12. UNBALANCED DELTA CONNECTED LOAD

When the load impedance in the three phases are not equal in magnitude or phase or both, the load is said to be *unbalanced.* If three unequal loads are connected to form a delta and connected across a 3-phase supply the currents in the three loads will not be equal in magnitude and/or phase. The three-phase currents and the line currents will also be unbalanced.

Example 40. *A 440 V, 50 Hz, 3-phase supply has delta-connected load having 50 Ω between R and y,* 159 *mH between Y and Band* 15.9 µF *between B and R. Find;*

(i) The line current for the sequence RYB.

(ii) The value of star-connected balanced resistors for the same power.

Solution. Phase voltage (E_{ph}) = line voltage (E_L) = 440 V
The potential difference across three-phases are given by
$E_{RY} = 440(1 + j0) = 4400°$ V

$E_{YB} = 440(-0.5 - j0.866)$ or 440 – 120° V
$E_{BR} = 440(-0.5 + j0.866)$ or 440 120° V

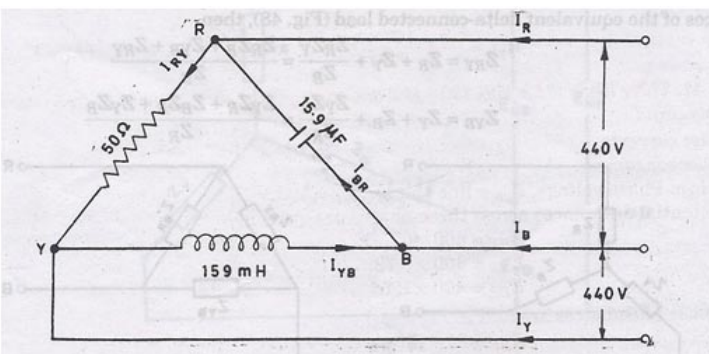


Fig. 50

Impedance , $Z_{RY} = (50 + j0) = 50\ 0°\ \Omega$

Impedance , $Z_{YB} = (0 + j2\pi fL) = (0 + j2\pi \times 50 \times 159 \times 10^{-3})$
$= (0 + j50)$ or $50\ 90°\ \Omega$

$$Impedance \quad Z_{BR} = \left(0 - \frac{j}{2\pi fC}\right)$$

$$= \left(0 - \frac{1}{2\pi \times 50 \times 15.9 \times 10^{-6}}\right) = (0 - j200) = 200\angle -90°$$

(i) ***Phase currents :***

$$Phase\ current, \quad I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{440\angle 0°}{50\angle 0°} = 8.1(1 + j0)A$$

$$Phase\ current, \quad I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{440\angle -120°}{50\angle 0°} = 8.8\angle -120°$$

$$Phase\ current, \quad I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{440\angle 120°}{200\angle -90°} = 2.2\angle 210° = 2.2(-0.866 - j0.5)A.$$

Line currents:

Line current $I_R = I_{RY} - I_{BR} = 8.8(1 + j0) - 2.2(-0.866 - j0.5)$

$= 8.8 + j0 + 1.905 + j1.1 = 10.7 + j1.1 =$ **10.75 ∠5.8°A. (Ans.)**

Line current $I_Y = I_{YB} - I_{RY} = 8.8(-0.866 + j0.5) - 8.8(1 + j0)$

$= -7.62 + j4.4 - 8.8 - j0$

$= -16.42 + j4.4 =$ **16.99 ∠*164.9°* A. (Ans.)**

Line current $I_B = I_{BR} = I_{YB} = 2.2(-0.866 - j0.5) - 8.8(-0.866 + j0.5)$

$= -1.905 - j1.1 + 7.62 - j4.4 = 5.715 - j5.5$

= 7.93 ∠- 43.9° A. (Ans.)

(ii) **Power supplied:**

$P = I_{RY}^2 \times R = 8.8^2 \times 50 = 3872$ W

Let the resistance per phase be R_{ph} which when connected in star across 440 V, 3-phase supply take power of 3872 W.

$$Line\ current, \quad I_L = I_{ph} = \frac{E_L/\sqrt{3}}{R_{ph}} = \frac{440/\sqrt{3}}{R_{ph}}$$

$$Power\ supplied, \quad P = \sqrt{3}E_L I_L$$

$$\therefore \qquad 3872 = \sqrt{3} \times 440 \times \frac{440/\sqrt{3}}{R_{ph}}$$

$$\therefore \qquad \boldsymbol{R_{ph}} = \frac{\sqrt{3} \times 440 \times 440/\sqrt{3}}{3872} = \frac{440 \times 440}{3872} = 50\ \Omega \quad (Ans.)$$

Example 41. *A 440 V, 50 Hz, 3-phase supply has delta-connected load having 50 Ω between R and y, 159 mH between Y and Band 15.9 μF between B and R. Find;*

(i) The line current for the sequence RYB.

(ii) The value of star-connected balanced resistors for the same power.

Solution.

Phase voltage(E_{ph}) = line voltage, (E_L) = 400 V
The potential differences across three-phases are given by:

$E_{RY} = 400\ 0°$
$E_{YB} = 400 – 120°$
$E_{BR} = 400\ 120°$

The phase impedances are:

$Z_{RY} = 31 + j59 = 66.6\ 62.3°$
$Z_{YB} = 30 – j40 = 50 – 53.1°$
$Z_{BR} = 80 + j60 = 100\ 36.9°$

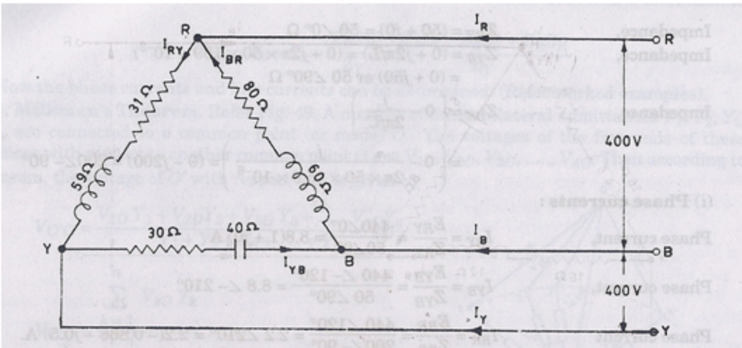


Fig. 51

Phase currents:

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{400\angle 0^0}{66.6\angle 62.3^0} = \mathbf{6\angle -62.3^0}\ \boldsymbol{A.}\quad \boldsymbol{Ans.}$$

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{400\angle -120^0}{50\angle -53.1^0} = \mathbf{8\angle -66.9^0}\ \boldsymbol{A.}\quad \boldsymbol{Ans.}$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{400\angle 120^0}{100\angle 36.9^0} = 4\angle \mathbf{83.1}\ \boldsymbol{A.}\quad \boldsymbol{Ans.}$$

Line currents:

