Ans. $\Rightarrow i_1 = 34/15, \quad i_2 = 19/9$

$i_3 = 41/30$

Q2. Ans $\Rightarrow i_1 = 0.245$

$i_2 = 0.111$

$i_3 = 0.117$

Time & frequency response of circuits

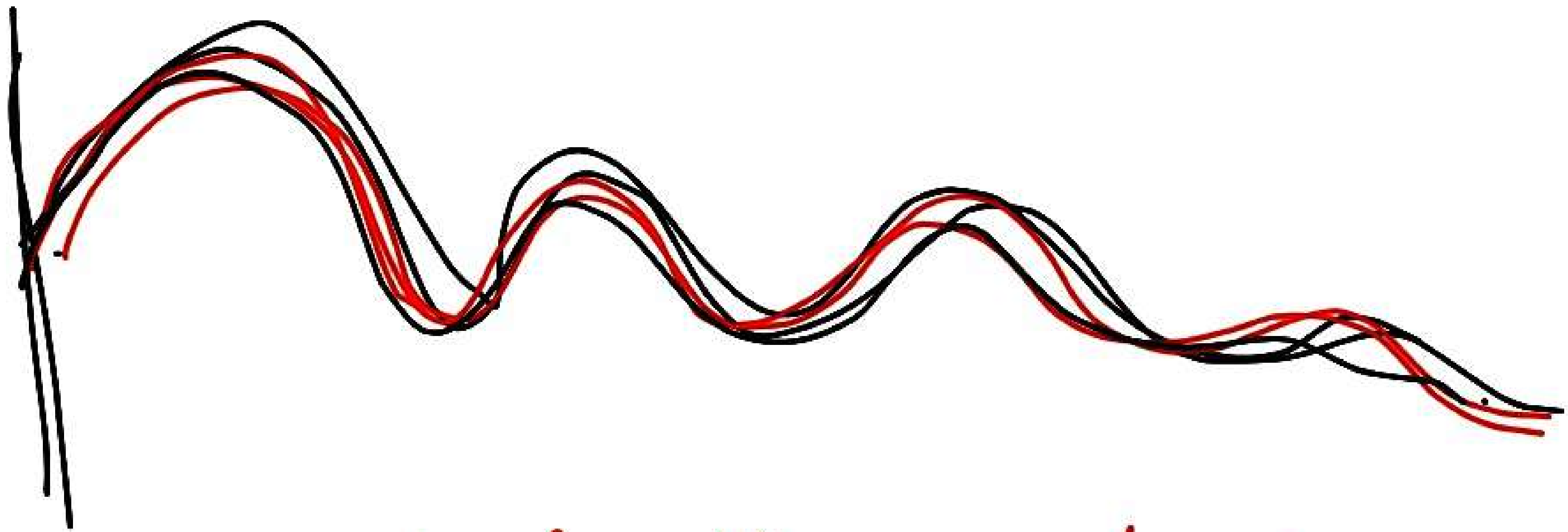
1) first & second order differential eqn

2) Transient & steady state response

③ Transfer function.

④ Concept of poles & zeros.

⑤ frequency response.



Transient in the system because of presence of $\boxed{L \,\&\, C}$

Energy stored in memory element
cannot change instantaneously.
ie within zero time.
due to the integration of
energy variables i_L & v_C

L & C will oppose sudden changes.

Results unstability.

When the network consist of only resistance (memoryless element) no transient occur in system.

$V = iR$ & $i = \frac{V}{R}$

Transient effects are more severe for DC as compared to AC. The transient free condition is possible to only for AC excitations. (At zero crossings of sin or cos)

Laplace transform :-

$R \xrightarrow{LT} R\ \Omega$

$L \xrightarrow{LT} SL\ \Omega$

$C \xrightarrow{LT} \frac{1}{SC}\ \Omega$

$v(t) \xrightarrow{LT} V(s)$

$i(t) \xrightarrow{LT} I(s)$

$\delta(t) \xrightarrow{LT} 1$

$u(t) \xrightarrow{LT} 1/s$

$K u(t) \xrightarrow{LT} K/s$

$$R(t) \xrightarrow{LT} 1/s^2$$

$$p(t) \xrightarrow{LT} 1/s^3$$

$$e^{-at}\,u(t) \xrightarrow{LT} \frac{1}{s+a}$$

$$=$$

$$\Downarrow$$

$$e^{-3t}\,u(t) \xrightarrow{LT} \frac{1}{s+3} \Leftarrow I(s)$$

$$\boxed{\frac{V(s)}{I(s)} = Z(s)\ \Omega \quad \text{ohm}}$$

$$V(s) = Z(s) \cdot I(s)$$

$$I(s) = \frac{V(s)}{Z(s)}$$

$$\boxed{\frac{I(s)}{V(s)} = Y(s)\ \mho \quad \text{mho}}$$

$$V(s) = \frac{I(s)}{Y(s)}$$

$$I(s) = Y(s) \cdot V(s)$$

$Z(s)$

$R \; \Omega \quad \Omega$

$sL \quad \Omega$

$\frac{1}{sC} \quad \Omega$

$Y(s)$

$\frac{1}{R} \quad \mho$

$\frac{1}{sL} \quad \mho$

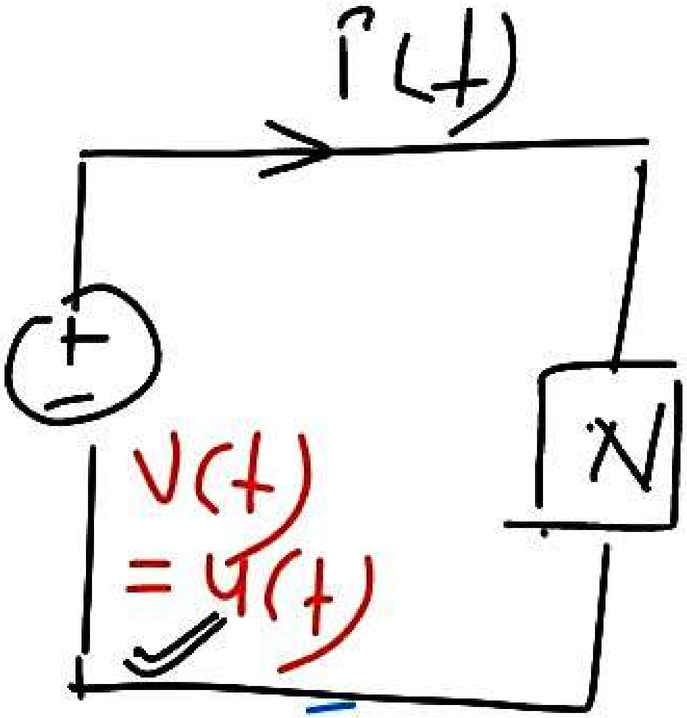
$sC \quad \mho$

Note :-

Impedance ($Z(s)$) will be in series.

Admittance in parallel.

we can add.



Network N contains only two elements & response to the unit step excitation

$i(t) = e^{-3t} u(t)$ Determine the elements & their connect.

$$\text{System Transfer function} = \frac{\text{Response LT}}{\text{Excitation LT}}$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)}$$

Admittance (mho)

(ohm)

$$\frac{I(s)}{V(s)} = \frac{1/s+3}{1/s} = \frac{s}{s+3} \Rightarrow Y(s)$$

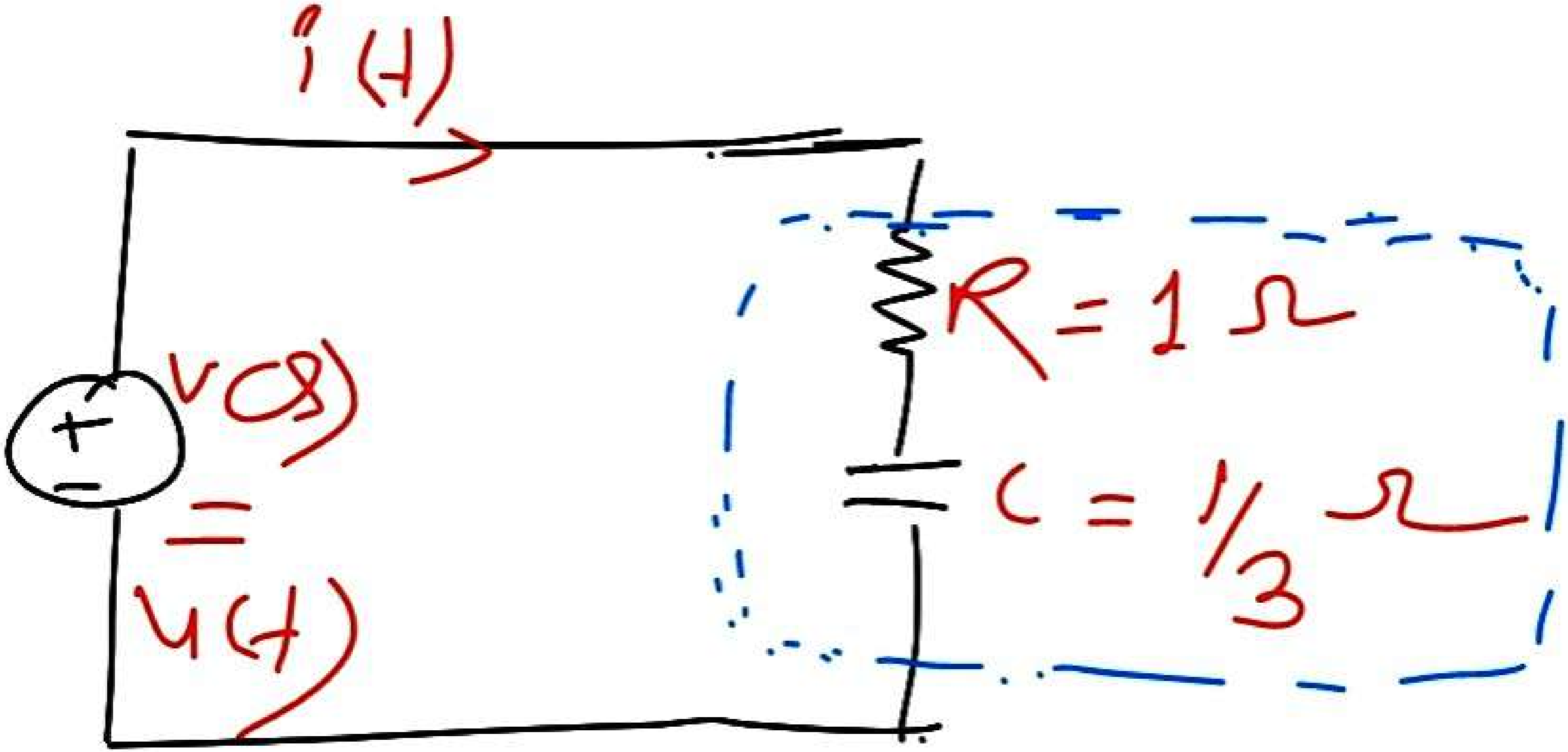
$$I(t) = e^{-3t}u(t) \xrightarrow{LT} I(s) = \frac{1}{s+3}$$

$$\underline{\underline{V(t)}} = u(t) \xrightarrow{LT} V(s) = \underline{\underline{1/s}}$$

$$Z(s) = \frac{1}{Y(s)} = \frac{s+3}{s}$$

$$= 1 + 3/s$$

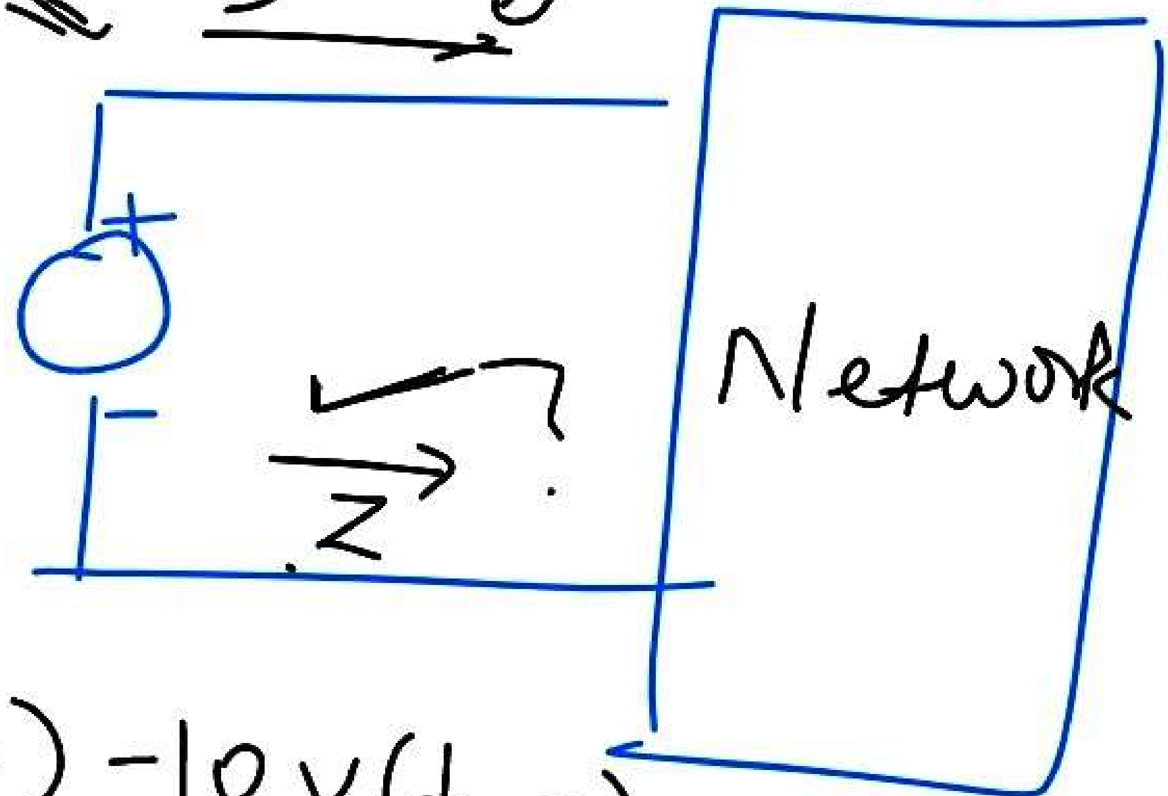
$$= R + \frac{1}{s \cdot 1/3} \Rightarrow R + \frac{1}{sC}$$



i(t)
v(t)
=
u(t)
R = 1 Ω
C = 1/3 Ω

$i(t) = e^{-t} - e^{-(t-2)}$

$v(t) =$



$10v(t) - 10v(t-2)$

find out the Z of N/w. using LT.

$v(t) = 10u(t) - 10u(t-2)$

1st step evaluate the LT of $v(t)$

$v(t) = 10U(t) - 10U(t-2)$

$V(s) = 10/s - 10\frac{1}{s}e^{-2s}$

$$= \frac{10}{s}\left(1 - e^{-2s}\right)$$

Current $i(t) = e^{-t} - e^{-(t-2)}$

$$I(s) = \frac{1}{s+1} - \frac{1}{s+1} \cdot e^{-2s} = \frac{1-e^{-2s}}{s+1}$$

$$Z(s) = \frac{V(s)}{I(s)}$$

$$= \frac{\frac{10}{s}\left(1-e^{-2s}\right)}{\frac{1-e^{-2s}}{s+1}} \Rightarrow \frac{\frac{10}{s}\left(1-e^{-2s}\right)}{\frac{1}{s+1}\left(1-e^{-2s}\right)}$$

$$= \frac{10/s}{1/(s+1)} = 10 \cdot \frac{s+1}{s}$$

$$= 10\left(1 + 1/s\right)$$

$$\underline{\underline{10 + \frac{10}{s}}} \Rightarrow \underline{\underline{10}} + \frac{1}{\frac{1}{10}s}$$

We know that Laplace transform of R i.e. value is unchanged.

& LT of capacitor $\frac{1}{sC}$

$$Z(s) = R + \frac{1}{sC} \Rightarrow \begin{matrix} R = 10\,\Omega \\ C = 1/10 \end{matrix}$$

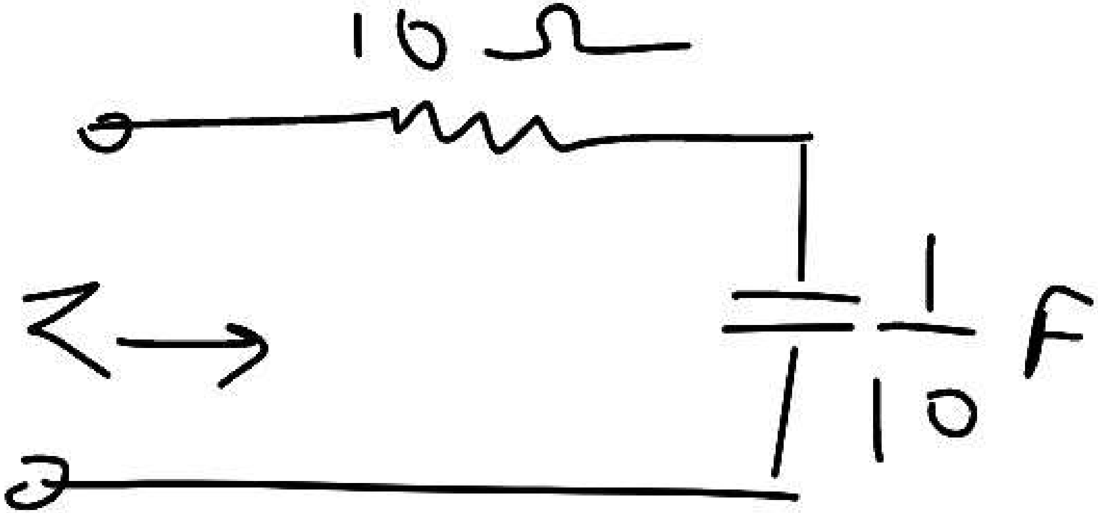
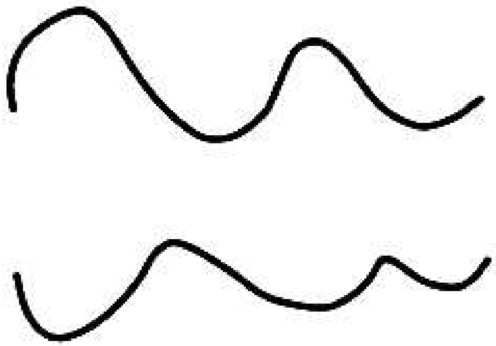


fig :- Impedance of the 1/ω

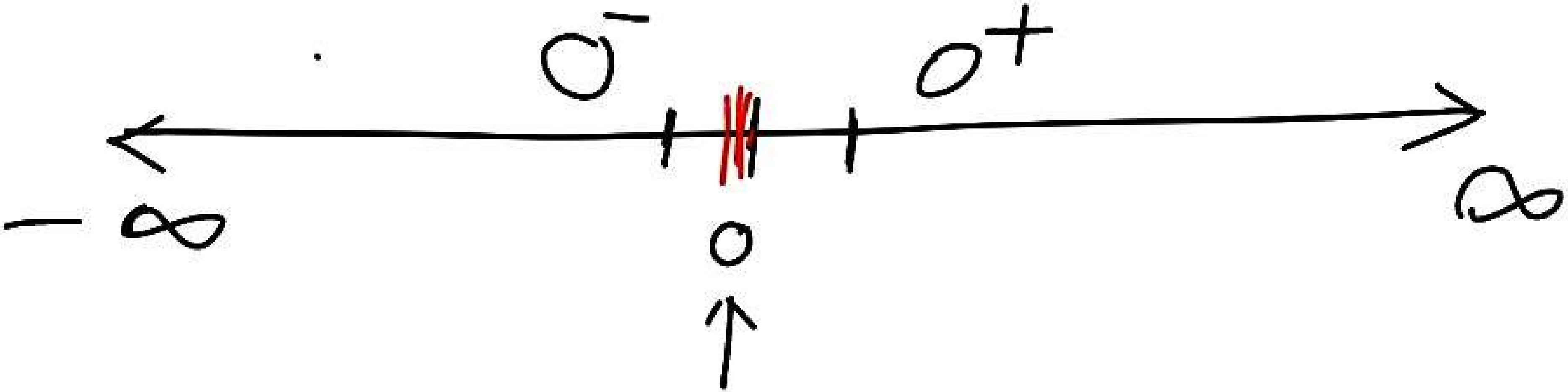
$\boxed{L \,\&\, C} \rightarrow$ Memory element.

$\underline{\underline{R}} \rightarrow$ Memoryless element.



$V = Ri \qquad i = V/R$

Behaviour of L & C elements at $t = 0$ and at $t \to \infty$



$$t = 0^+ \Rightarrow s = \infty \Rightarrow Z_L = \infty$$

$$L \rightarrow O.C$$

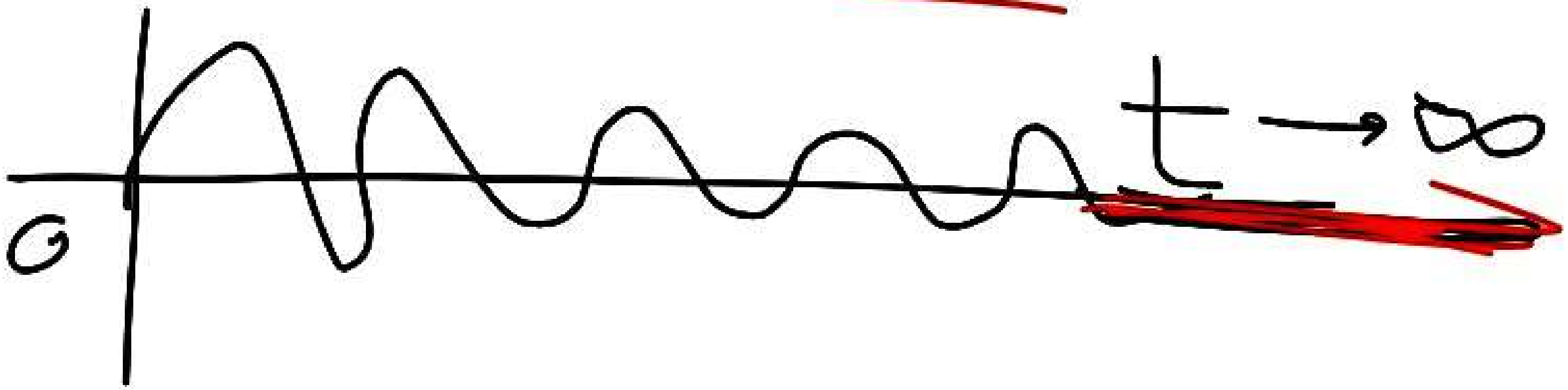
$$Z_C = 0 \Rightarrow C = S.C$$

$$Z_L = sL \qquad Z_C = \frac{1}{sC}$$

$t \to \infty \Rightarrow s = 0 \Rightarrow$

$Z_L \to 0 \Rightarrow L \to SC.$

$Z_C \to \infty \Rightarrow C \to OC$



In long time after the switching action $(t \to \infty)$ is steady state

In steady state → Inductor (L) ⇒ SC

Capacitor (C) ⇒ OC.

DC source is connected to N/w for long time. Then n/w. is said to be in Steady state when the energy stored in N/w is max. & constant. So, the energy stored in memory element will be max & constant.

$$I_L = \frac{1}{2} L i_L^2 = \text{Max \& constant}$$

$$i_L \Rightarrow \text{Max \& constant}$$

Inductor will behave as current source in steady state

$$V_L = L\frac{di_L}{dt} \Rightarrow V_L \Rightarrow 0 \qquad L = SC.$$

$$\frac{1}{2}CV_C^2 \Rightarrow \text{Max \& constant}$$

$$V_C \Rightarrow \text{Max \& constant}$$

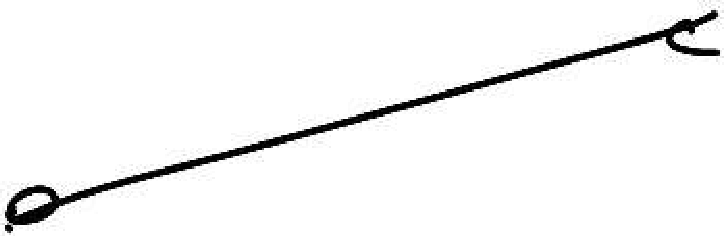
Capacitor will act as Constant Voltage source in steady state.

Note :- Discussion is valid only for DC excitations/source

→

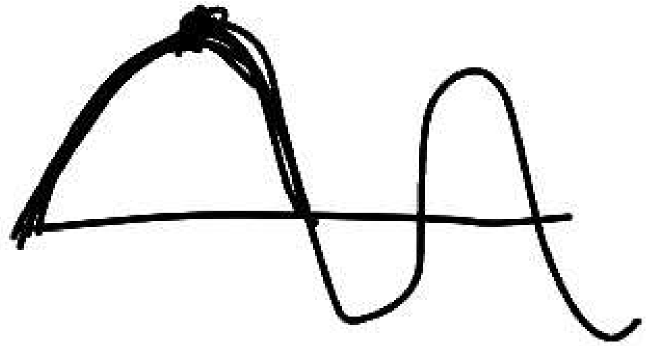
AC. exitations

L & C are charging



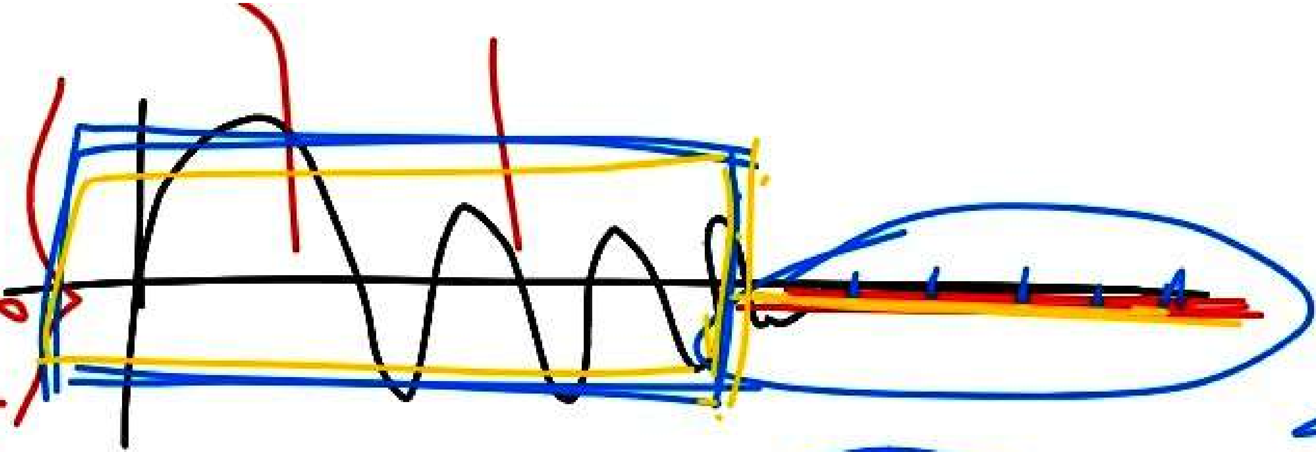
store

energing



$t : 0$

$s = \infty$



$t \to \infty$

$-\infty$ $\quad 0^-$ $\quad 0$ $\quad 0^+$ $\quad \infty$

$Z_L = sL \ \Omega$

$Z_C = \dfrac{1}{sC} \ \Omega$

$t \to \infty \Rightarrow Z_L = 0$

Inductor $\to s \to 0$

L = SC

C → OC

AC excitations

1. for AC, during positive slop of time period both L & C are charging state.

(2) During the negative slop of time period they (L, C) are discharging

(Delivering the energy)

Blind Approach $(s=j\omega)$

In steady state

$$Z_L = j\omega L \quad \& \quad Z_C = \frac{1}{j\omega C}\ \Omega$$

$$Z_L = sL$$

Blind Approach $(s=j\omega)$

In steady state

$$Z_L = j\omega L \quad \& \quad Z_C = \frac{1}{j\omega C}\ \Omega$$

$$Z_L = sL$$

for $\underline{\underline{A}}$ c $\quad \omega \neq 0$

$$Z_L \neq 0$$

$$\underline{\underline{Z_L}} = \overline{\overline{j\omega L \,\Omega}}$$

$$\underline{\underline{Z_C}} = \frac{1}{j\omega C}\,\Omega \quad (Z_C \neq \infty)$$

The impedances are finite and hence L & C elements are always present for AC excitation

$0 \neq 0$ ∞

$Z_L \neq 0$ $Z_C \neq \infty$

$Z_L = j\omega L$ $Z_C = \frac{1}{j\omega C}$

For DC

only in steady state $\omega = 0$

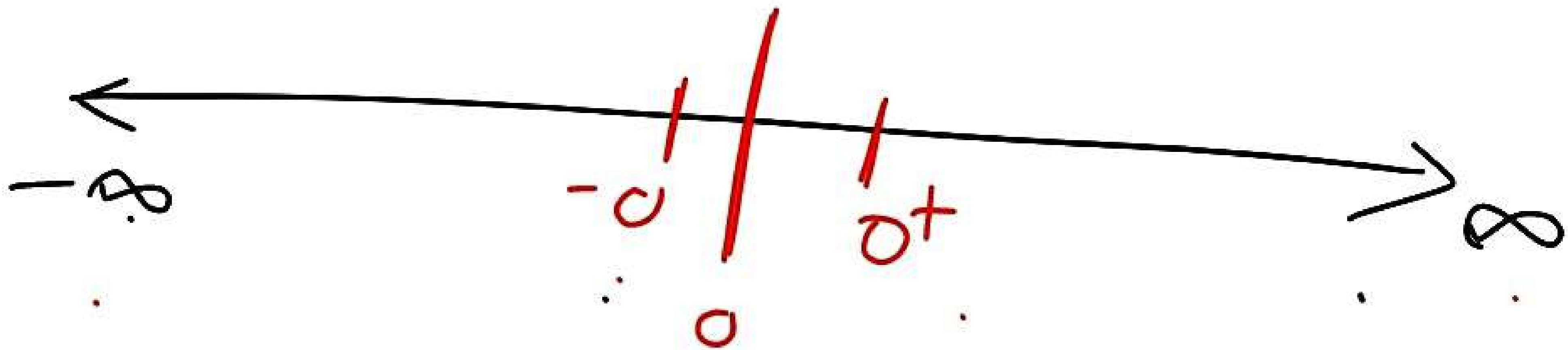
$Z_L = j\omega L = 0$ $\quad$ $L = SC$

$Z_C = \frac{1}{j\omega C} = \infty$ $\quad$ $C \to OC$

No inductor, capacitor operations in steady state. for DC excitati i.e. L & C elements will dies for d.c excitations.

(L & C elements will experiance a deep sleep)

Inductor current at $t = 0^-$ and at $t = 0^+$ instants



$$L \Rightarrow i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$$

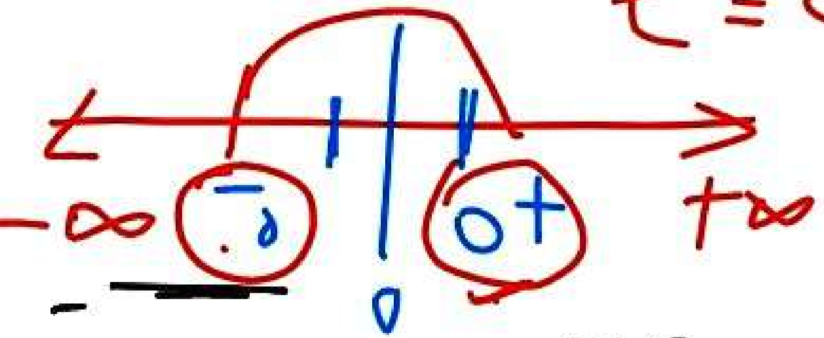
$$= \frac{1}{L}\int_{-\infty}^{0^-} v_L(t)\cdot dt + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt$$

$$= i_L(0^-) + \frac{1}{L}\int_{-0}^{t} v_L(t)\,dt$$

At $t = 0^+ \Rightarrow i_L(0^+)$

$$= i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} V_L$$

$t = 0^-$



$$L \Rightarrow i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$$

$$\Rightarrow \frac{1}{L}\int_{-\infty}^{0^-} v_L(t)\,dt + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt$$

$$\Rightarrow i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt$$

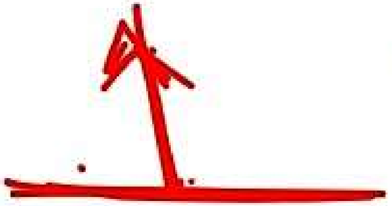
At $t = 0^+$

~~At~~ $t = 0^+$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L \rightarrow 0$$

$$i_L(0^+) = i_L(0^-) \text{ Amp}$$

$$E(0^+) = E(0^-) \rightarrow \text{Joule}$$



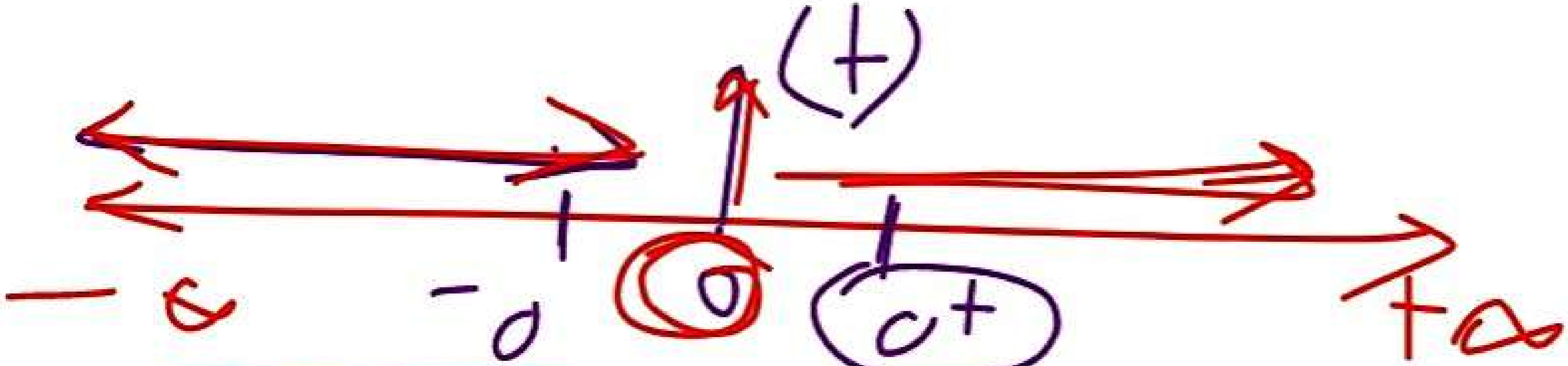
$\delta(t)$

$\boxed{v(t) = \delta(t)}$ $\int \delta(t) = 1$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{-0}^{+0} \underbrace{\delta(t)}_{=1}\, dt$$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \text{ Amp.}$$

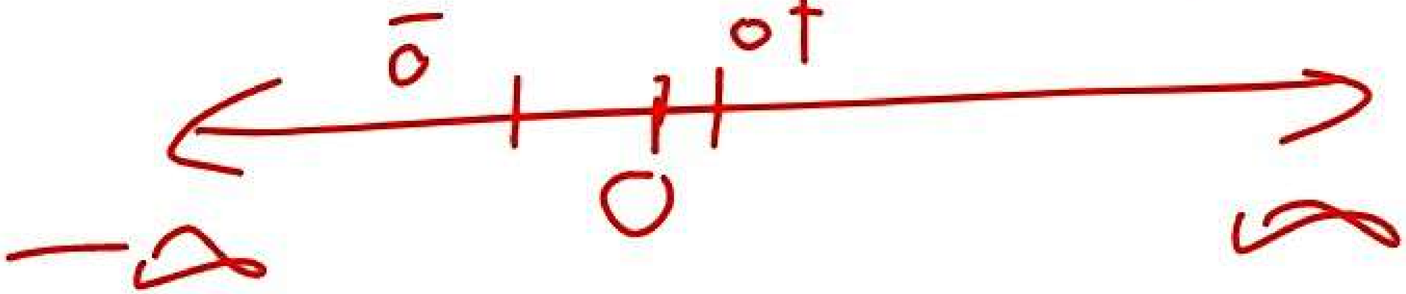
$$E_L(0^+) > E_L(0^-) \text{ J}$$



$$V_L(t) = \delta(t) = 0 \quad \text{for} \quad -\infty \le t \le 0^-$$

$$i_L(0^+) = \frac{1}{L}$$

$t = (0) \Rightarrow \underline{L = 0c}$ $\quad \underline{C = s \cdot c}$



$t \to \infty \quad L = s \cdot c \quad C = 0 \cdot c$

Blint Approach

In steady state $Z_L = j\underline{\underline{\omega}} L \ \Omega$

& $Z_C = \dfrac{1}{j\underline{\underline{\omega}} C} \ \Omega$

Blint Approach

In steady state $Z_L = j\underline{\underline{\omega}} L \ \Omega$

& $Z_C = \dfrac{1}{j\underline{\underline{\omega}} C} \Omega$

for AC $\quad \omega \neq 0$

$Z_L = j\omega L \neq 0$

$Z_C = \dfrac{1}{j\omega C} \neq \infty$

L & C elements always present & they are busy.

DC $\Rightarrow$

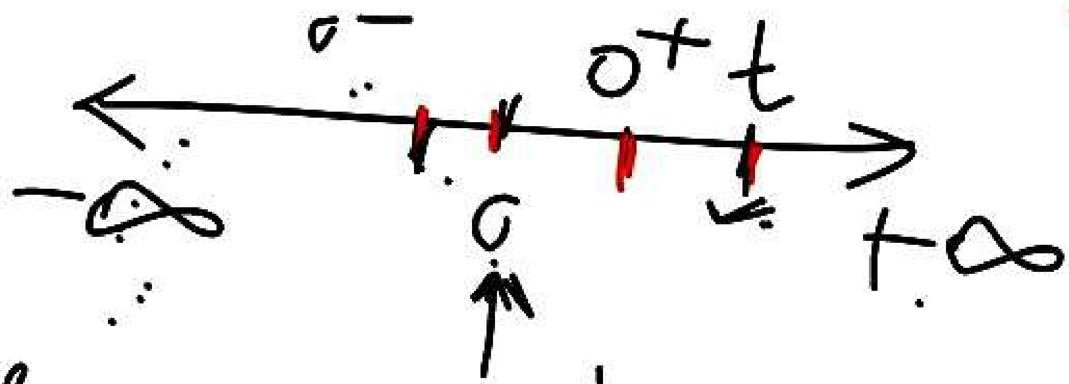
$\therefore \omega = 0$

$Z_L = 0 \rightarrow L \Rightarrow SC$

$Z_C = \infty$

$C \rightarrow O.C$

L & C elements will dies for DC



$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$$

$$= \frac{1}{L}\int_{-\infty}^{0^-} v_L(t)\,dt + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt$$

The inductor current & capacitor voltage at $t = 0^-$ & at $t = 0^+$

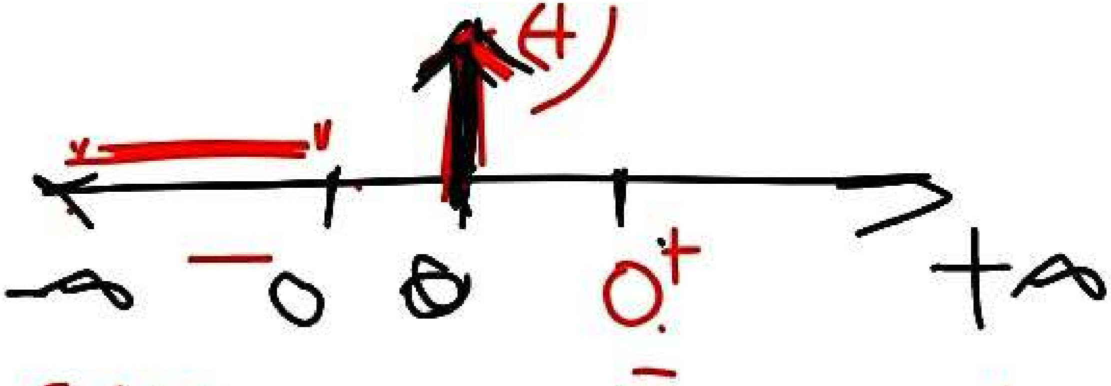
$i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(\cancel{t})\,dt$ Ohs

At $t = 0^+ \Rightarrow$

$i_L(0^+) = I_L(0^-)$ Amp.

$E_L(0^+) = E_L(0^-)$ J

Inductor current (i_L) cannot change within 0 time)

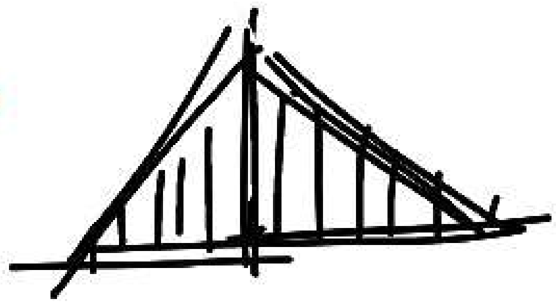


$$v_L(t) = \delta(t)$$

$$-\infty \le t < 0^+$$

$$\delta(t) = 1$$

$$i_L(0^+) = \frac{1}{L}\int_{-\infty}^{0^+} v_L(t)\,dt$$



$$i_L(0^+) = \frac{1}{L}$$

$$E_L(0^-) = 0\,J$$

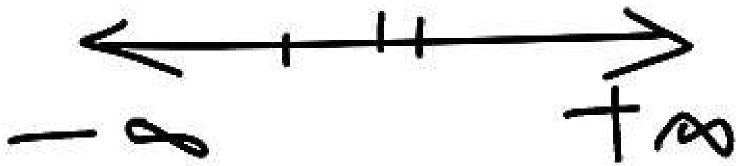
$$E_L(0^+) = \frac{1}{2} L i^2 (0^+)$$

$$= \frac{1}{2} \times L \times \frac{1}{L^2}$$

$$\boxed{E_L(0^+) > E_L(0^-)}$$

$$E_L(0^+) = \frac{1}{2L}$$

C: $V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c \, dt$

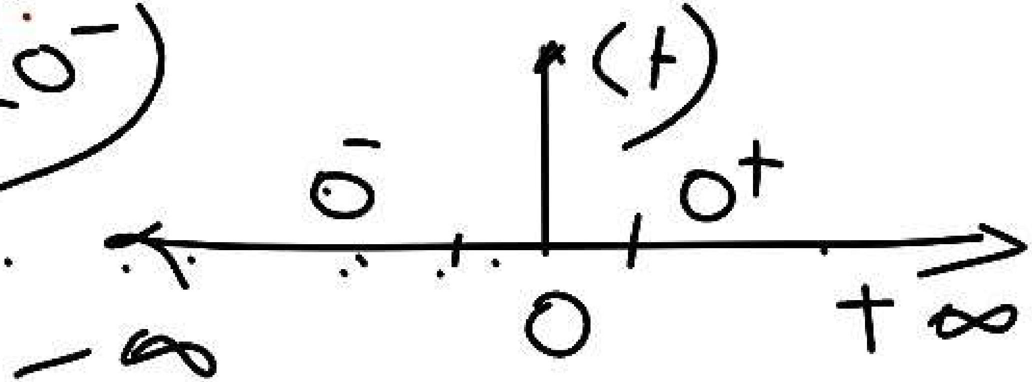


At 0^-

$V_c(0^+) = V_c(0^-)$

$E_c(0^+) = E_c(0^-)$

for impulse



$i_c = \delta(t)$

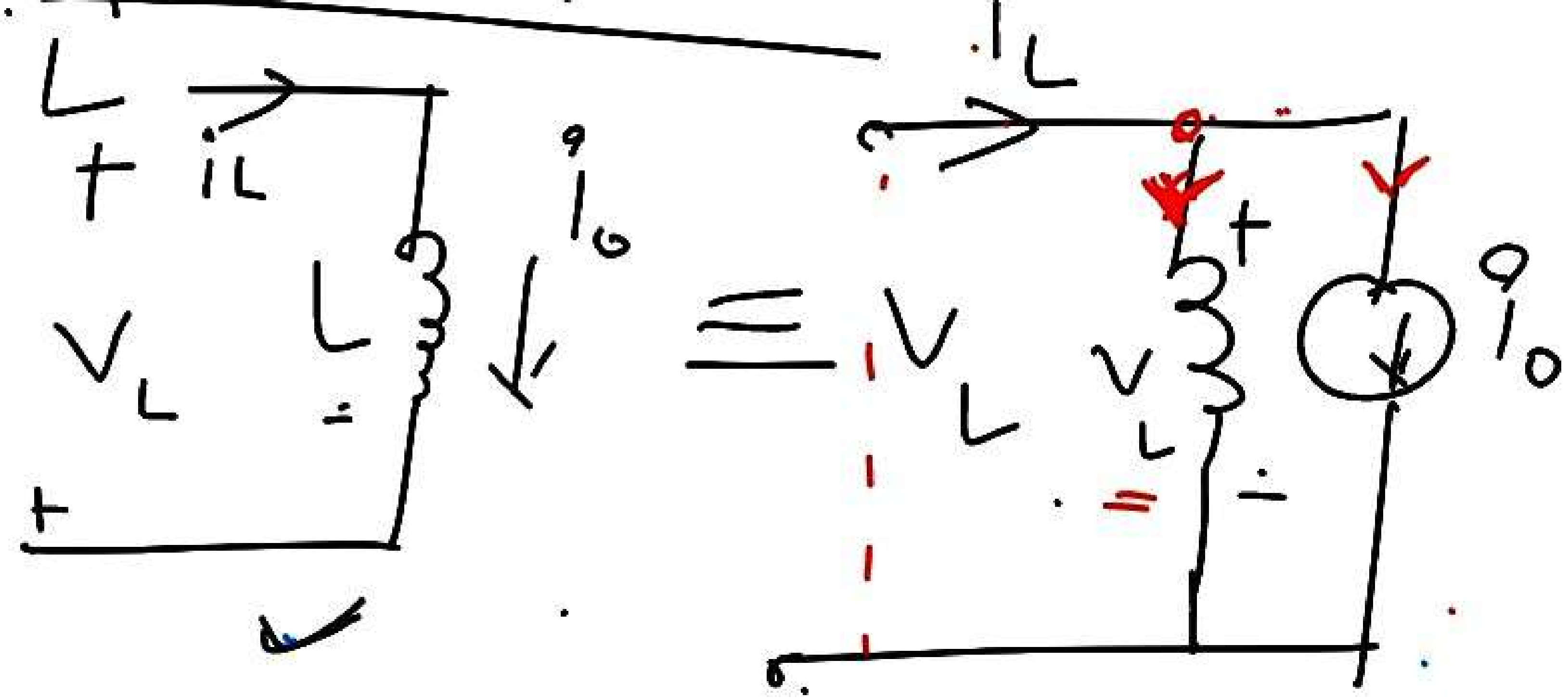
$\boxed{V_c(0^+) = \frac{1}{C}}$ $\boxed{i_L = \frac{1}{L}}$

$E_c(0^+) = \frac{1}{2} C V^2 (0^+)$ $\frac{1}{2L}$

$= \frac{1}{2} \times C \times \frac{1}{C^2}$ $\frac{1}{2C}$

$E_c(0^+) = \frac{1}{2C}$

Equivalent ckt



$$i_L = \frac{1}{L}\int_{-\infty}^{t} v_L (dt)$$

$$\boxed{i_L = i_L(0) + \frac{1}{L}\int v_L\, dt}$$

$$i_L(0) = i_0$$

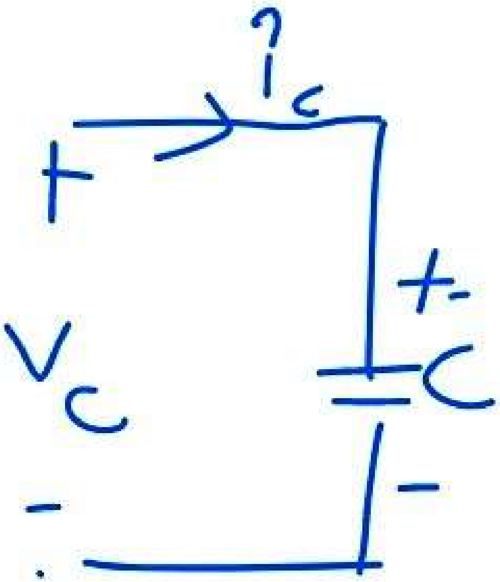
Proof

By KCL :-

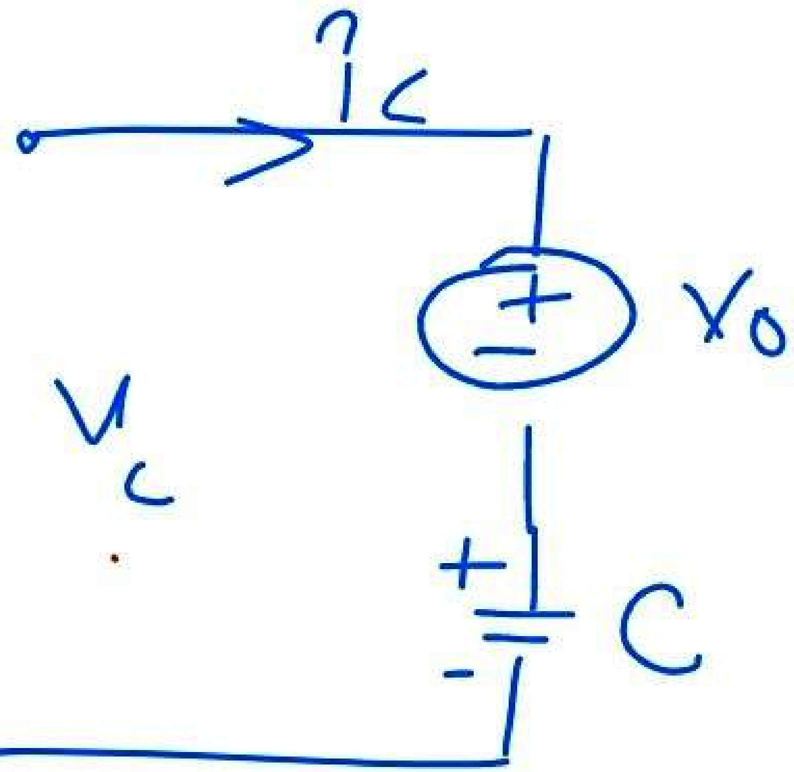
$$-i_L + I_L + i_0$$

$$= -i_L + \frac{1}{L}\int v_L (dt) + i_0 = 0$$

$$\Rightarrow i_L = \frac{1}{L}\int v_L (dt) + i_0$$



$V_0 \equiv V_c$



$$v_c = \frac{1}{C}\int_{-\infty}^{t} i_c\,dt \Rightarrow v_c(0) + \frac{1}{C}\int_{0}^{t} i_c\,(dt)$$

$$V_c = V_0 + \frac{1}{C}\int i_c\,dt$$

Proof

By KVL :-

$$V_c - V_0 - \frac{1}{C}\int_0^t i_c\,dt = 0$$

$$V_c = V_0 + \frac{1}{C}\int_0^t i_c\,dt$$

Classification of Transient

1) DC Transient ✓
2) AC Transient

(1) DC Transient

(1) Source free ckt ?–

(Without Independent sources)

- RL
  - $L \rightarrow i_0$
- RC
  - $C \rightarrow V_0$
- RLC
  - $L \rightarrow i_0$ or Both
  - $C \rightarrow V_0$

In all source free ckt, stored energies in the memory elements are maximum at (t=0) (initially). These stored energies will be delivered to memoryless element in exponentially decayed manner. Hence the energy $(t \to \infty)$ is zero.

Classification Transients

- DC
- AC

Source free circuit : (Without Independent source)

- RL → L → i_0
- RC → C → V_0.
- RLC → Both.

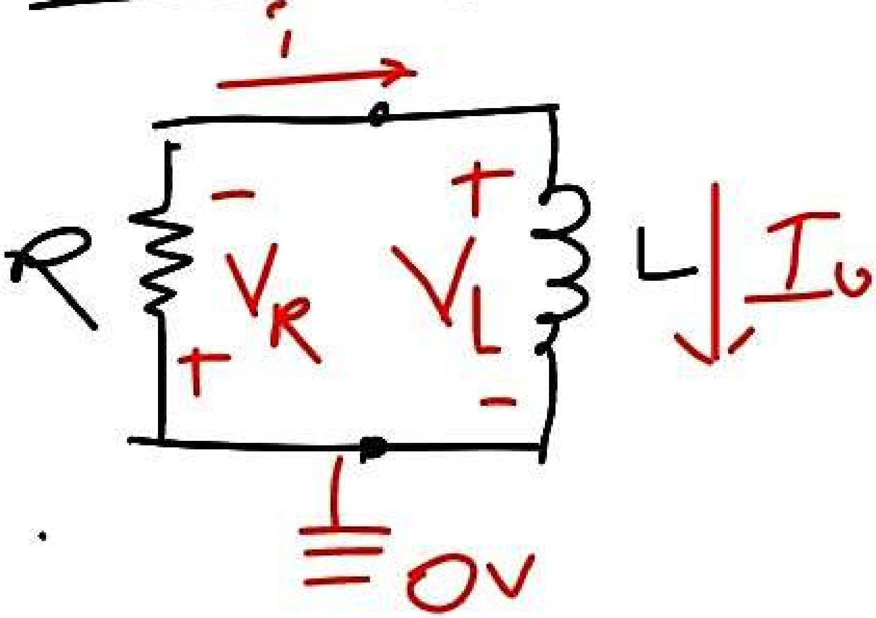
Property:-

$t = 0 \Rightarrow$ Energy max.

$t \to \infty \Rightarrow$ steady state $\to$ Energy $= 0$

LC $\xrightarrow{\text{Transfer}}$ R

Source free RL circuit:-



for $t \geq 0$

By KVL

$$-V_R - V_L = 0$$

$$-Ri - L\frac{di}{dt} = 0$$

$$Ri + L\frac{di}{dt} = 0$$

Divide by L

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

let $\boxed{D = \frac{d}{dt}} \Rightarrow \left(D + \frac{R}{L}\right)i = 0$

$D + \frac{R}{L} \Rightarrow$ Characteristic eqn.

$D = -\frac{R}{L}$

Here $s \Leftarrow D = \frac{d}{dt}$

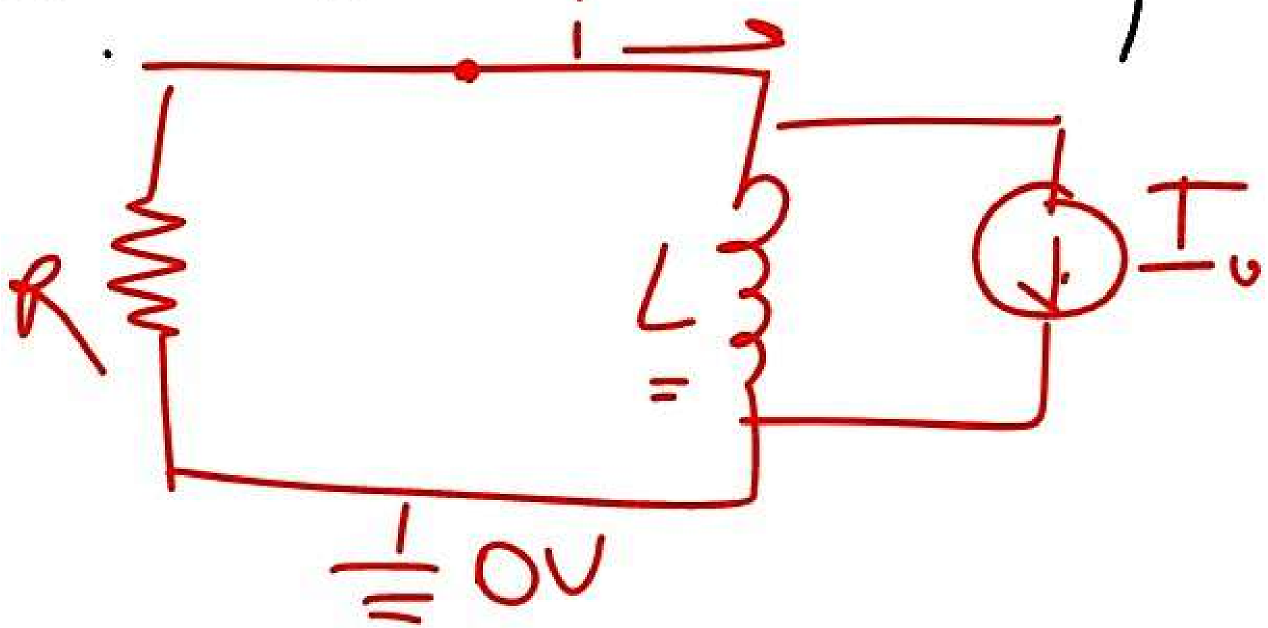
It is like $s = \alpha$

$\alpha = -\frac{R}{L}$

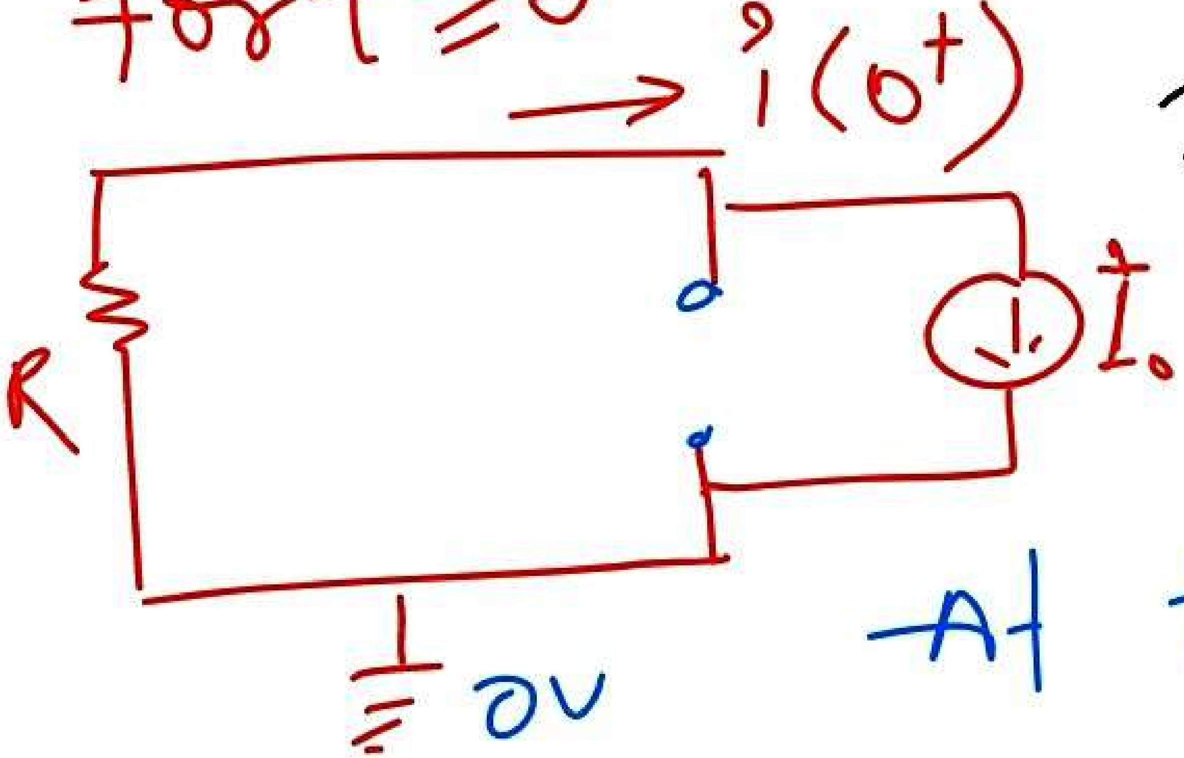
$\underbrace{i(t) = K e^{\alpha t}}$ for $0 \leq t \leq \infty$

$$Ke^{-\frac{R}{L}t} \text{ for } 0 \le t \le \infty$$

$$i(t) = \underline{K} e^{-R/L\, t} \text{ Amp.}$$



for $t \geq 0$



By KCL

$i(0^+) = I_0$

At $t = 0^+$ A Resistive circuit.

$t = 0^+$

$Z_L = sL$

$Z_C = \frac{1}{sC}$

$s \rightleftharpoons \infty \quad Z_L = \infty$

$L \longrightarrow$ o.c.

