Unit 1. L.D.E

* Linear Diff eqn with constant coeff.

$$a_0\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + a_2\frac{d^{n-2}y}{dx^{n-2}} + \dots + a_{n-1}\frac{dy}{dx}$$
$$+ a_n y = X, \quad a_0 \neq 0.$$

when $a_0, a_1, a_2, \dots a_n$ are all consts.
& X is funn of x, $a_0 \neq 0$.

$$D = \frac{d}{dx}.$$

$$a_0 D^n y + a_1 D^{n-1} y + a_2 D^{n-2} y + \dots + a_{n-1} Dy$$
$$+ a_n y = X.$$

$$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-1} + \dots + a_{n-1} D) y = X.$$

$$f(D) y = X$$

when $f(D)$ is a poly. in D of deg. n
& X is any funn of x.
$f(D) y = 0$ is Associated eqn.
$f(D) = 0$ is known as auxillary eqn.
The soln of eqn involves two parts
1) complementary funns (y_c or C.F)
2) Particular Integral (P.I)

$$\underline{y = y_c + y_p.}$$

* Methods for finding C.F.

To find y_c. find the roots of Auxillary eqn $f(D) = 0$.

I) If the roots are real & distinct
If $m_1, m_2, m_3, \dots$ are the real roots then. $y_c = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \dots$

II) If the roots are real & repeated.
If m_1, m_1 are repeated roots of $f(D) = 0$ then $y_c = (c_1 + c_2 x) e^{m_1 x}$.

iii) If the roots are complex (Complex roots always occur in conjugate pair) i.e. if $\alpha+i\beta$ is one root then $\alpha-i\beta$ will be another roots.

$$y_c = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x).$$

iv) If the roots are complex repeated. $\alpha \pm i\beta$ & $\alpha \pm i\beta$ are repeated roots.

$$\cancel{y_c = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)}..$$

$$y_c = e^{\alpha x}[(C_1+C_2x)\cos\beta x + (C_3+C_4x)\sin\beta x]$$

Q. Find the C.F. of $f(D)=0$.

1) $D^3-7D-6=0$.

$D=-1,\ D=3,\ D=-2$.

$$y_c = C_1e^{-x} + C_2e^{3x} + C_3e^{-2x}.$$

2) $D^3+7D^2+16D+10=0$.

$D=-1,\ \alpha=-3,\ \beta=1 \qquad D=-3\pm i$

$$y_c = C_1e^{-x} + e^{-3x}(C_2\cos x + C_3\sin x)$$

3) $D^3+3D^2+3D+1=0$.

$D=-1,-1,-1$

$$y_c = (C_1+C_2x+C_3x^2)e^{-x}.$$

4) $D^3+D^2+D+1=0$

$D=-1,\ \cancel{D=-1,-1},\ D=0\pm i$

$$y_c = C_1e^{-x} + e^{0x}(C_2\cos x + C_3\sin x)$$

5) $D^3 - 2D + 4 = 0$

$D = -2, \quad D = 1 \pm i$

$y_c = c_1 e^{-2x} + e^{x}(c_2 \cos x + c_3 \sin x)$

6) $D^3 - D^2 + 3D + 5 = 0$

or $D = -1, \quad D = 1 \pm 2i$

$y_c = c_1 e^{-x} + e^{x}(c_2 \cos 2x + c_3 \sin 2x)$

7) $D^3 + 7D^2 + 16D + 10 = 0$.

$D = -1, \quad D = -3 \pm i$

$y_c = c_1 e^{-x} + e^{-3x}(c_2 \cos x + c_3 \sin x)$.

8) $D^3 + 3D = 0$

$D(D^2 + 3) = 0$

$D = 0, \quad D^2 = -3$

$D = 0 \pm i\sqrt{3}$.

$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x)$.

9) $D^2 + 9 = 0$

$D = \pm 3i$

$y_c = c_1 \cos 3t + c_2 \sin 3t$.

10) $(D^2 - 2D + 2)y = 0$

$D = -1 \pm i$

$y = e^{-x}(c_1 \cos x + c_2 \sin x)$.

11) $D^2 + 5D + 6 = 0$

$D = -2, -3$

$y_c = c_1 e^{-2x} + c_2 e^{-3x}$.

Ex. $D^4 - a^4 = 0$

$(D^2 - a^2)(D^2 + a^2) = 0$

$D^2 - a^2 = 0, \quad D^2 + a^2 = 0$

$D^2 = a^2 \qquad D^2 = -a^2$

$D = \pm a \qquad D = \pm ia.$

$D = a, -a, \; 0 \pm ia$

$$y_c = c_1 e^{ax} + c_2 e^{-ax} + e^{0x}(c_3 \cos ax + c_4 \sin ax).$$

Ex. $(D^2 - 1)^2 (D^2 + 1)^2 = 0$

$(D-1)^2 = 0, \quad (D^2+1)^2 = 0$

$(D-1)(D-1) = 0, \quad (D^2+1) = 0 \quad (D^2+1) = 0$

$D = 1, 1 \qquad D^2 = -1, \qquad D^2 = -1$

$D = 1, 1 \qquad D = 0 \pm i \qquad D = 0 \pm i$

$$y_c = (c_1 + c_2 x) e^x + e^{0x}[(c_2 + c_3 x)\cos x + (c_4 + c_5 x)\sin x].$$

$$y_c = (c_1 + c_2 x) e^x + (c_2 + c_3 x)\cos x. + (c_4 + c_5 x)\sin x.$$

Ex $D^4 + 5D^2 + 4 = 0 \qquad D^2 = x$

$x^2 + 5x + 4 = 0$

$D = 0 \pm i, \; 0 \pm 2i$

$$y_c = e^{0x}(c_1 \cos x + c_2 \sin x) + e^{0x}(c_3 \cos 2x + c_4 \sin 2x.$$

Ex. $D^4 + 10D^2 + 9 = 0$

$D = \pm i \qquad D = \pm 3i$

$$y_c = c_1 \cos x + c_2 \sin x + c_3 \cos 3x + c_4 \sin 3x$$

Ex $D^4 - 2D^3 + D^2 = 0$,

$D^2(D^2 - 2D + 1) = 0$.

$D = 0, 0$, $D = 1, 1$

$y_c = (c_1 + c_2 x)e^{0x} + (c_3 + c_4 x)e^{x}$.

Ex $D^3 + 7D^2 + 16D + 10 = 0$.

$D = -1, -3 \pm i$

$y_c = c_1 e^{-x} + e^{-3x}(c_2 \cos x + c_3 \sin x)$.

Ex $D^3 + D^2 + D + 1 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 1 & 1 \\ & & & & \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$(D+1)(D^2 + 0D + 1) = 0$

$D = -1$, $D^2 + 1 = 0$

$D^2 = -1$

$D = \pm i$

$y_c = c_1 e^{-x} + e^{0x}(c_2 \cos x + c_3 \sin x)$.

Ex $D^3 - 5D^2 + 8D - 4 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 8 & -4 \\ & & & & \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array}$$

$D = 1, 2, 2$.

$y_c = c_1 e^{x} + (c_2 + c_3 x)e^{2x}$.

Ex $D^4 + 2D^3 + 3D^2 + 2D + 1 = 0$

$-\cancel{2}$	1	2	3	2	1
		-7	-9	-36	
	1	9	3		

$\underline{(D^2)^2 + 2D^2 \cdot D + \cancel{3}D^2} + 2D^2 + 2D + 1 = 0$

$(D^2 + D)^2 + 2(D^2 + D) + 1 = 0$

$x^2 + 2x + 1$
$(x+1)^2$

$(D^2 + D + 1)^2 = 0$

$D^2 + D + 1 = 0 \quad , \quad D^2 + D + 1 = 0$

$D = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2} \qquad D = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$\cancel{y_c} = \cancel{c_1}e^{-\frac{1}{2}x}[\cancel{c_1\cos}$

$y_c = e^{-\frac{1}{2}x}\left[(c_1 + c_2x)\cos\frac{\sqrt{3}}{2}x + (c_3 + c_4x)\sin\frac{\sqrt{3}}{2}x\right]$

Ex $D^5 - D^4 + 2D^3 - 2D^2 + D - 1 = 0$

$D^4(D-1) + 2D^2(D-1) + (D-1) = 0$

$(D-1)(D^4 + 2D^2 + 1) = 0$

$D = 1 \quad , \quad D^4 + 2D^2 + 1 = 0$

$D = 1 \quad , \quad (D^2 + 1)^2 = 0$

$D = 1 \quad , \quad D^2 = -1 \quad , \quad D^2 = -1$

$D = 0 \pm i \quad , \quad 0 \pm i$

$y_c = c_1e^x + e^{0x}[(c_2 + c_3x)\cos x + (c_3 + c_4x)\sin x]$

Inverse Operator :—

$f(D)$ is simple operator, &

$\frac{1}{f(D)}$ is the inverse operator of $f(D)$

~~Method to find~~

To ~~find~~ the Particular ~~method~~ Integral

① Short-cut method.

② General method.

③ Method of variation of parameters.

① ~~Short cut method~~ :-

for finding y_p :

$$y_p = \frac{1}{\phi(D)} f(x)$$

e.g. $(D^2+1)\, y = e^{2x}$.

To find y_p.

$$y_p = \frac{1}{D^2+1} \cdot e^{2x}$$

Engineering Mathematics III

Unit 1 : Linear Diff. Eq^n With Constant Coefficients

The form of L.D.E. with const. coeff. is

$\phi(D)y = f(x)$ & sol^n is

$\boxed{y = y_c + y_p}$ or $\boxed{y = C.F + P.I}$

*** Methods for finding Particular Integral (P.I or y_p)**

$$\boxed{y_p = \frac{1}{\phi(D)} \cdot f(x)}$$

There are 3 methods.

1) Short-cut method.
2) Method of variation of parameter
3) General method.

1) Short-cut method :—

There are 8 cases on the basis of R.H.S function $f(x)$.

i) If $f(x) = e^{ax}$ (a-constant)

ii) If $f(x) = a^x$

iii) If $f(x) = K$. (k-constant)

iv) If $f(x) = \cos(ax+b)$ or $\sin(ax+b)$

v) If $f(x) = \cosh(ax+b)$ or $\sinh(ax+b)$

vi) If $f(x) = x^m$ (poly. fun^n)

vii) If $f(x) = e^{ax}.V$ (V is any fun^n of x)

viii) If $f(x) = x.V$

(1) Short cut method :-

case i) When $f(x) = e^{ax}$

$$y_p = \frac{1}{\phi(D)} e^{ax}.$$

$D \to a$. provided $\phi(a) \neq 0$.

If $\phi(a) = 0$ then case of failure occu[rs] then

$$y_p = \frac{x}{\phi'(D)} e^{ax}.$$

$D \to a$.

$$y_p = \frac{x}{\phi'(a)} e^{ax}, \text{ provided } \phi'(a) \neq 0$$

If $\phi'(a) = 0$.

$$y_p = \frac{x^2}{\phi''(a)} e^{ax}, \text{ provided } \phi''(a) \neq 0$$

& so on.

Case (2) If $f(x) = a^x$ (a is any const)
then we know $\boxed{a^x = e^{x\log a}}$

$$f(x) = e^{x\log a}.$$

$$y_p = \frac{1}{\phi(D)} e^{x\log a}.$$

$\boxed{D \to \log a}$

$$y_p = \frac{1}{\phi(\log a)} e^{x\log a} \text{ or } a^x.$$

If $\phi(\log a) = 0$ case of failuar occurs & so on

* If $f(x) = a^{-x}$ then we use.

$$a^{-x} = e^{-x\log a}.$$

$$a^{-x} = e^{x(-\log a)}$$

$$y_p = \frac{1}{\phi(D)} a^{-x}. \qquad D \to -\log a.$$

$$y_p = \frac{1}{\phi(-\log a)} a^{-x}.$$

provide $\phi(-\log a) \neq 0$.

case (3) If $f(x) = K$ (K is any const)

$$y_p = \frac{1}{\phi(D)} K.$$

$$y_p = \frac{1}{\phi(D)} K \cdot e^{0x}$$

$$y_p = K \frac{1}{\phi(D)} e^{0x}$$

$\boxed{D \to 0}$

$$y_p = K \cdot \frac{1}{\phi(0)} (1)$$

provided $\phi(0) \neq 0$.

① Solve $(D^3 - D^2 + 4D - 4)\, y = e^x$.

$\Rightarrow$ To find y_c, consider A.E.

$D^3 - D^2 + 4D - 4 = 0$.

$D = 1, \quad 0 \pm 2i$

$y_c =$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3 - D^2 + 4D - 4} e^x$$

$D \to 1$

$$y_p = \frac{1}{1 - 1 + 4 - 4} e^x$$

$$y_p = \frac{1}{0} e^x$$ case of failuar

$$y_p = \frac{1}{3D^2 - 2D + 4} e^x$$

$D \to 1$

$$y_p = \frac{1}{3 - 2 + 4} e^x$$

$$y_p = \frac{1}{5} e^x$$

② $$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^{2x}$$

$(D^2 - 7D + 6)\, y = e^{2x}$.

$D^2 - 7D + 6 = 0$

$D = 6, 1$

$y_c = c_1 e^{6x} + c_2 e^x$

③ $(D^3 - 5D^2 + 8D - 4)y = e^{2x} + 2e^{x} + 3e^{-x} + 2$

$\Rightarrow$ $D = 1, 2, 2$

$$y_p = \frac{1}{D^3 - 5D^2 + 8D - 4} \left[e^{2x} + 2e^{x} + 3e^{-x} + 2 \right]$$

$$y_p = \frac{1}{D^3 - 5D^2 + 8D - 4} e^{2x} + 2 \frac{1}{D^3 - 5D^2 + 8D - 4} e^{x}$$

$$+ 3 \frac{1}{D^3 - 5D^2 + 8D - 4}$$

④ $(D^3 - 5D^2 + 8D - 4)y = e^{2x} + 2e^{x}$.

$\Rightarrow$ $D = 1, 2, 2$

$y_c = c_1 e^{x} + (c_2 + c_3 x) e^{2x}$.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3 - 5D^2 + 8D - 4} e^{2x} + 2e^{x}.$$

$$y_p = \frac{1}{D^3 - 5D^2 + 8D - 4} e^{2x} + 2 \frac{1}{D^3 - 5D^2 + 8D - 4} e^{x}$$

$D \to 2$ $\qquad$ $D \to 1$

$$y_p = \frac{1}{8 - 20 + 16 - 4} e^{2x} + 2 \cdot \frac{1}{1 - 5 + 8 - 4} e^{x}.$$

$$= \frac{1}{0} e^{2x} + 2 \cdot \frac{1}{0} e^{x}$$

case of failuar.

$$y_p = \frac{x}{3D^2 - 10D + 8} e^{2x} + 2 \cdot \frac{x}{3D^2 - 10D + 8} e^{x}.$$

$D \to 2$ $\qquad$ $D \to 1$

$$y_p = \frac{x}{12 - 20 + 8} e^{2x} + 2 \cdot \frac{x}{3 - 10 + 8} e^{x}$$

$$y_p = \frac{1}{0} e^{2x} + 2 \frac{x}{1} e^{x}$$

case of failuar

$y_p = \frac{x}{\quad}$

$(D^4+1)=0$

$D^4=-1$

$D^4+\cancel{D^2}-D^2 \; \cancel{+1} = 0$

$D^4+D^2+1 \cancel{-D^2=0}$

$D^4-1=0$

$(D^2-1)(D^2+1)=0$

$(D-1)(D-1)=0$

$\cancel{D^4+1=0}$

$\cancel{D^4-2D^2+2D^2+1=0}$

$\cancel{D^4+2D^2+1=2D^2}$

$\cancel{D^4-\frac{D^2}{2}+\frac{D^2}{2}+1}$

$\cancel{(D^2+2D+\frac{1}{4})(D^2+2D+\frac{1}{2})=0}$

$\cancel{D^2-\frac{1}{4}}$

$(D^2+\frac{1}{4})^2$

$D^2+2\cdot D\frac{1}{4}$

D^4+2D^2

$D^4+1=0$

$D^4-2D^2+2D^2+\frac{1}{4}-\frac{1}{4}=0$

$D^4-2D^2+\frac{1}{4}$

$D^4+1=0$

$D^4+2D^2+1-2D^2=0$

$(D^2+1)^2=(\sqrt{2}D)^2$

$(D^2+1-\sqrt{2}D)(D^2+1+\sqrt{2}D)=0$

$D^2-\sqrt{2}D+1=0, \quad D^2+\sqrt{2}D+1=0.$

$D=\frac{1}{\sqrt{2}}\pm\frac{1}{\sqrt{2}}i \qquad D=\frac{-1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}.$

$y_p=\frac{x^2}{6D-10}e^{2x}+2xe^x$

$D\to 2$

$y_p=\frac{x^2}{2}e^{2x}+2xe^x.$

$y=y_c+y_p$

$2^x=e^{x}$

(5) $(D^3-5D^2+8D-4)y=2^x+3$

$y_p=\frac{1}{D^3-5D^2+8D-4}2^x+\frac{1}{D^3-5D^2+8D-4}3$

$D\to\log 2 \qquad D\to 0$

$y_p=\frac{1}{(\log 2)^3-5(\log 2)^2+8\log 2-4}2^x+\frac{1}{0+0+0-4}3$

Rule 4: If $f(x) = \sin(ax+b)$ or $\cos(ax+b)$ then

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{\phi(D^2)} \sin(ax+b) \text{ or } \cos(ax+b)$$

Replace $D^2 \longrightarrow -a^2$

$$y_p = \frac{1}{\phi(-a^2)} \sin(ax+b)$$

provided $\phi(-a^2) \neq 0$ if $\phi(-a^2) = 0$ we get case of failur then solve further procedure.

formulae

1. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$.
2. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$.

3. $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$
4. $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$
5. $2 \cos A \cos B = \cos(A-B) + \cos(A+B)$

Ex 1) $(D^3 + D^2 + D + 1) y = \cos^2 x$ [use $\cos^2 x = \frac{1+\cos 2x}{2}$

$\Rightarrow$ A.E : $D^3 + D^2 + D + 1 = 0$

$D = -1,\ 0 \pm i$

$$y_c = c_1 e^{-x} + c_2 \cos x + c_3 \sin x.$$

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3+D^2+D+1} \cos^2 x.$$

$$y_p = \frac{1}{D^3+D^2+D+1} \left(\frac{1+\cos 2x}{2} \right).$$

$$y_p = \frac{1}{2} \left[\underset{D \to 0}{\frac{1}{D^3+D^2+D+1}}\, 1 + \underset{D^2 \to -4}{\frac{1}{D^3+D^2+D+1}} \cos 2x \right]$$

$$y_p = \frac{1}{2}\left[\frac{1}{0+0+0+1} + \frac{1}{-4D-4+D+1}\cos x\right]$$

$$y_p = \frac{1}{2}\left[+1 + \frac{1}{-3D-3}\cos x\right]$$

$$y_p = \frac{1}{2}\left[1 - \frac{1}{3}\left(\frac{1}{D+1}\right)\cos 2x\right]$$

$$y_p = \frac{1}{2}\left[1 - \frac{1}{3}\left(\frac{1}{D+1}\times\frac{D-1}{D-1}\right)\cos 2x\right]$$

$$y_p = \frac{1}{2}\left[1 - \frac{1}{3}\left(\frac{D-1}{D^2-1}\right)\cos 2x\right]$$

$$y_p = \frac{1}{2}\left[1 - \frac{1}{3}\frac{(D-1)}{D^2-1}\cos 2x\right]$$

$$D^2 \to -4$$

$$y_p = \frac{1}{2}\left[1 - \frac{1}{3}\frac{(D-1)}{-4-1}\cos 2x\right]$$

$$y_p = \frac{1}{2}\left[1 + \frac{1}{15}(-2\sin 2x - \cos 2x)\right]$$

$$y_p = \frac{1}{2}\left[1 - \frac{1}{15}(2\sin 2x + \cos 2x)\right]$$

$$y_p = \frac{1}{2} - \frac{1}{30}(2\sin 2x + \cos 2x)$$

$$y = y_c + y_p.$$

② $(D^3+D^2-D-1)y = x\sin x\cos x$. [use $\sin 2x = 2$

$\Rightarrow D = 1, -1, -1$

$$y_c = c_1e^{x} + (c_2+c_3x)e^{-x}$$

$$y_p = \frac{1}{D^3+D^2-D-1}\, x\sin x\cos x.$$

$$y_p = \frac{1}{D^3+D^2-D-1}\, \sin(x+x) + \sin x \,/\, \sin$$

$$D^2 \to -4$$

$$y_p = \frac{1}{-4D-4-D-1}\sin 2x.$$

$$y_p = \frac{1}{-5D-5}\sin 2x.$$

$$y_p = \frac{1}{-5(D+1)} \sin 2x$$

$$y_p = \frac{-1}{5} \frac{1}{D+1} \times \frac{D-1}{D-1} \sin 2x$$

$$y_p = \frac{-1}{5} \frac{D-1}{D^2-1} \sin 2x. \qquad D^2 \to -4$$

$$y_p = \frac{-1}{5} \frac{D-1}{-4-1} \sin 2x.$$

$$y_p = -\frac{1}{5}\left(\frac{-1}{5}\right)(D \sin 2x - \sin 2x)$$

$$y_p = \frac{1}{25}(2\cos 2x - \sin 2x).$$

$$y = y_c + y_p.$$

(3) $$\operatorname{cosec} x \frac{d^4y}{dx^4} + (\operatorname{cosec} x)\, y = \sin 2x.$$

