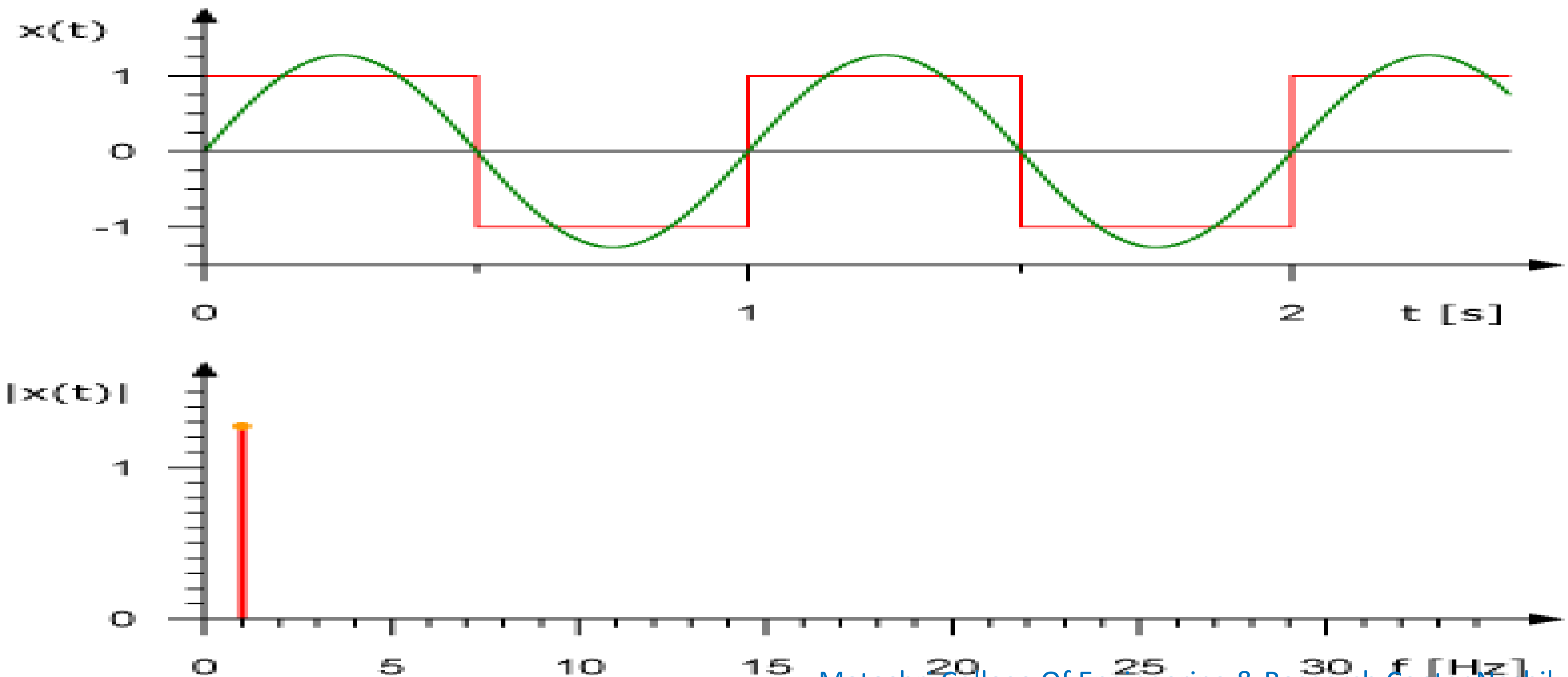