$Z_C = 0 =$ sc

At $t = 0^+$

$$i(0^+) = K e^{-0}$$

$$i(0^+) = K = I_0$$

$$0 \le t \le \infty$$

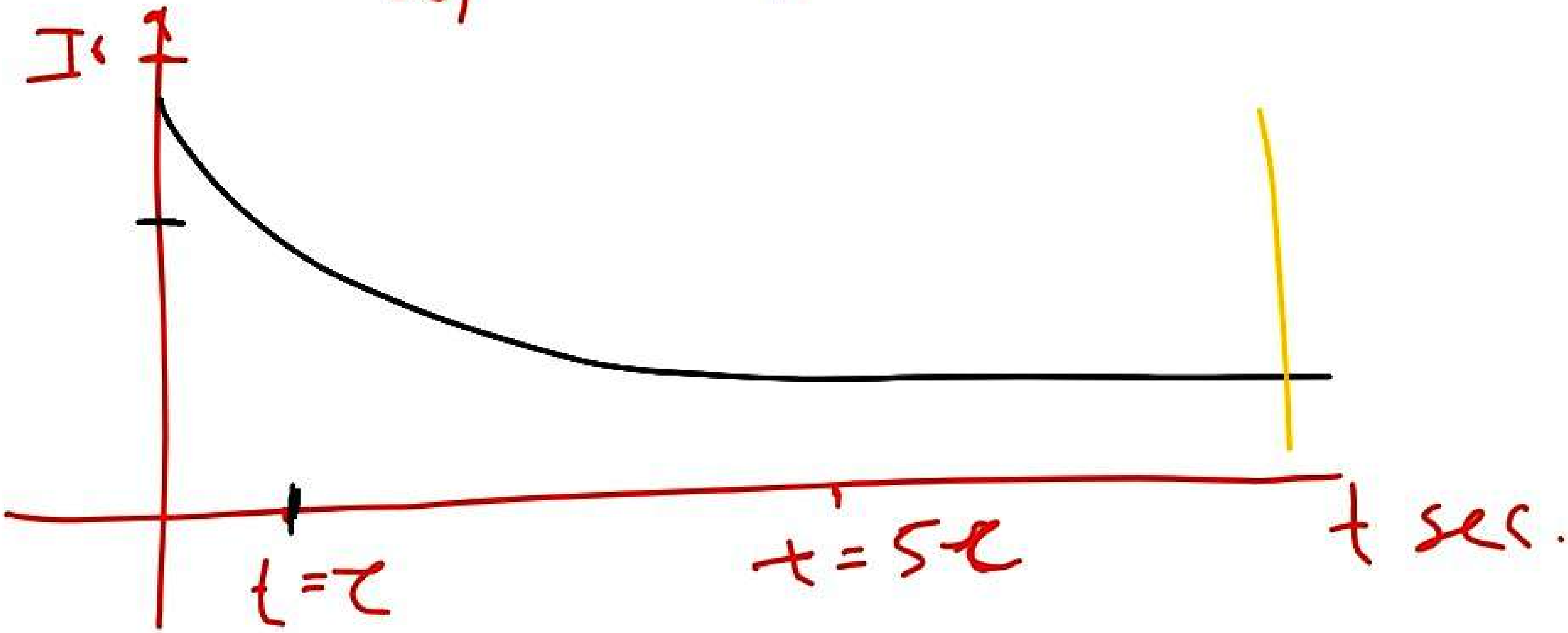
$$i(t) = I_0 e^{-R/L\, t} \text{ Amp.}$$

$$i(t) = I_0 e^{-t/\tau} \text{ Amp.}$$

$$\tau = \frac{L}{R} \text{ sec.} = \text{Time constant of the circuit.}$$

$$\boxed{i(t) = I_0 e^{-\frac{R}{L}t}}$$

$$V_L(t) = L\frac{di}{dt}(t) \Rightarrow \text{Ohms Law.}$$



$$i(t) = I_0 e^{-t/\tau}$$

At $t = \tau$

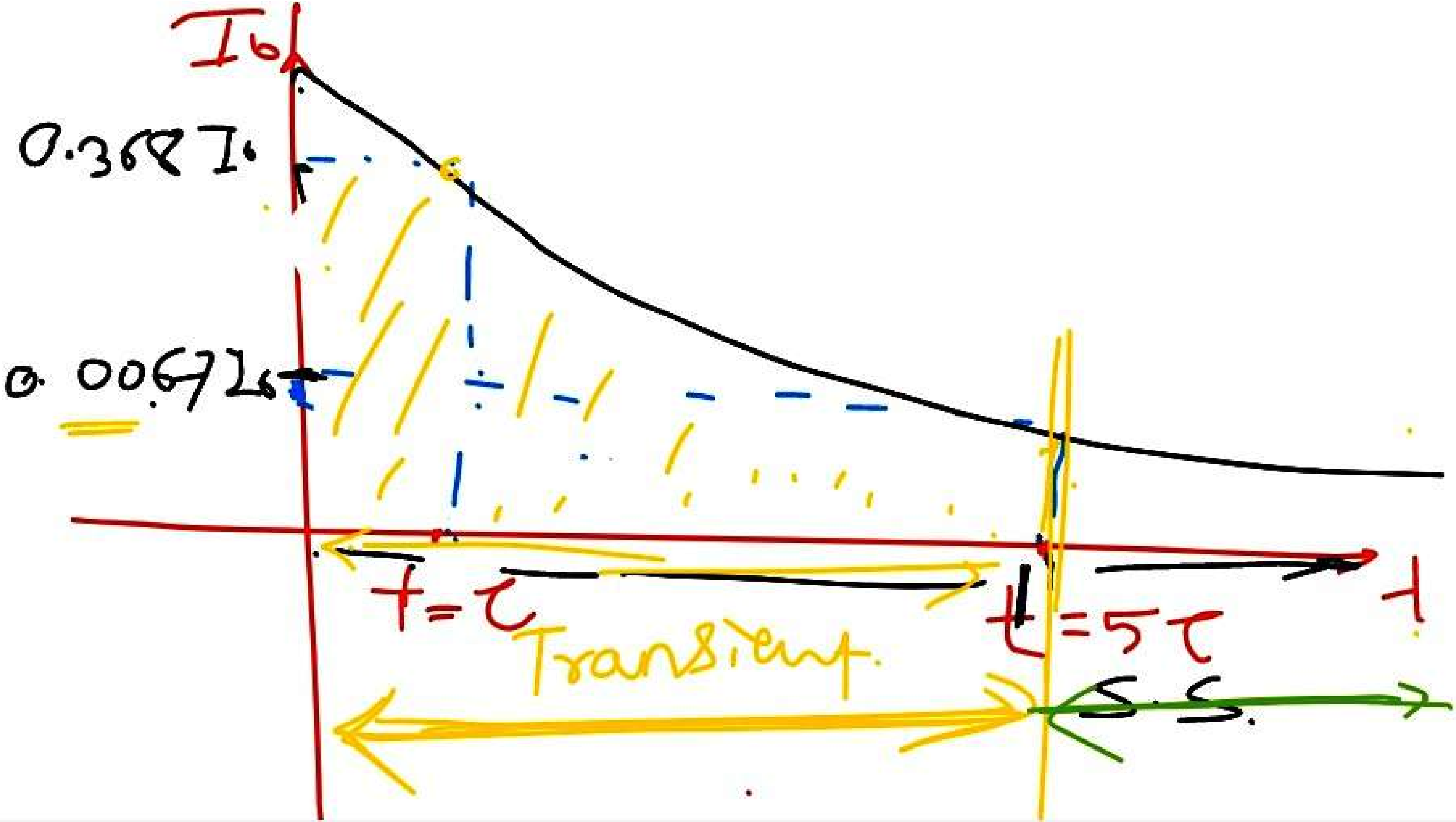
$$i(t) = I_0 e^{-1}$$

$$i(t) = \frac{I_0}{e} = 0.368 I_0$$

$$i(t) = I_0 e^{-t/\tau}$$

$$t = 5\tau$$

$$i(t) = \frac{I_0}{e^5} = 0.0067\, I_0 \approx 0$$



Io
0.368 Io
0.00672Io
t = τ
Transient
t = 5τ
S.S.
t

$$E_L(t) = \frac{1}{2} L i^2(t) \text{ Joule}$$

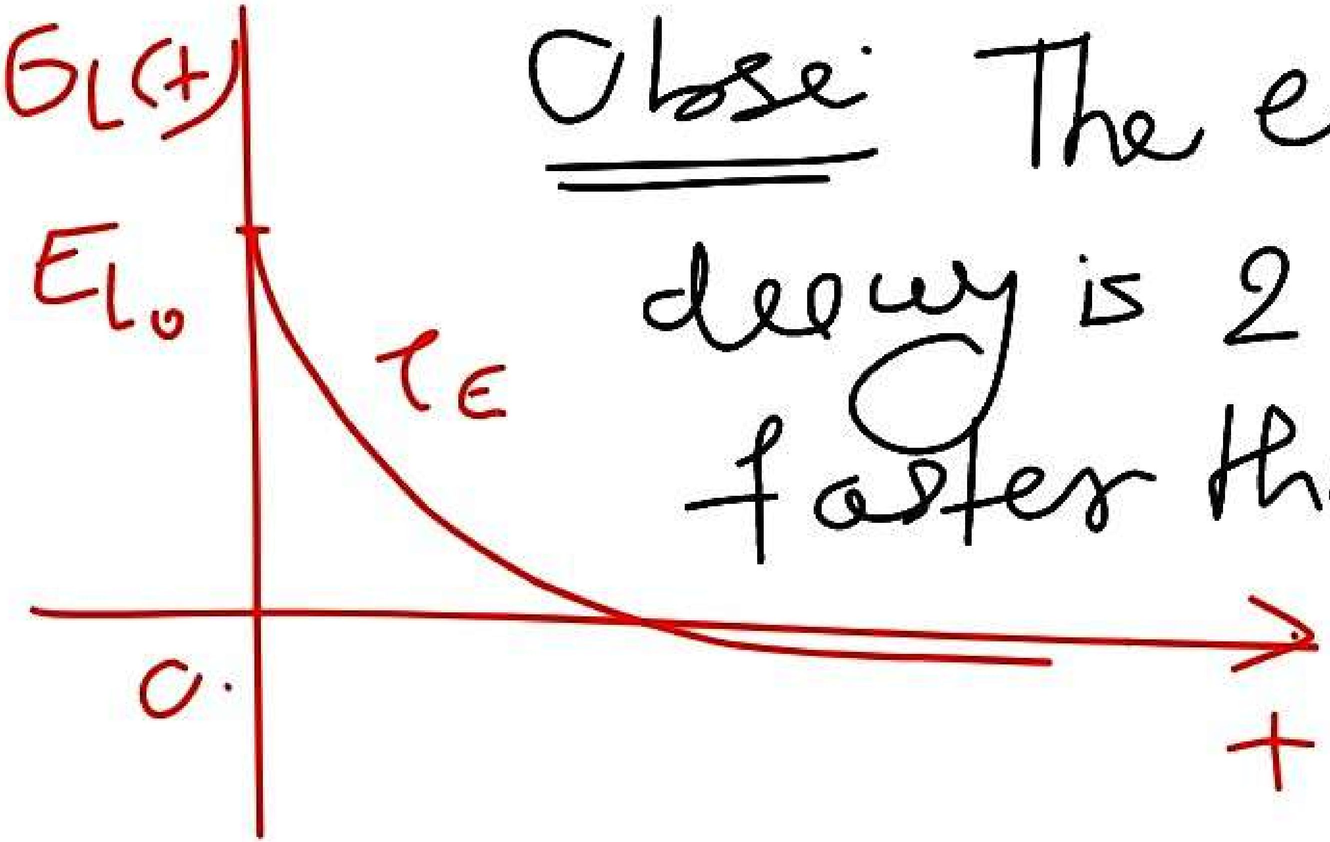
$$\tau_E = L/2R$$

$$= \frac{1}{2} L \left(I_0 e^{-t/\tau} \right)^2$$

$$\boxed{\tau_E = \frac{\tau_1}{2}}$$

$$= \frac{1}{2} L I_0^2 \cdot e^{-2t/\tau_1}$$

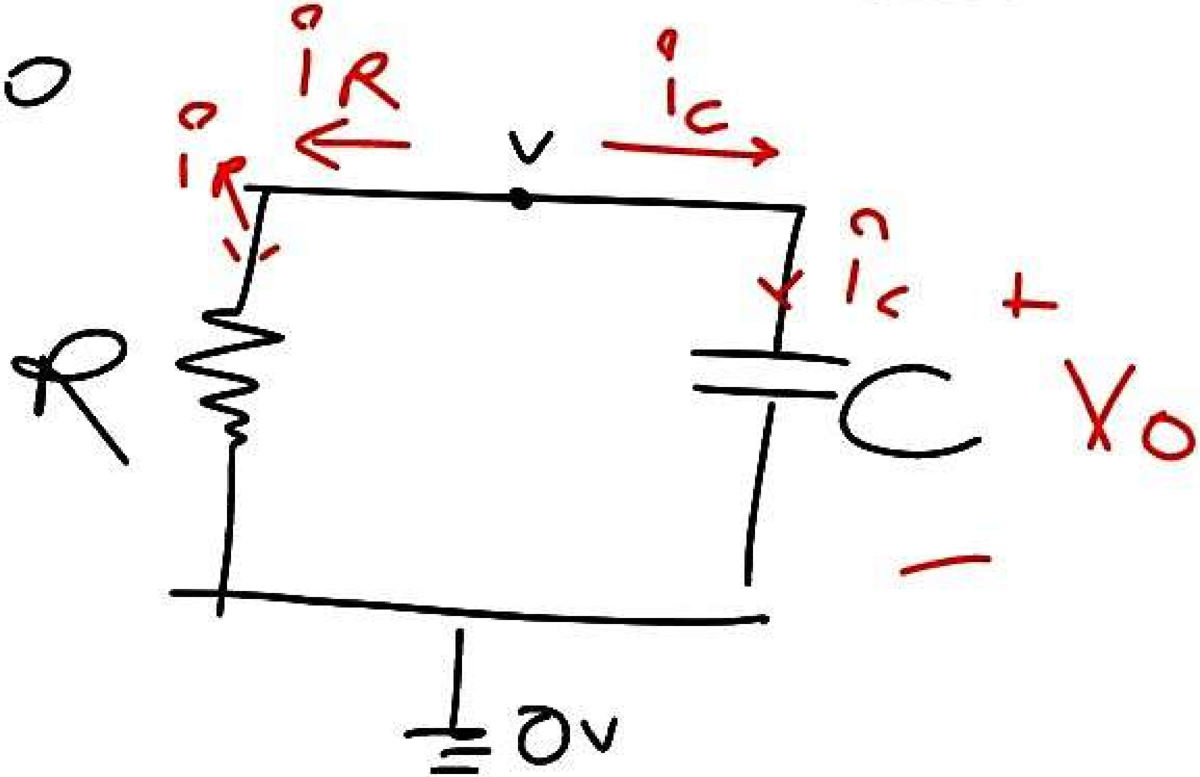
$$= E_{L0} \cdot e^{-t/\tau_E} \text{ Joule}$$



<u>Obse</u>: The energy decay is 2 times faster than current decay.

Source free RC circuit :-

for $t \geq 0$



By KCL

$$\left(\frac{d}{dt}+\frac{1}{RC}\right)v=0$$

$$\frac{d}{dt}=D$$

$$i_R+i_C=0$$

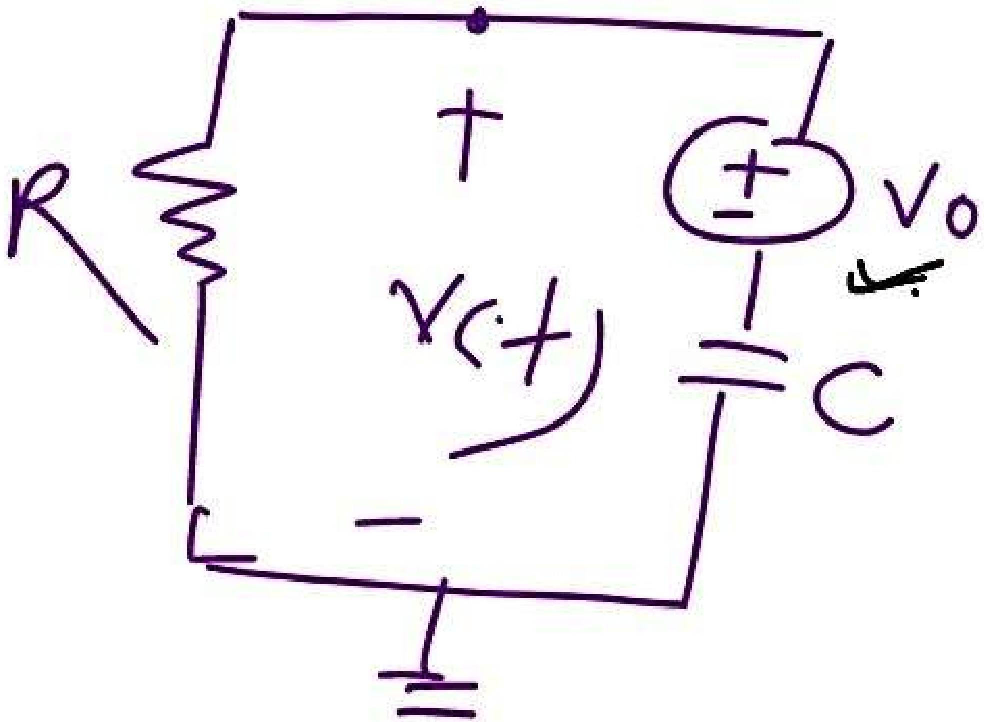
$$\frac{v}{R}+C\frac{dv}{dt}=0 \Rightarrow \frac{dv}{dt}+\frac{v}{RC}=0$$

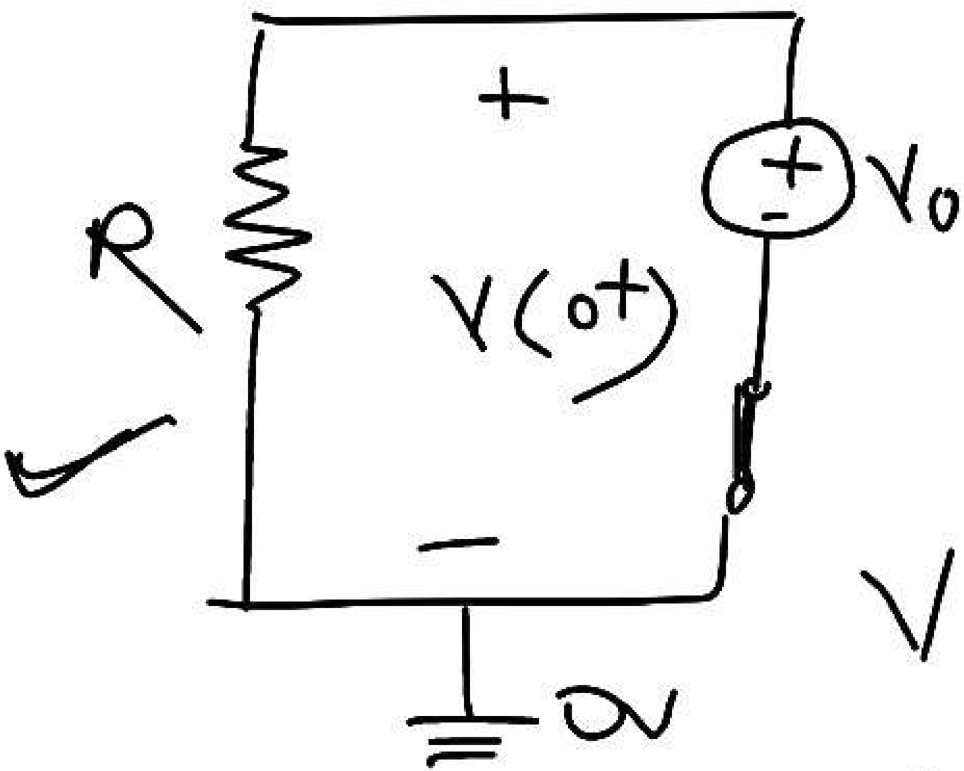
$$\left(D+\frac{1}{RC}\right)v=0$$

$$S=\alpha$$

$$v(t) = K e^{-t/RC} \text{ volt.} \quad 0 \le t \le \infty$$

for $t \geq 0$





At $t = 0^+$;

A resistive ckt.

$$v(0^+) = V_0 \rightarrow K$$

At $t = 0^+$ $\quad v(0^+) = Ke^0$

$$K = V_0 = V(0^+)$$

$$V(t) = V_0 e^{-t/RC} \qquad \tau = RC \text{ sec.}$$

$$V(t) = V_0 e^{-t/\tau}. \qquad 0 \leq t \leq \infty$$

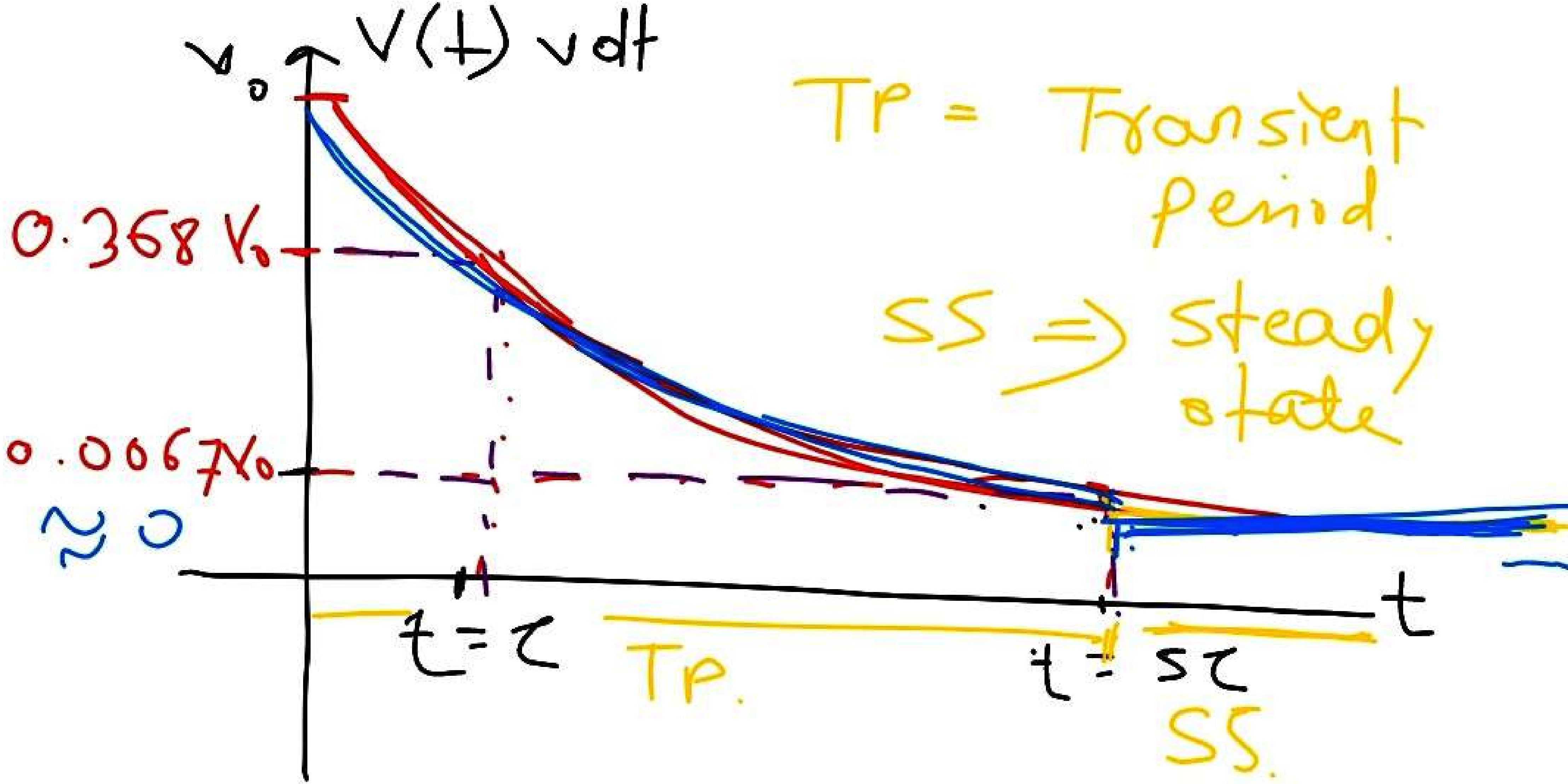
$$v(t) = v_0 \, e^{-t/\tau}$$

① for $t = \tau$

$$v(t) = \frac{v_0}{e} = 0.368\, v_0$$

② for $t = 5\tau$

$$v(t) = v_0/e^5 = 0.0067\, v_0$$



V(t) vdt
V_0
0.368 V_0
0.0067 V_0
≈ 0
t
t = τ
Tp.
t = 5τ
SS.
Tp = Transient period.
SS ⇒ steady state

$$E_c(t) = \frac{1}{2} C V^2(t) \text{ Joule}$$

$$= \frac{1}{2} C \left(V_0 e^{-t/\tau} \right)^2 \qquad (a^m)^n = a^{mn}$$

$$= \boxed{\frac{1}{2} C V_0^2} \cdot e^{-2t/\tau}$$

$$= E_{C0} \, e^{-2t/\tau_v}$$

$$E_c(t) = E_{co}\, e^{-2t/\tau_v}$$

$$\tau_E = \frac{\tau_v}{2}$$

$$E_c(t) = E_{co}\, e^{-t/\tau_E} \text{ Joule}$$

$$\tau_E = \frac{RC}{2} \text{ sec.}$$

Here the energy decay is 2 times faster than voltage decay.

$$\tau_i = \tau_v = \frac{L}{R} = RC$$

$$\frac{L}{CR^2} = 1 \Rightarrow \text{unitless.}$$

$$\frac{CR^2}{L} = 1 \Rightarrow \text{unitless.}$$

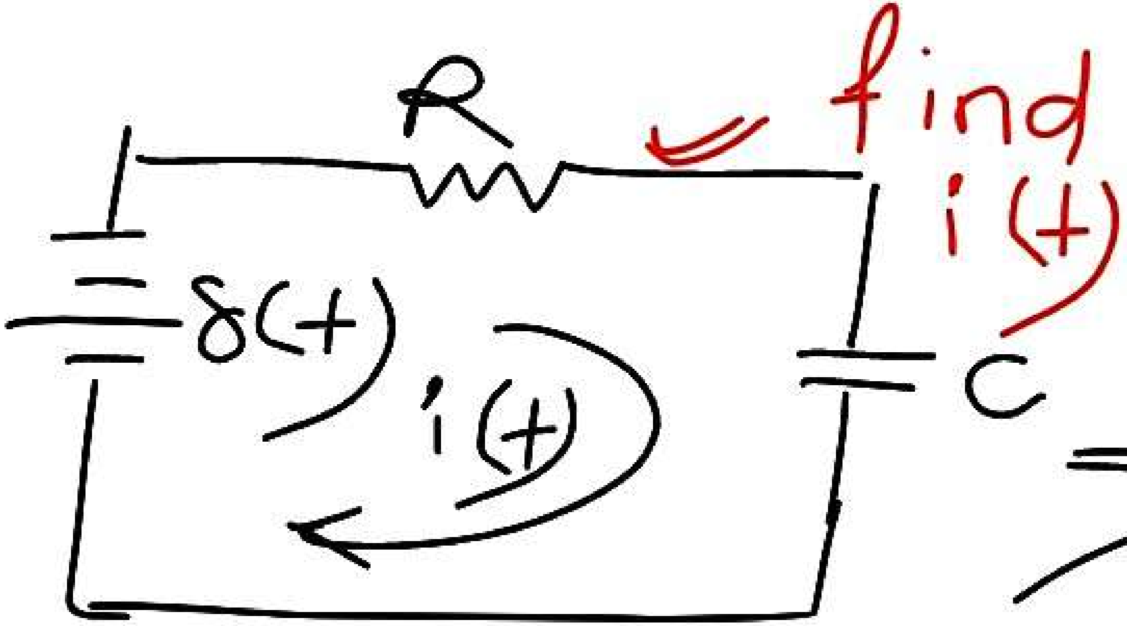
$$CR^2 = L \Rightarrow \text{Henry.}$$

$$C = \frac{L}{R^2} = \text{farad}.$$

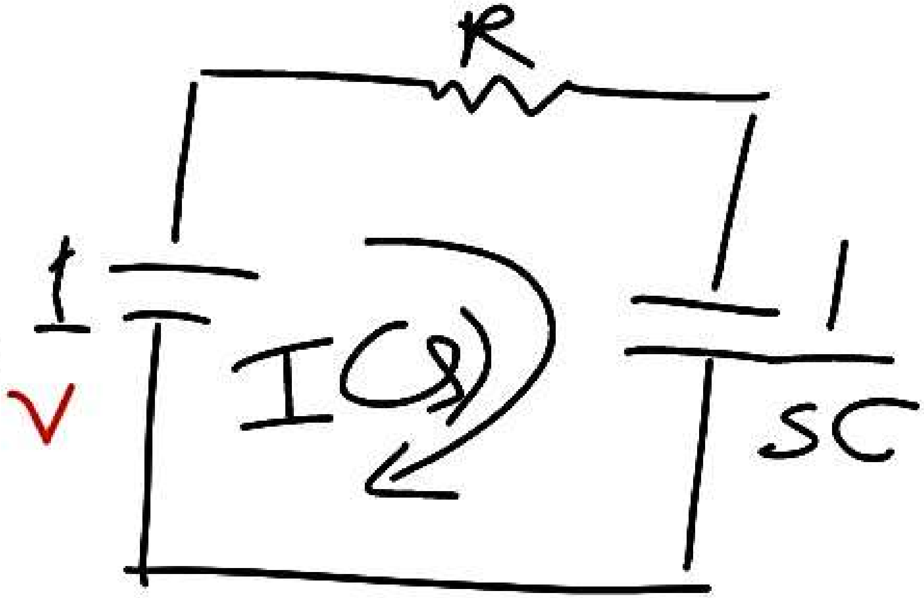
$$\frac{L}{RC} = R = \text{Ohm}.$$

$$\frac{L}{C} = R^2 = \text{Ohm}^2$$

$$\sqrt{\frac{L}{C}} = R = \text{ohm}$$



$\Rightarrow$



$$I(s) = \frac{1}{R + \frac{1}{sC}} = \frac{sC}{sRC + 1}$$

$$I(s) = \frac{V}{R}$$

Divide Num. & Den. by RC.

$$I(s) = \frac{SC/RC}{S + \frac{1}{RC}} \Rightarrow \frac{S/R}{S + \frac{1}{RC}}$$

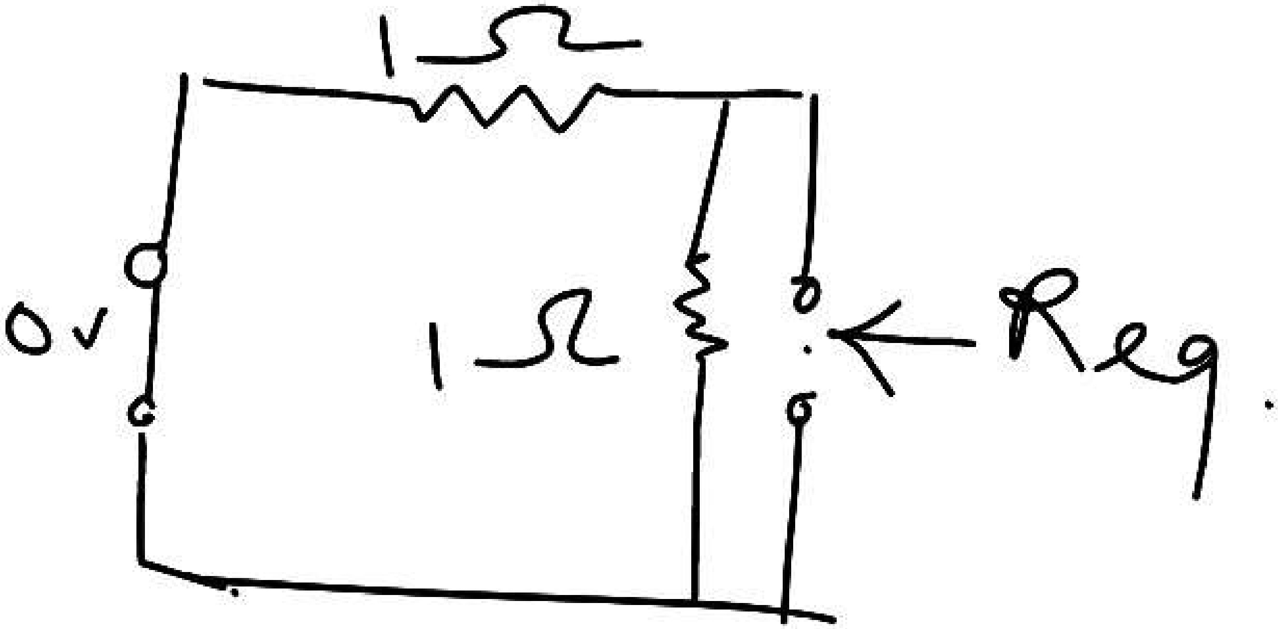
$$I(S) = \frac{1}{R}\left(\frac{S + \frac{1}{RC} - \frac{1}{RC}}{S + 1/RC}\right)$$

$$I(s) = \frac{1}{R}\left(1 - \frac{1/RC}{s + 1/RC}\right)$$

$$i(t) = \frac{1}{R}\left(\delta(t) - \frac{1}{RC}e^{-t/RC}\right)$$

Determine the time constants for following ckts.

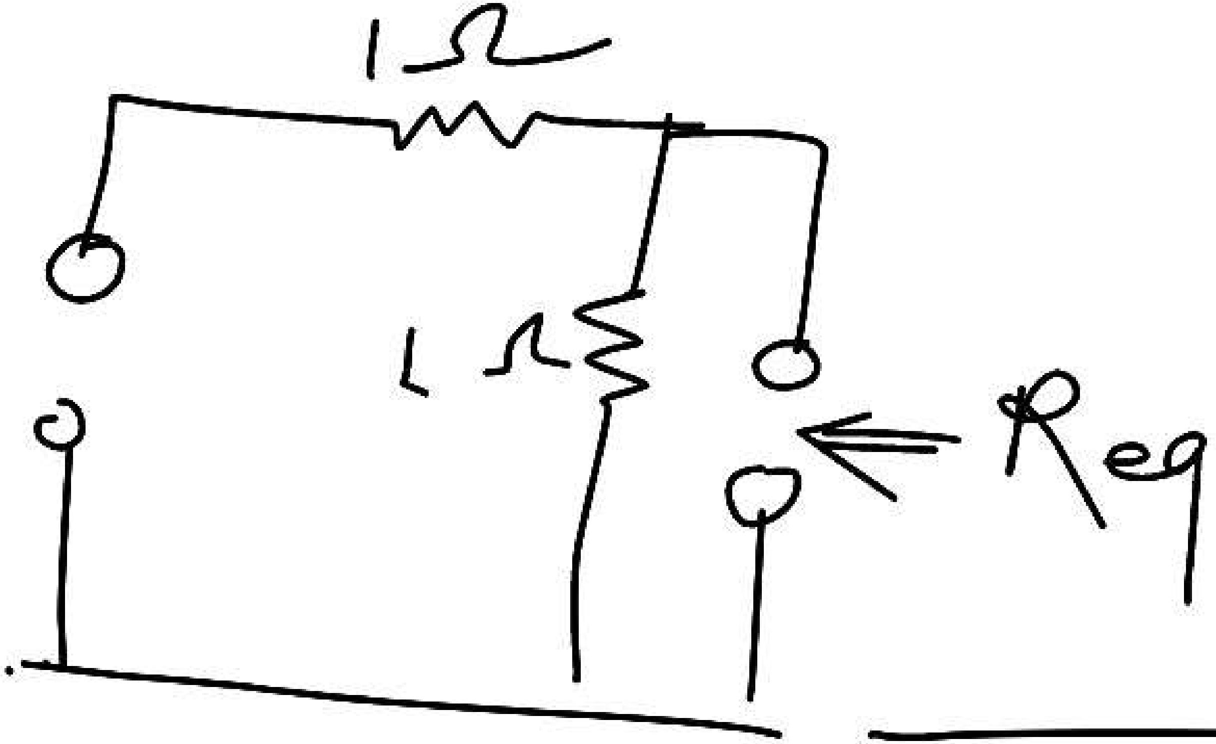
$$T = \frac{1}{R_{eq}}.$$



$$R_{eq} = \frac{1}{2}\,\Omega$$

$$\tau = \frac{1}{R_{eq}} = \frac{1}{1/2} = 2$$

$$\tau = 2\,\text{sec.}$$

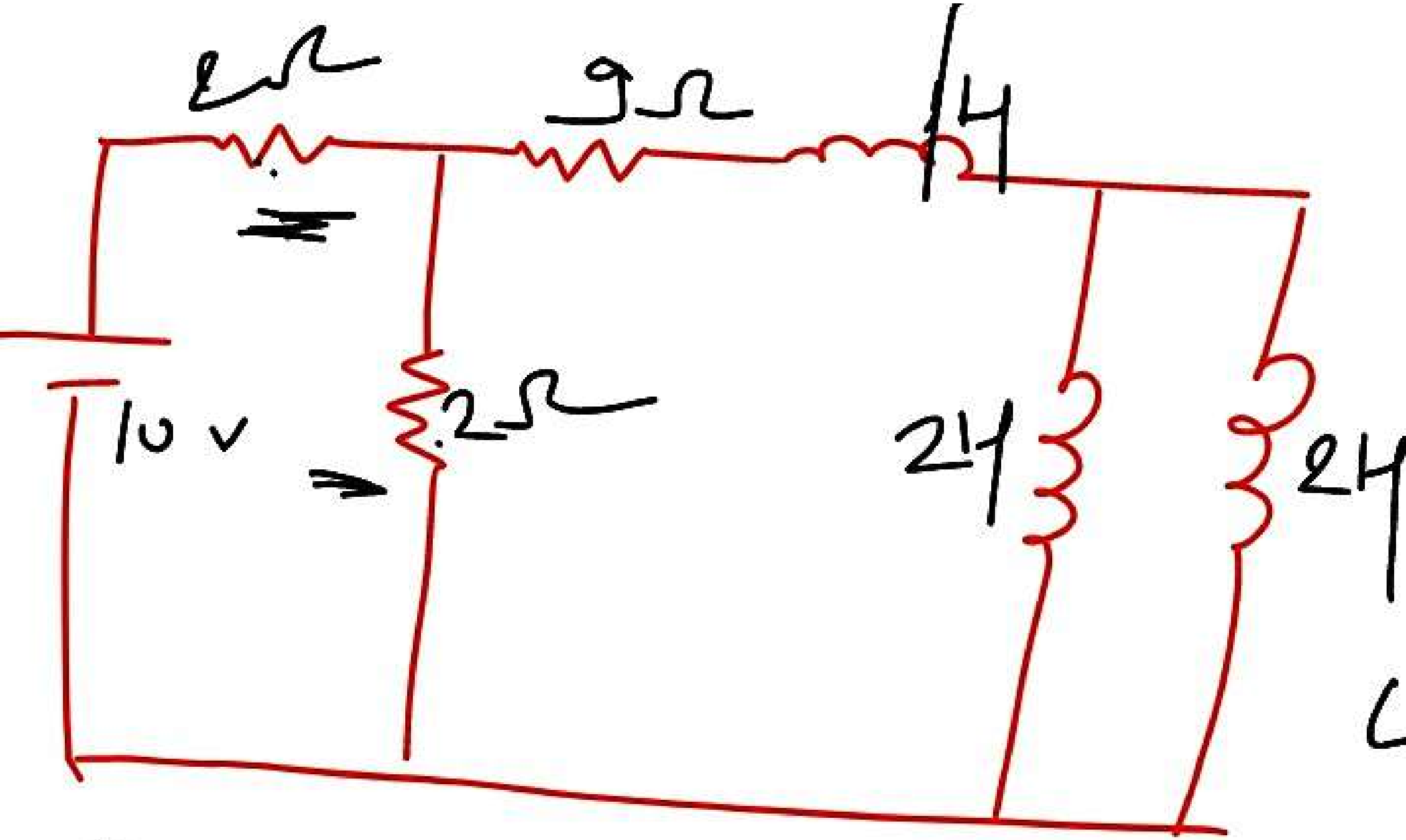


$$R_{eq} = 1\,\Omega$$

$$\tau = \frac{1}{R_{eq}}$$

$$\boxed{\tau = 1}$$

Obs:- Here R_{eq} is the equivalent resistance experience by the source current connected across the open inductor load terminals after eliminating internal independent source

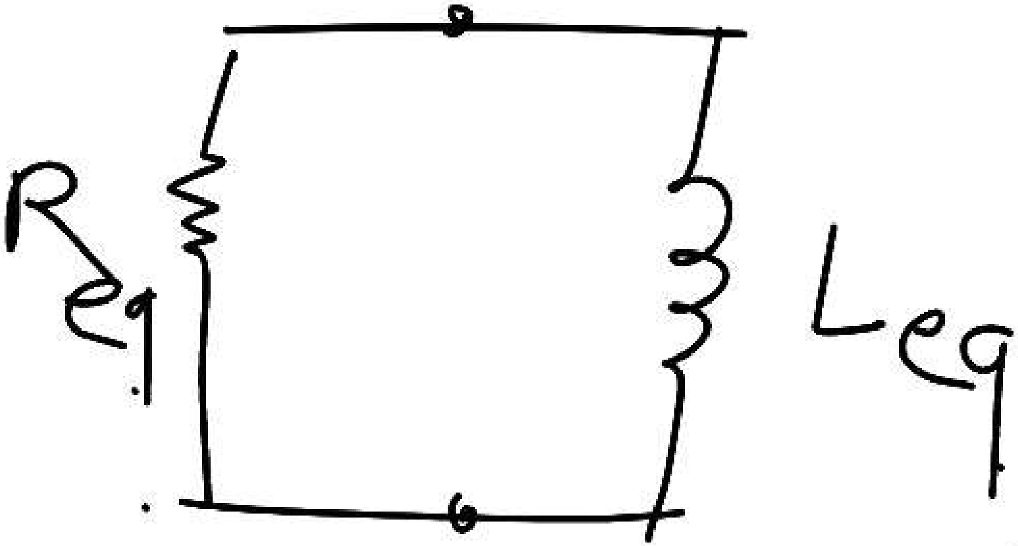


$$L_{eq.} = 1 + 2 \parallel 2$$

$$= 1 + \frac{2 \times 2}{2+2}$$

$$L_{eq} = 1$$

$$R_{eq} = 9 + 2 \parallel 2 \Rightarrow 9 + \frac{2 \times 2}{2+2} = 10$$



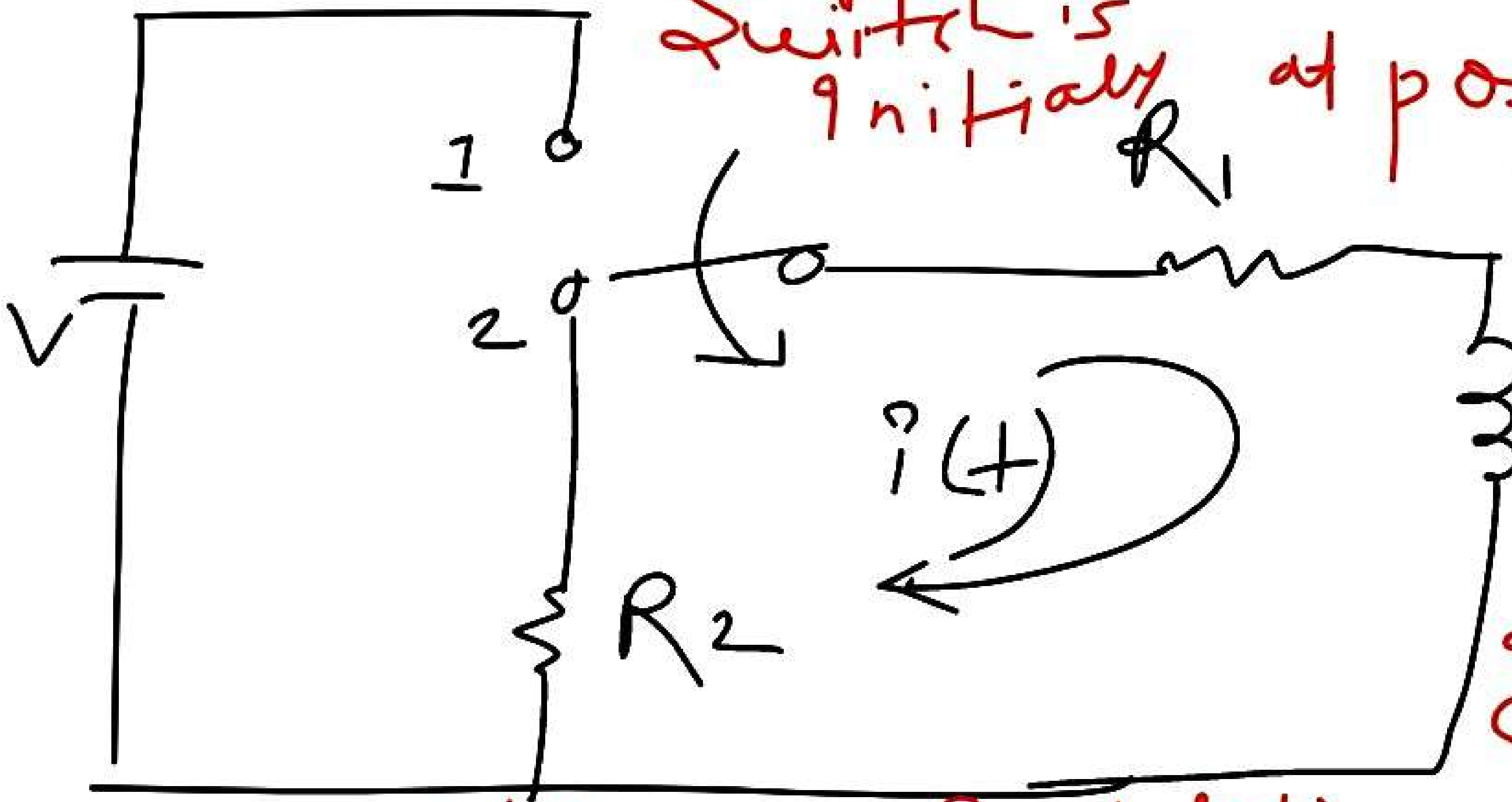
$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$\tau = \frac{2}{10}$$

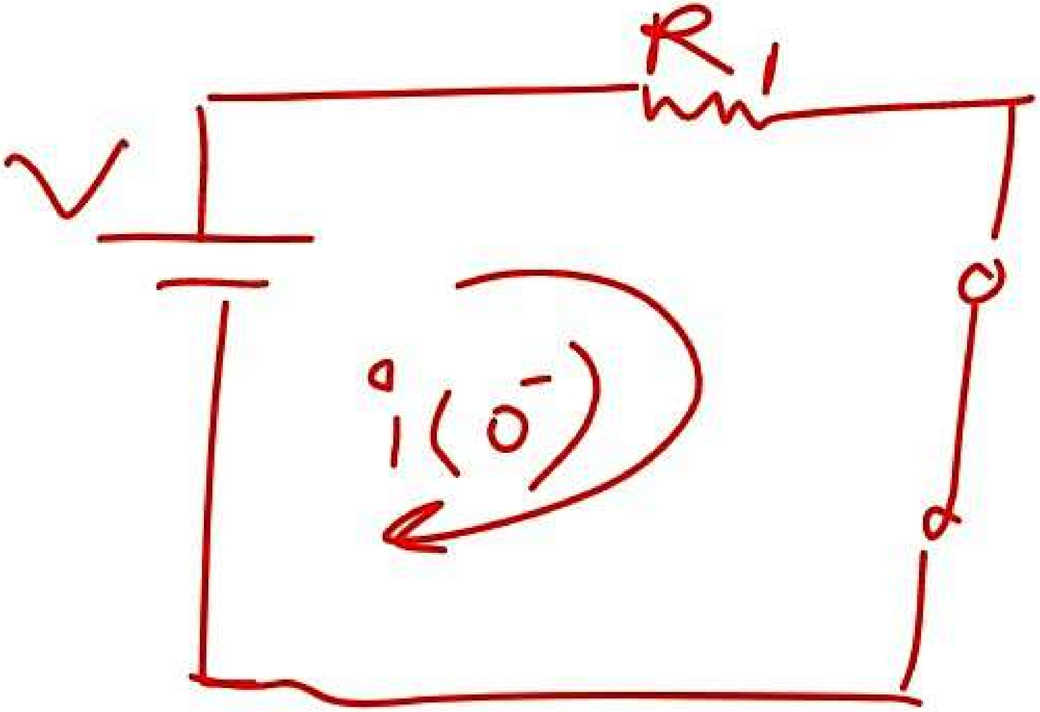
$$= \tau = 0.2\,\text{sec}.$$

Note When Resistor & inductor are not seperable, then circuit will have multiple time constants & the laplace transform approach (LTA) is used to evaluate the constants.

Switch is initially at position 1 on the steady state is reached switch changed to



switch 2 find i(t)

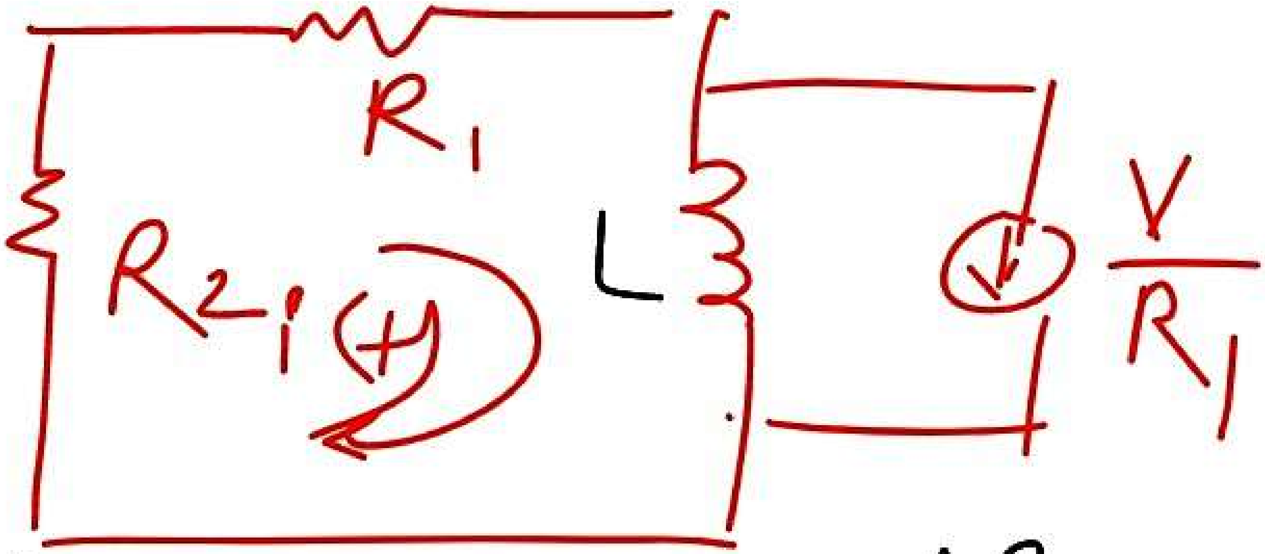


At $t = 0^-$
Nw. attained
SS. Hence
L acts as SC.

$$i(0^-) = \frac{V}{R_1}$$

Since inductor does not allow sudden change.

$$i(0^+) = \frac{V}{R_1}$$



$$(R_1+R_2)i + L\frac{di}{dt} = 0 \Rightarrow$$

$$\frac{di}{dt} + \left(\frac{R_1 + R_2}{= L}\right) i = 0$$

$$i(t) = K e^{-\left(\frac{R_1 + R_2}{L}\right) t}$$

At $t = 0$

$$i(0) = \frac{V}{R_1} = K$$

$$i(t) = \frac{V}{R_1} e^{-\left(\frac{R_1+R_2}{L}\right)t}$$

The capacitor in the circuit as shown below is initially charged to 12 volt with S_1 & S_2 open. S_1 is closed at $t=0$ while S_2 is closed at $t=3$. The waveform of capacitor current will be.

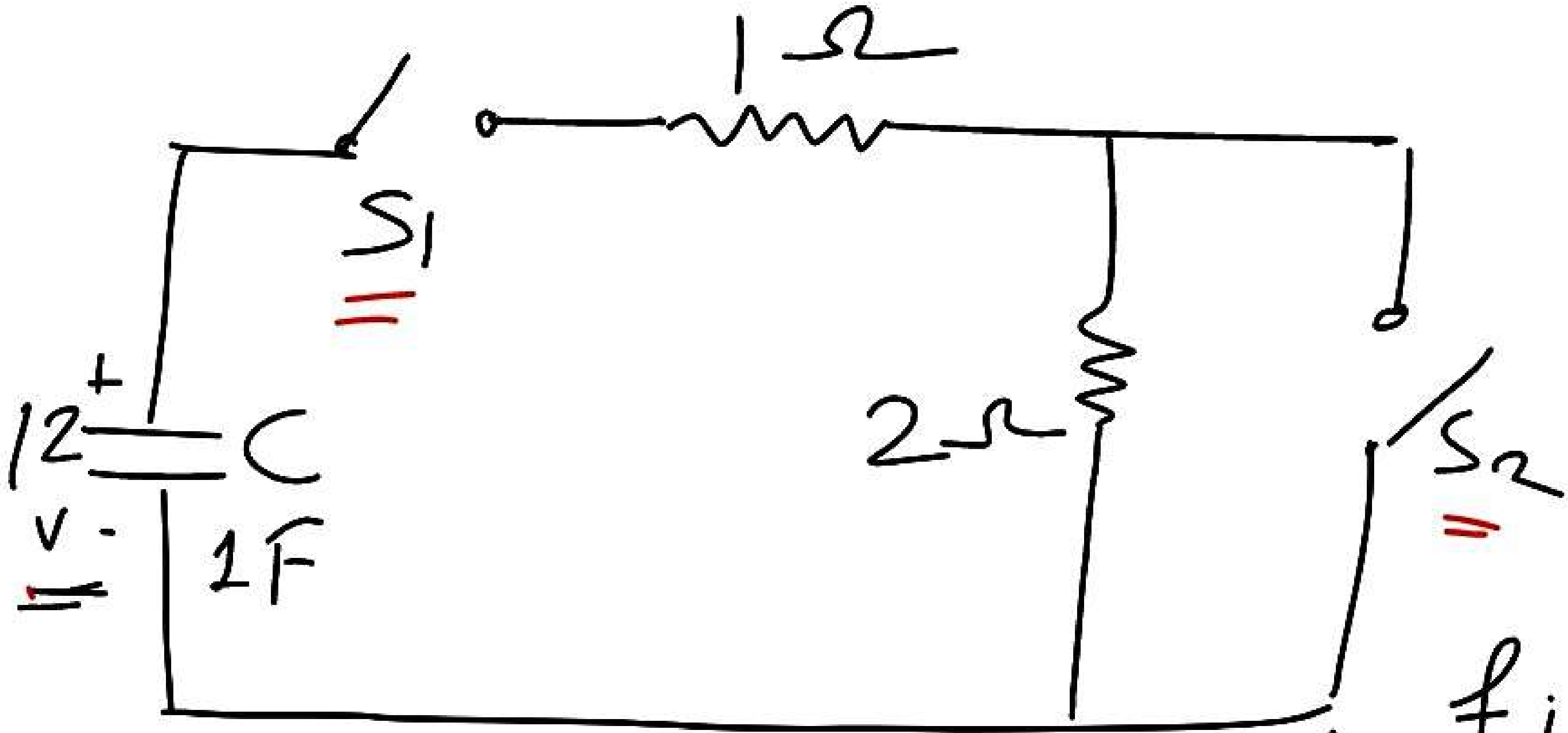
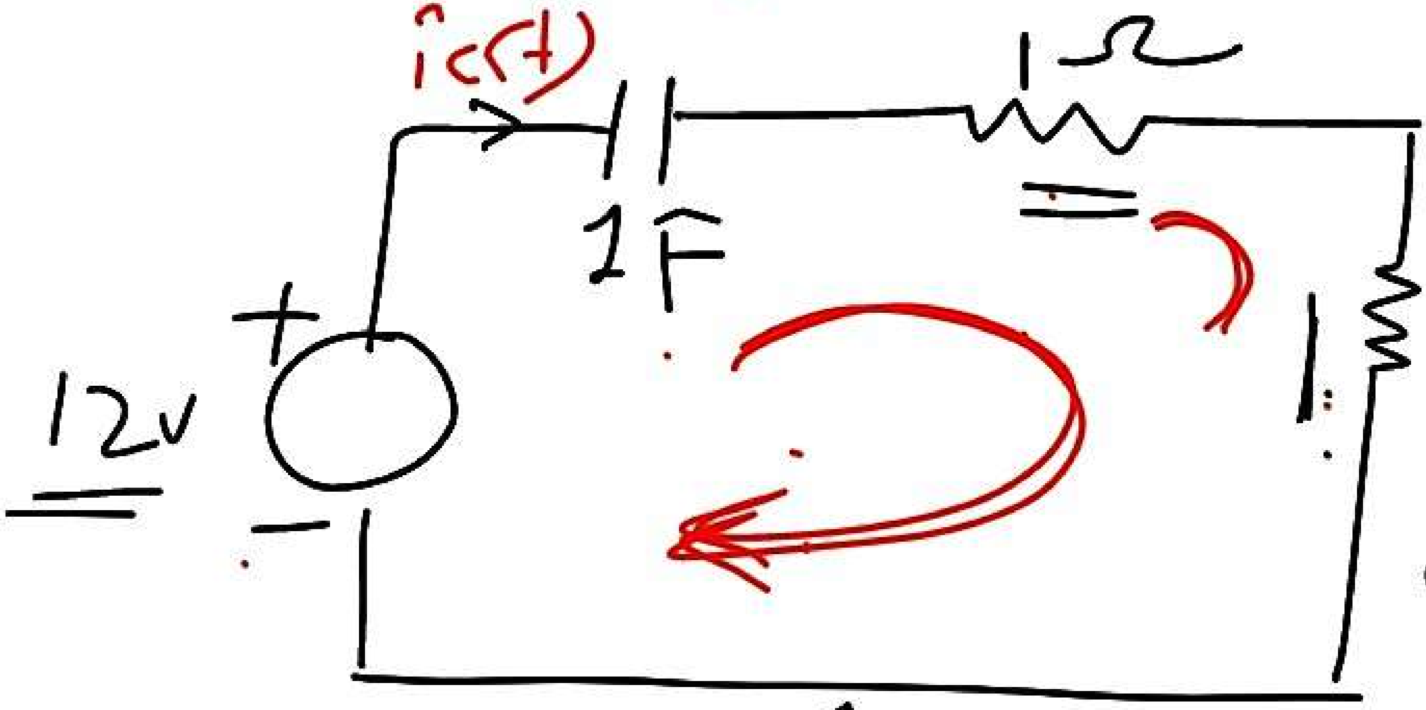


fig ①

for. $0 \le t < 3$

for S_1 closed & S_2 closed.



$0 < t < 3$ fig ②

$i_c(0^+) = \frac{12}{3}$

$= 4$ Amp

$\tau = RC = 3 \times 1$

$= 3$ sec.

$$i(t) = 4e^{-t/3} \quad i(0^+) \cdot e^{-t/3} \quad i(0^+) = 4 = K$$

$$\tau = 3$$

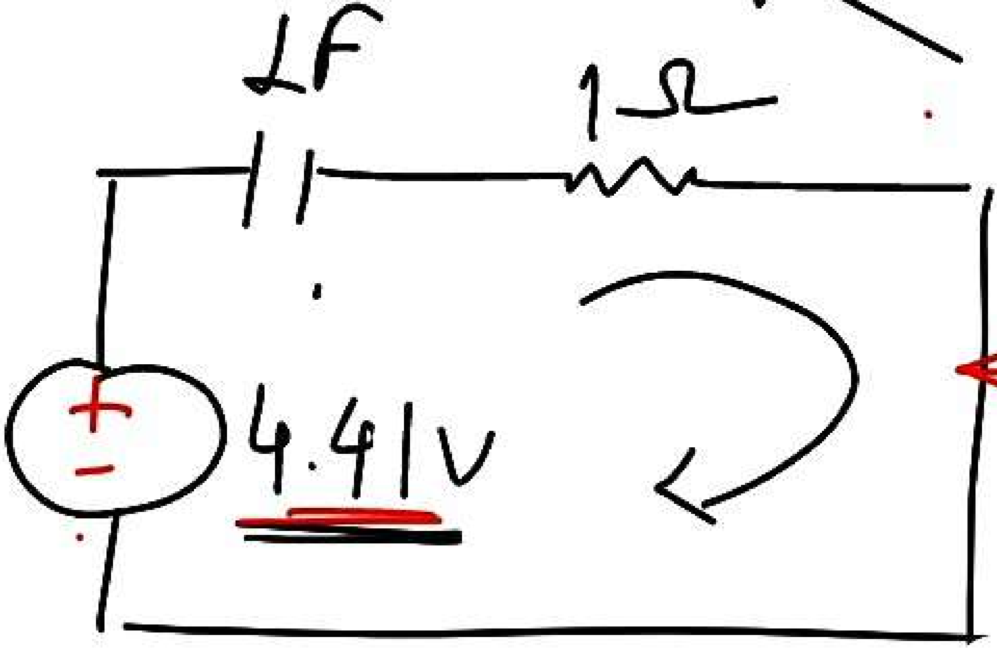
$$i(3_-) = 4e^{-1} \quad (\text{At } t = 3)$$

$$= 1.47$$

$$V_c(3_-) = V_c(3^+) = i(3^-) \cdot R$$

$$= 4e^{-1} \times 3 = 4.41$$

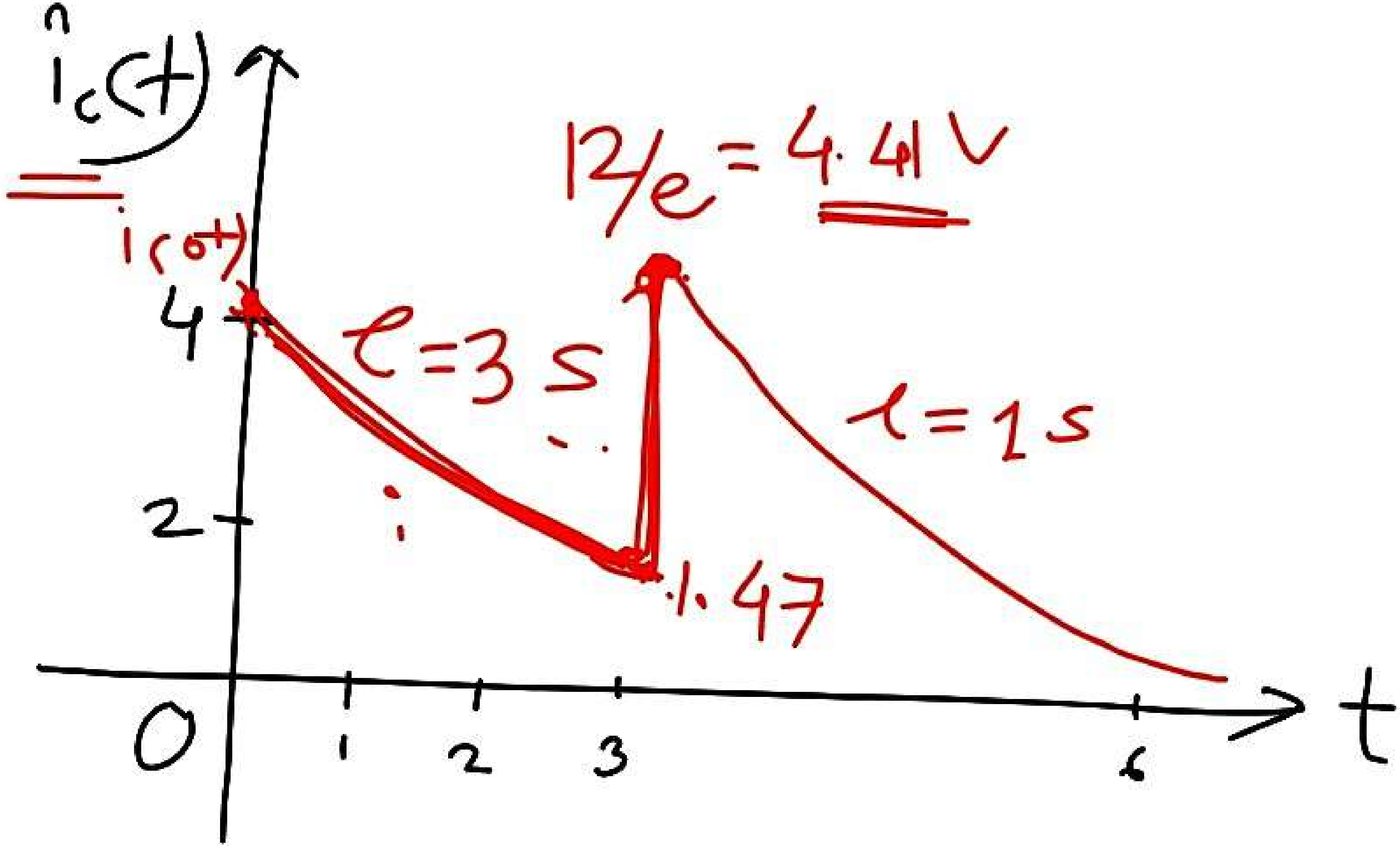
$$i_c(3^+) = \frac{v_c(3^+)}{R} = \frac{4.41}{1} = 4.41$$



$0 < t < 3$

for $3 < t < \infty$

$i_c(\infty) = 0$



$i_c(t)$
$i(0^+)$
$12/e = 4.41V$
$\tau = 3s$
$\tau = 1s$
1.47
4
2
0
1
2
3
6
t

With source circuit & using Laplace Transform Analysis.

Behaviour of inductor & capacitor with sources initial ($t = 0^+$) & final ($t \to \infty$)

r.

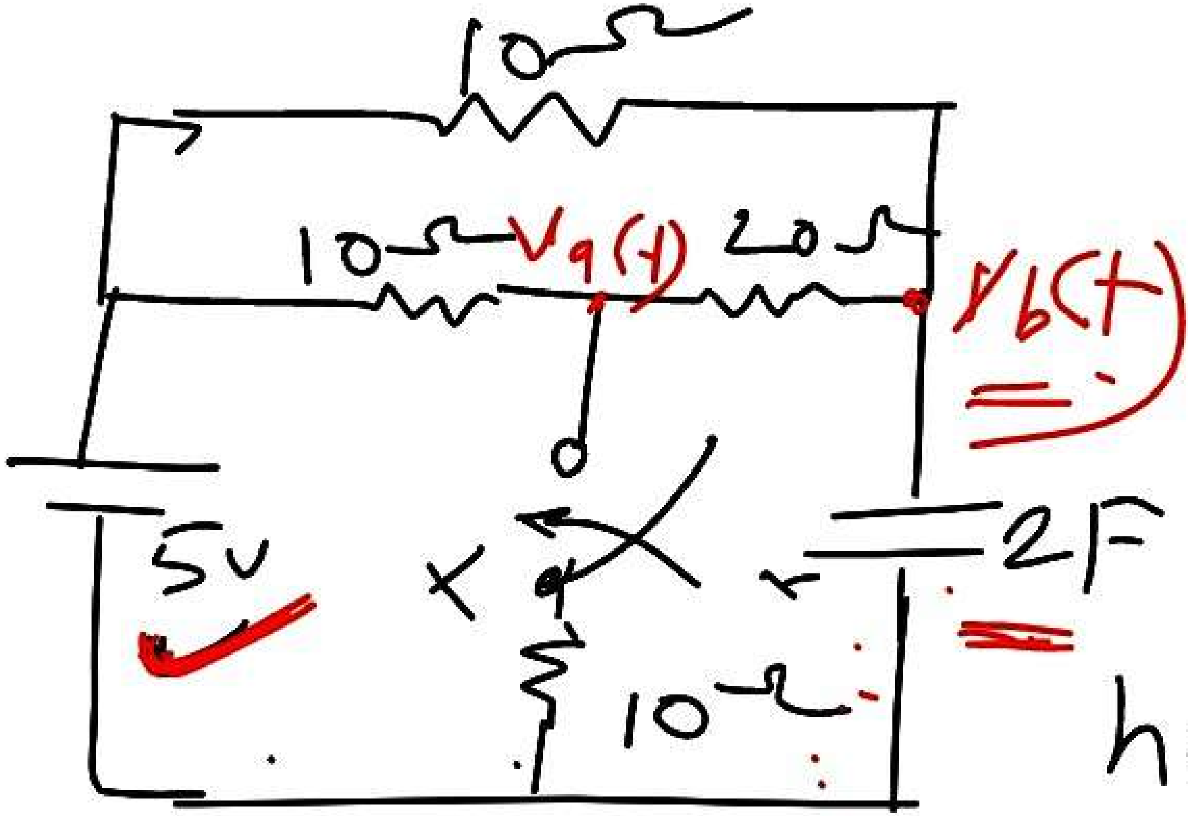
At $t = 0^+ \Rightarrow L = OC \Rightarrow C = S.C$

At $t \rightarrow \infty \Rightarrow L = SC \Rightarrow C = OC$

L & C elements will lose their significance & hence the circuit nature is resistive.