$\Rightarrow$ Divide by $\operatorname{cosec} x$.

$$\frac{d^4y}{dx^4} + y = \frac{\sin 2x}{\operatorname{cosec} x}.$$

$$(D^4+1)y = \sin 2x \cdot \sin x.$$

To find y_c $\left[D = \frac{+1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}},\ \frac{-1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}\right]$

$$y_c = e^{x/\sqrt{2}}\left[c_1 \cos\frac{x}{\sqrt{2}} + c_2 \sin\frac{x}{\sqrt{2}}\right] + e^{-x/\sqrt{2}}\left[c_3 \cos\frac{x}{\sqrt{2}} + c_4 \sin\frac{x}{\sqrt{2}}\right].$$

$$y_p = \frac{1}{g(D)} f(x)$$

$$y_p = \frac{1}{D^4+1} \sin 2x \sin x.$$

$$y_p = \frac{1}{D^4+1} \frac{1}{2}(2 \sin 2x \cdot \sin x).$$

$$y_p = \frac{1}{D^4+1} \frac{1}{2}\left[\cos(2-1)x - \cos(2+1)x\right]$$

$$y_p = \frac{1}{2} \frac{1}{D^4+1}(\cos x - \cos 3x)$$

$$y_p = \frac{1}{2}\frac{1}{D^4+1}(\cos x - \cos 3x)$$

$$y_p = \frac{1}{2}\left[\frac{1}{D^4+1}\cos x - \frac{1}{D^4+1}\cos 3x\right]$$

$$y_p = \quad D^2 \to -1 \qquad\qquad D^2 \to -9$$

$$y_p = \frac{1}{2}\left[\frac{1}{1+1}\cos x - \frac{1}{81+1}\cos 3x\right]$$

$$y_p = \frac{1}{2}\left[\frac{1}{2}\cos x - \frac{1}{82}\cos 3x\right]$$

$$y_p = \frac{1}{4}\cos x - \frac{1}{164}\cos 3x$$

$$y = y_c + y_p.$$

<u>H.W</u>

1) $(D^3-5D^2+8D-4)y = 4e^{2x}+e^x+2^x+3.$
2) $(D^4-1)y = e^x+1$
3) $(D^2+4D+4)y = e^{-2x}+2^x+3$
4) $(D^2-4)y = (1+e^x)^2+3$
5) $(D^2+13D+36)y = e^{-4x}+\sinh x.$

use sinhx
$\cosh x = \frac{e^x+}{2}$

1) $(D^4+m^4)y = \sin mx.$

2) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x = 0.$

3) $(D^3-D^2+3D+5)y = 2\sin x\cos x.$

4) $(D^4+8D^2+16)y = \cos^2 x$ $\qquad \boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$

5) $(D^4+2D^2+1)y = \cos x.$

6) $(D^2+4)y = \sin x\cos 3x.$ $\qquad 2\sin A\cos B = \sin(A+B)$ (A-
$y_c = c_1\cos 2x + c_2\sin 2x.$ $\qquad$ sin
$y_p = -\frac{1}{24}\sin 4x$

Rule 5 : If $f(x) = \cosh(ax+b)$ or $\sinh(ax+b)$

$$y_p = \frac{1}{\phi(D)} \cosh(ax+b)$$

$$D^2 \to a^2.$$

$$y_p = \frac{1}{\phi(a^2)} \cosh(ax+b)$$

provided $\phi(a^2) \neq 0$.

$y_p =$

Ex 1) $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 2\cosh 2x.$

$\Rightarrow \quad (D^3 - 4D)y = 2\cosh 2x.$

$D = 0, 2, -2.$

$y_c = c_1 e^{0x} + c_2 e^{2x} + c_3 e^{-2x}.$

$$y_p = \frac{1}{\phi(D)} 2\cosh 2x.$$

$$D^2 \to +4$$

$$y_p = 2\frac{1}{D^3 - 4D} \cosh 2x.$$

$$y_p = 2 \cdot \frac{1}{4D - 4D} \cosh 2x.$$

case of failuar

$$y_p = 2 \cdot \frac{x}{3D^2 - 4} \cosh 2x.$$

$$D^2 \to 4$$

$$y_p = 2 \cdot \frac{x}{12 - 4} \cosh 2x.$$

$$y_p = 2 \cdot \frac{x}{8} \cosh 2x.$$

$$y_p = \frac{x}{4} \cosh 2x.$$

$$y = y_c + y_p.$$

2) $(D^2+1)y = \sinh 4x$.

$\Rightarrow$ $D = \pm i$

$y_c = c_1\cos x + c_2 \sin x$.

$y_p = \frac{1}{D^2+1} \sinh 4x$.

$D^2 \rightarrow 4$

$y_p = \frac{1}{4+1} \sinh 4x$.

$y_p = \frac{1}{5} \sinh 4x$.

Rule 6 :- When $f(x) = x^m$

$y_p = \frac{1}{\phi(D)} f(x)$

$y_p = \frac{1}{\phi(D)} x^m$

$y_p = \frac{1}{1+\phi(D)} x^m$

$y_p = [1+\phi(D)]^{-1} x^m$.

Expand $[1+\phi(D)]^{-1}$ in ascending power of D.

$\cancel{\frac{1}{1+x} = (1+x)^{-1} = 1+x+x^2}$

$\frac{1}{1+x} = (1+x)^{-1} = 1-x+x^2-x^3+\ldots$

$\frac{1}{1-x} = (1-x)^{-1} = 1+x+x^2+x^3+\ldots$

Ex 1) $(D^4 - 2D^3 + D^2)y = x^3$.

$\Rightarrow$ A.E

$$D^4 - 2D^3 + D^2 = 0$$

$$D^2(D^2 - 2D + 1) = 0$$

$$D^2 = 0, \quad D^2 - 2D + 1 = 0$$

$$D = 0, 0, \quad D = 1, 1.$$

$$y_c = (c_1 + c_2 x)e^{0x} + (c_3 + c_4 x)e^{x}.$$

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^4 - 2D^3 + D^2} x^3$$

$D(x^3) = 3x^2$

$D^2(\ ) = 6x$

$D^3(\ ) = 6$

$D^4 = 0$

$2D^4 . 2D$

$2D^2 4D^2$

$$y_p = \frac{1}{D^2[1 + (D^2 - 2D)]} x^3$$

$$y_p = \frac{1}{D^2}[1 + (D^2 - 2D)]^{-1} x^3.$$

$-(D^2-2D)^3$

$$y_p = \frac{1}{D^2}\left[1 - (D^2 - 2D) + (D^2 - 2D)^2 - \cancel{(D^2 - 2D)^3} \ldots\right] x^3$$

$$y_p = \frac{1}{D^2}\left[x^3 - (D^2 - 2D)x^3 + (D^4 - 4D^3 + 4D^2)x^3\right.$$

$$- CD \quad D^6 - 4D^5 + 8D^4$$

$3x^2$

$6x$

$$y_p = \frac{1}{D^2}x^3 - \left(1 - \frac{2}{D}\right)x^3 + \left(\frac{D^2}{\cancel{D^2}} - \frac{4D}{4} + 4\right)x^3$$

$\frac{x^5}{...}$

$$y_p = \frac{x^5}{20} - x^3 + 2\frac{x^4}{4} + 6x - 4(3x^2) + 4x^3$$

$$y_p = \frac{x^5}{20} + \frac{x^4}{2} + 3x^3 - 12x^2$$

Ex $(D^3 - 6D^2 + 12D - 8)y = x^2 + 1$

$\Rightarrow$ $D = 2, 2, 2$

$y_c = (c_1 + c_2 x + c_3 x^2) e^{2x}$.

$y_p = \frac{1}{\phi(D)} f(x)$

$y_p = \frac{1}{D^3 - 6D^2 + 12D - 8} x^2 + 1$

$y_p = \frac{1}{-8\left[1 - \left(\frac{D^3 - 6D^2 + 12D}{8}\right)\right]} x^2 + 1$

$y_p = \frac{-1}{8}\left[1 - \left(\frac{D^3 - 6D^2 + 12D}{8}\right)\right]^{-1} (x^2 + 1)$

$y_p = \frac{-1}{8}\left[1 + \left(\frac{D^3 - 6D^2 + 12D}{8}\right) - \left(\frac{D^3 - 6D^2 + 12D}{8}\right)\right.$
$(x^2 + 1)$

$y_p = \frac{-1}{8}\left\{1 + \frac{1}{8}(D^3 - 6D^2 + 12D) - \frac{1}{64}(D^3 - 6D^2 + 12D\right.$
$\left\{(x^2 + 1)\right\}$

$y_p = \frac{-1}{8}\left\{x^2 + 1 + \frac{1}{8}(D^3 - 6D^2 + 12D) x^2 + 1\right.$
$- \frac{1}{64}(\cancel{D^6} - \cancel{12D^5} \cancel{+} 144D^2)(x^2 + 1)$

$y_p = -\frac{1}{8}\left\{x^2 + 1 + \frac{1}{8}\left[-6(2) + 12(2x)\right]\right.$
$\left. - \frac{1}{64} 144(2)\right\}$ $+2$

$y_p = -\frac{1}{8}\left\{x^2 + 1 - \frac{12}{8} + \frac{24}{8} x - \cancel{\frac{+45}{32}}\right.$

$y_p = -\frac{1}{8}\left\{x^2 + 3x + 4\right\}$.

$y = y_c + y_p$.

Ex $(D^2-2D+5)\,y = 25x^2$

$\Rightarrow$ $D = 1 \pm 2i$

$y_c = e^{x}\,[c_1\cos 2x + c_2 \sin 2x]$

$y_p = \frac{1}{\phi(D)}\, f(x)$

$y_p = \frac{1}{D^2-2D+5}\, 25x^2$

$y_p = 25\,\frac{1}{D^2-2D+5}\, x^2$

$y_p = 25\,\frac{1}{5\left[1+\left(\frac{D^2-2D}{5}\right)\right]}\, x^2$

$D(x^2) = 2x$
$D^2(x^2) = 2$
$D^3(x^2) = 0$

$y_p = \frac{25}{5}\left[1+\left(\frac{D^2-2D}{5}\right)\right]^{-1} x^2$

$y_p = 5\left[1+\frac{1}{5}(D^2-2D)+\frac{1}{25}(D^2-2D)^2 + \frac{1}{125}(D^2-2D)^3\right] x^2$

$y_p = 5\left[x^2+\frac{1}{5}(D^2-2D)x^2+\frac{1}{25}(D^4-4D^3+4D^2)x^2 + \frac{1}{125}(D^6(0)\right]$

$y_p = 5\left[x^2+\frac{1}{5}[2-2(2x)]+\frac{1}{25}[0+0+4(2)]\right]$

$y_p = 5\left(x^2+\frac{2}{5}-\frac{4x}{5}+\frac{8}{25}\right)$

$y_p = 5x^2+2-4x+\frac{8}{5}$

+2 $y_p = 5x^2+4x-\frac{2}{5}$

Ex $(D^3-2D+4)y = 3x^2-5x+2$.

$\Rightarrow$ $D^3-2D+4=0$.

$D = -2,\ 1\pm i$

$y_c = c_1e^{-2x} + e^{x}(c_2\cos x + c_3\sin x)$

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3-2D+4}(3x^2-5x+2)$$

$$y_p = \frac{1}{4\left[1+\left(\frac{D^3-2D}{4}\right)\right]}(3x^2-5x+2)$$

$$y_p = \frac{1}{4}\left[1+\left(\frac{D^3-2D}{4}\right)\right]^{-1}(3x^2-5x+2)$$

$$y_p = \frac{1}{4}\left\{1-\left(\frac{D^3-2D}{4}\right)+\left(\frac{D^3-2D}{4}\right)^2 - \cdots\right\}(3x^2-5x+2)$$

$$y_p = \frac{1}{4}\left\{3x^2-5x+2 - \frac{1}{4}(D^3-2D)(3x^2-5x+2) + \frac{1}{16}(D^3-2D)^2(3x^2-5x+2)\right\} \ldots$$

$D(3x^2-5x+2) = 6x-5$

$D^2(\quad) = 6$

$D^3(\quad) = 0$

$$y_p = \frac{1}{4}\left\{3x^2-5x+2 - \frac{1}{4_2}[0-2(6x-5)]\right.$$

$$+ \cancel{\frac{1}{16}}\left(\cancel{3x^2-5x+2x} - \frac{1}{2}(\cancel{8x-8x})\right)$$

$\cancel{0} - x$

$\frac{3}{16}\ \frac{6^3}{4^2}$ $$+ \frac{1}{16}\left(\cancel{0} + \ldots \frac{1}{16}(0-0+4(6)\right).$$

$$y_p = \frac{1}{4}\left\{3x^2-5x+2+\frac{1}{2}(6x-5)\right\}+\frac{3}{2}$$

$2-\frac{5}{2}$

$-\frac{5}{2}$ $$y_p = \frac{1}{4}\left\{3x^2-5x+2+3x-\frac{5}{2}+\frac{3}{2}\right\}$$

$$y_p = \frac{1}{4}\left\{3x^2-2x+1\right\}$$

Ex 2: Solve $(D^3+3D^2-4)y = x^2+x+1$

$\rightarrow$ To find y_c

Consider A.E. $D^3+3D^2-4=0$

$(D-1)(D^2+4D+4)=0$

$(D-1)(D+2)^2=0$

$D-1=0, \quad (D+2)^2=0$

$D=1, \quad D=-2, \quad D=-2$

$\therefore D = 1, -2, -2$

1	1	3	0
		1	4
	1	4	4

$$\therefore \boxed{y_c = c_1 e^{x} + (c_2+c_3 x)e^{-2x}}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3+3D^2-4}(x^2+x+1)$$

$$y_p = \frac{1}{-4\left[1-\left(\frac{D^3+3D^2}{4}\right)\right]}(x^2+x+1)$$

$$y_p = -\frac{1}{4}\left[1-\left(\frac{D^3+3D^2}{4}\right)\right]^{-1}(x^2+x+1)$$

$$y_p = -\frac{1}{4}\left[1+\left(\frac{D^3+3D^2}{4}\right)+\left(\frac{D^3+3D^2}{4}\right)^2+\dots\right](x^2+x+1)$$

$$y_p = -\frac{1}{4}\left[1+\frac{1}{4}(D^3+3D^2)+\frac{1}{16}(D^3+3D^2)^2+\dots\right] x^2+x+1$$

$$y_p = -\frac{1}{4}\left[x^2+x+1+\frac{1}{4}(D^3+3D^2)(x^2+x+1)+\dots\right]$$ Neglect higher order

$$y_p = -\frac{1}{4}\left[x^2+x+1+\frac{1}{4}(0+3(2))\right]$$

$$\boxed{y_p = -\frac{1}{4}\left(x^2+x+\frac{5}{2}\right)}$$

$D(x^2+x+1) = 2x+1$

$D^2(\ ) = 2$

$D^3(\ ) = 0$

$(1-z)^{-1} = 1+z+z^2+z^3+\dots$

Ex 3: Solve $(D^2+2D+2)y = x^3-4x$

→ To find y_c

Consider A.E.

$(D^2+2D+2) = 0$

$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$D = -1 \pm i$

$$\boxed{y_c = e^{-x}(c_1 \cos x + c_2 \sin x)}$$

To find

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2+2D+2} \cdot (x^3-4x)$$

$$y_p = \frac{1}{2\left[1+\left(\frac{D^2+2D}{2}\right)\right]}(x^3-4x)$$

$$y_p = \frac{1}{2}\left[1+\left(\frac{D^2+2D}{2}\right)\right]^{-1}(x^3-4x)$$

$(1+z)^{-1} = 1-z+z^2-z^3+\dots$

$$y_p = \frac{1}{2}\left\{1-\left(\frac{D^2+2D}{2}\right)+\left(\frac{D^2+2D}{2}\right)^2-\left(\frac{D^2+2D}{2}\right)^3 + \dots\right\}(x^3-4x)$$

$D(x^3-4x) = 3x^2-4$

$$y_p = \frac{1}{2}\left\{1-\frac{1}{2}(D^2+2D)+\frac{1}{4}(D^4+4D^3+4D^2) - \frac{1}{8}(8D^3)\right\}(x^3-4x)$$

$D^2(\ ) = 6x$

$D^3(\ ) = 6.$

$D^4(\ ) = 0$

$$y_p = \frac{1}{2}\left\{x^3-4x-\frac{1}{2}(6x+2(3x^2-4)) + \frac{1}{4}(0+4(6)+4(6x)) - \frac{1}{8}(8\cdot(6))\right\}$$

$$y_p = \frac{1}{2}\left\{x^3-4x-3x-3x^2+4+6+6x-6\right\}$$

$$y_p = \frac{1}{2}\left\{x^3-3x^2-x+4\right\}.$$

$$y = y_c + y_p.$$

$\frac{1}{4}(3x^2-2x+1)$

Rule 7 : If $f(x) = \cancel{x}\cdot e^{ax}\cdot V$ form.

$y_p = \frac{1}{\phi(D)} f(x)$ — V is any funn of x.

$y_p = \frac{1}{\phi(D)} e^{ax}\cdot V$

$y_p = e^{ax}\cdot\frac{1}{\phi(D+a)}\cdot V$ — V is any funn of x.

① $(D^3-7D-6)y = e^{2x}(1+x)$

$\Rightarrow D^3-7D-6=0$

$D =$

$y_c =$

$y_p = \frac{1}{\phi(D)} f(x)$

$y_p = \frac{1}{D^3-7D-6} e^{2x}(1+x)$

$y_p = e^{2x}\frac{1}{\cancel{D^3-7D-6}\ (D+2)^3-7(D+2)-6}(1+x)$

$y_p = e^{2x}\frac{1}{D^3+6D^2+12D-7D-14-6}(1+x)$

$3D^4$

$y_p = e^{2x}\frac{1}{D^3+6D^2+5D-12}(1+x)$

$y_p = \frac{e^{2x}}{-12}\frac{1}{\left[1-\left(\frac{D^3+6D^2+5D}{12}\right)\right]}(1+x)$

$y_p = \frac{e^{2x}}{-12}\left[1-\left(\frac{D^3+6D^2+5D}{12}\right)\right]^{-1}(1+x)$

$y_p = \frac{e^{2x}}{-12}\left[1+\left(\frac{D^3+6D^2+5D}{12}\right)\cdots\right](1+x)$

$y_p = \frac{e^{2x}}{-12}\left[1+x+\frac{1}{12}(0+0+5(1))\right]$

$\frac{5}{12}$

$y_p = \frac{e^{2x}}{-12}\left[1+x+\frac{5}{12}\right]$

$y_p = \frac{e^{2x}}{-12}\left[x+\frac{17}{12}\right]$

(2) $(D^2-4D+3)y = x^3e^{2x}$.

$D = 1, 3$.

$y_c = c_1e^x + c_2e^{3x}$.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2-4D+3} x^3 e^{2x}.$$

$$y_p = e^{2x}\cdot\frac{1}{(D+2)^2-4(D+2)+3}\cancel{e^{2x}}\cdot x^3$$

$$y_p = e^{2x}\frac{1}{D^2+\cancel{4D}+4-\cancel{4D}-8+3}x^3$$

$$y_p = e^{2x}\frac{1}{D^2-1}x^3$$

$$y_p = \frac{e^{2x}}{-1}\left[1-D^2\right]^{-1}x^3$$ $D(x^3)=$ $D^2(x^3)=$

$$y_p = \frac{e^{2x}}{-1}\left[1+D^2+D^4+\dots\right]x^3$$ $D^3(x^3)$.

$$y_p = \frac{e^{2x}}{-1}\left[x^3+6x\right]$$

$$y_p = -e^{2x}(x^3+6x).$$

(3) $(D^4-1)y = \cos x\cosh x$.