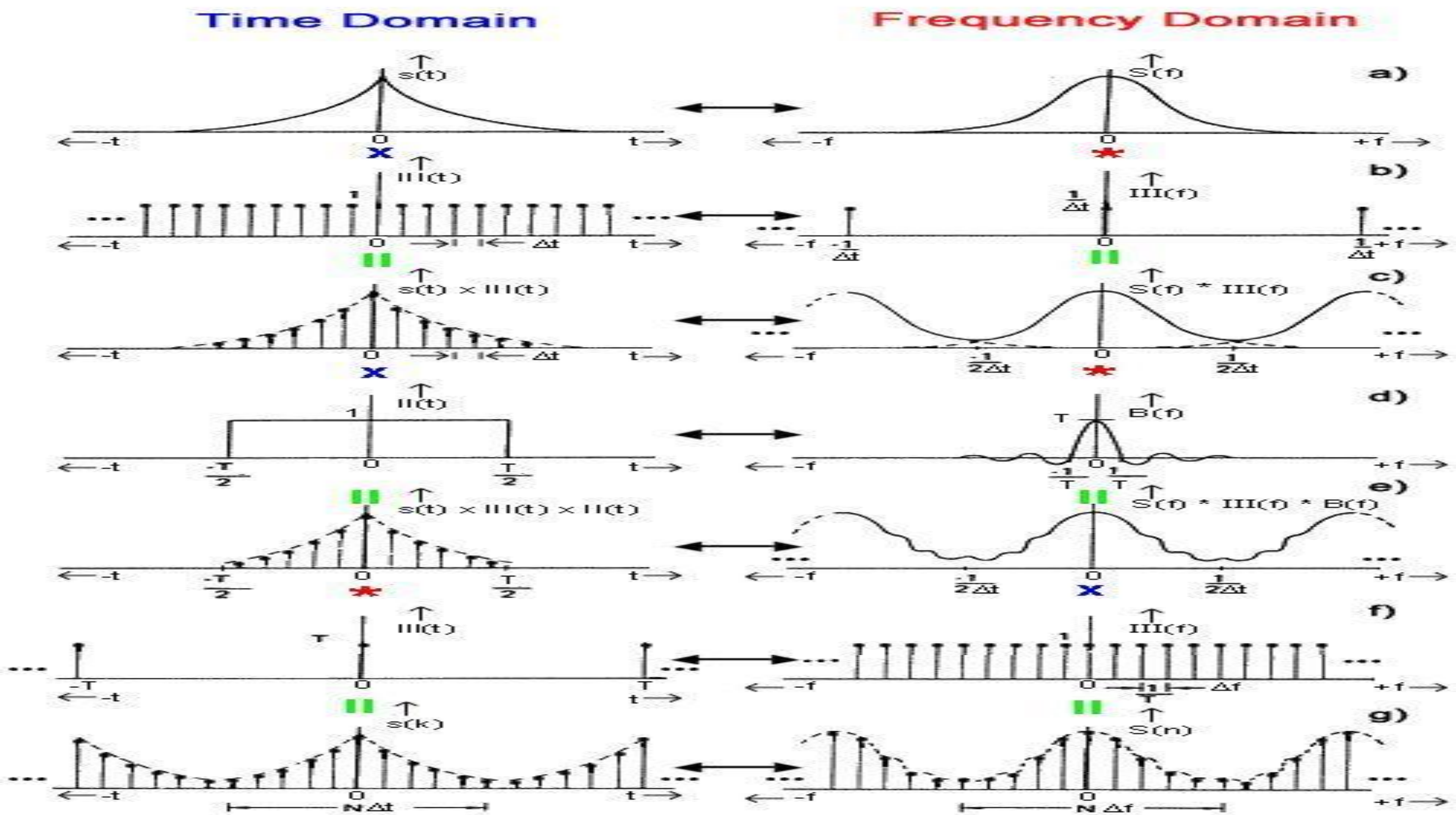
UNIT 2 : Transform