In a N/w. which is a steady state is reached with switch open At $t=0$, switch is closed. Determine

$V_a(0^-)$ $V_a(0^+)$ & $V_b(0^+)$ & $V_b(0^-)$

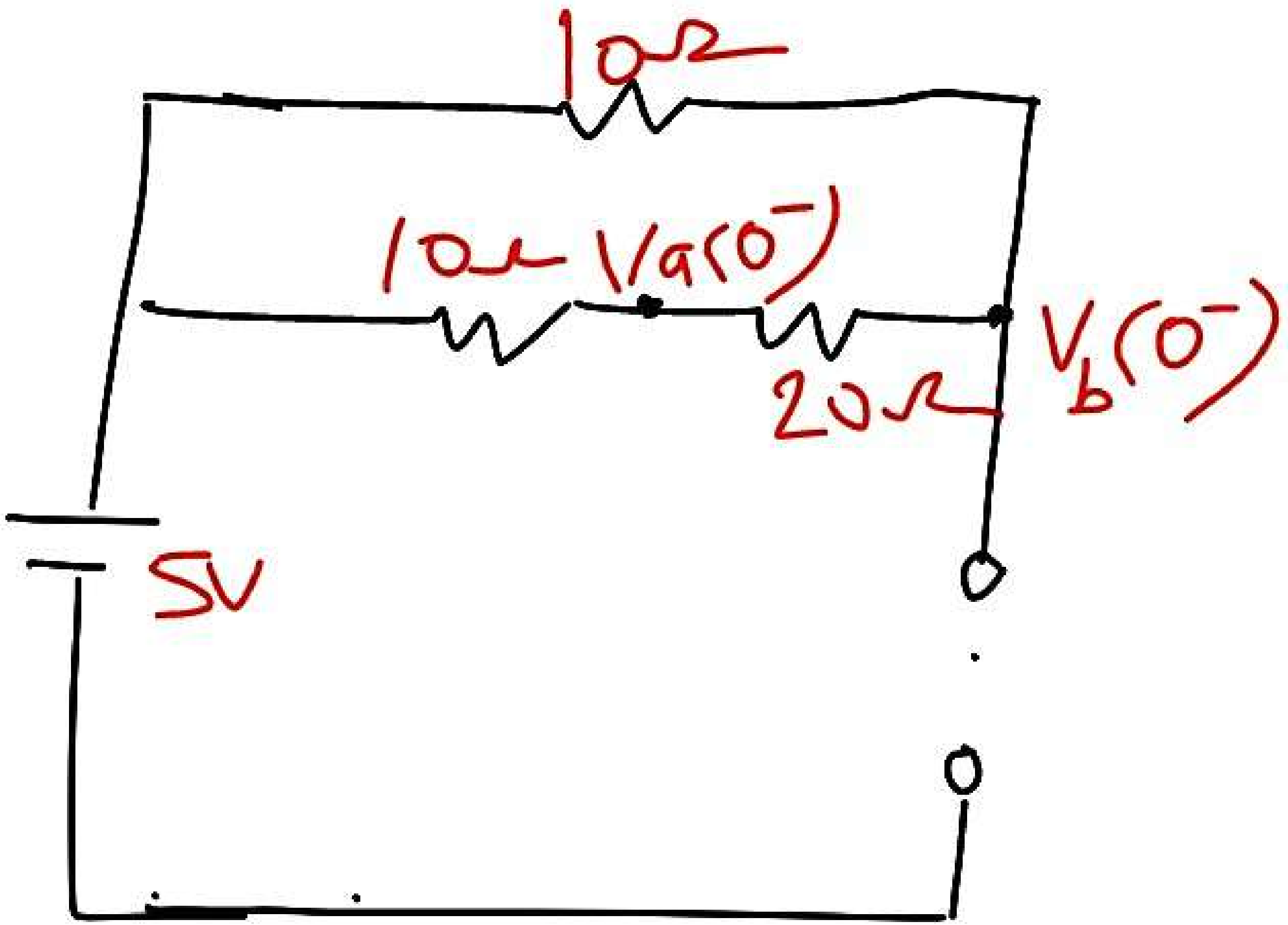


At $t = 0^-$

N/w. attain the SS. &

hence capacitor

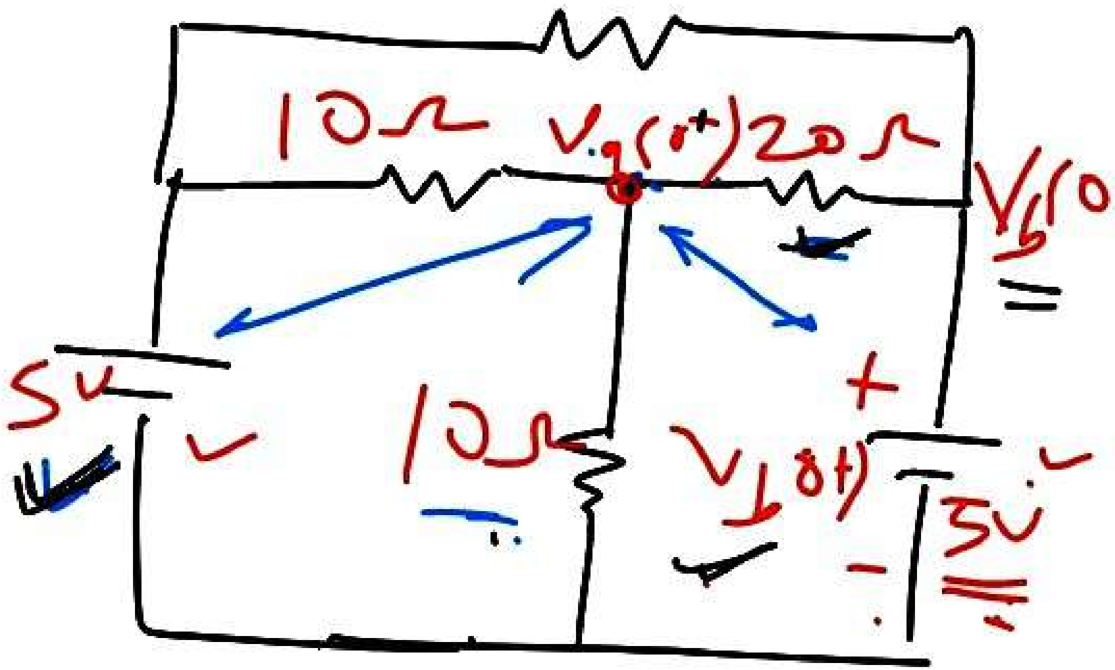
will be OC



$$V_a(0^-) = 5V$$

$$V_b(0^-) = 5V$$

At $(t = 0^+)$ The capacitor acts as voltage source of $\underline{\underline{5}}$ V



$V_b(0^-) = V_b(0^+) = 5v$

$0^+ \rightarrow 0^-$

find $V_a(0^+)$

writing KCL eqn at $t = 0^+$

$$\frac{V_a(0^+) - 5}{10} + \frac{V_a(0^+)}{10} + \frac{V_a(0^+) - 5}{20} = 0$$

$$V_a(0^+) = \frac{2V_a(0^+) - 10 + 2V_a(0^+) + V_a(0^+) - 5}{20}$$

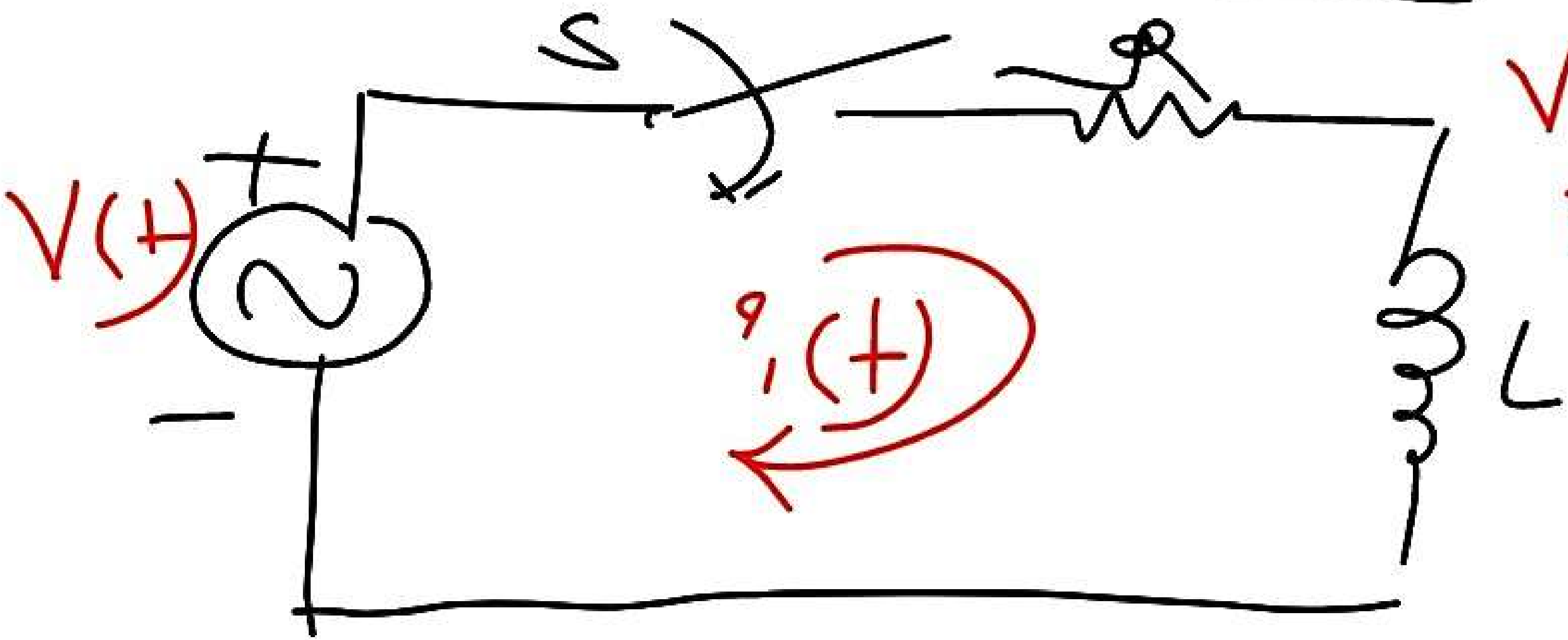
$$\frac{5V_a(0^+)-15}{20}=0$$

$$\boxed{V_a(0^+) = 15/5 = 3 \text{ volt}}$$

$$0^- \leftarrow \underline{\underline{0^+}}$$

Ac Transients :-

(1) Series RL circuit :-



$V(t) = V_m \sin(\omega t + \phi)$

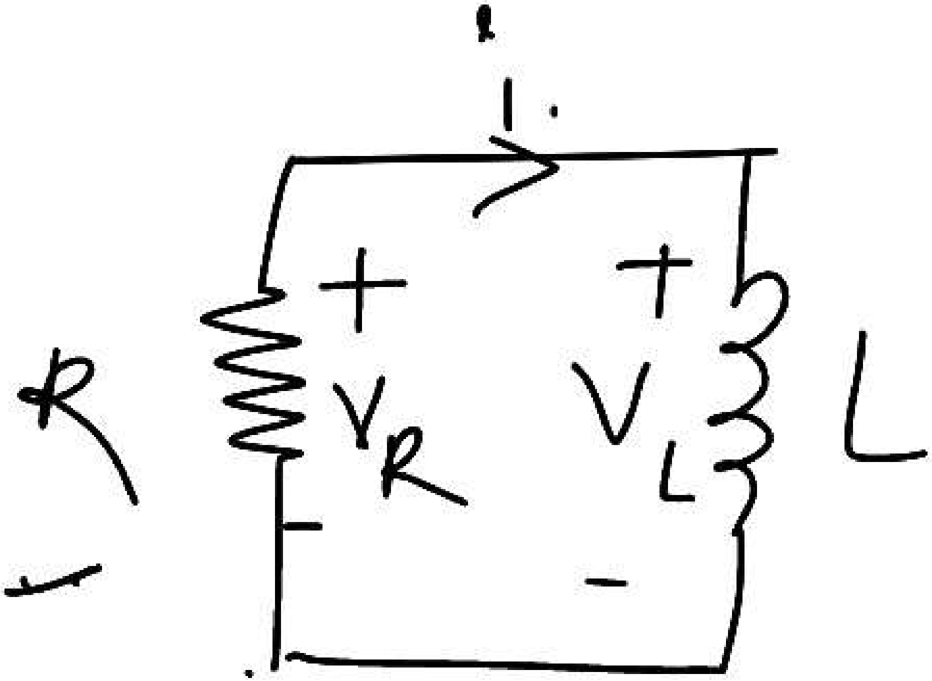
$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$i_{tr}(t) \Rightarrow$ Transient ✓ Natural Response

$i_{ss}(t) \Rightarrow$ Steady state. ✓ forced response.

$$i(t) = \underline{Ke^{-t/\tau}} + \underline{i_{ss}(t)}$$

$$= \underbrace{\boxed{Ke^{-R/L\,t}}}_{\text{Transient Response}} + \underbrace{i_{ss}(t)}_{\text{steady response.}}$$



By KVL

$$-V_R - V_L = 0$$

$$-Ri - L\frac{di}{dt} = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = 0$$

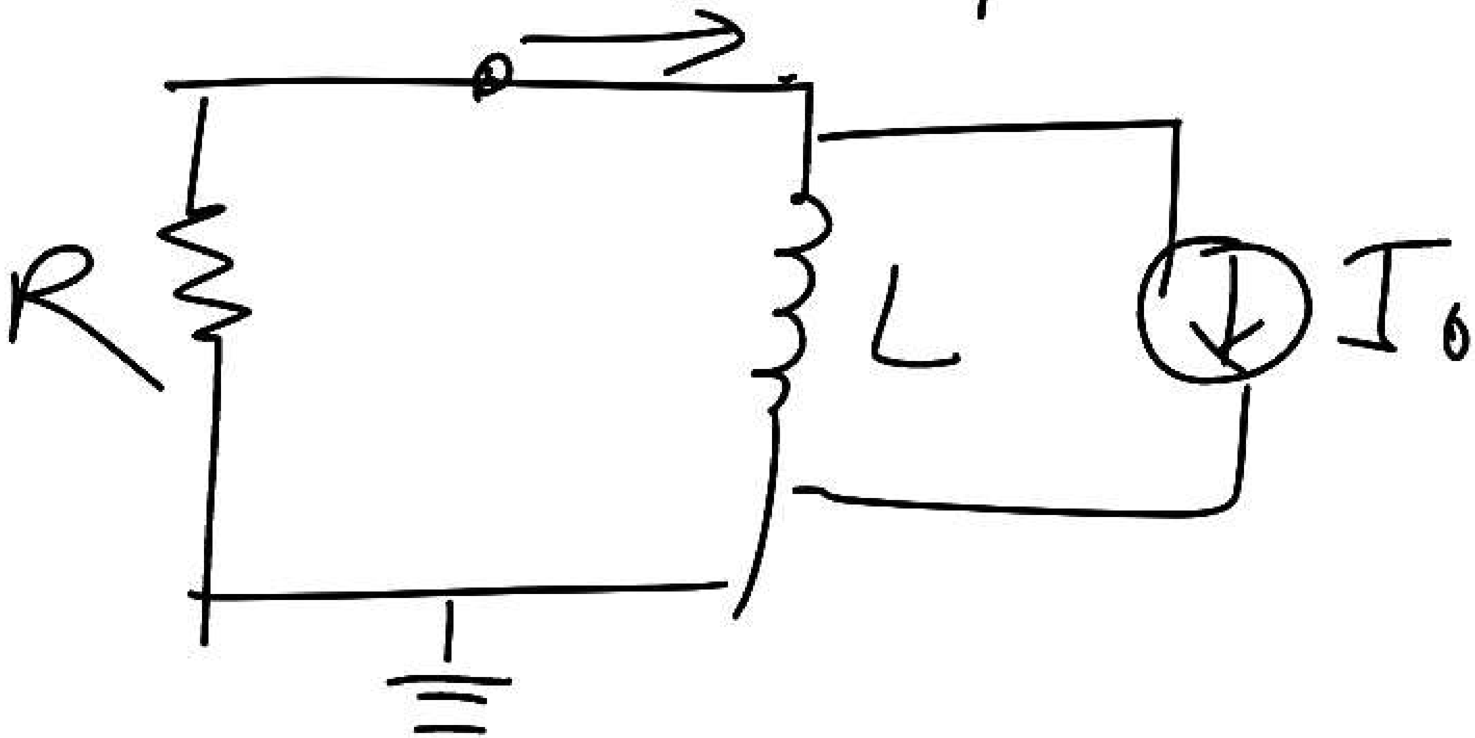
let

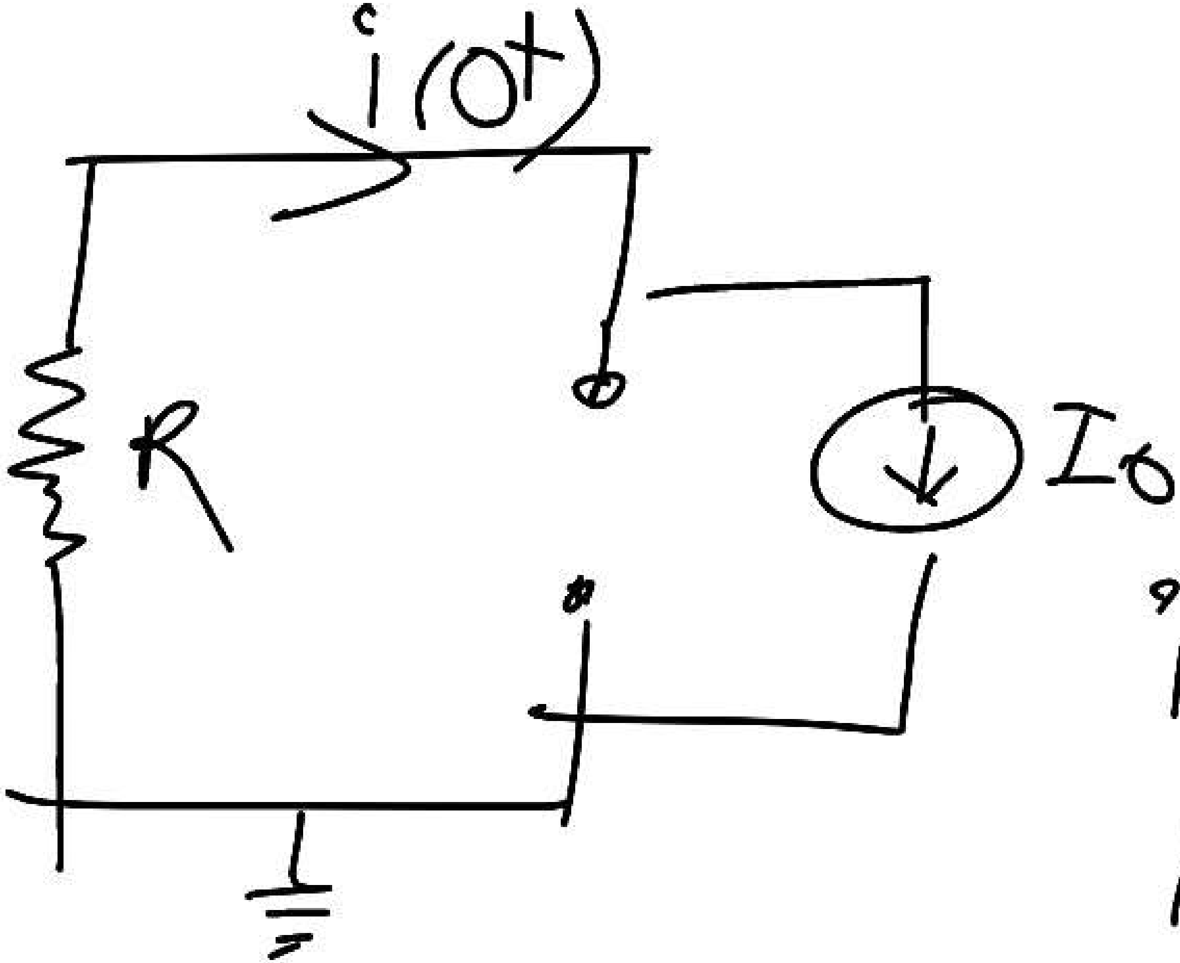
$D = \frac{d}{dt} \Rightarrow \left(D + \frac{R}{L}\right) i = 0$

$D = -\frac{R}{L}$ $\qquad S = \alpha$

$i(t) = Ke^{\alpha t}$ for $0 \leq t \leq \infty$

$= k e^{-R/L\, t}$ Amp for $0 \le t \le \infty$





$$-\quad i(0^+) = I_0$$

$$i(0^+) = Ke^0$$

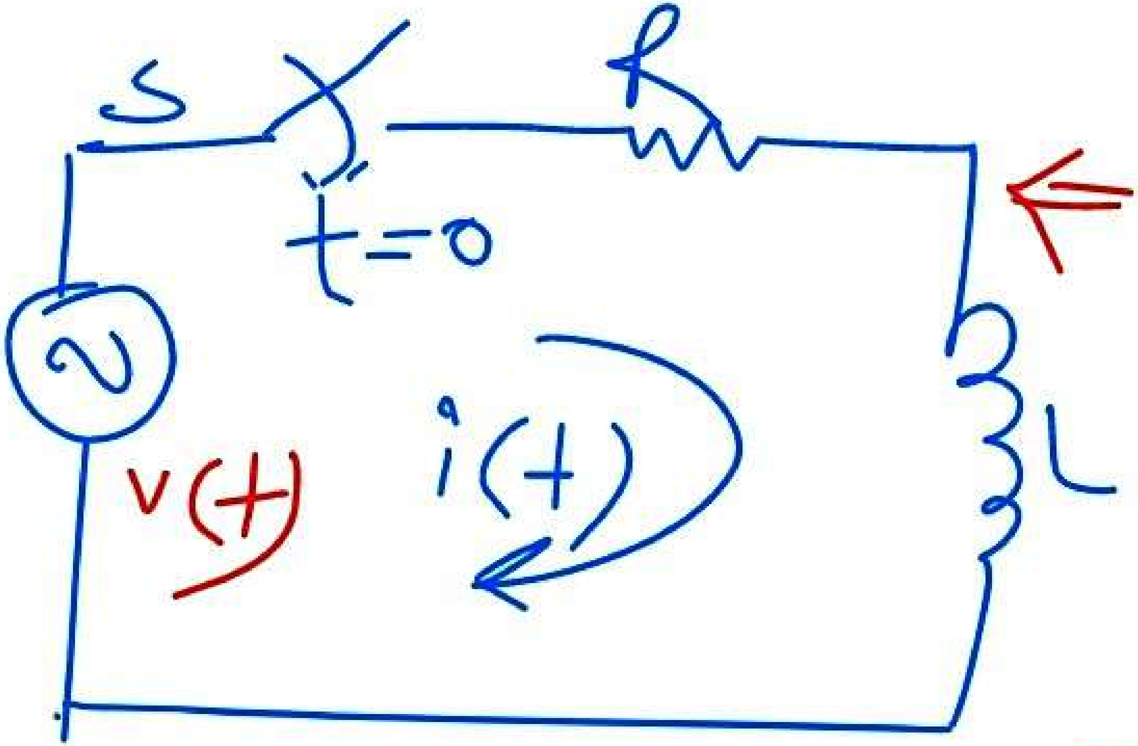
$$K = I_0$$

$$i(0^+) = \underline{\underline{k}} e^{-0} = I_0$$

$$I(t) = I_0 e^{-R/L\, t}$$

$$i(t) = I_0 e^{-t/\tau}$$

$$\tau = L/R \Rightarrow \text{Time constant}$$



← series RL ckt.

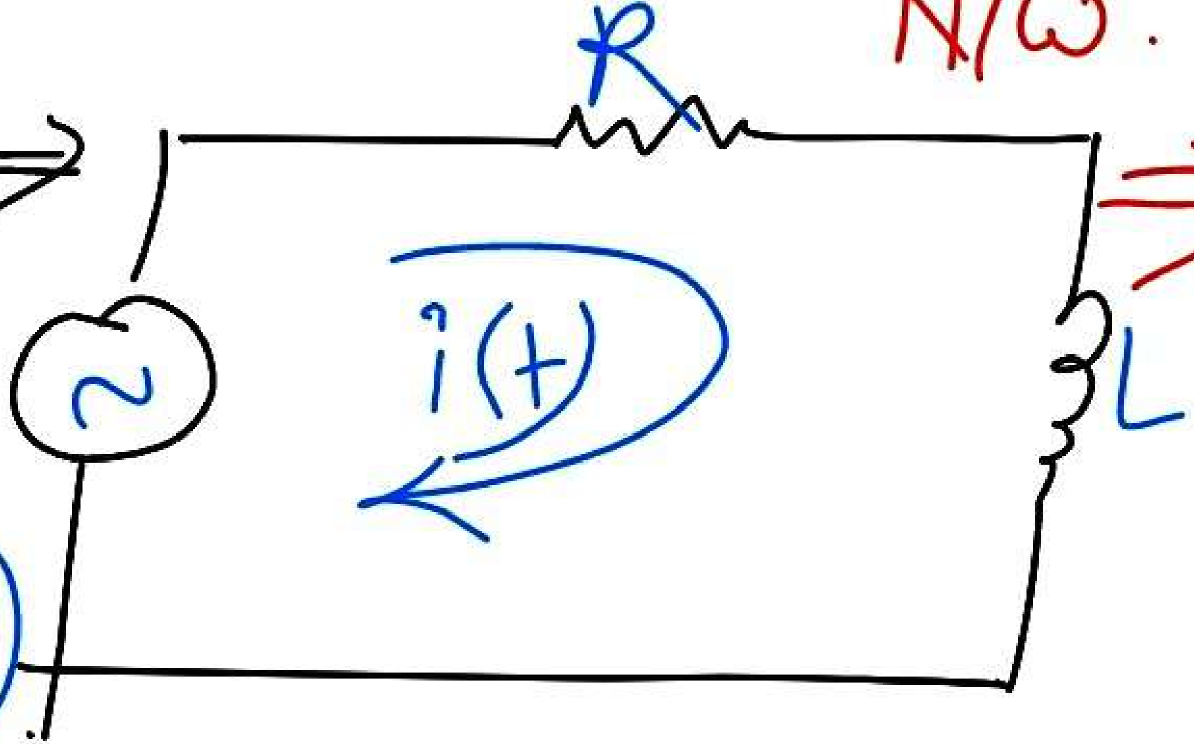
$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$i_{tr}(t) = K e^{-t/\tau}$$

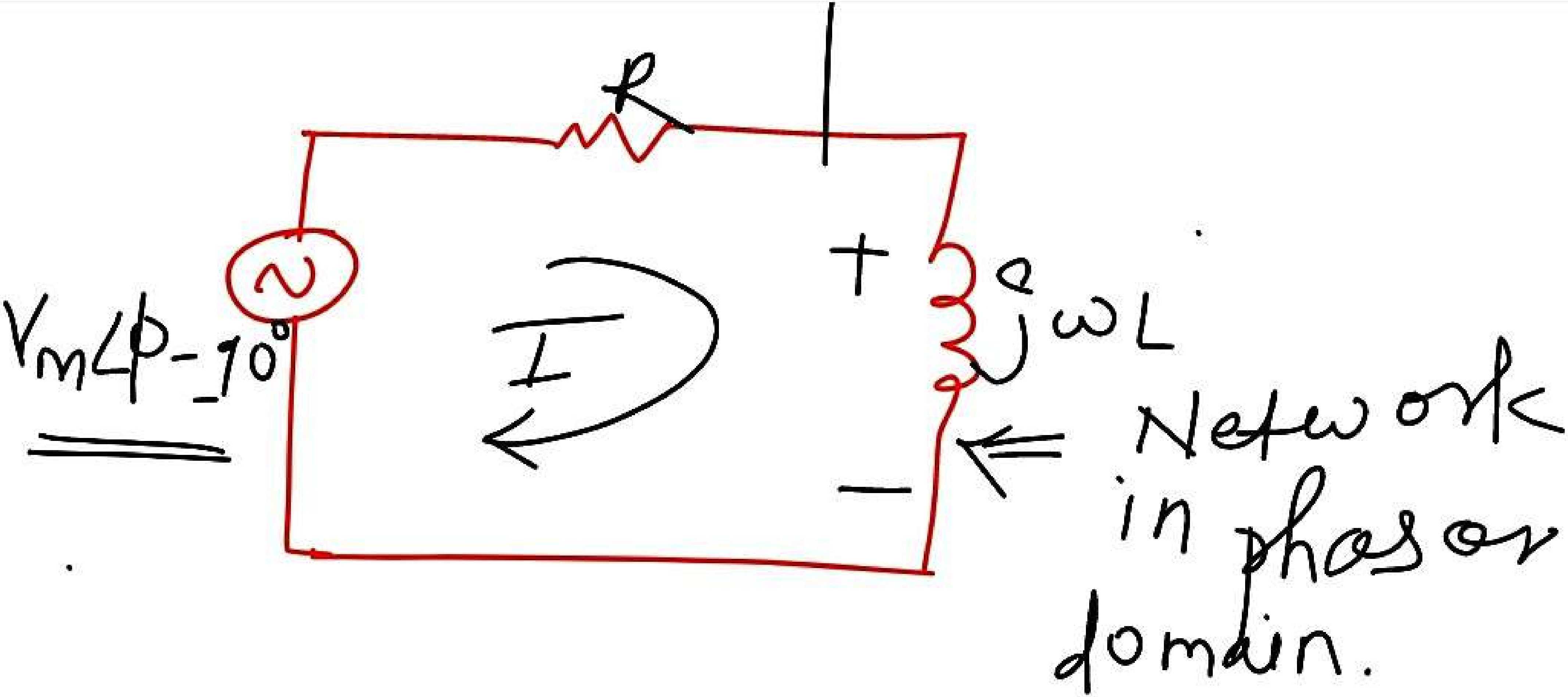
$i_{ss}(t)$

$v(t)$

$V_M \sin(\omega t + \phi)$



$\Rightarrow$ N/w. is in steady state.


R
+
−
jωL
I
Vm∠Φ−90°
Network
in phasor
domain.

$\therefore v(t) = V_m \cos(\omega t + \phi)$

$$V = V_m e^{j\phi} \Leftarrow \text{exponential.}$$

$$= V_m \angle \phi \Leftarrow \text{Polar}$$

$$= V_m(\cos\phi + j\sin\phi)$$

Rectangular.

$$i(t) = I_m \cos(\omega t + \theta)$$

$$\bar{I} \Rightarrow I_m \angle \theta \Rightarrow \text{polar.}$$

$$= I_m e^{j\theta} \Rightarrow \text{Exponential.}$$

$$= I_m(\cos\theta + j\sin\theta) \Rightarrow \text{Rect.}$$

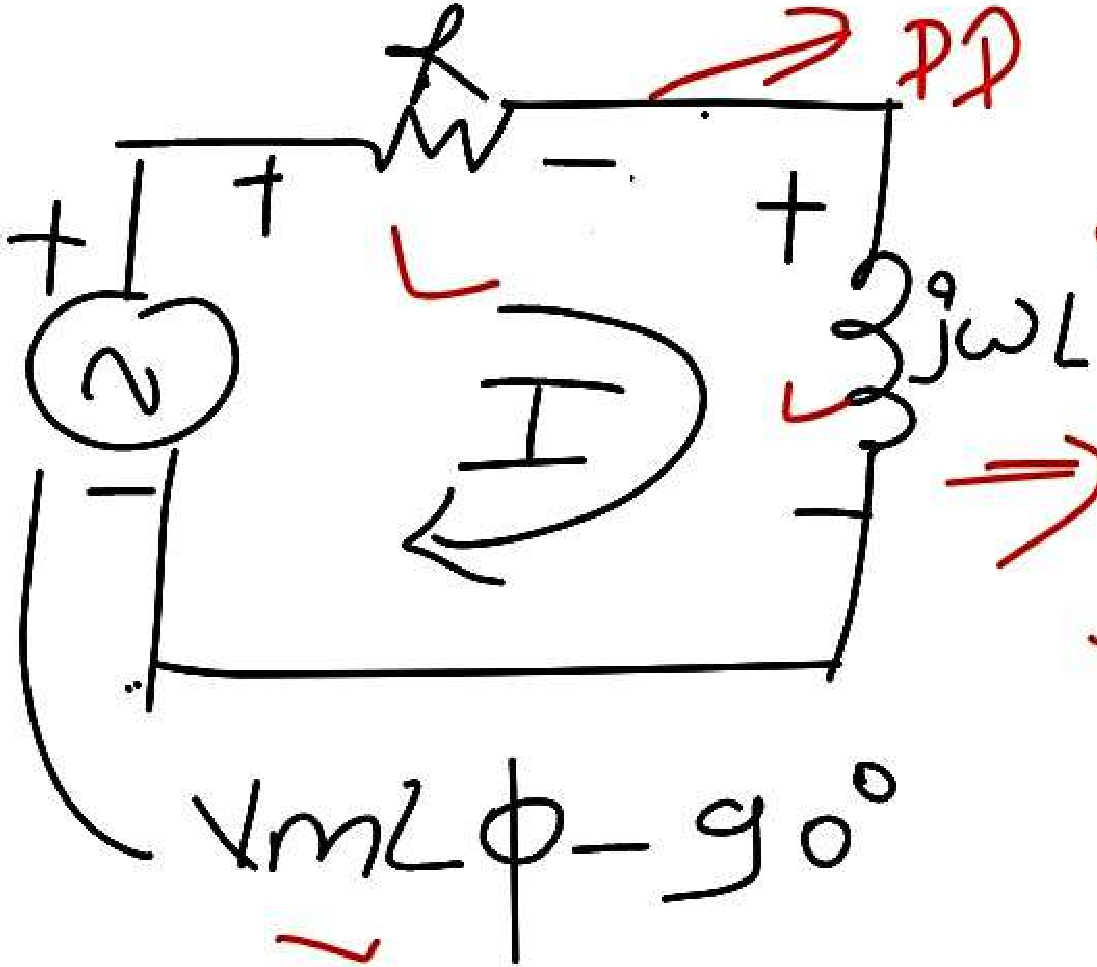
$$i(t) = 10\sin(2t + 30^\circ) \text{ Amp.}$$

$$= 10\cos(2t + 30^\circ - 90^\circ) \text{ Amp}$$

$$= 10\cos(2t - 60^\circ) \text{ Amp.}$$

$$I = 10\angle{-60^\circ} \Rightarrow P \Rightarrow 10(\cos 60 - j\sin 60^\circ)$$

$$= 10e^{-j60^\circ} \Rightarrow E \qquad \text{Rect.}$$



By KVL in PP.

$$\Rightarrow V_m\angle\phi-90^\circ - RI - j\omega L I = 0$$

By KVL

$$V_m\angle\phi - 90^\circ - RI - j\omega LI = 0$$

$$V_m\angle\phi - 90^\circ = RI + j\omega LI$$

$$= I(R + j\omega L)$$

$$\therefore I = \frac{V_m\angle\phi - 90^\circ}{R + j\omega L} \Rightarrow$$

$$I = \frac{V_m \angle \phi - 90^\circ}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle \phi - \tan^{-1}\left(\frac{\omega L}{R}\right) - 90^\circ$$

$$= \alpha \angle \beta$$

$$\alpha = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}$$

$$\beta = \underbrace{\phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ}$$

$$=$$

$\alpha e^{j\beta}$

$$i(t) = R.P.\left[I.e^{j\omega t} \right]$$

$$= \alpha \cos(\omega t + \beta)$$

$$= \underbrace{\alpha \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ\right)}$$

$$v(t) = V_m \cos(\omega t + \phi)$$

$$= \text{Real part}\left[V_m e^{j(\omega t + \phi)}\right]$$

$$= \text{Real part}\left[\underline{V_m e^{j\phi}} \cdot e^{j\omega t}\right]$$

$$= \text{Real part}\left[\underline{\underline{V}} \cdot e^{j\omega t}\right]$$

$$\frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{Amp.}$$

$$\Rightarrow \boxed{i_{ss}(t)}$$

$$t\text{-}D \leftrightarrow P\cdot D$$

$$i(t) \Longleftrightarrow I \qquad =$$

$$v(t) \Longleftrightarrow V$$

$$R \Longleftrightarrow R$$

$$L \Longleftrightarrow j\omega L \quad (s = j\omega)$$

$$C \Longleftrightarrow \frac{1}{j\omega C}$$

$$V_t = V_m \sin(\omega t + \phi)$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{R+sL}$$

$$s = j\omega$$

$$H(j\omega) = \frac{1}{R+j\omega L}$$

Method (2) By LT-A

$v(t) = V_m \sin(\omega t + \phi)$ $\quad \boxed{s = j\omega}$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{R+sL}$$

$V = IR \Rightarrow \frac{V}{I} = Z \qquad \frac{V(s)}{I(s)} = Z(s)$

$$H(j\omega) = Y(j\omega) = \frac{1}{Z(j\omega)} = \frac{1}{R + j\omega L}$$

$$= \frac{1}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1} \frac{\omega L}{R}$$

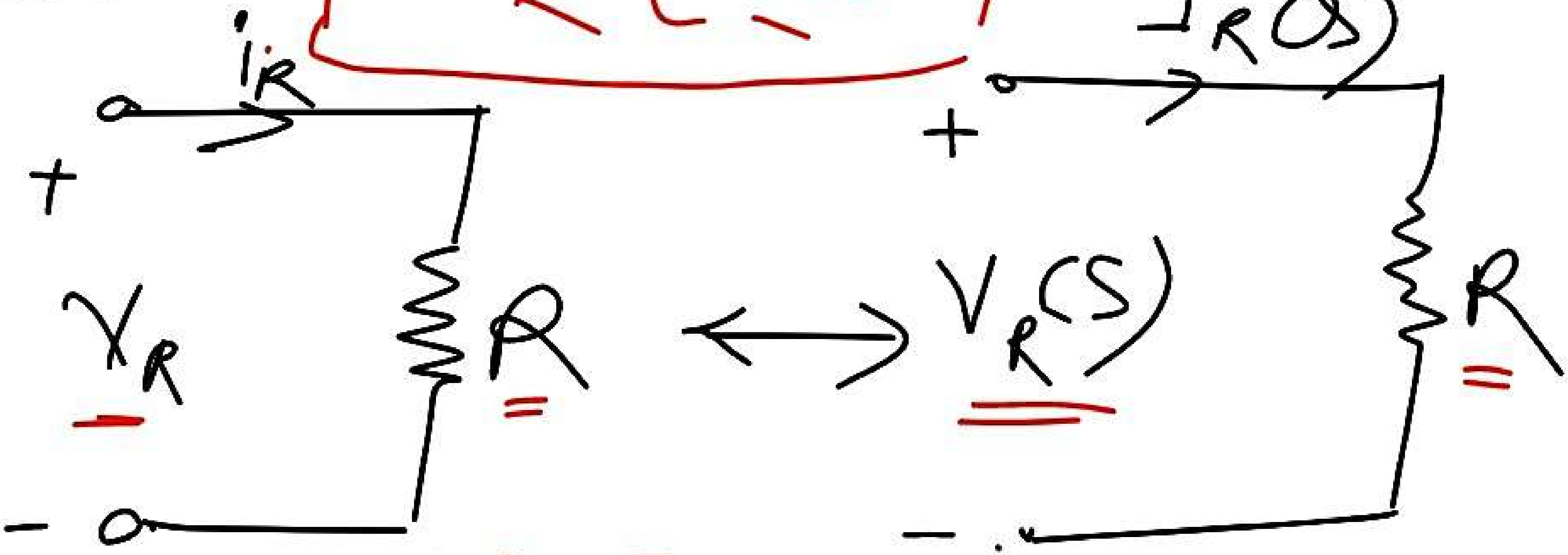
$$I(s) = H(s) \cdot V(s)$$

$$i(t) = \underbrace{\frac{1}{\sqrt{R^2+(\omega L)^2}} V_m \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{ Amp.}}_{i_{ss}(t)}$$

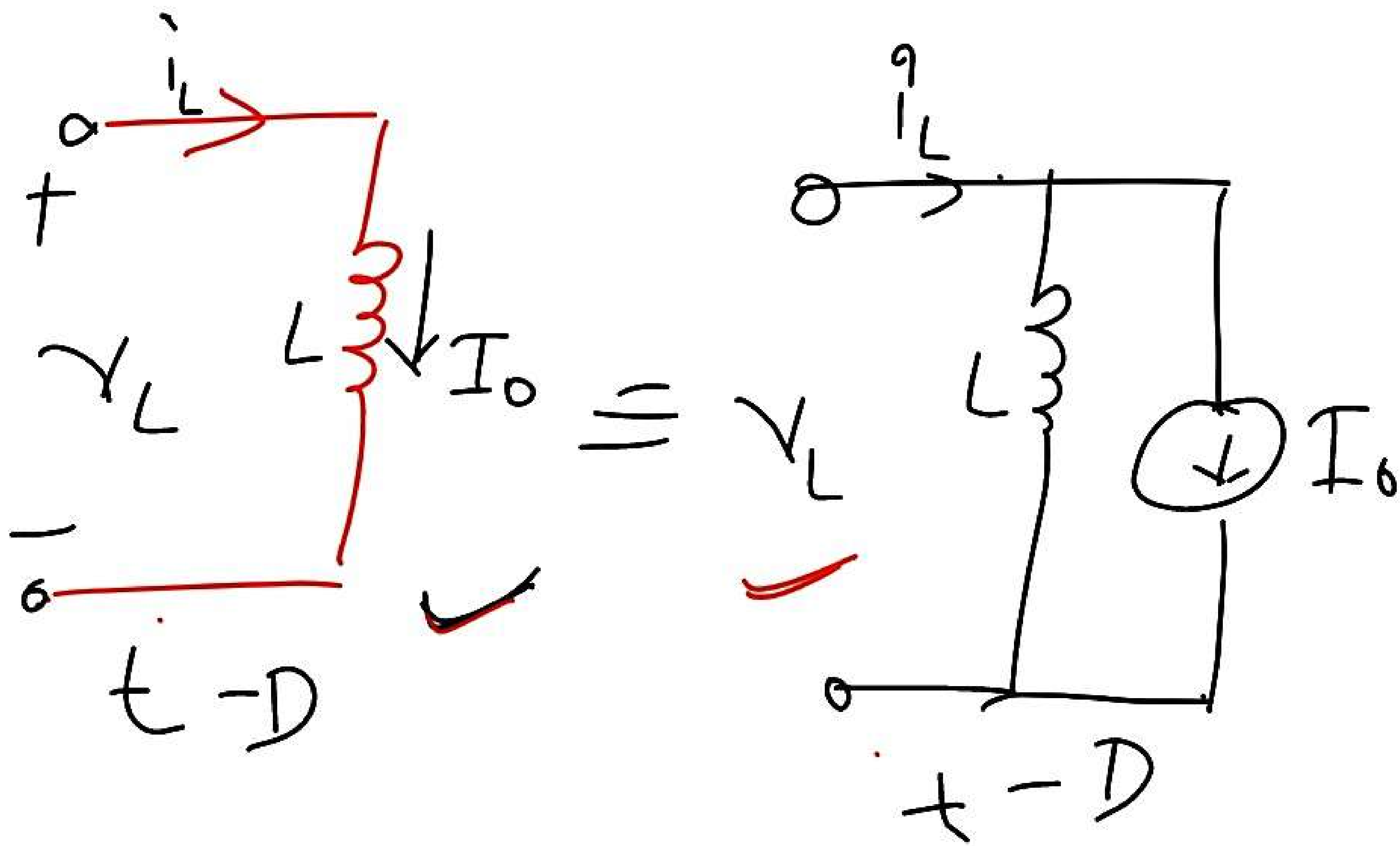
$$i(t) = i_{ss}(t)$$

R :-

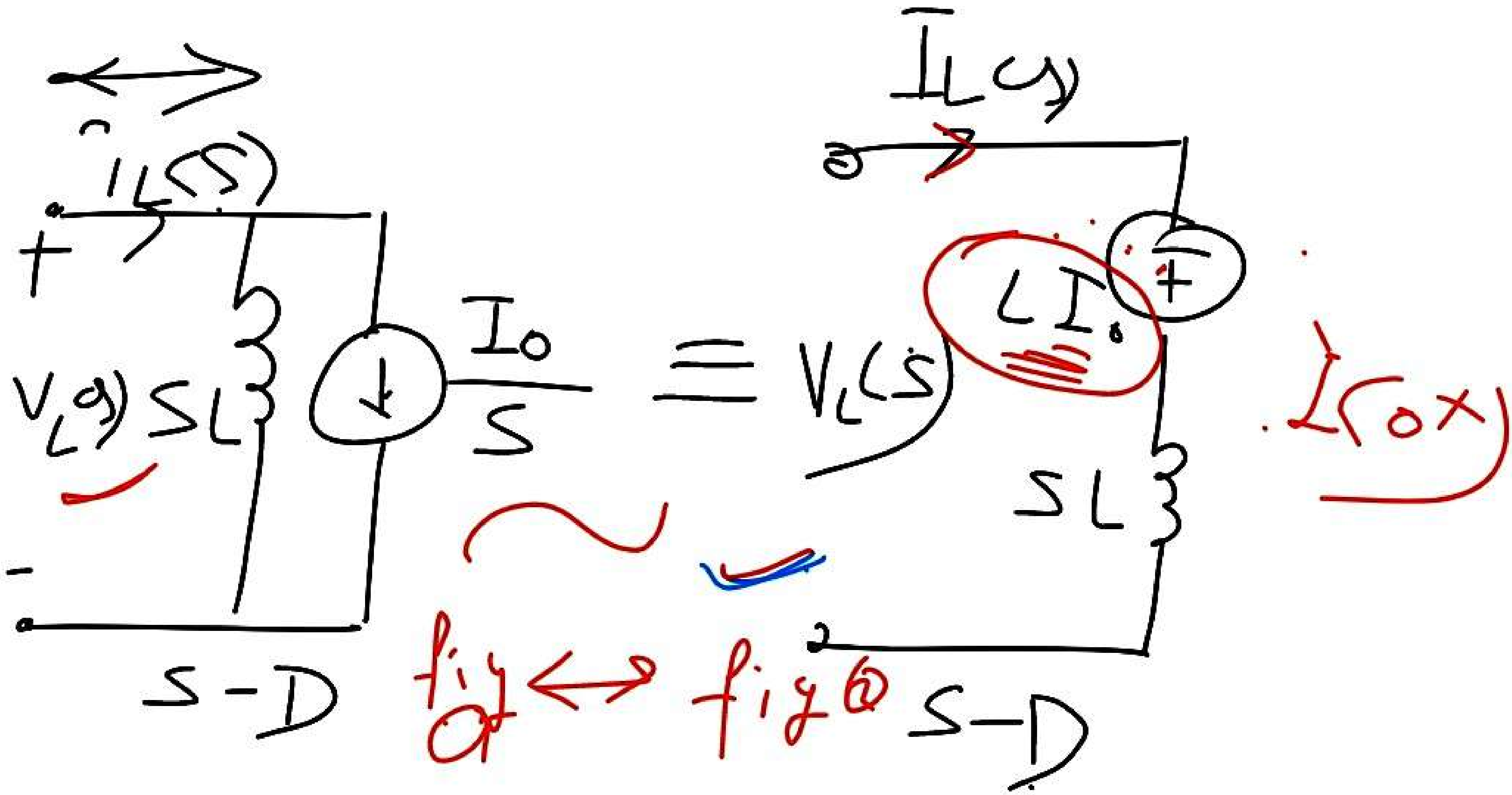
$$\boxed{0 \le t \le \infty}$$



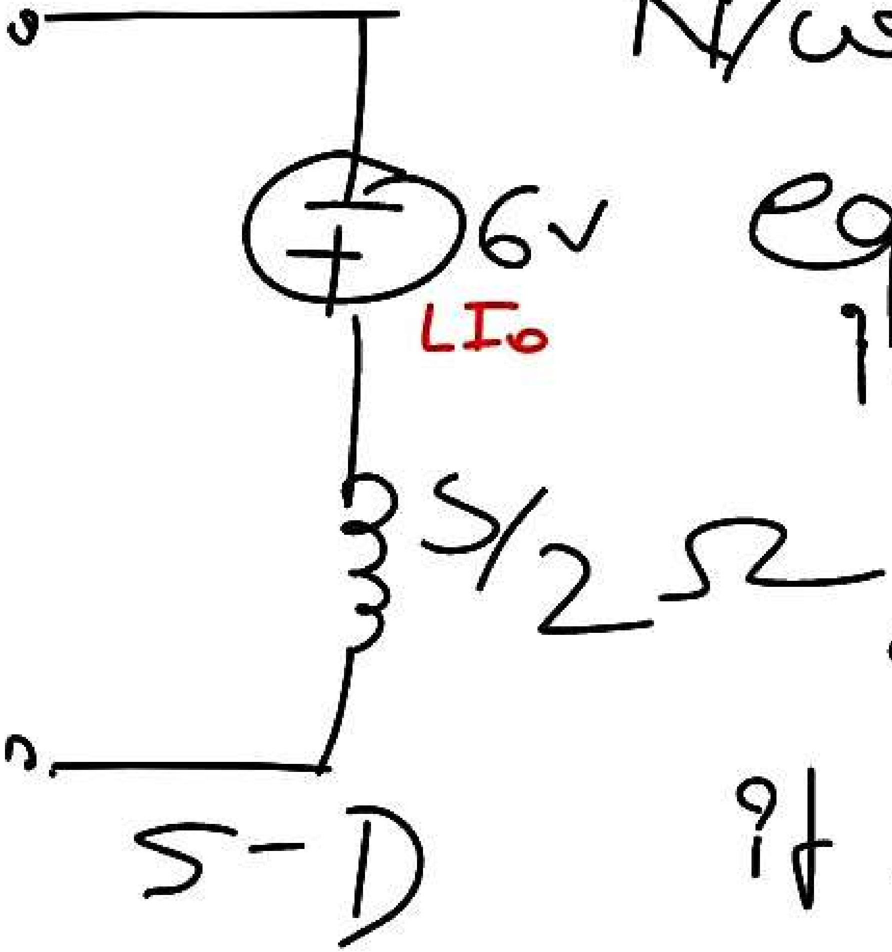
t-D ① $\xleftrightarrow{LT}$ ② s-D



i_L
+
v_L
L
I_0
−
t-D
$\equiv$
v_L
i_L
L
I_0
t-D



$I_L(s)$
$V_L(s)$
SL
$\frac{I_0}{S}$
$\equiv$
LI_0
$I_L(0)$
$S-D$
fig a ⟷ fig b
$S-D$



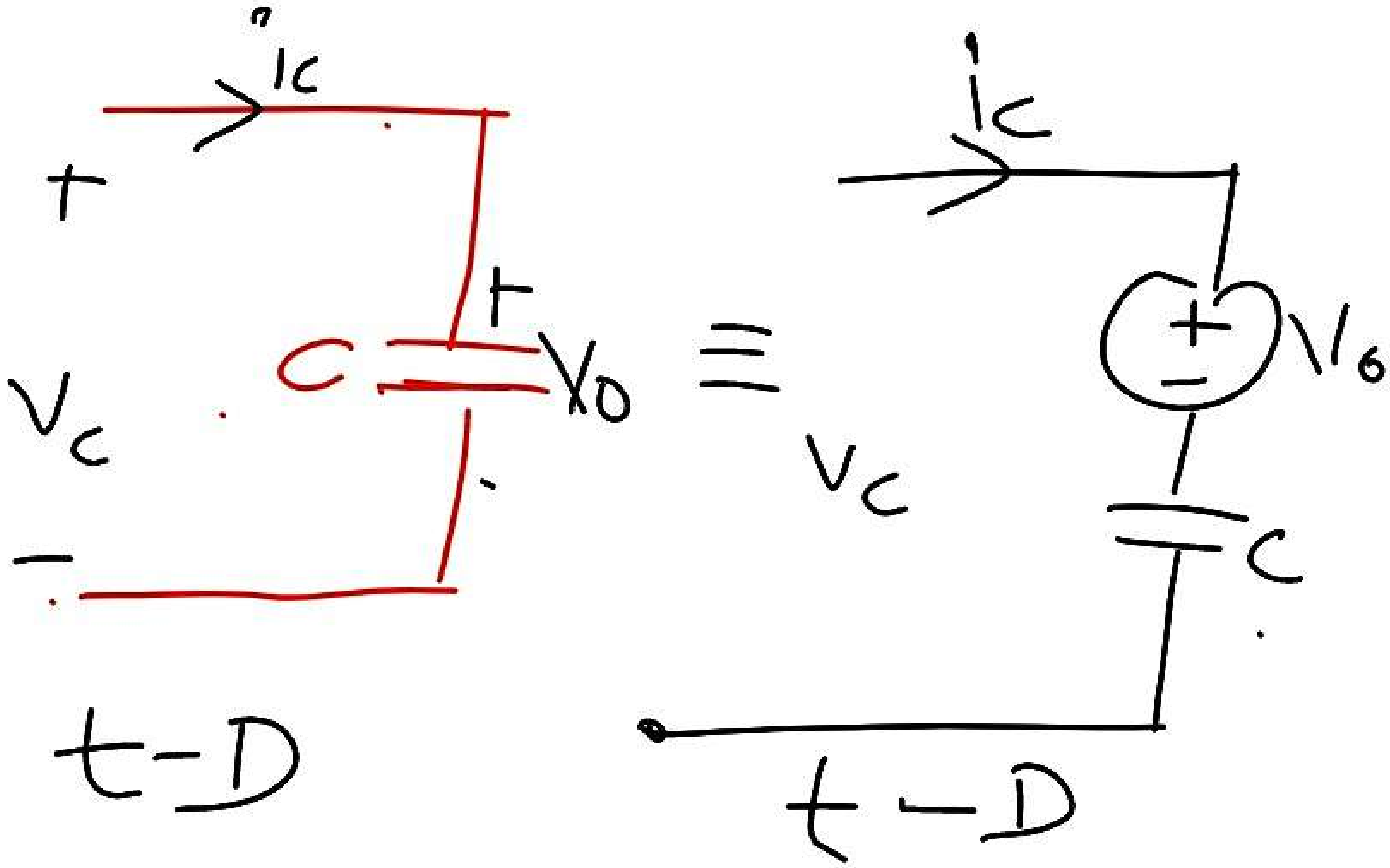
N/w. represents the equivalent of Inductor in Laplace domain Determine initial current through it.

Since $LI_0 = 6$ volt.

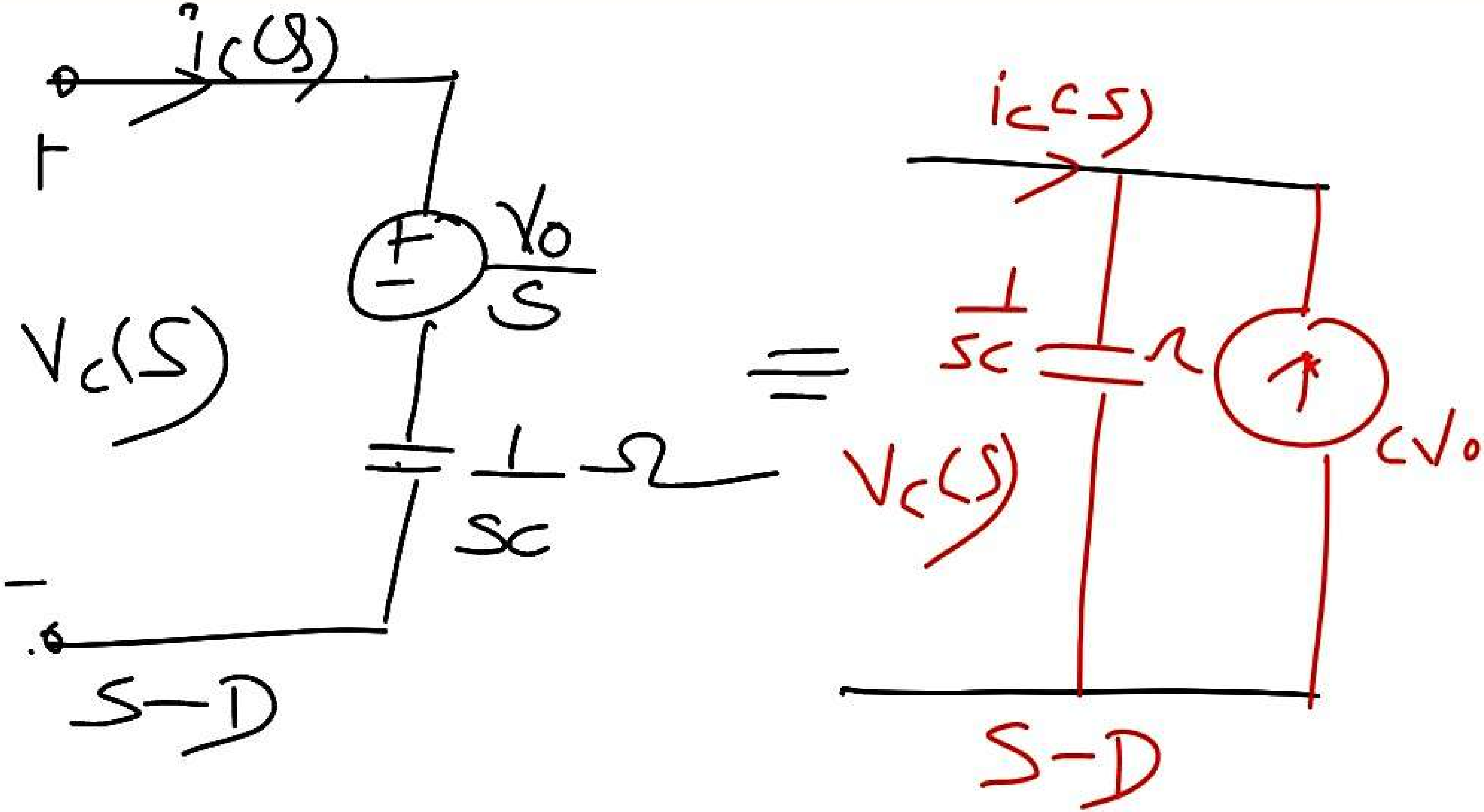
$L = \frac{1}{2} H$

$$\boxed{I_0 = 12 \text{ Amp.}}$$

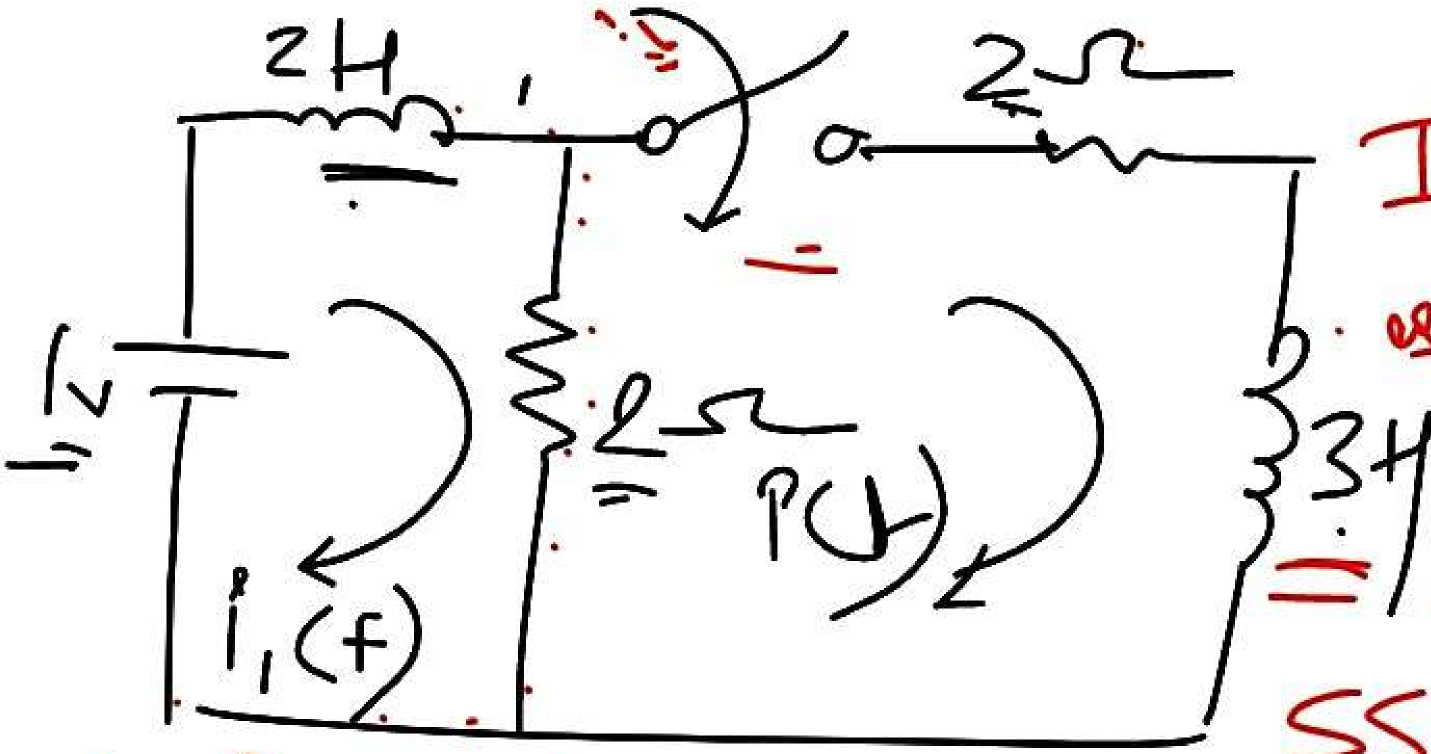
$I_0 = \frac{6}{L} = \frac{6}{1/2} = 12$ Amp.



i_C
+
v_C
−
C
+
V_0
≡
i_C
V_0
v_C
C
t−D
t−D

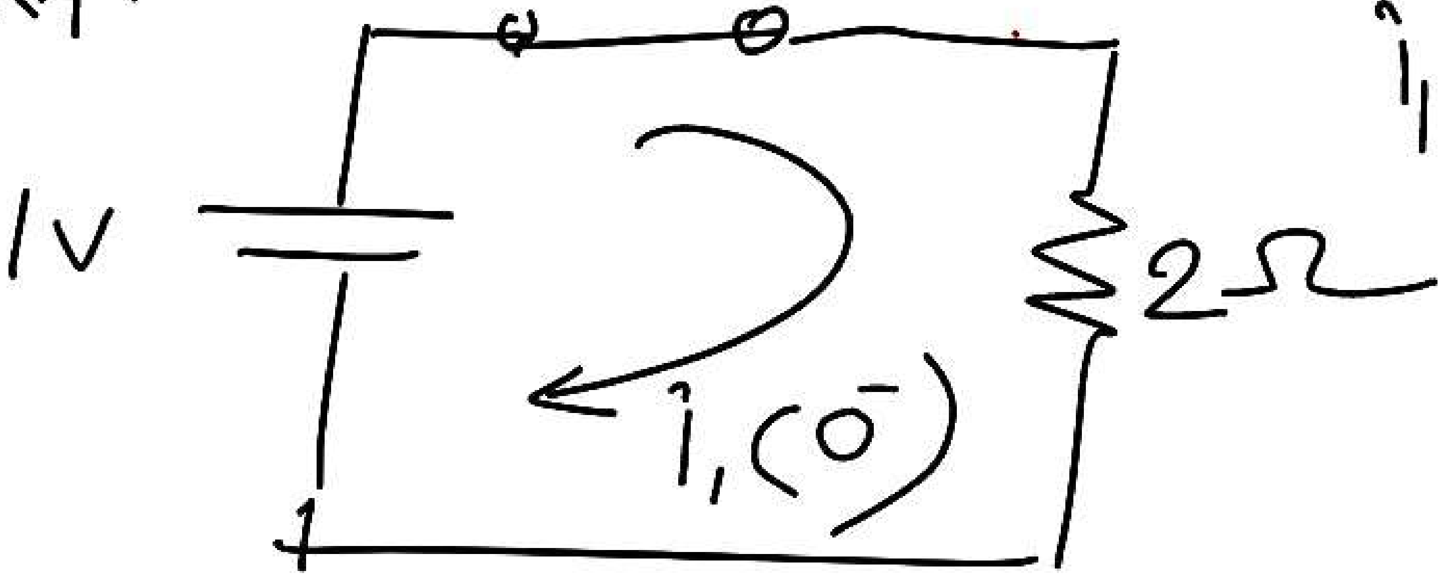


$i_c(s)$
+
$V_c(S)$
−
$\frac{V_0}{S}$
$\frac{1}{sc}\ \Omega$
S−D
≡
$i_c(s)$
$\frac{1}{sc}\ \Omega$
$V_c(S)$
cV_0
S−D



In the n/w switch is closed at $t=0$ the SS being reached before $\underline{t=0}$ Determine current through inductor 3H

At $t=0^-$, ss condition is reached Hence, inductor of 2H will act.



$i_1(0^-) = \frac{1}{2}$ Amp.

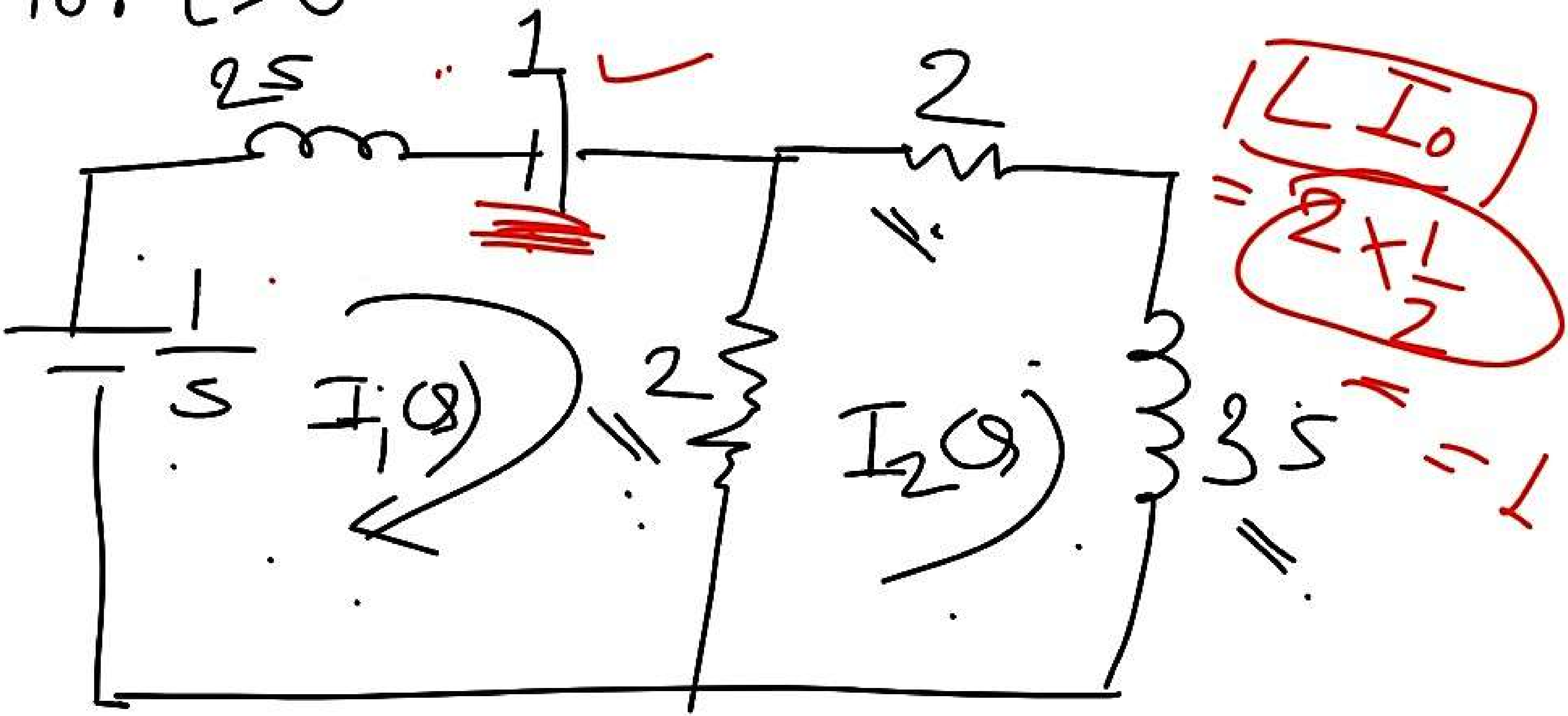
$i_2(0^-) = 0$

Since the current through the inductor cannot change instantaneously.

$$i_1(0^+) = \tfrac{1}{2}\ \text{Amp}$$

$$i_2(0^+) = 0\ \text{Amp.}$$

for $t>0$



$$\frac{1}{s} - 2sI_1(s) + 1 - 2\left(I_1(s) - I_2(s)\right) = 0$$

$$\Rightarrow \left(2 + \underline{\underline{2s}}\right)I_1(s) - 2I_2(s) = 1 + \underline{\underline{\frac{1}{s}}}$$

$$1$$

Apply KVL to mesh 2

$$-2\left(I_2(s)-I_1(s)\right)-2I_2(s)-3sI_2(s)=0$$

$$\Rightarrow -2I_1(s)+(4+3s)I_2(s)=0 \quad (2)$$

Cramer's rule

$$ax+by=e$$
$$cx+dy=f$$

$$x=\frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \qquad y=\frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

By Cramer's rule:

$$I_2(s) = \frac{\begin{vmatrix} 2+2s & 1+1/s \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} 2+2s & -2 \\ -2 & 4+3s \end{vmatrix}} \Rightarrow$$

$$\frac{\frac{2}{s}(s+1)}{(2s+2)(4+3s)-4} \Rightarrow$$

$$\Rightarrow \frac{s+1}{s(2s^2+7s+2)} = \frac{s+1}{3s\left(s+\frac{1}{3}\right)(s+2)}$$

$$= \frac{\frac{1}{3}(s+1)}{s(s+2)\left(s+\frac{1}{3}\right)}$$

By partial fraction.

$$I_2(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+\frac{1}{3}}$$

$$A = \left.\frac{\frac{1}{3}(s+1)}{(s+2)\left(s+\frac{1}{3}\right)}\right|_{s=0}$$

$$A = \frac{1}{2}$$

$$B = \left. \frac{\frac{1}{3}(s+1)}{s\left(s+\frac{1}{3}\right)} \right|_{s=-2}$$

$$B = -\frac{1}{10}$$

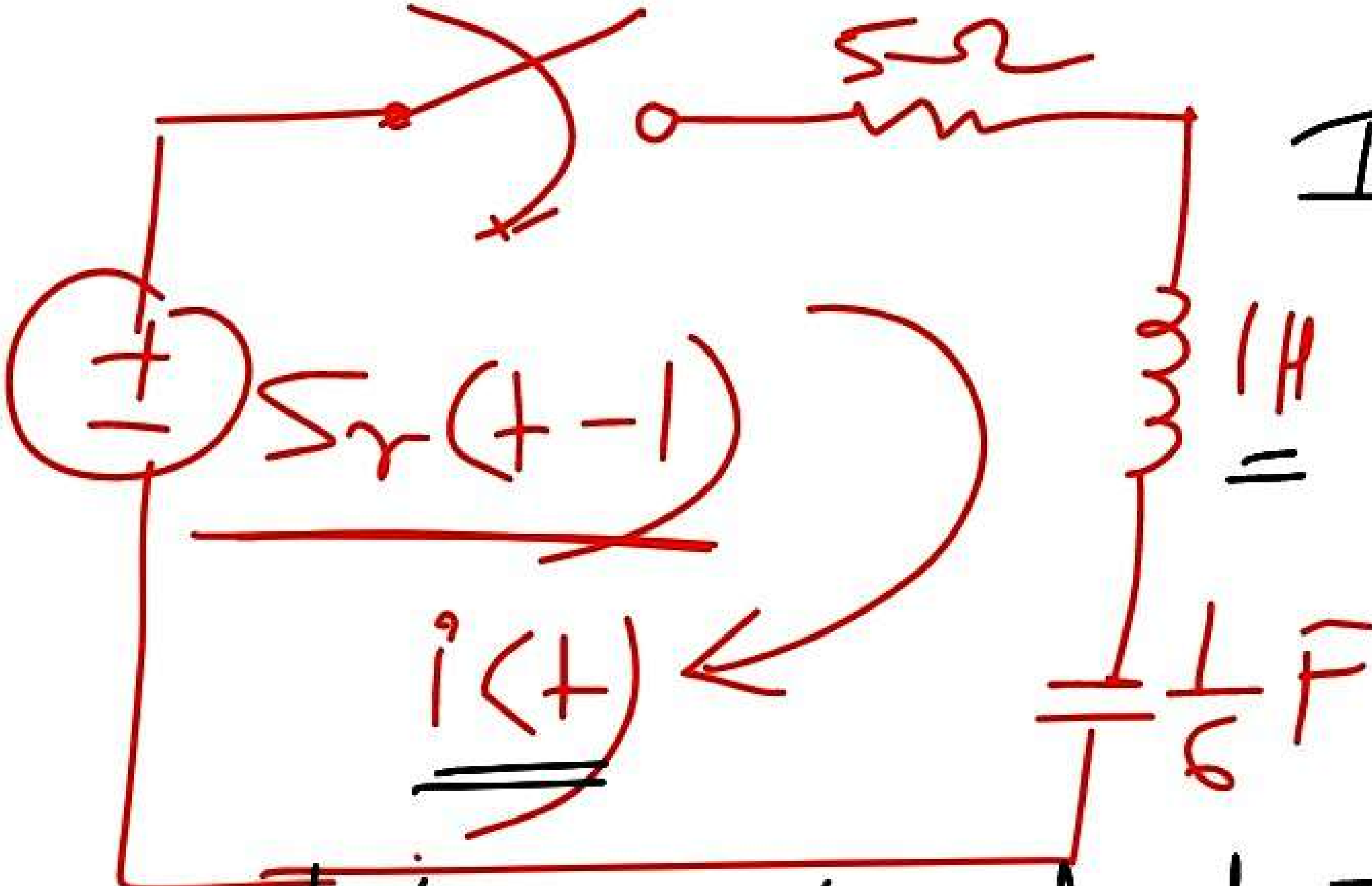
$$c = \left.\frac{\frac{1}{3}(s+1)}{s\cdot(s+2)}\right|_{s=-\frac{1}{3}} = -\frac{2}{5}$$

$$I_2(s) = \frac{1}{2s} - \frac{1}{10(s+2)} - \frac{2}{5\left(s+\frac{1}{3}\right)}$$

Taking Inverse Laplace T.