$\mathbf{I_R} = I_{RY} - I_{BR} = 6\angle -62.3° - 4\angle 83.1°$

$= 6(0.465 - j0.885) - 4(0.12 + j0.993)$

$= 2.79 - j5.31 - 0.48 - j3.97 = 2.31 - j\,9.28 =$ **9.56 ∠- 76° A. (Ans.)**

$\mathbf{I_Y} = I_{YB} - I_{RY} = 8\angle -66.9° - 6\angle -62.3°$

$= 8(0.392 - j0.92) - 6(0.465 - j0.885)$

$= (3.136 - j7.36) - (2.79 - j5.31)$

$= 0.346 - j2.05 =$ **2.07 ∠- 80.4° A. (Ans.)**

$\mathbf{I_B} = I_{BR} - I_{YB} = 4\angle 83.1° - 8\angle -66.9°$

$= 4(0.12 + j0.993) - 8(0.392 - j0.92)$

$= 0.48 + j3.97 - 3.136 + j7.36$

$= -2.656 + j11.33 =$ **11.637 ∠103.2° A. (Ans.)**

Total power is the sum of power in different phases

$P = E_{RY} \times I_{RY} + E_{YB} \times I_{YB} + E_{BR} \times I_{BR}$

$= 400\angle 0° \times 6\angle -62.3° + 400\angle -120° \times 8\angle -66.9°$

$+ 400\angle 120° \times 4\angle 83.1°$

$= 400 \times 6\cos(-62.3°) + 400 \times 8\cos(53.1°) + 400 \times 4\cos(-36.9°)$

$= 1115.6 + 1921.3 + 1279.5 = 4316.4$ W

$OR\quad P = I_{RY}^2 R_{RY} + I_{YB}^2 \times R_{YB} + I_{BR}^2 \times R_{BR}$

$= 6^2 \times 31 + 8^2 \times 30 + 42 \times 80 = 1116 + 1920 + 1280 = 4316$ W

Hence, **total power = 4316.4 W. (Ans.)**

Example 42. *For the unbalanced delta-connected load of Fig 52, find the phase currents, line currents and total power consumed by the load, when phase sequence is:*

(i) RYB
(ii) RBY

(AMIE Advanced Electrical M/c Winter, 1998)

Solution.

(i) **Phase sequence RYB :**

The potential differences across three phases are:

$E_{RY} = 200\ 0°$
$E_{YB} = 200\text{-}120°$
$E_{BR} = 200\ 120°$

Impedance, $Z_{RY} = 12 + j16 = 20\ 53.13°$
Impedance, $Z_{YB} = 16 + j12 = 20\ 36.8°$
Impedance, $Z_{BR} = 8 – j6 = 10 – 36.8°$

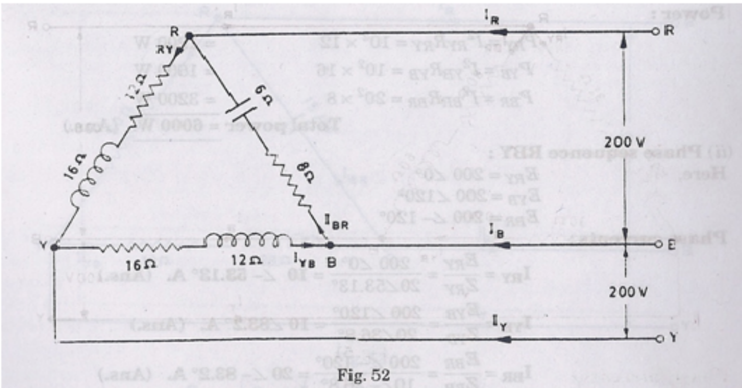


Fig. 52

Phase current, $I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{200\angle 0°}{20\angle 53.13°} = \mathbf{10\angle -53.13°\ A.}$ (Ans.)

Similary, $I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{200\angle -120°}{20\angle 36.8°} = \mathbf{10\angle -156.8°\ A.}$ (Ans.)

$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{200\angle -120°}{10\angle -36.8°} = \mathbf{20\angle 156.8°\ A.}$ (Ans.)

Line currents:

Line current, $\mathbf{I_R} = I_{RY} - I_{BR} = 10\angle -53.13° - 12\angle 156.8°$

$= 10(0.60 - j0.8) - 20(-0.72 + j0.394) = 6.0 - j8 + 18.4 - j7.88$

$= 24.4 - j15.88 =$ **29.11 ∠- 33° A. (Ans.)**

Similarly, $\mathbf{I_Y} = I_{YB} - I_{RY} = 10\angle -156.8° - 10\angle -53.13°$

$= 10(-0.919 - j0.394) - 10(0.6 - j0.8) = -9.19 - j3.94 - 6 + j8$

$= 15.19 + j1.06 =$ **15.72 ∠*165*° A. (Ans.)**

$\mathbf{I_B} = I_{BR} - I_{YB} = 20\angle 156.8° - 10\angle -156.8$

$= 20(0.919 - j0.394) - 10(-0.919 - j0.394)$

$= -18.38 + j7.88 + 9.19 + j3.94 = -9.19 + j11.82 = 14.97\angle 127.9°$

[Check $\Sigma I = (24.4 - j15.88) + (-15.19 + j4.06) + (-9.19 + j11.82) = 0$]

Power:

$P_{RY} = I_{RY}^2 R_{RY} = 10^2 \times 12 \qquad = 1200\ W$

$P_{YB} = I_{YB}^2 R_{YB} = 10^2 \times 16 \qquad = 1600\ W$

$P_{BR} = I_{BR}^2 R_{BR} = 20^2 \times 8 \qquad = 3200\ W$

$\textbf{\textit{Total power}} \quad \mathbf{= 6000\ W}$ (*Ans.*)

Phase sequence RBY:

Here $E_{RY} = 200$
$E_{YB} = 200$

$E_{BR} = 200$

Phase currents:

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{200\angle 0°}{20\angle 53.13°} = \mathbf{10\angle -53.13° \text{ A.}}\ \text{(Ans.)}$$

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{200\angle 120°}{20\angle 36.8°} = \mathbf{10\angle 83.2° \text{ A.}}\ \text{(Ans.)}$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{200\angle -120°}{10\angle -36.8°} = \mathbf{20\angle -83.2° \text{ A.}}\ \text{(Ans.)}$$

Line currents:

$I_R = I_{RY} - I_{BR} = 10\angle -53.13° - 20\angle -83.2°$

$= 10(0.6 - j0.8) - 20(0.118 - j0.993) = 6 - j8 - 2.36 + j19.86$

$= 3.64 + j11.86 =$ **12.4 ∠72.9° A. (Ans.)**

$I_Y = I_{YB} - I_{RY} = 10\angle 83.2° - 10\angle -53.13°$

$= 10(0.118 + j\,0.993) - 10(0.6 - j0.8)$

$= 1.18 + j9.93 - 6 + j8 = -4.82 + j17.93 =$ **18.56 ∠105° A. (Ans.)**

$I_B = I_{BR} - I_{YB} = 20\angle -83.2° - 10\angle 83.2°$

$= 20(0.118 - j0.993) - 10(0.118 + j0.993)$

$= 2.36 - j19.86 - 1.18 - j9.93 = 1.18 - j29.79 =$ **29.81 ∠- 87.7°. (Ans.)**

$P_{RY} = 10^2 \times 12 = 1200$ W

$P_{YB} = 10^2 \times 16 = 1600$ W

$P_{BR} = 20^2 \times 8 = 3200$ W

Total power = 6000 W. (Ans.)

Example 43. *A 3-phase, 400 V, 50 Hz system has the following load connected in delta:*
Between R and Y lines: a non-reactive resistor of 50 Ω
Between Y and B lines: a coil having a resistance of 10 Ω and reactance of 30 Ω
Between B and R lines: a loss free capacitor of 15 μ F.
Calculate: (i) The phase currents (ii) The line currents.
Assume phase sequence to be RYB.

Solution.

