4.8.3

Let π_k : overall probability that a randomely chosen observation comes from the kth class.

$f_k(x)$: Probability density function of x for an observation that is in k^{th} class.

$f_k(x) \rightarrow N(\mu_k, \sigma_k^2)$

$x^2 - 2\mu_k x + \mu_k^2$

$$f_k(x) = \frac{1}{\sqrt{2\pi}\,\sigma_k} e^{-\frac{1}{2\sigma_k^2}(x-\mu_k)^2}$$

$$p_k(x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{k} \pi_l f_l(x)}$$

$$\log(p_k(x)) = \log(f_k(x)) + \log(\pi_k) - \log\left(\sum_{l=1}^{k} \pi_l f_l(x)\right)$$

$$\Rightarrow = x^2\left(\frac{-1}{2\sigma_k^2}\right) + x\left(\frac{\mu_k}{\sigma_k^2}\right) - \frac{\mu_k^2}{2\sigma_k^2} - \log(\sigma_k\sqrt{2\pi})$$

$\log(p_k(x))$ can be seen as a function which is quadratic in x.

Hence, quadratic discriminant analysis.

4.8.7

Let $k=1$ for companies which issues a dividend and let $k=2$ for companies that don't issue a divident.

% profits follows

for class k = 1 $\approx N(\hat{\mu}_1 = 10, \hat{\sigma}^2 = 36)$

class k=2 $\approx N(\hat{\mu}_2 = 0, \hat{\sigma}_2^2 = 36)$

$\therefore$

$\pi_1 = 0.8$

$\pi_2 = 0.2$

$$P_k(x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{k} \pi_l f_l(x)} = \frac{\pi_1 f_1(4)(0.8)}{0.8 f_1(4) + 0.2 f_2(4)}$$

$$P_1(x=4) = \frac{0.8(0.04033)}{0.8 \times 0.04033 + 0.2 \times 0.05324}$$

$$= 0.75185$$