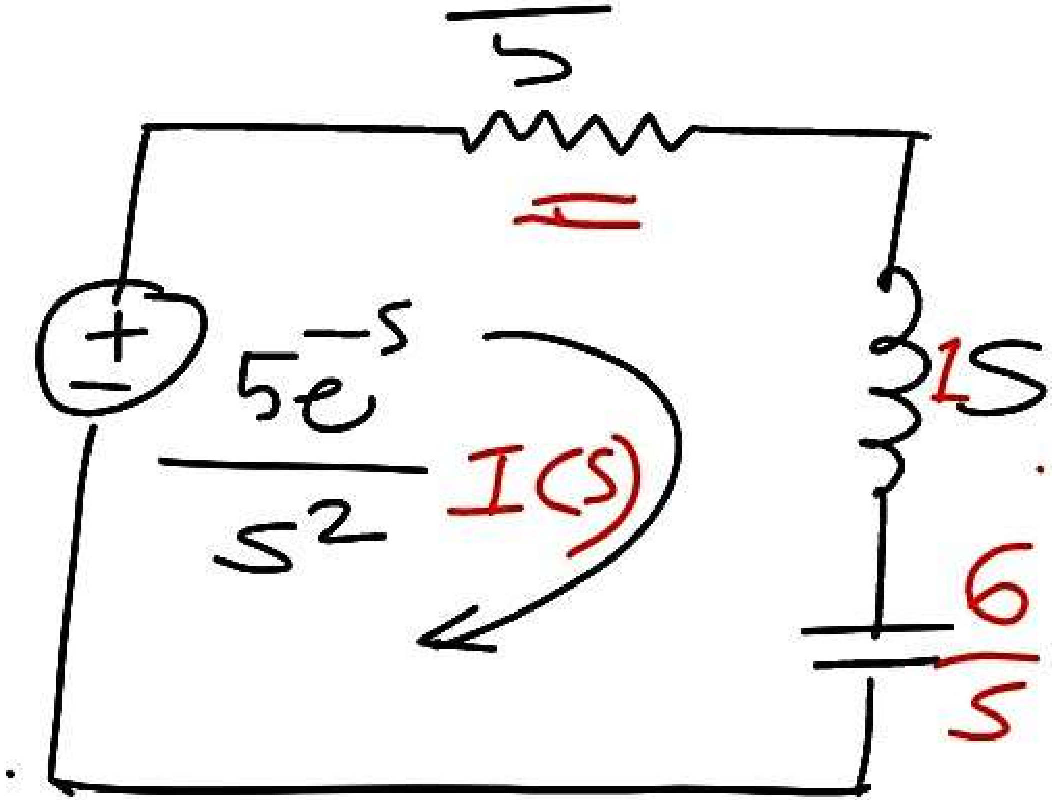
$$\boxed{i_2(t) = \frac{1}{2} - \frac{1}{10}e^{-2t} - \frac{2}{5}e^{-\left(\frac{1}{3}\right)t}}$$

for $t>0$



In N/ω. Determine the current $i(t)$ when the switch is closed at $t = 0$ with 0 initial condition.

Transformed N/w.



$$5r(t-1) \xleftrightarrow{LT} \frac{5e^{-s}}{s^2}$$

$$r(t) \xleftrightarrow{LT} \frac{1}{s^2}$$

-Apply KVL to mesh for $t>0$

$$\frac{5e^{-s}}{s^2} = 5I(s) + sI(s) + \frac{6}{s}I(s)$$

$$I(s) = \frac{5e^{-s}}{s(s^2+5s+6)} = \frac{5e^{-s}}{s(s+3)(s+2)}$$

By partial fraction expansion

$$\frac{1}{s(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$$

By solving

$$A = \left.\frac{1}{(s+3)(s+2)}\right|_{s=0} = \frac{1}{6}$$

$$B = \left.\frac{1}{s(s+2)}\right|_{s=-3} = \frac{1}{3}$$

$$C = \left.\frac{1}{s(s+3)}\right)_{s=-2} = \frac{-1}{2}$$

$$I(s) = 5e^{-s}\left[\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}\right]$$

$$\frac{5e^{-s}}{6s} + \frac{5e^{-s}}{3(s+3)} - \frac{5e^{-s}}{2(s+2)}$$

Taking inverse LT

$$i(t) = \frac{5}{6}u(t-1) + \frac{5}{3}e^{-3(t-1)}u(t-1) - \frac{5}{2}e^{-2(t-1)}u(t-1) \quad \text{for } t>0$$

$u(t) = \frac{1}{s}$ $\delta(t) = 1$

$r(t) = 1/s^2$ $p(t) = 1/s^3$

$s \xrightarrow{ILT} t$

Phasor Representation:

⇒ A phasor is a complex no. that represents amplitude & phase of sinusoid. It is defined only for the cosinusoidal signals.

⇒ All sinusoidal signals are converted into cosinusoids by subtracting 90° from phase

⇒ A Phasor may be expressed in exponential form, polar form, rectangular form.

⇒ phasor has magnitude & direction. it behaves as vector

$$\underline{\underline{V(t)}} = V_m \cos(\omega t + \phi)$$

$$= \text{Real part}\left[V_m e^{j(\omega t + \phi)}\right]$$

$$= \text{Real part}\left[V_m \cdot e^{j\omega t} \cdot e^{j\phi}\right]$$

$$= \text{Real part}\left[\dot{V} e^{j\omega t}\right]$$

$$\underline{\underline{V = V_m e^{j\phi}}} \Rightarrow \text{Exponential form of phasor representation}$$

$$= \underline{\underline{V_m \angle \phi}} \rightarrow \text{Polar form of the phasor representation.}$$

$V_m(\cos\phi + j\sin\phi) \Rightarrow$

Rectangular form of phasor representation.

Note :- The add. or sub. of two phasors are generally carried out only in rectangular form. when as division & multiplication of phasors are carried out either in polar or exponential form.

$$i(t) = I_m \cos(\omega t + \theta)$$

$$I = I_m \angle \theta \Rightarrow \text{polar}$$

$$= I_m e^{j\theta} \Rightarrow \text{expo.}$$

$$= I_m(\cos\theta + j\sin\theta) \Rightarrow \text{Rect.}$$

$$v(t) = \overset{V_m}{10}\cos(2t + \overset{\phi}{30^\circ}) \text{ volt.}$$

$$V = 10\angle 30^\circ \Rightarrow \text{polar.}$$

$$= 10\, e^{j30} \Rightarrow \text{expo.}$$

$$= 10(\cos 30^\circ + j\sin 30^\circ) \Rightarrow$$

$$= 10\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) \text{ Rect.} \rightarrow \text{Phasor R.}$$

$$i(t) = 10\sin(2t + 30^\circ) \text{ Amp.}$$

$$\Rightarrow 10\cos(2t + 30^\circ - 90^\circ) \text{ Amp.}$$

$$\Rightarrow 10\cos(2t - 60^\circ)$$

$$I = 10\angle -60^\circ \text{ polar.}$$

$$= 10\, e^{-j60} \text{ exp.}$$

$$10(\cos 60^\circ - j\sin 60^\circ)$$

$$= 10\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \rightarrow \text{phasor representation}$$

$$v(t) = 10\sin(2t + 30^\circ) \text{ volt.}$$

$$= 10\cos(2t + 30^\circ - 90^\circ)$$

$$v = 10\cos(2t - 60^\circ)$$

$$v = 10\angle -60^\circ \text{ — polar}$$

$$= 10 e^{-j60^\circ} \Rightarrow \text{expo.}$$

$$v = 10(\cos 60^\circ - j\sin 60^\circ) \rightarrow \text{Rect.}$$

$$= 10\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \Rightarrow \text{phasor representation}$$

$$\boxed{\frac{V}{I} = Z \ \Omega} \rightleftharpoons \boxed{\frac{I}{V} = Y} \ \mho$$

$V = ZI$ $\quad \Omega$

$I = VY$ $\quad \mho$

$I = \frac{V}{Z}$

ohm

mho

$V = IR$

$$\boxed{R = \frac{V}{I}}$$

$$\boxed{Y = \frac{1}{Z}} \ \mho$$

$$\boxed{V = \frac{I}{Y}}$$

$R \neq Z$

$\boxed{LC}$

$$t\text{-}D \longleftrightarrow P \cdot D$$

$$i(t) \longleftrightarrow I$$

$$v(t) \longleftrightarrow V$$

$$R \longleftrightarrow R\,\Omega$$

$$L \longleftrightarrow j\omega L\,\Omega \quad (\text{where } j\omega = s)$$

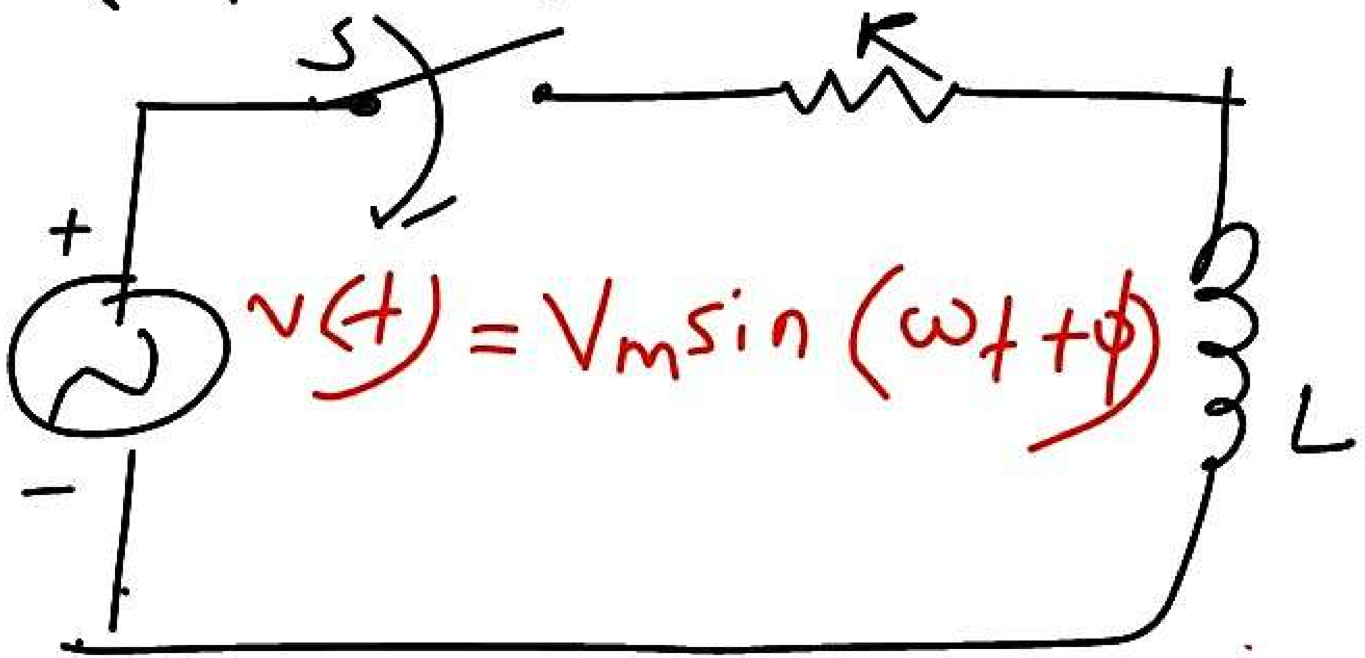
$$= \frac{1}{j\omega C}\,\Omega$$

Note:-

- Analysis of AC in steady state is generally carried out in phasor domain i.e. KCL, KVL, Ohm's law, Nodal, Mesh, ST, are written only in phasor domain.

Ac Transients :-

① Series RL ckt.



$v(t) = V_m \sin(\omega t + \phi)$

Complex situation

= Complex function + perticular Integration.

$$CS = CF + PI$$

= Natural Respons. + Forced Response

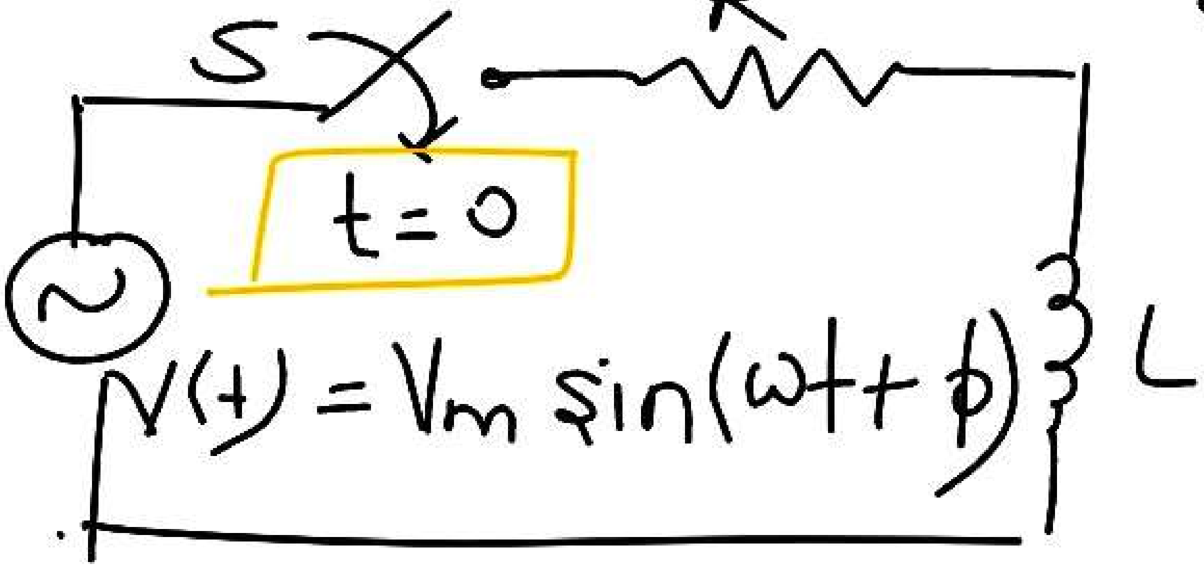
Phasors

→ phasor is a complex number that represent amplitude & phase of sinusoid.

$$\boxed{V(t) = V_m \cos(\omega t + \phi)}$$

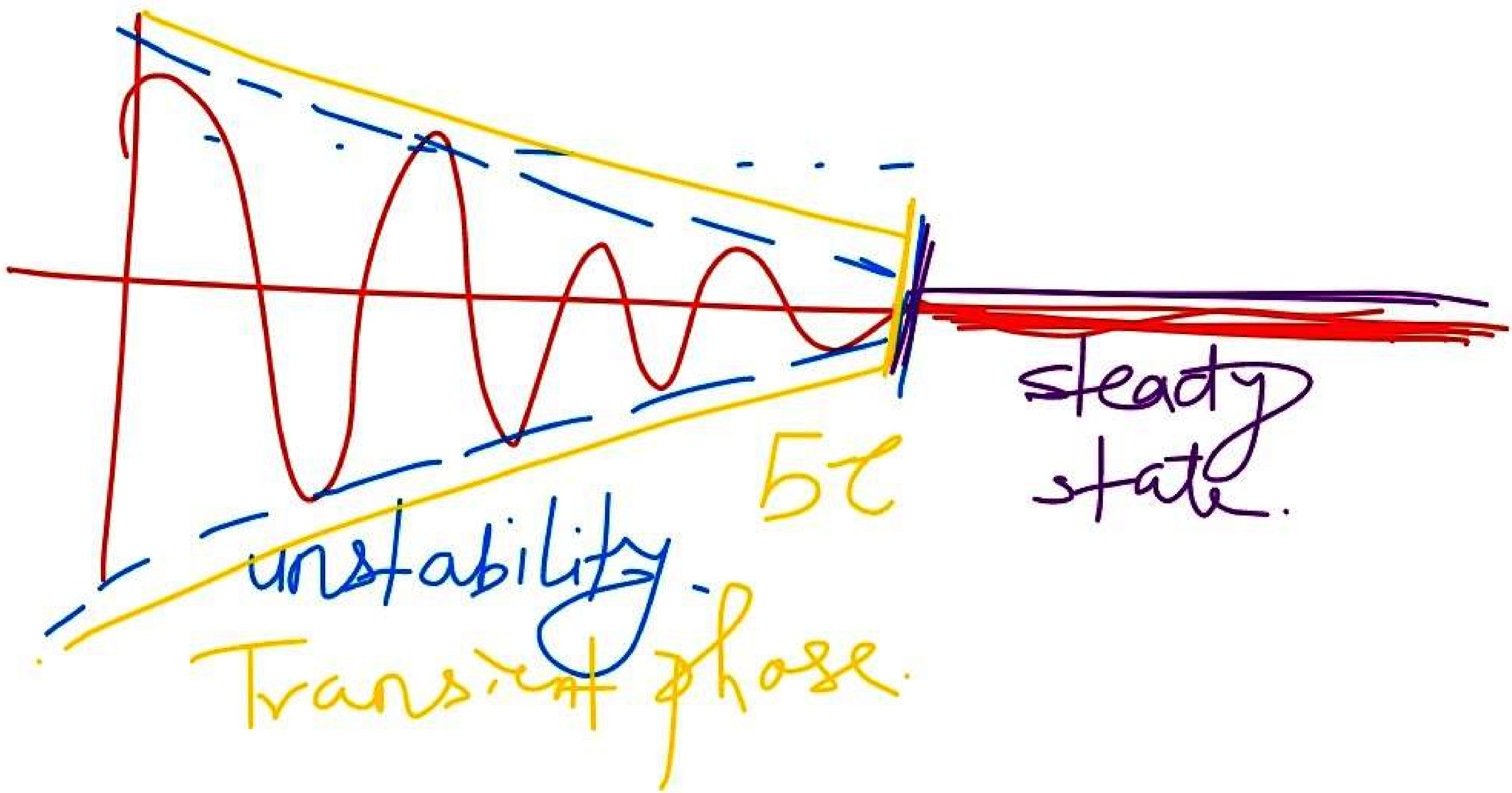
AC transients :-

1) Series RL circuit :-



$$v(t) = V_m \sin(\omega t + \phi)$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$



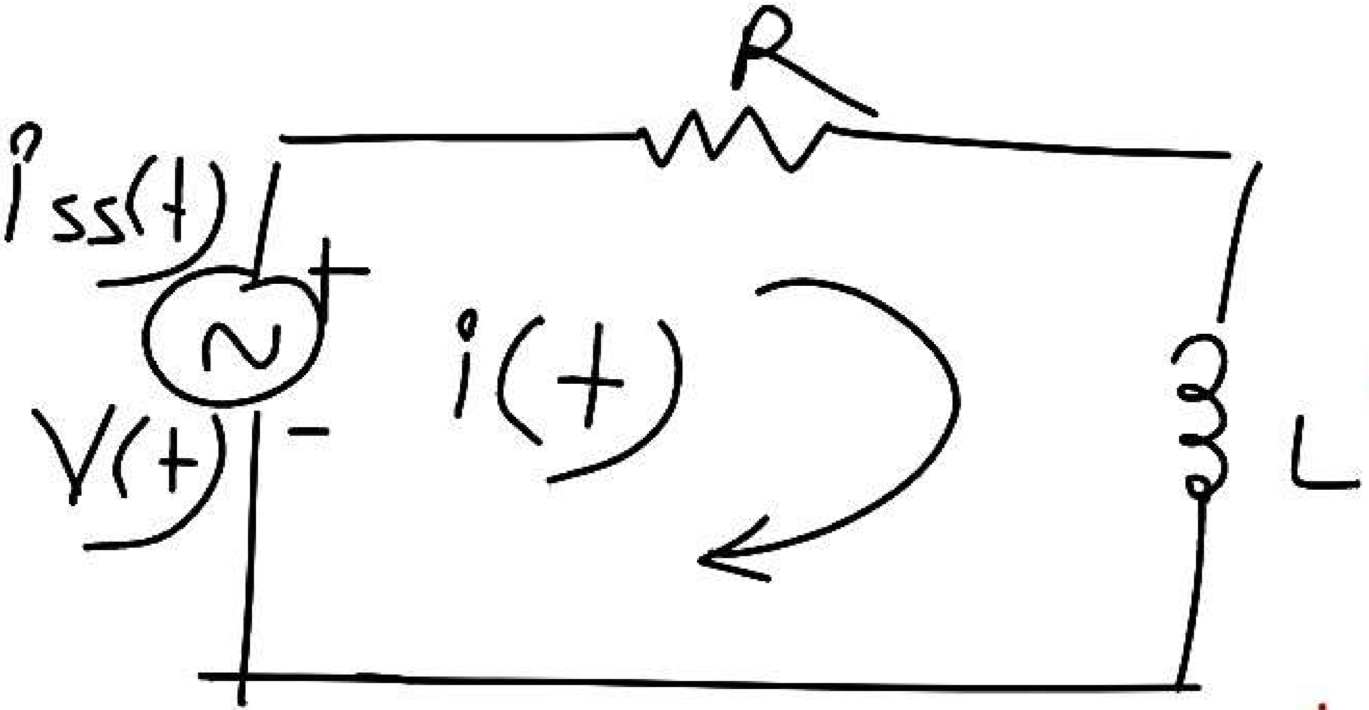
5τ
steady state.
unstability
Transient phase.

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$i_{tr}(t) = Ke^{-t/\tau}$$

$$= Ke^{-\frac{R}{L}t}$$

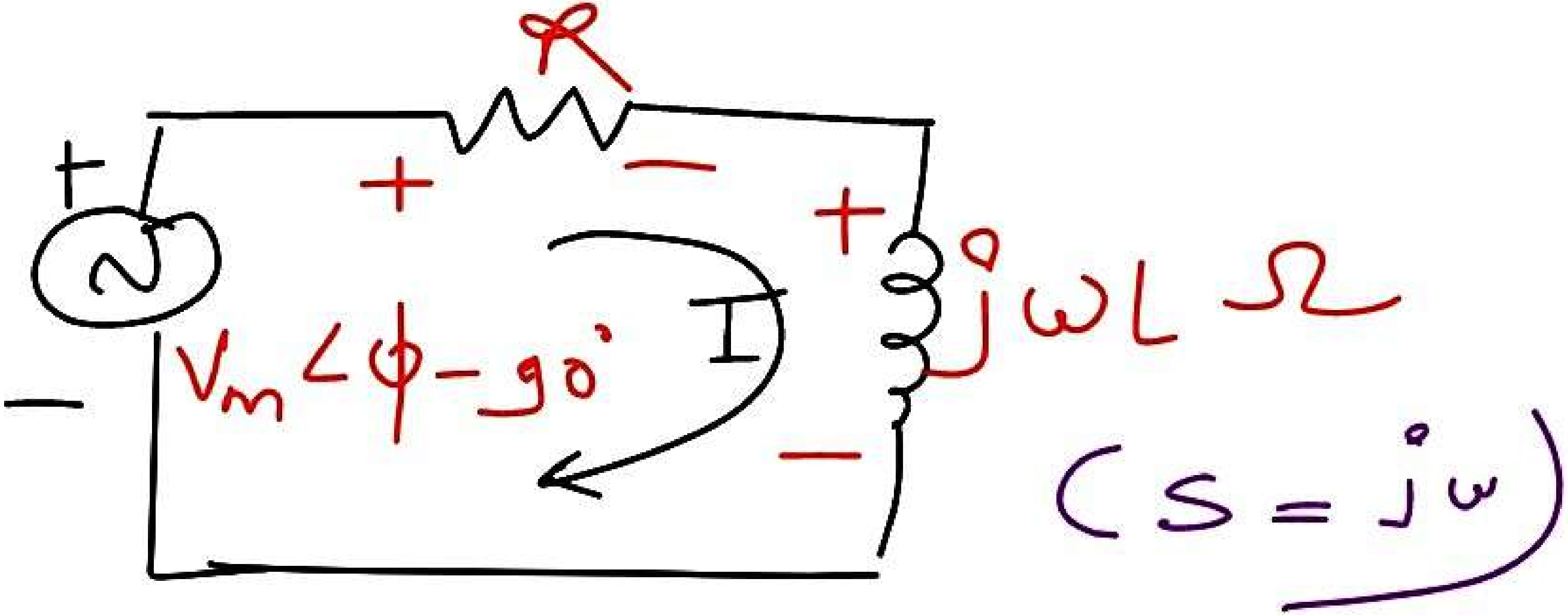
$$\boxed{i_{ss}(t) = ?}$$



N/w in steady state

$$V(t) = V_m \sin(\omega t + \phi)$$

By phasor Approch



By KVL in P→D

$$V_m \angle \phi - 90^\circ - RI - j\omega L I = 0$$

$$V_m \angle \phi - 90^\circ = RI + j\omega L I = 0$$

$$= I(R + j\omega L)$$

$$\therefore \; I = \frac{V_m \angle \phi - 90^\circ}{R + j\omega L}$$

$$I = \frac{V_m \angle \phi - 90^\circ}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$

$$I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ$$

$$\alpha \angle \beta \Rightarrow \alpha e^{j\beta}$$

$$i(t) = \text{Real part}\left[I e^{j\omega t}\right] \text{ Amp.}$$

$$= \alpha \cos(\omega t + \beta)$$

$$= \alpha \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ\right)$$

$$= \frac{V_m}{\sqrt{R^2 + \omega L^2}} \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ\right)$$

$$\frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\omega t+\phi-\tan^{-1}\frac{\omega L}{R}\right)$$

Amp.

$$= i_{ss}(t)$$

$$t\text{-}D \longleftrightarrow P\text{-}D$$

$$i(t) \longleftrightarrow I$$

$$v(t) \longleftrightarrow V$$

$$R \longleftrightarrow R$$

$$L \longleftrightarrow j\omega L$$

$$C \longleftrightarrow \frac{1}{j\omega C}$$

Method (2) By LTA

$$v(t) = V_m \sin(\omega t + \phi)$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{R + sL}$$

$s \rightarrow j\omega$

$$H(j\omega) = \frac{1}{R + j\omega L}$$

$$\frac{1}{\sqrt{R^2+(\omega L)^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$

$$I(s) = H(s) \cdot V(s)$$

$$I_{ss}(t)$$

$$i(t) = \frac{1}{\sqrt{R^2+(\omega L)^2}} V_m \cdot \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$$I(t) = i_{tr}(t) + i_{ss}(t)$$

$$= \boxed{K e^{-\frac{R}{L}t}}_{t=0} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\omega \underbrace{t}_{t=0} + \phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i(0^-) = i(0^+) \Rightarrow \underline{0A}.$$

$t=0$ $\quad t=0^-$ $\quad t=0^+$ $\quad t=0$

$$0 = Ke^{-0} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(0+\phi-\tan^{-1}\frac{\omega L}{R}\right)$$

$$\therefore K = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\underbrace{\phi-\tan^{-1}\frac{\omega L}{R}}_{0}\right)$$

As. $K < 0 \Rightarrow K << 1$,

Transient effects are less severe for A.C

Suppose $\phi - \tan^{-1}\frac{\omega L}{R} = 0$

$K = 0$

$i_{tr}(t) = 0 \Rightarrow i(t) = i_{ss}(t)$

i.e. A transient free response

So, condition for transient free response at $t = 0$

$$\phi - \tan^{-1}\frac{\omega L}{R} = 0$$

$$\therefore \boxed{\phi = \tan^{-1}\frac{\omega L}{R}} \Rightarrow$$

$(\omega t + \phi)$

$t = 0$

$$= \frac{\tan^{-1}\omega L}{L}$$

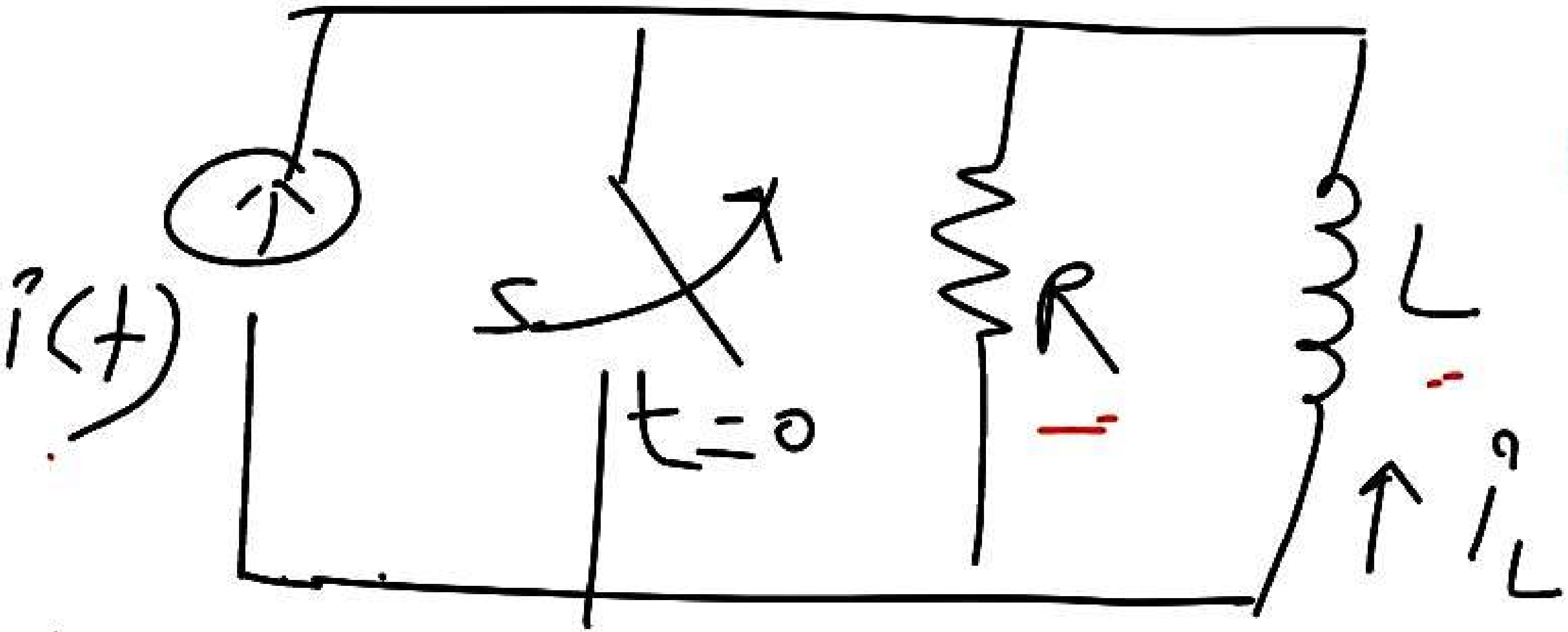
Note:. If the total phase of excitation at the time of switching is equal to $\tan^{-1}\left(\frac{\omega L}{R}\right)$ then no transient will result at the time of switching for sinusoidal excitation.

⇒ If the switch is closed at $t = t_0$ then condition for transient free response at $t = t_0$ ⇒

$$\boxed{\omega t + \phi = \tan^{-1}\frac{\omega L}{R}}$$

AC Transient :-

Parallel RL Circuit :-



$L \rightarrow C$

$\tau = L/R$

$\tau = RC$

$$i(t) = I_m \sin(\omega t + \phi)$$

$i_{Lss}(t)$ by LTA

$$\frac{I_L(s)}{I(s)} = H(s) = \left(\frac{\text{Desired LT}}{\text{Excitation LT}}\right)$$

$$I_L(s) = \frac{I(s) \cdot R}{R + sL}$$

(By current Division in S Domain)

$$\frac{I_L(s)}{I(s)} = \frac{R}{R+sL}$$

T.F. $H(s)$

$$H(s) = \frac{R}{R+sL}$$

$$H(j\omega) = \frac{R}{R+j\omega L} \Rightarrow \frac{1}{1+\frac{j\omega L}{R}}$$

$$\Rightarrow \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i_L(t) = \underbrace{i_{L\,tr}(t)} + i_{L\,ss}(t)$$

$$= \underline{K e^{-R/L\,t}} + \frac{I_m}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$$\boxed{t=0}$$

$$i_L(0^-) = i_L(0^+) = 0 \qquad t=0$$

$$K = \frac{-I_m}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \sin\left(\phi - \underline{\tan^{-1}\frac{\omega L}{R}}\right)$$

$$\Leftarrow \quad \ll 1$$

Suppose $\underbrace{\phi - \tan^{-1}\frac{\omega L}{R}}$ $= 0$

$\boxed{K = 0}$ $\Rightarrow i_{tr}(t) = 0$

$i_L(t) = i_{Lss}(t)$

$\Rightarrow$ Transient free ~~res~~ response

$$\boxed{\phi - \tan^{-1}\left(\frac{\omega L}{R}\right) = 0}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \quad \text{at } t = 0$$

$$\boxed{\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R}} \quad \text{at } t = t_0$$

If $i(t) = I_m \cos(\omega t + \phi)$ then

$\phi = \tan^{-1} \frac{\omega L}{R} + \frac{\pi}{2}$ at $t = 0$

$\omega t_0 + \phi = \tan^{-1} \frac{\omega L}{R} + \pi/2$

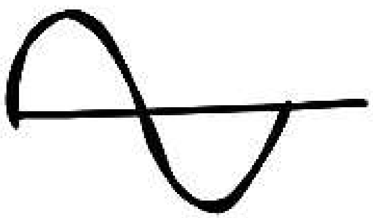
at $t = t_0$

Note :- Transient free condition is not possible for network with both the energy Storing elements ie RLC Network.

$i(t) = i_{tr}(t) + i_{ss}(t)$ $\qquad i_L(0^+)$

$s_1, s_2 = \alpha \pm j\beta$ $\qquad v_L(0^+)$

K_1 & $K_2 \Rightarrow$



$$e^{\alpha t}\left(K_1 \cos\beta t + K_2 \sin\beta t\right) + i_{ss}(t)$$

Obs → Transient free (+ve value) in RL & RC ckt with AC excitation will depend on the source freq (ω) initial phase ϕ, ckt constant (R, L, C) & on nature of excitation (sine or cosine)

But Not on the maximum value of the excitation.
(V_m or I_m)

1) $L(K) = \frac{K}{s}$

2) $L(e^{at}) = \frac{1}{s-a}$

3) $L(e^{-at}) = \frac{1}{s+a}$

4) $L(\sin at) = \frac{a}{s^2+a^2}$

5) $L(\cos at) = \dfrac{s}{s^2+a^2}$ $\quad s>0$

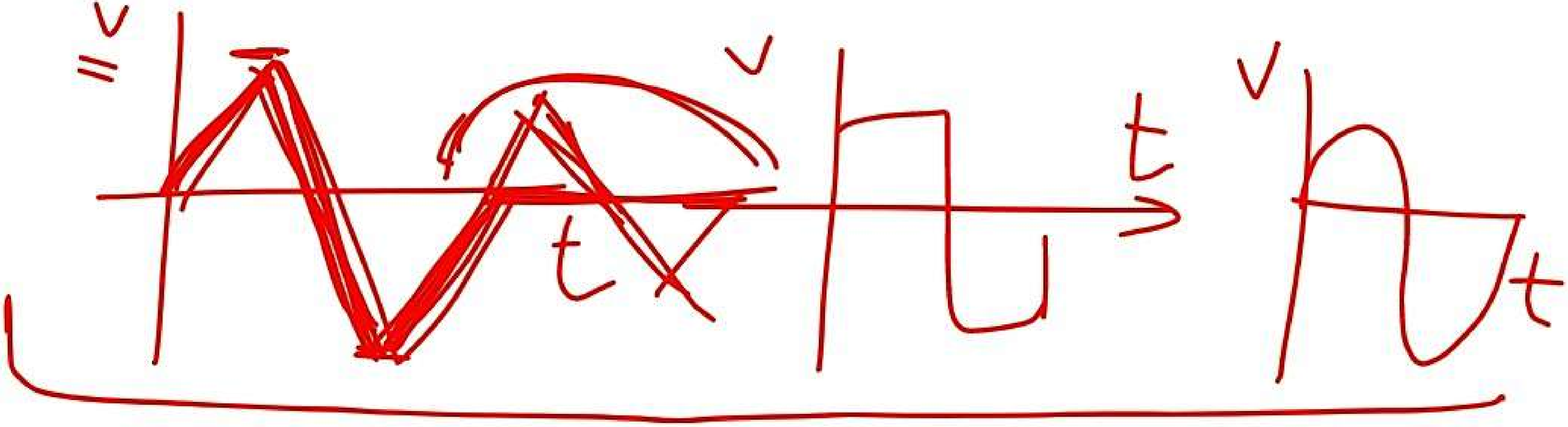
6) $L(\sinh at) = \dfrac{a}{s^2-a^2}$ $\quad s>|a|$

7) $L(\cosh at) = \dfrac{s}{s^2-a^2}$ $\quad s>|a|$

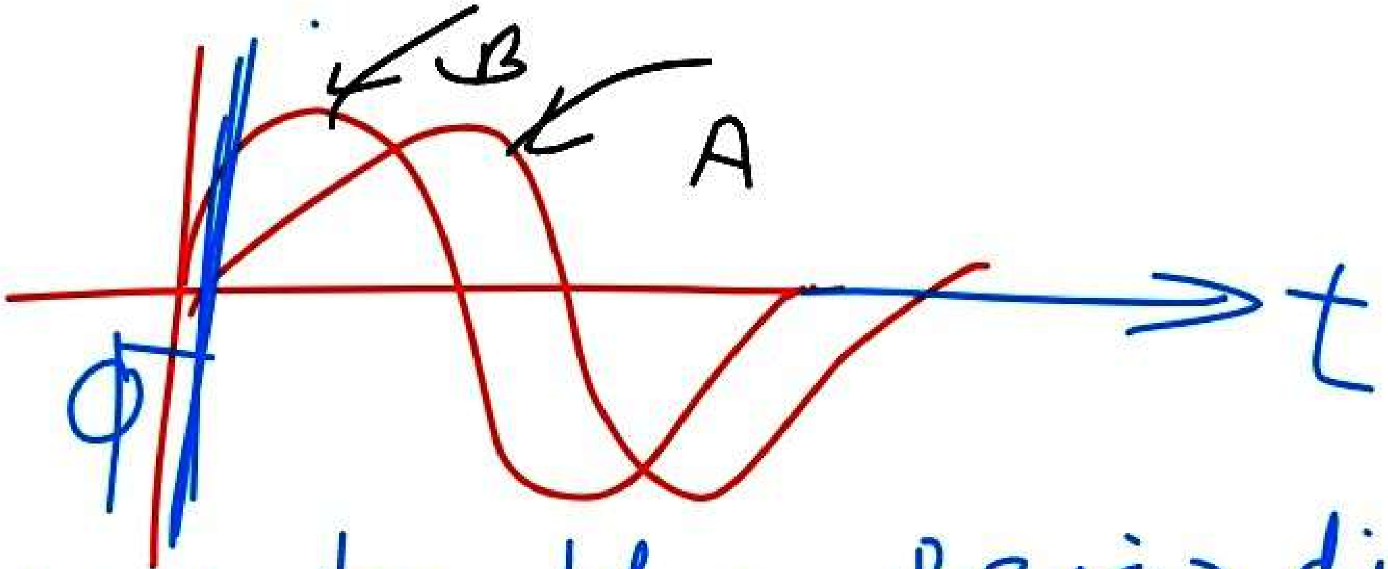
(8) $L(t^n) = \dfrac{\Gamma(n+1)}{s^{n+1}} =$ $\qquad n > -1$

$$\boxed{\frac{n!}{s^{n+1}} \text{ when } n \in \mathbb{N}}$$

AC circuit Analysis:-



$$v = V_m \sin\theta$$
$$= V_m \sin \omega t$$
$$= V_m \sin 2\pi f t$$
$$= V_m \sin \frac{2\pi}{T} t$$



$x(t)$ be the periodic signal of period T

$$x(t \pm T) = x(t)$$

Avg value = DC value

$$x_{avg} = x_{DC} = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{\text{Area of signal over 1 period}}{\text{period}}.$$

Transient Circuit Analysis

Laplace Transform Application for solving steady state AC circuits

$f(t)$	$F(s)$	$F(t)$	$F(s)$
$i(t)$	$I(s)$	$\delta(t)$	1
$v(t)$	$V(s)$	$u(t)$	$1/s$
R	R	$r(t)$	$1/s^2$
L	sL	$p(t)$	$1/s^3$
C	$\frac{1}{sC}\,\Omega$	$e^{-at}u(t)$	$1/s+a$
		$Ku(t)$	K/s.

$$\frac{V(s)}{I(s)} = Z(s)\ \Omega$$

$$\Rightarrow V(s) = Z(s) \cdot I(s) \Rightarrow I(s) = \frac{V(s)}{Z(s)}$$

$$\frac{I(s)}{V(s)} = Y(s)\ \mho \Rightarrow V(s) = \frac{I(s)}{Y(s)}$$

$$\Rightarrow I(s) = Y(s) \cdot V(s)$$

$$Y(s) = \frac{1}{Z(s)}\ \mho$$

$Z(s)$	$Y(s)$
$R\ \Omega$	$1/R\ \mho$
$sL\ \Omega$	$1/sL\ \mho$
$1/sC\ \Omega$	$sC\ \mho$

We can add impedances in series and admittances in parallel.

Ex. 1 Find out the laplace transform of the shifted gate pulse shown below.

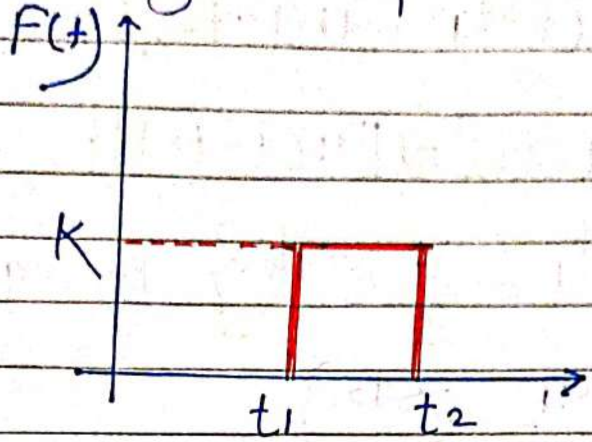


fig. A shifted gate function.

→ This shifted gate function can be synthesized with the help of two step functions as follows.

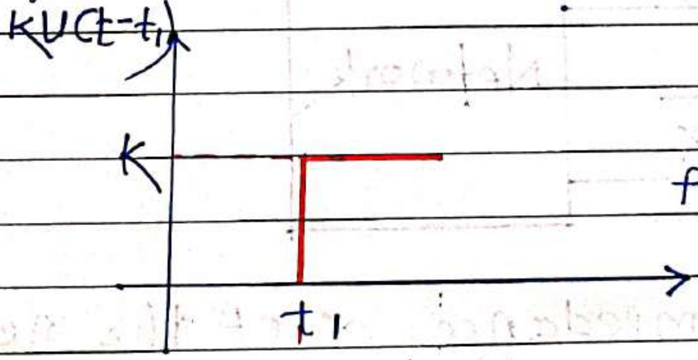


fig 1 A step function starting at t_1

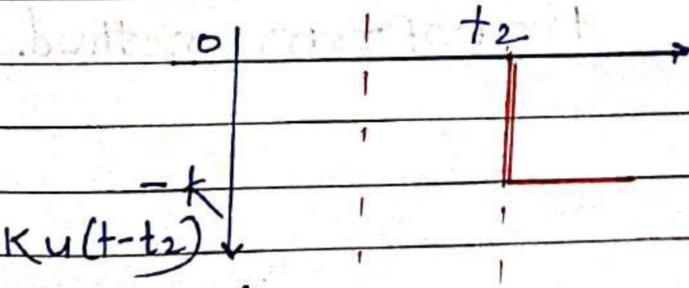


Fig 2 A step function starting at t_2

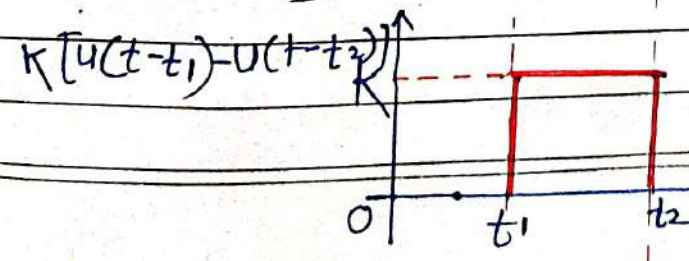


Fig. A gate pulse obtained by subtracting signal of Fig 1 & 2

The above waveform show that the gate pulse can be represented as

$$f(t) = K[U(t-t_1) - U(t-t_2)]$$

$$\mathcal{L}[f(t)] = \mathcal{L}\{K[U(t-t_1) - U(t-t_2)]\}$$

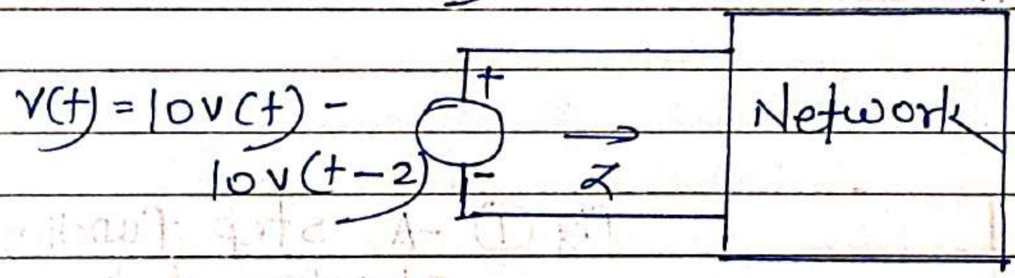
$$= K\{\mathcal{L}[U(t-t_1)] - \mathcal{L}[U(t-t_2)]\}$$

$$= K\left\{\frac{1}{s}e^{-st_1} - \frac{1}{s}e^{-st_2}\right\} \text{ By equation}$$

$$= \frac{K}{s}\left(e^{-st_1} - e^{-st_2}\right)$$

Ex. (2) Figure shows an impedances whose voltage and current are as shown.

$$i(t) = e^{-t} - e^{-(t-2)}$$



Find out the impedance Z of the network using Laplace transform method.

→ Let us first evaluate the laplace transform of v(t) & i(t)

$$v(t) = 10\,u(t) - 10\,u(t-2)$$

By using Laplace transform of above eqn we get.

$$V(s) = 10\frac{1}{s} - 10\frac{1}{s}e^{-2s}$$

$$= \frac{10}{s}(1-e^{-2s})$$

The current is given as

$$i(t) = e^{-t} - e^{-(t-2)}$$

Taking Laplace transform of above eqn,

$$I(s) = \frac{1}{s+1} - \frac{1}{s+1}e^{-2s} = \frac{1-e^{-2s}}{s+1}$$

The driving point impedance Z(s) is given as

$$Z(s) = \frac{V(s)}{I(s)}$$

$$= \frac{\frac{10}{s}(1-e^{-2s})}{\frac{1}{s+1}(1-e^{-2s})} = 10\,\frac{s+1}{s}$$

$$= 10\left(1+\frac{1}{s}\right)$$

$$= 10 + \frac{10}{s}$$

$$10 + \frac{1}{\frac{1}{10}s}$$

We know that Laplace transform of R i.e. the value is unchanged and laplace transform of capacitor is $\frac{1}{sC}$. Thus, the above impedance appears to be series connection of resistor and capacitor i.e.

$$Z(s) = R + \frac{1}{sC}$$

$R = 10\ \Omega$ and

$$C = \frac{1}{10} \text{ Farad}$$

∴ The circuit will be:

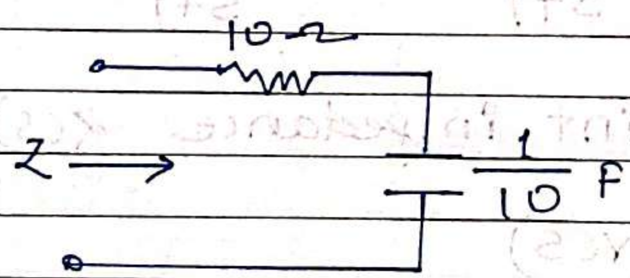
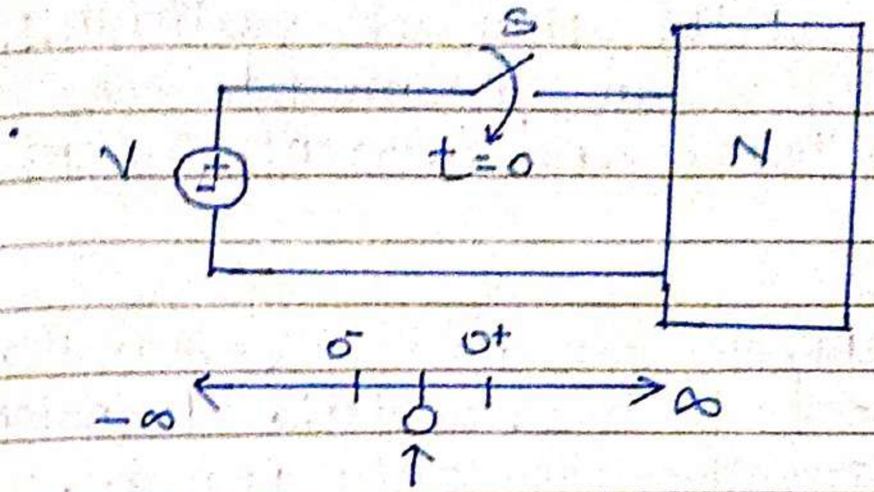


fig: Impedance of Network

Introduction to Transient Analysis :-



$0^- = -0.000 \ldots 01$

$0^+ = +0.000 \ldots 01$

$0^+ - 0^- = 0.000 \ldots 02$

$= 0.0$

i.e. With in zero time.

Introduction :-

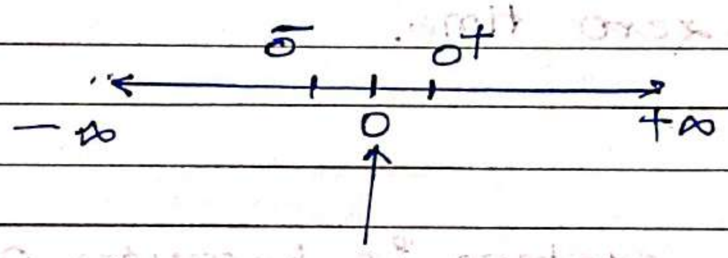
→ Transient in the system is because of the presence of energy storing elements called the inductor & the capacitor.

→ Since the energy stored in a memory element cannot change instantaneously i.e. within zero time. (due to integrations of energy variables i_L and V_c). L & C elements will oppose sudden changes in the system, which results in unstability of the system due to severe oscillations.

→ When the network consists of only resistances (a memoryless system), no transient occur in the system at the time of switching, since the resistor can accomodate any amount of voltages and currents ($v = Ri$ or $i = v/R$)

→ The transient effects are more severe for d.c. as compared to a.c. and the transient free condition is possible to only for a.c. excitations (at zero crossings of sine or cosine)

Behaviour of L and C elements at $t = 0^+$ and as $t \to \infty$ instants



$Z_L = sL \ \Omega$

$Z_C = \frac{1}{sC} \ \Omega$

$t = 0^+ \longrightarrow s = \infty \longrightarrow L \to O.C.$

$Z_C = 0 \longrightarrow C \longrightarrow SC$

$t \longrightarrow \infty \longrightarrow s \to 0 \Rightarrow Z_L \to 0 \quad L \to SC.$

$Z_C \to \infty \quad C \to O.C.$

- A long time after the switching action ($t \to \infty$) is the steady state. In steady state, the inductor behaviour is a short circuit behaviour and the capacitor behaviour is an open circuit behaviour.

Steady state OR Drunked state :-

Whenever an independant DC source is connected to the network for a long time. (ie. ideally infinite amount of time and practically upto 5 time constants), then the network is said to be in steady state. In steady state, then energy stored in the network is maximum and constant. So, energy stored in the memory elements is maximum and constant.

ie. $\frac{1}{2} L i_L^2$ = Maximum & constant.

$\Rightarrow i_L$ = Maximum & constant

So, inductor will act as a constant current source in steady state

since $V_L = L \cdot \frac{di_L}{dt} \Rightarrow V_L \to 0 \Rightarrow L \to SC.$

Similarly $\frac{1}{2} C V_C^2$ = Maximum & constant

$\Rightarrow V_C$ = Maximum & constant

So, capacitor will acts as a constant voltage source in steady state.

Since $i_c = C\frac{d}{dt}V_c \Rightarrow i_c \to 0 \Rightarrow C \to OC$

Observation :-

The above discussion is valid only for dc excitations / sources.

What about AC excitations / sources ?

→ for AC, during the positive slop of the period, both L & C elements are charging [i.e. storing the energy] and during the negative slope of the period, they are discharging [i.e. delivering the energy] So, the net energy observed over the period is zero. And hence, L & C elements are always present for a.c. excitations / sources.

Blind Approach :-

→ In steady state, $Z_L = j\omega L \ \Omega$ &

$$Z_C = \frac{1}{j\omega C} \ \Omega$$

→ for AC, $\omega \neq 0$ and hence $Z_L = j\omega L \ (\neq 0)$ and $Z_C = \frac{1}{j\omega C} \ \Omega \ (\neq \infty)$ i.e. The impedances are finite and hence, L and C elements are always present for a.c excitations/sources.

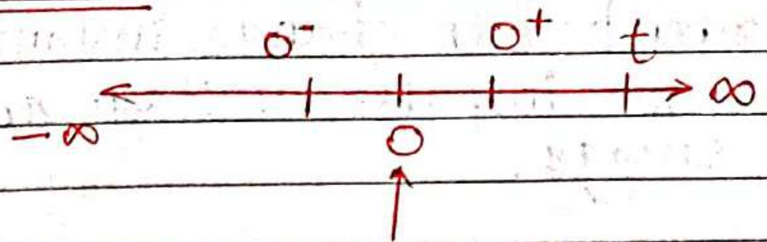
→ for dc. $\omega = 0$ and hence

$X_L = 0\,\Omega \Rightarrow L \rightarrow$ S.C.

$X_C = \infty\,\Omega \Rightarrow C \rightarrow$ O.C.

i.e. The L & C elements will dies for d.c. excitations/sources in steady state.

Inductor current at $t = 0^-$ and at $t = 0^+$ instants



$$L: \; i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$$

$$= \frac{1}{L}\int_{-\infty}^{0^-} v_L(t)\,dt + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt$$

$$= i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v_L(t)\,dt.$$

At $t = 0^+ \Rightarrow i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} \cancel{v_L} = 0.dt$

$$\boxed{i_L(0^+) = i_L(0^-) \text{ Ampere}}$$

$$\boxed{E_L(0^+) = E_L(0^-) \text{ Joule}}$$

So, inductor current cannot change instantaneous i.e. with zero time for all the practical excitations. Similarly the energy.

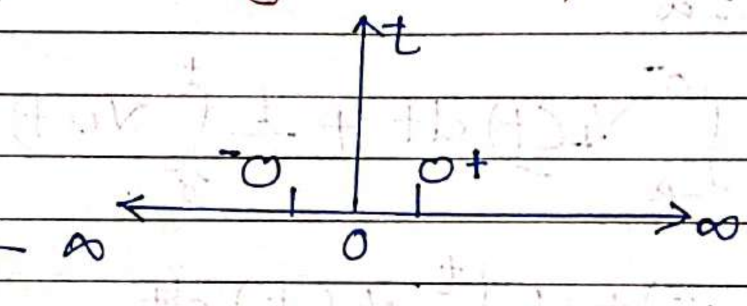
If $v_L(t) = \delta(t)$, then $i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} \delta(t)\,dt$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \text{ Ampere}$$

$$E_L(0^+) > E_L(0^-) \text{ Joule.}$$

So, inductor current can change instantaneously i.e. within zero time for impulse voltage across it. Similarly the energy.

More Specifically for Impulse :-



$v_L(t) = \delta(t) = 0$ for $-\infty \le t < 0^-$

$$\Rightarrow i_L(0^-) = \frac{1}{L}\int_{-\infty}^{0^-} v_L(t)\,.dt = 0 \text{ Ampere}$$

$$i_L(0^+) = \frac{1}{L} \text{ Ampere.}$$

So, $E_L(0^+) = \frac{1}{2L}$ Joule. $E_L(0^-) = 0\,J$

$$E_L(0^+) = \frac{1}{2} L\, i_L^2(0^+)$$

$$= \frac{1}{2} L \left(\frac{1}{L}\right)^2$$

Capacitor Voltages at $t = 0^-$ & at $t = 0^+$ instants:-

C: $$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C \cdot dt$$

$$= V_C(0^-) + \frac{1}{C}\int_{0^-}^{t} i_C\, dt$$

At $t = 0^+ \Rightarrow V_C(0^+) = V_C(0^-) + \frac{1}{C}\underbrace{\int_{0^-}^{0^+} i_C\, dt}_{=0}$

$$V_C(0^+) = V_C(0^-) \text{ volt.}$$

for all practical inputs

$$E_C(0^+) = E_C(0^-) \text{ Joule.}$$

More specially for impulse:

$i_C = \delta(t) = 0$ for $-\infty \le t < 0^-$

$$V_C(0^-) = \frac{1}{C}\int_{-\infty}^{0^-} i_C \cdot dt = 0 \text{ volt.}$$

$$v_c(0^+) = \frac{1}{C} \text{ volt.}$$

So $E_c(0^-) = 0\,J$ & $E_c(0^+)$

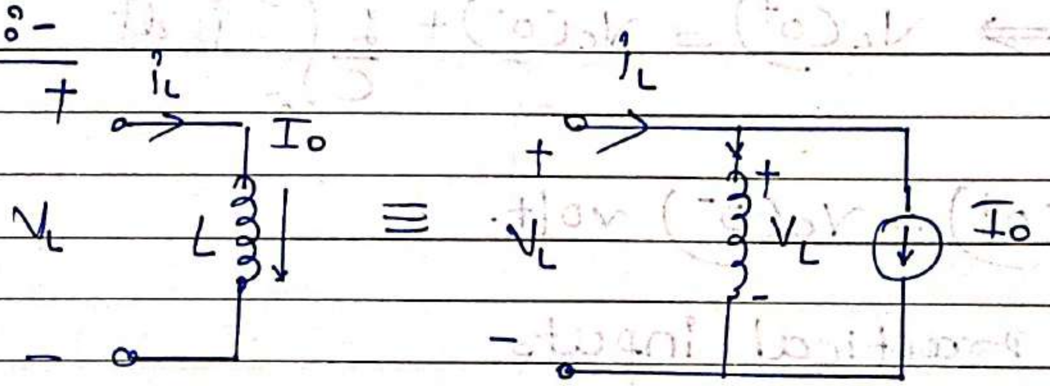
$$= \frac{1}{2} C v_c^2(0^+)$$

$$= \frac{1}{2} C \cdot \left(\frac{1}{C}\right)^2$$

$$E_c(0^+) = \frac{1}{2C} \text{ Joule.}$$

Equivalent Circuits :-

L :-



$$i_L = \frac{1}{L} \int_{-\infty}^{t} V_L \, dt$$

$$i_L = i_L(0) + \frac{1}{L} \int_0^t V_L \, dt \quad \Big| \quad i_L(0) = I_0$$

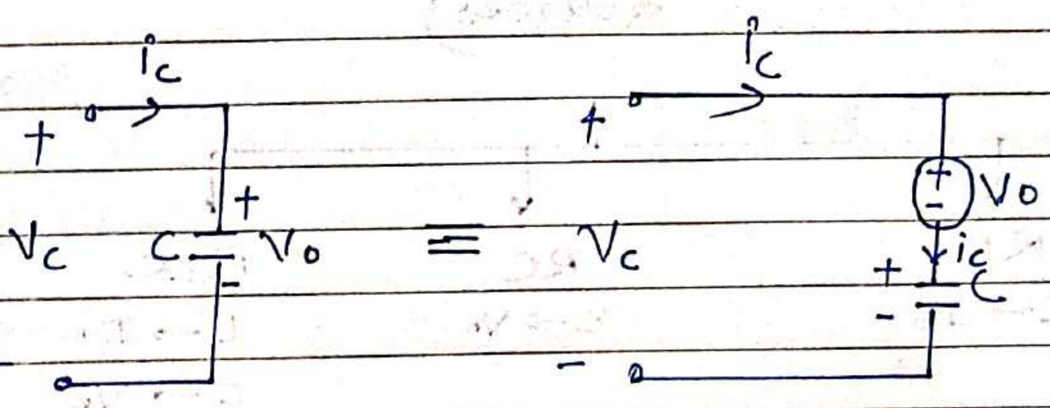
Proof :

By KCL,

$$\Rightarrow -i_L + \frac{1}{L}\int_0^t V_L \cdot dt + I_0 = 0$$

$$\Rightarrow i_L = I_0 + \frac{1}{L}\int_0^t V_L \, dt$$

C :



$$V_c = \frac{1}{C}\int_{-\infty}^t i_c \, dt$$

$$V_c = V_c(0) + \frac{1}{C}\int_0^t i_c \cdot dt \quad \Big| \; V_C(0) = V_0$$

Proof :

By KVL $\quad V_c - V_0 - \frac{1}{C}\int_0^t i_c \cdot dt = 0$

$$V_c = V_0 + \frac{1}{C}\int_0^t i_c \cdot dt$$

Classification of Transient

1. DC Transient
2. AC Transient

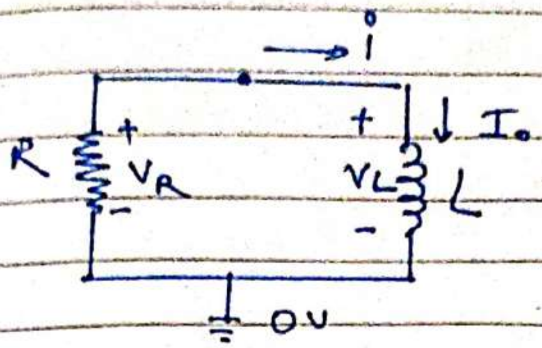
DC Transient :-

1) Source free circuit (without independent sources)

RL	RC	RLC
$L \rightarrow I_0$	$C \rightarrow V_0$	$L \rightarrow I_0$
		$C \rightarrow V_0$
		(OR) Both

Property :- In all the source free circuits, stored energies in the memory elements are maximum at $t = 0$ and as a function of time, these stored energies will be delivered to the memoryless resistances in an exponentially decayed manner and hence, the energy present in the network at steady state $(t \rightarrow \infty)$ is zero.