$E_{ph} = E_L = 400$ V

The potential difference across the three phases are:

$E_{RY} = 400\ 0°$ V
$E_{YB} = 400 - 120°$ V
$E_{BR} = 400\ 120°$ V

$E_{YB} = 400 \angle -120°$ V

$E_{BR} = 400 \angle 120°$ V

Impedance

$Z_{RY} = 50 + j0 = 50 \angle 0° \Omega$

$Z_{YB} = 10 + j30 = 31.62 \angle 71.56° \Omega$

$$Z_{BR} = \left(0 - \frac{j}{2\pi f C}\right) = \left(0 - \frac{j}{2\pi \times 50 \times 15 \times 10^{-6}}\right)$$

Phase currents:

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{400\angle 0°}{50\angle 0°} = \mathbf{8\angle 0° \ A.} \ \ \text{(Ans.)}$$

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{400\angle -120°}{31.62\angle 71.56°} = \mathbf{12.65\angle -191.56° \ A.} \ \ \text{(Ans.)}$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{400\angle 120°}{212\angle -90°} = \mathbf{1.886\angle 210° \ A.} \ \ \text{(Ans.)}$$

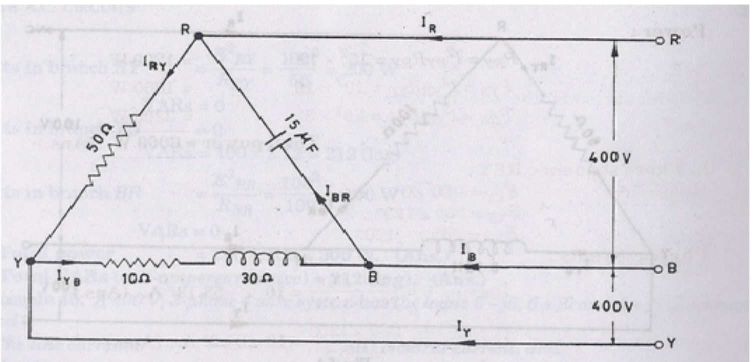


Fig. 53

Line Current

$\mathbf{I_R} = I_{RY} - I_{BR} = 8\ 0° - 1.886\ 210°$

$= 8 - 1.886\ (-0.866 - j0.5) = 8 + 1.633 + j0.943$
$= 9.633 + j0.943 = \mathbf{9.68\ 5.6° \ A. \ (Ans.)}$

$\mathbf{I_Y} = I_{YB} - I_{RY} = 12.65 - 191.56° - 8\ 0°$

$= 12.65(-0.979 + j0.2) - 8 = -12.38 + j2.53 - 8$
$= -20.38 + j2.53 = \mathbf{20.53\ 7.07° \ A. \ (Ans.)}$

$I_B = I_{BR} - I_{YB} = 1.886\ 210° - 12.65 - 191.56°$
$= 1.886\ (-0.866 - j0.5) - 12.65(-0.979 + j0.2)$
$= -1.633 - j\ 0.943 + 12.38 - j2.53$

$= 10.747 - j3.473 = \mathbf{11.29\ 17.4°. \ (Ans.)}$

Example 45. *A* 400 *V, 3-phase* 4 *wire system has the loads* 6 – *j8,* 6 + j0 *and* 4 + j3 Ω *connected* in *star. Find:*
(i) *The line currents,*

(ii) *Neutral current, and*

(iii) *Total power.*

Solution. The connections are shown in Fig. 56.

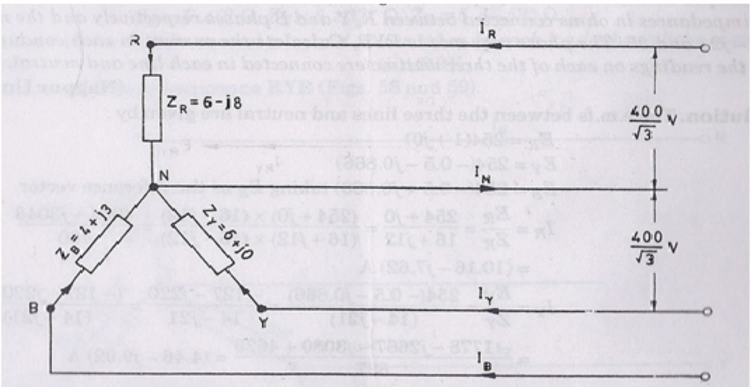


Fig. 56

The potential differences across the three phases are:

$$E_{RN} = \frac{400}{\sqrt{3}}\angle 0^0 = 230.9\angle 0^0\ V$$

$$E_{YN} = 230.9\angle -120^0\ V$$

$$E_{BN} = 230.9\angle 120^0\ V$$

Impedances:

$$Z_R = 6 - j8 = 010\angle -53.13^0$$

$$Z_Y = 6 + j0 = 6\angle 0^0$$

$$Z_B = 4 + j3 = 5\angle 36.9^0$$

(i) ***Line currents:***

$$I_R = I_{RN} = \frac{E_R}{Z_R} = \frac{230.9\angle 0^0}{10\angle -53.13^0} = \mathbf{23.09\angle 53.13^0\ A.}\ (\mathbf{Ans}).$$

$$I_Y = I_{YN} = \frac{E_Y}{Z_Y} = \frac{230.9\angle -120^0}{6\angle 0^0} = \mathbf{38.48\angle -120^0\ A.}\ (\mathbf{Ans}).$$

$$I_B = I_{BN} = \frac{E_B}{Z_B} = \frac{230.9\angle 120^0}{5\angle 36.9^0} = \mathbf{46.18\angle 83.1^0\ A.}\ (\mathbf{Ans}).$$

(ii) ***The current in the neutral wire:***

$\mathbf{I_N} = -(I_R + I_Y + I_B)$

= – [23.09 53.13° + 38.48-120°+ 46.18 83.1°

= – [23.09(0.6 + *j0.8)* + 36.18 (- 0.5 -j0.866) – 46.18 (0.12 + j0.99)] = – [13.85 + *j*18.47 – 18.24 – *j*31.59 + 5.54 +*j*45.72]

= – (1.15 + *j*32.6*)* = -1.15. -*j*32.6 = **32.62 -92° A. (Ans.)**

Power:

Power taken by phase *R*,

$$P_B = I_R^2 \times R_R = 23.09^2 \times 6 = 3199\ W\ (app.).$$

Similarly, $P_Y = I_Y^2 \times R_Y = 38.48^2 \times 6 = 8884\ W\ (app.).$

and $P_B = I_B^2 \times R_B = 46.18^2 \times 4 = 8530\ W\ (app.).$

Total power taken $P_R + P_Y + P_B = 3199 + 8884 + 8530 = \mathbf{20613\ W\ (Ans).}$

Example 46. *A 440/254 volt,* 3 *phase, 4-core supplies an unbalanced load represented by the following impedance in ohms connected between R, Y and B phases respectively and the neutral 16 + j12;* 14 – *j21 and* 25. *The phase sequence is RYE. Calculate the current in each conductor of the cable and the readings on each of the three wattmeters connected in each line and neutral.*

Solution. The e.m.fs between the three lines and neutral are given by :

$E_R = 254(1 + j0)$

$E_Y = 254(- 0.5 – j0.866)$

$E_B = 254(- 0.5 + j0.866)$ taking ER as the reference vector

$$I_R = \frac{E_R}{Z_R} = \frac{254 + j0}{16 + j12} = \frac{(254 + j0) \times (16 - j12)}{(254 + j12) \times (16 - j12)} = \frac{4064 - j3048}{400}$$

$$= (10.16 - j7.62)A$$

$$I_Y = \frac{E_Y}{Z_Y} = \frac{254(-0.5 - j0.866)}{(14 - j21)} = \frac{-127 - j220}{14 - j21} = \frac{(-127 - j220) \times (14 + j21)}{(14 - j21)(14 + j21)}$$

$$= \frac{-1778 - j2667 - j3080 + 4620}{637} = (4.46 - j9.02)A$$

$$I_B = \frac{E_B}{Z_B} = \frac{254(-0.5 + j0.866)}{25} = \frac{-127 + j220}{25} = (-5.08 + j8.8)A$$