⇒ $D = 1, -1, 0\pm i$

$y_c = c_1e^x + c_2e^{-x} + c_3\cos x + c_4\sin x$.

$$y_p = \frac{1}{D^4-1}\cos x\cdot\cosh x$$

1 n=0

$$y_p = \frac{1}{D^4-1}\cos x\left(\frac{e^x+e^{-x}}{2}\right).$$

1 1 n=1

1 2 1 n=2

$$y_p = \frac{1}{2}\left[\frac{1}{D^4-1}e^x\cos x + \frac{1}{D^4-1}e^{-x}\cdot\cos x\right]$$

1 3 3 1 n=3

1 4 6 4 1 n=4

$$y_p = \frac{1}{2}\left[e^x\frac{1}{(D+1)^4-1}\cos x + e^{-x}\frac{1}{(D-1)^4-1}\cos x\right]$$

$$y_p = \frac{1}{2}\left[e^x\cdot\frac{1}{D^4+4D^3+6D^2+4D+\cancel{1}-1}\cos x\right.$$

$-x$ … $\sin x$]

$$y_p = \frac{1}{2}\left[e^{x}\,\frac{1}{D^4+4D^3+6D^2+4D}\cos x + e^{-x}\,\frac{1}{D^4-4D^3+6D^2-4D}\cos x\right]$$

$D^2 \to -1$ $\qquad\qquad D^2 \to -1$

$$y_p = \frac{1}{2}\left[e^{x}\,\frac{1}{1-4\cancel{D}-6+4\cancel{D}}\cos x + e^{-x}\cdot\frac{1}{1+4\cancel{D}-6-4\cancel{D}}\cos x\right]$$

$$y_p = \frac{e^{x}}{2}\left(\frac{\cos x}{-5}\right) + \frac{e^{-x}}{2}\left(\frac{\cos x}{-5}\right)$$

$$y_p = \frac{e^{x}}{-10}\cos x - \frac{e^{-x}}{-10}\cos x$$

$$y_p = \frac{e^{x}}{-10}\cos x + \frac{e^{-x}}{10}\cos x$$

$$y_p = \frac{1}{10}\left[e^{x} + e^{-x}\right]\cos x$$

$$y_p = \frac{1}{5}\left[\frac{e^{x}+e^{-x}}{2}\right]\cos x$$

$$\boxed{y_p = \frac{1}{5}\cos x\cdot\cosh x}$$

(4) $(D^2-4)\,y = e^{3x}x^2$

$\Rightarrow\ D^2-4=0$

$D = \pm 2$

$\cancel{y_c = c_1\cos 2x\,(c_1+c_2x)\,e}$

$y = c_1e^{2x} + c_2e^{-2x}$

$$y_p = \frac{1}{D^2-4}\,e^{3x}x^2$$

$$y_p = e^{3x}\,\frac{1}{(D+3)^2-4}\,x^2$$

$$y_p = e^{3x}\,\frac{1}{D^2+6D+\underbrace{9-4}_{5}}\,x^2$$

$$y_p = \frac{e^{3x}}{5}\,\frac{1}{\left[1+\left(\frac{D^2+6D}{5}\right)\right]}\,x^2$$

$D(x^2) = 2x$
$D^2(x^2) = 2$

$$\cancel{y_p = \frac{e^{3x}}{-4}\left[1-\left(\frac{D^2+6D+9}{4}\right)\right]^{-1} x^2}$$

$$\cancel{y_p = \frac{e^{3x}}{-4}\left[1+\left(\frac{D^2+6D+9}{4}\right)+\left(\frac{D^2+6D+9}{4}\right)^2+\right.}$$

$$y_p = \frac{e^{3x}}{5}\left[1+\left(\frac{D^2+6D}{5}\right)\right]^{-1} x^2$$

$$y_p = \frac{e^{3x}}{5}\left[1-\left(\frac{D^2+6D}{5}\right)+\left(\frac{D^2+6D}{5}\right)^2-\dots\right] x^2$$

$$y_p = \frac{e^{3x}}{5}\left[x^2-\frac{1}{5}(2+6(2x)) + \cancel{\frac{1}{25}(x+x}\right.$$
$$\left. + \frac{1}{25}\left(D^4+12D^3+6D^2\right)x^2\right]$$

$$y_p = \frac{e^{3x}}{5}\left[x^2-\frac{1}{5}(2+12x)+\frac{1}{25}\left(6(2)\right)\right]$$

$$-\frac{2}{5}+\frac{12}{25} \qquad \frac{-10+12}{25} \qquad \frac{2}{25}$$

$$y_p = \frac{e^{3x}}{5}\left[x^2-\frac{2}{5}+\frac{12}{5}x+\frac{12}{25}\right]$$

$$y_p = \frac{e^{3x}}{5}\left[x^2+\frac{12}{5}x+\frac{2}{25}\right]$$

$$y_p = \frac{e^{3x}}{5}\left[\frac{25x^2+60x+2}{25}\right]$$

$$y_p = \frac{e^{3x}}{125}\left[25x^2-60x+62\right]$$

$$y_p = \frac{e^{3x}}{125}\left[25x^2-60x+62\right]$$

Ex 1) Solve $(D^2-4D+4)y = e^{2x}\cdot\sin 3x$

→ To find y_c

Consider A.E.

$$(D^2-4D+4)=0$$
$$(D-2)^2=0$$
$$D=2,2$$
$$\boxed{y_c=(c_1+c_2x)e^{2x}}$$

To find y_p.

$$y_p=\frac{1}{\phi(D)}f(x).$$
$$y_p=\frac{1}{D^2-4D+4}\cdot e^{2x}\cdot\sin 3x$$

Replace $D \to D+2$

$$y_p=e^{2x}\frac{1}{(D+2)^2-4(D+2)+4}.\sin 3x.$$
$$y_p=e^{2x}\cdot\frac{1}{D^2+\cancel{4D}+\cancel{4}-\cancel{4D}-\cancel{8}+\cancel{4}}\sin 3x.$$
$$y_p=e^{2x}\frac{1}{D^2}\cdot\sin 3x.$$

Replace $D^2 \to -9$

$$y_p=e^{2x}\cdot\frac{1}{-9}\sin 3x.$$
$$\boxed{y_p=\frac{-e^{2x}}{9}\sin 3x}$$
$$y=y_c+y_p$$
$$\boxed{y=(c_1+c_2x)e^{2x}-\frac{e^{2x}}{9}\sin 3x.}$$

Ex 2> Solve $(D^2-4D+3)y = x^3 \cdot e^{2x}$

$\rightarrow$ To find y_c

Consider A.E.

$$D^2-4D+3=0$$
$$D^2-3D-D+3=0$$
$$D(D-3)-1(D-3)=0$$
$$(D-3)(D-1)=0$$
$$D=3,1$$
$$\therefore \boxed{y_c = c_1 e^{3x} + c_2 e^{x}}$$

To find y_p.

$$y_p = \frac{1}{\phi(D)} f(x)$$
$$y_p = \frac{1}{D^2-4D+3} \cdot x^3 \cdot e^{2x}$$

$D \rightarrow D+2$

$$y_p = e^{2x} \cdot \frac{1}{(D+2)^2-4(D+2)+3} x^3$$
$$y_p = e^{2x} \frac{1}{D^2+\cancel{4D}+4-\cancel{4D}-8+3} x^3$$
$$y_p = e^{2x} \cdot \frac{1}{D^2-1} x^3$$
$$y_p = e^{2x} \frac{1}{-1[1-D^2]} x^3$$
$$y_p = -e^{2x} (1-D^2)^{-1} \cdot x^3$$
$$y_p = -e^{2x} [1+D^2+D^4+\dots] x^3$$
$$y_p = -e^{2x} (x^3+6x+0)$$
$$\boxed{y_p = -e^{2x}(x^3+6x)}$$

$D(x^3) = 3x^2$

$D^2(x^3) = 6x$

$D^3(x^3) = 6$

$D^4(x^3) = 0$

$(1-z)^{-1} = 1+z+z^2+\dots$

$$y = y_c + y_p$$

$$\boxed{y = c_1 e^{3x} + c_2 e^{x} - e^{2x}(x^3 + 6x)}$$

Ex 3) Solve $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = e^{2x}(1+x)$

$\rightarrow$ $D = \frac{d}{dx}$

$$(D^3 - 7D - 6)y = e^{2x}(1+x)$$

To find y_c

Consider A.E.

$D^3 - 7D - 6 = 0$

$(D+1)(D^2 - D - 6) = 0$

$D = -1, \quad D^2 - D - 6 = 0$

$D = -2, 3$

$\therefore D = -1, -2, 3$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$$\therefore \boxed{y_c = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x}}$$

To find y_p.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3 - 7D - 6} e^{2x}(1+x)$$

$$y_p = e^{2x} \frac{1}{(D+2)^3 - 7(D+2) - 6} (x+1) \qquad D \rightarrow D+2$$

$$y_p = e^{2x} \frac{1}{D^3 + 6D^2 + 12D + 8 - 7D - 14 - 6} (x+1)$$

$$y_p = e^{2x} \frac{1}{D^3 + 6D^2 + 5D - 12} (x+1)$$

$$y_p = e^{2x} \cdot \frac{1}{-12\left[1 - \left(\frac{D^3 + 6D^2 + 5D}{12}\right)\right]} (x+1)$$

$$y_p = \frac{e^{2x}}{-12} \left[1 - \left(\frac{D^3 + 6D^2 + 5D}{12}\right)\right]^{-1} (x+1).$$

$D(x+1) = 1$

$D^2(x+1) = 0$

$$y_p = -\frac{e^{2x}}{12}\left[1 + \left(\frac{D^3+6D^2+5D}{12}\right) + \cdots\right](x+1)$$

$$y_p = -\frac{e^{2x}}{12}\left\{x+1 + \frac{1}{12}\left[0 + 6(0) + 5(1)\right]\right\}$$

$$y_p = -\frac{e^{2x}}{12}\left(x+1+\frac{5}{12}\right)$$

$$\boxed{y_p = -\frac{e^{2x}}{12}\left(x+\frac{17}{12}\right)}$$

$$y = y_c + y_p$$

$$\therefore\ y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x} - \frac{e^{2x}}{12}\left(x+\frac{17}{12}\right)$$

Ex 4) Solve $(D^4-1)y = \cos x \cdot \cosh x$

$\rightarrow$ To find y_c

$\boxed{\cosh x = \frac{e^x+e^{-x}}{2}}$

Consider A.E.

$D^4-1=0$

$D^4 - a^4 =$

$(D^2-1)(D^2+1)=0$

$D^2-1=0 \qquad D^2+1=0$

$D=\pm 1 \qquad D=0\pm i$

$D = 1, -1, 0\pm i$

$$\boxed{y_c = c_1 e^x + c_2 e^{-x} + c_3\cos x + c_4 \sin x}$$

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^4-1} \cos x \cdot \cosh x.$$

$$y_p = \frac{1}{D^4-1} \cos x \cdot \left(\frac{e^x+e^{-x}}{2}\right).$$

$$y_p = \frac{1}{2}\left[\frac{1}{D^4-1}\left(e^x \cos x + e^{-x}\cdot \cos x\right)\right]$$

$$y_p = \frac{1}{2}\left\{\frac{1}{D^4-1} e^x\cdot \cos x + \frac{1}{D^4-1} e^{-x}\cdot \cos x\right\}$$

$D \to D+1 \qquad D \to D-1$

$$y_p = \frac{1}{2}\left\{e^x \frac{1}{(D+1)^4-1}\cos x + e^{-x}\frac{1}{(D-1)^4-1}\cos x\right\}.$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

$$y_p = \frac{1}{2}\left\{e^x \cdot \frac{1}{(D^4+4D^3+6D^2+4D+\cancel{1})-\cancel{1}}\cos x \right.$$
$$\left. + e^{-x}\frac{1}{(D^4-4D^3+6D^2-4D+\cancel{1})-\cancel{1}}\cos x\right\}$$

Rule 8

$f(x) = x \cdot V$ / $y_p = \left[x - \frac{\phi'(D)}{\phi(D)}\right]\frac{1}{\phi(D)}$

Ex. $(D^2 + 3D + 2)y = xe^{-x}\sin x$.

$D = -1, -2$

$y_c = c_1e^{-x} + c_2e^{-2x}$.

$y_p = \frac{1}{\phi(D)} f(x)$

$y_p = \frac{1}{D^2+3D+2} e^{-x} \cdot x\sin x$

$y_p = e^{-x} \frac{1}{(D-1)^2 + 3(D-1) + 2} x\sin x$.

$y_p = e^{-x} \frac{1}{D^2 - 2D + \cancel{1} + 3D - 3 + \cancel{2}} x\sin x$.

$y_p = e^{-x} \frac{1}{D^2 + D} x\sin x$.

$y_p = e^{-x}\left[x - \frac{2D+1}{D^2+D}\right]\frac{1}{D^2+D}\sin x$.

$D^2 \to -1$

$y_p = e^{-x}\left[x - \frac{(2D+1)}{-1+D}\right]\frac{1}{-1+D}\sin x$.

$y_p = e^{-x}\left[x - \frac{(2D+1)}{D-1} \times \frac{D+1}{D+1}\right]\frac{1}{D-1} \times \frac{D+1}{D+1}\sin x$

$y_p = e^{-x}\left[x - \frac{(2D+1)(D+1)}{D^2-1}\right]\frac{D+1}{D^2-1}\sin x$.

$D^2 \to -1$ $\qquad D^2 \to -1$

$y_p = e^{-x}\left[x - \frac{(2D+1)(D+1)}{-1-1}\right]\frac{D+1}{-1-1}\sin x$.

$y_p = e^{-x}\left[x - \frac{[2D^2 + 2D + D + 1]}{-2}\right]\frac{D+1}{-2}\sin x$

$y_p = e^{-x}\left[x - \frac{(-2 + 3D + 1)}{-2}\right]\frac{D+1}{-2}\sin x$.

$y_p = \frac{e^{-x}}{-2}\left[x - \frac{(3D-1)}{-2}\right](\cos x + \sin x)$

$y_p = \frac{e^{-x}}{-2}\left[x\cos x + x\sin x + \frac{1}{2}(3D-1)\cos x + \frac{(3D-1)}{2}\sin x\right]$

$+\frac{3}{2}$

$y_p = \frac{e^{-x}}{-2}\left[x\cos x + x\sin x - \frac{1}{2}3\sin x - \frac{1}{2}\cos x + \frac{1}{2}3\cos x - \sin x\right]$

$y_p = \frac{e^{-x}}{2}[(x + 2$

$$= \frac{e^{-x}}{-2}\left\{x + \frac{(3D-1)}{2}\right\}(D+1)\sin x$$

$$= \frac{e^{-x}}{-2}\left\{x(D+1)\sin x + \frac{1}{2}(3D-1)(D+1)\sin x\right.$$

$$= \frac{e^{-x}}{-2}\left\{x(D+1)\sin x + \frac{1}{2}(3D^2+3D-D-1)\sin x\right.$$

$$= \frac{e^{-x}}{-2}\left\{x(D+1)\sin x + \frac{1}{2}(-3+2D-1)\sin x\right.$$

$$= \frac{e^{-x}}{-2}\left\{x(\cos x+\sin x) + \frac{1}{2}(2D-4)\sin x\right.$$

$$= \frac{e^{-x}}{-2}\left\{x(\cos x+\sin x) + \frac{1}{\cancel{2}}\cancel{2}(D-2)\sin x\right\}$$

$$= \frac{e^{-x}}{-2}\left\{x(\cos x+\sin x) + \cos x - 2\sin x\right\}$$

$$\boxed{y_p = \frac{e^{-x}}{2}\left\{(x+1)\cos x + (x-2)\sin x\right\}}$$

Ex $(D^3+3D+2)y = x\cos 2x$.

$D =$

y_c

$$y_p = \frac{1}{\phi(D)}f(x)$$

$$y_p = \frac{1}{D^3+3D+2}\,x\cos 2x$$

$$y_p = \left[x - \frac{3D^2+3}{D^3+3D+2}\right]\frac{1}{D^3+3D+2}\cos 2x$$

$$D^2 \to -4$$

$$y_p = \left[x - \frac{3(-4)+3}{-4D+3D+2}\right]\frac{1}{-4D+3D+2}\cos 2x$$

$$y_p = \left[x + \frac{9}{2-D}\right]\frac{1}{2-D}\cos 2x.$$

$$y_p = \left[x + \frac{9}{2-D}\times\frac{2+D}{2+D}\right]\frac{1}{2-D}\times\frac{2+D}{2+D}\cos 2x$$

$$y_p = \left[x + \frac{9(2+D)}{4-D^2}\right]\frac{(2+D)}{4-D^2} \cos 2x$$

$$y_p = \left[x + \frac{9(4+6D+D^2)}{4-D^2}\right]\frac{1}{4-D^2} \cos x$$

$$D^2 \to -4$$

$$y_p = \left[x + 9\frac{(\cancel{4}+6D-\cancel{4})}{4+4}\right]\frac{1}{4+4} \cos 2x$$

$$y_p = \frac{1}{8}\left[x + \frac{\overset{27}{\cancel{54}}D}{4\cancel{8}}\right] \cos 2x$$

$$y_p = \frac{1}{8}\left[x + \frac{27}{4}D\right] \cos 2x$$

$$y_p = \frac{1}{8}\left[x\cos 2x + \frac{27}{4}2(-\sin 2x)\right]$$

$$y_p = \frac{1}{8}\left[x\cos x - \frac{27}{2}\sin 2x\right]$$

Ex 1) Solve $\frac{d^2y}{dx^2} + 4y = x\sin x$

$\rightarrow$ $D = \frac{d}{dx}$

$$(D^2+4)y = x\sin x$$

To find y_c,

Consider A.E. $D^2+4=0$

$D = 0 \pm 2i$

$$\therefore \boxed{y_c = c_1\cos 2x + c_2\sin 2x.}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2+4} x\cdot\sin x$$

$$y_p = \left[x - \frac{2D}{D^2+4}\right]\frac{1}{D^2+4}\sin x$$

Replace $D^2 \rightarrow -1$

$$y_p = \left[x - \frac{2D}{-1+4}\right]\frac{1}{-1+4}\sin x.$$

$$y_p = \frac{1}{3}\left[x - \frac{2D}{3}\right]\sin x$$

$$y_p = \frac{1}{3}\left(x\sin x - \frac{2}{3}\cos x\right).$$

$$\boxed{y_p = \frac{x}{3}\sin x - \frac{2}{9}\cos x}$$

$$y = y_c + y_p$$

$$y = c_1\cos 2x + c_2\sin 2x + \frac{x}{3}\sin x - \frac{2}{9}\cos x.$$

SHREE

Date: / / Page No.

2) Solve $(D^2-2D+1)y = xe^x \sin x$

→ To find y_c

Consider A.E. $D^2-2D+1=0$

$$(D-1)^2=0$$

$$D=1,1$$

$$\therefore \boxed{y_c=(c_1+c_2x)e^x}$$

To find y_p

$$y_p=\frac{1}{\phi(D)}f(x)$$

$$y_p=\frac{1}{D^2-2D+1}x.e^x.\sin x$$

$$y_p=\frac{1}{D^2-2D+1}e^x.(x\sin x)$$

$D \to D+1$

$$y_p=e^x.\frac{1}{(D+1)^2-2(D+1)+1}x\sin x$$

$$y_p=e^x\frac{1}{D^2+\cancel{2D}+\cancel{1}-\cancel{2D}-\cancel{2}+\cancel{1}}x\sin x$$

$$y_p=e^x.\frac{1}{D^2}x\sin x$$

$$y_p=e^x\left[x-\frac{2D}{D^2}\right]\frac{1}{D^2}\sin x$$

Replace $D^2 \to -1$

$$y_p=e^x\left[x-\frac{2D}{-1}\right]\frac{1}{-1}\sin x$$

$$\boxed{y_p=-e^x(x\sin x+2\cos x)}$$

$$y=y_c+y_p$$

$$y=(c_1+c_2x)e^x-e^x(x\sin x+2\cos x)$$

SHREE

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3) Solve $(D^2-1)y = x\sin x + (1+x^2)e^x$

$\Rightarrow$ To find y_c

Consider A.E.

$$D^2-1=0$$

$$D=\pm 1$$

$$\therefore \boxed{y_c = c_1e^x + c_2e^{-x}}$$

To find y_p.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2-1}\left[x\sin x + (1+x^2)\cdot e^x\right]$$

$$y_p = \frac{1}{D^2-1} x\sin x + \frac{1}{D^2-1}(1+x^2)\cdot e^x.$$

$$y_p = \left[x - \frac{2D}{D^2-1}\right]\frac{1}{D^2-1}\sin x + e^x \frac{1}{(D+1)^2-1}(x^2+1)$$

$$D^2 \to -1$$

$$y_p = \left[x - \frac{2D}{-1-1}\right]\frac{1}{-1-1}\sin x + e^x\frac{1}{D^2+2D+\cancel{1}-\cancel{1}}(x^2+1)$$

$$y_p = \left[x - \frac{\cancel{2}D}{-\cancel{2}}\right]\frac{1}{-2}\sin x + e^x\frac{1}{D^2+2D}(x^2+1)$$

$$y_p = \frac{-1}{2}(x+D)\sin x + e^x\frac{1}{2D(1+D/2)}(x^2+1)$$

$$y_p = \frac{-1}{2}(x\sin x + \cos x) + \frac{e^x}{2D}\left[1+\frac{D}{2}\right]^{-1}(x^2+1)$$

$$D(x^2+1) = 2x$$

$$D^2(x^2+1) = 2$$

$$D^3(x^2+1) = 0$$

$$y_p = -\frac{1}{2}(x\sin x + \cos x) + \frac{e^x}{2D}\left[1 - \frac{D}{2} + \frac{D^2}{4} + \dots\right](x^2+1)$$

$$y_p = -\frac{1}{2}(x\sin x + \cos x) + \frac{e^x}{2D}\left[x^2+1 - \frac{1}{\cancel{2}}(\cancel{2}x) + \frac{1}{4}(2)\right]$$

$$y_p = -\frac{1}{2}(x\sin x + \cos x) + \frac{e^x}{2D}\left(x^2 - x + \frac{3}{2}\right)$$

$$y_p = -\frac{1}{2}\left(x\sin x + \cos x\right) + \frac{e^x}{2}\left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{3}{2}x\right).$$

$$y = y_c + y_p.$$

SHREE

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4) Solve $(D^2-1)y = x\sin 3x$

To find y_c.

Consider A.E. $D^2-1=0$

$D=\pm 1$

$\therefore y_c = c_1e^{x}+c_2e^{-x}$

To find y_p.

$y_p = \frac{1}{\phi(D)} f(x)$

$y_p = \frac{1}{D^2-1} x\cdot\sin 3x$

$y_p = \left[x - \frac{2D}{D^2-1}\right]\frac{1}{D^2-1}\cdot\sin 3x.$

$D^2 \to -9$

$y_p = \left[x - \frac{2D}{-9-1}\right]\frac{1}{-9-1}\sin 3x$

$y_p = \frac{1}{-10}\left[x - \frac{2D}{-10}\right]\sin 3x.$

$y_p = \frac{-1}{10}\left[x+\frac{1}{5}D\right]\sin 3x$

$y_p = \frac{-1}{10}\left[x\sin 3x + \frac{1}{5}(3\cos 3x)\right]$

$\boxed{y_p = \frac{-x}{10}\sin 3x - \frac{3}{50}\cos 3x}$

$y = y_c + y_p$

$\boxed{y = c_1e^{x}+c_2e^{-x} - \frac{x}{10}\sin 3x - \frac{3}{50}\cos 3x}$

Ex 5) $(D^3+3D+2)y = x\cos 2x$.