Source free RL circuit :-



For $t \geq 0$.
By KVL

$-V_R - V_L = 0$

$-R.i - L.\frac{di}{dt} = 0$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

Let

$$D = \frac{d}{dt} \Rightarrow \left[D + \frac{R}{L}\right] i = 0$$

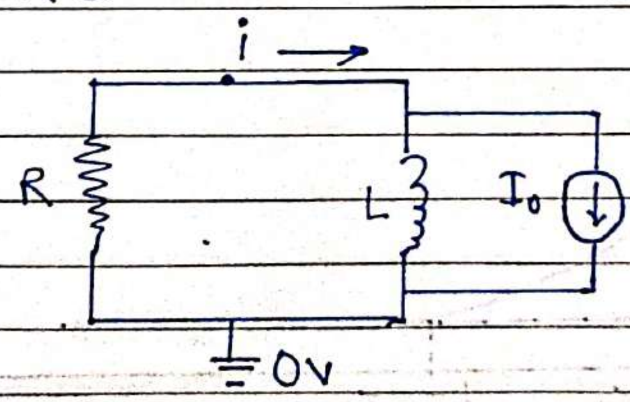
Characteristic equation $D + \frac{R}{L} = 0$

$$D = \frac{-R}{L}$$

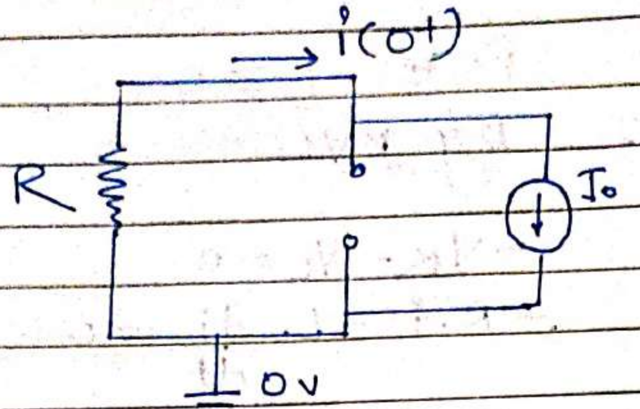
It is like $s = \alpha$

$$i(t) = Ke^{\alpha t} \quad \text{for} \quad 0 \leq t \leq \infty$$

$$= Ke^{-\frac{R}{L}t} \text{ A} \quad \text{for} \quad 0 \leq t \leq \infty$$



for $t \geq 0$



At $t = 0^+$: A resistive circuit

By KCL $\Rightarrow -i(0^+) + I_0 = 0$

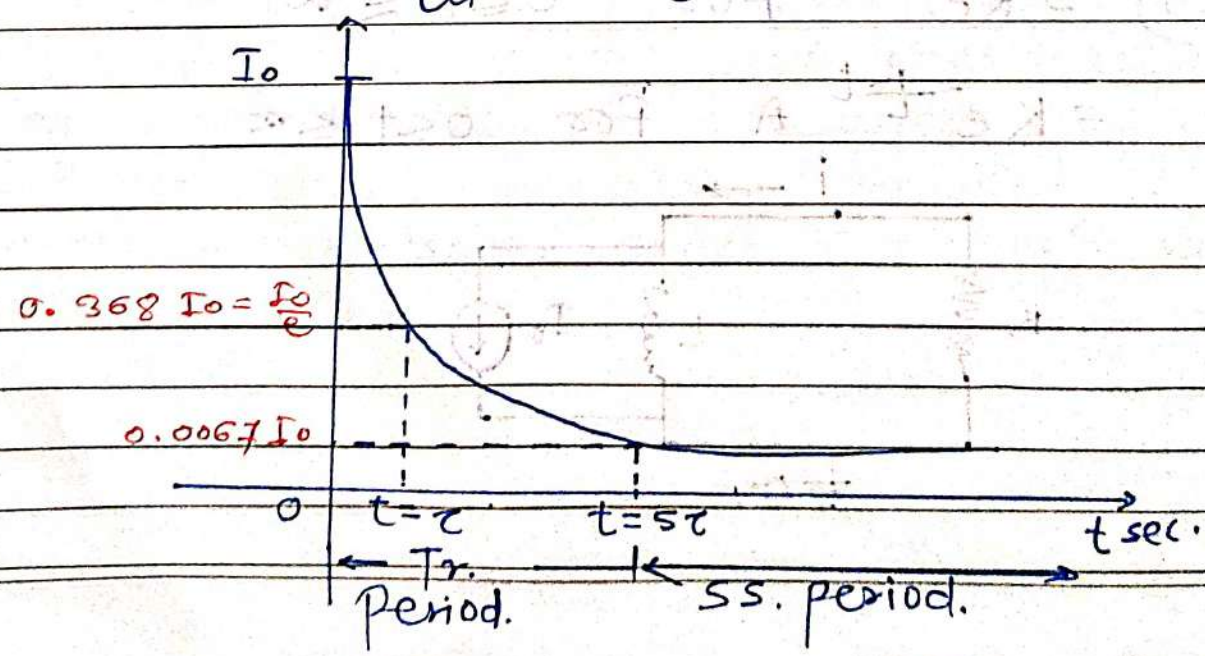
$\Rightarrow i(0^+) = I_0$

So $t = 0^+ \Rightarrow i(0^+) = ke^{-0} \Rightarrow k = i(0^+) = I_0$

$\Rightarrow i(t) = I_0 e^{-\frac{R}{L}t}$ Ampere, for $0 \leq t \leq \infty$

$\Rightarrow \tau = \frac{L}{R}$ seconds = Time constant of the circuit

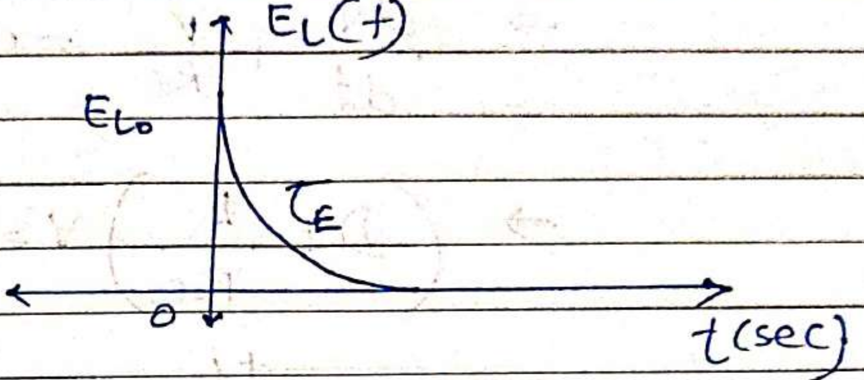
$v_L(t) = L \frac{di(t)}{dt}$ By Ohm's law



At $t = 5\tau$,
$i(t) = I_0 e^{-5} = \frac{I_0}{e^5} = 0.0067\, I_0 \cong 0.0$

$E_L(t) = \frac{1}{2} L i^2(t)$ Joule $= \frac{1}{2} L \cdot \left[I_0 e^{-\frac{t}{\tau_i}} \right]^2$

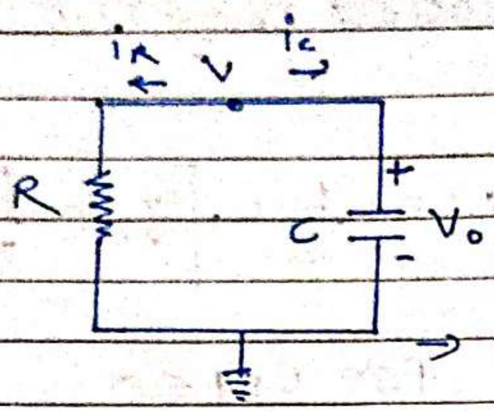
$= E_{L_0} e^{-\frac{t}{\tau_E}}$ Joule for $0 \leq t \leq \infty$



$\tau_E = \frac{\tau_i}{2} = \frac{L}{2R}$ sec.

So the energy is 2 times faster than the current decay

Source Free RC circuit



For $t \geq 0$

By KCL

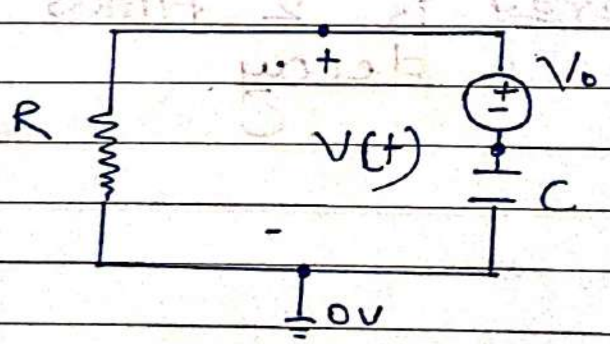
$$i_R + i_c = 0$$

$$\Rightarrow \frac{v}{R} + C\frac{dv}{dt} = 0$$

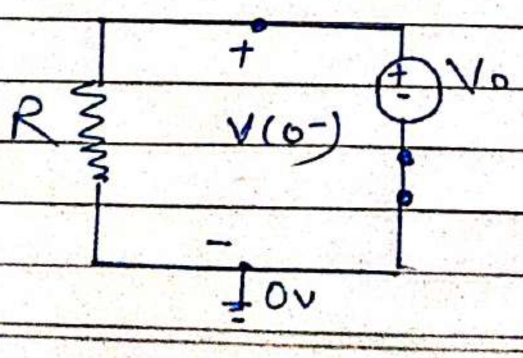
$$\Rightarrow \frac{dv}{dt} + \frac{v}{RC} = 0$$

$$\Rightarrow \left(D + \frac{1}{RC}\right)v = 0$$

$$\Rightarrow v(t) = ke^{-t/RC} \text{ volt for } 0 \leq t \leq \infty$$



for $t \geq 0$



At $t = 0^+$; A resistive circuit

$\Rightarrow v(0^+) = v_0$

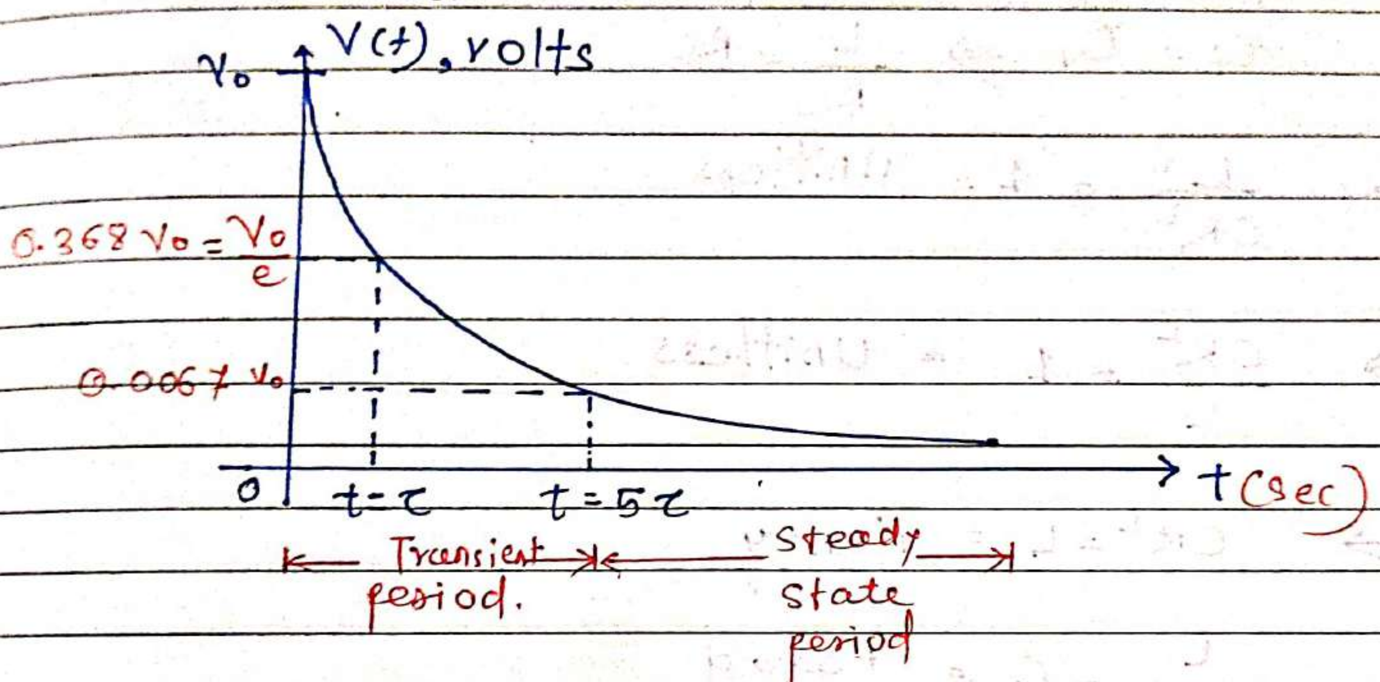
so, $t = 0^+ \Rightarrow v(0^+) = ke^{0} \Rightarrow k = v(0^+) = v_0$

$$v(t) = V_0 e^{\frac{-t}{RC}} \quad V \text{ for } 0 \leq t \leq \infty$$

$$v(t) = v_0 e^{t/\tau} \quad v \quad \text{for } 0 \leq t \leq \infty$$

$\tau = RC = \text{sec.} =$ Time constant of the circuit

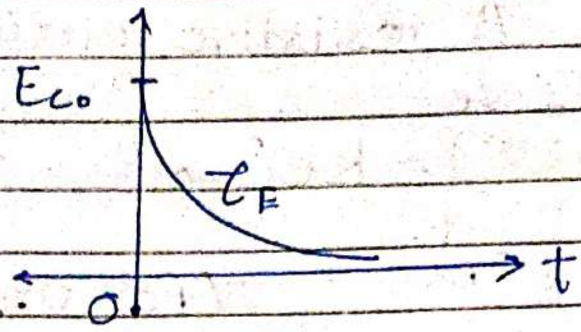
$i_c = C \dfrac{dv(t)}{dt}$ By Ohm's law



At $t = 5\tau$, $v(t) = v_0 e^{-5} = \dfrac{I_0}{e^5} = 0.0067\,V_0 \cong 0.0$

$$E_c(t) = \frac{1}{2} C v^2(t) \text{ Joule} = \frac{1}{2} C \left(V_0 e^{-t/\tau_v} \right)^2$$

$$= E_c\, e^{-t/\tau_E} \text{ Joule for } 0 \leq t \leq \infty$$



$$\tau_E = \frac{\tau_v}{2} = \frac{RC}{2} \text{ sec.}$$

So the energy decay is 2 times faster than the voltage decay.

$\rightarrow$ Second = second

$\tau_i = \tau_v \Rightarrow \frac{L}{R} = RC$

$\rightarrow \frac{L}{CR^2} = 1 =$ Unitless

$\rightarrow \frac{CR^2}{L} = 1 =$ Unitless

$\rightarrow CR^2 = L =$ Henry

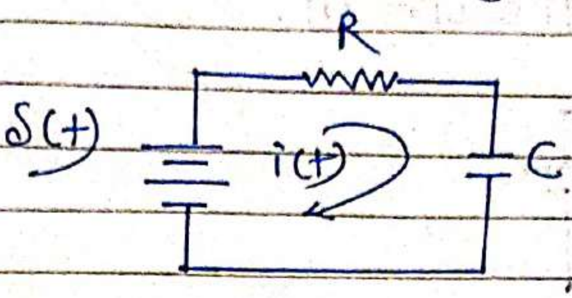
$\rightarrow \frac{L}{R^2} = C =$ Farad

$\rightarrow \frac{L}{RC} = R =$ ohm

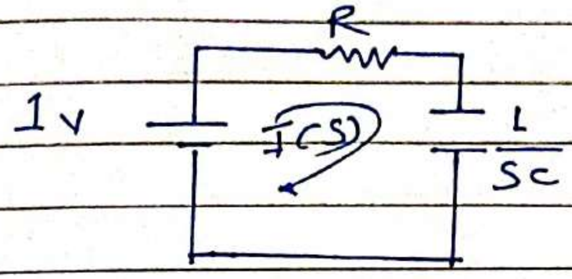
$\rightarrow \frac{L}{C} = R^2 = \text{ohm}^2$

$\rightarrow \sqrt{\frac{L}{C}} = R^2 =$ ohm.

Example :- The circuit shown below, find $i(t)$ if the impulse voltage is applied to the circuit



→



$$I(S) = \frac{1}{R + \frac{1}{SC}} = \frac{SC}{SRC+1}$$

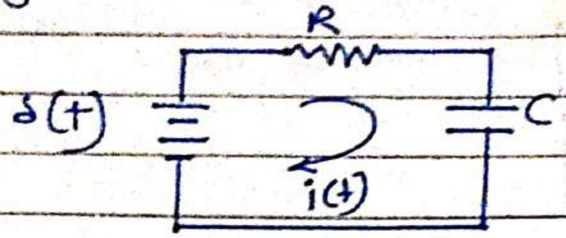
$$I(S) = \frac{SC/RC}{S + \frac{1}{RC}} = \frac{S/R}{S + 1/RC}$$

$$I(S) = \frac{1}{R}\left[\frac{S + \frac{1}{RC} - \frac{1}{RC}}{S + \frac{1}{RC}}\right]$$

$$= \frac{1}{R}\left[1 - \frac{1/RC}{S + 1/RC}\right] \Rightarrow i(t) = \frac{1}{R}\left[\delta(t) - \frac{1}{RC}e^{-t/RC}\right]$$

Example:

The circuit is shown. find $i(t)$ if the impulse voltage is applied to the circuit.



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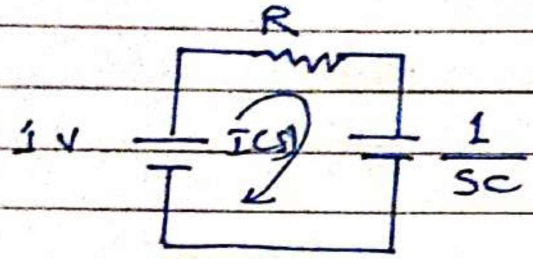
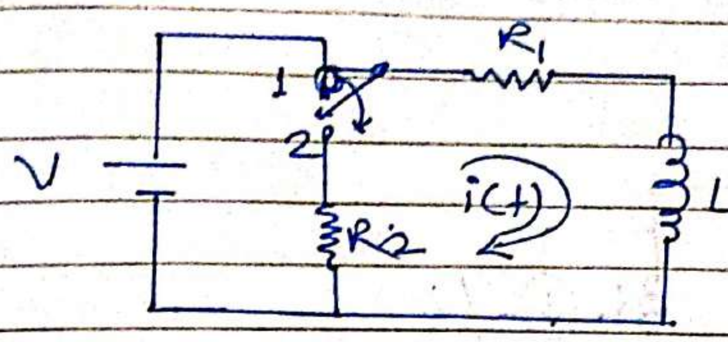
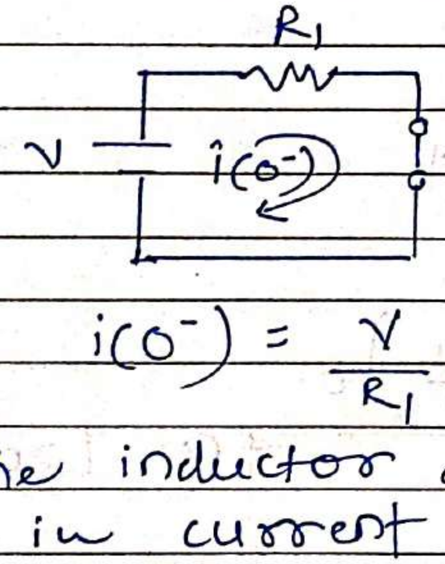


Example :- In the network of Figure, the switch is initially at position 1. On the steady state having reached, the switch is changed to position 2. Find current $i(t)$

→



At $t=0^-$, the network has attained steady state condition, Hence, the inductor acts as a short circuit



$$i(0^-) = \frac{V}{R_1}$$

Since the inductor does not allow sudden change in current

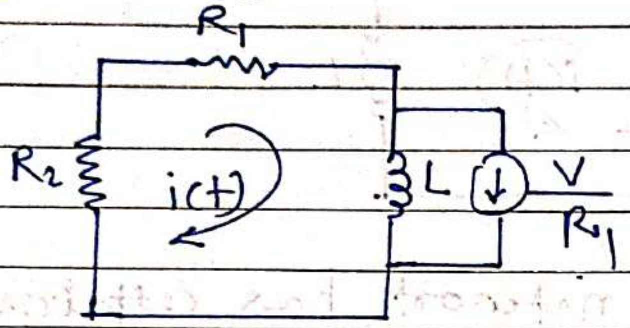
$$i(0^+) = \frac{V}{R_1}$$

Writing KVL equation for $t>0$

$$(R_1+R_2)\cdot i + L\frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{(R_1+R_2)}{L}i = 0$$

The solution of this differential equation is given by



$$i(t) = Ke^{-\left(\frac{R_1+R_2}{L}\right)t}$$

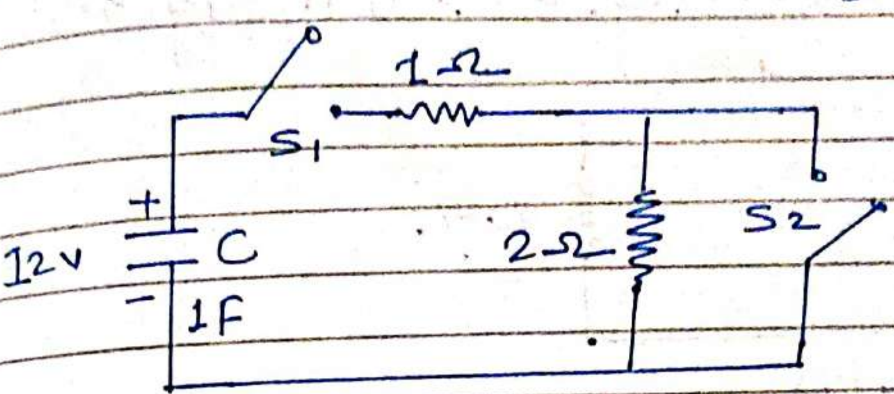
At $t=0$, $i(0) = \frac{V}{R_1}$

$$\frac{V}{R_1} = ke^{0} = k$$

$$i(t) = \frac{V}{R_1}e^{-\left(\frac{R_1+R_2}{L}\right)t} \quad \text{for } t>0$$

Example :-

The capacitor in the circuit as shown below is initially charged to 12 volt with S_1 & S_2 open. S_1 is closed at $t=0$ while S_2 is closed at $t=3$. The waveform of the capacitor current is represented by



→ for $0<t<3$, the circuit is shown in fig.

$$i(0^+) = \frac{12}{3} = 4 \text{ Amp.}$$

$$\tau = RC = 1 \times 3 = 3 \text{ second}$$

$$i(t) = 4e^{-t/3}$$

$$i(3_-) = 4e^{-1} = 1.48$$

$$V_c(3_-) = V_c(3_+) = 4e^{-1} \times 3 = 4.44$$

At $t=3$, S_2 is closed. 2Ω is short circuited. Current jumps to a higher value and then decays to zero quickly with a reduced time constant.

For $3 < t < \infty$ The circuit is shown in Fig (2)

$$i_c(3_+) = \frac{v_c(3_+)}{1} = 4.44$$

$i_c(\infty) = 0$

$i_c(t)$ for $0 < t < \infty$ is shown in Fig (3)

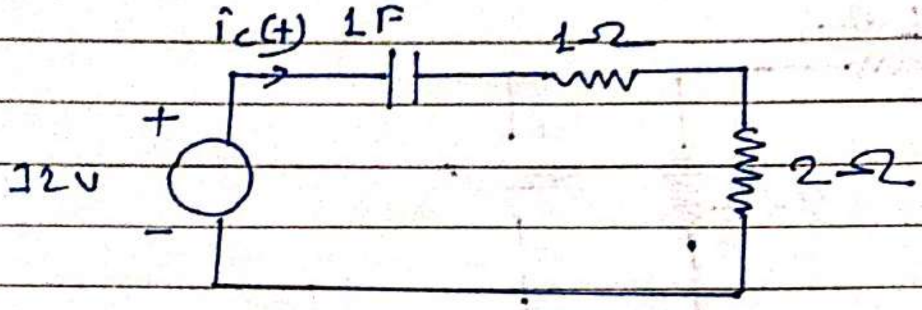


Fig (1)

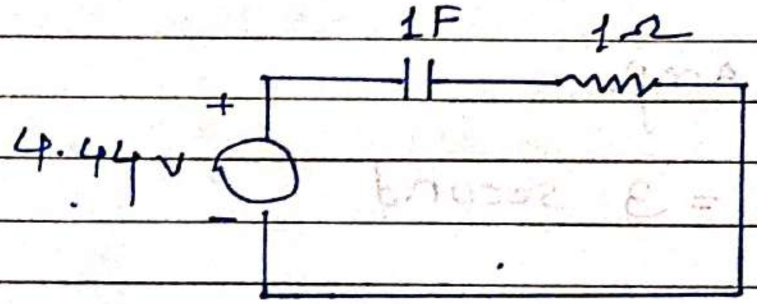


Fig (2)

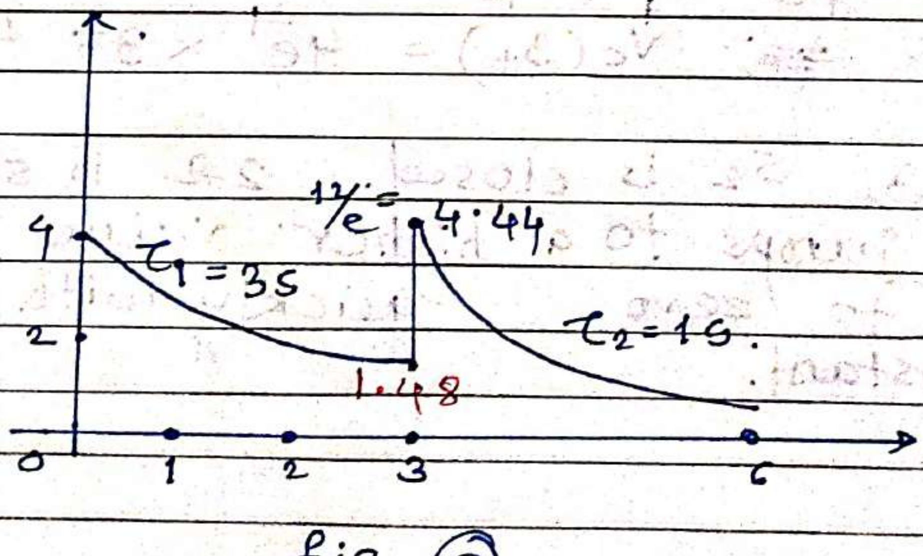


fig (3)

Note :- With source circuits and using Laplace Transform Analysis

Behaviour of inductor and capacitor with sources, initial ($t = 0^+$) and final ($t \to \infty$) conditions

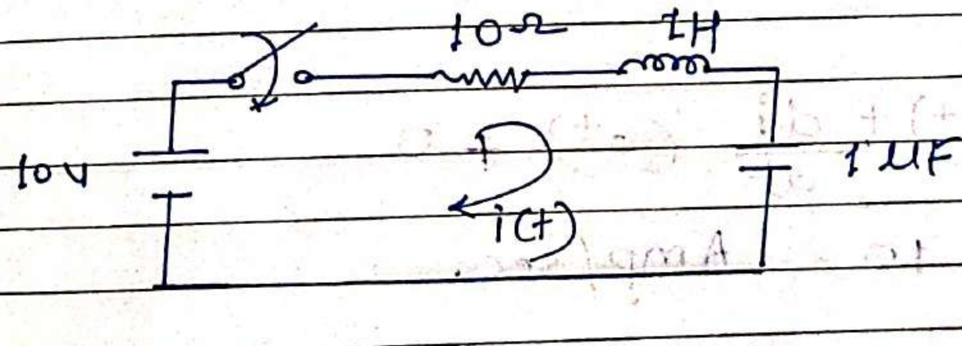
At $\to t = 0^+ \Rightarrow L = OC \Rightarrow C = SC.$

As $\to t \to \infty \Rightarrow L = SC. \Rightarrow C = OC.$

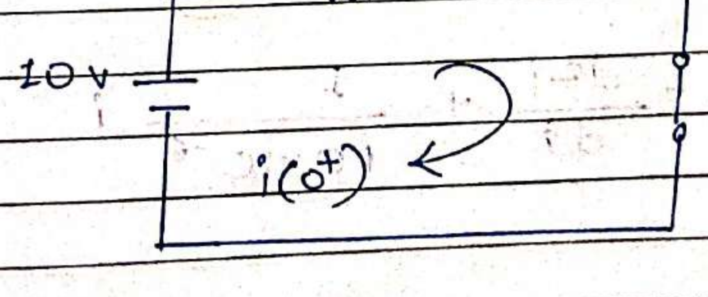
Property :- At these two instants, L & C elements will lose their significance and hence the nature of the circuit is resistive

Example :- In the network shown in figure the switch is closed. Assuming all initial conditions as zero

Find i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.



Soln :-



At $t = 0^-$

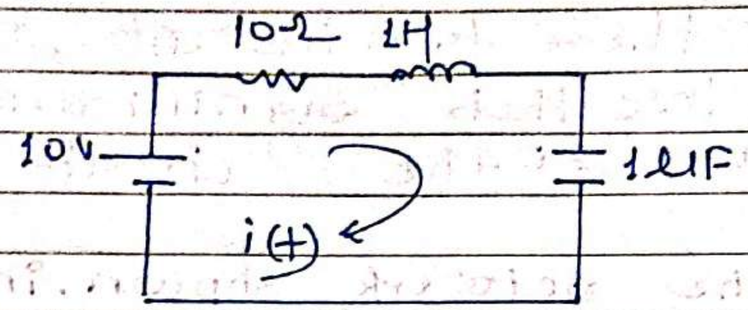
$i(0^-) = 0$, $v_c(0^-) = 0$

At $t = 0^+$, the inductor acts as an open circuit and the capacitor acts as a short circuit

$i(0^+) = 0$

$v_c(0^+) = 0$

Writing KVL equation For $t > 0$



$$10 - 10i - 1\frac{di}{dt} - \frac{1}{10\times10^{-6}}\int_0^t i\,dt = 0$$

$$10 = 10i + \frac{di}{dt} - \frac{1}{10\times10^{-6}}\int_0^t i\,dt \quad \text{--------} \quad (1)$$

At $t = 0^+$

$$10 = 10\,i(0^+) + \frac{di}{dt}(0^+) + 0$$

$$\frac{di}{dt}(0^+) = 10 \text{ Amp/sec.}$$

Differentiating the equation (1)

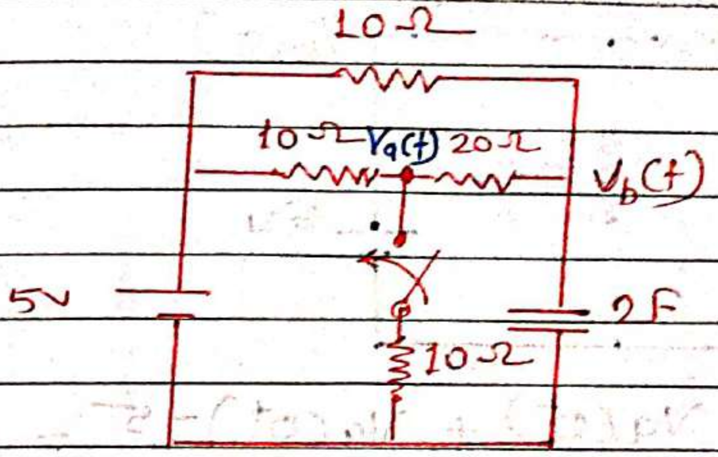
$$0 = 10\frac{di}{dt} + \frac{d^2i}{dt} + \frac{1}{10\times10^{-6}}\,i$$

At $t = 0^+$

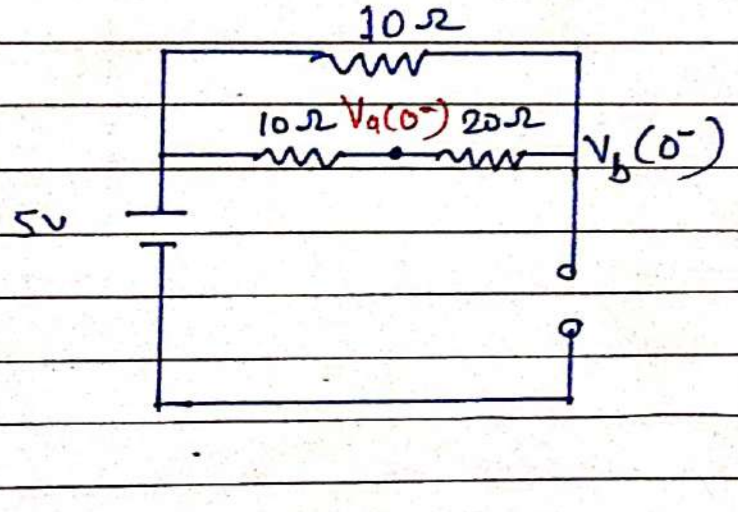
$$0 = 10\,\frac{di}{dt}(0^+) + \frac{d^2i}{dt^2}(0^+) + \frac{1}{10^{-5}}\,i(0^+)$$

$$\frac{d^2i}{dt^2}(0^+) = -10\times10 = -100\ \text{Amp/sec.}$$

Example:- In the accompanying fig. is shown a network in which a steady state is reached with switch open. At $t=0$, switch is closed. Determine $V_a(0^-)$, $V_a(0^+)$, $V_b(0^-)$ & $V_b(0^+)$



At $t=0^-$, the network attains steady state condition. Hence, the capacitor acts as an open circuit.



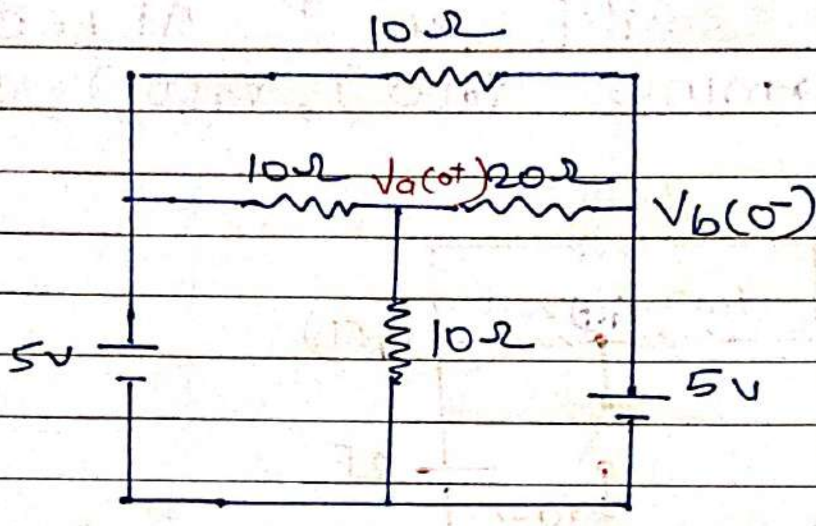
$V_a(0^-) = 5$ volt.

$V_b(0^-) = 5$ volt.

At $t = 0^+$, the capacitor acts as a voltage source of 5 volt.

$V_b(0^+) = 5$ volt.

Writing KCL equation at $t = 0^+$



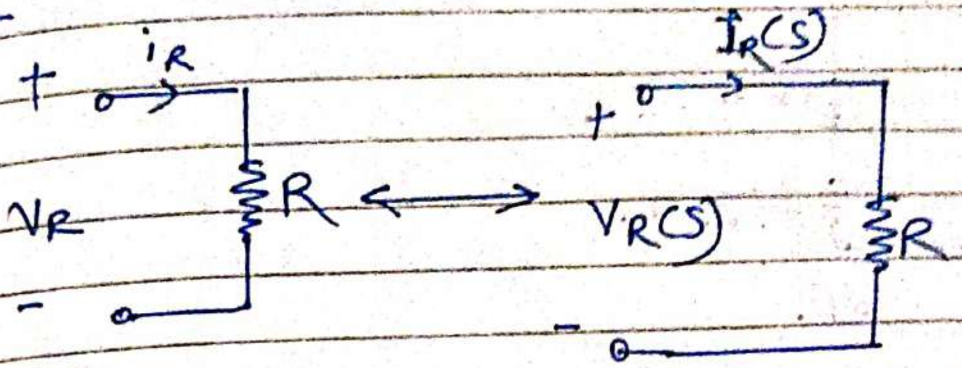
$$\frac{V_a(0^+) - 5}{10} + \frac{V_a(0^+)}{10} + \frac{V_a(0^+) - 5}{20} = 0$$

$$0.25\, V_a(0^+) = 0.75$$

$$V_a(0^+) = 3 \text{ volt.}$$

With sources, Laplace transform approach of solving transient problems for $0 \le t < \infty$

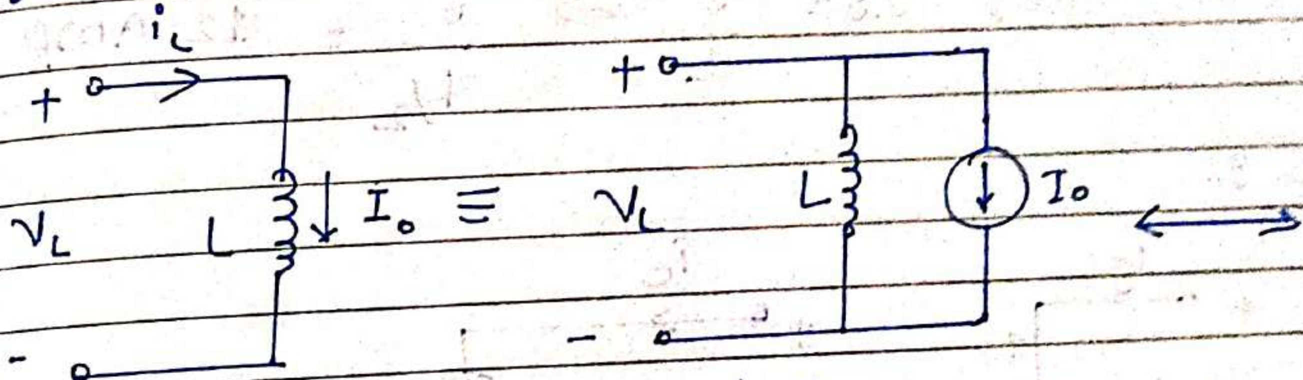
R :-



t − D

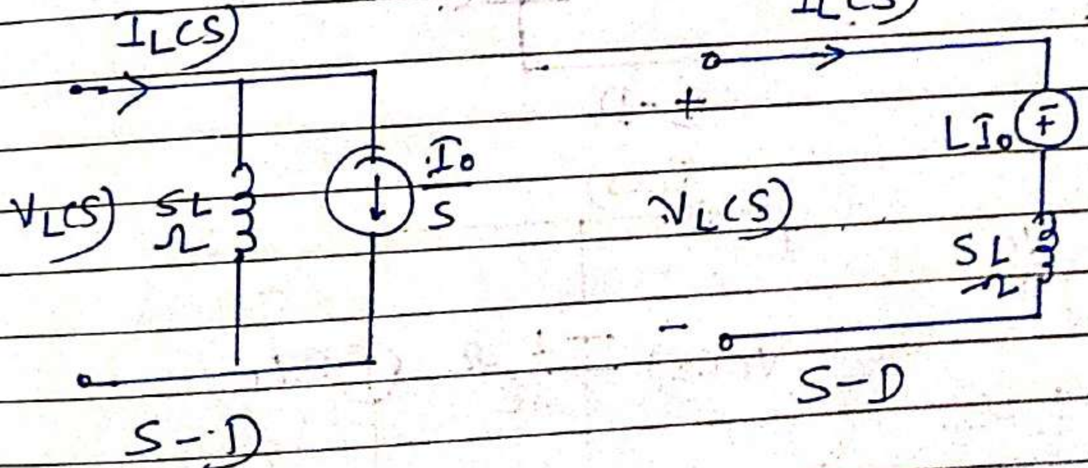
S D

L :-



t − D

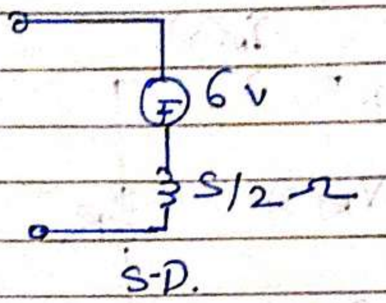
t − D



S − D

S − D

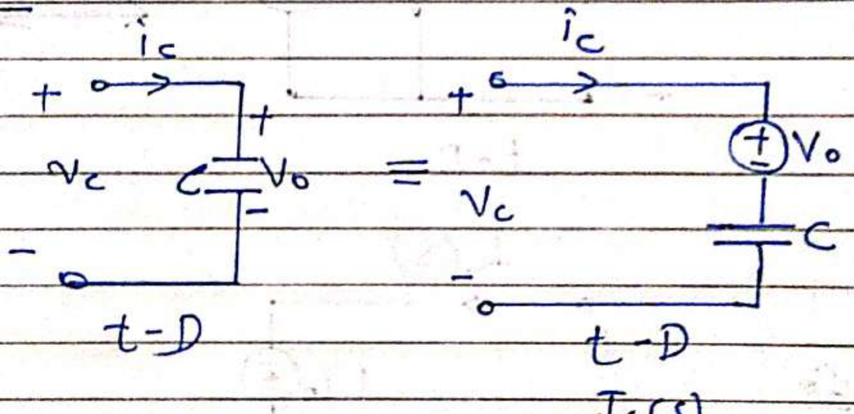
Example:- The network below, represents the equivalent of an inductor in Laplace domain. Determine initial current through it

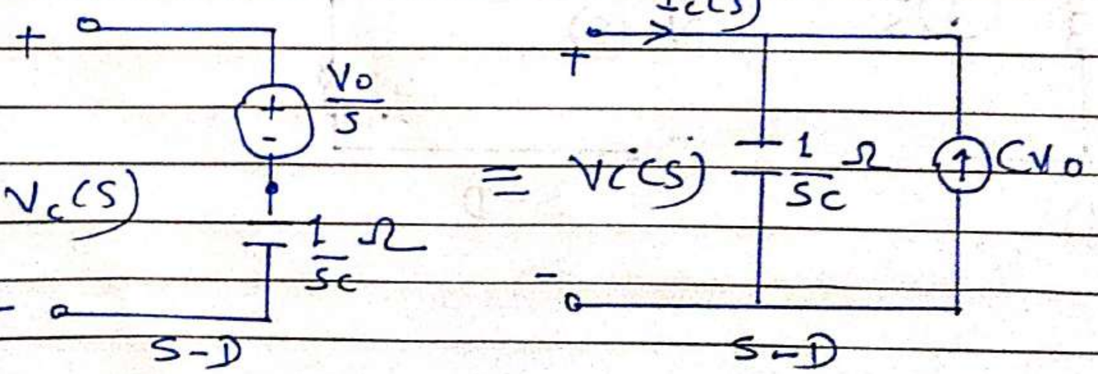


Since $LI_o = 6$ v & $L = 1/2$ Henry.

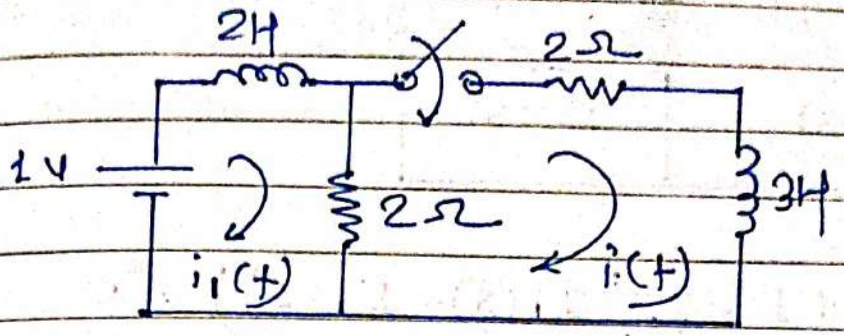
So, $I_o = \frac{6}{L} \Rightarrow \frac{6}{1/2} = 12$ Amp

C :-

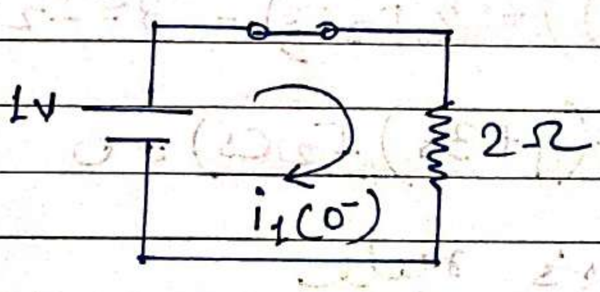




Example: In the network shown, in Figure, the switch is closed at $t=0$, the steady state being reached before $t=0$. Determine current through inductor of 3 H.



→ At $t = 0^-$, steady state condition is reached. Hence, the inductor of 2H acts as a short circuit.



$$i_1(0^-) = 1/2 \text{ Amp.}$$

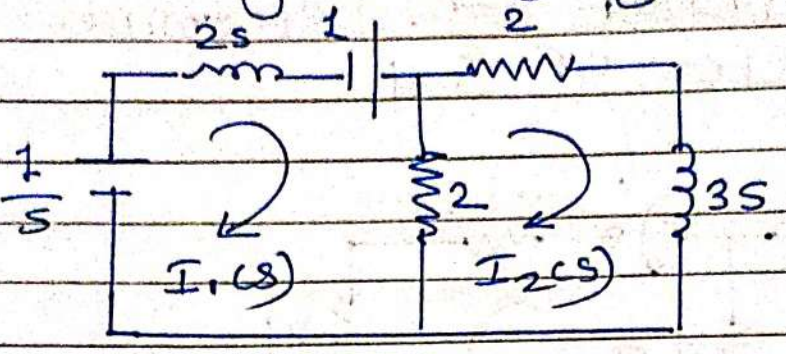
$$i_2(0^-) = 0$$

since current through the inductor cannot change instantaneously

$$i_1(0^+) = \frac{1}{2} \text{ Amp.}$$

$$i_2(0^+) = 0$$

For $t>0$, the transformed network is shown in fig. Applying KVL to Mesh 1



$$\frac{1}{s} - 2sI_1(s) + 1 - 2\left(I_1(s) - I_2(s)\right) = 0$$

$$(2+2s)I_1(s) - 2I_2(s) = 1 + \frac{1}{s}$$

Applying KVL to mesh 2

$$-2\left(I_2(s) - I_1(s)\right) - 2I_2(s) - 3sI_2(s) = 0$$

$$-2I_1(s) + (4+3s)I_2(s) = 0$$

By cramer's rule

$$I_2(s) = \frac{\begin{vmatrix} 2+2s & 1+1/s \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} 2+2s & -2 \\ -2 & 4+3s \end{vmatrix}}$$

$$= \frac{\frac{2}{s}(s+1)}{(2s+2)(4+3s) - 4}$$

$$= \frac{s+1}{s(2s^2+7s+2)} = \frac{s+1}{3s\left(s+\frac{1}{3}\right)(s+2)}$$

$$= \frac{\frac{1}{3}(s+1)}{s(s+2)\left(s+\frac{1}{3}\right)}$$

By partial fraction expansion,

$$I_2(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1/3}$$

$$A = \left.\frac{1/3\,(s+1)}{(s+2)(s+1/3)}\right|_{s=0} = 1/2$$

$$C = \left.\frac{1/3\,(s+1)}{s(s+2)}\right|_{s=-1/3} = -2/5$$

$$B = \left.\frac{1/3\,(s+1)}{s(s+1/3)}\right|_{s=2} = \frac{-1}{10}$$

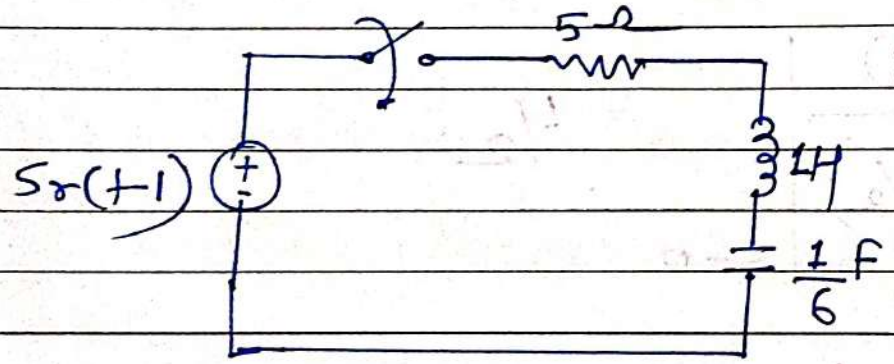
$$I_2(s) = \frac{1}{2s} - \frac{1}{10(s+2)} - \frac{2}{5\left(s+\frac{1}{3}\right)}$$

Taking inverse Laplace transform

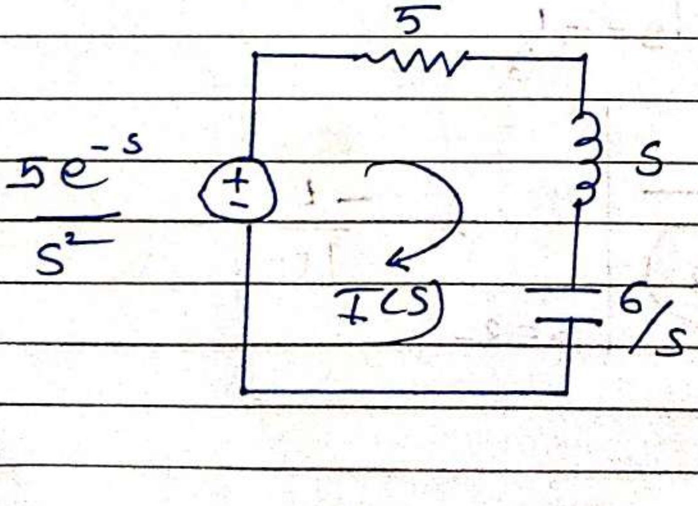
$$i_2(t) = \frac{1}{2} - \frac{1}{10}e^{-2t} - \frac{2}{5}e^{-(1/3)t} \quad \text{for } t>0$$

~~For~~ example:-

For the network shown, determine the current $i(t)$ when the switch is closed at $t=0$ with zero initial conditions.



The transformed network is shown in fig.



Applying KVL to the mesh for $t>0$

$$\frac{5e^{-s}}{s^2} - 5I(s) - \frac{6}{s}I(s) = 0$$

$$\cancel{5e}\; 5I(s) + sI(s) + \frac{6}{s}I(s) = \frac{5e^{-s}}{s^2}$$

$$I(s) = \frac{5e^{-s}}{s(s^2+5s+6)} = \frac{5e^{-s}}{s(s+3)(s+2)}$$

By partial fraction expansion

$$\frac{1}{s(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$A = \left.\frac{1}{(s+3)(s+2)}\right|_{s=0} = 1/6$$

$$B = \left.\frac{1}{s(s+2)}\right|_{s=-3} = \frac{1}{3}$$

$$C = \left.\frac{1}{s(s+3)}\right|_{s=-2} = -1/2$$

$$I(s) = 5e^{-s}\left[\frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}\right]$$

$$= \frac{5e^{-s}}{6s} + \frac{5e^{-s}}{3(s+3)} - \frac{5e^{-s}}{2(s+2)}$$

Taking inverse Laplace transform

$$i(t) = \frac{5}{6} u(t-1) + \frac{5}{3} e^{-3(t-1)} u(t-1) - \frac{5}{2} e^{-2(t-1)} u(t-1)$$

Phasor Representation and its Importance in steady state AC circuit Analysis

A phasor is a complex number that represents amplitude and phase of a sinusoid. It is defined only for the cosinusoidal signals. All sinusoidal signals are converted into cosinusoids by subtracting 90° from phase.

- As a complex quantity a phasor may be expressed in exponential form, polar form, rectangular form. Since phasor has magnitude & phase ("direction"), it behaves as a vector.

$$v(t) = V_m \cos(\omega t + \phi)$$

$$= \text{Real part}\left[V_m e^{j(\omega t + \phi)}\right]$$

$$= \text{Real part}\left[V_m e^{j\phi} \cdot e^{j\omega t}\right]$$

$$= \text{Real part}\left[V \cdot e^{j\omega t}\right]$$

$v = V_m e^{j\phi}$ → Exponential form of phasor. representation

$= V_m \angle \phi$ → Polar form of the phasor representation.

$= V_m(\cos\phi + j\sin\phi)$ → Rectangular form of Phasor representation.

$i(t) = I_m \cos(\omega t + \theta)$

$I = I_m \angle \theta$ → Polar

$= I_m e^{j\theta}$ → Expo

$= I_m(\cos\theta + j\sin\theta)$ → Rect.

eg. $v(t) = 10\cos(2t + 30^\circ)$ volt.

$v = 10\angle 30^\circ$ → Polar

$= 10 e^{j30^\circ}$ → Expo

$= 10(\cos 30^\circ + j\sin 30^\circ)$ → Rect

$= 10\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right]$ i.e. phasor is a representation of complex number

eg. $i(t) = 10\sin(2t + 30^\circ)$ Ampere

$= 10\cos(2t + 30^\circ - 90)$ Ampere

$= 10\cos(2t - 60^\circ)$ Ampere

$I = 10\angle -60^\circ$.

$= 10e^{-j60^\circ}$

$= 10(\cos 60^\circ - j\sin 60^\circ)$

$= 10\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$

$$\boxed{\frac{V}{I} = Z\ \Omega}$$

$V = Z.I \Rightarrow I = \frac{V}{Z}$

$\frac{I}{V} = Y \quad \mho$

$I = Y.V \Rightarrow V = \frac{I}{Y}$

$$\boxed{Y = \frac{1}{Z}}\ \mho$$

$t - D \longleftrightarrow P - D$

$i(t) \longleftrightarrow I$

$v(t) \longleftrightarrow V$

$R \longleftrightarrow R\ \Omega$

$L \longleftrightarrow j\omega L\ \Omega \quad (s = j\omega)$

$C \longleftrightarrow \frac{1}{j\omega C}\ \Omega$

Note:

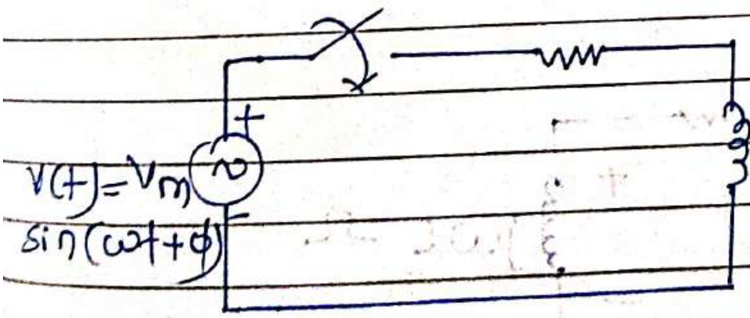
→ Analysis of AC in steady state is generally carried out in the phasor domain i.e. KCL, KVL, Ohm's Law, Nodal, Mesh, and source transformation are written only in the phasor domain.

- Addition & subtraction operations of phasors are carried out only in the rectangular form, whereas division & multiplication operations of phasors are carried out either in polar or in exponential form...

AC Transient:

Steady State Response and Transient free condition for RL, RC & RLC Circuits.

Series RL Circuit:-



$C.S = C.F + P.I$

Complex situation = complex function + perticular integration.

= Natural response + forced Response

→ Transient Response + Steady state response

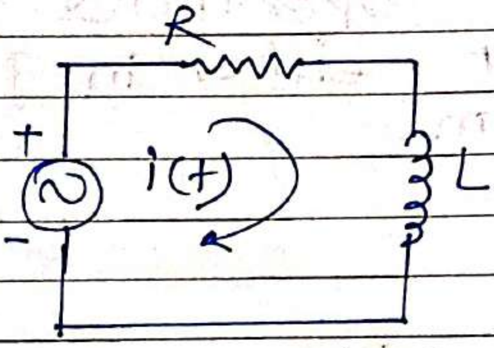
⇒ Response upto 5τ + Response after 5τ

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$i(t) = Ke^{-t/\tau} + i_{ss}(t)$$

$$= Ke^{-R/L\,t} + i_{ss}(t)$$

$i_{ss}(t)$

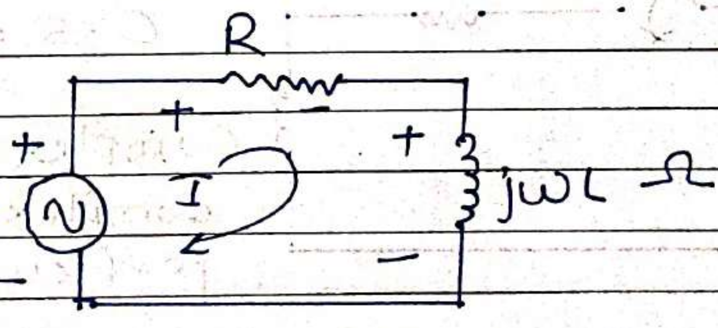


$v(t) = V_m \sin(\omega t + \phi)$

Network is in steady state

Method (1): By Phasor approach :-

Transform above network into phasor domain



NW is in P.D

$V = R \cdot I$ / By ohm's law in P.D.

By KVL in P-D

$\Rightarrow V_m \angle \phi - 90° - RI - j\omega L I = 0$

$$I = \frac{V_m \angle \phi - 90°}{R + j\omega L} = \frac{V_m \angle \phi - 90°}{\sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\left(\frac{\omega L}{R}\right)}$$

$$\Rightarrow I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle \phi - \tan^{-1}\left(\frac{\omega L}{R}\right) - 90°$$

$$= \alpha \angle \beta = \alpha e^{j\beta}$$

$i(t) = R.P\,[I \cdot e^{j\omega t}]$ Ampere

$$= \alpha \cos(\omega t + \beta)$$

$$= \alpha \cos\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R} - 90°\right)$$

$$= \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{ Amp.}$$

$$= i_{ss}(t)$$

Method (2): By LTA:

$$v(t) = V_m \sin(\omega t + \phi)$$

$$\frac{I(S)}{V(S)} = H(S) = Y(S) = \frac{1}{Z(S)} = \frac{1}{R+SL}$$

$$H(j\omega) = \frac{1}{R + j\omega L}$$

$$= \frac{1}{\sqrt{R^2+(\omega L)^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$

$$\Rightarrow I(S) = H(S) \cdot V(S)$$

$$i(t) = \frac{1}{\sqrt{R^2+(\omega L)^2}} V_m \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right) \text{Amp}$$

$$= i_{ss}(t)$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= Ke^{-R/L\,t} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$i(0^-) = 0A = i(0^+)$

$$\Rightarrow 0 = Ke^{-0} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(0+\phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$$\Rightarrow K = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right) << 1$$

As $K << 1$, the transient effects are less severe for AC.

$v_L(t) = L\frac{di(t)}{dt}$ By ohm's law.

Suppose $\phi - \tan^{-1}\frac{\omega L}{R} = 0 \Rightarrow K = 0$

$\Rightarrow i_{tr}(t) = 0 \Rightarrow i(t) = i_{ss}(t)$

ie A transient free response

So, condition for transient free response at $t=0$ is:

$$\boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$$

ie. $\omega t + \phi \Big|_{t=0} = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Note :-

→ So, if the total phase of excitation at the time of switching is equal to $\tan^{-1}\left(\frac{\omega L}{R}\right)$ then no transient will result in the system at the time of switching for sinusoidal excitation.

→ If the switch is closed at $t = t_0$ then the condition for the transient free response at $t = t_0$ is

$$\omega t_0 + \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

→ If the excitation is $v(t) = V_m \cos(\omega t + \phi)$ then the sine is replaced by cosine in the steady state response and hence 'k' is a function of cosine.

→ Suppose $\phi - \tan^{-1}\frac{\omega L}{R} = \frac{\pi}{2}$

$$\Rightarrow k = 0 \Rightarrow i_{tr}(t) = 0$$

$$\Rightarrow i(t) = i_{ss}(t)$$

i.e. A transient-free response

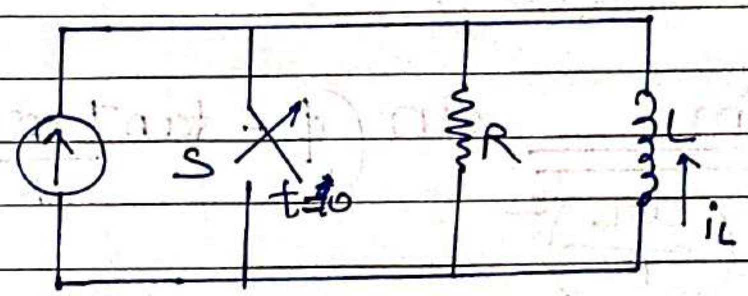
$$\phi - \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2} \quad \text{at } t=0$$

$$\omega t_0 + \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2} \quad \text{at } t = t_0$$

In the above cases if 'L' is replaced by 'C' then $\tau = \frac{L}{R}$ should be replaced by

$\tau = RC$:

Parallel RL Circuit :-



$i_{Lss}(t)$ by LTA :-

$$\frac{I_L(s)}{I(s)} = H(s) = \left(\frac{\text{desired L.T}}{\text{excitation LT}}\right)$$

$$I_L(s) = \frac{I(s) . R.}{R+sL}$$

(By current division in s-domain)

$$\frac{I_L(s)}{I(s)} = H(s) = \frac{R}{R+sL}$$

$$H(j\omega) = \frac{1}{1+\frac{j\omega L}{R}} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i_L(t) = i_{Ltr}(t) + i_{Lss}(t)$$

$$= Ke^{-R/L\,t} + \frac{I_m}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i_L(0^-) = 0 = i_L(0^+)$$

$$K = \frac{-I_m}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right) << 1$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$ By ohm's Law

Suppose $\phi - \tan^{-1}\frac{\omega L}{R} = 0$

$\Rightarrow K = 0 \Rightarrow i_{Ltr}(t) = 0$

$\Rightarrow i_L(t) = i_{Lss}(t)$

ie. A transient - free response.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ at } t=0$$

$$\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R} \text{ at } t=t_0$$

(same as series RL circuit)

If $i(t) = I_m \cos(\omega t + \phi)$, then

$$\phi = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2} \text{ at } t=0$$

$$\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R} + \frac{\pi}{2} \text{ at } t=t_0$$

In the above cases if 'L' is replaced by 'C' the $\tau = \frac{L}{R}$ should be replaced by $\tau = RC$.

Note :- Transient free condition is not possible for network with both the energy storing elements. ie. for RLC networks.

Reason :- $S_1, S_2 = \alpha \pm j\beta$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= e^{\alpha t}(K_1 \cos\beta t + K_2 \sin\beta t) + i_{ss}(t)$$

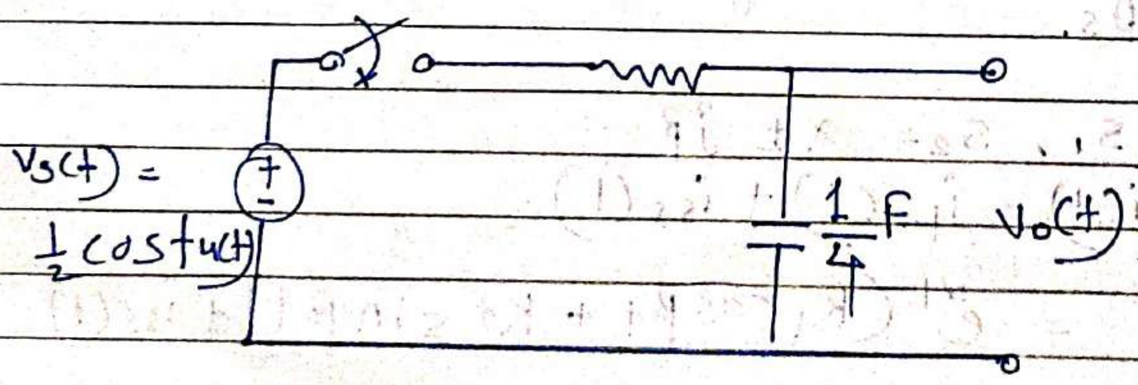
Here by applying initial conditions $i_L(0^+)$ and $v_c(0^+)$, K_1 is a function of sine and K_2 is a function of cosine (vice versa), hence no time will satisfy both K_1 & K_2 simultaneously to zero and so the transient term is always present in the complete response.

Observation :-

So, the transient - free time (t_0 value) in RL and RC circuits with AC excitations will depend on the source frequency (ω), its initial phase (ϕ), on the circuit constants (R, L, C) and on the nature of the excitation (sine or cosine), but not on the maximum value of the excitation (V_m or I_m)

Practice Problem :-

For the network shown in fig find the response $v_o(t)$



Laplace transform of some elementary functions.

The direct application of the definition gives the following formulae.