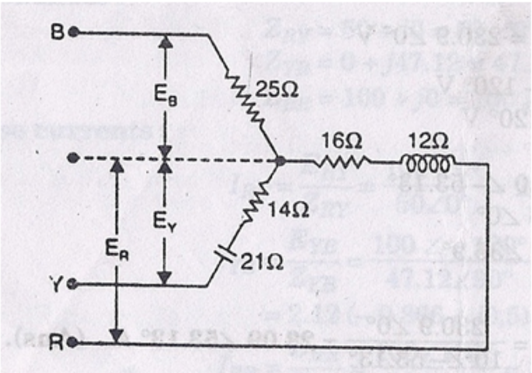


Fig.57 (a) Circuit Diagram

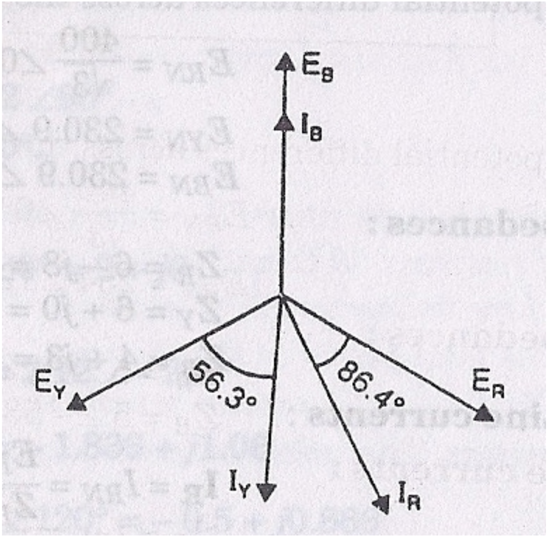


Fig.57 (b) Phasor Diagram

Now current in neutral wire,

$I_N = -(I_R + I_Y + I_B)$

$= -[(10.16 - j7.62) + (4.46 - j9.02) + (-5.08 + j8.8)]$

$= -9.54 + j7.84$

Evaluating these currents, we get

$I_R = \sqrt{10.16^2 + (-7.62)^2} = \mathbf{12.7\ A.}\ (\boldsymbol{Ans.})$

$I_Y = \sqrt{4.46^2 + (-9.02)^2} = \mathbf{10.06\ A.}\ (\boldsymbol{Ans.})$

$I_B = \sqrt{(-5.08)^2 + (8.8)^2} = \mathbf{10.16\ A.}\ (\boldsymbol{Ans.})$

$I_N = \sqrt{(-9.54)^2 + (7.84)^2} = \mathbf{12.35\ A.}\ (\boldsymbol{Ans.})$

$Power\ in\ R-phase \quad = I_R^2 \times R_R = (-12.7)^2 \times 16 = \mathbf{2580.6\ W.}\ (\boldsymbol{Ans.})$

$Power\ in\ Y-phase \quad = I_Y^2 \times R_Y = (10.06)^2 \times 14 = \mathbf{1416.8\ W.}\ (\boldsymbol{Ans.})$

$Power\ in\ B-phase \quad = I_B^2 \times R_B = (10.16)^2 \times 25 = \mathbf{2580.6\ W.}\ (\boldsymbol{Ans.})$

Example 47. *A 3-phase, 4-wire system having a 440 V line to line has the following loads connected between the respective lines and neutral:*
Z_R = 5 0° Ω, Z_Y = 5 37° Ω, Z_B = 5 -53° Ω.
Calculate the current in neutral wire and power taken by each load when phase sequence is:
(i) RYB
(ii) RBY.

Solution. *(i)* **Phase sequence RYB** (Figs. 58 and 59).

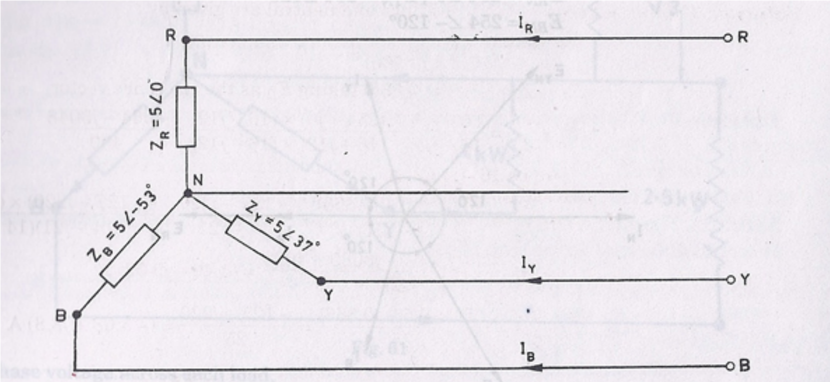


Fig. 58

The potential differences across the three phases are:

$$E_{RN}=\frac{440}{\sqrt{3}}\angle 0^0=254\angle 0^0, E_{YN}=254\angle -120^0$$

$$E_{BN}=254\angle 120^0$$

Impedances:

$$Z_R=5\angle 0^0,\ \ Z_Y=5\angle 37^0$$

$$Z_B=5\angle -53^0$$

Line currents:

$$I_R=I_{RN}=\frac{E_{RN}}{Z_R}=\frac{254\angle 0^0}{5\angle 0^0}=50.8\angle 0^0$$

$$I_Y=I_{YN}=\frac{E_{YN}}{Z_Y}=\frac{254\angle -120^0}{5\angle 37^0}=50.8\angle -157^0$$

$$I_B=I_{BN}=\frac{E_{BN}}{Z_B}=\frac{254\angle 120^0}{5\angle -53^0}=50.8\angle 173^0$$

Current in the neutral:

$I_N = -(I_R + I_Y + I_B)$

$= -(50.8\ 0° + 50.8\ -157° + 50.8\ 173°)$
$= -[50.8 + 50.8\ (-0.92 - j0.39) + 50.8(-0.99 + j0.122)]$
$= -[50.8 - 46.74 - j19.81 - 50.3 + j6.2]\}$

$= -(-46.24 - j13.61) = 46.24 + j13.61$

= 48.2 16.4°. (Ans.)

Now $R_R = 5\ \Omega$, $R_Y = 5 \cos 37° = 4\ \Omega$,

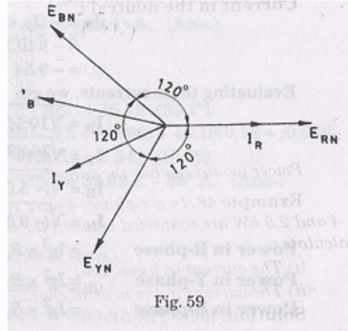


Fig. 59

R_B = 5 cos (- 53°) = 3 Ω
Power taken by phase *R*,

$P_R = 50.8^2 \times 5 =$ **12903 W. (Ans.)**

$P_Y = 50.8^2 \times 4 =$ **10322 W. (Ans.)**

$P_B = 50.8^2 \times 3 =$ **7742 W. (Ans.)**

Phase Sequence RBY (Fig. 60):

E_{RN} = 254 0°, E_{YN} = 254 120°
E_{BN} = 254 -120°

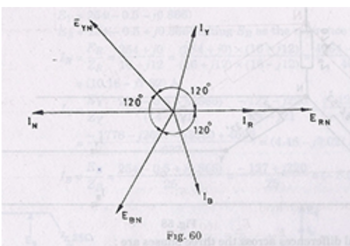


Fig. 60

Line currents:

$$I_R = I_{RN} = \frac{E_{RN}}{Z_R} = \frac{254\angle 0^0}{5\angle 0^0} = 50.8\angle 0^0$$

$$I_Y = I_{YN} = \frac{E_{YN}}{Z_Y} = \frac{254\angle 120^0}{5\angle 37^0} = 50.8\angle 83^0$$

$$I_B = I_{BN} = \frac{E_{BN}}{Z_B} = \frac{254\angle -120^0}{5\angle -53^0} = 50.8\angle -67^0$$

Current in the neutral:

$I_N = -(I_R + I_Y + I_B)$

= – (50.8 0° + 50.8 83° + 50.8 – 67°)

= – [50.8 + 50.8(0.122 + j0.992) + 50.8(0.39 – j0.92)] = – (50.8 + 6.197 + j50.4 + 19.81- j46.74)

= – (76.81 + j3.66) = – 76.81 – j3.66

= 76.89 -177.3° A. (Ans.)