$\Rightarrow$ To find y_c

Consider A.E : $D^3+3D+2=0$

$D = -0.596,\ 0.298 \pm i(8073)$

$$y_1 = c_1 e^{-0.569x} + e^{0.298x}\left[c_2\cos(8073)x + c_3\sin(8073)x\right]$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^3+3D+2} x\cos 2x.$$

$$y_p = \left[x - \frac{3D^2+3}{D^3+3D+2}\right]\frac{1}{D^3+3D+2}\cos 2x. \qquad D^2 \to -4$$

$$y_p = \left[x - \frac{3(-4)+3}{(-4)D+3D+2}\right]\frac{1}{(-4)D+3D+2}\cos 2x$$

$$y_p = \left[x - \frac{(-9)}{-D+2}\right]\frac{1}{-D+2}\cos 2x$$

$$y_p = \left[x + \frac{9}{2-D}\right]\frac{1}{2-D}\cos 2x$$

$$y_p = \left[x + \frac{9}{2-D}\times\frac{2+D}{2+D}\right]\frac{1}{2-D}\times\frac{2+D}{2+D}\cos 2x$$

$$y_p = \left[x - \frac{9(2+D)}{4-D^2}\right]\frac{(2+D)}{4-D^2}\cos 2x. \qquad D^2 \to -4$$

$$y_p = \left[x - \frac{9(2+D)}{4-(-4)}\right]\frac{(2+D)}{4-(-4)}\cos 2x$$

$$y_p = \left[x - \frac{9(2+D)}{8}\right]\frac{(2+D)}{8}\cos 2x$$

$$y_p = \frac{1}{8}\left[x - \frac{(18+9D)}{8}\right]\left[2\cos 2x + (-2\sin 2x)\right]$$

$$y_p = \frac{1}{8}\left[x - \frac{1}{8}(\right.$$

H.W

1) $(D^2-1)y = xe^x \sin x$.

2) $(D^2+1)^2 y = 24x \cos x$

3) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x \sin 2x$.

4) $(D^2+6D+9)y = \frac{1}{x^3} e^{-3x}$.

5) $(D^4-3D^3-2D^2+4D+4)y = x^2 e^x$.

6) $(D^2+2D+1)y = e^{-x} \log x$.

7) $(D^3-D^2+D+1)y = \cosh x \sin x$.

8) $(D^2-4)y = x \sinh x$.

9) $\frac{d^3y}{dx^3} + 8y = x^4 + 2x + 1$

10) $(D^2-1)y = e^x + x^3$.

* General method :-

This method used when short-cut method fails.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D-m} f(x).$$

$$y_p = e^{mx} \int e^{-mx} f(x)\, dx.$$

$$y_p = \frac{1}{D+m} f(x) = e^{-mx} \int e^{mx} f(x)\, dx.$$

$$y_p = \frac{1}{(D-m_1)(D-m_2)} f(x)$$

$$= \frac{1}{(D-m_1)} e^{m_2 x} \int e^{-m_2 x} f(x)\, dx.$$

$$= e^{m_1 x} \int e^{-m_1 x} \left[e^{m_2 x} \int e^{-m_2 x} f(x)\, dx \right] dx$$

$$\frac{1}{(D+m_1)(D+m_2)} f(x) = \frac{1}{(D+m_1)} e^{-m_2 x} \int e^{m_2 x}\, dx.$$

$$= e^{-m_1 x} \int e^{+m_1 x} \left[e^{-m_2 x} \int e^{m_2 x}\, dx \right] dx$$

Ex. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}.$

$\Rightarrow (D^2 + 3D + 2)\, y = e^{e^x}.$

$D = -2, -1$

$y_c = c_1 e^{-2x} + c_2 e^{-x}.$

$$y_p = \frac{1}{\phi(D)} f(x).$$

$$y_p = \frac{1}{D^2 + 3D + 2} e^{e^x}.$$

$$y_p = \frac{1}{(D+2)(D+1)} e^{e^x}$$

$$y_p = \frac{1}{(D+2)} \cdot e^{-x} \int e^{x} \cdot e^{e^x}\, dx.$$

put $e^x = t$

$e^x dx = dt.$

$$y_p = \frac{1}{(D+2)} e^{-x} \int e^{t} \cdot dt$$

$$y_p = \frac{1}{(D+2)} e^{-x} \; e^{t}$$

$$y_p = e^{-2x} \int e^{2x}\, e^{-x} \cdot e^{e^x}\, dx$$

$$y_p = e^{-2x} \int e^{x} \cdot e^{e^x}\, dx.$$

$$y_p = e^{-2x} \int e^{t}\, dt$$

$$y_p = e^{-2x}\; e^{t}$$

$$y_p = e^{-2x} \cdot e^{e^x}.$$

$$y = y_c + y_p$$

Ex $(D^2-1)y = e^{-x} \sin e^{-x} + \cos e^{-x}.$

$\Rightarrow$ $D^2 - 1 = 0$

$D = \pm 1$

$y_c = c_1 e^{x} + c_2 e^{-x}.$

$$y_p = \frac{1}{\phi(D)} f(x).$$

$$y_p = \frac{1}{D^2-1} \cdot e^{-x} \sin e^{-x} + \cos e^{-x}$$

$$y_p = \frac{1}{(D-1)(D+1)} e^{-x} \cdot \sin e^{-x} + \cos e^{-x}$$

~~$y_p = e^{-x} \int e^{x}$~~

$$y_p = \frac{1}{D-1} e^{-x} \int e^{x} \left(e^{-x} \cdot \sin e^{-x} + \cos e^{-x}\right) dx$$

put $e^{-x} = t$

$$y_p = \frac{1}{D-1} e^{-x} \int$$

$$y_p = \frac{1}{(D+2)} \cdot e^{-x} \int e^{x} \cdot e^{e^x} \, dx.$$

put $e^x = t$

$e^x dx = dt.$

$$y_p = \frac{1}{(D+2)} e^{-x} \int e^{t} \cdot dt$$

$$y_p = \frac{1}{(D+2)} e^{-x} \quad e^{t} \quad \cancel{+x}.$$

$$y_p = e^{-2x} \int e^{2x} \, e^{-x} \cdot e^{e^{x}} \, d\cancel{x}$$

$$y_p = e^{-2x} \int e^{x} \cdot e^{e^{x}} \, d\cancel{x}.$$

$$y_p = e^{-2x} \int e^{t} \, dt$$

$$y_p = e^{-2x} \quad e^{t}$$

$$y_p = e^{-2x} \cdot e^{e^{x}}.$$

$$y = y_c + y_p$$

Ex $(D^2 - 1) y = e^{-x} \sin e^{-x} + \cos e^{-x}.$

$\Rightarrow$ $D^2 - 1 = 0$

$D = \pm 1$

$y_c = c_1 e^{x} + c_2 e^{-x}.$

$$y_p = \frac{1}{\phi(D)} f(x).$$

$$y_p = \frac{1}{D^2 - 1} \, e^{-x} \sin e^{-x} + \cos e^{-x}$$

$$y_p = \frac{1}{(D-1)(D+1)} \, e^{-x} \cdot \sin e^{-x} + \cos e^{-x}$$

~~$y_p = e^{-x} \int e^{x}$~~

$$y_p = \frac{1}{D-1} \, e^{-x} \int e^{x} \left(e^{-x} \cdot \sin e^{-x} + \cos e^{-x} \right) dx$$

put $e^{-x} = t$

$$y_p = \frac{1}{D-1} \, e^{-x} \int$$

$f(x) = \cos e^{-x}$
$f'(x) = -\sin e^{-x}(-e^{-x})$
$= e^{-x} \cdot \sin e^{-x}$

$y_p = \{ e^{-x} \int$

$$y_p = \frac{1}{D-1} \left\{ e^{-x} \int e^{x} \left(\cos e^{-x} + e^{-x} \sin e^{-x} \right) dx \right.$$

use $\int e^{x} (f(x) + f'(x))\, dx = e^{x} f(x)$.

$$y_p = \frac{1}{D-1} \left\{ e^{-x} \cdot e^{x} \cos e^{-x} \right\}$$

$$y_p = \frac{1}{D-1} \cos e^{-x}.$$

$$y_p = e^{x} \cdot \int e^{-x} \cos e^{-x}\, dx$$

put $e^{-x} = t$ $\quad -e^{-x} dx =$

$$y_p = -e^{x} \int \cos t\, dt$$

$$y_p = -e^{x} [\sin t]$$

$$\boxed{y_p = -e^{x} \sin e^{-x}}.$$

Ex $(D^2 + 3D + 2)y = \sin e^{x}$.

$\Rightarrow$ $D = -2, -1$.

$y_c = c_1 e^{-x} + c_2 e^{-x}$

$$y_p = \frac{1}{(D+1)(D+2)} \sin e^{-x}.$$

$$y_p = \frac{1}{D+2} \cdot e^{-x} \cdot \int e^{x} \sin e^{-x}\, dx.$$

$$y_p = \frac{1}{D+2} e^{-x} \cdot \int \sin t\, dt.$$

$$y_p = \frac{1}{(D+1} e^{-2x} \int e^{2x} \sin e^{-x}\, dt$$

$$y_p = \frac{1}{D+1} e^{2x}$$

$$y_p = \frac{1}{(D+2)(D+1)} \sin e^x$$

$$y_p = \frac{1}{D+2} e^{-x} \int e^x \sin e^x \, dx$$

$$y_p = \frac{1}{D+2} e^{-x} \int \sin t \, dt$$

$$y_p = \frac{1}{D+2} e^{-x} (-\cos e^x)$$

$$y_p = -e^{-2x} \int e^{2x} e^{-x} \cos e^x \, dx$$

$$y_p = -e^{-2x} \int e^x \cdot \cos e^x \, dx$$

$$y_p = -e^{-2x} \int \cos t \, dt$$

$$y_p = -e^{-2x} \sin t$$

$$y_p = -e^{-2x} \sin e^x$$

Ex. $(D^2+3D+2)y = e^{e^x} + \cos e^x$

$\Rightarrow$ $D^2+3D+2 = (D+2)(D+1) = 0$

$y_c = c_1 e^{-2x} + c_2 e^{-x}$

$$y_p = \frac{1}{(D+2)(D+1)} \left(e^{e^x} + \cos e^x\right)$$

$$y_p = \frac{1}{D+2} e^{-x} \cdot \int e^x \left(e^{e^x} + \cos e^x\right) dx$$

put $e^x = t$

$e^x dx = dt$

$$y_p = \frac{1}{D+2} e^{-x} \int (e^t + \cos t) \, dt$$

$$y_p = \frac{1}{D+2} e^{-x} [e^t + \sin t]$$

$$y_p = \frac{1}{D+2} e^{-x} \left(e^{e^x} + \sin e^x\right)$$

$$y_p = e^{-2x} \cdot \int e^{2x} \cdot e^{-x} \left(e^{e^x} + \sin e^x\right)$$

$$y_p = e^{-2x} \int e^x \left(e^{e^x} + \sin e^x\right) dx$$

$$y_p = e^{-2x} \int (e^t + \sin t)\, dt$$

$$y_p = e^{-2x} \left[e^t + (-\cos t) \right] dt$$

$$y_p = e^{-2x} \left[e^{e^x} - \cos e^x \right]$$

$$y = y_c + y_p.$$

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Ex 3: Solve $(D^2+3D+2)y = \sin e^x$.

⇒ To find y_c,

Consider A.E.

$$D^2+3D+2=0$$

$$D=-2,-1$$

$$\boxed{y_c = c_1 e^{-2x} + c_2 e^{-x}}$$

To find.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2+3D+2} \sin e^x$$

$$y_p = \frac{1}{(D+2)(D+1)} \cdot \sin e^x .$$

$$y_p = \frac{1}{D+2}\left[\frac{1}{D+1} \sin e^x\right]$$

$$y_p = \frac{1}{D+2}\left\{ e^{-x} \cdot \int e^x \cdot \sin e^x \, dx \right\}. \qquad a=1$$

Put $e^x = t$

$e^x dx = dt$

$$y_p = \frac{1}{D+2}\left\{ e^{-x} \cdot \int \sin t \, dt \right\}$$

$$y_p = \frac{1}{D+2}\left\{ e^{-x} (-\cos t) \, dt \right.$$

$$y_p = -\frac{1}{D+2}\left\{ e^{-x} \cdot \cos e^x \right\}.$$

$$y_p = -e^{-2x} \cdot \int e^{2x} \cdot e^{-x} \cdot \cos e^x \, dx. \qquad a=2$$

$$y_p = -e^{-2x} \cdot \int e^x \cdot \cos e^x \, dx \qquad \text{put } e^x = t, \; e^x dx = dt$$

$$y_p = -e^{-2x} \int \cos t \, dt$$

$$y_p = -e^{-2x} \sin t$$

$$\boxed{y_p = -e^{-2x} \cdot \sin e^x}$$

$$y = y_c + y_p.$$

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Ex 2: Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

$\Rightarrow$ $D = \frac{d}{dx}$

$$(D^2 + 3D + 2)y = e^{e^x}.$$

To find y_c

Consider A.E. $D^2 + 3D + 2 = 0$

$$D = -2, -1$$

$$\therefore \boxed{y_c = c_1 e^{-2x} + c_2 e^{-x}.}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(x).$$

$$y_p = \frac{1}{D^2+3D+2} \cdot e^{e^x}$$

$$y_p = \frac{1}{(D+2)(D+1)} e^{e^x}.$$

$$y_p = \frac{1}{D+2} \left[\frac{1}{D+1} e^{e^x} \right]$$

$a = 1$

$$y_p = \frac{1}{D+2} \left\{ e^{-x} \cdot \int e^x \cdot e^{e^x} dx \right\}.$$

put $e^x = t$

$e^x dx = dt$

$$y_p = \frac{1}{D+2} \left\{ e^{-x} \int e^t \cdot dt \right\}$$

$$y_p = \frac{1}{D+2} \left\{ e^{-x} \cdot e^t \right\}.$$

$$y_p = \frac{1}{D+2} \left(e^{-x} \cdot e^{e^x} \right).$$

$a = 2$

$$y_p = e^{-2x} \cdot \int e^{2x} \cdot e^{-x} \cdot e^{e^x} dx.$$

$$y_p = e^{-2x} \cdot \int e^x \cdot e^{e^x} dx$$

put $e^x = t$

$e^x dx = dt$

$$y_p = e^{-2x} \cdot \int e^t dt$$

$y_p = e^{-2x} \cdot e^t$

$$\boxed{y_p = e^{-2x} \cdot e^{e^x}}$$

$y = y_c + y_p.$

Ex 3: Solve $(D^2-9D+18)y = e^{e^{-3x}}$

$\rightarrow$ To find y_c

Consider A.E. $D^2-9D+18=0$

$D = 6, 3$

$\therefore \boxed{y_c = c_1 e^{6x} + c_2 e^{3x}}$

To find y_p.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2-9D+18} e^{e^{-3x}}$$

$$y_p = \frac{1}{(D-6)(D-3)} e^{e^{-3x}}$$

$$y_p = \frac{1}{D-6}\left[\frac{1}{D-3} e^{e^{-3x}}\right]$$

$$y_p = \frac{1}{D-6}\left[e^{3x} \cdot \int \underline{e^{-3x}} \cdot e^{e^{-3x}} \underline{dx}\right]$$

$a=3$

Put $e^{-3x} = t$

$-3e^{-3x}dx = dt$

$e^{-3x}dx = -\frac{dt}{3}$

$$y_p = \frac{1}{D-6}\left[e^{3x}\int e^{t} \cdot \left(-\frac{dt}{3}\right)\right]$$

$$y_p = -\frac{1}{3} \cdot \frac{1}{(D-6)}\left(e^{3x} \cdot \int e^t dt\right).$$

$$y_p = -\frac{1}{3}\frac{1}{(D-6)}\left(e^{3x} \cdot e^{t}\right)$$

$$y_p = -\frac{1}{3}\frac{1}{D-6}\left(e^{3x} \cdot e^{e^{-3x}}\right)$$

$$y_p = -\frac{1}{3} e^{6x} \cdot \int e^{-6x} \cdot e^{3x} \cdot e^{e^{-3x}} dx.$$

$$y_p = -\frac{1}{3} e^{6x} \cdot \int e^{-3x} e^{e^{-3x}} dx.$$

$$y_p = -\frac{1}{3} e^{6x} \int e^{t} \left(-\frac{dt}{3}\right).$$

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$$y_p = +\frac{1}{9} e^{6x} \int e^{t}\, dt$$

$$y_p = \frac{1}{9} e^{6x} \cdot e^{t}$$

$$\boxed{y_p = \frac{1}{9} e^{6x} \cdot e^{e^{-3x}}}$$

$$y = y_c + y_p$$

$$y = c_1 e^{6x} + c_2 e^{3x} + \frac{1}{9} e^{6x} \cdot e^{e^{-3x}}$$

Ex 4: Solve $(D^2+3D+2)y = e^{e^x} - \sin(e^x)$

→ To find y_c

Consider A.E. $D^2+3D+2=0$

$D = -1, -2$

$D = -2, -1$

$\therefore \boxed{y_c = c_1 e^{-2x} + c_2 e^{-x}}$

To find y_p.

$y_p = \frac{1}{\phi(D)} f(x)$

$y_p = \frac{1}{D^2+3D+2} (e^{e^x} - \sin e^x)$

$y_p = \frac{1}{(D+2)(D+1)} (e^{e^x} - \sin e^x)$.

$y_p = \frac{1}{D+2} \left[\frac{1}{D+1} (e^{e^x} - \sin e^x) \right]$

$a=1$

$y_p = \frac{1}{D+2} \left[e^{-x} \cdot \int e^{+x} \cdot (e^{e^x} - \sin e^x)\, dx \right]$

put $e^x = t$

$e^x dx = dt$

$y_p = \frac{1}{D+2} \left[e^{-x} \cdot \int (e^t - \sin t)\, dt \right]$.

$y_p = \frac{1}{D+2} \left[e^{-x} \cdot (e^t + \cos t) \right]$

$y_p = \frac{1}{D+2} \left[e^{-x} \cdot (e^{e^x} + \cos e^x) \right]$

$y_p = e^{-2x} \cdot \int e^{2x} \cdot e^{-x} (e^{e^x} + \cos e^x)\, dx$ $\boxed{a=2}$

$y_p = e^{-2x} \int e^x (e^{e^x} + \cos e^x)\, dx$.

$y_p = e^{-2x} \int (e^t + \cos t)\, dt$

$y_p = e^{-2x} (e^t + \sin t)$

$\boxed{y_p = e^{-2x} (e^{e^x} + \sin e^x)}$

$y = y_c + y_p$

Ex5: Solve $(D^2+3D+2)y = e^{e^x} + \cos e^x$

$\rightarrow$ To find y_c

A.E : $D^2+3D+2=0$

$D = -2, -1$

$\therefore y_c = c_1 e^{-2x} + c_2 e^{-x}$

To find y_p.

$$y_p = \frac{1}{\phi(D)} f(x)$$

$$y_p = \frac{1}{D^2+3D+2}\left(e^{e^x} + \cos e^x\right)$$

$$y_p = \frac{1}{(D+2)(D+1)}\left(e^{e^x} + \cos e^x\right)$$

$$y_p = \frac{1}{D+2}\left[\frac{1}{D+1}\left(e^{e^x} + \cos e^x\right)\right]$$

$$y_p = \frac{1}{D+2}\left[e^{-x}\int e^{x}\left(e^{e^x} + \cos e^x\right)dx\right] \qquad a=1$$

Put $e^x = t$

$e^x dx = dt$

$$y_p = \frac{1}{D+2}\left[e^{-x}\int \left(e^{t} + \cos t\right)dt\right]$$

$$y_p = \frac{1}{D+2}\left[e^{-x}\left[e^{t} + \sin t\right]\right]$$

$$y_p = \frac{1}{D+2}\left[e^{-x}\cdot\left(e^{e^x} + \sin e^x\right)\right]$$

$$y_p = e^{-2x}\int e^{2x}\cdot e^{-x}\left(e^{e^x} + \sin e^x\right)dx \qquad a=2$$

$$y_p = e^{-2x}\int e^{x}\left(e^{e^x} + \sin e^x\right)dx.$$

$$y_p = e^{-2x}\int \left(e^{t} + \sin t\right)dt$$

$$y_p = e^{-2x}\left[e^{t} - \cos t\right]$$

$$\boxed{y_p = e^{-2x}\left(e^{e^x} - \cos e^x\right)}$$

$y = y_c + y_p$

H.W

1) $(D^2+5D+6)y = e^{e^x}$

2) $(D^2-3D+2)y = \frac{1}{e^{e^{-x}}} + \cos\left(\frac{1}{e^x}\right)$.

3) $(D^2-9D+18)y = e^{e^{-3x}}$.

4) $(D^2-2D-3)y = 3e^{-3x}.\sin(e^{-3x}) + \cos(e^{-3x}$

$y_p = \frac{1}{(D+}$

* Method of Variation of Parameters when eqn of type $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = X$

when a, b, c are all consts. & X is a funn ~~x~~.