① $L\{k\} = \frac{k}{s}$ $\quad s > 0$

② $L\{e^{at}\} = \frac{1}{s-a}$ $\quad s > a$

③ $L\{e^{-at}\} = \frac{1}{s+a}$ $\quad s > -a$

④ $L\{\sin at\} = \frac{a}{s^2+a^2}$ $\quad s > 0$

⑤ $L\{\cos at\} = \frac{s}{s^2+a^2}$ $\quad s > 0$

⑥ $L\{\sinh at\} = \frac{a}{s^2-a^2}$ $\quad s > |a|$

⑦ $L\{\cosh at\} = \frac{s}{s^2-a^2}$ $\quad s > |a|$

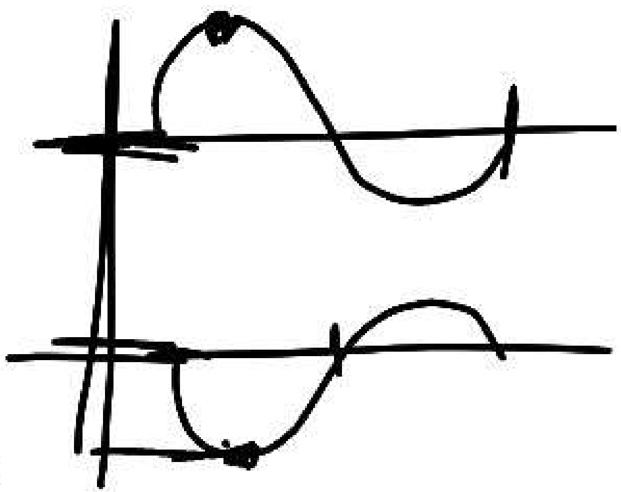
⑧ $L\{t^n\} = \frac{\Gamma n+1}{s^{n+1}}$ $\quad (n > -1)$

$$\begin{cases} \frac{n!}{s^{n+1}}, & \text{where } n \in N \\ \frac{n\Gamma n}{s^{n+1}} & (n \notin z) \end{cases}$$

AC Circuit Analysis

Sinusoidal voltage signal.

$$v = V_m \sin\theta$$
$$= V_m \sin \omega t$$
$$= V_m \sin 2\pi f t$$
$$= V_m \sin \frac{2\pi}{T} t$$

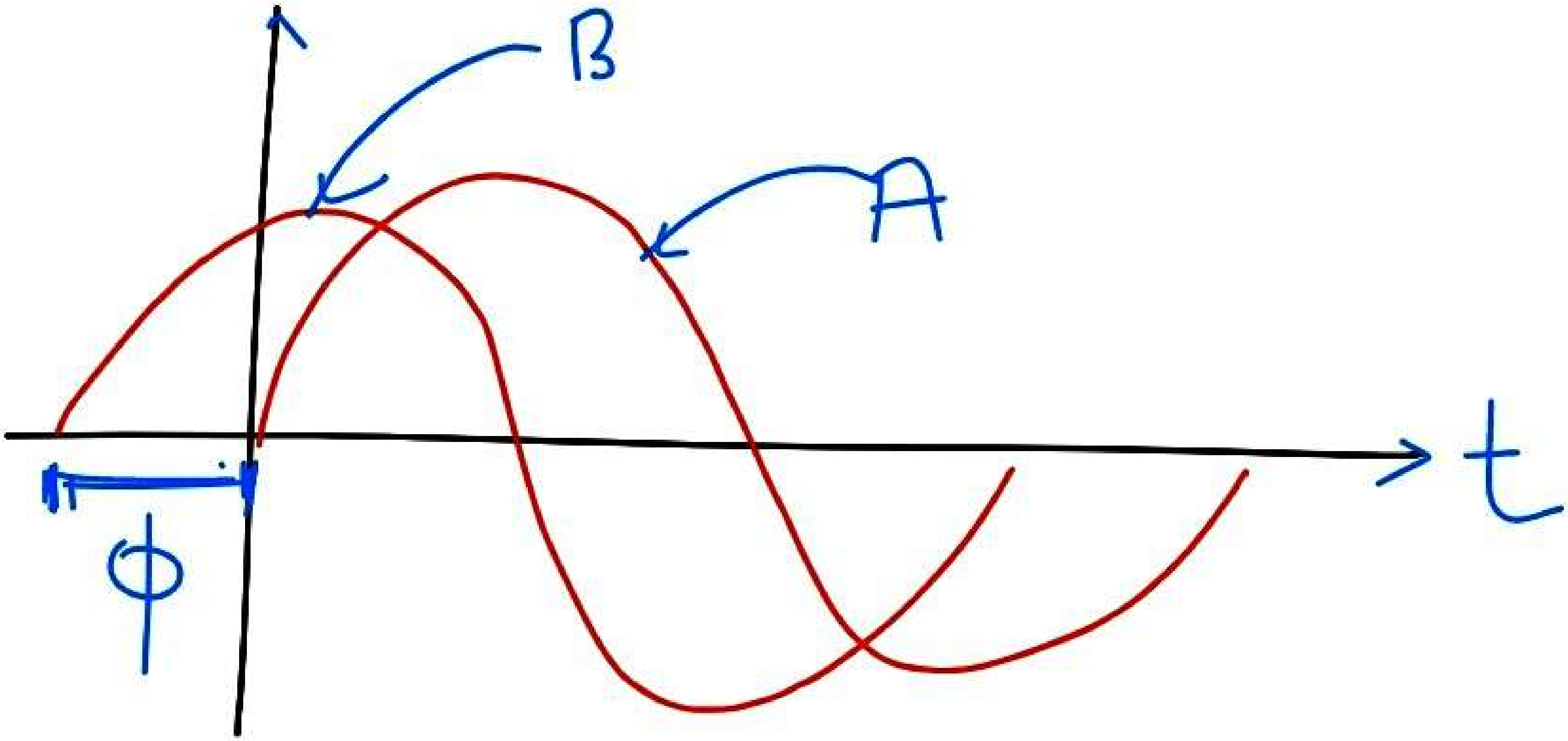


Note: (+) sign when used in connection with the phase difference denotes lead.

(−) sign means lag.

$$V_A = V_m \sin \omega t$$
$$V_B = V_m \sin (\omega t + \phi)$$

$\Rightarrow$ quantity B is leads A by a phase angle ϕ


B
A
t
ϕ

Let $x(t)$ be the periodic signal of period T.

then $x(t \pm T) = x(t)$

Average value = DC value.

$$\boxed{X_{avg} = X_{dc} = \frac{1}{T}\int_0^T x(t)\,dt}$$

$$= \frac{\text{Area of the signal over one period}}{\text{Period.}}$$

RMS value $= X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\, dt}$

3) Peak Factor $= \dfrac{x(t)_{max}}{x(t)_{avg}}$.

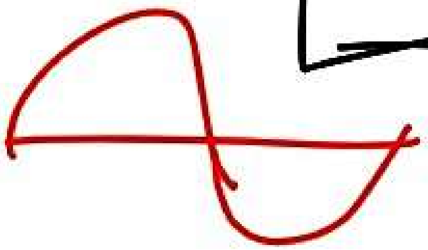
$$= \frac{X_{max}}{X_{Avg}}.$$

(4) $\text{Form factor} = \dfrac{X_{rms}}{X_{avg}}$.

Note:- The avg. of value of sine or cosine function of any phase & freq. is zero.

And its rms value

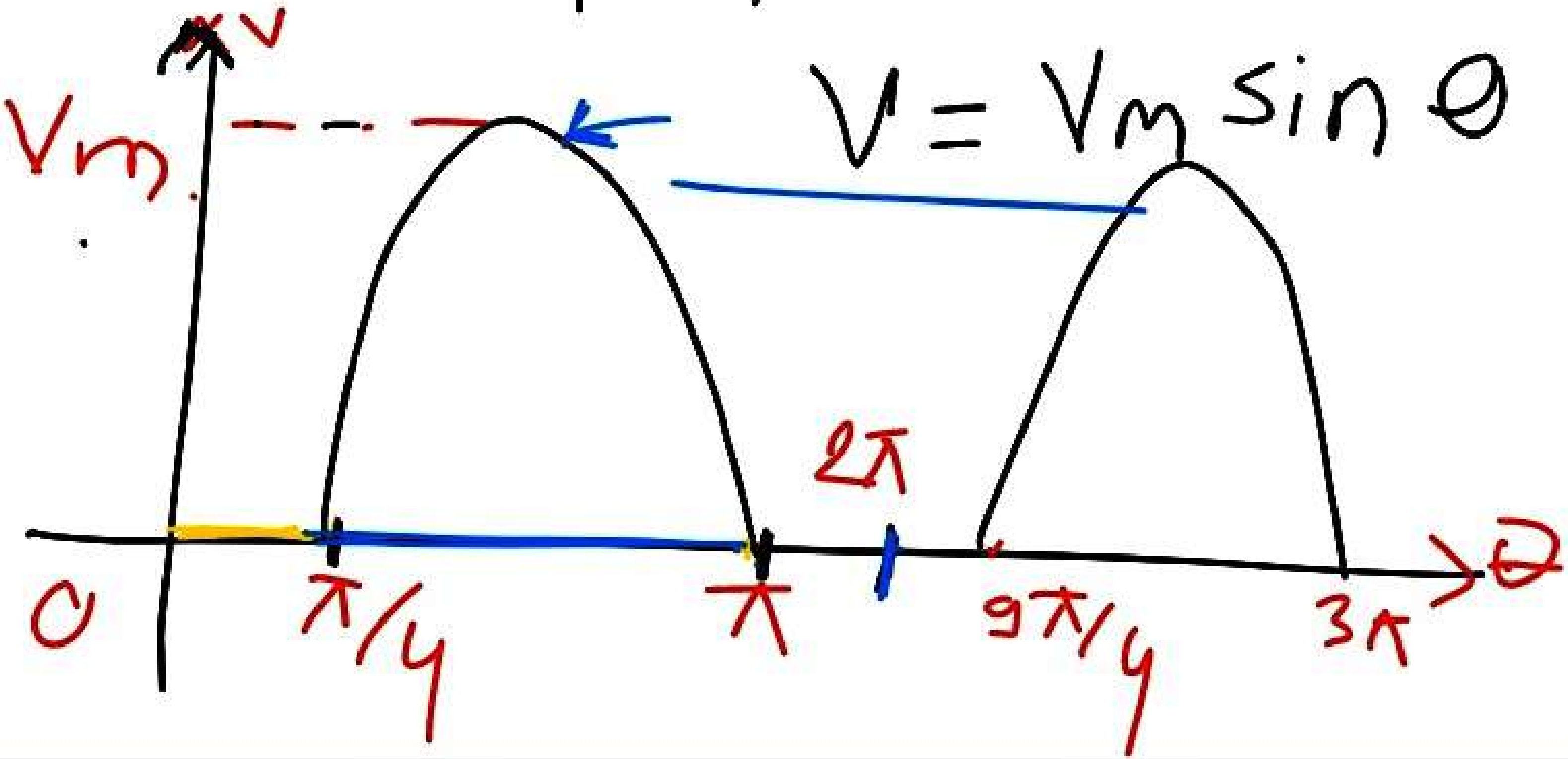
$$\boxed{\frac{\text{Max value}}{\sqrt{2}}}$$



$$f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots \ldots b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots$$

$f(t)$ RMS value $= \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 \ldots + b_1^2 + b_2^2}$

Find the avg. value & rms value of the waveform.



$$\dot{V} = 0$$

$$= V_m \sin\theta$$

$$= 0$$

$$0 < \theta < \pi/4$$

$$\pi/4 < \theta < \pi$$

$$\pi < \theta < 2\pi$$

$$V_{avg} = \frac{1}{2\pi} \int_{0}^{2\pi} v(\theta)\, d\theta \qquad T = 2\pi$$

$$= \frac{1}{2\pi} \int_{\pi/4}^{\pi} V_m \sin\theta \, d\theta$$

$=$

$$\Rightarrow \frac{V_m}{2\pi}\left(-\cos\theta\right)_{\pi/4}^{\pi}$$

$$\Rightarrow \frac{3}{4} \frac{V_m}{2\pi}$$

$$\frac{V_m}{2\pi} \times \left(1 + 0.707\right)$$

$$= 0.272\, V_m$$

$\underline{\underline{V_{avg}}}$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta \, d\theta}$$

$$=$$

$$\sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi}\sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

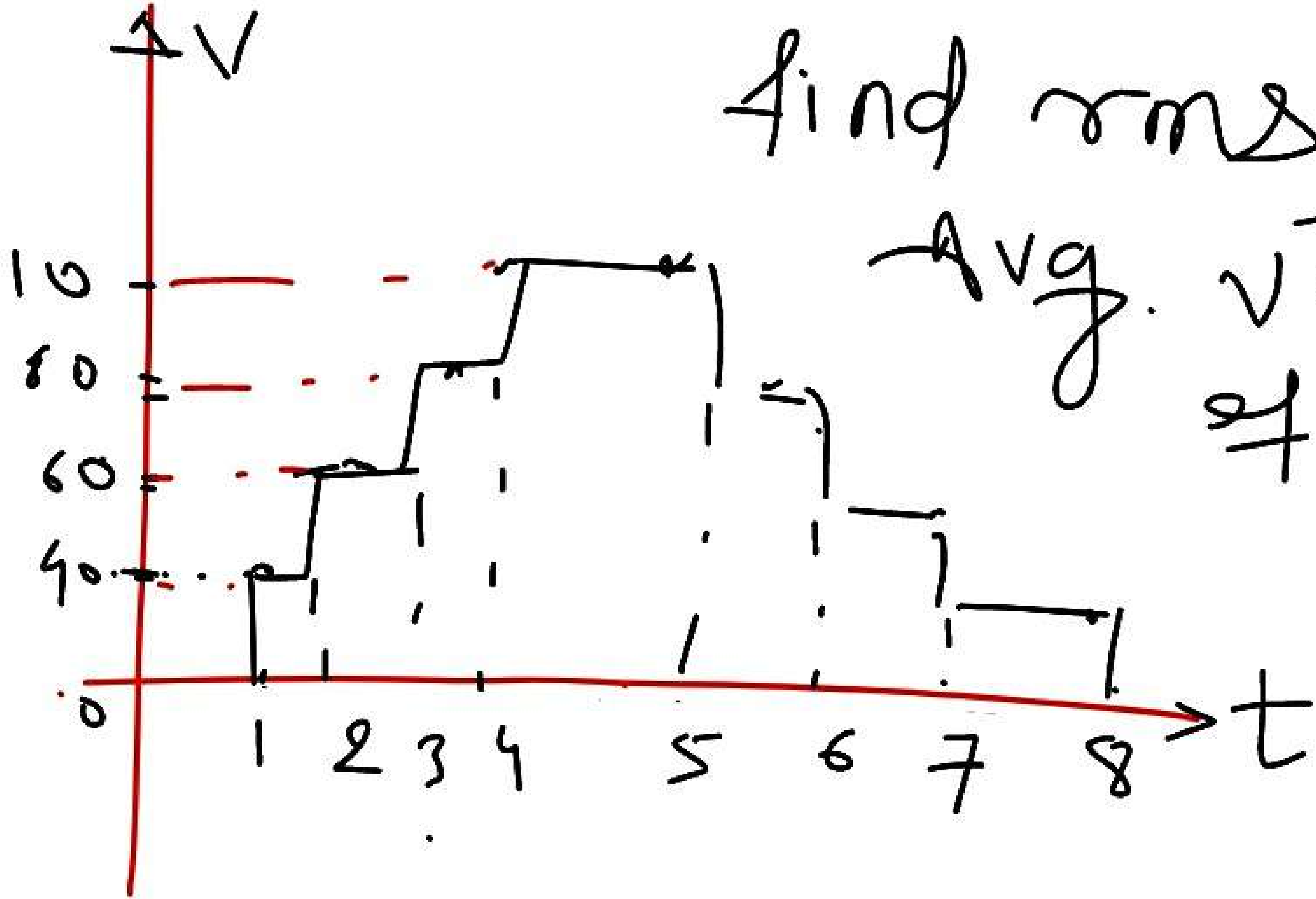
$$\sqrt{\frac{V_m^2}{2\pi}\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left(\frac{\cancel{\pi}}{2}-\frac{\sin 2\pi}{4}-\frac{\pi}{8}+\frac{\sin \pi/2}{4}\right)}$$

$$\sqrt{0.227 V_m^2}$$

$$= 0.476 V_m.$$

$$\boxed{V_{rms} = 0.476 V_m.}$$



find rms & Avg. value of wave form.

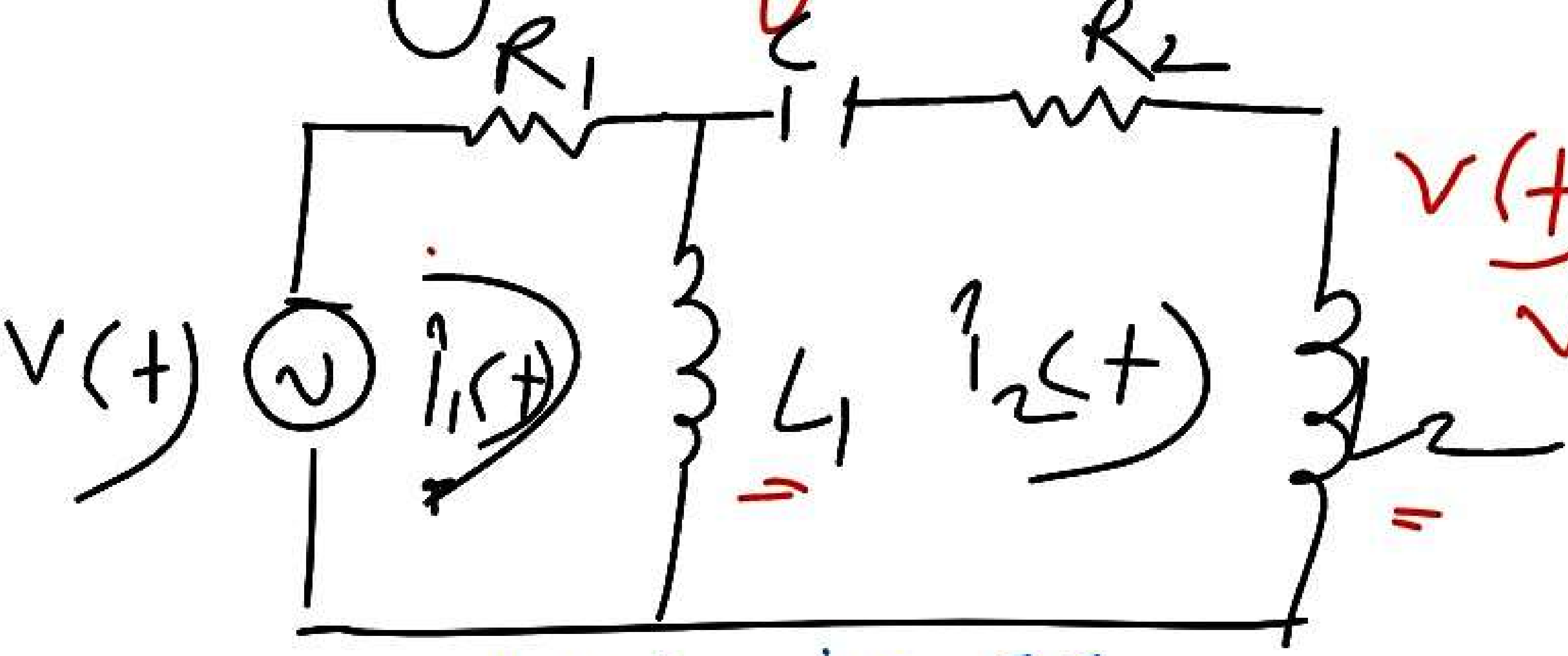
$$\underline{\underline{V_{avg}}} = \frac{0+40+60+80+100+80+60+40}{8}$$

$$\Rightarrow \boxed{57.5\,V.}$$

$$V_{rms} = \sqrt{\frac{0^2+40^2+60^2+80^2+100^2+80^2+60^2+40^2}{8}}$$

$$\Rightarrow \boxed{64.42\,V}$$

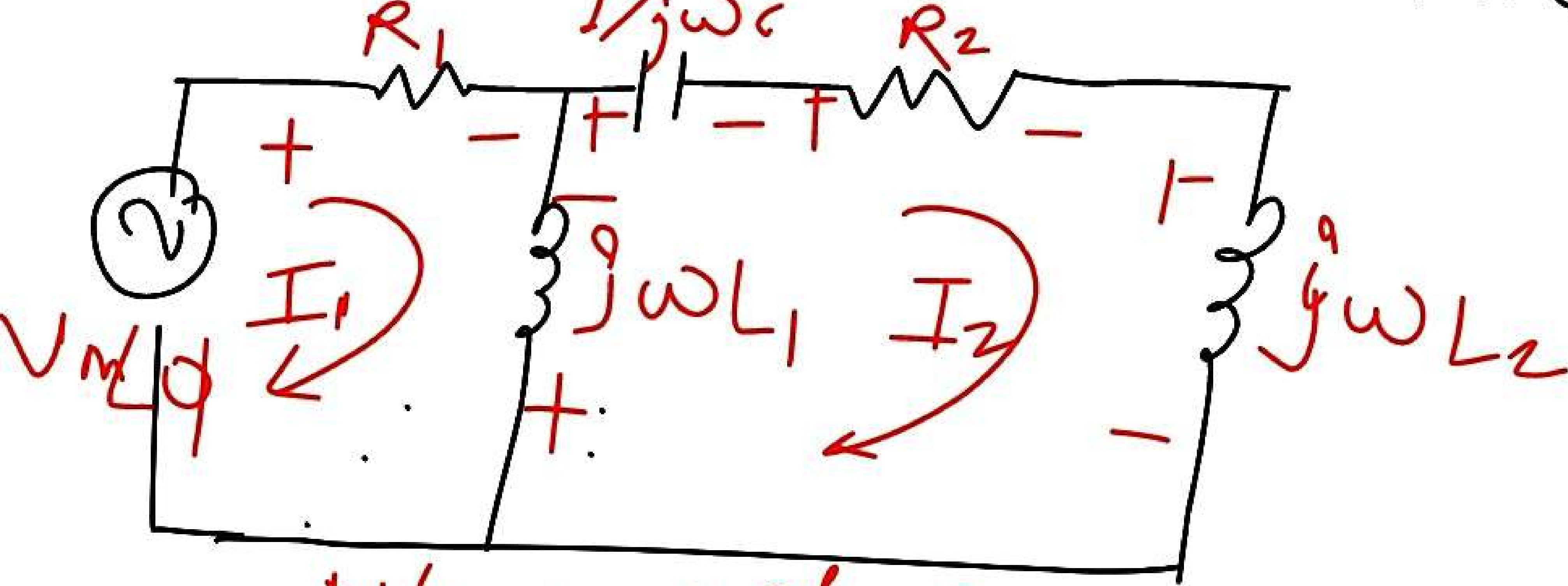
(2) Sinusoidal Steady state Analysis By using Phasor.



$v(t) = V_m \cos(\omega t + \phi)$

NW. in SS.

Transform → Phasor form



N/w in Phasor.

$$V = ZI$$

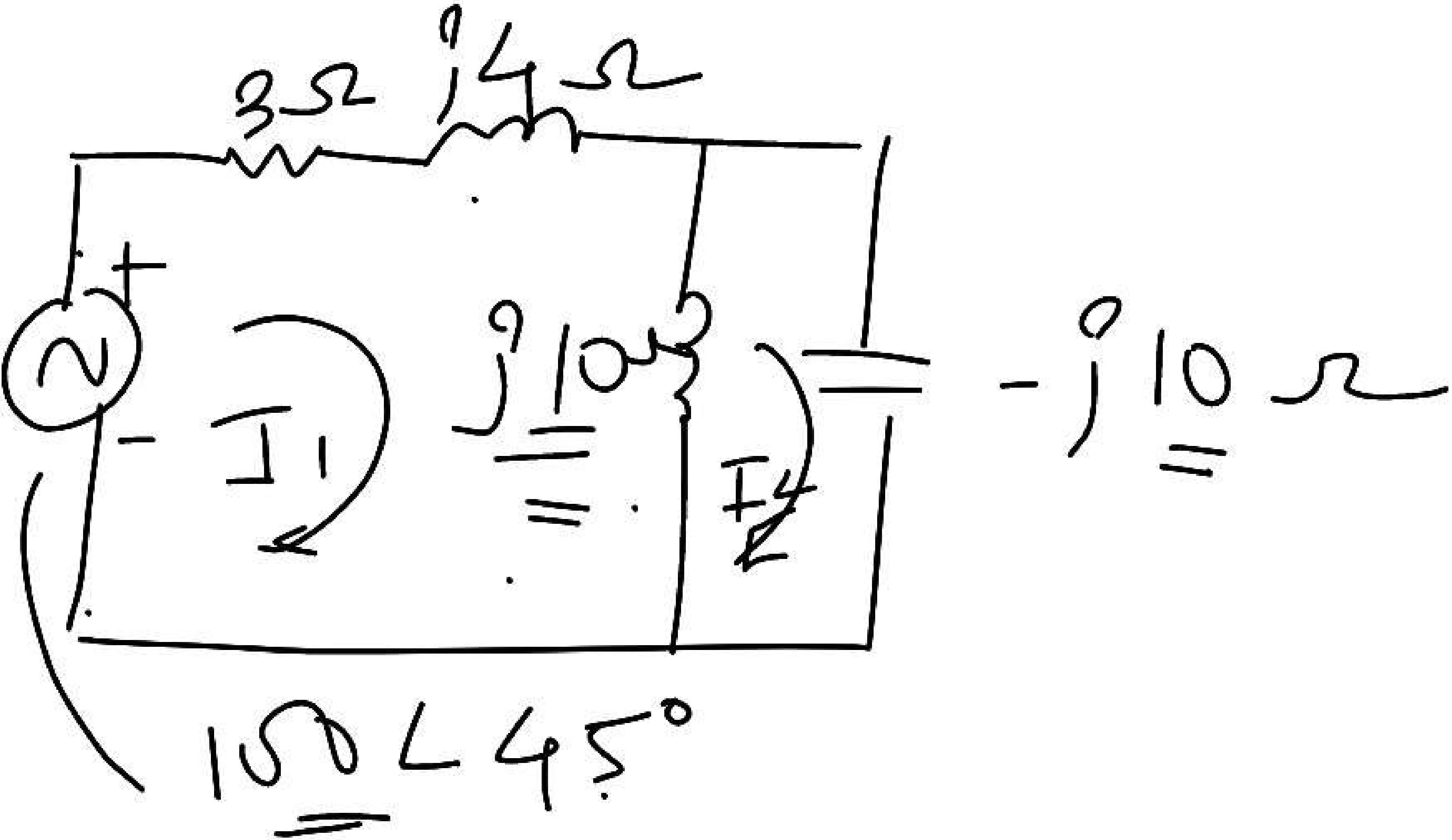
By KVL

$$\Rightarrow V_m \angle \Phi - R_1 I_1 - j\omega L (I_1 - I_2) = 0 \quad \text{— (1)}$$

$$\Rightarrow \frac{-1}{j\omega c} I_2 - R_2 I_2 - j\omega L_2 I_2 - j\omega L_1 (I_2 - I_1) = 0$$

$$I_1 = \frac{\Delta_1}{\Delta}\Bigg|_{i_1(t)} =$$

$$I_2 = \frac{\Delta_2}{\Delta}\Bigg|_{i_2(t)}$$


3Ω
j4Ω
+
-
I1
j10Ω
I2
-j10Ω
100∠45°

$\Rightarrow$ for mesh 1

$$\Rightarrow 100\angle 45^\circ - (3+j4)I_1 - j10(I_1 - I_2) = 0$$

$$(3+j14)I_1 - j10 I_2 = 100\angle 45^\circ \quad \text{--- (1)}$$

Mesh 2

$$\Rightarrow -j10(I_2 - I_1) + j10 I_2 = 0$$

$$j10 I_1 = 0$$

$$I_1 = 0$$

substitute in 1

$$I_1 = 0$$

$$-j10I_2 = 100\angle 45^\circ$$

$$I_2 = \frac{100\angle 45}{10\angle -90^\circ}$$

$$\boxed{I_2 = 10\angle 135^\circ \text{ Amp.}}$$

AC Circuit Analysis

Sinusoidal voltage signal.

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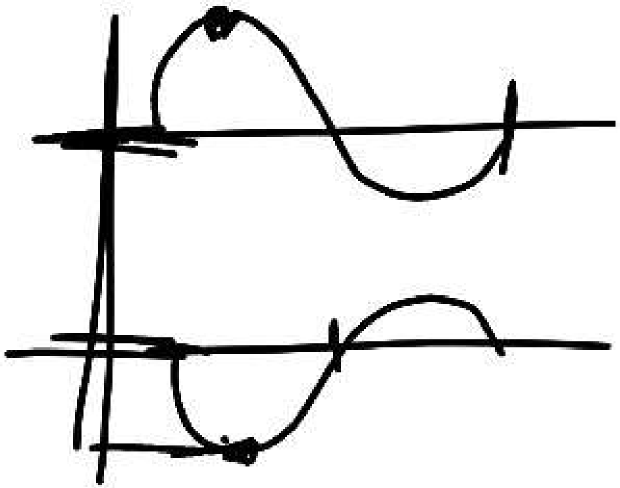
$$= V_m \sin \omega t$$

$$= V_m \sin 2\pi f t$$

$$= V_m \sin \frac{2\pi}{T} t$$



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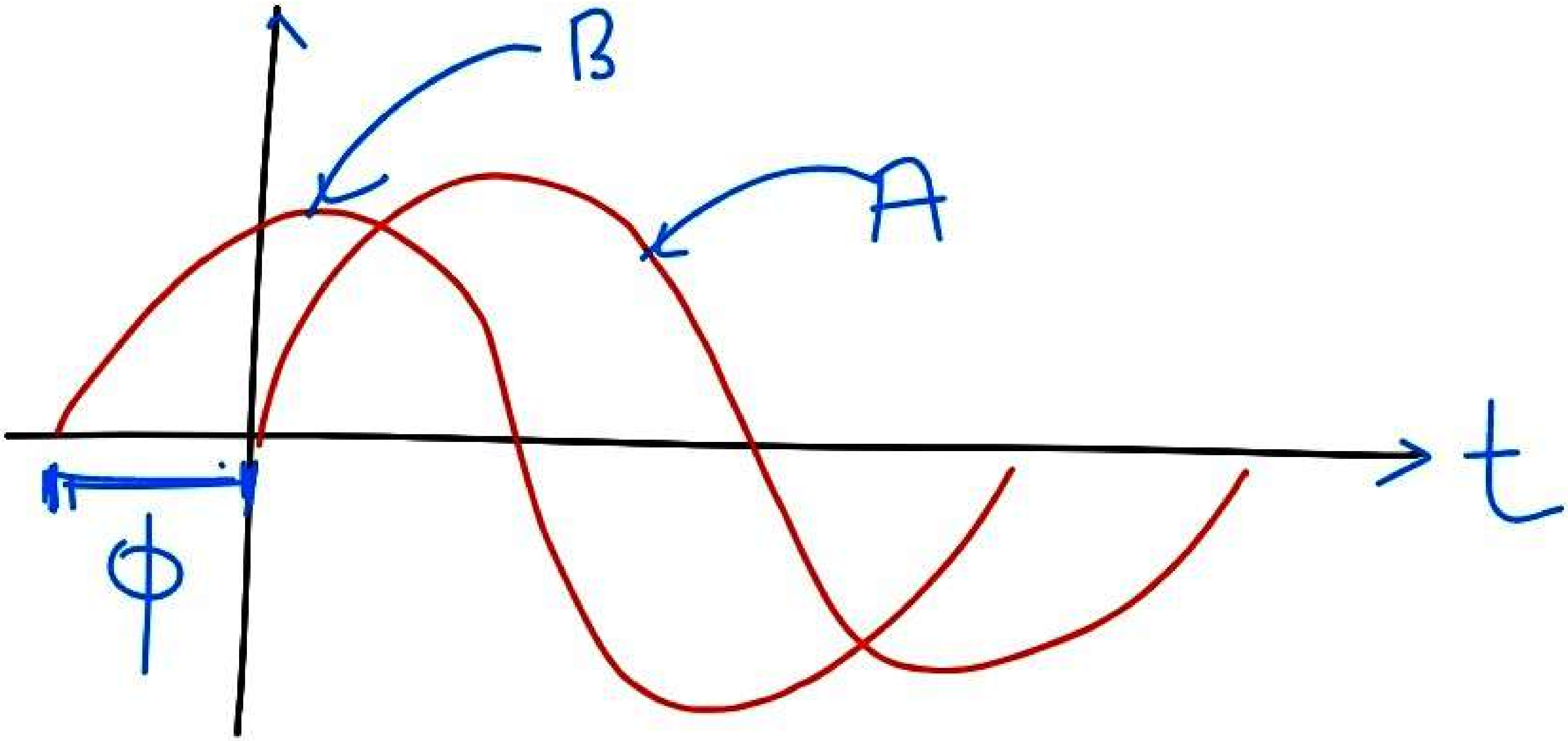
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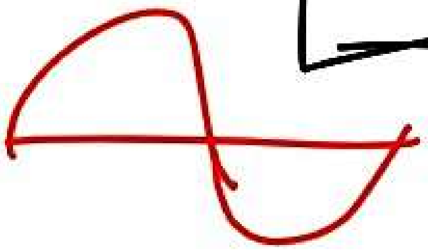

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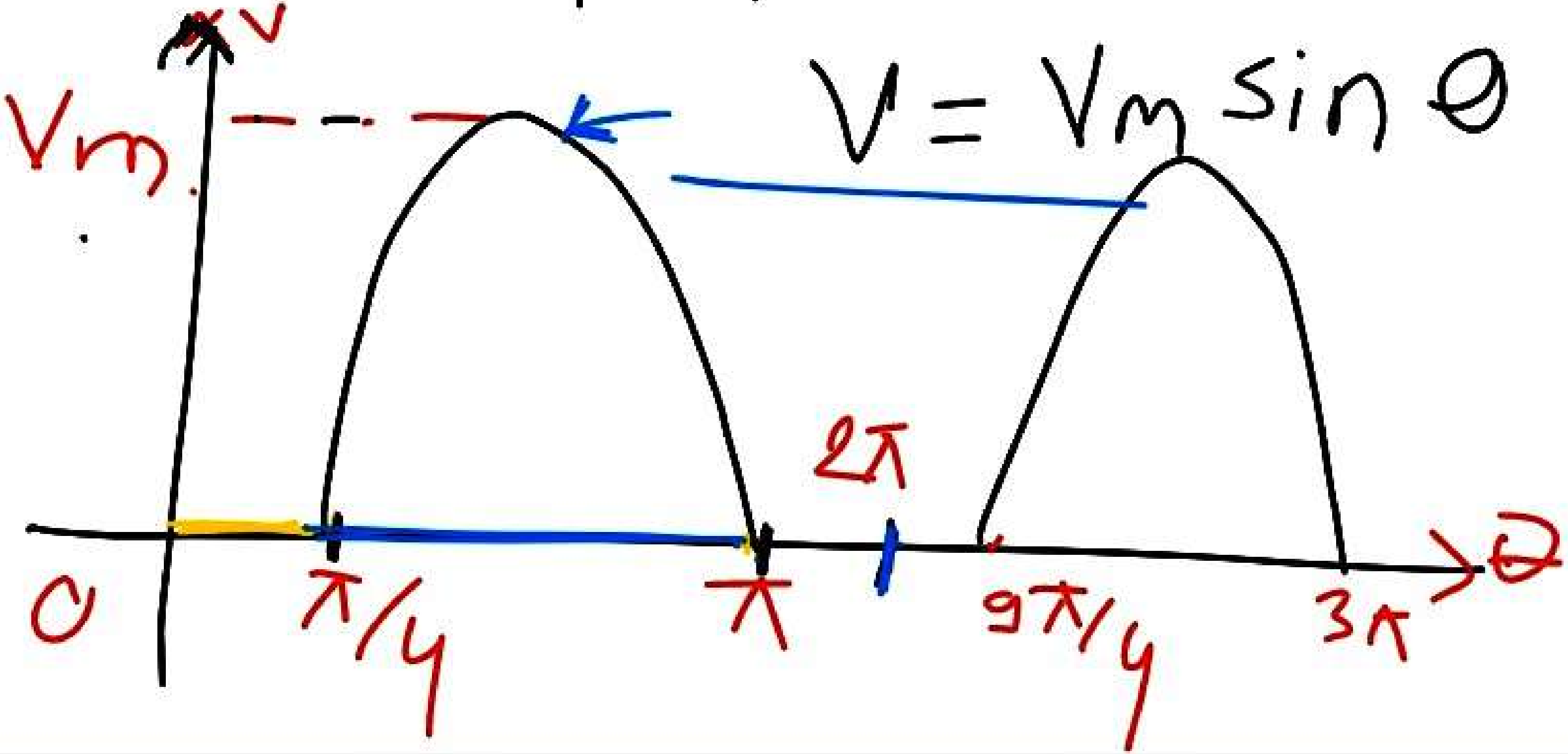
$$b_2 \sin 2\omega t + \ldots$$

$f(t)$

RMS value $= \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 \ldots + b_1^2 + b_2^2}$

Find the avg. value & rms value of the waveform.



$$V = 0$$

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$=$

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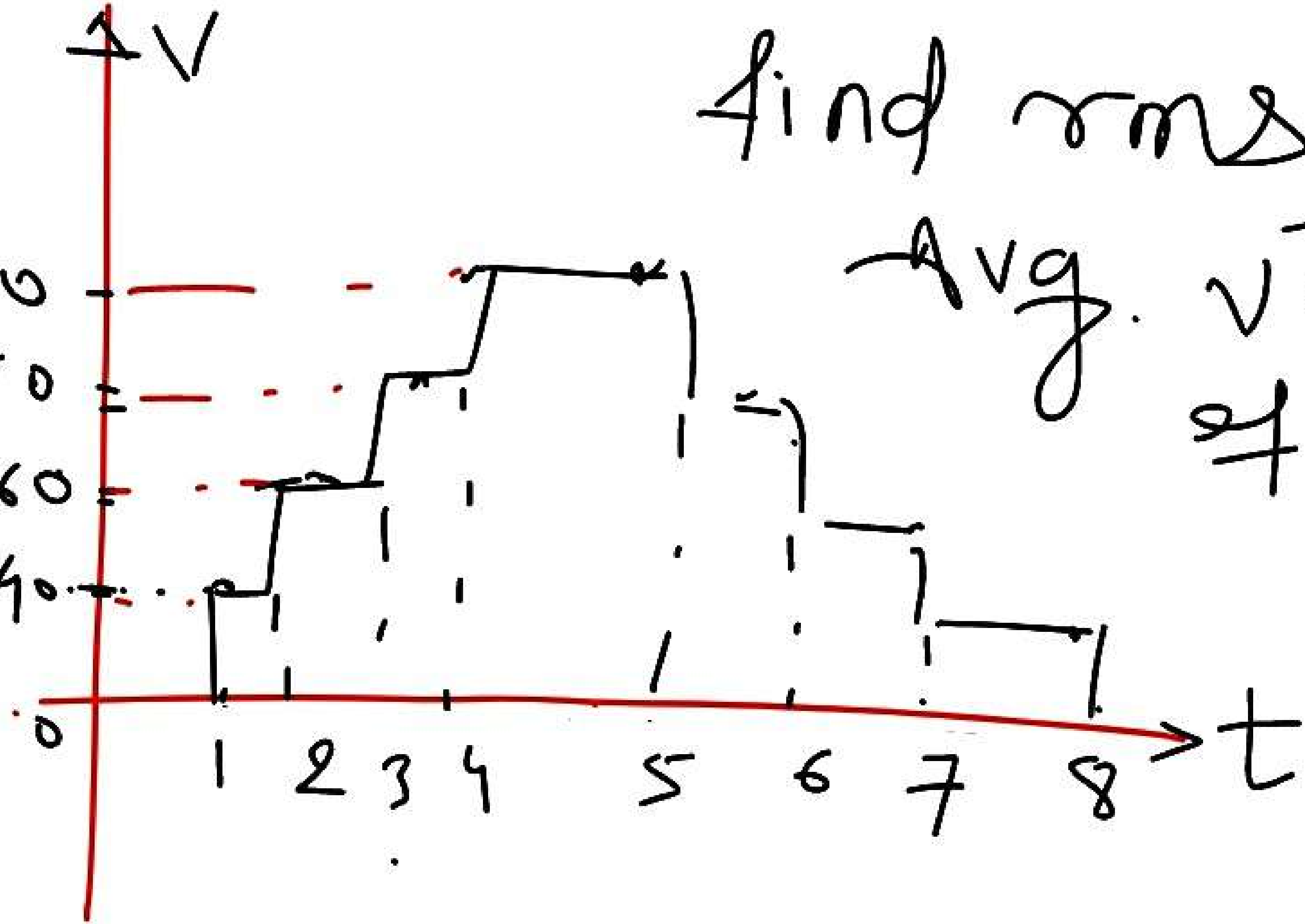

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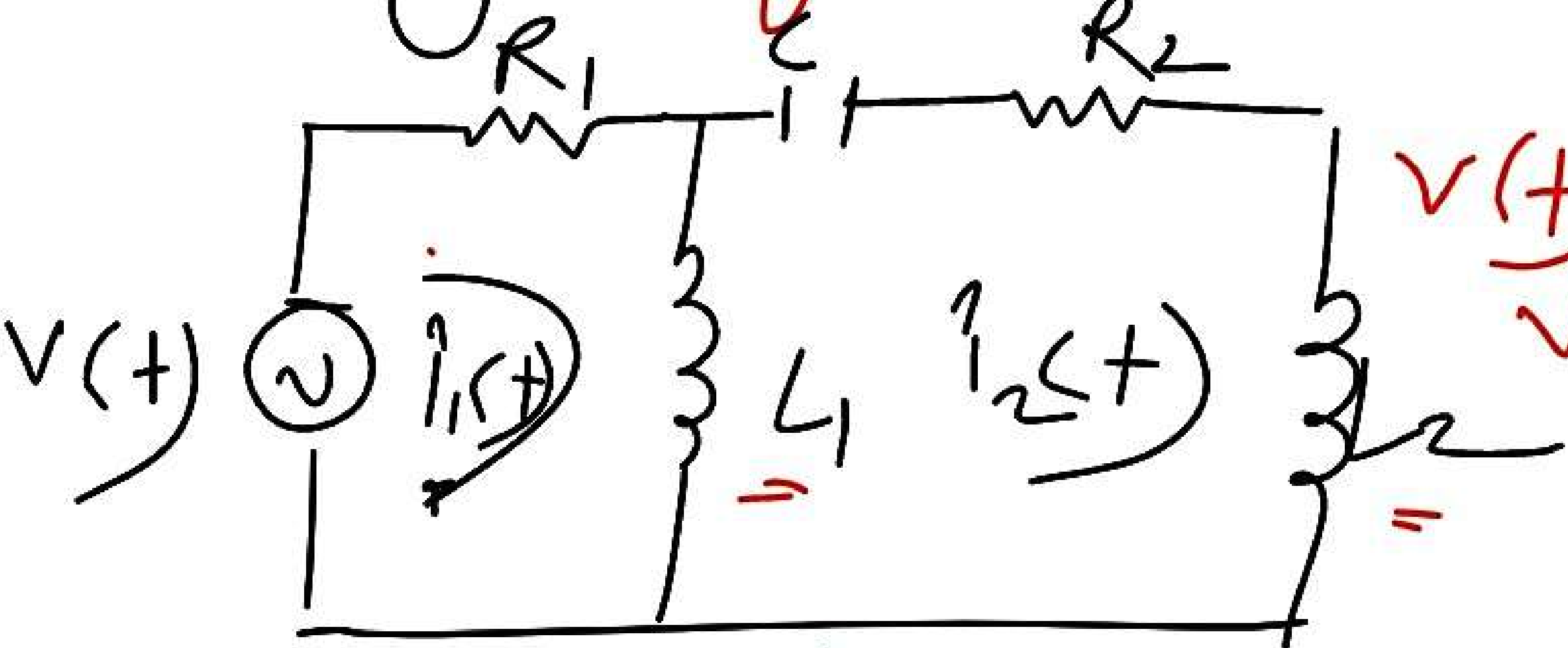
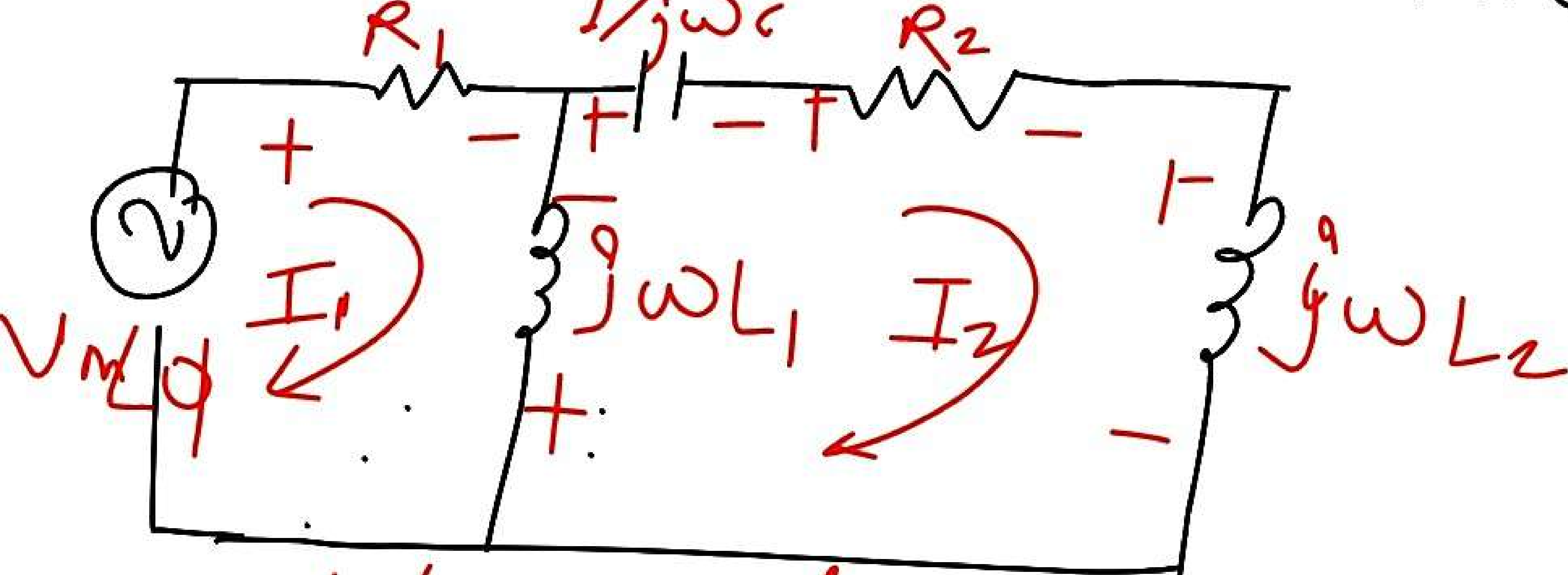


$v(t) = V_m \cos(\omega t + \phi)$

NW. in SS.

Transform $\rightarrow$ phasor form



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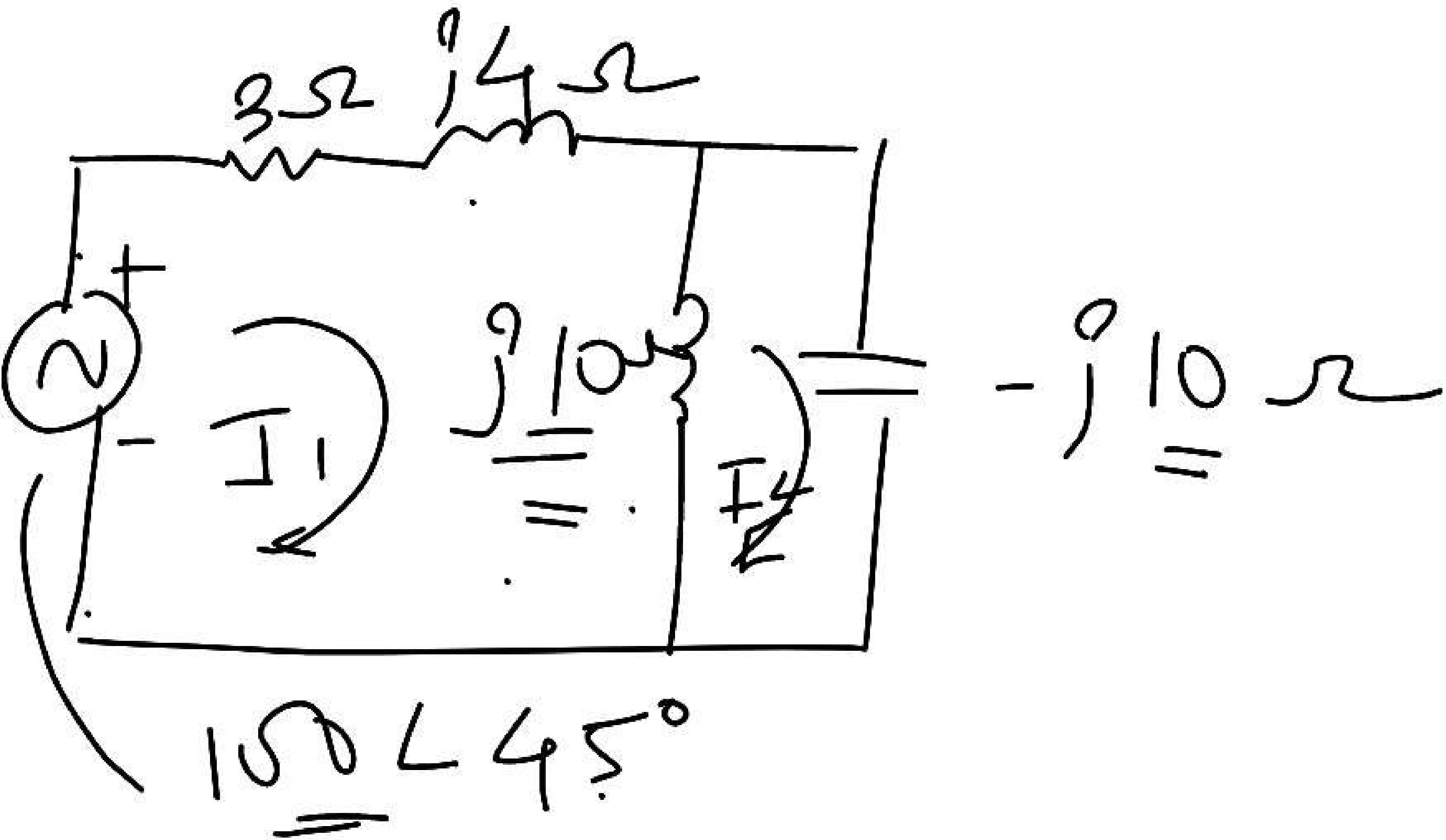
$$\Rightarrow V_m\angle\phi - R_1 I_1 - j\omega L(I_1 - I_2) = 0 \quad \text{--- (1)}$$

$$\Rightarrow \frac{-1}{j\omega c} I_2 - R_2 I_2 - j\omega L_2 I_2 - j\omega L_1 (I_2 - I_1) = 0$$

$$I_1 = \frac{\Delta_1}{\Delta}\Big|_{i_1(t)} =$$

$$I_2 = \frac{\Delta_2}{\Delta}\Big|_{i_2(t)}$$



3Ω
j4Ω
+
-
I1
j10Ω
I2
-j10Ω
100∠45°

$\Rightarrow$ for mesh 1

$$\Rightarrow 100\angle 45^\circ - (3+j4)I_1 - j10(I_1 - I_2) = 0$$

$$(3+j4)I_1 - j10 I_2 = 100\angle 45^\circ \quad \text{--- } (1)$$

Mesh 2

$$\Rightarrow -j10(I_2 - I_1) + j10 I_2 = 0$$

$j10 I_1 = 0$

$I_1 = 0$

substitute in 1

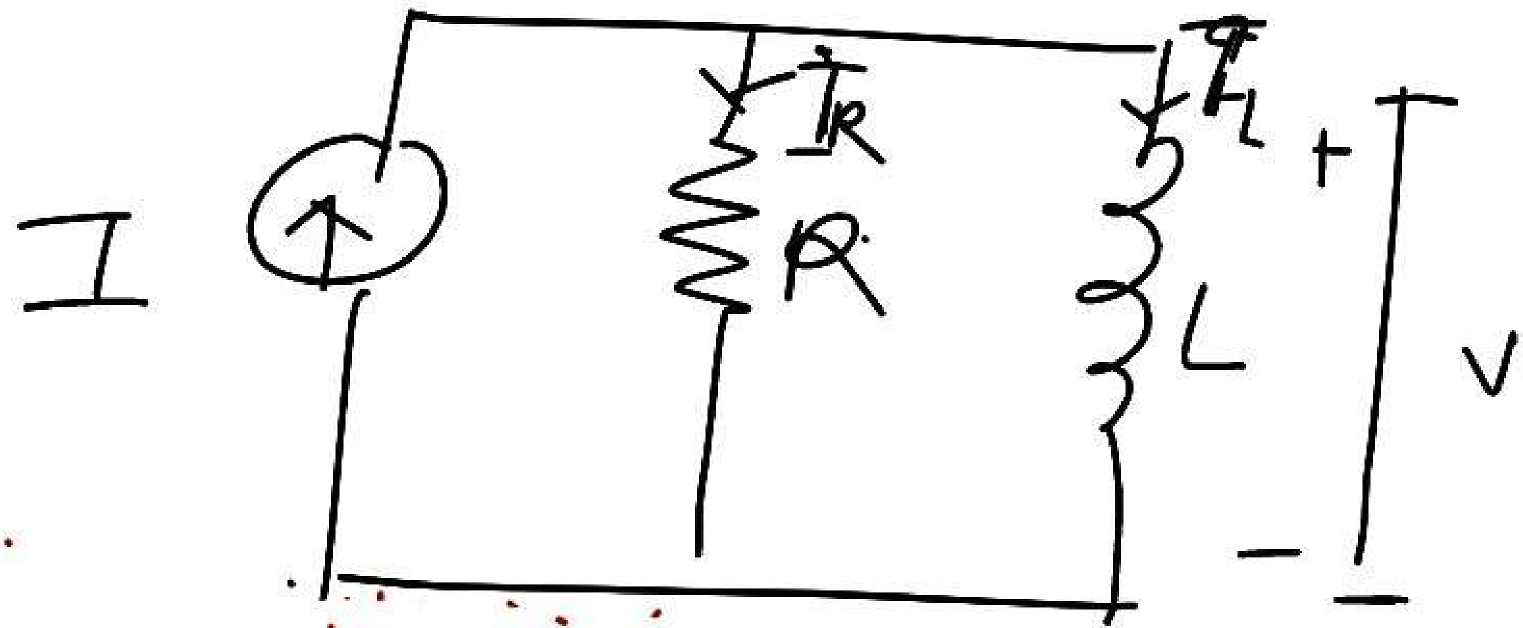
$I_1 = 0$

$$-j10I_2 = 100\angle 45^\circ$$

$$I_2 = \frac{100\angle 45}{10\angle -90^\circ}$$

$$\boxed{I_2 = 10\angle 135^\circ \text{ Amp.}}$$

Phasor for ||el RL ckts:



$$I_R = \frac{V}{R}$$

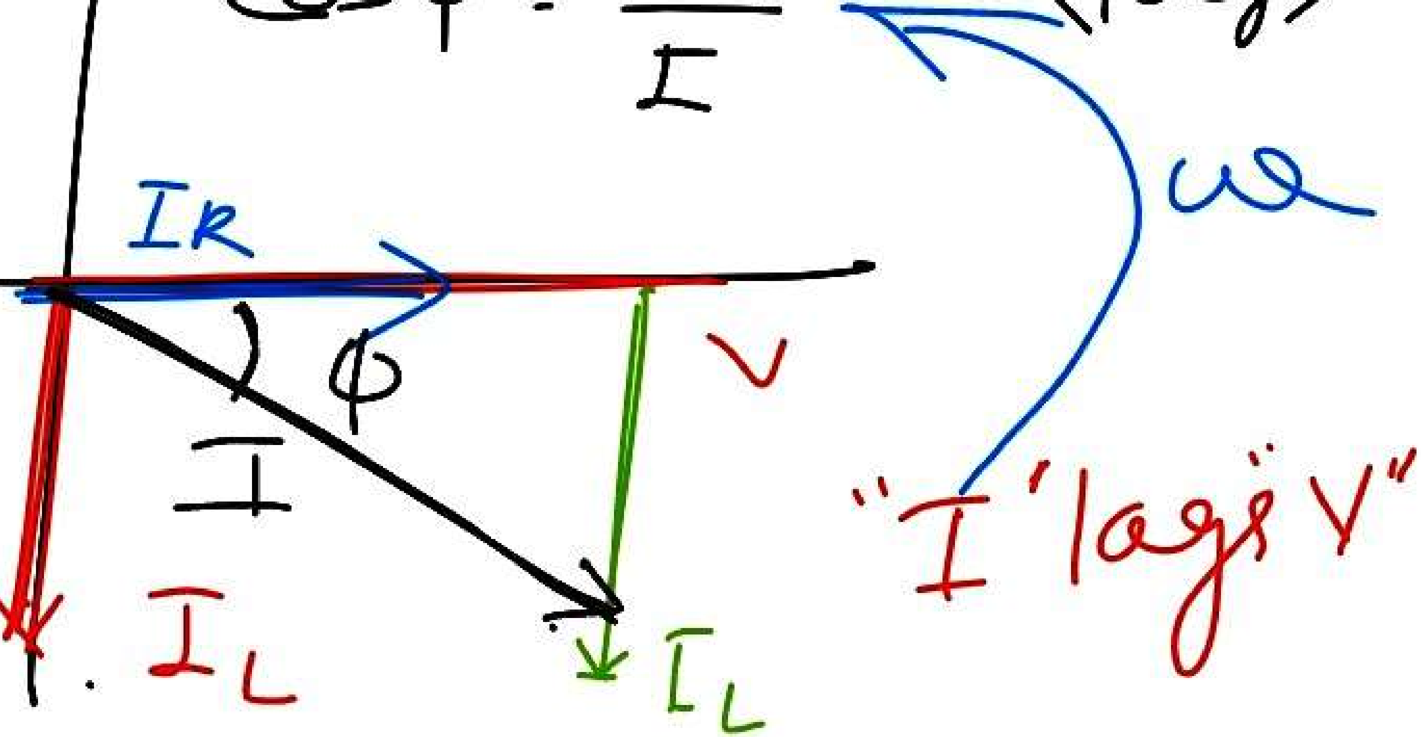
$$I_L = \frac{V}{X_L} \angle -90$$

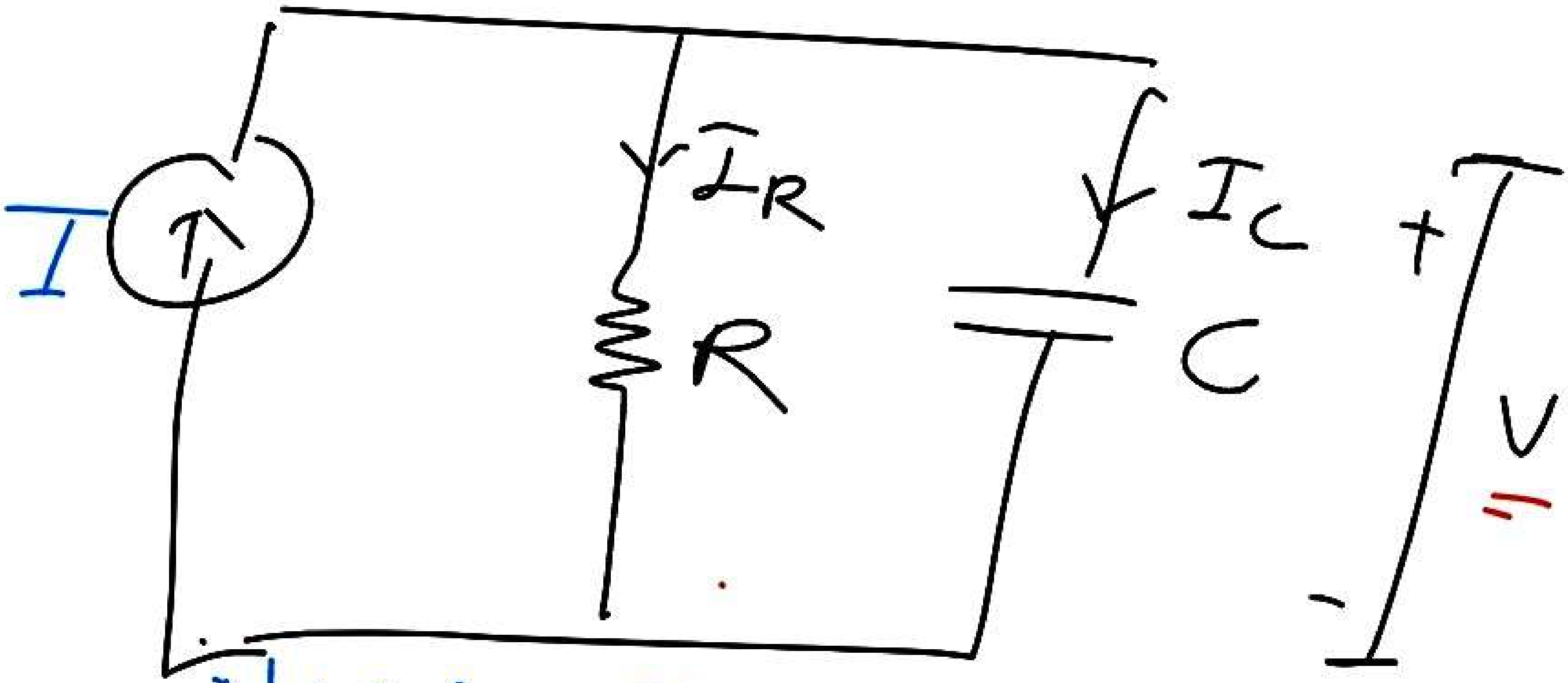
N/w in SS.

$$I = \sqrt{I_R^2 + I_L^2}$$

$$\cos\phi = \frac{I_R}{I} \quad (\text{lag})$$

$$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right)$$





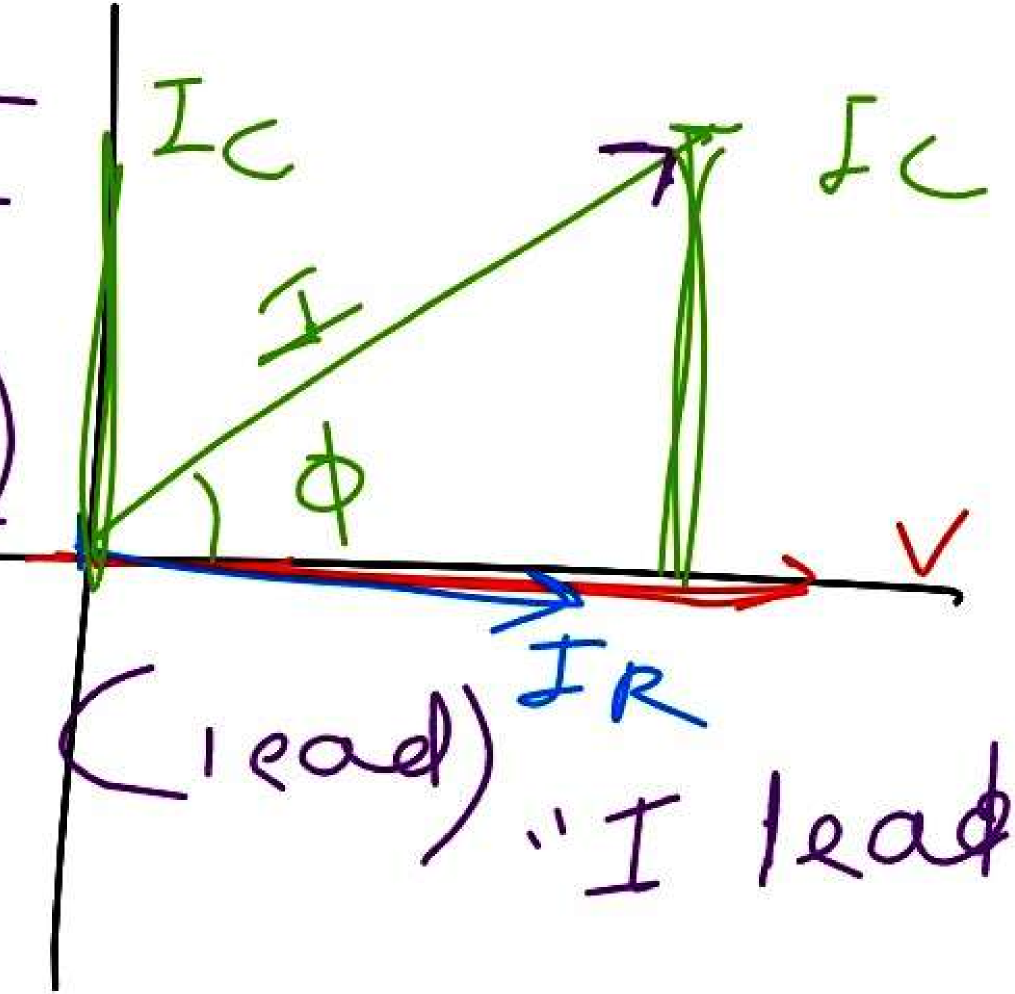
N/w in SS.

$I_R = \frac{V}{R}$ & $I_C = \frac{V}{X_C} \angle 90^\circ$

$$I = \sqrt{I_R^2 + I_C^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

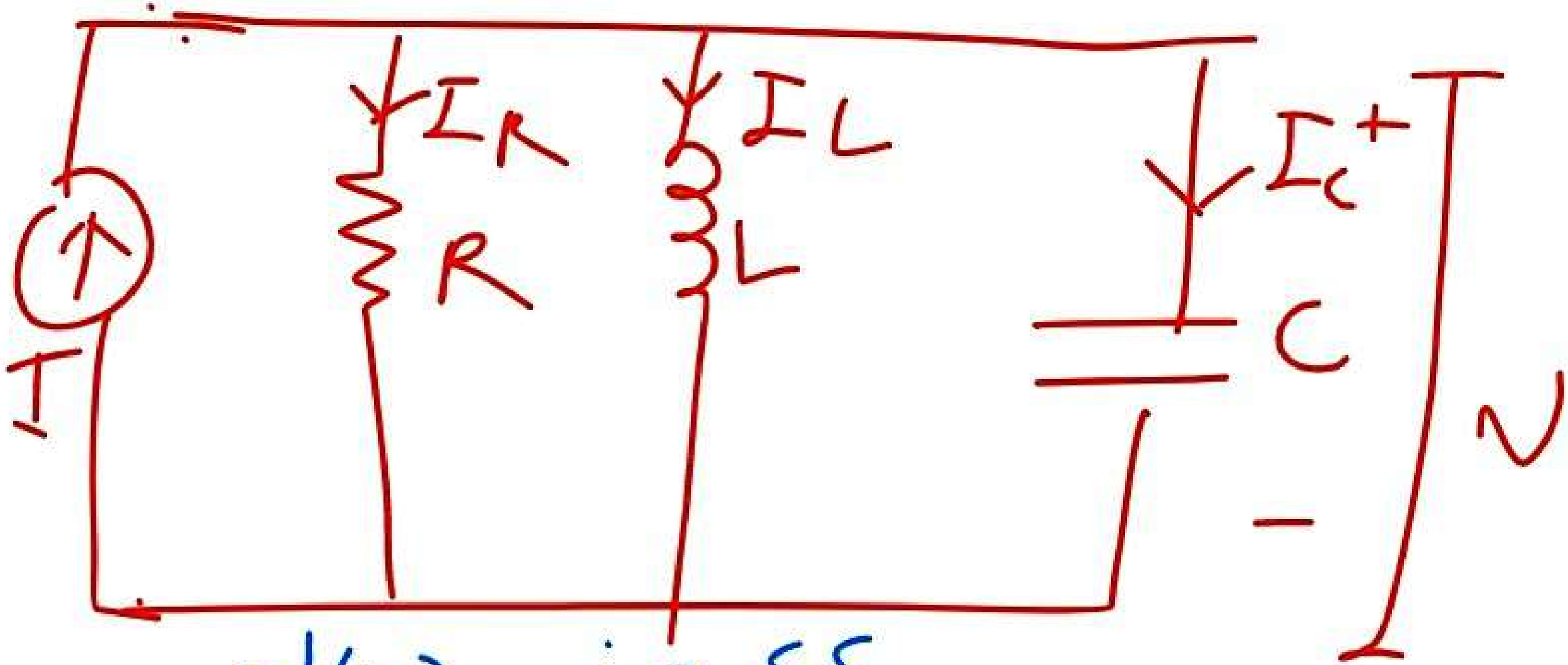
$$\cos\phi = \frac{I_R}{I}$$



(lead)

"I lead V"

||el RLC ckt.