Power would remain the same because magnitude of branch currents is unaltered.

Example 48. *In a 3-phase, 4-wire system, the line voltage is 400 V and non-inductive loads of 5, 4 and 2.5 kW Ware connected between the three lines conductors and the neutral, as shown in Fig. 61.Calculate*
i. The current in lines; and
ii. The current in the neutral conductor.

Solution. Refer Fig. 61.

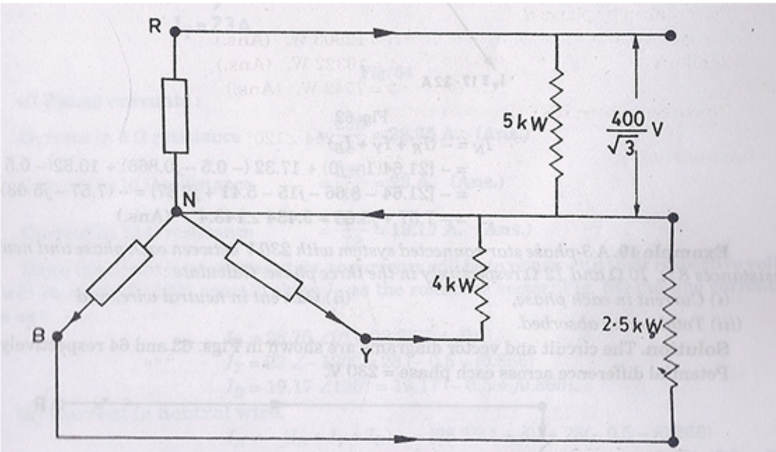


Fig. 61

Phase voltage across each load,

$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\ V$$

(i) Current in phase R,

$$I_R = (line\ current) = \frac{kW \times 1000}{E_{ph}\cos\phi} = \frac{5 \times 1000}{231 \times 1} = \mathbf{21.64\ A.}\ \ (\boldsymbol{Ans.})$$

$$Current\ in\ phase\ Y,\ \ I_Y = \frac{4 \times 1000}{231 \times 1} = \mathbf{17.32\ A.}\ \ (\boldsymbol{Ans.})$$

$$Current\ in\ phase\ B,\ \ I_B = \frac{2.5 \times 1000}{231 \times 1} = \mathbf{10.82\ A.}\ \ (\boldsymbol{Ans.})$$

These currents are mutually 120° out of phase as the circuit is purely resistive. Taking I_R as the reference vector, line currents may be given as

$\mathbf{I_R}$ = 21.64 0° = 21.64 (1 + j0)
$\mathbf{I_Y}$ = 17.32 -120° = 17.32 (- 0.5 – j0.866)
$\mathbf{I_B}$ = 10.82 120° = 10.82 (- 0.5 + j0.866)

(ii) **Current in the neutral wire** (Refer Fig. 62):

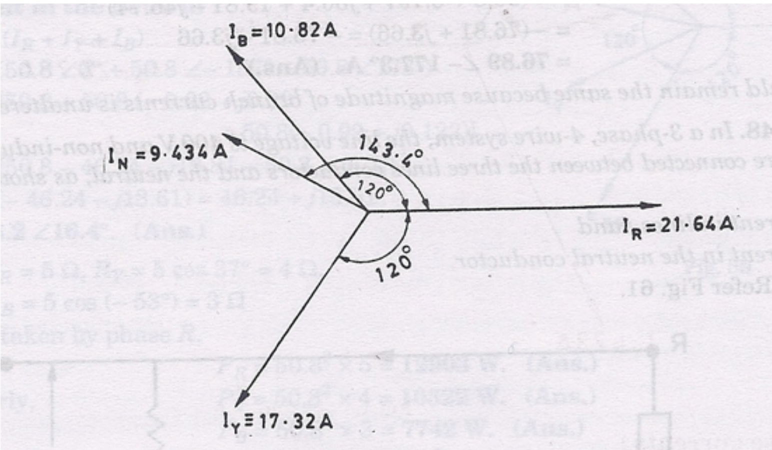


Fig. 62

$\mathbf{I_N}$ =- $(I_R + I_Y + I_B)$

= – [21.64(1- j0) + 17.32 (- 0.5 – j0.866) + 10.82(- 0.5 + j0.866)]

= – [21.64 – 8.66 – j15 – 5.41 + j9.371 = – (7.57 – j5.63)

= -7.57 + j5.63 = **9.434 143.4°. (Ans.)**

Example 49. *A 3-phase star-connected system with 230 V between each phase and neutral has resistances 8 Ω, 10 Ω and 12 Ω respectively in the three phase. Calculate:*

i. *Current in each phase,*
ii. *Current in neutral wire, and*
iii. *Total power absorbed.*

Solution. The circuit and vector diagrams are shown in Figs. 63 and 64 respectively.

Potential difference across each phase = 230 V

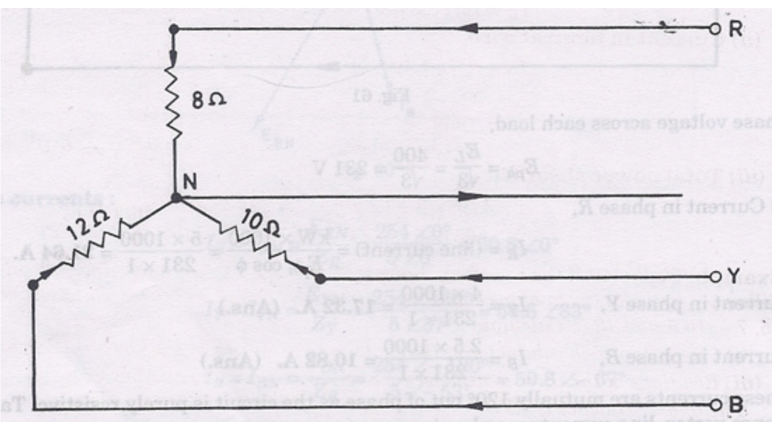


Fig. 63

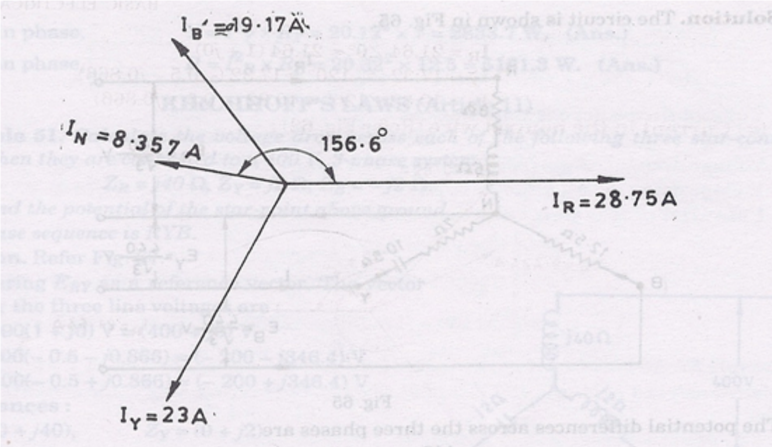


Fig. 64

(i) **Phase currents:**

$$\text{Current in } 8\ \Omega \text{ resistance} = \frac{230}{8} = \mathbf{28.75\ A}.\ \ (\mathbf{Ans.})$$

$$\text{Current in } 10\ \Omega \text{ resistance} = \frac{230}{10} = \mathbf{23\ A}.\ \ (\mathbf{Ans.})$$

$$\text{Current in } 12\ \Omega \text{ resistance} = \frac{230}{12} = \mathbf{19.17\ A}.\ \ (\mathbf{Ans.})$$