Let the complementary funn is $y_c = Ay_1 + By_2$ then the P.I is,

$$y_p = uy_1 + vy_2 \quad \text{where}$$

$$u = -\int \frac{y_2 f(x)}{W}\,dx, \quad v = \int \frac{y_1 f(x)}{W}\,dx.$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}.$$

W is called Wronskian.

$$\underline{W = (y_1 y_2' - y_1' y_2)}$$

Ex $(D^2+4)y = \sec 2x$.

$\Rightarrow$ $D = \pm 2i$

$y_c = \cos 2x + \sin 2x$.

~~$yp = \frac{1}{D^2+4}$~~ $y_1 = \cos 2x$, $y_2 = \sin 2x$.

~~y_c~~

~~$u = -\int \sin 2x$.~~

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}.$$

$$W = 2\cos^2 2x + 2\sin^2 2x.$$

$$\underline{W = 2}.$$

$$u = -\int \frac{\sin 2x \sec 2x}{2} \cdot dx.$$

$$u = -\frac{1}{2}\int \sin 2x \cdot \frac{1}{\cos 2x}\, dx.$$

$$u = -\frac{1}{2}\int \tan 2x \; dx.$$

$$u = -\frac{1}{2} \log \frac{(\sec 2x)}{2}.$$

$$u = -\frac{1}{4} \log \sec 2x.$$

$$v = \int \frac{y_1 f(x)}{\omega}\, dx.$$

$$v = \int \frac{\cos 2x \cdot \sec 2x}{2}\, dx.$$

$$v = \frac{1}{2}\int \frac{\cos 2x}{\cos 2x}\, dx.$$

$$v = \frac{1}{2} x$$

$$v = \frac{x}{2}$$

$$y_p = u y_1 + v y_2$$

$$y_p = \left(-\frac{1}{4} \log \sec 2x\right) \cos 2x + \frac{x}{2} \sin 2x$$

$$y_p = \frac{x}{2} \sin 2x - \frac{1}{4} \log \sec 2x.$$

2) $$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

$$\Rightarrow \quad (D^2-1)\, y = \frac{2}{1+e^x}$$

$$D = 1, -1$$

$$y_c = c_1 e^x + c_2 e^{-x}$$

$$y_c = c_1 y_1 + c_2 y_2$$

$$\therefore y_1 = e^x, \quad y_2 = e^{-x}$$

$$\omega = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\omega = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix}$$

$$\omega = -e^{x}\cdot e^{-x} - e^{x}\cdot e^{-1}$$

$$\omega = -1 - 1 = -2$$

$$\boxed{\omega = -2}$$

$$u = \int \frac{-y_2 f(x)}{W} dx.$$

$$\frac{1}{t(1+t)} = \frac{A}{t} + \frac{B}{1+t}$$

$$u = -\int \frac{e^{-x}}{(-\cancel{2})} \frac{\cancel{2}}{1+e^{x}} dx.$$

$$1 = A(1+t) + Bt$$

$$A = 1, \ B = -1$$

$$u = +\int \frac{e^{-x}}{1+e^{x}} dx.$$

put $e^{x} = t$

$e^{x} dx = dt$.

$$u = +\int \frac{1}{e^{x}(1+e^{x})} dx.$$

$$u = +\int \frac{A}{e^{x}} + \frac{B}{1+e^{x}} dx$$

$$u = +\int \frac{A}{t} + \frac{B}{1+t} dx =$$

$$u = +\int \left(\frac{1}{t} + \frac{(-1)}{1+t}\right) dx$$

$$u = +\int \left(\frac{1}{e^{x}} - \frac{1}{1+e^{x}}\right) dx.$$

$$u = +\int e^{-x} dx + \int \frac{e^{-x}}{e^{-x}+1} dx$$

$$\int \frac{f'(x)}{f(x)} dx = \log x$$

$$u = +\frac{e^{-x}}{-1} + \int \frac{-e^{-x}}{e^{-x}+1} dx.$$

$f(x) = e^{-x} + 1$

$f'(x) = -e^{-x}$

$$u = -e^{-x} - \log(e^{-x}+1)$$

$$v = \int \frac{y_1 f(x)}{W} dx.$$

$$v = \int \frac{e^{x}}{-\cancel{2}} \cdot \frac{\cancel{2}}{1+e^{x}}.$$

$f(x) = 1+e^{x}$

$f'(x) = e^{x}$

$$v = -\int \frac{e^{x}}{1+e^{x}} dx.$$

$$v = -\log(1+e^{x})$$

$y_p = uy_1 + vy_2$

$y_p = (-e^{-x} + \log(1+e^{-x}))e^{x} + (-\log(1+e^{x}))e^{-x}$

$\underline{y_p = -1 + e^{x}.\log(1+e^{-x}) - e^{-x}\log(1+e^{x})}$

$y = y_c + y_p.$

3) $(D^2-2D+2)y = e^{x}\tan x.$

$\Rightarrow \; D = 1 \pm i$

$y_c = e^{x}(c_1\cos x + c_2\sin x).$

$y_1 = e^{x}\cos x \qquad y_2 = e^{x}\sin x$

$$\omega = \begin{vmatrix} e^{x}\cos x & e^{x}\sin x \\ e^{x}\cos x - e^{x}\sin x & e^{x}\sin x + e^{x}\cos x \end{vmatrix}$$

$$\omega = e^{x}.e^{x}\begin{vmatrix} \cos x & \sin x \\ \cos x - \sin x & \sin x + \cos x \end{vmatrix}.$$

$\omega = e^{2x}[\sin x\cos x + \cos^2 x - \sin x\cos x + \sin^2 x]$

$\omega = e^{2x}(1)$

$\boxed{\omega = e^{2x}}$

$u = -\int \frac{y_2 f(x)}{\omega}\,dx.$

$u = -\int \frac{e^{x}\sin x}{e^{2x}} e^{x}\tan x\,dx.$

$u = -\int \sin x.\frac{\sin x}{\cos x}\,dx.$

$u = -\int \frac{\sin^2 x}{\cos x}\,dx$

$u = -\int \frac{(1-\cos^2 x)}{\cos x}\,dx.$

$u = -\int (\sec x - \cos x)\,dx.$

$u = -\log(\sec x + \tan x) + \sin x.$

$$v = \int \frac{y_1 f(x)}{\omega} dx$$

$$v = \int \frac{e^x \cos x \; e^x \tan x}{e^{2x}} . dx$$

$$v = \int \cos x . \frac{\sin x}{\cos x} dx$$

$$v = -\cos x$$

$$y_p = u y_1 + v y_2$$

$$y_p = [\sin x - \log(\sec x + \tan x)] e^x \cos x + (-\cos x) e^x \sin x$$

$$y_p = e^x [\sin x - \log(\sec x + \tan x) \cos x - \sin x \cos x]$$

$$y_p = e^x [\cancel{\sin x} \cos x - \cos x . \log(\sec x + \tan x) - \cancel{\sin x} \cos x]$$

$$\underline{y_p = e^x \cos x \log(\sec x + \tan x)}$$

$$y = y_c + y_p .$$

4) $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$

$\Rightarrow (D^2 + 4) y = \tan 2x .$

$D = \pm 2i$

$y_c = c_1 \cos 2x + c_2 \sin 2x .$

$\therefore y_1 = \cos 2x \qquad y_2 = \sin 2x .$

$$\omega = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$\omega = 2\cos^2 2x + 2\sin^2 2x$

$\boxed{\omega = 2}$

$u = \int \frac{-y_2 f(x)}{\omega} dx .$

$u = -\int \frac{\sin 2x \tan 2x}{2} . dx$

$v = \int \frac{y_1 f(x)}{\omega} dx$

$v = \int \frac{\cos 2x . \tan 2x}{2} dx$

$v = \frac{1}{2} \int \cos 2x . \frac{\sin 2x}{\cos 2x} dx$

$v = \frac{1}{2} \int \sin 2x \, dx$

$v = \frac{1}{2} \left(\frac{-\cos 2x}{2} \right) .$

$$u = -\frac{1}{2}\int \sin 2x \cdot \frac{\sin 2x}{\cos 2x}\, dx.$$

$$u = -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x}\, dx.$$

$$u = -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x}\, dx.$$

4) $(D^2+1)y = \sec x \tan x.$

$\Rightarrow \quad D = \pm i.$

$y_c = c_1\cos x + c_2 \sin x.$

$y_1 = \cos x \qquad y_2 = \sin x.$

$$\omega = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\omega = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$\omega = \cos^2 x + \sin^2 x$$

$$\boxed{\omega = 1}$$

$$u = -\int \frac{\sin x \cdot \sec x \tan x}{1}\, dx. \qquad \because \tan^2 x = \sec^2 x$$

$$u = -\int \frac{\sin x}{\cos x}\cdot \tan x\, dx.$$

$$u = -\int \tan^2 x\, dx. \quad u = -\int (1+\sec^2 x)\, dx$$

$$u = x - \tan x$$

$$v = \int \frac{y_1 f(x)}{\omega}\, dx.$$

$$v = \int \cancel{\cos x}\cdot \cancel{\sec x}\cdot \tan x\, dx.$$

$$v = \int \tan x\, dx$$

$$v = \log \sec x$$

$y_p = uy_1 + vy_2$.

$y_p = (+x - \tan x)\cos x + \overset{\log(\sec x)}{\cancel{\tan x}}\sin x$.

$y_p = + x\cos x - \sin x + \sin x \cdot \log(\sec x)$

5). $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$.

$\Rightarrow$ $(D^2+3D+2)y = e^{e^x}$.

$D^2+3D+2=0$

$D = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$.

$y_1 = e^{-x} \qquad y_2 = e^{-2x}$.

$$w = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$$

$w = -2e^{-3x} + e^{-3x}$.

$w = -e^{-3x}$.

$u = -\int \frac{y_2 f(x)}{w}\,dx$.

$u = -\int \frac{e^{-2x}\, e^{e^x}}{-e^{-3x}}\,dx$.

$u = \int e^{x} \cdot e^{e^x}\,dx$.

put $e^x = t$

$e^x dx = dt$

$u = \int e^t\,dt$

$u = e^t$

$\boxed{u = e^{e^x}}$

$v = \int \frac{y_1 f(x)}{w}\,dx$.

$v = \int \frac{e^{-x}\, e^{e^x}}{-e^{-3x}}\,dx$.

$v = -\int e^{2x} \cdot e^{e^x}\,dx$.

$$v = -\int t \cdot e^{t}\, dt$$

$$v = -[t \cdot e^{t} - e^{t}] \qquad -e^{t}[t-1]$$

$$v = \cancel{e^{x} \cdot e^{e^{x}} - e^{x}}$$

$$v = \cancel{e^{x}(e^{e^{x}} - 1)} \qquad = -e^{e^{x}}(e^{x}-1)$$

$$y_p = u y_1 + v y_2$$

$$y_p = e^{e^{x}} e^{-x} + e^{x}(e^{e^{x}} - 1) e^{-2x}$$

$$y_p = e^{e^{x}} e^{-x} + e^{-x}(e^{e^{x}} - 1)$$

$$y_p = e^{e^{x}} \cdot e^{-x} + e^{-x} e^{e^{x}} - e^{-x}$$

$$y_p = 2 e^{e^{x}} \cdot e^{-x} - e^{-x}$$

$$y_p = 2 e^{e^{x}} \cdot e^{-x}$$

$$y_p = e^{-x} \cdot e^{e^{x}} - e^{e^{x}}(e^{x}-1) e^{-2x}$$

$$y_p = e^{e^{x}}[\cancel{e^{-x}} - \cancel{e^{-x}} + e^{-2x}]$$

$$y_p = e^{e^{x}}[e^{-2x} + e^{-2x}]$$

$$y_p = e^{-2x} e^{e^{x}}$$

Ex on Variation by Parameters.

1: Solve $(D^2+4)y = \tan 2x$ by method of variation of parameters.

Ans:

$$(D^2+4)y = \tan 2x$$

To find y_c,

Consider A.E. $D^2+4=0$

$$D = 0 \pm 2i$$

$$y_c = e^{0x}(c_1\cos 2x + c_2 \sin 2x)$$

$$\boxed{y_c = c_1\cos 2x + c_2\sin 2x}$$

To find y_p,

$$y_1 = \cos 2x \quad , \; y_2 = \sin 2x$$

$$\omega = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$\omega = 2\cos^2 2x + 2\sin^2 2x$$

$$\omega = 2(\cos^2 2x + \sin^2 2x)$$

$$\boxed{\omega = 2}$$

$$u = -\int \frac{y_2 f(x)}{\omega}\,dx$$

$$u = -\int \frac{\sin 2x \tan 2x}{2}\,dx$$

$$u = -\frac{1}{2}\int \sin 2x \cdot \frac{\sin 2x}{\cos 2x}\,dx$$

$$u = -\frac{1}{2}\int \frac{\sin^2 2x}{\cos 2x}\,dx$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$$u = -\frac{1}{2}\int \frac{1-\cos^2 2x}{\cos 2x}\,dx$$

$$u = -\frac{1}{2}\int (\sec 2x - \cos 2x)\,dx.$$

$$u = -\frac{1}{2}\left[\int \sec 2x\,dx - \int \cos 2x\,dx\right]$$

$$u = -\frac{1}{2}\left[\frac{\log(\sec 2x + \tan 2x)}{2} - \frac{\sin 2x}{2}\right]$$

$$\underline{u = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right]}$$

$$v = \int \frac{y_1 f(x)}{\omega}\,dx$$

$$v = \int \frac{\cos 2x \cdot \tan 2x}{2}\,dx$$

$$v = \frac{1}{2}\int \cancel{\cos 2x} \cdot \frac{\sin 2x}{\cancel{\cos 2x}}\,dx$$

$$v = \frac{1}{2}\int \sin 2x\,dx$$

$$v = \frac{1}{2}\left[-\frac{\cos 2x}{2}\right]$$

$$\underline{v = -\frac{1}{4}\cos 2x}$$

$$y_p = uy_1 + vy_2$$

$$y_p = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x) - \sin 2x\right]\cos 2x - \frac{1}{4}\cos 2x \sin 2x.$$

$$y_p = -\frac{1}{4}\left[\log(\sec 2x + \tan 2x)^{\cos 2x} - \sin 2x \cdot \cancel{\cos 2x} + \cos 2x \cancel{\sin 2x}\right]$$

$$y_p = -\frac{1}{4}\log(\sec 2x + \tan 2x)$$

Solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by method of variation of parameters.

Ans:- $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

$D = \frac{d}{dx}$

$(D^2+1)y = \operatorname{cosec} x$

To find y_c:

Consider A.E. $D^2+1=0$

$D^2 = -1$

$D = 0 \pm i$

$\boxed{y_c = c_1\cos x + c_2\sin x}$

To find y_p

$y_1 = \cos x \qquad y_2 = \sin x.$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos^2 x + \sin^2 x = 1$

$\boxed{W = 1}$

$u = -\int \frac{y_2 f(x)}{W} dx$

$u = -\int \frac{\sin x \operatorname{cosec} x}{1} dx.$

$u = -\int \cancel{\sin x} \cdot \frac{1}{\cancel{\sin x}} dx$

$u = -\int 1\, dx$

$\boxed{u = -x}$

$$v = \int \frac{y_1 f(x)}{\omega}\, dx$$

$$v = \int \frac{\cos x \cdot \operatorname{cosec} x}{1}\, dx$$

$$v = \int \cos x \cdot \frac{1}{\sin x}\, dx$$

$$v = \int \cot x \, dx$$

$\int \cot x \, dx = \log(\sin x)$

$$\boxed{v = \log(\sin x)}$$

$$y_p = u y_1 + v y_2$$

$$y_p = -x \cdot \cos x + \log(\sin x)\sin x$$

$$y = y_c + y_p$$

$$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \cdot \log(\sin x)$$

Solve $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$ by method of variation of parameters.

Ans: To find y_c

Consider A.E. $D^2-6D+9=0$

$$D = 3, 3$$

$$\boxed{y_c = (c_1 + c_2 x)e^{3x}}$$

To find y_p.

$$y_c = c_1 \underline{e^{3x}} + c_2 \underline{xe^{3x}}$$

$$\therefore y_1 = e^{3x} \quad y_2 = xe^{3x}$$

$$\omega = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$\omega = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & 3xe^{3x} + e^{3x} \end{vmatrix}$$

$d(uv) = udv + vdu$

$$\omega = e^{3x}(3xe^{3x} + e^{3x}) - 3xe^{3x} \cdot e^{3x}$$

$$\omega = \cancel{3xe^{6x}} + e^{6x} - \cancel{3xe^{6x}}$$

$$\boxed{\omega = e^{6x}}$$

$$u = -\int \frac{y_2 f(x)}{\omega} \, dx$$

$$u = -\int \frac{x\cancel{e^{3x}} \cdot \frac{\cancel{e^{3x}}}{x^2}}{\cancel{e^{6x}}} \, dx.$$

$$u = -\int \frac{1}{x}\,dx$$

$$\int \frac{1}{x}\,dx = \log x.$$

$$\boxed{u = -\log x}$$

$$v = \int \frac{y_1 f(x)}{W}\,dx$$

$$v = \int \frac{\cancel{e^{3x}} \cdot \frac{\cancel{e^{3x}}}{x^2}}{\cancel{e^{6x}}}\,dx$$

$$v = \int \frac{1}{x^2}\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1}$$

$$\boxed{v = -\frac{1}{x}}$$

$$y_p = u y_1 + v y_2$$

$$y_p = -\log x\, e^{3x} + \left(-\frac{1}{\cancel{x}}\right) \cancel{x} e^{3x}$$

$$\underline{y_p = -e^{3x} \cdot \log x - e^{3x}}$$

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x) e^{3x} - e^{3x} \cdot \log x - e^{3x}$$

$$\underline{y = (c_1 + c_2 x) e^{3x} - e^{3x} (\log x + 1)}$$

Q: Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$ by method of variation of parameters.

Ans :- $D = \frac{d}{dx}$

$$D^2y - 2Dy + 2y = e^x \cdot \tan x$$

$$(D^2 - 2D + 2)\,y = e^x \cdot \tan x$$

To find y_c,

Consider A.E. $D^2 - 2D + 2 = 0$

$$D = 1 \pm i$$

$$\boxed{y_c = e^x (c_1 \cos x + c_2 \sin x)}$$

To find y_p

$$y_1 = \cancel{c_1}\, e^x \cos x \qquad y_2 = \cancel{c_2}\, e^x \cdot \sin x.$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^x \cos x & e^x \sin x \\ -e^x \sin x + e^x \cos x & e^x \cos x + e^x \sin x \end{vmatrix}$$

$$W = e^x e^x \begin{vmatrix} \cos x & \sin x \\ -\sin x + \cos x & \cos x + \sin x \end{vmatrix}$$

$$W = e^{2x} (\cos^2 x + \cancel{\sin x \cos x} + \sin^2 x - \cancel{\sin x \cos x})$$

$$W = e^{2x} (1)$$

$$\boxed{W = e^{2x}}$$

$$u = -\int \frac{y_2 f(x)}{W}\, dx$$

$$u = -\int \frac{\cancel{e^{x}}\cdot \sin x \;\cancel{e^{x}}\cdot \tan x}{\cancel{e^{2x}}}\,dx$$

$$u = -\int \sin x \cdot \frac{\sin x}{\cos x}\,dx$$

$$\cos^2 x + \sin^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$

$$u = -\int \frac{\sin^2 x}{\cos x}\,dx$$

$$u = -\int \frac{1-\cos^2 x}{\cos x}\,dx$$

$$u = -\int (\sec x - \cos x)\,dx$$

$$u = -\left[\int \sec x\,dx - \int \cos x\,dx\right]$$

$$\int \sec x\,dx = \log(\sec x + \tan x)$$

$$u = -\left[\log(\sec x + \tan x) - \sin x\right]$$

$$\underline{u = -\log(\sec x + \tan x) + \sin x.}$$

$$v = \int \frac{y_1 f(x)}{\omega}\,dx$$

$$v = \int \frac{\cancel{e^{x}}\cdot \cos x \;\cancel{e^{x}}\cdot \tan x}{\cancel{e^{2x}}}\,dx$$

$$v = \int \cancel{\cos x}\cdot \frac{\sin x}{\cancel{\cos x}}\,dx$$

$$v = \int \sin x\,dx$$

$$\boxed{v = -\cos x}$$

$$y_p = u y_1 + v y_2$$

$$y_p = [-\log(\sec x + \tan x) + \sin x]\, e^{x}\cos x + (-\cos x)\cdot e^{x}\sin x$$

$$y_p = -e^{x} \cdot \cos x \log(\sec x + \tan x) + e^{x} \cdot \sin x \cos x$$
$$- e^{x} \sin x \cdot \cos x$$

$$\boxed{y_p = -e^{x} \cos x \log(\sec x + \tan x)}$$

$$y = y_c + y_p$$

$$y = e^{x}(c_1 \cos x + c_2 \sin x) - e^{x} \cos x \log(\sec x + \tan x)$$

Solve $(D^2-2D)y = e^x.\sin x$ by method of variation of parameters.