N/w. in SS.

$$I_R = \frac{V}{R}$$

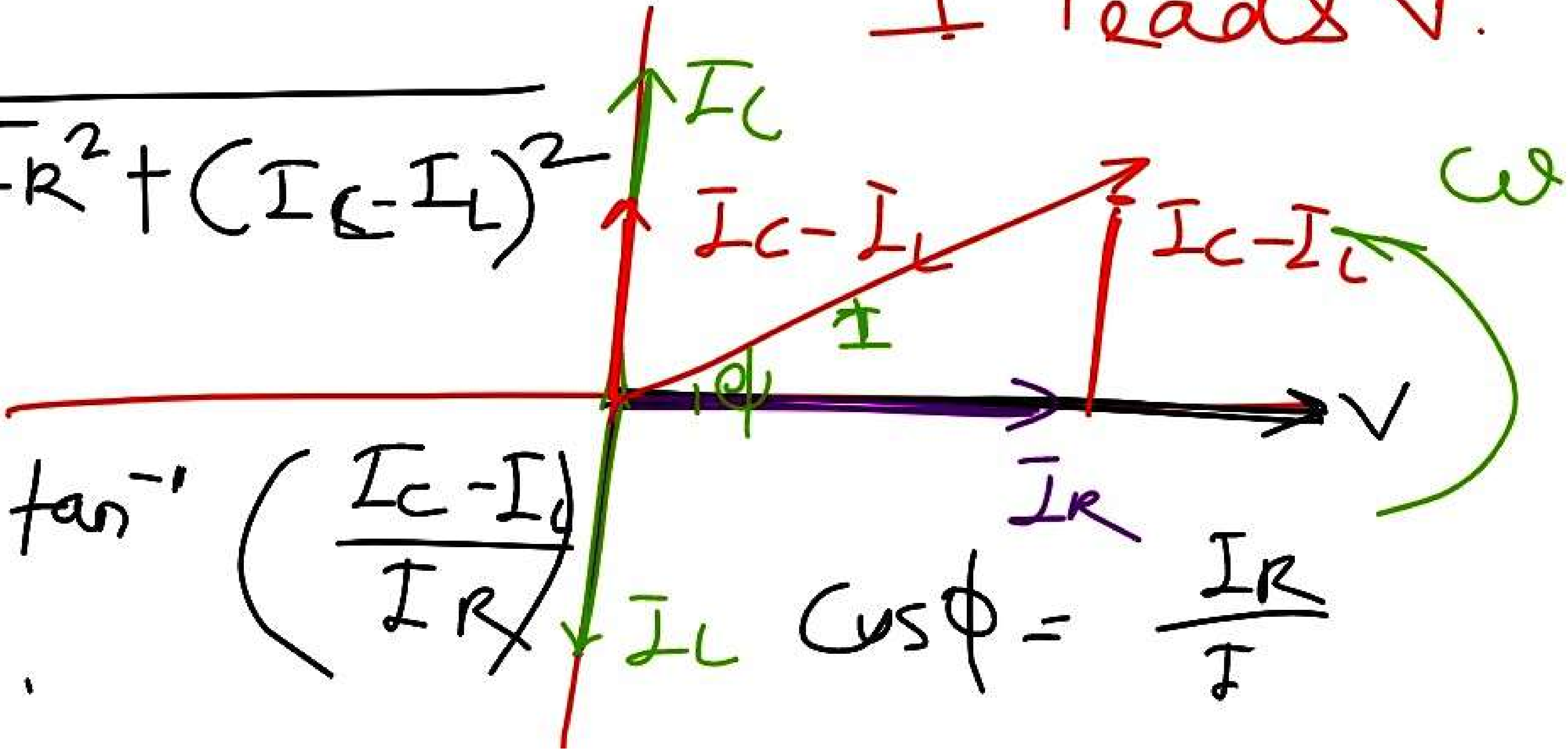
$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

(1) $I_C > I_L$

I leads V.

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$



$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I}$$

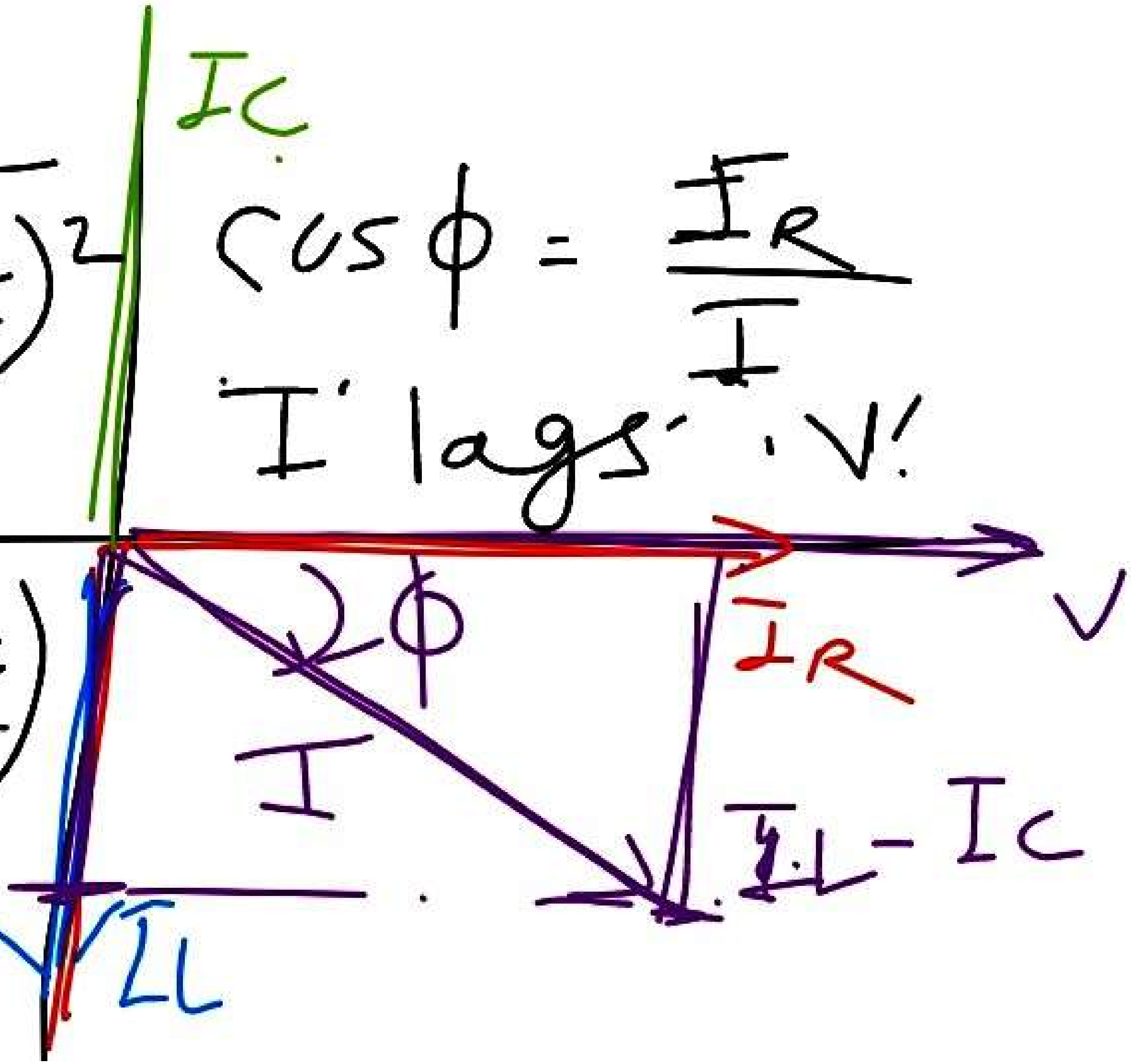
2) $I_L > I_C$

$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

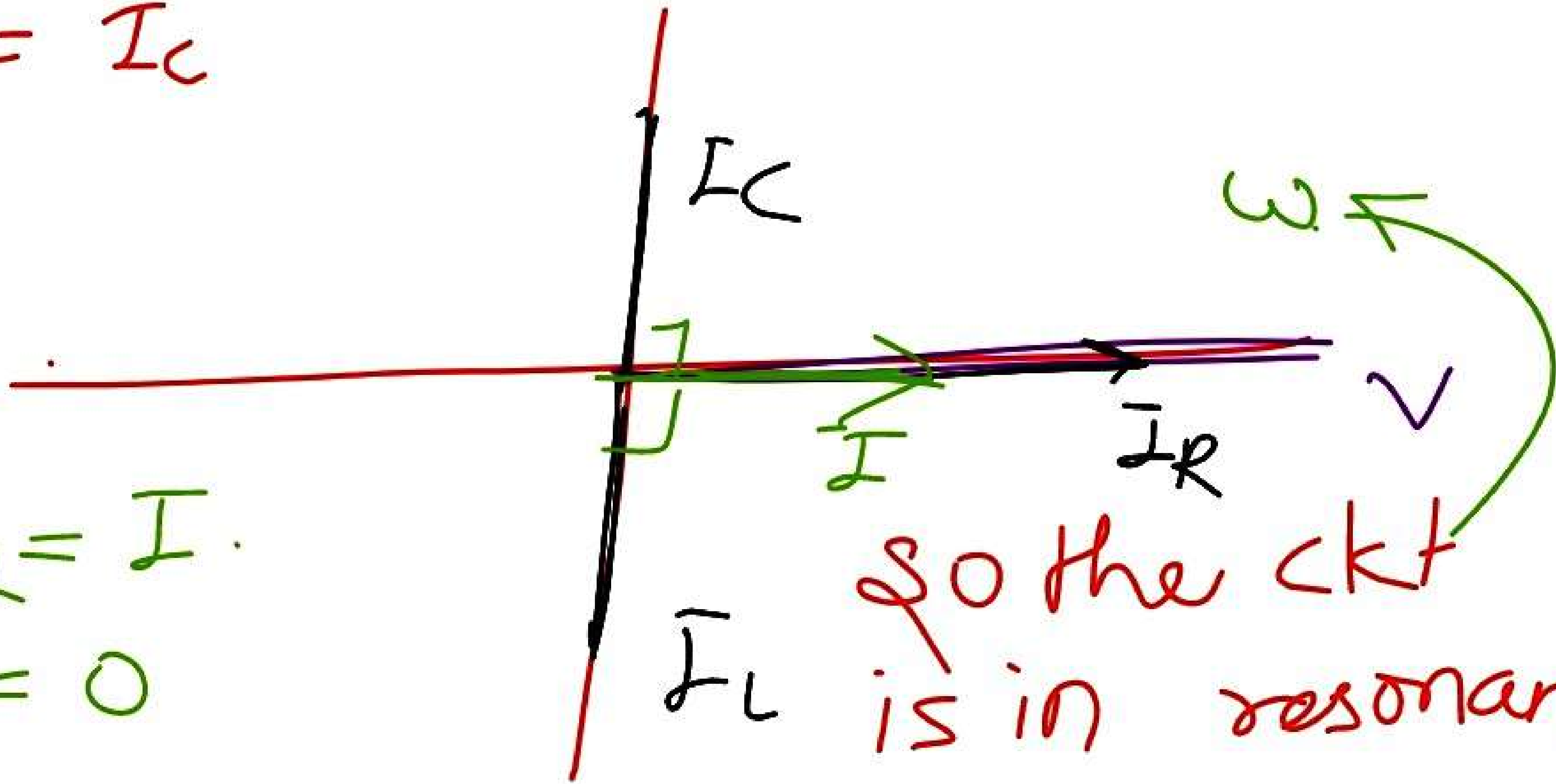
$$\cos\phi = \frac{I_R}{I}$$

I lags V!

$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$



$I_L = I_C$



$I_R = I$.

$\phi = 0$

so the ckt is in resonance

Power calculations in AC.

For DC circuit.

$$P_{DC} = VI = \frac{V^2}{R} = I^2R$$

Power in any instant in AC ckt is called instantaneous power. & it is defined by

$$p = vi$$

$$\left.\begin{aligned} P &= \frac{1}{T}\int_0^T vi\,dt. \\ P &= \frac{1}{2\pi}\int_0^{2\pi} vi\,dt \end{aligned}\right\} \text{Avg power}$$

Power in pure resistive ckt

$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

$$P = vi$$

$$= V_m I_m \sin^2 \omega t$$

$$= \frac{1}{2} V_m I_m (1 - \cos 2\omega t)$$

Active power is given by

$$P = \frac{1}{T} \int_0^T v i \, dt$$

$$\frac{V_m I_m}{2T}\int_0^T (1-\cos^2\omega t)\,dt$$

$$= \frac{1}{2} V_m I_m \Rightarrow \left(\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\right)$$

$$=$$

$\Rightarrow V_{rms} I_{rms}$ Watt.

Note:. Power waveform is +ve throughout the cyle.

$\Rightarrow$ As I & V are in phase.

⇒ power flows only in one direction.

⇒ It is consumed in R & appears in the form of heat.

Power in pure Inductive ckt.

V leads I by 90°.

$\Rightarrow$ $V = V_m \sin(\omega t + 90^\circ)$

$i = I_m \sin \omega t$

$= P = Vi$

$$V_m I_m \sin(\omega t + 90^\circ)\sin \omega t$$

$$= \frac{1}{2} V_m I_m (2\cos \omega t \sin \omega t)$$

$$= \boxed{\frac{1}{2} V_m I_m \sin 2\omega t}$$

$$P_L = -\frac{V_m I_m}{4\omega L}\left(\cos 4\pi - \cos 0\right)$$

$$P_L = 0$$

$\Rightarrow$ for pure Inductive ckt the active power is zero.

Reactive Power

→ It is the power supplied to Reactance (Inductive or Capacitive)

Q.

$$Q_L = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$= V_{rms} I_{rms}.$$

$$V_L = V_{rms} = X_L I_L$$

$$Q_L = V_L I_L$$

$$Q_L = I_L^2 X_L$$

$$Q_L = \frac{V_L^2}{X_L}$$

$Q_L \Rightarrow$ Rate of change of Reactive energy betn Load & Source

Power in capacitive ckt

voltage lag current by 90°

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t + 90^\circ)$

$p = vi$

$\frac{1}{2} V_m I_m \underline{2 \sin \omega t} \sin(\omega t \pm 90^\circ)$

$$\boxed{= \frac{1}{2} V_m I_m \sin 2\omega t}$$

$$P_c \Rightarrow \frac{1}{T_0} \int_0^T p \, dt = \frac{V_m I_m}{2 T_0} \int_0^T \sin 2\omega t \, dt$$

$\overline{\text{Avg power}} \Rightarrow$ Active power.

$$\frac{V_m I_m}{4\omega t}(1-1)=0$$

⇒ In pure capacitive ckt the active or Avg power is supplied over one complete cycle is zero.

$$Q_c = \frac{V_c^2}{X_c} \quad \text{or} \quad I_c^2 X_c$$

(Leading reactive power)

Q_c is rate of change of reactive power beth load & source Q_c is taken as -ve.

Power in general series ckt

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \phi)$

ϕ is phase diff betn V & I.

$\phi \Rightarrow -ve$. current leads the V

$$P_{ins.} \Rightarrow v\,i$$

$$\Rightarrow V_m I_m \sin\omega t \sin(\omega t - \phi)$$

$$\Rightarrow \frac{1}{2} V_m I_m \left(2 \sin\omega t \sin(\omega t - \phi)\right)$$

$$P_{ins.} = \frac{1}{2} V_m I_m \left(\cos\phi - \cos(2\omega t - \phi)\right)$$

Avg power

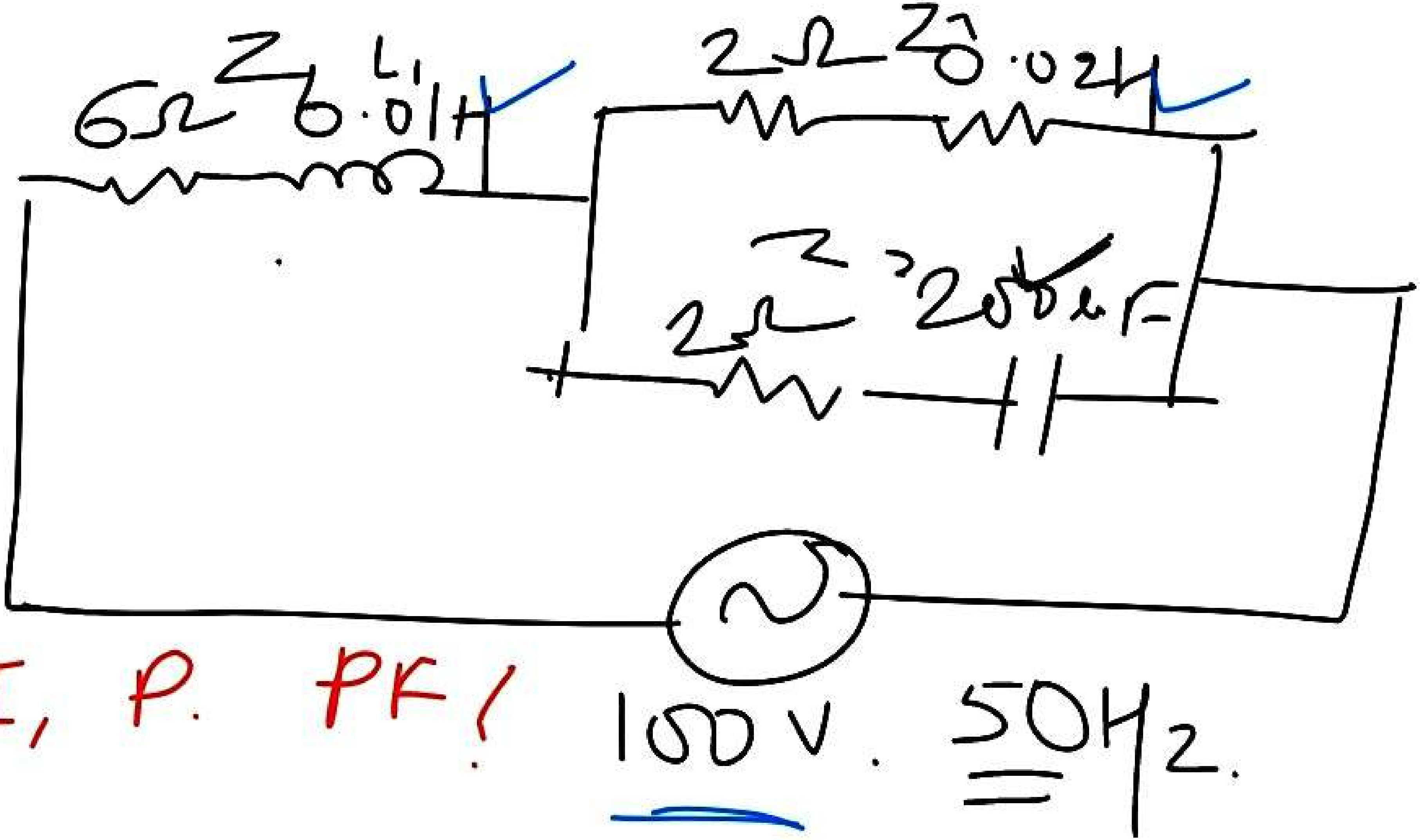
$$\Rightarrow P_L = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$= V_{rms} I_{rms} \cos\phi$$

$$P_L = \frac{1}{2T}\int_0^T \sin \omega t \, dt.$$

$P =$ Apparent power $\times \cos\phi$

$Q = V_{rms} I_{rm} \sin\phi$



I, P. PF?

$$X_{L1} = 2\pi \times 50 \times 0.01 = 3.14\,\Omega$$

$$X_{L2} = 2\pi \times 50 \times 0.02 \Rightarrow 6.28\,\Omega$$

$$X_C = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\,\Omega$$

$$\left.\begin{aligned} \bar{Z}_1 &= R_1 + j\omega L \\ \bar{Z}_2 &= R_2 + j\omega L \\ \bar{Z}_3 &= R - j\omega C \end{aligned}\right\} \begin{aligned} &6 + j3.14\ \Omega \\ &4 + j6.28\ \Omega \\ &2 - j15.92 \end{aligned}$$

$$\bar{Z} = Z_1 + \frac{\bar{Z}_2 \times \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$= (6 + j3.14) + \frac{(4 + j6.28) + (2 - j15.92)}{(4 + j6.28) + (2 - j15.92)}$$

$$\boxed{= 17.27 \angle 30.75^\circ \, \Omega}$$

$$I = \frac{\bar{V}}{Z} = \frac{100\angle 0^\circ}{17.27\angle 30.75^\circ}$$

$$= 5.79\angle -30.75^\circ \text{ Amp.}$$

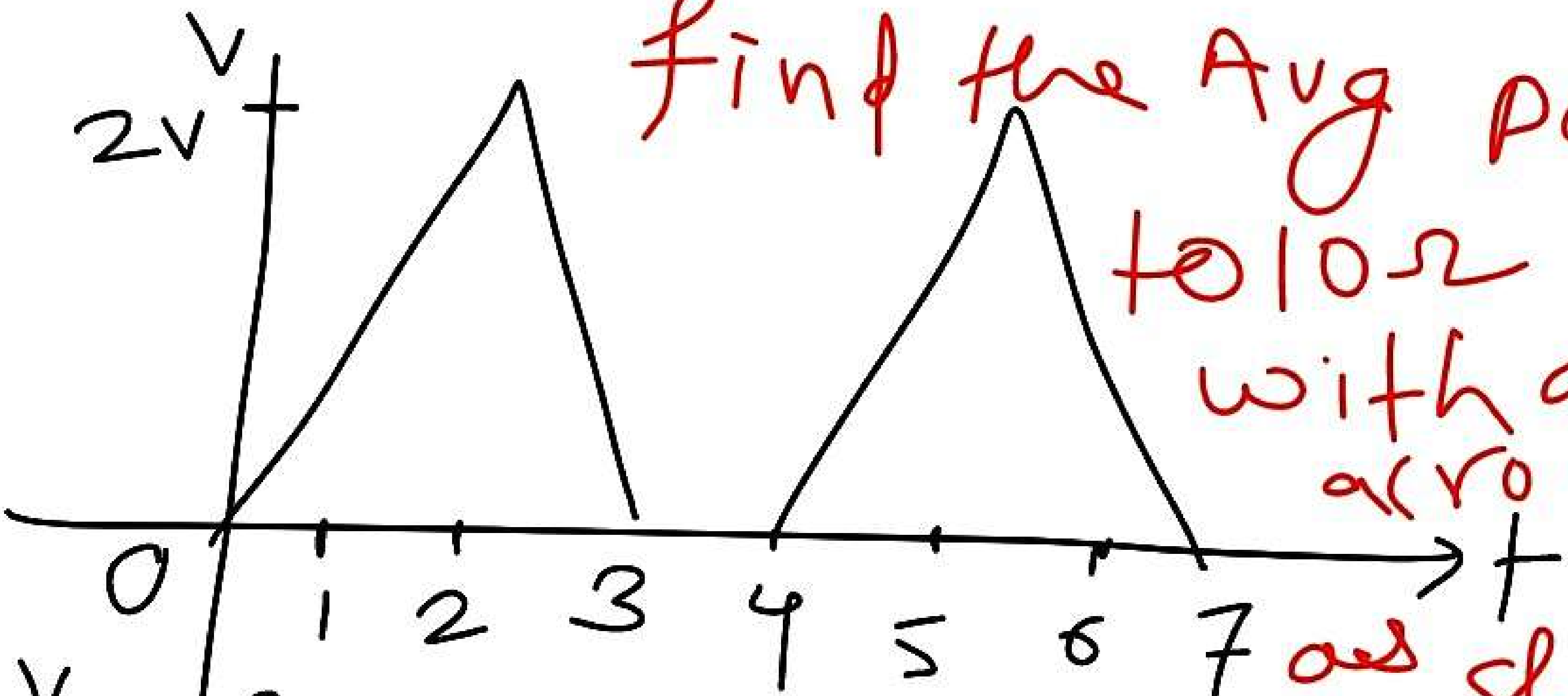
$$P = VI\cos\phi = 100 \times 5.79 \times \cos(30.75^\circ)$$

$$= \underline{\underline{497.94\,W}}.$$

$$PF = \cos\phi = \cos(30.75^\circ)$$

$$\doteq 0.86 \text{ (Lagging)}$$

find the Avg power to 10Ω res. with a voltage across it as shown in fig.



V_{rms}.

$P = (V_{rm})^2 / R$

Open circuit impedance parameters

(Z parameter) → It represents → relation betn voltage & current)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow$$

Matrix eqⁿ.

$V_1 = Z_{11} I_1 + Z_{12} I_2 \leftarrow (I_2 = 0)$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

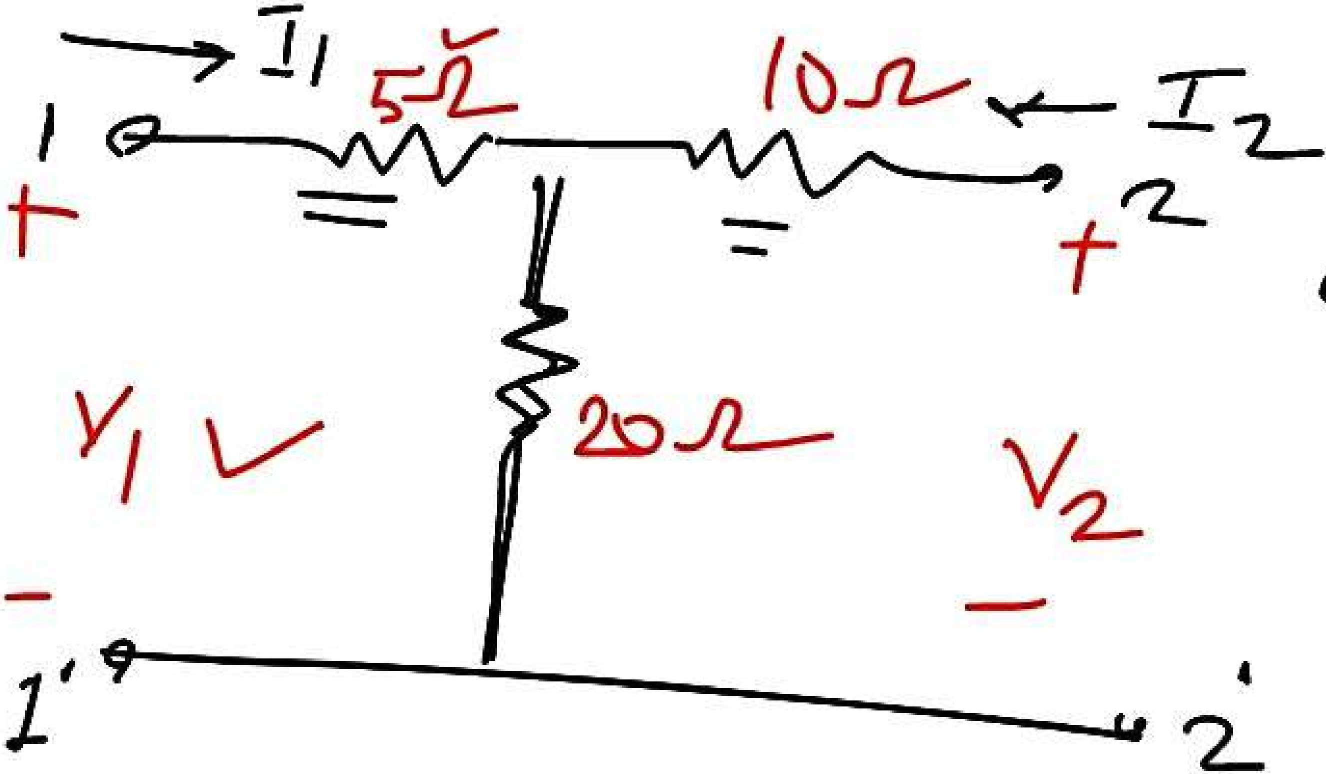
$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \Rightarrow$ Driving point at port1.

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Leftarrow \text{Transfer Impedance}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \Leftarrow \text{Transfer Impedance}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \Leftarrow$$ Driving point Impedance at port 2.

$$Z_{12} = \left|\frac{V_1}{I_2}\right|_{I_1=0} \Rightarrow$$



Determine the z parameter

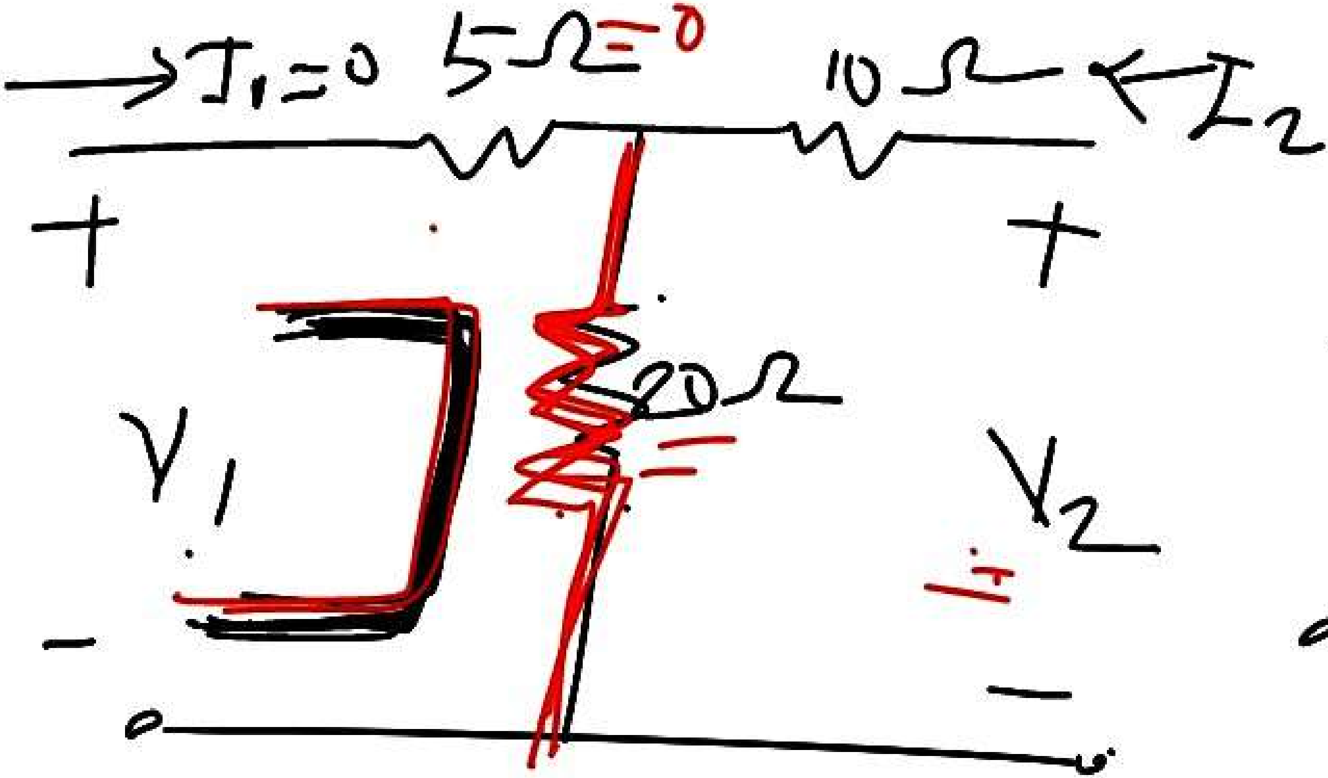
→ (a) When $(I_1 = 0)$

Port 1 ⇒ OC.

⇒ By KVL ⇒ $10I_2 + 20I_2 - V_2 = 0$

$= 30I_2 - V_2 = 0$

$\therefore \left|\frac{V_2}{I_2}\right| = 30\,\Omega$ ⇒ Z_{22}



$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = 20\,\Omega$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = 20\,\Omega$$

$V_2 = 20 I_1$ $\quad$ $V_1 = 20 I_2$

Case 2

When $I_2 = 0 \Rightarrow$ Port 2 oc.

$\Rightarrow$ KVL $\Rightarrow 5I_1 + 20I_1 - V_1 = 0$

$$\therefore \quad 25I_1 - V_1 = 0$$

$$\therefore \left|\frac{V_1}{I_1}\right| = Z_{11} = \underline{\underline{25\,\Omega}}$$

$$[K] = \begin{bmatrix} 25 & 20 \\ 20 & 30 \end{bmatrix}$$

Short Circuit Admittance parameters

(Y parameters)

⇒ Relationship betn current & voltages.

Admittance Matrix.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

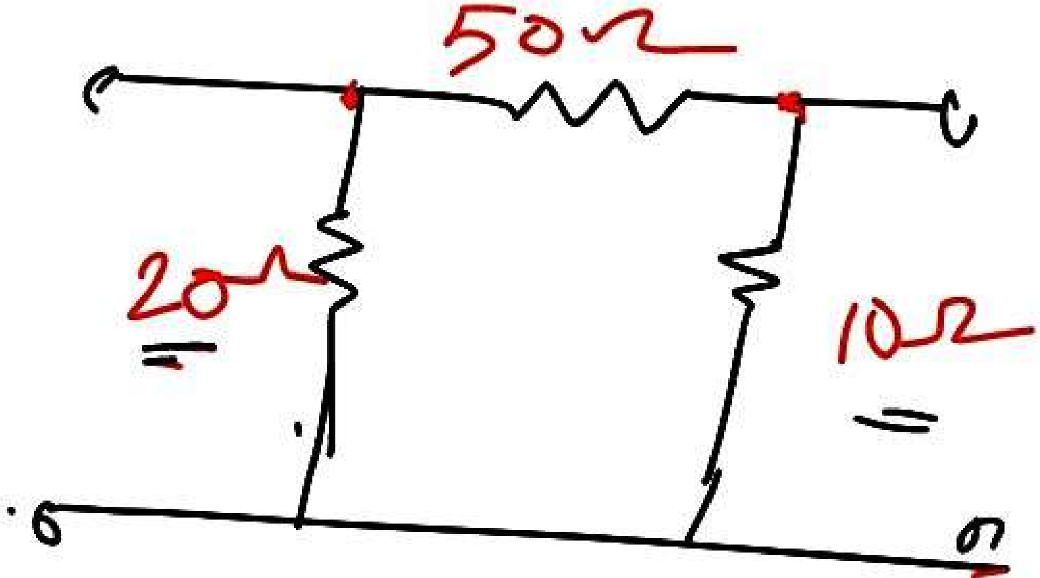
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$ $\Leftarrow$ Driving point admittance at port 1.

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$ $\Leftarrow$ Transfer Admittance

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$ $\Leftarrow$ Transfer Admittance

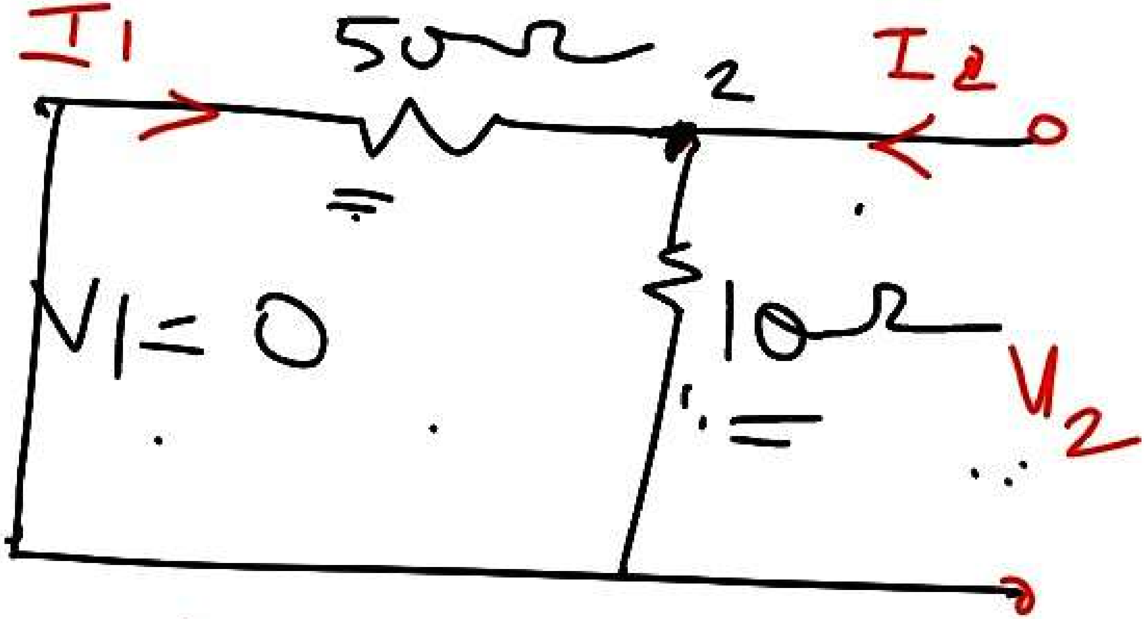
$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$ $\Leftarrow$ Driving point admittance at port 2



find the
y parameters

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

① $V_1 = 0$ Port 1 S.C.



when $V_1 = 0$

By KCL

$$\frac{V_2 - V_1}{10} + \frac{V_2 - V_1}{50} = I_2$$

$V_1 = 0$

$$\frac{V_2 - 0}{10} + \frac{V_2 - 0}{50} = I_2$$

$$V_2\left(\frac{1}{10}+\frac{1}{50}\right) = I_2$$

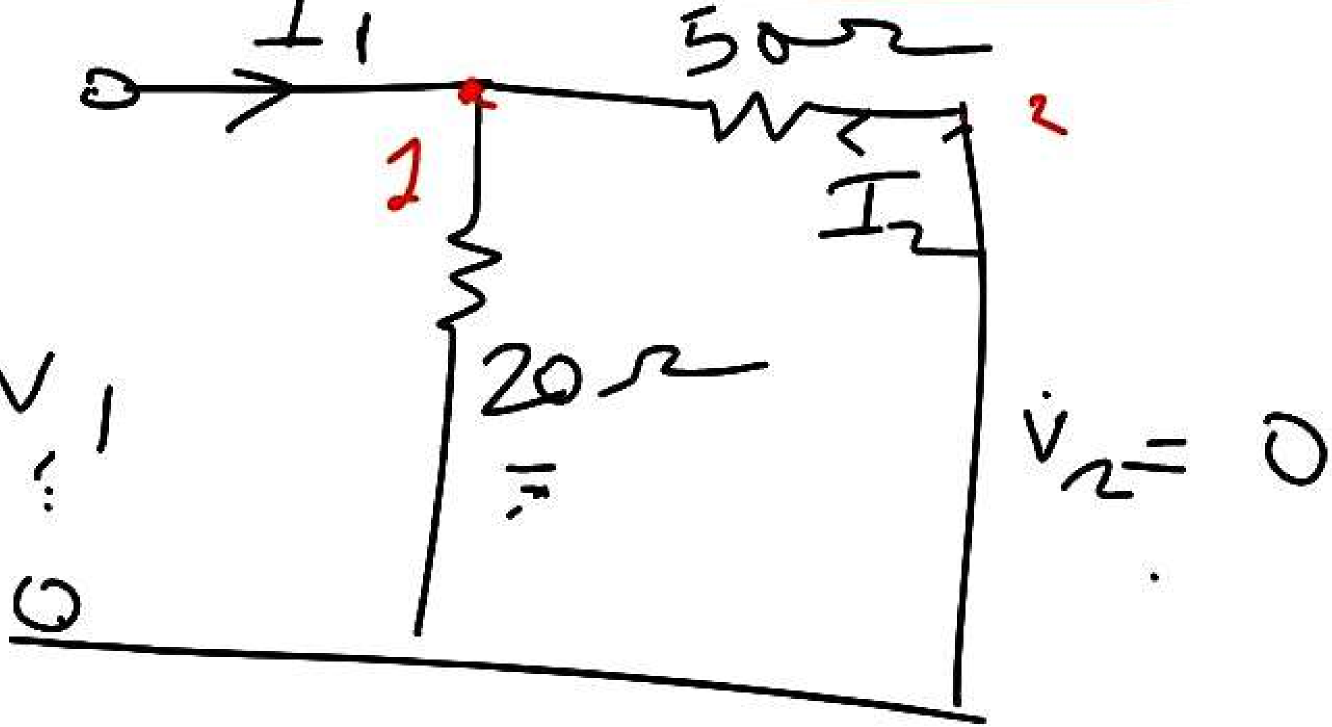
$$\therefore \frac{I_2}{V_2} = \frac{3}{25}\,℧ = 0.12\,℧$$

$$Y_{22} = 0.12\,℧$$

$$I_1 = \frac{V_1 - V_2}{50} = \frac{0 - V_2}{50}$$

$$= y_{12} = \left| \frac{I_1}{V_2} \right|_{V_1 = 0} = \frac{1}{50} = 0.02$$

$V_2 = 0$ $\Leftarrow$ Case 2



KCL at node 1.

$$\frac{V_1 - V_2}{20} + \frac{V_1 - V_2}{50} = I_1 \qquad (V_2 = 0)$$

$$\frac{V_1 - 0}{20} + \frac{V_1 - 0}{50} = I_1$$

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2 = 0} \Rightarrow \frac{1}{20} + \frac{1}{50} = 0.07 \mho$$

$$I_2 = \frac{V_2 - V_1}{50} \quad (V_2 = 0)$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{1}{50} = 0.02 \, \mho$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} 0.07 & 0.02 \\ 0.02 & 0.42 \end{bmatrix} \mho$$

ABCD Parameter or Transmission Param

It gives relation betn i/p quantities & o/p quantities in 2 port N/w.

(Thus it is voltage-current pair)

Transmission parameter matrix

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

I1
V1
port1
Linear
passive
N/w.
I2
port 2
V2

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \Leftarrow$$ open ckt reverse voltage gain

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \Leftarrow$$ Short ckt transfer Impedance

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$ open ckt Transfer admittance

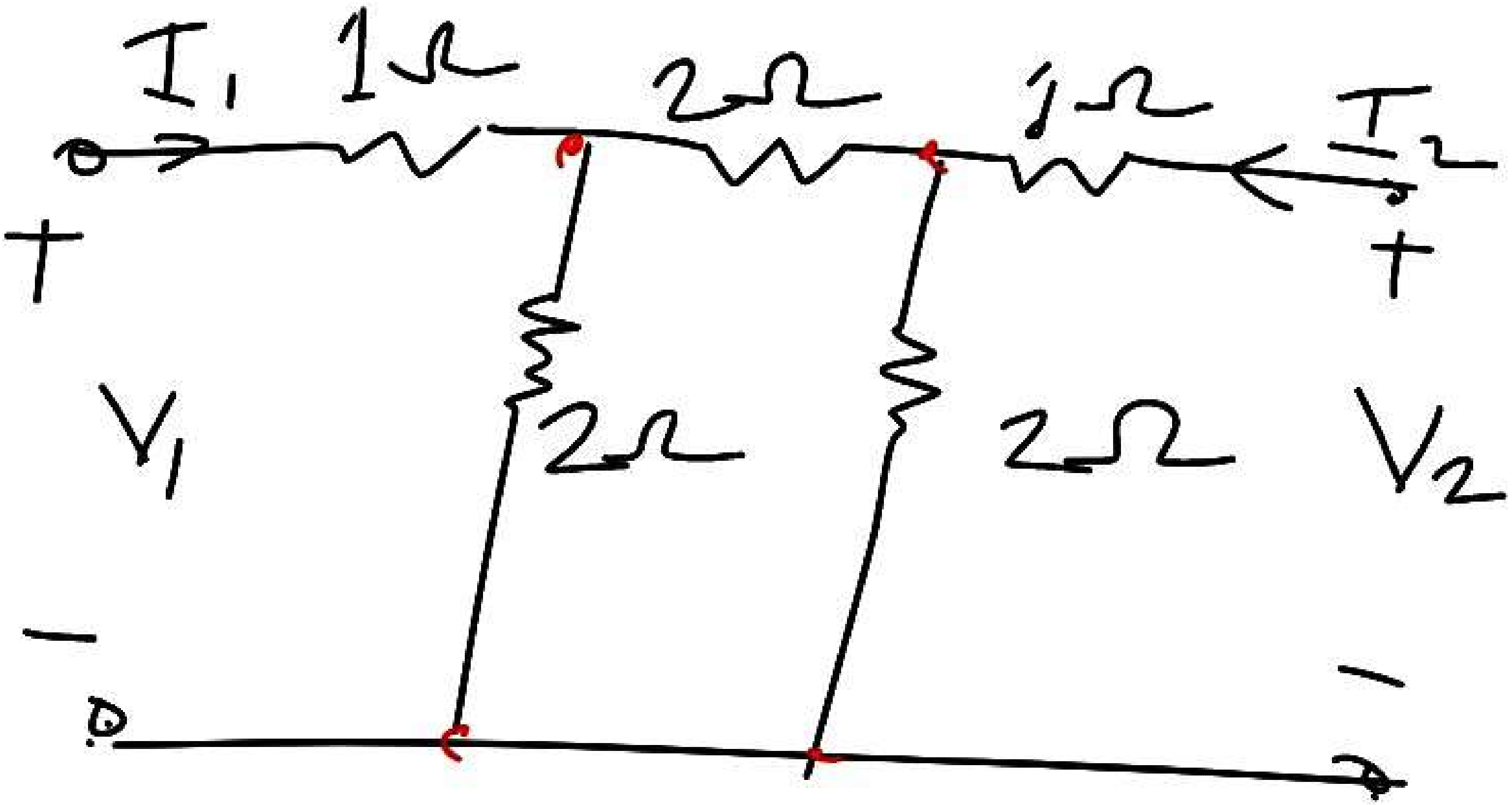
$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$ ⇐ short ckt. reverse current gain

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$B = \left.-\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

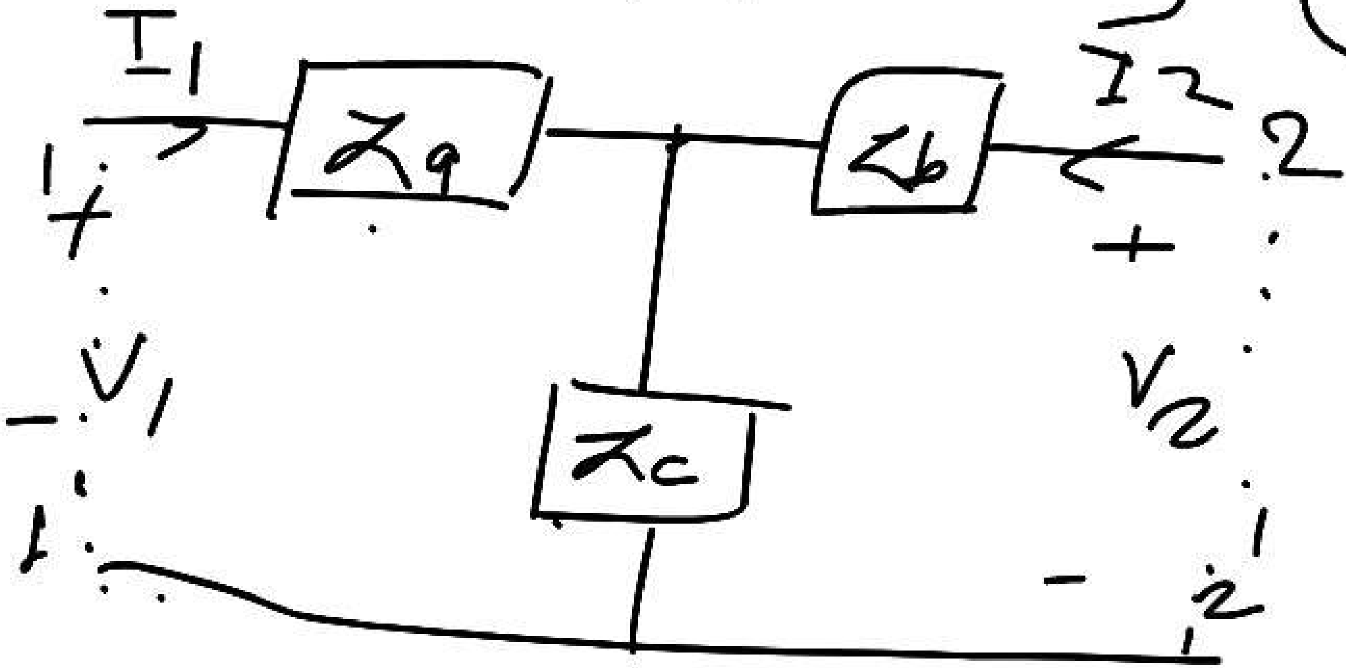
$$D = \left.-\frac{I_1}{I_2}\right|_{V_2=0}$$



find ABCD parameter

T N/W:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$



$(Z_a + Z_b) I_1 + Z_c I_2 = V_1$

$Z_c I_1 + (Z_b + Z_c) I_2 = V_2$

$$A = \frac{z_{11}}{z_{21}} \qquad B = \frac{\Delta z}{z_{21}}$$

$$C = \frac{1}{z_{12}} \qquad D = \frac{z_{22}}{z_{12}}$$

$$y_{11} = \frac{z_{22}}{\Delta z}$$

$$y_{12} = y_{21} = -\frac{z_{12}}{\Delta z}$$

$$y_{22} = \frac{z_{11}}{\Delta z}$$

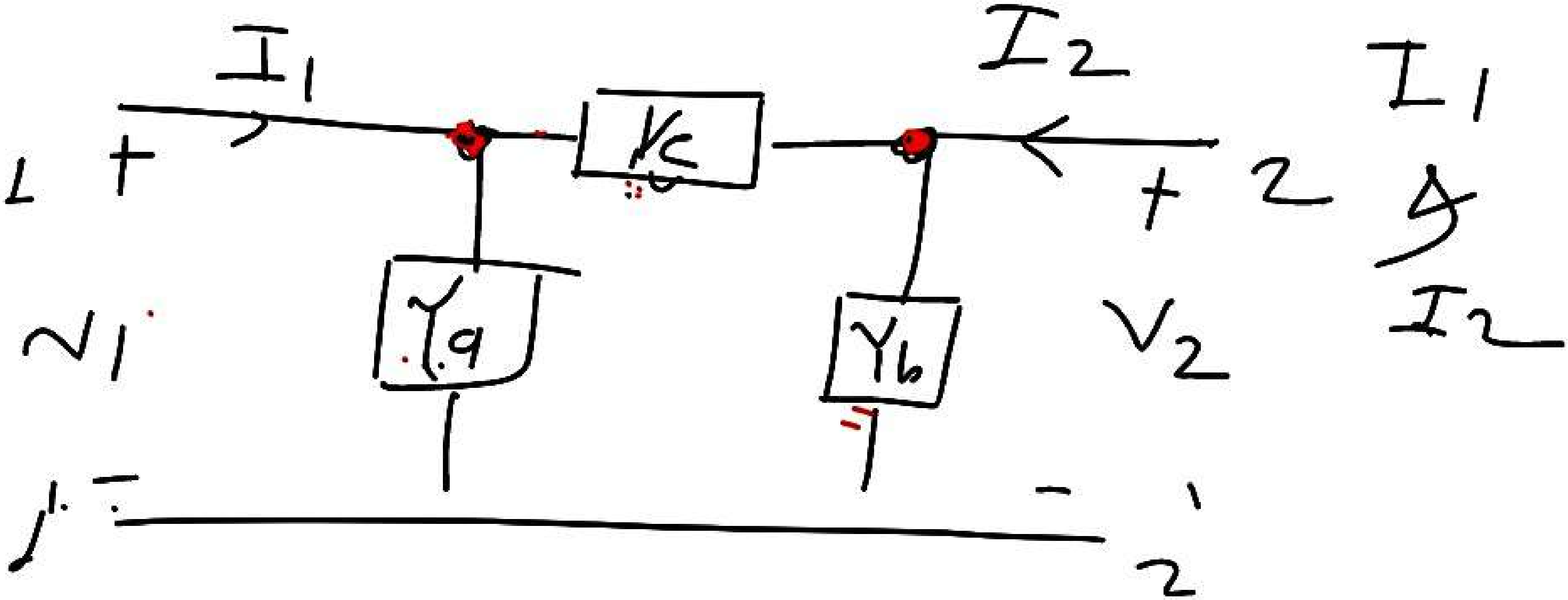
→ Write KVL eqn.

→ Find Z parameters.

→ Give the relation betn ABCD & Z parameters & find ABCD.

→ Give the relation betn Y & Z parameter & find Y paramete

(2) π or delta N/w.



→ Write KCL eqn

→ find Y parameters.

→ $A = -\frac{Y_{22}}{Y_{21}}$ $B = -\frac{1}{Y_{21}}$

$C = -\frac{\Delta Y}{y_{21}}$ $D = -\frac{y_{11}}{y_{21}}$

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$$I_1 = (V_1 - V_2)Y_c + V_1 Y_a$$

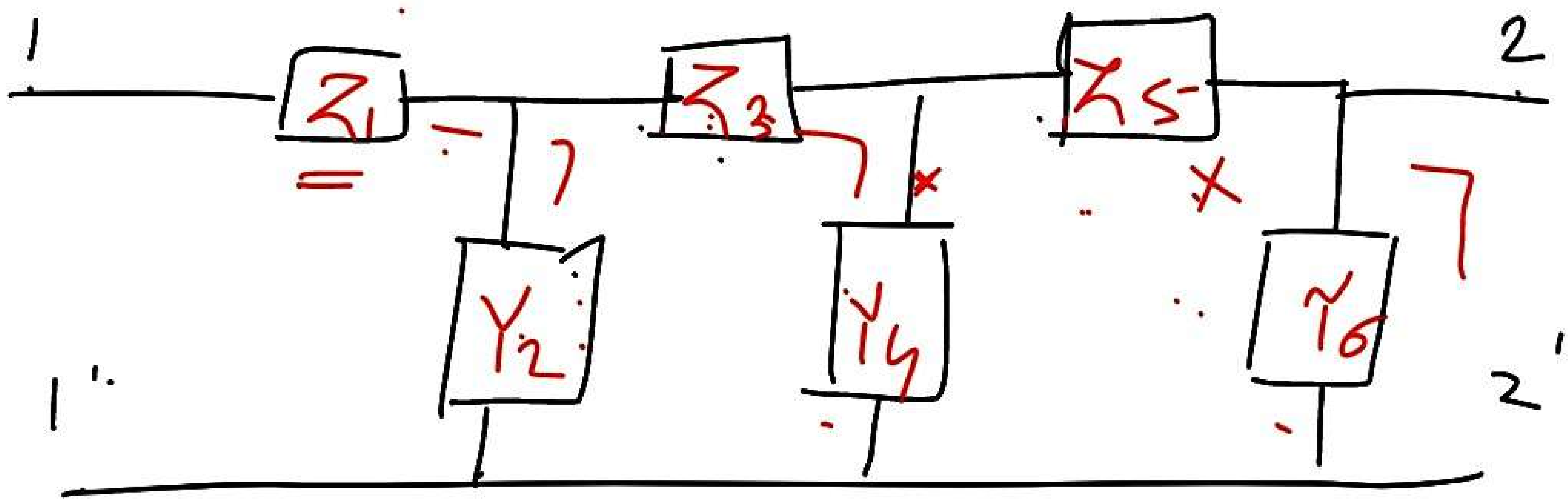
$$\Rightarrow (Y_a + Y_c)V_1 - Y_c V_2 = I_1$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$I_2 \Rightarrow V_2 Y_b + (V_2 - V_1)Y_c$$

$$\Rightarrow -Y_c V_1 + (Y_b + Y_c)V_2$$

Ladder N/w

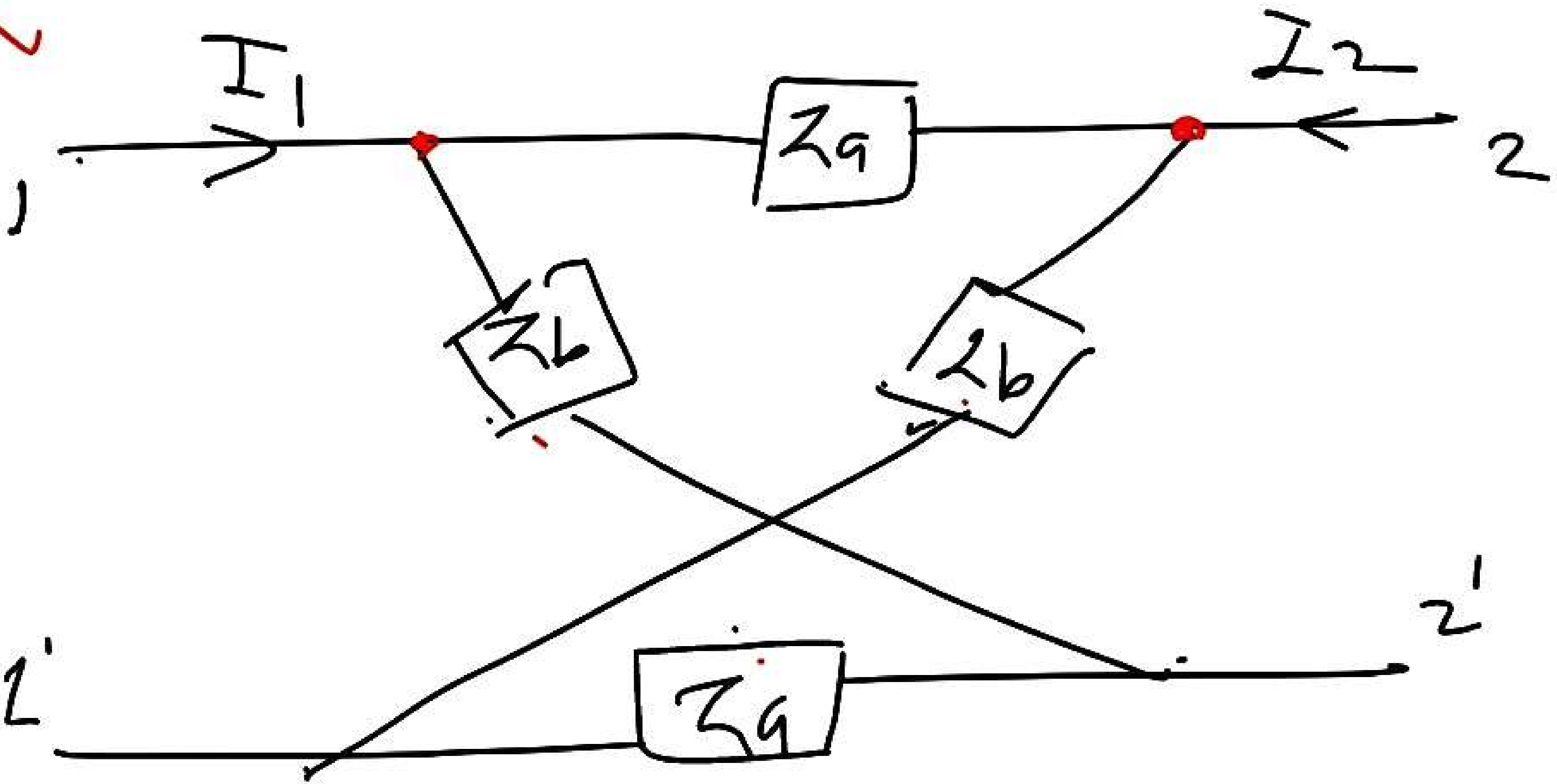


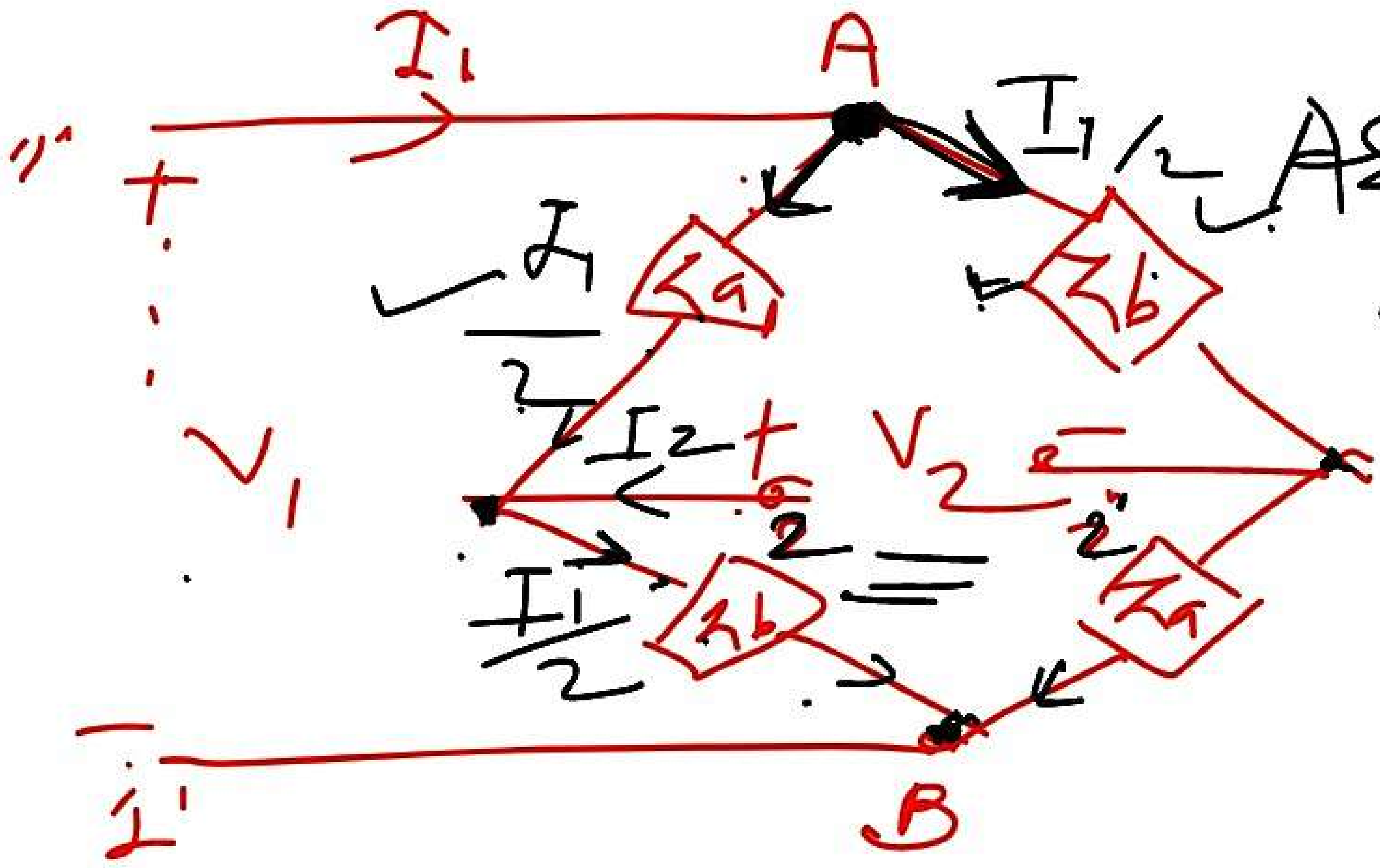
Driving point Impedance at port 1

Continued fraction.

$$Z_{11} = Z_1 + \cfrac{1}{Y_2 + \cfrac{1}{Z_3 + \cfrac{1}{Y_4 + \cfrac{1}{Z_5 + \cfrac{1}{Y_6}}}}}$$

Lattice N/w





Assume $I_2 = 0$

$$\frac{I_1}{2} Z_a + V_2 = \frac{I_1}{2} Z_b$$

$$V_2 = \frac{I_1}{2} Z_b - \frac{I_1}{2} Z_a$$

$$V_2 = \frac{I_1}{2} \left(Z_b - Z_a \right)$$

$$V_1 = \frac{I_1}{2} Z_a + \frac{I_1}{2} Z_b$$

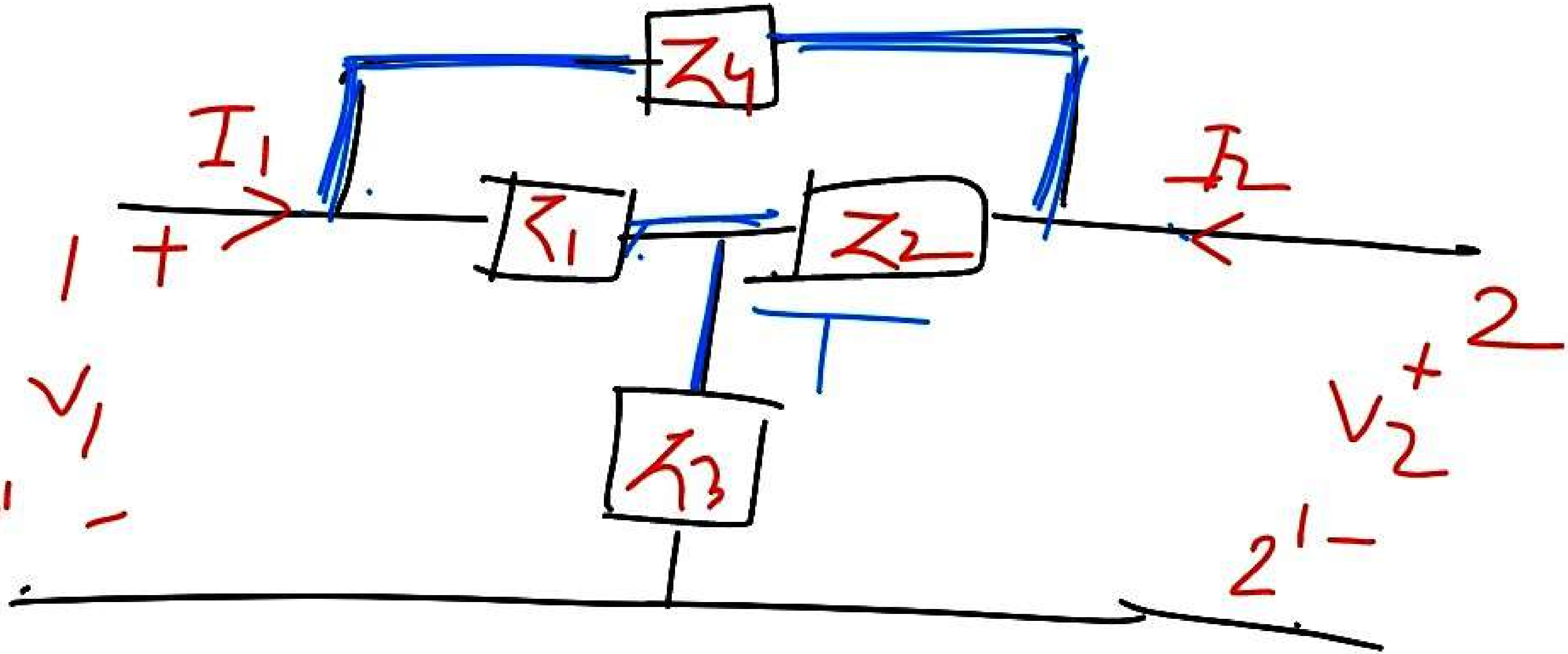
$$V_2 = \frac{I_1}{2} Z_b - \frac{I_1}{2} Z_a$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$z_{12} = z_{21} \Rightarrow \frac{z_b - z_a}{2}$$

$$z_{11} = z_{22} \Rightarrow \frac{z_b + z_a}{2}$$

Bridge T N/w



$$Y_{jk} = \frac{\Delta_{jk}}{\Delta} \Rightarrow Y_{11} = \frac{\Delta_{11}}{\Delta}$$