- **Transform :** A given function or signal can be converted between time and frequency domain with pair of mathematical operator or tool is called transform.



Time domain signal

Frequency domain signal



Type of Transform :

- **Fourier Transform (FT)** :

Use for periodic and non periodic function and for stable signal.

- **Z Transform (ZT)** :

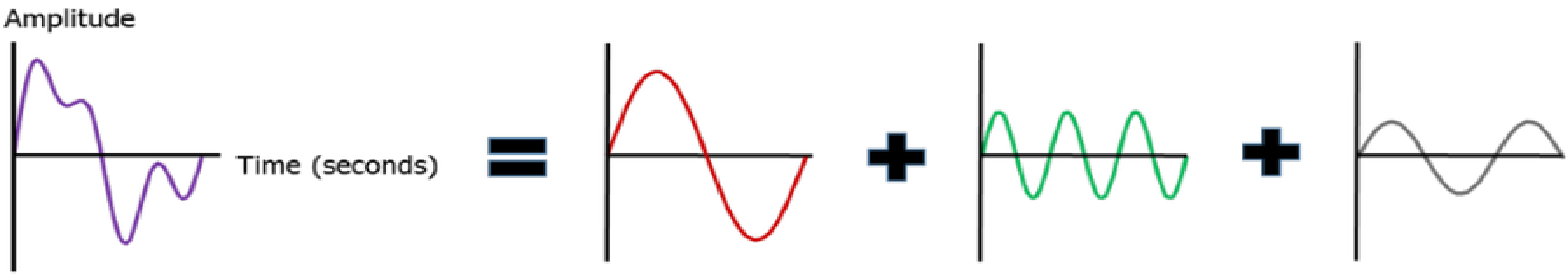
Use for real or complex number sequence and for discrete signal .

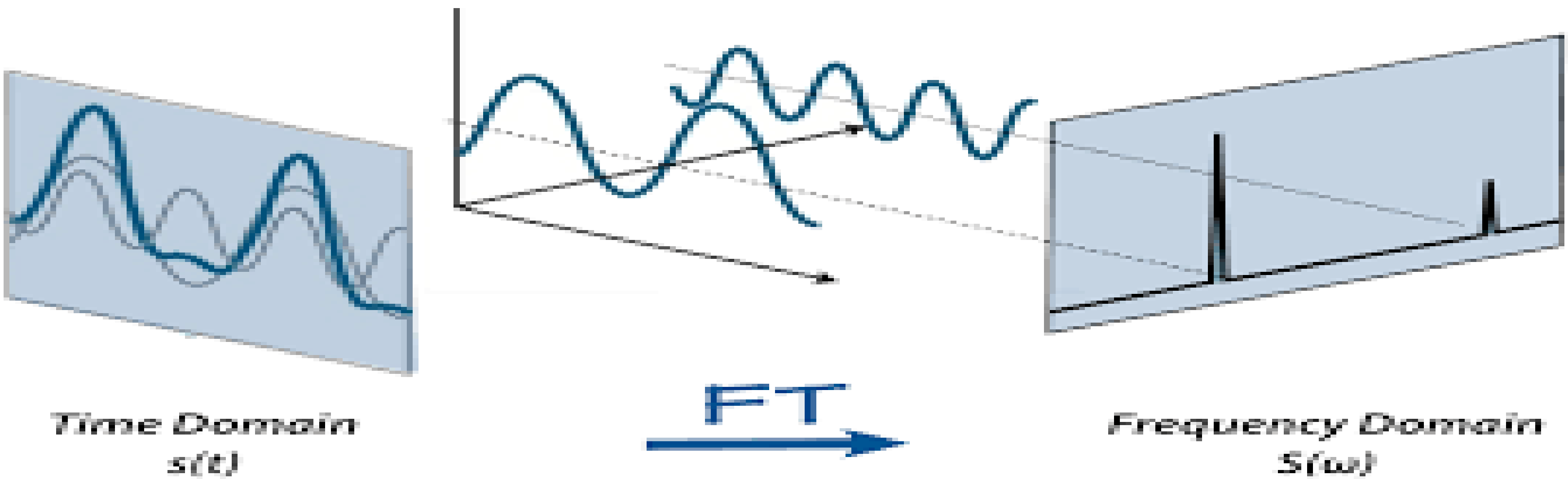
- **Laplace Transform(LT)** :

Use for periodic and non periodic function and for all type of signal.

Fourier Transform

Time domain signal → **Fourier Transform** → Frequency domain signal





Fourier transform convert non periodic function <u>f(t)</u> in time domain into a function <u>$F(\lambda)$</u> in frequency domain

Use of Fourier Transform

- Solving Boundary value problems in science and engineering

eg. Conduction of heat, wave propagation

- Theory of communication
- Use in the filtering, modulation and sampling of the signal, which is the most important application of Fourier transform in signal processing.
- Very little information is lost from the signal during the transformation, the **Fourier transform** maintains information on amplitude, harmonics, and phase and uses all parts of the waveform to translate the signal into the frequency domain.