Ans: To find

Consider A.E. $D^2-2D=0$

$$D(D-2)=0$$

$$D=0,\ D=2$$

$$D=0,2$$

$$y_c = c_1 e^{0x} + c_2 e^{2x}$$

$$\boxed{y_c = c_1 + c_2 e^{2x}}$$

To find y_p

$$y_1 = 1 \quad , \quad y_2 = e^{2x}$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$w = \begin{vmatrix} 1 & e^{2x} \\ 0 & 2e^{2x} \end{vmatrix}$$

$$w = 2e^{2x} - 0$$

$$\boxed{w = 2e^{2x}}$$

$$u = -\int \frac{y_2 f(x)}{w}\, dx$$

$$u = -\int \frac{\cancel{e^{2x}} e^x.\sin x}{2\cancel{e^{2x}}}\, dx$$

$$u = -\frac{1}{2}\int e^x.\sin x\, dx$$

$a=1$
$b=1$

$$\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$u = \frac{-1}{2}\left[\frac{e^x}{1+1}(\sin x - \cos x)\right]$$

$$\underline{u = -\frac{1}{4}e^x(\sin x - \cos x)}$$

$$v = \int \frac{y_1 f(x)}{\omega}\,dx$$

$$v = \int \frac{1 \cdot e^x \cdot \sin x}{2e^{2x}}\,dx$$

$$v = \frac{1}{2}\int e^{-x}\sin x\,dx$$

$a = -1$
$b = 1$

$$v = \frac{1}{2}\left[\frac{e^{-x}}{1+1}(-\sin x - \cos x)\right]$$

$$\underline{v = -\frac{e^{-x}}{4}(\sin x + \cos x)}$$

$$y_p = uy_1 + vy_2$$

$$y_p = \left[-\frac{e^x}{4}(\sin x - \cos x)\right]\cdot 1 + \left[-\frac{e^{-x}}{4}(\sin x + \cos x)\right] e^{2x}$$

$$y_p = -\frac{e^x}{4}(\sin x - \cos x) - \frac{e^x}{4}\left[(\sin x + \cos x)\right]$$

$$y_p = \frac{-e^x}{4}\left[\sin x - \cancel{\cos x} + \sin x + \cancel{\cos x}\right]$$

$$\underline{y_p = \frac{-e^x}{4}(2\sin x)} \qquad \boxed{y_p = -\frac{e^x}{2}\sin x.}$$

$$y = y_c + y_p$$

$$\underline{y = c_1 + c_2 e^{2x} - \frac{1}{2}e^x \sin x.}$$

H.W

1) $(D^2+D)y = (1+e^x)^{-1}$

2) $(D^2-4D+4)y = e^{2x} \cdot \sec^2 x$.

3) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$.

4) $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$

* Cauchy's or Euler's Homo. Linear Differential Equation.

A Eqn of the type.

$$(a_0 x^n D^n + a_1 x^{n-1} D^{n-1} + \dots + a_{n-1} x D + a_n) y = f(x).$$

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} x \frac{dy}{dx} + a_n y = P(x). \quad (1)$$

It can be reduced to L.D.E by putting $x = e^z$ or $z = \log x$.

$$x \frac{dy}{dx} = Dy$$

$$x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3 \frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

& so on.

Ex 1) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$

$\Rightarrow$ Put $x = e^z \Rightarrow z = \log x$.

$$x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

$$x\frac{dy}{dx} = Dy.$$

$$D(D-1)y - Dy + 4y = \cos z + e^z \sin z$$

$$(D^2 - D - D + 4)y = \cos z + e^z \sin z.$$

$$(D^2 - 2D + 4)y = \cos z + e^z \sin z.$$

which is L.D.E in y & z.

$$D^2 - 2D + 4 = 0$$

$$D = 1 \pm i\sqrt{3}.$$

$$y_c = e^z (c_1 \cos\sqrt{3}z + c_2 \sin\sqrt{3}z).$$

* Cauchy's or Euler's Homo. Linear Differential Equation.

A Eqn of the type.

$$(a_0 x^n D^n + a_1 x^{n-1} D^{n-1} + \dots + a_{n-1} x D + a_n) y = f(x).$$

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} x \frac{dy}{dx} + a_n y = P(x) \quad (1)$$

It can be reduced to L.D.E by putting $x = e^z$ or $z = \log x$.

$$x \frac{dy}{dx} = Dy$$

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2)y$$

& so on.

Ex 1) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x)$

$\Rightarrow$ Put $x = e^z \Rightarrow z = \log x$.

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$x \frac{dy}{dx} = Dy.$$

$$D(D-1)y - Dy + 4y = \cos z + e^z \sin z$$

$$(D^2 - D - D + 4)y = \cos z + e^z \sin z.$$

$$(D^2 - 2D + 4)y = \cos z + e^z \sin z.$$

which is L.D.E. in y & z.

$$D^2 - 2D + 4 = 0$$

$$D = 1 \pm i\sqrt{3}.$$

$$y_c = e^z (c_1 \cos\sqrt{3} z + c_2 \sin\sqrt{3} z).$$

$$y_p = \frac{1}{D^2-2D+4}\cos z + \frac{1}{D^2-2D+4} e^z \sin z.$$

$$y_p = \quad D^2 - 1 - 1$$

$$y_p = \frac{1}{-1-2D+4}\cos z + e^z \frac{1}{(D+1)^2 - 2(D+1)+4}$$

$$y_p = \frac{1}{3-2D}\cos z + e^z \frac{1}{D^2+\cancel{2D}+1-\cancel{2D}-2+4} \sin z$$

$$y_p = \frac{1}{3-2D} \times \frac{3+2D}{3+2D}\cos z + e^z \frac{1}{D^2+3}\sin z.$$

$$y_p = \frac{(3+2D)\cos z}{9-4D^2} + e^z \frac{1}{-1+3}\sin z$$

$$y_p = \frac{(3+2D)\cos z}{9+4} + \frac{e^z}{2}\sin z.$$

$$y_p = \frac{1}{13}\left[3\cos z + 2(-\sin z)\right] + \frac{e^z}{2}\sin z$$

$$y_p = \frac{1}{13}\left[3\cos z - 2\sin z\right] + \frac{e^z}{2}\sin z.$$

$$y = y_c + y_p.$$

$$y = e^z\left(c_1\cos\sqrt{3}z + c_2\sin\sqrt{3}z\right) + \frac{1}{13}(3\cos z - 2\sin z) + \frac{e^z}{2}\sin z$$

$$y = x\left(c_1\cos\sqrt{3}(\log x) + c_2\sin\sqrt{3}(\log x)\right). + \frac{1}{13}\left(3\cos(\log x) - 2\sin(\log x)\right) + \frac{x}{2}\sin$$

2) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2$

$\Rightarrow \quad x = e^z \Rightarrow z = \log x.$

$$\cancel{x^2\frac{d^2y}{dx^2} = D(D-1)y - 3Dy + 5y}$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x\frac{dy}{dx} = Dy.$$

$$D(D-1)y - 3Dy + 5y = e^{2z}$$

$$(D^2 - D - 3D + 5)y = e^{2z}$$

$(D^2-4D+5)y = e^{2z}$ which is L.D.E in y & z

$D^2-4D+5=0$

$D=$

y_c

$$y_p = \frac{1}{\phi(D)} f(z)$$

$$y_p = \frac{1}{D^2-4D+5} e^{2z}$$

$D \to 2$

$$y_p = \frac{1}{2^2-4(2)+5} e^{2z}$$

$$y_p = \frac{1}{4-4+5} e^{2z}$$

$$y_p = \frac{1}{5} e^{2z}$$

$y = y_c + y_p$

$y =$

$y =$

3) $$x^3\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} + xy = \sin(\log x)$$

$\Rightarrow$ divide by x.

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{x}\sin(\log x)$$

Put $x = e^z \Rightarrow z = \log x$.

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x\frac{dy}{dx} = Dy$$

$D(D-1)y + 3Dy + y = e^{-z}\sin z$

$(D^2-D+3D+1)y = e^{-z}\sin z$.

$(D^2+2D+1)y = e^{-z}\sin z$.

$D = -1, -1$

$y_c = (c_1+c_2 z)e^{-z}$

$$y_p = \frac{1}{D^2+2D+1} e^{-z} \sin z$$

$$y_p = e^{-z} \frac{1}{(D-1)^2+2(D-1)+1} \sin z$$

$$y_p = e^{-z} \frac{1}{D^2-\cancel{2D}+\cancel{1}+\cancel{2D}-\cancel{2}+\cancel{1}} \sin z$$

$$y_p = e^{-z} \frac{1}{D^2} \sin z$$

$$D^2 \to -1$$

$$y_p = -e^{-z} \sin z$$

$$y = (c_1 + c_2 z) e^{-z} - e^{-z} \sin z$$

$$y = (c_1 + c_2 \log x) \frac{1}{x} - \frac{1}{x} \sin(\log x)$$

$$y = \frac{1}{x} \left[c_1 + c_2 \log x - \sin(\log x) \right]$$

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Ex 1) : Solve

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$$

$\Rightarrow$ Given D.E is Cauchy's Homo. Linear D.E

To reduce it to L.D.E

put $x = e^z \Rightarrow z = \log x$

$$x\frac{dy}{dx} = Dy$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$D(D-1)y - Dy + 4y = \cos z + e^z \sin z.$$

$$(D^2 - D - D + 4)y = \cos z + e^z \cdot \sin z.$$

$$(D^2 - 2D + 4)y = \cos z + e^z \cdot \sin z.$$

which is L.D.E. in y & z.

To find y_c

Consider A.E

$$D^2 - 2D + 4 = 0$$

$$D = 1 + i\sqrt{3}$$

$$y_c = e^z(c_1\cos\sqrt{3}z + c_2\sin\sqrt{3}z).$$

$$\boxed{y_c = x[c_1\cos\sqrt{3}(\log x) + c_2\sin\sqrt{3}(\log x)]}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(z)$$

$$y_p = \frac{1}{D^2 - 2D + 4}(\cos z + e^z \cdot \sin z)$$

$$y_p = \frac{1}{D^2-2D+4}\cos z + \frac{1}{D^2-2D+4} e^z \cdot \sin z$$

Replace $D^2 \to -1$ $\qquad D \to D+1$

$$y_p = \frac{1}{-1-2D+4}\cos z + e^z \cdot \frac{1}{(D+1)^2 - 2(D+1) + 4}\sin z$$

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$$y_p = \frac{1}{3-2D}\cos z + e^z \cdot \frac{1}{D^2+2\cancel{D}+1-2\cancel{D}-2+4}\sin z$$

$$y_p = \frac{1}{3-2D} \times \frac{3+2D}{3+2D}\cos z + e^z \frac{1}{D^2+3}\sin z$$

$$y_p = \frac{(3+2D)\cos z}{9-4D^2} + e^z \cdot \frac{1}{D^2+3}\sin z$$

Replace $D^2 \to -1$ $\qquad D^2 \to -1$

$$y_p = \frac{(3+2D)\cdot\cos z}{9-4(-1)} + e^z \frac{1}{-1+3}\sin z$$

$$y_p = \frac{1}{13}\left[3\cos z + 2(-\sin z)\right] + \frac{e^z}{2}\sin z$$

$$y_p = \frac{1}{13}\left(3\cos z - 2\sin z\right) + \frac{e^z}{2}\sin z.$$

$$\therefore \boxed{y_p = \frac{1}{13}\left[3\cos(\log x) - 2\sin(\log x)\right] + \frac{x}{2}\sin(\log x)}$$

$$y = y_c + y_p$$

$$y = x\left[c_1\cos\sqrt{3}(\log x) + c_2\sin\sqrt{3}(\log x)\right] + \frac{1}{13}\left[3\cos(\log x) - 2\sin(\log x)\right] + \frac{x}{2}\sin(\log x)$$

Ex 2 : Solve

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2\sin(\log x)$$

→ which is Cauchy's homo. linear D.E.
Reduce to L.D.E. by putting
$x = e^z \Rightarrow z = \log x$
Let $x\frac{dy}{dx} = Dy$

$x^2\frac{d^2y}{dx^2} = D(D-1)y$

$x^2 = x \cdot x = e^z \cdot e^z = e^{2z}$

$D(D-1)y - 3Dy + 5y = e^{2z}\sin z$

$(D^2 - 4D + 5)y = e^{2z}\sin z$.

which L.D.E in y & z

To find y_c

Consider A.E. $D^2 - 4D + 5 = 0$

$D = 2 \pm i$

$\therefore y_c = e^{2z}(c_1\cos z + c_2\sin z)$

$y_c = x^2[c_1\cos(\log x) + c_2\sin(\log x)]$

To find y_p.

$y_p = \frac{1}{\phi(D)} f(z)$

$y_p = \frac{1}{D^2 - 4D + 5} e^{2z}\sin z$

$D \to D+2$

$y_p =$

$y_p = e^{2z}\frac{1}{(D+2)^2 - 4(D+2) + 5}\sin z$

$y_p = e^{2z}\frac{1}{D^2 + \cancel{4D} + 4 - \cancel{4D} - 8 + 5}\sin z$

$y_p = e^{2z}\frac{1}{D^2+1}\sin z$

$D^2 \to -1$

$$y_p = e^{2z} \cdot \frac{1}{(-1)+1} \sin z$$

case fails.

$$y_p = e^{2z} \frac{z}{2D \cancel{-4}} \cdot \sin z.$$

$$y_p = \frac{e^{2z}}{2} z \left(\frac{1}{D} \sin z\right).$$

$$y_p = \frac{e^{2z}}{2} \cdot z \, (-\cos z).$$

$$y_p = -\frac{1}{2} z e^{2z} \cdot \cos z$$

$$y_p = -\frac{1}{2} (\log x) x^2 \cdot \cos(\log x)$$

$$y = y_c + y_p.$$

$$y = x^2 \left[c_1 \cos(\log x) + c_2 \sin(\log x) \right] - \frac{1}{2} x^2 (\log x) \cdot \cos(\log x).$$

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Ex 3: Solve

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

→ which Cauchy homo. Linear D.E.
Reduce to L.D.E. by putting.

$x = e^z \Rightarrow z = \log x$.

$x^2 \frac{d^2y}{dx^2} = D(D-1)y$

$x^3 \frac{d^3y}{dx^3} = D(D-1)(D-2)y$

$D(D-1)(D-2)y + 2D(D-1)y + 2y = 10\left(e^z + \frac{1}{e^z}\right)$

$(D^3 - D^2 + 2)y = 10(e^z + e^{-z})$

which is L.D.E in y & z.

To find y_c

Consider A.E.

$D^3 - D^2 + 2 = 0$

$D = -1, \ 1 \pm i$

$y_c = c_1 e^{-z} + e^z(c_1 \cos z + c_2 \sin z)$.

$\underline{y_c = c_1 x^{-1} + x[c_1 \cos(\log x) + c_2 \sin(\log x)]}$

To find y_p.

$y_p = \frac{1}{\phi(D)} f(z)$

$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10(e^z + e^{-z})$

$y_p = 10 \cdot \left[\frac{1}{D^3 - D^2 + 2} e^z + \frac{1}{D^3 - D^2 + 2} e^{-z}\right]$

$D \to -1$

Replace $D \to 1$

$y_p = 10\left[\frac{1}{1^3 - 1^2 + 2} e^z + \frac{1}{(-1)^3 - (-1)^2 + 2} e^{-z}\right]$

$y_p = 10\left[\frac{1}{2} e^z + \frac{1}{-1-1+2} e^{-z}\right]$

$y_p = 10\left(\frac{e^z}{2} + \frac{1}{0} e^{-z}\right)$

Ex 4) Solve $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^5$

→ This is Cauchy homo. Linear Diff Eqn.
To reduce L.D.E by putting
$x = e^z \Rightarrow z = \log x$.

$x\frac{dy}{dx} = Dy$

$x^2\frac{d^2y}{dx^2} = D(D-1)y$

$D(D-1)y - 4Dy + 6y = e^{5z}$.

$(D^2 - D - 4D + 6)y = e^{5z}$.

$(D^2 - 5D + 6)y = e^{5z}$.

which is L.D.E in y & z.

To find y_c

Consider A.E. $D^2 - 5D + 6 = 0$.

$D = 3, 2$

$\therefore y_c = c_1e^{3z} + c_2e^{2z}$.

$$\boxed{y_c = c_1x^3 + c_2x^2}$$

To find y_p.

$y_p = \frac{1}{\phi(D)} f(z)$

$y_p = \frac{1}{D^2 - 5D + 6} e^{5z}$.

Replace $D \to 5$

$y_p = \frac{1}{(5)^2 - 5(5) + 6} e^{5z}$.

$y_p = \frac{1}{\cancel{25} - \cancel{25} + 6} e^{5z}$

$y_p = \frac{1}{6} e^{5z}$

$$\boxed{y_p = \frac{1}{6}x^5}$$

$y = y_c + y_p$

$$\boxed{y = c_1x^3 + c_2x^2 + \frac{1}{6}x^5}$$

Ex 5: Solve

$$x^3\frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} - 2y = x^2 + x^{-3}$$

$\rightarrow$ Put $x = e^z \Rightarrow z = \log x$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

$$D(D-1)(D-2)y + D(D-1)y - 2y = e^{2z} + e^{-3z}.$$

$$(D^3 - 2D^2 + D - 2)\,y = e^{2z} + e^{-3z}.$$

which L.D.E. in y & z.

To find y_c.

Consider A.E. $D^3 - 2D + D - 2 = 0$.

$$D = 2,\ 0 \pm i$$

$$\therefore y_c = c_1e^{2z} + c_2\cos z + c_3\sin z.$$

$$\boxed{y_c = c_1x^2 + c_2\cos(\log x) + c_3\sin(\log x)}$$

To find y_p.

$$y_p = \frac{1}{\phi(D)}\,f(z).$$

$$y_p = \frac{1}{D^3 - 2D^2 + D - 2}\left(e^{2z} + e^{-3z}\right).$$

$$y_p = \frac{1}{D^3 - 2D^2 + D - 2}e^{2z} + \frac{1}{D^3 - 2D^2 + D - 2}e^{-3z}.$$

Replace $D \rightarrow 2$ $\qquad D \rightarrow -3$

$$y_p = \frac{1}{(2)^3 - 2(2)^2 + (2) - 2}e^{2z} + \frac{1}{(-3)^3 - 2(-3)^2 + (-3) - 2}e^{-3z}$$

$$y_p = \frac{1}{0}e^{2z} + \frac{1}{-50}e^{-3z}.$$

case fails.

$$y_p = \frac{z}{3D^2 - 4D + 1}e^{2z} - \frac{1}{50}e^{-3z}.$$

Replace $D \rightarrow 2$

*. Legendre's Differential Eqn :- (3rd order)

$$a_0 (ax+b)^3 \frac{d^3y}{dx^3} + a_1 (ax+b)^2 \frac{d^2y}{dx^2}$$

$$+ a_2 (ax+b) \frac{dy}{dx} + a_3 y = f(x)$$

To reduce it into L.D.E by putting

$ax+b = e^z \Rightarrow z = \log(ax+b)$

Put $(ax+b)^3 \frac{d^3y}{dx^3} = a^3 D(D-1)(D-2)y$

$$(ax+b)^2 \frac{d^2y}{dx^2} = a^2 D(D-1)y$$

$$(ax+b) \frac{dy}{dx} = aDy.$$

__Ex 1)__ $(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x.$

$\Rightarrow$ Put $(2x+1) = e^z \Rightarrow z = \log(2x+1)$

$(2x+1)\frac{dy}{dx} = 2Dy$

$$(2x+1)^2 \frac{d^2y}{dx^2} = 2^2 D(D-1)y$$

$$= 4 D(D-1)y$$

$$4D(D-1) - 2\cdot 2Dy - 12y = 6\left(\frac{e^z-1}{2}\right)$$

$$(4D^2 - 4D - 4D - 12)y = 3(e^z-1)$$

$$(4D^2 - 8D - 12)y = 3(e^z-1)$$

$$(D^2 - 2D - 3)y = \frac{3}{4}(e^z-1)$$

which is L.D.E in y & z.

$$D^2 - 2D - 3 = 0$$

$$D = 3, -1$$

$$y_c = c_1 e^{3z} + c_2 e^{-z}$$

$$y_p = \frac{1}{\phi(D)} f(z)$$

$$y_p = \frac{1}{D^2-2D-3} \frac{3}{4}(e^z-1)$$

$$y_p = \frac{3}{4}\left[\frac{1}{\underset{D\to 1}{D^2-2D-3}} e^z - \frac{1}{\underset{D\to 0}{D^2-2D-3}} 1\right]$$

$$y_p = \frac{3}{4}\left[\frac{1}{1-2-3} e^z - \frac{1}{0-0-3}\right]$$

$$y_p = \frac{3}{4}\left[\frac{1}{-4} e^z + \frac{1}{3}\right]$$

$$y_p = -\frac{3}{16} e^z + \frac{1}{4}$$

$$y = y_c + y_p$$

$$y = c_1 e^{3z} + c_2 e^{-z} - \frac{3}{16} e^z + \frac{1}{4}$$

$$y = c_1 (2x+1)^3 + c_2 (2x+1)^{-1} - \frac{3}{16}(2x+1) + \frac{1}{4}$$

2) $$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$

$\Rightarrow$ Put $(3x+2) = e^z \Rightarrow z = \log(3x+2)$

$$(3x+2)^2 \frac{d^2y}{dx^2} = 3^2 D(D-1)y$$

$3x = e^z - 2$

$x = \frac{e^z-2}{3}$

$$(3x+2)\frac{dy}{dx} = 3Dy$$

$$\therefore 9D(D-1) + 3\cdot 3Dy - 36y =$$

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$$3x^2+4x+1 = 3\left(\frac{e^z-2}{3}\right)^2+4\left(\frac{e^z-2}{3}\right)+1$$

$$= \frac{(e^z-2)^2}{3} + \frac{4}{3}(e^z-2)+1$$

$$= \frac{1}{3}(e^{2z}-\cancel{4e^z}+4+\cancel{4e^z}-8+3)$$

$$= \frac{1}{3}(e^{2z}-1).$$

$$\therefore (9D^2-\cancel{9D}+\cancel{9D}-36)y = \frac{1}{3}(e^{2z}-1).$$

$$(9D^2-36)y = \frac{1}{3}(e^{2z}-1)$$

$$(D^2-4)y = \frac{1}{27}(e^{2z}-1).$$

which is L.D.E in y & z

$D^2-4=0$

$D=\pm 2$

$y_c = c_1e^{2z}+c_2e^{-2z}.$

$$y_p = \frac{1}{D^2-4}\cdot\frac{1}{27}(e^{2z}-1).$$

$$y_p = \frac{1}{27}\left[\frac{1}{D^2-4}e^{2z} - \frac{1}{D^2-4}1\right]$$

$$y_p = \frac{1}{27}\left[\frac{1}{4-4}e^{2z} - \frac{1}{-4}\right]$$

$$y_p = \frac{1}{27}\left[\frac{z}{2D}e^{2z}+\frac{1}{4}\right]$$

$$y_p = \frac{1}{27}\left[\frac{z}{4}e^{2z}+\frac{1}{4}\right]$$

$y = y_c + y_p$

$$y = c_1e^{2z}+c_2e^{-2z}+\frac{1}{27}\left[\frac{z}{4}e^{2z}+\frac{1}{4}\right]$$

$$y = c_1(3x+2)^2+c_2(3x+2)^{-2} \cancel{+\frac{1}{27}\{}$$

$$+\frac{1}{108}\left[\frac{\log(3x+2)}{4}(3x+2)^2+1\right]$$

$27 \times 4 = 108$

③ $(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y$

$= 5 \log(5+2x)$

$\Rightarrow$ Put $(5+2x) = e^z \Rightarrow z = \log(5+2x)$

$(5+2x)^2 \frac{d^2y}{dx^2} = 4D(D-1)y$

$(5+2x)\frac{dy}{dx} = 2Dy$

$4 \cdot D(D-1)y - 6 \cdot 2 \cdot Dy + 8y = 5z$.