(Mesh)

$\Delta \rightarrow$ loop basis determinant

$\Delta_{jk} \Rightarrow$ Cofactor. (O/P is shorted)

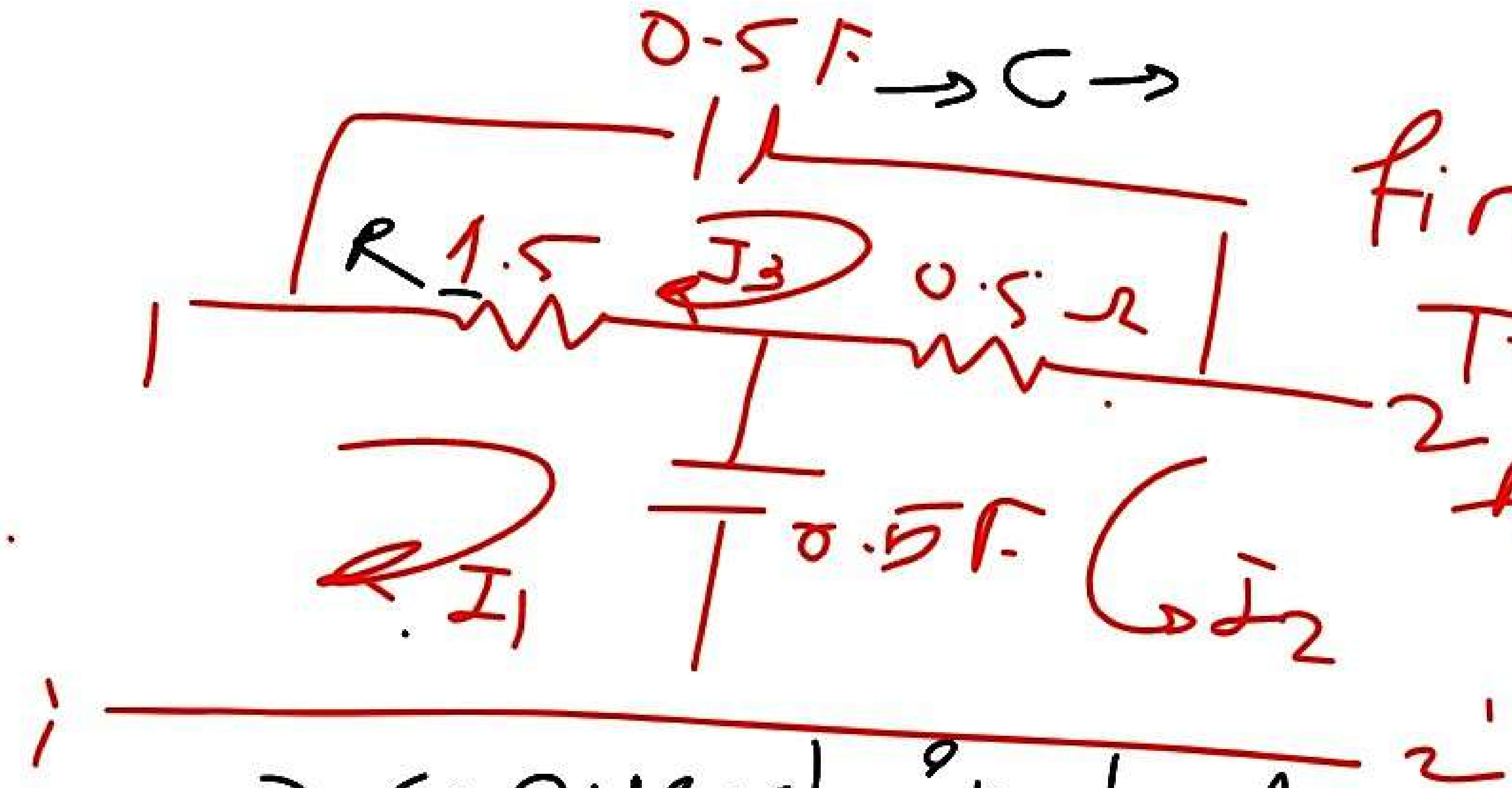
$$z_{jk} = \frac{\Delta'_{kj}}{\Delta'}$$

(Node basis calculation)

Δ' = loop basis determinant

Δ'_{kj} => cofactor. (o/p is open circuited)

$$G_{21} = \frac{I_2}{I_1} = \frac{\Delta_{21}}{\Delta_{11}}$$



find Transfer Admittance Y_{21}

⇒ convert in L domain

$$\left(1+\frac{2}{5}\right)I_1+\frac{2}{5}I_2-I_3=V_1$$

$$\frac{2}{5}I_1+\left(\frac{1}{2}+\frac{2}{5}\right)I_2+\frac{1}{2}I_3=V_2$$

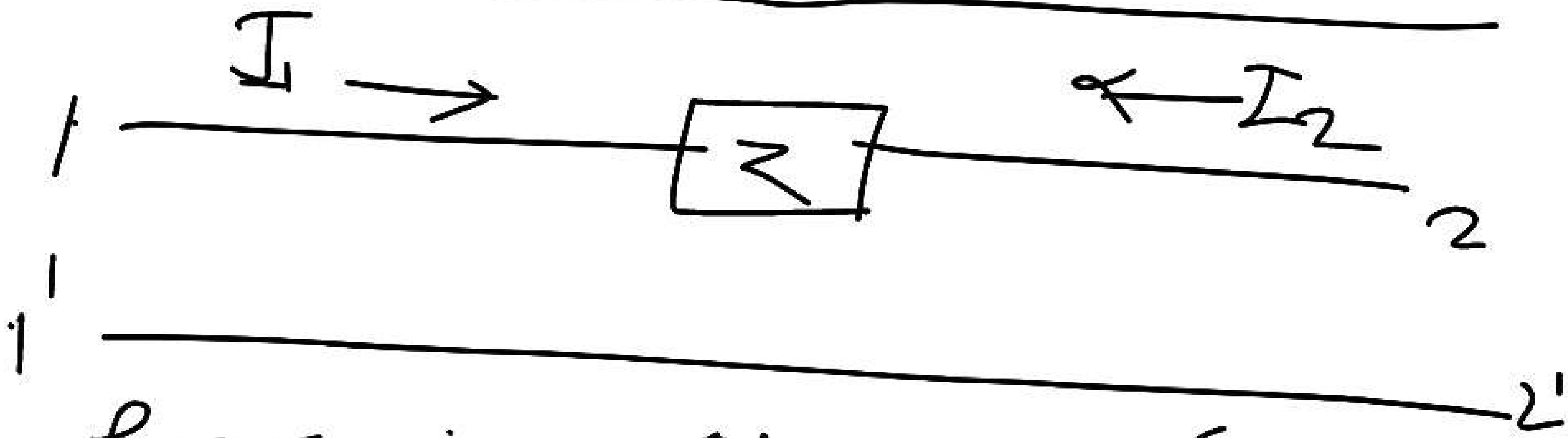
$$-I_1+\frac{1}{2}I_2+\left(\frac{3}{2}+\frac{2}{5}\right)I_3=0$$

$$\Delta = \begin{vmatrix} \left(1+\frac{2}{s}\right) & \frac{2}{s} & -1 \\ \frac{2}{s} & \left(\frac{1}{2}+\frac{2}{s}\right) & \frac{1}{2} \\ -1 & \frac{1}{2} & \left(\frac{3}{2}+\frac{2}{s}\right) \end{vmatrix} \cdot \begin{matrix} V_1 \\ V_2 \\ 0 \end{matrix}$$

$$\Delta_{12} \Rightarrow (-1)^{1+2} \begin{bmatrix} \frac{2}{s} & \frac{1}{2} \\ -1 & \end{bmatrix}$$

$$\Delta_{12} \Rightarrow (-1)^{1+2} \left| \quad \right|$$

for series & shunt element



for series element $(I_1 = -I_2)$

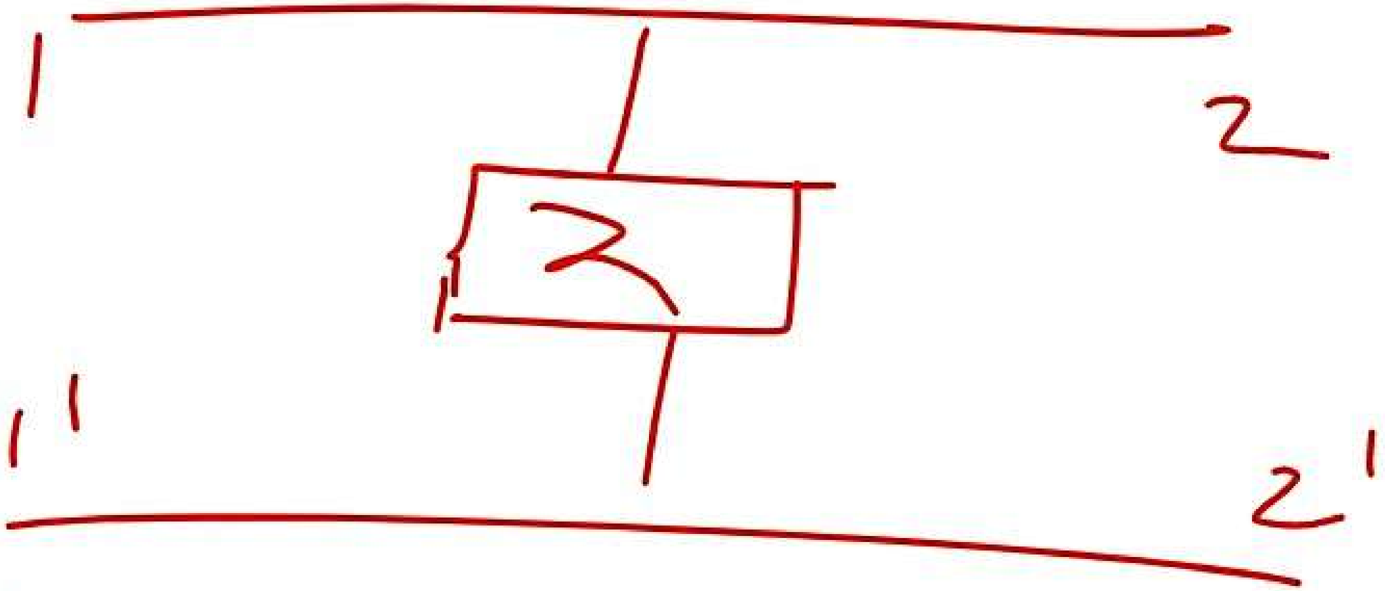
Reciprocal & symm.

for series - element ($I_1 = -I_2$)
Z parameters are not defined.
($\because$ If $I_2 = 0$ then I_1 also zero.

$$[Y] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

for shunt element $V_1 = V_2$



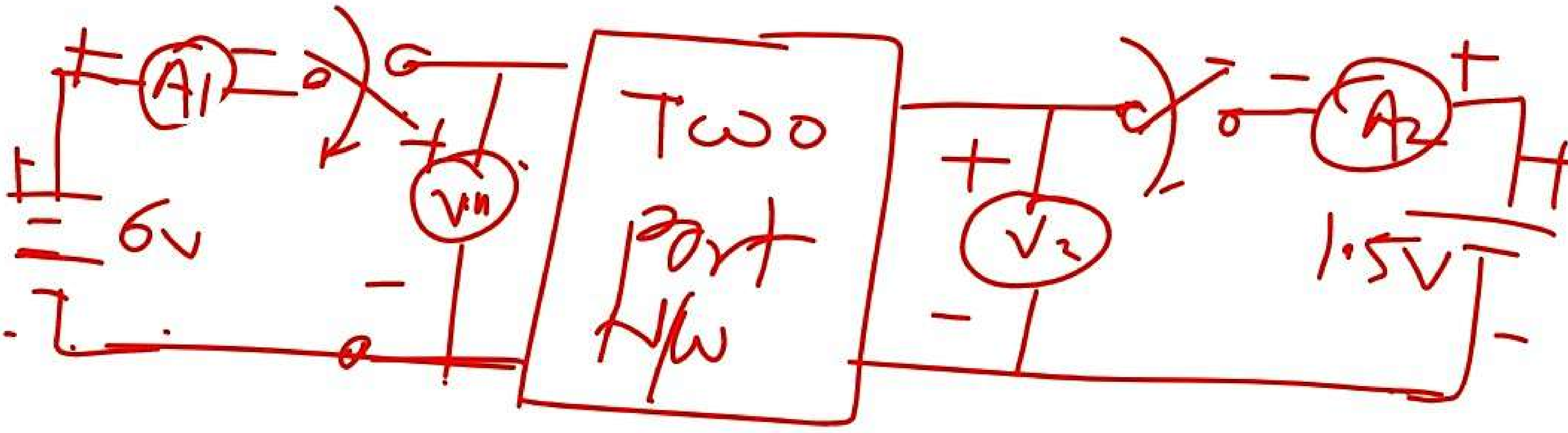
Y parmeter will not define

$\because V_2 = 0$ then $V_1 = 0$

$$[Z] = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

$$ABCD = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$$

Reciprocal & symemetric


A1
6V
Vin
Two
port
N/w
V2
A2
1.5V

1 S_1 = open.

$A_1 = 0\ A$

$V_2 = 1.5\ V$

S_2 = closed.

$V_1 = 4.5\ V$

$A_2 = 1\ A$.

2 S_1 closed

$A_1 = 4\ A$

$V_2 = 6\ V$

S_2 = open.

$V_1 = 6\ V$

$A_2 = 2\ A$.

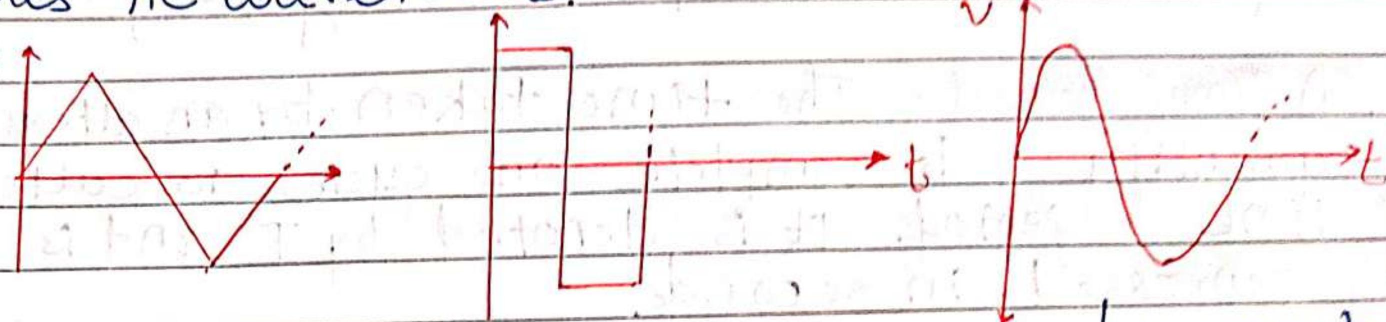
$$\left.\begin{array}{l} Z \text{ parmeter matrix} \\ h \text{ parmeter matrix} \end{array}\right\} \text{find}$$

Time And Frequency Response of circuit.

AC Circuit Analysis:-

Average and RMS values of Periodic Signal

An alternating waveform changes its magnitude and direction periodically. Fig. shows various AC waveforms.



Many times, alternating voltages and currents are represented by a sinusoidal waveforms. A sinusoidal voltage canbe represented as

$$v = V_m \sin\theta$$
$$i = V_m \sin \omega t$$
$$= V_m \sin 2\pi f t$$
$$= V_m \sin \frac{2\pi}{T} t$$

Terms related with alternating Quantity :-

(1) **Waveform :-** A waveform is a graph in which the instantaneous value of any quantity is plotted againts time. Fig. shows a few waveforms.

② Cycle :- One complete set of positive and negative values of an alternating quantity is termed as cycle.

③ Frequency :- The number of cycles per second of an alternating quantity is known as frequency. It is denoted by F and is expressed in hertz (H_z) or cycles per second.

④ Time period :- The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by T and is expressed in seconds.

$$\boxed{T = \frac{1}{f}}$$

⑤ Amplitude :- The maximum positive or negative value of an alternating quantity is called the amplitude.

⑥ Phase :- The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.

⑦ Phase difference :- This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude

A leading alternating quantity is one which reaches its maximum or zero value earlier ~~ex~~ as compared to the other quantity.

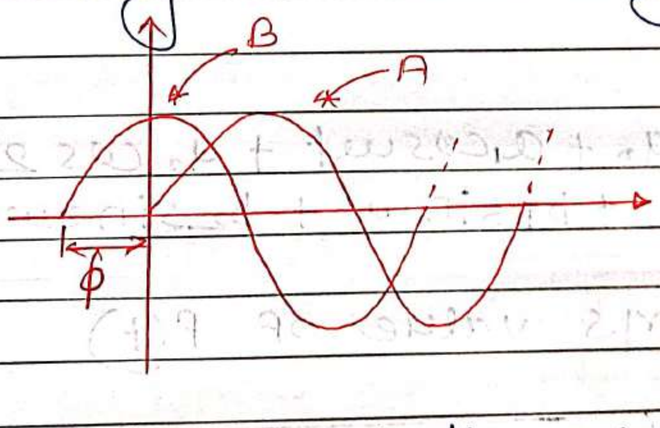
A laggin alternating quantity is one which attains its maximum or zero value later than the other quantity.

A plus(+) sign when used in connection with the phase difference denotes 'lead', whereas a minus(-) sign denotes 'lag'.

$v_A = V_m \sin \omega t$

$v_B = V_m \sin(\omega t + \phi)$

Here, quantity B leads A by a phase angle ϕ



Let $x(t)$ be the periodic signal of period 'T'

then $x(t \pm T) = x(t)$

Average value = DC value

$= X_{avg} = X_{dc} = \frac{1}{T}\int_0^T x(t)\,dt$

$= \frac{\text{Area of the signal over one period}}{\text{period}}$

RMS value $= X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$

Peak Factor $= \frac{x(t)_{max}}{x(t)_{avg}} = \frac{X_{max}}{X_{avg}}$

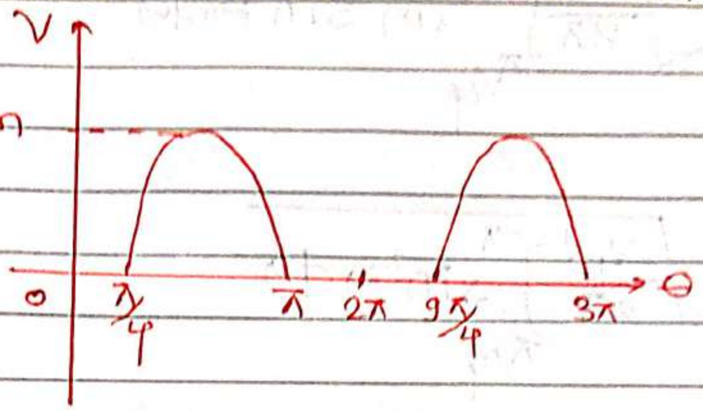
→ The average and value of sine or cosine function of any phase and frequency is zero, and its rms value is $\frac{\text{Maximum value}}{\sqrt{2}}$

→ If $f(t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \ldots$
$\ldots + b_1\sin\omega t + b_2\sin 2\omega t + \ldots$

then the RMS value of $f(t)$

$f(t)_{RMS} = \sqrt{a_0^2 + \frac{1}{2}(a_1^2 + a_2^2 + \ldots + b_1^2 + b_2^2 + \ldots)}$

Example :- Find the average value and rms value of the waveform shown in Fig.



$$v = 0 \qquad 0 < \theta < \pi/4$$
$$= V_m \sin\theta \qquad \pi/4 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

$$v_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} v_m \sin\theta\, d\theta$$

$$= \frac{v_m}{2\pi}\left(-\cos\theta\right)_{\pi/4}^{\pi}$$

$$= \frac{V_m}{2\pi}(1 + 0.707) = 0.272\ V_m.$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} v_m^2 \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{v_m^2}{2\pi}\int_{\pi/4}^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{v_m^2}{2\pi}\int_{\pi/4}^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

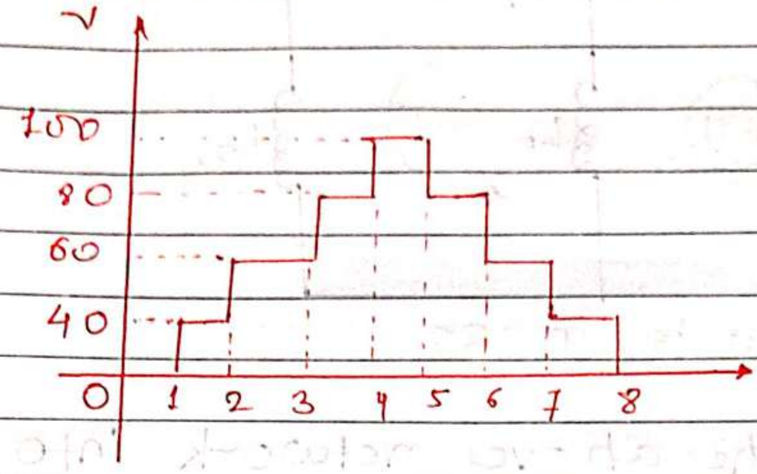
$$= \sqrt{\frac{v_m^2}{2\pi}\left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{v_m^2}{2\pi}\left(\frac{\pi}{2}-\frac{\sin 2\pi}{4}-\frac{\pi}{8}+\frac{\sin \pi/2}{4}\right)}$$

$$= \sqrt{0.227 v_m^2}$$

$$= 0.476 v_m$$

Example :- Find the rms and average value of the waveform shown in figure.



$$V_{avg.} = \frac{0+40+60+80+100+80+60+40}{8}$$

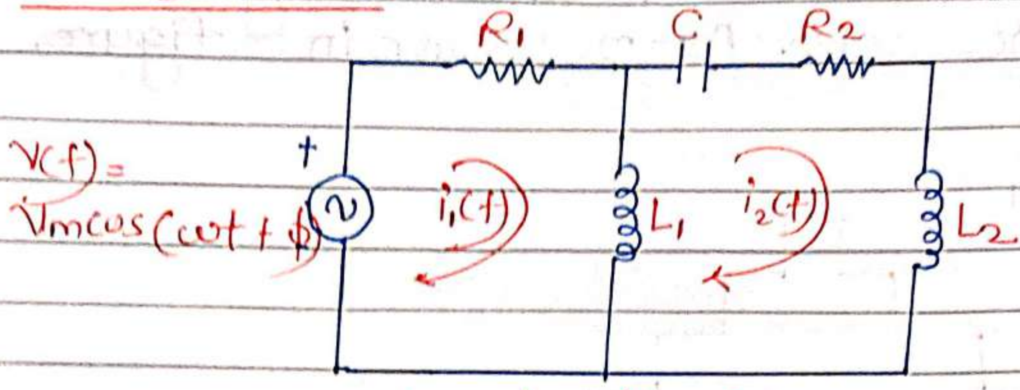
$$V_{rms} = \sqrt{\frac{(0)^2+(40)^2+(60)^2+(80)^2+(100)^2+(80)^2+(60)^2+(40)^2}{8}}$$

$= 64.42$ volt.

Sinusoidal Steady state Analysis (S^3A) by Using phasors

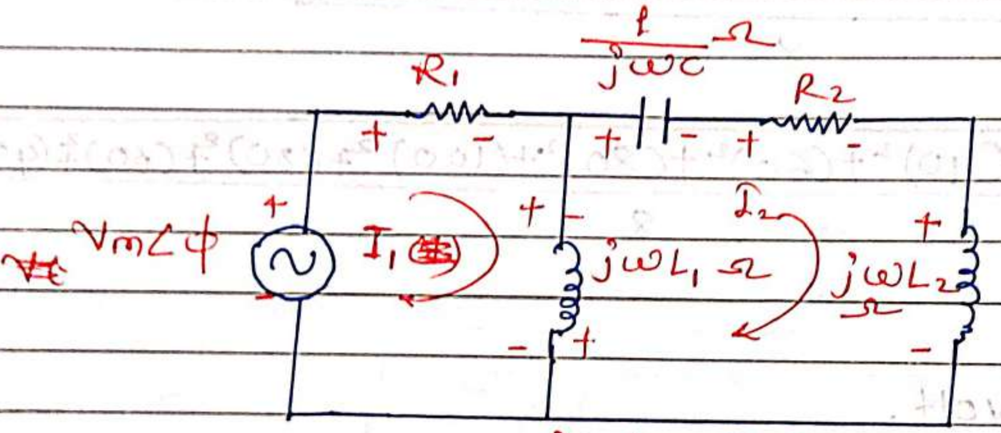
Steady state indicates absence of transients. It is achieved practically after 5 time constants of switching action & analysis in steady state is generally carried out by using phasors.

Procedure :-



N/w. is in SS.

Transform the above network into phasor domain.



N/w is in P.D.

$V = Z \cdot I$

By KVL in Phasor domain

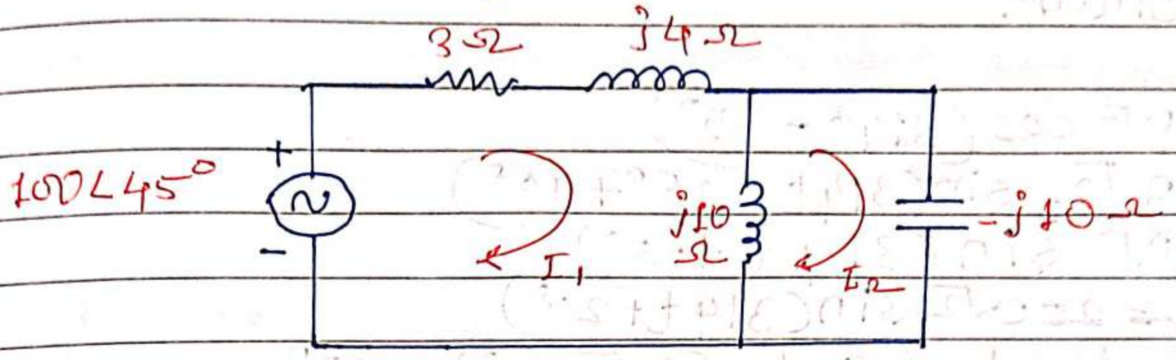
$V_m \angle \phi - R_1 I_1 - j\omega L_1 (I_1 - I_2) = 0$ ——— (1)

$\frac{-1}{j\omega C} I_2 - R_2 I_2 - j\omega L_2 I_2 - j\omega L_1 (I_2 - I_1) = 0$ ——— (2)

$I_1 = \frac{\Delta_1}{\Delta}$ | $i_1(t) = \text{Real Part}[I_1 \cdot e^{j\omega t}]$ Amp.

$$I_2 = \frac{\Delta_2}{\Delta} \Big|_{i_2(t)} = \text{Real part}\left[I_2 \cdot e^{j\omega t} \right] \text{ Amp.}$$

Example :- Find mesh currents I_1 & I_2 in the network of figure.



Applying KVL to Mesh 1

$$100\angle 45^\circ - (3+j4)I_1 - j10(I_1 - I_2) = 0$$

$$(3+j14)I_1 - j10 I_2 = 100\angle 45^\circ \quad \text{——— (1)}$$

Applying KVL to mesh 2,

$$-j10(I_2 - I_1) + j10(I_2) = 0$$

$$j10 I_1 = 0$$

$$I_1 = 0$$

Substituting I_1 in eqn (1) we get

$$-j10 I_2 = 100\angle 45^\circ$$

$$I_2 = \frac{100\angle 45^\circ}{10\angle -90^\circ} = 10\angle 135^\circ \text{ Amp.}$$

Example:

A two element series circuit is connected across an ac source given by $e = 200\sqrt{2}\sin(314t + 20^\circ)$. The current in the circuit then is found to be $i = 10\sqrt{2}\cos(314t - 25^\circ)$. Determine the parameters of the circuit.

$\rightarrow$ $i = 10\sqrt{2}\cos(314t - 25^\circ)$

$= 10\sqrt{2}\sin(314t - 25^\circ + 90^\circ)$

$i = 10\sqrt{2}\sin(314t + 65^\circ)$

Also, $e = 200\sqrt{2}\sin(314t + 20^\circ)$

Current lead emf by $(65^\circ - 20^\circ) = 45^\circ$.

power Factor $= \cos 45^\circ = 0.707$ (leading)

Now,

$$Z = \frac{V_m}{I_m} \frac{200\sqrt{2}}{10\sqrt{2}} = 20\Omega$$

$R = Z\cos\phi = 20(0.707) = 14.14\Omega$ &

$X_C = Z\sin\phi = 20(0.707) = 14.14\Omega$

$$X_C = \frac{1}{2\pi f C} = 14.14 \Rightarrow C = \frac{1}{2\pi f \times 14.14}$$

$$= \frac{1}{2 \times 3.14 \times 50 \times 14.14}$$

$= 2.2523 \times 10^{-4}$ F

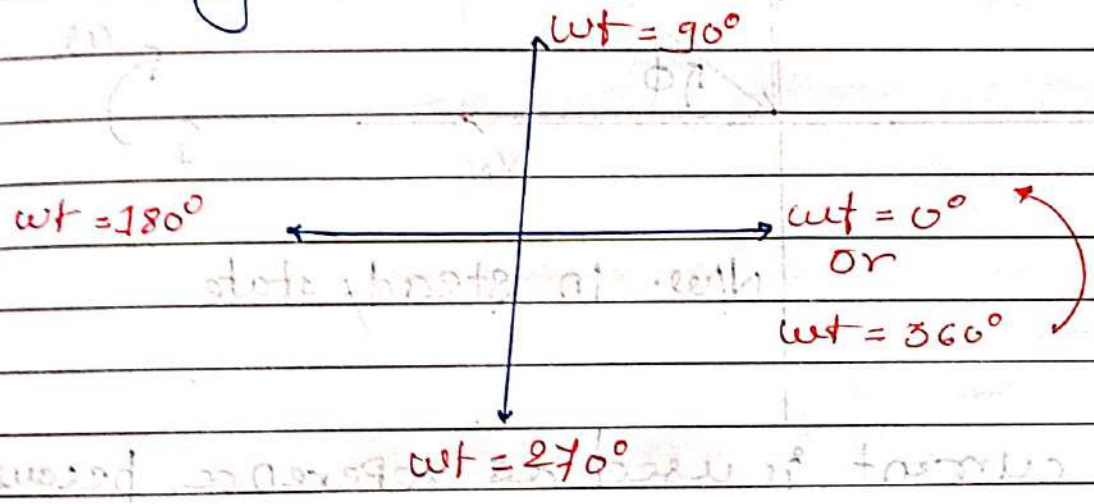
$= 225.23\ \mu F$

Phasor Diagram :-

- A graphical or pictorial representation of phasors is known as phasor diagram.

- Phasor diagram is a pictorial representation of all the phase voltages and their respective currents in a network.

- Phasor is a rotating vector, which rotates in the anticlockwise direction with angular frequency 'ω' in the time domain



$$Z_R = R\,\Omega = R\angle 0^\circ\,\Omega$$

$$Z_L = j\omega L\,\Omega = jX_L\,\Omega = X_L\angle 90^\circ\,\Omega$$

where

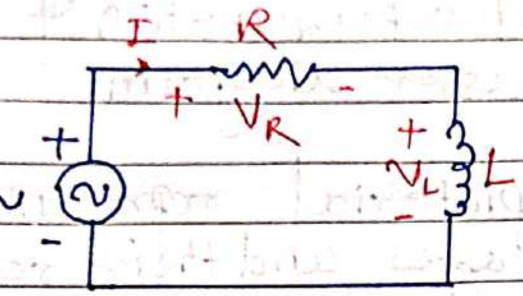
$$X_L = \omega L$$

$$Z_C = \frac{1}{j\omega C}\,\Omega = -j\frac{1}{\omega C}$$

$$= -jX_C = X_C\angle -90^\circ\,\Omega$$

where $X_C = \frac{1}{\omega C}$

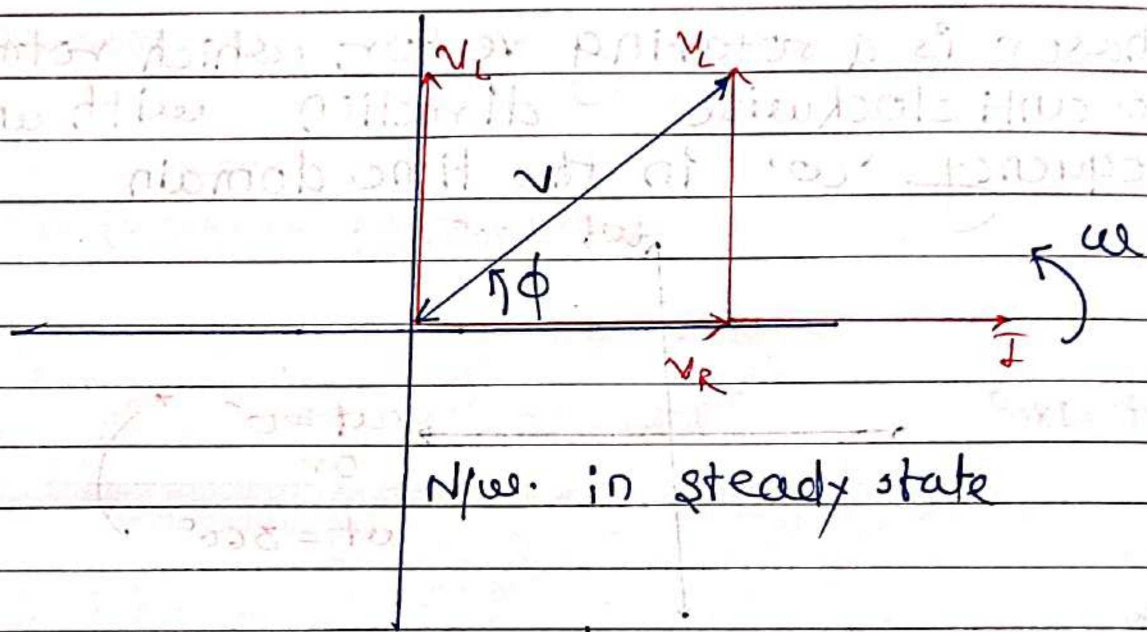
Series RL circuit :-



$V_R = I.R$

$V_L = I.Z_L$

$= I.X_L \angle 90°$



N/w. in steady state

In series current is used as reference. because it is same.

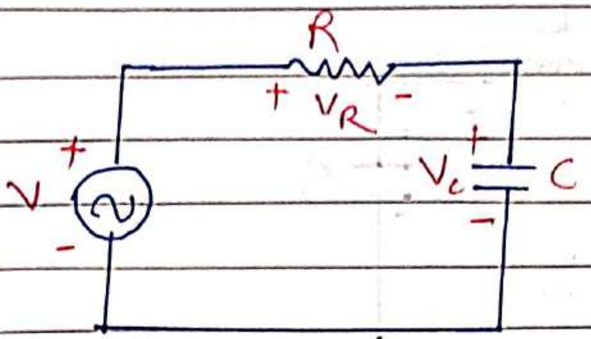
Current lags the voltage by 90°.

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \text{Impedance Angle} = \text{Admittance Angle}$$

$\cos\phi$ - PF (power factor) = $\frac{V_R}{V}$ (lag) i.e. "I" lags "V".

Series RC circuit :-

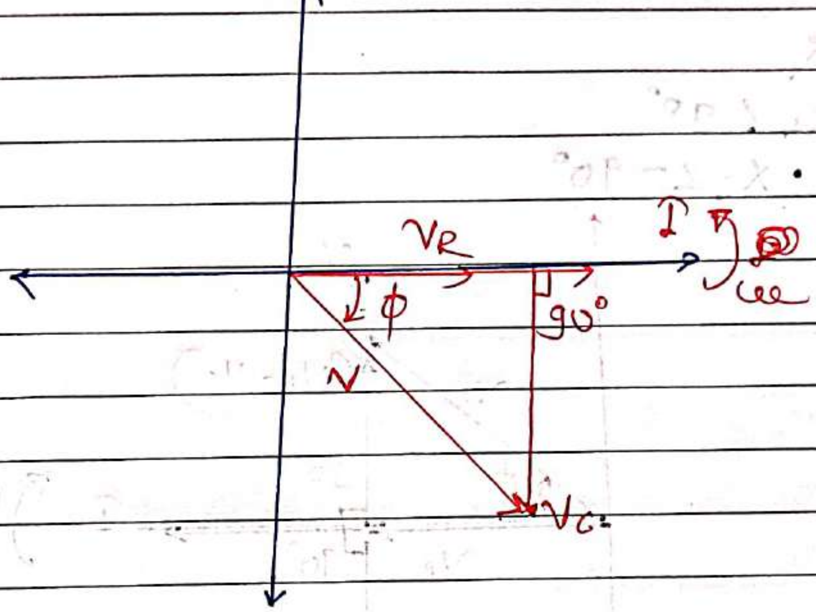


$$V_R = I \cdot R$$

$$V_C = I \cdot Z_C$$

$$= I \cdot X_C \angle -90^\circ$$

Network in steady state



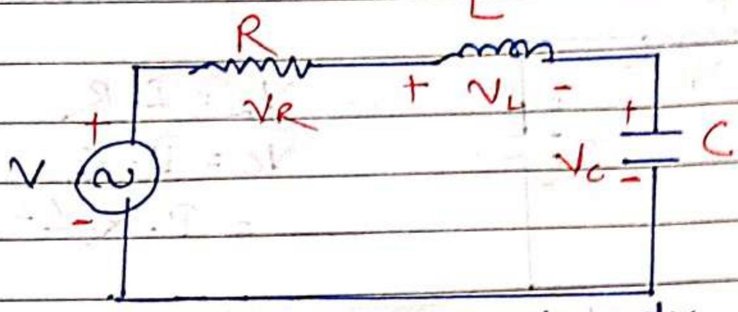
$$V = \sqrt{V_R^2 + V_C^2}$$

$$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right) = \text{Impedance Angle}$$

$$= \text{Admittance Angle}$$

i.e. "I" leads "V"

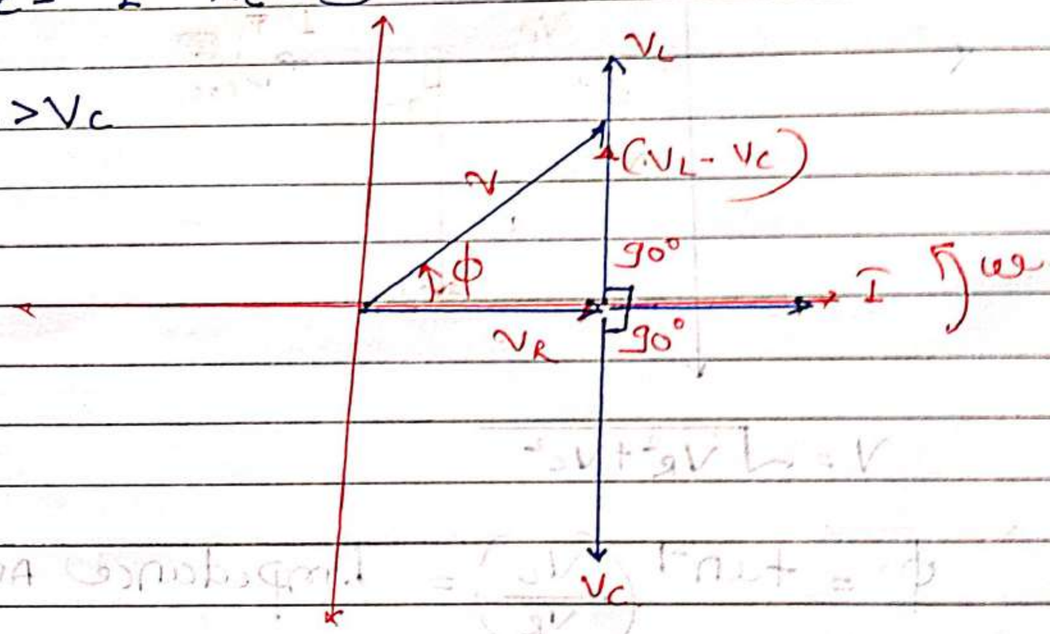
Series RLC circuit :-



N/w. in steady state.

$V_R = I \cdot R$
$V_L = I \cdot X_L \angle 90^\circ$
$V_C = I \cdot X_C \angle -90^\circ$

① $V_L > V_C$



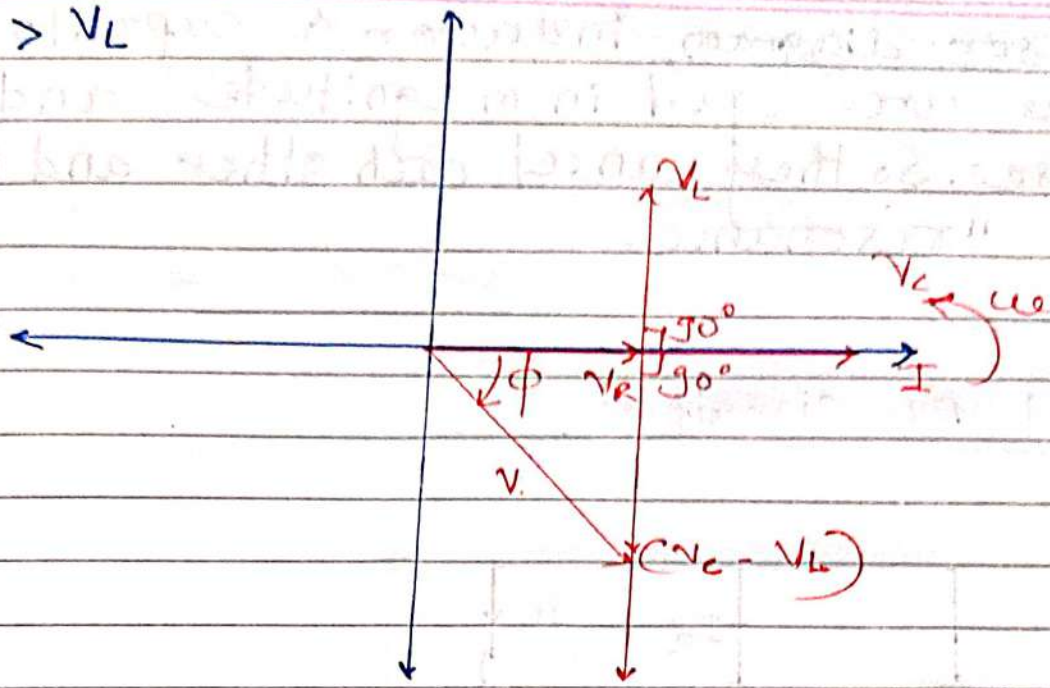
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

= Impedance Angle (ie. "I" lags "V")
= Admittance Angle

$$\cos\phi = P.F. = \left(\frac{V_R}{V}\right) \text{ (lag)}$$

(2) $V_C > V_L$



$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

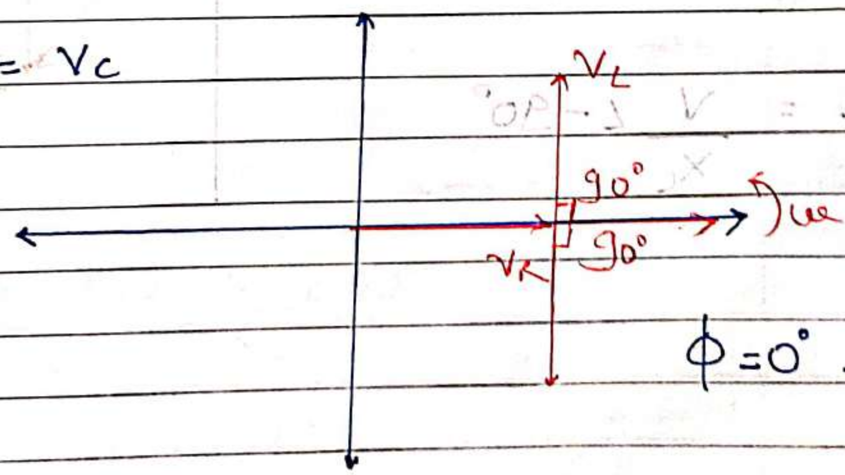
$$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$$

= Impedance Angle
= Admittance Angle

$\cos\phi$ = Power factor $= \left(\frac{V_R}{V}\right)$ (lead)

"I" leads "V"

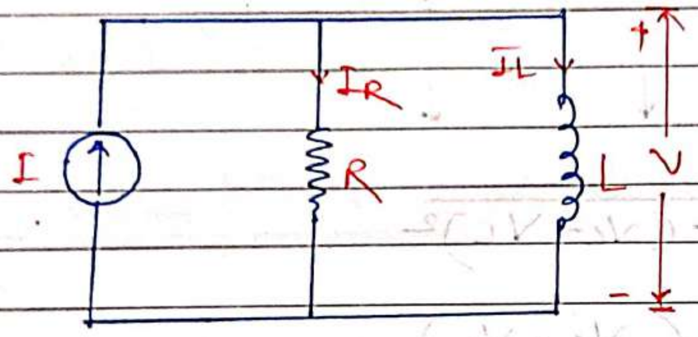
(3) $V_L = V_C$



$\phi = 0° \Rightarrow$ unity power factor.

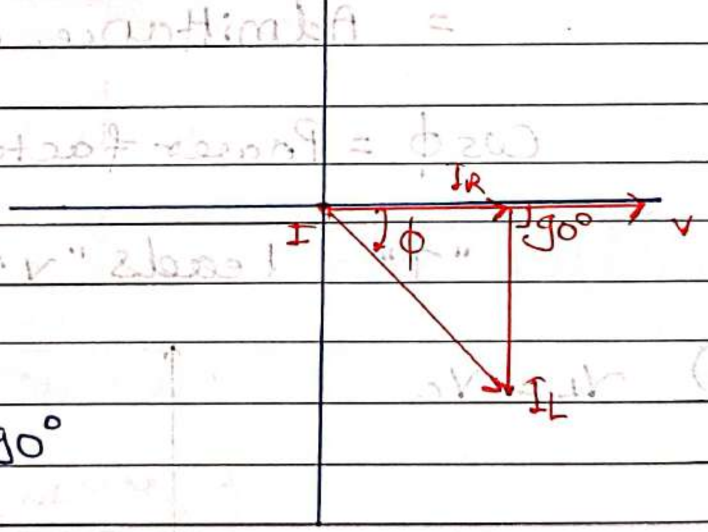
In phasor diagram, inductor & capacitor voltagas are equal in magnitude and opposite in phase. So they cancel each other and this is called "resonance.

Parallel RL Circuit :-



N/w. in steddy state

$$I_R = \frac{V}{R}$$



$$I_L = \frac{V}{Z_L}$$

$$\Rightarrow \quad I_L = \frac{V}{X_L} \angle -90°$$

$$I = \sqrt{I_R^2 + I_L^2}$$

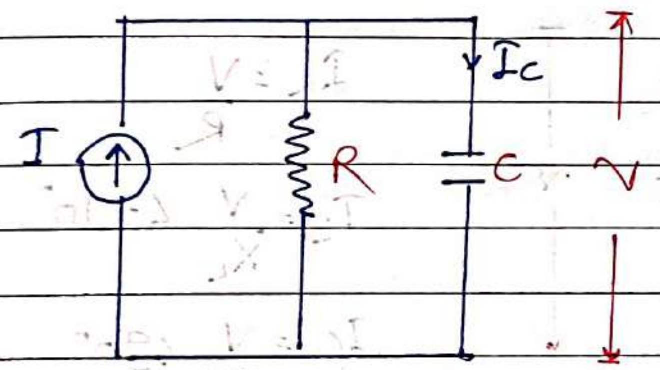
$$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right) = \text{Impedance Angle}$$

$$= \text{Admittance Angle}$$

$$\cos\phi = \text{P.F.} = \left(\frac{I_R}{I}\right) \text{(lag)}$$

i.e. "I" lags "V".

Parallel RC circuit :-

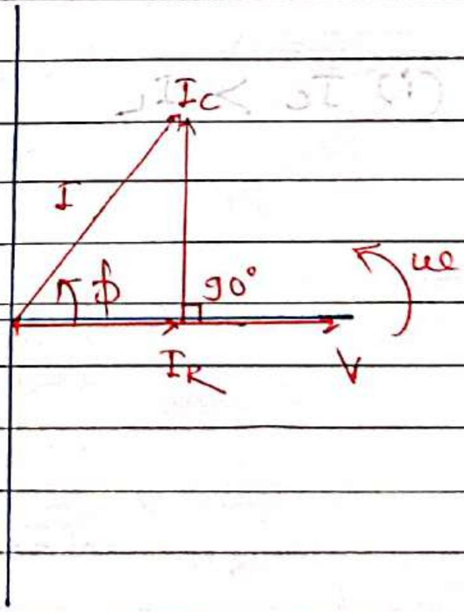


N/w. is in steady state

$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{Z_C}$$

$$\Rightarrow I_C = \frac{V}{X_C} \angle 90^\circ$$



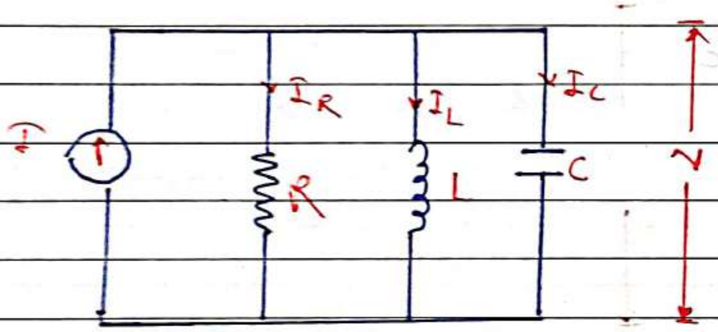
$$I = \sqrt{I_R^2 + I_C^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right) = \text{Impedance Angle}$$
$$= \text{Admittance Angle}$$

$$\cos\phi = P.F. = \left(\frac{I_R}{I}\right) \text{ (lead)}$$

i.e. "I" leads "V"

Parallel RLC circuit :-



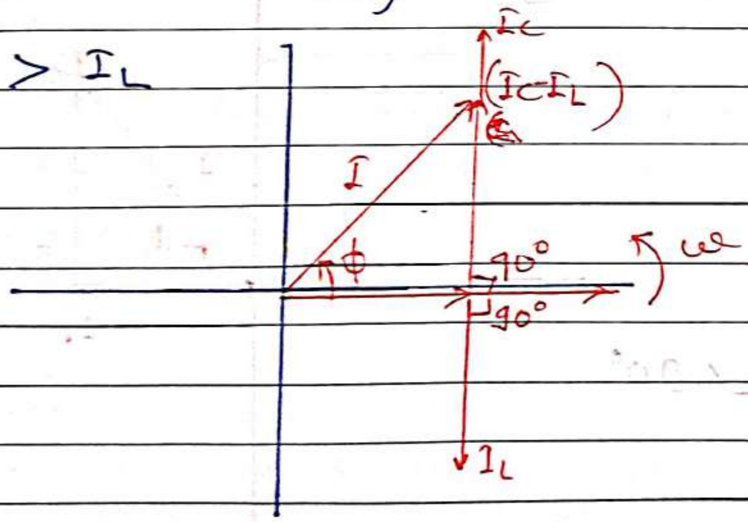
$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

N/w. is in steady state

(i) $I_C > I_L$



$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

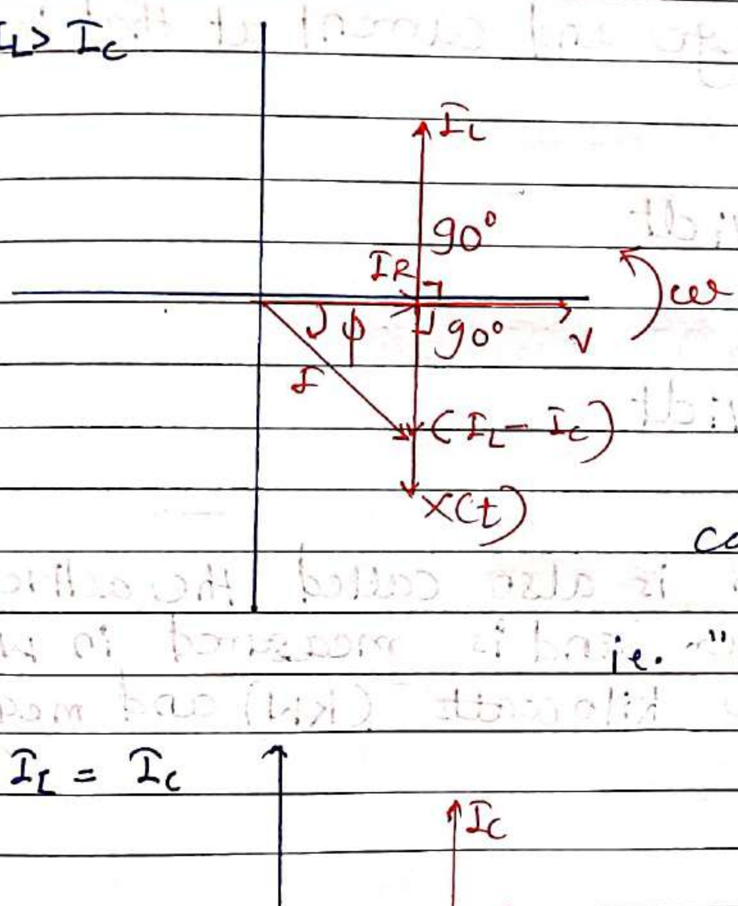
$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$

= Impedance Angle
= Admittance Angle

$$\cos\phi = P.F. = \left(\frac{I_R}{I}\right) \quad \text{(lead)}$$

i.e. "I" leads "V"

(ii) $I_L > I_C$



$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

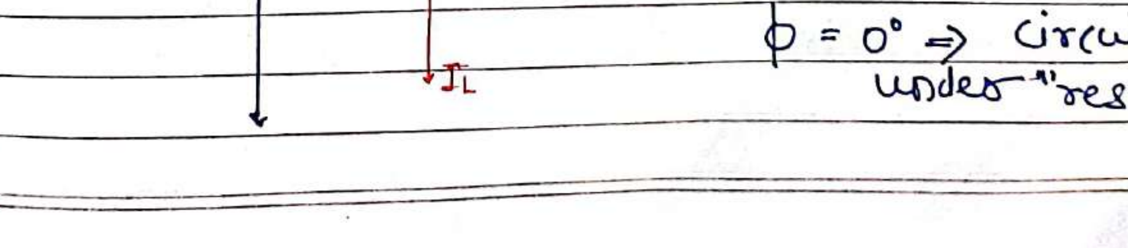
$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

= Impedance Angle
= Admittance Angle

$$\cos\phi = PF = \left(\frac{I_R}{I}\right) \quad \text{(lag)}$$

i.e. "I" lags "V"

(iii) $I_L = I_C$



$\phi = 0° \Rightarrow$ circuit is under "resonance"

Power Calculations :-

Power relations in AC circuits :-

For a DC circuit, the power is given by

$$P_{dc} = VI = I^2R = \frac{V^2}{R}$$

The power at any instant in an AC circuit is called instantaneous power. It is defined by the product of voltage and current at that instant.

i.e. $P = vi$

$$P = \frac{1}{T}\int_0^T vi\,dt$$

Also

$$P = \frac{1}{2\pi}\int_0^{2\pi} vi\,dt$$

The average power is also called the active power or true power and is measured in watts (W). The bigger units are kilowatts (kW) and megawatt (MW)

Power in Pure resistive circuits

The voltage and current in a pure resistive circuits are in phase and may be respectively represented by

$$v = V_m \sin \omega t$$

$$\& \quad i = I_m \sin \omega t$$

The instantaneous power is given by

$$P = vi$$

$$= V_m I_m \sin^2 \omega t$$

$$= \frac{1}{2} V_m I_m (1 - \cos 2\omega t)$$

∴ The active power is given by

$$P = \frac{1}{T}\int_0^T vi\,dt = \frac{V_m I_m}{2T}\int_0^T (1 - \cos 2\omega t)\,dt$$

$$= \frac{1}{2} V_m I_m$$

$$\left(\because \int_0^T \cos 2\omega t\,dt = 0\right)$$

$$= \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) = V_{rms} I_{rms} \text{ Watt.}$$

The power waveform is positive throughout the cycle. This is because voltage & current are in phase.

The power flows only in one direction. It is consumed in R & appears in the form ~~of~~ of heat.

Power in pure Inductive Circuit :-

The voltage leads the current by 90° in a pure inductive circuit. Let the voltage and current at any instant be respectively represented by

$$v = V_m \sin(\omega t + 90^\circ) \text{ and } i = I_m \sin \omega t$$

The instantaneous power is given by

$$P = vi$$
$$= V_m I_m \sin(\omega t + 90^\circ) \sin \omega t$$
$$= \frac{1}{2} V_m I_m (2 \cos \omega t \sin \omega t)$$
$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

Active Power (OR) Average Power :-

$$P_L = \frac{1}{2T} \int_0^T V_m I_m \sin 2\omega t \, dt$$
$$= \frac{1}{2T} V_m I_m \frac{1}{2\omega} (-\cos 2\omega t)_0^T$$
$$= \frac{-V_m I_m}{4\omega T} (\cos 2\omega T - \cos 0)$$
$$= \frac{-V_m I_m}{4\omega T} (\cos 4\pi - \cos 0)$$
$$= \frac{-V_m I_m}{4\omega T} (1 - 1) = 0$$

Therefore in a pure inductive circuit, the active power supplied over a complete cycle average out is zero.

Reactive Power :-

The power supplied to a reactance (Inductive or capacitive) is termed reactive power. Unlike active power denoted by symbol P, the reactive power is denoted by symbol Q and given the name volt-amp-reactive (var). Let us now see how the reactive power is calculated

The peak value of instantaneous power is given by

$$Q_L = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}.$$

for a pure inductive circuit

$$V_L = V_{rms} = X_L I_L$$

$$\therefore Q_L = V_L I_L$$

$$= I_L^2 X_L$$

Also $Q_L = \frac{V_L^2}{X_L}$

Here Q_L is rate of change of reactive energy between the load and the source. By convention, Q_L is taken as positive and is called the lagging reactive power.

Power in Pure Capacitive Circuit:-

The voltage lags the current by 90° in a pure capacitive circuit. Let the voltage & current at any instant be represented by

$$v = V_m \sin \omega t$$

$$\& \quad i = I_m \sin(\omega t + 90^\circ)$$

$\therefore$ The instantaneous power can be written as

$$P = vi = \frac{1}{2} V_m I_m 2 \sin \omega t \sin(\omega t + 90^\circ)$$

$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

Active power (OR) Average Power:-

$$P_c = \frac{1}{T}\int_0^T p\,dt = \frac{V_m I_m}{2T}\int_0^T \sin 2\omega t\,dt$$

$$= \frac{-V_m I_m}{4\omega T}\left(\cos 2\omega t\right)_0^T$$

$$= \frac{-V_m I_m}{4\omega T}(1-1) = 0$$

Reactive Power :-

The peak value of instantaneous power can be written as

$$P_c = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$$

For a purely capacitive circuit

$$V_{rms} = V_C = X_C I_C$$

$$Q_C = V_C I_C$$

$$= I_C^2 X_C$$

OR $$Q_C = \frac{V_C^2}{X_C}$$

Here Q_C is the rate of change of reactive power between the load & the source. By convention, Q_C is taken as negative & is called the leading reactive power.

Power in a general series circuit :-

For a passive ac circuit, let the voltage & current at any instant to be given by

$$v = V_m \sin \omega t$$
$$\& \quad i = I_m \sin(\omega t - \phi)$$

Where ϕ is the phase difference between the voltage and the current. If ϕ is -ve, current leads the voltage.

The instantaneous power P is given by

$$P = vi$$
$$= V_m I_m \sin(\omega t - \phi)$$
$$= \frac{1}{2} V_m I_m (2 \sin \omega t \sin(\omega t - \phi))$$
$$= \frac{1}{2} V_m I_m (\cos\phi - \cos(2\omega t - \phi))$$

The average power P is given by

$$P_L = \frac{1}{2T} \int_0^T \sin \omega t \; dt$$

$$= \frac{1}{T} \int_0^T \frac{V_m I_m}{2} (\cos\phi - \cos(2\omega t - \phi)) dt$$

$$= \frac{V_m I_m}{2T} \left[\int_0^T \cos\phi \, dt - \int_0^T \cos(2\omega t - \phi) dt \right]$$

$$= \frac{V_m I_m}{2T} \times \cos\phi \times T \quad \left(\because \int_0^T \cos(2\omega t - \phi)\,dt = 0 \right)$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = V_{rms} I_{rms} \cos\phi$$

The product of V_{rms} & I_{rms} is called that apparent power and P is called the true or real power.

$\therefore P =$ Apparent power $\times \cos\phi$

or

True (real) power = Apparent power × Power factor
That is, the product of rms values of voltage & current will the cosine of the angle between them (i.e. the power factor) is called the real power or true power) denoted by symbol P.

Reactive Power :-

We can now generalize our previous definition and state that the product of rms values of voltage & current with the sine of the angle between them is called the reactive power Q.

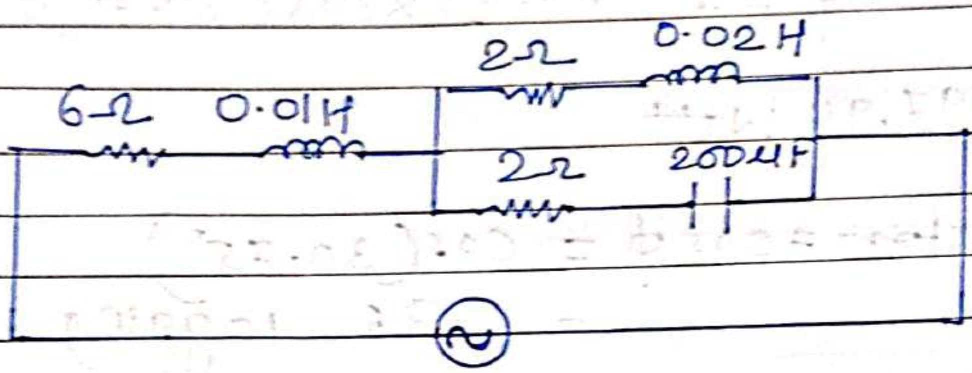
$$\therefore Q = V_{rms} I_{rms} \sin\phi$$

The reactive power is also called the wattless power or quadrature Power. The table represents the different expressions of power in ac series circuits.

Circuit	Power factor	Real Power $P = V_{rms} I_{rms} \cos\phi$	Reactive Power $Q = V_{rms} I_{rms} \sin\phi$	Apparent power $S = \sqrt{P^2+Q^2}$
R	1	$V_{rms} I_{rms}$	0	$V_{rms} I_{rms}$
L	0	0	$V_{rms} I_{rms}$ (lag)	$V_{rms} I_{rms}$
C	0	0	$V_{rms} I_{rms}$ (lead)	$V_{rms} I_{rms}$
RL	$\cos\phi$ (lag)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lag)	$V_{rms} I_{rms}$
RC	$\cos\phi$ (lead)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lead)	$V_{rms} I_{rms}$
	$\cos\phi$ (lag)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (Lag)	$V_{rms} I_{rms}$
RLC	$\cos\phi$ (lead)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lead)	$V_{rms} I_{rms}$
	$\cos\phi = 1$ (In phase)	$V_{rms} I_{rms}$	0	$V_{rms} I_{rms}$.

Example :-

Determine the current in the circuit of fig. Also, find the power consumed as well as power factor.



200 V. 50Hz.

$$X_{L1} = 2\pi \times 50 \times 0.01 = 3.14\,\Omega$$

$$X_{L2} = 2\pi \times 50 \times 0.02 = 6.28\,\Omega$$

$$X_c = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\,\Omega$$

$$\bar{Z}_1 = 6 + j3.14\,\Omega$$

$$\bar{Z}_2 = 4 + j6.28\,\Omega$$

$$\bar{Z}_3 = 2 - j15.92\,\Omega$$

$$\bar{Z} = \bar{Z}_1 + \frac{\bar{Z}_2 \times \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$= (6 + j3.14) + \frac{(4 + j6.28)(2 - j15.92)}{(4 + j6.28) + (2 - j15.92)}$$

$$= 17.27\angle 30.75^\circ\,\Omega$$

$$I = \frac{\bar{V}}{Z} = \frac{100\angle 0^\circ}{17.27\angle 30.75^\circ}$$

$$= 5.79\angle -30.75^\circ \text{ Amp.}$$

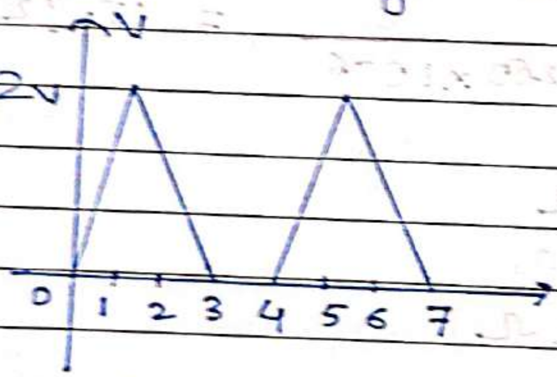
$$P = VI\cos\phi = 100 \times 5.79 \times \cos(39.75^\circ)$$

$$= 497.94 \text{ Watt}$$

$$\text{Power factor} = \cos\phi = \cos(30.75^\circ)$$
$$= 0.86 \text{ lagging.}$$

Example :-

Find the average power delivered to a $10\,\Omega$ resistor with a voltage across it as shown in the figure.



$$V_{rms} = \frac{1}{4}\int_0^4 (v)^2\,dt$$

$$= 2 \times \frac{1}{4}\int_0^{1.5} \left(\frac{3}{4}t\right)^2 dt$$

$$\frac{1}{2}\int_0^{1.5} \frac{9}{16} t^2 \, dt$$

$$= \frac{1}{2} \times \frac{9}{16} \times \frac{t^3}{3}\Big|_0^{1.5}$$

$$V_{rms} = \frac{3}{2 \times 16} \times (1.5)^3 = 0.316$$

$$P = \frac{(V_{rms})^2}{R} = \frac{(0.316)^2}{10} = 100 \text{ milliwatt.}$$