Since the circuit is purely resistive, so currents will be in phase with their respective voltages and will be mutually 120^0 apart. Taking I_R as the reference vector (Fig. 64) the line currents are given as:

$I_R = 28.75 \ = 28.75(1 + j0)$

$I_Y = 23\ -120° = 23\ (- 0.5 – j0.866)$

$I_B = 19.17\ 120° = 19.17\ (- 0.5 + j0.866)$.

(ii) **Current in neutral wire,**

$I_N = – (I_R + I_Y + I_B) = – [28.75(1 + j0) + 23(- 0.5 – j0.866) + 19.17(- 0.5 + j0.866)$
$= – [28.75 – 11.5 – j19.92 – 9.58 + j16.6)$

$= – [7.67 – j3.32) = – 7.67 + j3.32 =$ **8.357 156.6° A. (Ans.)**

(iii) **Total power absorbed**

$$= I_R^2 \times 8 + I_Y^2 \times 10 + I_B^2 \times 12 = 28.75^2 \times 8 + 23^2 \times 10 + 19.17^2 \times 12$$

$= 6612.5 + 5290 + 4409.8 =$ **16312.3 W. (Ans.)**

Example 50. *A 440 V (line-to-line), 3-phase, 4-core supplies an unbalanced load represented by the following impedances in ohms connected between R, Y and B phases respectively and the neutral 8 + j6, 7 – j10.5 and 12.5. Calculate:*

(i) Current in each conductor of the cable,
(ii) Current in the neutral wire, and
(iii) Readings of the three wattmeters connected in each line and neutral.

The phase sequence is RYB.

Solution. The potential differences across the three phases are:

$$E_{RN} = \frac{440}{\sqrt{3}} = 254\angle 0^0$$

$E_{YN} = 254$ -120°
$E_{BN} = 254$ 120°.

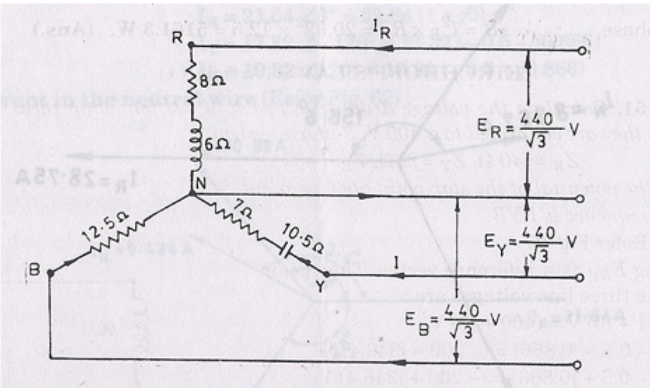


Fig. 65

Impedances:

$Z_R = 8 + j6 = 10$ 36.9°

$Z_Y = 7 – j10.5 = 12.62$ – 56.3°
$Z_B = 12.5 = 12.5$ 0°.

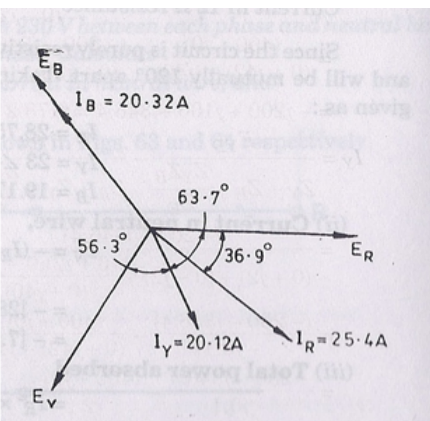


Fig. 66

(i) **Phase currents:**

$$I_R = I_{RN} = \frac{E_{RN}}{E_R} = \frac{254\angle 0^0}{10\angle 36.9^0}$$
$$= \mathbf{25.4\angle -36.9^0\ A.}\ \ (\boldsymbol{Ans.})$$
$$I_Y = I_{YN} = \frac{E_{YN}}{Z_Y} = \frac{254\angle -120^0}{12.62\angle -56.3^0}$$
$$= \mathbf{20.12\angle -63.7^0\ A.}\ \ (\boldsymbol{Ans.})$$
$$I_B = I_{BN} = \frac{E_{BN}}{Z_B} = \frac{254\angle 120^0}{12.5\angle 0^0}$$
$$= \mathbf{20.32\angle 120^0\ A.}\ \ (\boldsymbol{Ans.})$$

(ii) **Current in the neutral wire:**

$I_N = -(I_R + I_Y + I_B)$

= – [25.4 -36.9° + 20.12 – 63.7° + 20.32 120°]

= – [25.4(+ 0.8 – j0.6) + 20.12(0.44 – j0.896) + 20.32(- 0.5 + j0.866)]

= – (20.32 – j15.24 + 8.85 – j18.03 – 10.16 + j17.6)

= – (19.01 – j15.67) = – 19.01 + j15.67 = **24.64 39.5° A. (An s.)**

(iii) **Readings of wattmeters:**

Power in phase, $R = I_R^2 \times R_R = 25.4^2 \times 8 = \mathbf{5161.3\ W.}\ \ (\boldsymbol{Ans.})$

Power in phase, $Y = I_Y^2 \times R_Y = 20.12^2 \times 7 = \mathbf{2833.7\ W.}\ \ (\boldsymbol{Ans.})$

Power in phase, $B = I_B^2 \times R_B = 20.32^2 \times 12.5 = \mathbf{5161.3\ W.}\ \ (\boldsymbol{Ans.})$

KIRCHHOFF'S LAWS (Article 11)

Example 51. *Calculate the voltage drop across each of the following three star-connected impedances when they are connected to a 400 V, 3-phase system:*
$Z_R = j40\Omega$, $Z_Y = j2\Omega$, $Z_B = -j2\Omega$.
Also find the potential of the star-point above ground.
The phase sequence is RYB.

Solution. Refer Fig. 67.