$(4D^2 - 4D - 12D + 8)y = 5z$.

$(4D^2 - 16D + 8)y = 5z$.

$4(D^2 - 4D + 2)y = 5z$

$(D^2 - 4D + 2)y = \frac{5}{4}z$.

~~which~~ which is Linear in y & z.

$D^2 - 4D + 2 = 0$.

$D = 2 \pm \sqrt{2}$

$y_c = c_1 e^{(2+\sqrt{2})z} + c_2 e^{(2-\sqrt{2})z}$.

$y_p = \frac{1}{D^2-4D+2} \frac{5}{4}z$

$y_p = \frac{5}{4} \frac{1}{D^2-4D+2} z$

$y_p = \frac{5}{4} \frac{1}{2} \frac{1}{\left[1 + \left(\frac{D^2-4D}{2}\right)\right]} z$

$y_p = \frac{5}{8}\left[1 + \left(\frac{D^2-4D}{2}\right)\right]^{-1} z$.

$y_p = \frac{5}{8}\left[1 - \left(\frac{D^2-4D}{2}\right) + \right] z$

$y_p = \frac{5}{8}\left[z - \frac{1}{2}\left[0 - 4(1)\right]\right]\}$

$y_p = \frac{5}{8}\left[z + 2\right)$

$y = y_c + y_p$

Ex 1) Solve

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin[\log(1+x)]$$

$\Rightarrow$ This Legendre's form D.E.

To reduce it to L.D.E by putting

$(1+x)=e^z \Rightarrow z=\log(1+x)$.

$(1+x)\frac{dy}{dx}=Dy$ $\quad a=1$.

$(1+x)^2\frac{d^2y}{dx^2}=D(D-1)y$.

$D(D-1)y+Dy+y=2\sin z$.

$(D^2+1)y=2\sin z$

which is L.D.E. in y & z.

To find y_c,

Consider A.E $\quad D^2+1=0$

$D=0\pm i$

$y_c= c_2\cos z+c_3\sin z$.

$y_c=c_2\cos[\log(1+x)]+c_3\sin[\log(1+x)]$

To find y_p

$y_p=\frac{1}{\phi(D)}f(z)$.

$y_p=\frac{1}{D^2+1}2\sin z$. $\quad a=1$

Replace $D^2\to -1$

$y_p=2\cdot\frac{1}{\cancel{(-1)}+\cancel{1}}\sin z$

case fails.

$y_p=\cancel{2}\cdot\frac{z}{\cancel{2}D}\sin z$.

$y_p=z\cdot\left(\frac{1}{D}\sin z\right)$

$y_p=z\cdot(-\cos z)$.

$$\boxed{y_p=-\log(1+x)\cos[\log(1+x)}$$

Ex 1) Solve

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin[\log(1+x)]$$

→ This Legendre's form of D.E.
To reduce it to L.D.E. by putting
$1+x=e^z \Rightarrow z=\log(1+x)$

$a=1$

$$(1+x)\frac{dy}{dx}=Dy$$

$$(1+x)^2\frac{d^2y}{dx^2}=D(D-1)y$$

$$D(D-1)y+Dy+y=2\sin z$$

$$(D^2-\cancel{D}+\cancel{D}+1)y=2\sin z$$

$$(D^2+1)y=2\sin z.$$

which L.D.E. in y & z

To find y_c

Consider A.E : $D^2+1=0$

$$D=0\pm i$$

$$\therefore y_c=c_1\cos z+c_2\sin z$$

$$y_c=c_1\cos[\log(1+x)]+c_2\sin[\log(1+x)]$$

To find y_p

$$y_p=\frac{1}{\phi(D)}f(z)$$

$$y_p=\frac{1}{D^2+1}2\sin z.$$

$$y_p=2.\frac{1}{D^2+1}\sin z.$$

$a=1$

Replace $D^2\rightarrow -1$

$$y_p=2.\frac{1}{\cancel{-1}+\cancel{1}}\sin z.$$

case fails.

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$$y_p = \frac{z}{2D} \sin z$$

$$y_p = \frac{z}{2}\left(\frac{1}{D} \sin z\right).$$

$$y_p = \frac{z}{2}\left(-\cos z\right).$$

$$y_p = -\frac{z \cos z}{2}$$

$$\cancel{y_p = -(1+x)}$$

$$y_p = -\log(1+x)\frac{\cos \log(1+x)}{2}.$$

$$y = y_c + y_p.$$

Ex 2) Solve

$$(2x+1)^2\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$$

$\Rightarrow$ This is Legendre's Linear Eqn.
To reduce L.D.E. by putting
$(2x+1) = e^z \Rightarrow z = \log(2x+1)$

$$(2x+1)\frac{dy}{dx} = 2 \cdot Dy$$

$a = 2$
$2x+1 = e^z$
$2x = e^z - 1$
$x = \frac{e^z-1}{2}$

$$(2x+1)\frac{d^2y}{dx^2} = 4 \cdot D(D-1)y$$

$$4 \cdot D(D-1)y - 2 \cdot 2Dy - 12y = 6\left(\frac{e^z-1}{2}\right)$$

$$(4D^2 - 4D - 4D - 12)y = 3(e^z-1)$$

$$(4D^2 - 8D - 12)y = 3(e^z-1)$$

$$4(D^2 - 2D - 3)y = 3(e^z-1)$$

$$(D^2 - 2D - 3)y = \frac{3}{4}(e^z-1)$$

which is L.D.E in y & z.
To find y_c.
Consider A.E. $D^2 - 2D - 3 = 0$

$$D = 3, -1$$

$$y_c = c_1e^{3z} + c_2e^{-z}$$

$$\boxed{y_c = c_1(2x+1)^3 + c_2(2x+1)^{-1}}$$

To find y_p.

$$y_p = \frac{1}{\phi(D)} \cdot f(z)$$

$$y_p = \frac{z}{2D} \sin z$$

$$y_p = \frac{z}{2}\left(\frac{1}{D} \sin z\right).$$

$$y_p = \frac{z}{2}\left(-\cos z\right).$$

$$y_p = -\frac{z \cos z}{2}$$

$$y_p = -\cancel{(1+x)}$$

$$y_p = -\log(1+x) \frac{\cos \log(1+x)}{2}.$$

$$y = y_c + y_p.$$

Ex 2) Solve

$$(2x+1)^2 \frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x$$

→ This is Legendre's Linear Eqⁿ.
To reduce L.D.E. by putting

$(2x+1) = e^z \Rightarrow z = \log(2x+1)$

$(2x+1)\frac{dy}{dx} = 2 \cdot Dy$

$(2x+1)\frac{d^2y}{dx^2} = 4 \cdot D(D-1)y$

$a = 2$
$2x+1 = e^z$
$2x = e^z - 1$
$x = \frac{e^z - 1}{2}$

$4 \cdot D(D-1)y - 2 \cdot 2Dy - 12y = 6\left(\frac{e^z-1}{2}\right)$

$(4D^2 - 4D - 4D - 12)y = 3(e^z - 1)$

$(4D^2 - 8D - 12)y = 3(e^z - 1)$

$4(D^2 - 2D - 3)y = 3(e^z - 1)$

$(D^2 - 2D - 3)y = \frac{3}{4}(e^z - 1)$

which is L.D.E in y & z.
To find y_c.
Consider A.E. $D^2 - 2D - 3 = 0$

$D = 3, -1$

$y_c = c_1 e^{3z} + c_2 e^{-z}$

$$\boxed{y_c = c_1 (2x+1)^3 + c_2 (2x+1)^{-1}}$$

To find y_p.

$y_p = \frac{1}{\phi(D)} \cdot f(z)$

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3) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2+4x+1$

→ Put $3x+2 = e^z \Rightarrow z = \log(3x+2)$

$3x = e^z - 2$

$x = \frac{e^z-2}{3}$

$a=3$

$(3x+2)\frac{dy}{dx} = 3 \cdot Dy$

$(3x+2)^2\frac{d^2y}{dx^2} = 9 \cdot D(D-1)y$

$9 \cdot D(D-1)y + 3 \cdot 3Dy - 36y = 3\left(\frac{e^z-2}{3}\right)^2 + 4\left(\frac{e^z-2}{3}\right) + 1$

$(9D^2 - \cancel{9D} + \cancel{9D} - 36)y = \cancel{3}\frac{(e^{2z}-4e^z+4)}{\cancel{3} \cdot 3} + \frac{4}{3}(e^z-2)+1$

$(9D^2-36)y = \frac{e^{2z} - \cancel{4e^z} + 4 + \cancel{4e^z} - 8 + 3}{3}$

$(9D^2-36)y = \frac{e^{2z}-1}{3}$

$9(D^2-4)y = \frac{e^{2z}-1}{3}$

$(D^2-4)y = \frac{e^{2z}-1}{27}$

which is L.D.E. in y & z.

so sol^n is,

To find y_c

Consider A.E. $D^2-4=0$

$D = \pm 2$

$D = 2, -2$

$\therefore y_c = c_1e^{2z} + c_2e^{-2z}$

$\underline{y_c = c_1(3x+2)^2 + c_2(3x+2)^{-2}}$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(z)$$

$$y_p = \frac{1}{D^2-4} \frac{1}{27} (e^{2z}-1).$$

$$y_p = \frac{1}{27}\left[\frac{1}{D^2-4} e^{2z} - \frac{1}{D^2-4} 1\right]$$

Replace $D \to 2$ $\quad D \to 0$

$$y_p = \frac{1}{27}\left[\frac{1}{\cancel{4}-4} e^{2z} - \frac{1}{0-4}\right]$$

case fails

$$y_p = \frac{1}{27}\left[\frac{z}{2D} \cdot e^{2z} + \frac{1}{4}\right].$$

$$y_p = \frac{1}{27}\left[\frac{z}{2}\left(\frac{1}{D} e^{2z} + \frac{1}{4}\right)\right]$$

$$y_p = \frac{1}{27}\left[\frac{z}{2}\left(\frac{e^{2z}}{2}\right) + \frac{1}{4}\right]$$

$$y_p = \frac{1}{27}\left[\frac{ze^{2z}}{4} + \frac{1}{4}\right]$$

$$y_p = \frac{1}{108}\left(ze^{2z} + 1\right)$$

$$y_p = \frac{1}{108}\left[\log(3x+2)\,(3x+2)^2 + 1\right]$$

$$y = y_c + y_p$$

$$\therefore y = c_1(3x+2)^2 + c_2(3x+2)^{-2} + \frac{1}{108}\left[(3x+2)^2 \log(3x+2) + 1\right]$$

Ex4) $(x+2)^2\frac{d^2y}{dx^2}-(x+2)\frac{dy}{dx}+y=3x+4$

→ Put $x+2=e^z \Rightarrow z=\log(x+2)$

$x=e^z-2$

$(x+2)\frac{dy}{dx}=Dy$ $\qquad a=1$

$(x+2)^2\frac{d^2y}{dx^2}=D(D-1)y$.

$D(D-1)y-Dy+y=3(e^z-2)+4$

$(D^2-2D+1)y=3e^z-2$

which L.D.E. in y & z.

To find y_c

Consider A.E. $D^2-2D+1=0$

$D=1,1$

$\therefore y_c=(c_1+c_2z)e^z$.

$y_c=[c_1+c_2\log(x+2)](x+2)$

To find y_p

$y_p=\frac{1}{\phi(D)}f(z)$

$y_p=\frac{1}{D^2-2D+1}\cdot(3e^z-2)$

$y_p=3\cdot\frac{1}{D^2-2D+1}e^z-\frac{1}{D^2-2D+1}\cdot 2$

Replace $D\to 1$ $\qquad D\to 0$

$y_p=3\cdot\frac{1}{\cancel{1}-2+\cancel{1}}e^z-\frac{1}{0-0+1}2]$

case fails

$y_p=3\cdot\frac{z}{2D-2}e^z-2$

Replace $D\to 1$

$y_p=3\cdot\frac{z}{2(\cancel{1})-\cancel{2}}e^z-2$

case fails

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$$y_p = 3 \cdot \frac{z^2}{2} e^{z} - 2$$

③ $$y_p = \frac{3(x+2)}{2} \log(x+2)^2 - 2$$

$$y = y_c + y_p$$

$$y = (x+2)\left[c_1 + c_2 \log(x+2)\right] + \frac{3}{2}(x+2)\left[\log(x+2)\right]^2 - 2$$

Simultaneous Linear Diff. Eqns

If there are two or more dependent variable each of which is a function of one independent variables then the system of diff. eqn. is called a simultaneous linear diff. eqn.

for example : $\frac{dx}{dt} + 3\frac{dy}{dt} + y = t$

$$\frac{dy}{dt} - x - y = t^2$$

Here t is single independent and x & y are the two dependent variables.
such eqns are called Simultaneous D.E.

Note : The number of Eqns is same as the number of dependent variable.

* Method of Solution :—

Methods of solving these equations are similar to those of solving algebraic equations

$$x + y = 3$$
$$2x - y = 9$$

Ex 1) Solve $(D+2)x + (D+1)y = t$

$5x + (D+3)y = t^2$

$\Rightarrow$ $D = \frac{d}{dt}$

Solving for y, By Cramer's rule

$$\begin{vmatrix} D+2 & D+1 \\ 5 & D+3 \end{vmatrix} y = \begin{vmatrix} D+2 & t \\ 5 & t^2 \end{vmatrix}.$$

$$[(D+2)(D+3) - (D+1)5]y = (D+2)t^2 - 5t.$$

$$(D^2+5D+6-5D-5)y = Dt^2 + 2t^2 - 5t.$$

$$(D^2+1)y = 2t + 2t^2 - 5t$$

$$\boxed{(D^2+1)y = 2t^2 - 3t}$$

which L.D.E in y & t

To find y_c

Consider A.E. $D^2+1=0$

$D = \pm 1$

$D = 1, -1$

$$\boxed{y_c = c_1e^t + c_2e^{-t}}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(t)$$

$$y_p = \frac{1}{D^2+1}(2t^2-3t)$$

$$y_p = [1+D^2]^{-1}(2t^2-3t)$$

$$y_p = [1 - D^2 + D^4 + \dots](2t^2-3t).$$

$$y_p = [2t^2 - 3t - (4) + 0]$$

$$\boxed{y_p = 2t^2 - 3t - 4}$$

$$y = y_c + y_p.$$

$$\boxed{y = c_1e^t + c_2e^{-t} + 2t^2 - 3t - 4}$$

$D(2t^2-3t) = 4t - 3$

$D^2(\ ) = 4$

$D^3(\ ) = 0$

$(1+z)^{-1} = 1 - z + z^2 - z^3 + \dots$

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Ex 1) Solve $(D+2)x + (D+1)y = t$

$5x + (D+3)y = t^2$

→ Solving for y, using Cramers rule.

$$\begin{vmatrix} D+2 & D+1 \\ 5 & D+3 \end{vmatrix} y = \begin{vmatrix} D+2 & t \\ 5 & t^2 \end{vmatrix}$$

$$[(D+2)(D+3) - (D+1)5]\, y = (D+2)t^2 - 5t$$

$$(D^2 + \cancel{5D} + 6 - \cancel{5D} - 5)\, y = Dt^2 + 2t^2 - 5t$$

$$(D^2+1)\, y = 2t + 2t^2 - 5t$$

$$(D^2+1)\, y = 2t^2 - 5t$$

which L.D.E. in y & t.

To find y_c

Consider A.E. $D^2+1=0$

$D = \pm i$

$D = 1, -1$

$$\therefore \boxed{y_c = c_1 e^{t} + c_2 e^{-t}}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(t)$$

$$y_p = \frac{1}{D^2+1}(2t^2 - 5t)$$

$$y_p = \frac{1}{(1+D^2)}(2t^2 - 5t)$$

$$y_p = [1+D^2]^{-1}(2t^2 - 5t)$$

$(1+z)^{-1} = 1 - z + z^2 - z^3 + \ldots$

$$y_p = [1 - D^2 + D^4 - \ldots](2t^2 - 5t)$$

$D(2t^2 - 5t) = 4t - 5$

$D^2(\;) = 4$

$D^3(\;) = 0$

$$y_p = [2t^2 - 5t - 4 + 0]$$

$$y_p = 2t^2 - 5t - 4.$$

$$y = y_c + y_p$$

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x1) Solve $(D+2)x + (D+1)y = t$

$5x + (D+3)y = t^2$

$\Rightarrow \quad D = \frac{d}{dt}$

Solving for y, Using Cramer's rule

$$\begin{vmatrix} D+2 & D+1 \\ 5 & D+3 \end{vmatrix} y = \begin{vmatrix} D+2 & t \\ 5 & t^2 \end{vmatrix}$$

$$[(D+2)(D+3) - 5(D+1)]y = (D+2)t^2 - 5t$$

$$(D^2 + 5\cancel{D} + 6 - 5\cancel{D} - 5)y = 2t + 2t^2 - 5t$$

$$(D^2+1)y = 2t^2 - 3t$$

which is L.D.E. in y & t.

To find y_c,

Consider A.E. $\quad D^2+1=0$

$D^2 = -1$

$D = 0 + i$

$$y_c = e^{0t}(c_1\cos t + c_2 \sin t)$$

$$\boxed{y_c = c_1\cos t + c_2\sin t}$$

To find y_p

$$y_p = \frac{1}{\phi(D)} f(t)$$

$$y_p = \frac{1}{D^2+1}(2t^2-3t)$$

$$y_p = [1+D^2]^{-1}(2t^2-3t)$$

$$y_p = [1 - D^2 + \ldots](2t^2-3t)$$

$$y_p = 2t^2 - 3t - 4$$

$$y = y_c + y_p$$

$D(2t^2-3t) = 4t-3$

$D^2(\ ) = 4$

$D^3(\ ) = 0$

$(1+z)^{-1} = 1 - z + z^2 - z^3 + \ldots$

$(1+$
$(1-z)^{-1} = 1+z+$

$$y_p = \frac{4}{7}e^{2t} - \frac{3}{5}\left[1 + \frac{D^2+4D}{5} + \dots\right] t$$

$$y_p = \frac{4}{7}e^{2t} - \frac{3}{5}\left[t + \frac{1}{5}(0 + 4(1))\right]$$

$$y_p = \frac{4}{7}e^{2t} - \frac{3}{5}\left(t + \frac{4}{5}\right)$$

$$\boxed{y_p = \frac{4}{7}e^{2t} - \frac{3t}{5} - \frac{12}{25}}$$

$$y = y_c + y_p$$

$$\boxed{y = c_1 e^{-5t} + c_2 e^{t} + \frac{4}{7}e^{2t} - \frac{3}{5}t - \frac{12}{25}}$$

To find x,

$$-3x + (D+2)y = e^{2t}$$

$$3x = (D+2)y - e^{2t}$$

$$3x = (D+2)\left[c_1 e^{-5t} + c_2 e^{t} + \frac{4}{7}e^{2t} - \frac{3}{5}t - \frac{12}{25}\right] - e^{2t}$$

$$3x = -5c_1 e^{-5t} + c_2 e^{t} + \frac{8}{7}e^{2t} - \frac{3}{5} + 2c_1 e^{-5t} + 2c_2 e^{t} + \frac{8}{7}e^{2t} - \frac{6}{5}t - \frac{24}{25} - e^{2t}$$

$$3x = (-5c_1 + 2c_1)e^{-5t} + (c_2 + 2c_2)e^{t} + \left(\frac{8}{7} + \frac{8}{7} - 1\right)e^{2t} + \left(\frac{-3}{5} - \frac{24}{25}\right) - \frac{6}{5}t$$

$$3x = -3c_1 e^{-5t} + 3c_2 e^{t} + \frac{9}{7}e^{2t} - \frac{6}{5}t - \frac{39}{25}$$

$$\boxed{x = -c_1 e^{-5t} + c_2 e^{t} + \frac{3}{7}e^{2t} - \frac{2}{5}t - \frac{13}{25}}$$

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$\{y = c_1\cos t + c_2\sin t + 2t^2 - 3t - 4\}$.