* Fourier Integral :-

If function $f(x)$ satisfies the Dirichlet's Condition,

(1) $f(x)$ is absolutly integrable.

(2) $\int_{-\infty}^{\infty} |f(x)|\,dx$ is convergent.

then

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{-i\lambda(u-x)}\,du\,d\lambda.$$

This is called Fourier Integral.

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(1) **Fourier Integral**

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{-i\lambda(u-x)}\,du\,d\lambda.$$

(2) **Fourier cosine Integral**

$$f(x)=\frac{2}{\pi}\int_{0}^{\infty}\int_{0}^{\infty} f(u)\cos\lambda u\cos\lambda x\,du\,d\lambda$$

(3) **Fourier sine Integral**

$$f(x)=\frac{2}{\pi}\int_{0}^{\infty}\int_{0}^{\infty} f(u)\sin\lambda u\sin\lambda x\,du\,d\lambda.$$

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* Fourier Transform *

Function	Transform	Inverse Transform
(1) Function is neither even nor odd.	Fourier Transform $= F.T. = F(\lambda)$ $F(\lambda) = \int_{-\infty}^{\infty} f(u) e^{-i\lambda u} du$	Inverse Fourier Transform $= IFT$ $IFT = f(x)$ $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\lambda) e^{i\lambda x} d\lambda$
(2) Function is even	Fourier Cosine Transform $= FCT$ $FCT = F_c(\lambda)$ $F_c(\lambda) = \int_0^{\infty} f(u) \cos \lambda u \, du$	Inverse Fourier Cosine Transform $= IFCT = f(x)$ $f(x) = \frac{2}{\pi} \int_0^{\infty} F_c(\lambda) \cos \lambda x \cdot d\lambda$
(3) Function is odd	Fourier Sine Transform $= FST$ $FST = F_s(\lambda)$ $F_s(\lambda) = \int_0^{\infty} f(u) \sin \lambda u \, du$	Inverse Fourier Sine Transform $= IFST = f(x)$ $f(x) = \frac{2}{\pi} \int_0^{\infty} F_s(\lambda) \sin \lambda x \cdot d\lambda$

* NOTE :-

(1) $\int_{-\infty}^{\infty} f(x)dx = 0 \rightarrow f(x)$ is odd

$= 2\int_0^{\infty} f(x)dx \rightarrow f(x)$ is even

$f(-x) = -f(x) \Rightarrow f(x)$ is odd eg. $\sin x$

$f(-x) = f(x) \Rightarrow f(x)$ is even eg $\cos x$

$e^{f(x)}$ is neither even nor odd.

(2) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

$\int_0^{\infty} e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$

(3) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

$\int_0^{\infty} e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$

(4) $\int_0^{\infty} \frac{\sin ax}{x}dx = \begin{cases} \frac{\pi}{2} & a>0 \\ -\pi/2 & a<0 \end{cases}$

(5) $\int_a^b uv = [u\int v]_a^b - \int_a^b u'\int v.$

(6) $\int uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$

u = algebraic, $u', u'', u''' \ldots$ are derivative

v = trignometric, $v_1, v_2, v_3 \ldots$ are integration

(7) DUIS Rule, $I = \int_a^b f(x;\alpha)dx$

$\frac{d}{d\alpha}(I) = \int_a^b \frac{\partial}{\partial\alpha}(f(x,\alpha)\cdot dx).$

(8) $|x| \leq a \Rightarrow -a \leq x \leq a \rightarrow$ Check fun even or odd.

$|x| > a \Rightarrow x > a$ and $x < -a$

(9) $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

(10) $\sin n\pi = 0$

$\sin(2n\pi) = 0$

$\sin(n\pm1)\pi = 0$

$\cos n\pi = (-1)^n$

$\cos(2n\pi) = (-1)^{2n}$
$= 1.$

$\cos(n\pm1)\pi = (-1)^{n\pm1}$

$\sin\frac{n\pi}{2}$ and $\cos\frac{n\pi}{2}$ depend on the value of n.

(11) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B).$