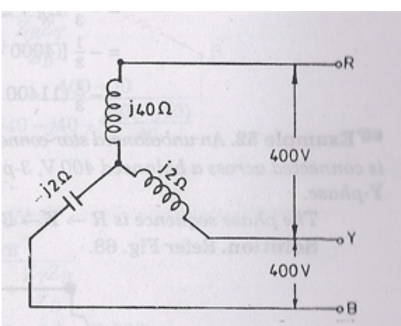


Fig. 67

Considering E_{RY} as a reference vector. The vector expressions for the three line voltages are:

E_{RY} = 400(1 + j0) V = (400 + j0) V

E_{YB} = 400(- 0.5 – j0.866) = (- 200 – j346.4) V

E_{BR} = 400(- 0.5 + j0.866) = (- 200 + j346.4) V

Impedances:

Z_R = (0 + j40),

Z_Y= (0 + j2)

Z_B = (0 – j2)

Now for finding I_R, I_Y and I_B, using the relations:

$$I_R = \frac{E_{RY}}{Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}} - \frac{E_{BR}}{Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}}$$
$$= \frac{400 + j0}{(0 + j40) + (0 + j2) + \frac{(0 + j40)(0 + j2)}{(0 - j2)}} - \frac{(-200 + j346.4)}{(0 - j2) + (0 + j40) + \frac{(0 - j2)(0 + j40)}{(0 + j2)}}$$
$$= \frac{(400 + j0)}{(0 + j42) - j40} - \frac{(-200 + j346.4)}{(0 + j38) - j40} = \frac{400}{j2} - \frac{(-200 + j346.4)}{j2}$$
$$= -j200 + j100 + 346.4 = (173.2 - j100)A \; or \; 200 \angle -30^0 A$$

$$I_Y = \frac{E_{YB}}{Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}} - \frac{E_{RY}}{Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}}$$
$$= \frac{(-200 - j346.4)}{(0 + j2) + (0 - j2) + \frac{(0 + j2)(0 - j2)}{(0 + j40)}} - \frac{(400 + j0)}{(0 + j40) + (0 + j2) + \frac{(0 + j40)(0 + j2)}{(0 - j2)}}$$
$$= \frac{(-200 - j346.4)}{-j0.1} - \frac{(400 + j0)}{(0 + j42) - j40}$$
$$= \frac{-200 - j346.4}{-j0.1} - \frac{(400 + j0)}{j2} = j2000 + 3464 + j200 = 3464 - j1800$$
$$= 3904\angle -27.45^0 A$$

$$I_B = \frac{E_{BR}}{Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}} - \frac{E_{YB}}{Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}}$$
$$= \frac{(-200 + j346.4)}{(0 - j2) + (0 + j40) + \frac{(0 - j2)(0 + j40)}{(0 + j2)}} - \frac{(-200 - j346.4)}{(0 + j2) + (0 - j2) + \frac{(0 + j2)(0 - j2)}{(0 + j40)}}$$
$$= \frac{(-200 - j346.4)}{j38 - j40} - \frac{(-200 - j346.4)}{-j0.1} = \frac{(-200 + j346.4)}{-j2} - \frac{(-200 - j346.4)}{-j0.1}$$
$$= j100 - 173.2 + j2000 - 3464 = -3637.2 + j1900 = 4101\angle 152.4^0 A$$

Voltage drops across impedances:

Voltage drop, $E_R = I_R Z_R$ = (173.2 – j100) × j40 = **(4000 + j6928) V. (Ans.)**

Voltage drop, $E_Y = I_Y Z_Y$ = (3464 – j1800)(j2) = **(3600 + j6928) V. (Ans.)**

Voltage drop, $E_B = I_B Z_B$ = (- 3637.2 + j1900) (- j2) = **(3800 + j7274.4) V. (Ans.)**

Potential of star point above ground:

$$= \frac{1}{3}(E_R + E_Y + E_B)$$

$$= \frac{1}{3}[(4000 + j6928) + (3600 + j6928) + (3800 + j7274.4)]$$

$$= \frac{1}{3}(11400 + j17656) = -(3800 + j7043.5) = \mathbf{-8003\ V.} \quad (\boldsymbol{Ans.})$$

Example 52. *An unbalanced star-connected load shown in Fig.* 68, Z_R == *j40* Ω, Z_Y = – *j40* Ω, Z_B = 40 Ω *is connected across a balanced 400 V, 3-phase supply. Find the reading of the wattmeter connected in Y-phase.*

The phase sequence is R → Y → B.

Solution. Refer Fig. 68.

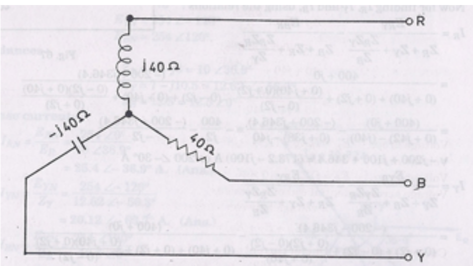


Fig. 68

Taking E_{RY} as reference vector

E_{Ry} = 400 0° = 400 (1 + j0)
E_{YB} = 400 -120^0 == 400 (- 0.5 – j0.866)
E_{BR} == 400 120° == 400(- 0.5 + j0.866)
Impedances are
Z_R == (0 + j40), Z_Y = (0 – j40), Z_B = (40 + j0)

$$I_R = \frac{E_{RY}}{Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}} - \frac{E_{BR}}{Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}}$$

$$= \frac{(400 + j0)}{j40 + j40 + \frac{j40 \times (-j40)}{40}} - \frac{400(-0.5 + j0.866)}{40 + j40 + \frac{40 \times j40}{-j40}}$$

$$= \frac{400}{40} - \frac{-200 + j346.4}{40 + j40 - 40}$$

$$= 10 - \frac{-200 + j346.4}{j40} = 10 - \frac{+j200 - j346.4}{40}$$

$$= 10 - j5 + 8.66 = (1.34 - j5)A$$

$$I_Y = \frac{E_{YB}}{Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}} - \frac{E_{RY}}{Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}}$$

$$= \frac{400(-0.5 + j0.866)}{-j40 + 40 + \frac{(-j40) \times 40}{j40}} - \frac{400 + j0}{j40 - j40 + \frac{j40(-j40)}{40}}$$

$$= \frac{-200 - j346.4}{-j40 + 40 - 40} - \frac{400 + j0}{40}$$

$$= j5 + 8.66 - 10 = (-1.34 - j5)A$$

$$I_B = \frac{E_{BR}}{Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}} - \frac{E_{YB}}{Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R}}$$

$$= \frac{400(0.5 + j0.866)}{40 + j40 + \frac{40 \times j40}{-j40}} - \frac{400(-0.5 - j0.866)}{-j40 + 40 + \frac{(-j40) \times 40}{j40}}$$

$$= \frac{200 + j346.4}{40 + j40 - 40} - \frac{-200 - j346.4}{-j40 + 40 - 40} = \frac{200 + j346.4}{j40} = \frac{-200 - j346.4}{-j40}$$

$$= 8.66 + j5 + j5 - 8.66 = j10\ A$$

Voltage drop, $E_R = I_R Z_R = (1.34 - j5) \times j40 = (200 + j53.6)$ V

Voltage drop, $E_Y = I_Y Z_Y = (-1.34 - j5) \times (-j40) = (-200 + j53.6)$ V

Voltage drop, $E_B = I_B Z_B = j10 \times 40 = (0 + j400)$ V

Voltage cross potential coil of wattmeter

= voltage between supply neutral and load neutral

$$= \frac{1}{3}(E_R + E_Y + E_B) = \frac{1}{3}(200 + j53.6 - 200 + j53.6 + j400) == j169\ V$$

Current through wattmeter current coil x conjugate of voltage across wattmeter potential coil
$= I_Y \times (-j169) = (-1.34 - j5) \times (-j169) = (-845 + j226.5)$

$\therefore$ Wattmeter reading == - 845 W. (Ans.)

STAR-DELTA CONVERSION

Example 53. *An unbalanced star-connected load has branch impedances of Z_R = 10 30° Ω, Z_Y = 20 45° Ω and Z_B = 20 60° Ω and is connected across a balanced 3-phase, 3-wire supply of 440 V. Find:*

i. Line currents, and
ii. Voltage across each impedance
Use star=delta conversion method.