Soln for finding x, use the eqn when coeff of x is simple.

$$5x + (D+3)y = t^2$$

$5x = t^2 - (D+3)y$

$5x = t^2 - (D+3)[c_1\cos t + c_2\sin t + 2t^2 - 3t - 4]$

$5x = t^2 - [-c_1\sin t + c_2\cos t + 4t - 3 + 3c_1\cos t + 3c_2\sin t + 6t^2 - 9t - 12]$

$5x = t^2 + c_1\sin t - c_2\cos t - 4t + 3 - 3c_1\cos t - 3c_2\sin t - 6t^2 + 9t + 12$

$5x = (c_1 - 3c_2)\sin t - (c_2 + 3c_1)\cos t - 5t^2 + 5t + 15$

$$x = \left(\frac{c_1 - 3c_2}{5}\right)\sin t - \left(\frac{c_2 + 3c_1}{5}\right)\cos t - t^2 + t + 3$$

$$\boxed{y = c_1\cos t + c_2\sin t + 2t^2 - 3t - 4}$$

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x3) Solve the simulteneous L.D.E with given conditions

$$\frac{du}{dx} + v = \sin x$$

$$\frac{dv}{dx} + u = \cos x$$

Given that when $x=0$ then $u=1$ & $v=0$

$\Rightarrow$ $Du + v = \sin x$

$Dv + u = \cos x$

$Du + v = \sin x$.

$u + Dv = \cos x$.

Solving for v. By Cramer's Rule

$$\begin{vmatrix} D & 1 \\ 1 & D \end{vmatrix} v = \begin{vmatrix} D & \sin x \\ 1 & \cos x \end{vmatrix}$$

$(D^2-1)v = -\sin x - \sin x$.

$(D^2-1)v = -2\sin x$.

which is L.D.E. in v & x.

To find y_c

Consider A.E $D^2-1=0$

$D = \pm 1$

$$\boxed{v_c = c_1 e^{x} + c_2 e^{-x}}$$

To find v_p

$$v_p = \frac{1}{\phi(D)} f(x)$$

$$v_p = \frac{1}{D^2-1}(-2\sin x).$$

$$v_p = -2 \cdot \frac{1}{D^2-1} \sin x.$$

Replace $D^2 \to -1$

$$v_p = -2 \cdot \frac{1}{-1-1} \sin x$$

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$\boxed{v_p = \sin x}$

$v = v_c + v_p$

$\boxed{v = c_1 e^{x} + c_2 e^{-x} + \sin x.}$

To find u,

$Dv + u = \cos x.$

$u = \cos x - Dv$

$u = \cos x - D(c_1 e^{x} + c_2 e^{-x} + \sin x).$

$u = \cancel{\cos x} - c_1 e^{x} + c_2 e^{-x} - \cancel{\cos x}.$

$\boxed{u = -c_1 e^{x} + c_2 e^{-x}}$

At $x = 0$, then $u = 1$ & $v = 0$

$u = -c_1 + c_2$ } on simplification.
$v = c_1 + c_2$

$c_2 = 1/2$ & $c_1 = -1/2$.

$u = \frac{1}{2} e^{x} + \frac{1}{2} e^{-x}.$

$u = \frac{e^{x} + e^{-x}}{2}.$

$\boxed{u = \cosh x},$

$v = -\frac{1}{2} e^{x} + \frac{1}{2} e^{-x} + \sin x.$

$v = -\left(\frac{e^{x} - e^{-x}}{2}\right) + \sin x.$

$v = -\sinh x + \sin x.$

$\boxed{v = \sin x - \sinh x}$

Ex 1) $(D+2)x + (D+1)y = t$

$5x + (D+3)y = t^2$

$\Rightarrow \quad D = \dfrac{d}{dt}$

Solving for y, by Cramer's rule.

$$\begin{vmatrix} D+2 & D+1 \\ 5 & D+3 \end{vmatrix} y = \begin{vmatrix} D+2 & t \\ 5 & t^2 \end{vmatrix}$$

$$[(D+2)(D+3) - 5(D+1)]y = (D+2)t^2 - 5t$$

$$(D^2 + 3\cancel{D} + 2\cancel{D} + 6 - 5\cancel{D} - 5)y = Dt^2 + 2t^2 - 5t$$

$$(D^2+1)\,y = (2t) + 2t^2 - 5t$$

$(D^2+1)y = 2t^2 - 3t$ which is L.D.E in y & t

To find y_c

Consider A.E. $D^2+1 = 0$

$D = 0 \pm i$ $\qquad \alpha = 0$, $\beta = 1$

$$y_c = c_1 \cos t + c_2 \sin t.$$

$$y_p = \frac{1}{\phi(D)} f(t)$$

$$y_p = \frac{1}{D^2+1}(2t^2 - 3t)$$

$$y_p = \frac{1}{1+D^2}(2t^2-3t)$$

$$y_p = [1+D^2]^{-1}(2t^2-3t)$$

$$y_p = \{1 - D^2 + D^4 + \cdots](2t^2-3t)$$

Neglect higher order term

$$y_p = 2t^2 - 3t - 4 + 0$$

$(1+z)^{-1} = 1 - z + z^2 - z^3 + \cdots$

$f(t) = 2t^2 - 3t$

$D(\ ) = 4t - 3$

$D^2(\ ) = 4$

$D^3(\ ) = 0$

$$\boxed{y_p = 2t^2 - 3t - 4}$$

$$y = y_c + y_p.$$

$$\boxed{y = c_1 \cos t + c_2 \sin t + 2t^2 - 3t - 4}$$

To find value of x, use the eqn when coef. x is simple.

i.e $\quad 5x + (D+3)y = t^2$

$$x = \frac{1}{5}\left[t^2 - (D+3)y\right]$$

$$x = \frac{1}{5}\left[t^2 - (D+3)(c_1 \cos t + c_2 \sin t + 2t^2 - 3t\right.$$

$$x = \frac{1}{5}\left[t^2 - [-c_1 \sin t + c_2 \cos t + 4t - 3 + 3c_1 \cos\right.$$
$$+ 3c_2 \sin t + 6t^2 - 9t -$$

$$x = \frac{1}{5}\left[\cancel{t^2} + c_1 \sin t - c_2 \cos t - \cancel{4t} + \cancel{3} - 3c_1 \cos\right.$$
$$- 3c_2 \sin t - \cancel{6t^2} + \cancel{9t} + \cancel{12}$$

$$x = \frac{1}{5}\left[-5t^2 + 5t + 15 + (c_1 - 3c_2)\sin t\right.$$
$$\left. + (-c_2 - 3c_1)\cos t\right]$$

$$\boxed{x = -t^2 + t + 3 + \left(\frac{c_1 - 3c_2}{5}\right)\sin t - \left(\frac{c_2 + 3c_1}{5}\right)\cos}$$

Ex 2). Solve the simulteneous L.D.E with given conditions.

$$\frac{du}{dx} + v = \sin x$$

$$\frac{dv}{dx} + u = \cos x$$

$D = \frac{d}{dx}$

$$\boxed{\begin{aligned} Du + v &= \sin x \\ Dv + u &= \cos x . \end{aligned}}$$

Given that when $x = 0$ then $u = 1$ & $v = 0$

Soln : Solving for v, by crammer's rule.

$$\begin{vmatrix} D & 1 \\ 1 & D \end{vmatrix} v = \begin{vmatrix} D & \sin x \\ 1 & \cos x \end{vmatrix}$$

$$(D^2 - 1) v = D \cos x - \sin x$$

$$(D^2 - 1) v = -\sin x - \sin x$$

$$(D^2 - 1) v = -2 \sin x \quad \text{L.D.E. in } v \text{ \& } x$$

To find v_c

Consider A.E. $D^2 - 1 = 0$

$$D = \pm 1$$

$$D = 1, -1$$

$$\boxed{v_c = c_1 e^{x} + c_2 e^{-x}}$$

To find v_p

$$v_p = \frac{1}{\phi(D)} f(x)$$

$$v_p = \frac{1}{D^2 - 1} (-2 \sin x)$$

$a = 1$

$$v_p = (-2) \frac{1}{D^2 - 1} \sin x$$

Replace $D^2 \rightarrow -1$

$$v_p = \cancel{-2} \ \cancel{\frac{1}{(-1)^2 - 1}} \cdot \cancel{\sin x}.$$

$$v_p = (-2) \cdot \frac{1}{-1 - 1} \sin x$$

$$v_p = (\cancel{-2}) \cdot \frac{1}{(\cancel{-2})} \sin x$$

$$\boxed{v_p = \sin x .}$$

$$v = v_c + v_p$$

$$\boxed{v = c_1 e^{x} + c_2 e^{-x} + \sin x}$$

for finding value of u. Consider eqn where coeff. of u is simple.

i.e

$$Dv + u = \cos x \,.$$

$$u = \cos x - Dv$$

$$u = \cos x - D\left(c_1 e^{x} + c_2 e^{-x} + \sin x\right)$$

$$u = \cos x - \left[c_1 e^{x} + c_2\left(-e^{-x}\right) + \cos x\right]$$

$$u = \cancel{\cos x} - c_1 e^{x} + c_2 e^{-x} - \cancel{\cos x}$$

$$\boxed{u = -c_1 e^{x} + c_2 e^{-x}}$$

At $x = 0$ then $u = 1$, & $v = 0$

$$1 = -c_1 + c_2 \Rightarrow -\cancel{c_1} + c_2 = 1$$

$$0 = c_1 + c_2 + 0 \Rightarrow + \quad \cancel{c_1} + c_2 = 0$$

$$2c_2 = 1$$

$$\boxed{c_2 = 1/2}$$

$$-c_1 + c_2 = 1$$

$$c_1 = c_2 - 1$$

$$c_1 = \frac{1}{2} - 1$$

$$\boxed{c_1 = -1/2}$$

$$u = -\left(-\frac{1}{2}\right)e^{x} + \frac{1}{2}e^{-x} \,.$$

$$u = \frac{e^{x}}{2} + \frac{e^{-x}}{2} \Rightarrow u = \frac{e^{x} + e^{-x}}{2}$$

$$\boxed{u = \cosh x} \quad \times$$

$$v = -\frac{1}{2}e^{x} + \frac{1}{2}e^{-x} + \sin x \,.$$

$$v = -\left(\frac{e^{x} - e^{-x}}{2}\right) + \sin x$$

$$\times \; \boxed{v = -\sinh x + \sin x}$$

Ex 3) Solve the system of equations

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t$$

Soln : $D = \frac{d}{dt}$

$$Dx + 4x + 3y = t$$

$$Dy + 2x + 5y = e^t$$

$$(D+4)x + 3y = t$$

$$2x + (D+5)y = e^t$$

Solving for y, by cramer's rule.

$$\begin{vmatrix} D+4 & 3 \\ 2 & D+5 \end{vmatrix} y = \begin{vmatrix} D+4 & t \\ 2 & e^t \end{vmatrix}$$

$$[(D+4)(D+5) - 6]y = (D+4)e^t - 2t.$$

$$(D^2 + 5D + 4D + 20 - 6)y = e^t + 4e^t - 2t$$

$$(D^2 + 9D + 14)y = 5e^t - 2t$$

which is L.D.E in y & t

To find y_c

Consider A.E.

$$(D^2 + 9D + 14) = 0$$

$$D = -2, -7$$

$$y_c = c_1 e^{-2t} + c_2 e^{-7t}$$

$$\boxed{y_c = c_1 e^{-2t} + c_2 e^{-7t}}$$

To find y_p.

$$y_p = \frac{1}{\phi(D)} f(t)$$

$$y_p = \frac{1}{D^2+9D+14} \cdot \left[5e^t - 2t\right]$$

$$y_p = \frac{1}{D^2+9D+14} \cdot 5e^t - \frac{1}{D^2+9D+14} 2t$$

$$y_p = 5 \cdot \frac{1}{D^2+9D+14} e^t - 2 \cdot \frac{1}{D^2+9D+14} t.$$

Replace $D \to 1$

$$y_p = 5 \cdot \frac{1}{1+9+14} e^t - 2 \cdot \frac{1}{14\left[1 + \left(\frac{D^2+9D}{14}\right)\right]} t$$

$$y_p = 5 \cdot \frac{1}{24} e^t - \frac{2}{14} \left[1 + \left(\frac{D^2+9D}{14}\right)\right]^{-1} t.$$

$$y_p = \frac{5}{24} e^t - \frac{1}{7} \left[1 + \left(\frac{D^2+9D}{14}\right)\right]^{-1} t$$

use $(1+z)^{-1} = 1 - z + z^2 - z^3 + \dots$

$D(t) = 1.$

$D^2(t) = 0.$

$$y_p = \frac{5}{24} e^t - \frac{1}{7} \left[1 - \left(\frac{D^2+9D}{14}\right) + \dots\right] t.$$

$$y_p = \frac{5}{24} e^t - \frac{1}{7} \left[t - \frac{1}{14}(D^2 t + 9Dt)\right]$$

$$y_p = \frac{5}{24} e^t - \frac{1}{7} \left[t - \frac{1}{14}(0 + 9(1))\right]$$

$$\boxed{y_p = \frac{5}{24} e^t - \frac{t}{7} + \frac{9}{98}}$$

$$y = y_c + y_p.$$

$$\boxed{y = c_1 e^{-2t} + c_2 e^{-7t} + \frac{5}{24} e^t - \frac{t}{7} + \frac{9}{98}}$$

To find x. consider sutaible eq^n for x.

$$2x + (D+5)y = e^t$$

$$2x = e^t - (D+5)y$$

$$2x = e^t - (D+5)\left(c_1 e^{-2t} + c_2 e^{-7t} + \frac{5}{24}e^t - \frac{t}{7} + \frac{9}{98}\right).$$

$$2x = e^t - \left[c_1(-2e^{-2t}) + c_2(-7e^{-7t}) + \frac{5}{24}e^t - \frac{1}{7} + 0\right.$$
$$\left. + 5c_1 e^{-2t} + 5c_2 e^{-7t} + \frac{25}{24}e^t - \frac{5t}{7} + \frac{45}{98}\right]$$

$\Rightarrow$

$$2x = e^t + 2c_1 e^{-2t} + 7c_2 e^{-7t} - \frac{5}{24}e^t + \frac{1}{7}$$
$$- 5c_1 e^{-2t} - 5c_2 e^{-7t} - \frac{25}{24}e^t + \frac{5t}{7} - \frac{45}{98}$$

$$2x = \left(1 - \frac{5}{24} - \frac{25}{24}\right)e^t + (2c_1 - 5c_1)e^{-2t}$$
$$+ (7c_2 - 5c_2)e^{-7t} + \frac{5t}{7} + \left(\frac{1}{7} - \frac{45}{98}\right)$$

$$2x = \left(\frac{-1}{4}\right)e^t + (-3c_1)e^{-2t} + (2c_2)e^{-7t}$$
$$+ \frac{5t}{7} + \left(\frac{14-45}{98}\right)$$

$$2x = -\frac{1}{4}e^t - 3c_1 e^{-2t} + 2c_2 e^{-7t} + \frac{5t}{7} + \frac{31}{98}$$

$$\boxed{x = \frac{-1}{8}e^t - \frac{3}{2}c_1 e^{-2t} + c_2 e^{-7t} + \frac{5}{14}t + \frac{31}{196}}$$

$\therefore$

$$x = \frac{-1}{8}e^t - \frac{3}{2}c_1 e^{-2t} + c_2 e^{-7t} + \frac{5}{14}t + \frac{31}{196} \quad \&$$

$$y = c_1 e^{-2t} + c_2 e^{-7t} + \frac{5}{24}e^t - \frac{t}{7} + \frac{9}{98}$$

4) Solve simulteneously.

$$\frac{dx}{dt} - 3x - 6y = t^2$$

$$\frac{dy}{dt} + \frac{dx}{dt} - 3y = e^t.$$

$\Rightarrow$ $D = \frac{d}{dt}$

$$Dx - 3x - 6y = t^2$$

$$Dy + Dx - 3y = e^t$$

$$\left.\begin{matrix}(D-3)x - 6y = t^2 \\ Dx + (D-3)y = e^t\end{matrix}\right\}.$$

Solving for y, by cramer's rule.

$$\begin{vmatrix} D-3 & -6 \\ D & D-3 \end{vmatrix} y = \begin{vmatrix} D-3 & t^2 \\ D & e^t \end{vmatrix}$$

$$[(D-3)(D-3) + 6D]\, y = (D-3)e^t - Dt^2$$

$$[D^2 - \cancel{3D} - \cancel{3D} + 9 + \cancel{6D}]\, y = \underline{e^t - 3e^t} - (2t).$$

$$(D^2 + 9)y = -2e^t - 2t$$

which is L.D.E in y & t.

To find yp

consider A.E.

$$(D^2+9) = 0 \qquad D^2 = -9$$

$$D = \pm 3i \qquad D = \pm\sqrt{-9}$$

$$D = 0 \pm 3i$$

$$y_c = e^{0t}(c_1 \cos 3t + c_2 \sin 3t)$$

$$\boxed{y_c = c_1 \cos 3t + c_2 \sin 3t}$$

To find yp.

$$y_p = \frac{1}{\phi(D)} f(t)$$

$$y_p = \frac{1}{D^2+9}\left(-2e^t - 2t\right).$$

$$y_p = \frac{1}{D^2+9}\left(-2e^t\right) + \frac{1}{D^2+9}\left(-2t\right).$$

$$y_p = -2 \cdot \frac{1}{D^2+9} e^t + (-2)\frac{1}{D^2+9} t.$$

Replace $D \to 1$

$$y_p = -2 \cdot \frac{1}{1+9} e^t - 2 \cdot \frac{1}{9\left[1+\frac{D^2}{9}\right]} t$$

$$y_p = -\frac{2}{10} e^t - \frac{2}{9}\left[1+\frac{D^2}{9}\right]^{-1} t$$

$D(t) = 1$

$D^2(t) = 0$

$$y_p = \frac{-2}{10} e^t - \frac{2}{9}\left[1 - \frac{D^2}{9} + \dots\right] t$$

$$y_p = \frac{-1}{5} e^t - \frac{2}{9}\left[t - \frac{D^2}{9} t\right]$$

$$\boxed{y_p = -\frac{e^t}{5} - \frac{2t}{9}}$$

$$y = y_c + y_p.$$

$$\boxed{y = c_1 \cos 3t + c_2 \sin 3t - \frac{e^t}{5} - \frac{2t}{9}}$$

To find x, Consider a eqn when coeff. of x is simple.

$$Dx + (D-3)y = e^t$$

$$Dx = e^t - (D-3)y$$

$$Dx = e^t - (D-3)\left(c_1 \cos 3t + c_2 \sin 3t - \frac{e^t}{5} - \frac{2t}{9}\right)$$

$$Dx = e^t - \Big[-3c_1 \sin 3t + 3c_2 \cos 3t - \frac{e^t}{5} - \frac{2}{9}$$
$$- 3c_1 \cos 3t - 3c_2 \sin 3t + 3e^t/5 + \frac{6t}{9}$$

$$Dx = e^t + 3c_1 \sin 3t - 3c_2 \cos 3t + \frac{e^t}{5} + \frac{2}{9}$$
$$+ 3c_1 \cos 3t + 3c_2 \sin 3t - \frac{3e^t}{5} - \frac{6t}{9}$$

$$Dx = (3c_1 + 3c_2)\sin 3t + (3c_1 - 3c_2)\cos 3t$$
$$\left(1 + \frac{1}{5} - \frac{3}{5}\right)e^t - \frac{6t}{9} + \frac{2}{9}.$$

$$Dx = (3c_1 + 3c_2)\sin 3t + (3c_1 - 3c_2)\cos 3t + \frac{3}{5}e^t$$
$$- \frac{2t}{3} + \frac{2}{9}$$

$$x = \frac{1}{D}\left[3(c_1 + c_2)\sin 3t + 3(c_1 - c_2)\cos 3t + \frac{3}{5}e^t - \frac{2t}{3} + \frac{2}{9} \right]$$

$$x = \cancel{3}(c_1 + c_2)\left(-\frac{\cos 3t}{\cancel{3}}\right) + \cancel{3}(c_1 - c_2)\left(\frac{\sin 3t}{\cancel{3}}\right) + \frac{3}{5}e^t$$
$$- \frac{\cancel{2}}{3}\left(\frac{t^2}{\cancel{2}}\right) + \frac{2}{9}t$$

$$x = -(c_1 + c_2)\cos 3t + (c_1 - c_2)\sin 3t + \frac{3}{5}e^t$$
$$- \frac{t^2}{3} + \frac{2}{9}t$$

$\therefore$ Sol^n is.

$$x = -(c_1 + c_2)\cos 3t + (c_1 - c_2)\sin 3t + \frac{3}{5}e^t - \frac{t^2}{3} + \frac{2}{9}t$$

&

$$y = c_1 \cos 3t + c_2 \sin 3t - \frac{e^t}{5} - \frac{2t}{9}$$