Solution. Taking E_{RY} are reference vector
Then E_{RY} = 440 0°
E_{YB} = 440 -120°
E_{BR} = 440 120°
Refer Fig.69

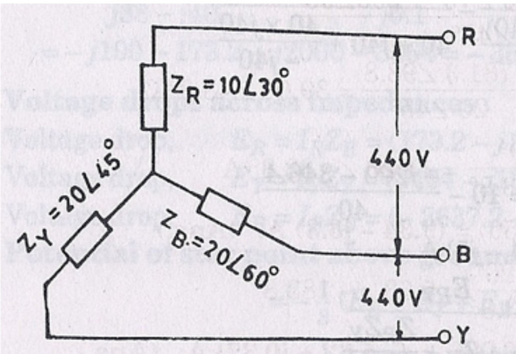


Fig. 69 (a)

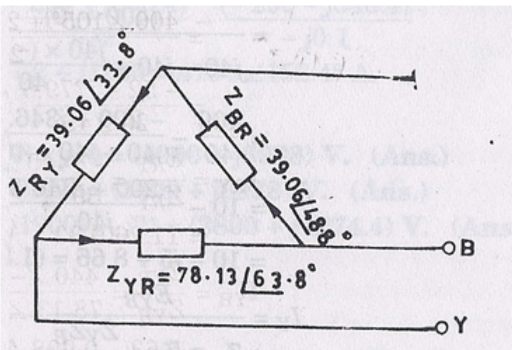


Fig. 69 (b)

Also Z_R = 10 30° (given)
Z_Y= 20 45° (given)
Z_B = 20 60° (given)

Impedances of equivalent delta-connected load (Refer Fig.69):

$$Now\ \ Z_{RY} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B} = \frac{Z_R Z_B + Z_Y Z_B + Z_R Z_Y}{Z_B}$$

$$= \frac{(10\angle 30^0)(20\angle 60^0) + (20\angle 45^0)(20\angle 60^0) + (10\angle 30^0)(20\angle 45^0)}{20\angle 60^0}$$

$$= \frac{200\angle 90^0 + 400\angle 105^0 + 200\angle 75^0}{20\angle 60^0}$$

$$= \frac{200(0 + j1) + 400(-0.26 + j0.966) + 200(0.26 + j0.966)}{20\angle 60^0}$$

$$= \frac{j200 - 104 + j386.4 + 52 + j193.2}{20\angle 60^0}$$

$$= \frac{-52 + j779.6}{20\angle 60^0} = \frac{731.3\angle 93.8^0}{20\angle 60^0} = 39.06\angle 33.8^0\ \Omega$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R} = \frac{Z_Y Z_R + Z_B Z_R + Z_Y Z_B}{Z_R}$$

$$= \frac{(20\angle 45^0)(10\angle 30^0) + (20\angle 60^0)(10\angle 30^0) + (20\angle 45^0)(20\angle 60^0)}{10\angle 30^0}$$

$$= \frac{200\angle 75^0 + 200\angle 90^0 + 400\angle 105^0}{10\angle 30^0}$$

$$= \frac{200(0.62 + j0.966) + j200 + 400(-0.26 + j0.966)}{10\angle 30^0}$$

$$= \frac{52 + j193.2 + j200 - 104 + j386.4}{10\angle 30^0}$$

$$= \frac{-52 + j779.6}{10\angle 30^0} = \frac{781.3\angle 93.8^0}{10\angle 30^0} = 78.13\angle 63.8^0\ \Omega$$

$$Z_{YB} = Z_Y + Z_B + \frac{Z_Y Z_B}{Z_R} = \frac{Z_Y Z_R + Z_B Z_R + Z_Y Z_B}{Z_R}$$

$$= \frac{(20\angle 45^0)(10\angle 30^0) + (20\angle 60^0)(10\angle 30^0) + (20\angle 45^0)(20\angle 60^0)}{10\angle 30^0}$$

$$= \frac{200\angle 75^0 + 200\angle 90^0 + 400\angle 105^0}{10\angle 30^0}$$

$$= \frac{200(0.62 + j0.966) + j200 + 400(-0.26 + j0.966)}{10\angle 30^0}$$

$$= \frac{52 + j193.2 + j200 - 104 + j386.4}{10\angle 30^0}$$

$$= \frac{-52 + j779.6}{10\angle 30^0} = \frac{781.3\angle 93.8^0}{10\angle 30^0} = 78.13\angle 63.8^0\ \Omega$$

$$Z_{BR} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y} = \frac{Z_B Z_Y + Z_R Z_Y + Z_B Z_R}{Z_Y}$$

$$= \frac{(20\angle 60^0)(20\angle 45^0) + (10\angle 30^0)(20\angle 45^0) + (20\angle 60^0)(10\angle 30^0)}{20\angle 45^0}$$

$$= \frac{400\angle 105^0 + 200\angle 75^0 + 200\angle 90^0}{20\angle 45^0}$$

$$= \frac{-52 + j779.6}{20\angle 45^0} = \frac{781.3\angle 93.8^0}{20\angle 45^0} = 39.06\angle 48.8^0\ \Omega$$

$$\boldsymbol{I_{RY}} = \frac{E_{RY}}{Z_{RY}} = \frac{440\angle 0^0}{39.06\angle 33.8^0} = 11.26\angle -33.8^0\ A$$

$= 11.26(0.83 - j0.56) = \boldsymbol{(9.34 - j6.3)A.}$ ***(Ans.)***

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{440\angle -120^0}{78.13\angle 63.8^0} = 5.63\angle{-183.8^0}\ A$$

$$= 5.63(-0.998 + j0.066) = (\mathbf{-5.62 - j0.37})\boldsymbol{A}.\ \ (\boldsymbol{Ans.})$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{440\angle 120^0}{39.06\angle 48.8^0} = 11.26\angle 71.2^0\ A$$

$$= 11.26(0.322 + j0.947) = (\mathbf{3.625 - j10.66})\boldsymbol{A}.\ \ (\boldsymbol{Ans.})$$

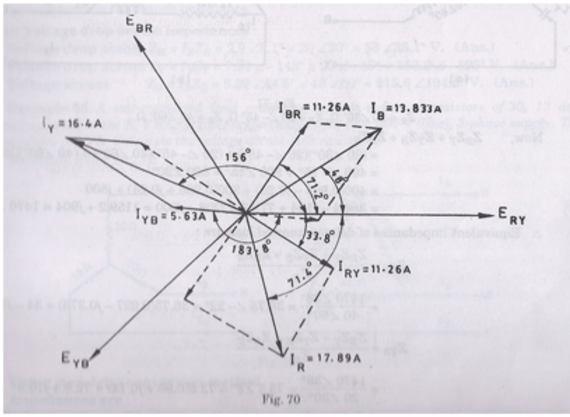


Fig. 70

(i) **Line currents:**

$\mathbf{I_R}$ = $I_{RY} – I_{BR}$ = (9.34 – j6.3) – (3.625 + j10.66)
= 5.715 – j16.96 = **17.89 -71.4° A. (Ans.)**
$\mathbf{I_Y}$ = $I_{YB} – I_{RY}$ = (- 5.62 + j0.37) – (9.34 – j6.3)

= -14.96 + j6.67 = **16.4 156° A. (Ans.).**

I_B = $I_{BR} – I_{YB}$ = (3.625 + j10.66) – (- 5.62 + j0.37) .
= 9.245 + j10.29 = **13.833 48° A. (Ans.)**

These are the currents in the phases of the star-connected unbalanced load. Let us find voltage drop across each star-connected branch impedance.

(ii) **Voltage across impedances :**

Voltage across $Z_R = I_R \times Z_R$

= 17.89 -71.4° × 10 30° = **178.9 – 41.4° V. (Ans.)**
Voltage across $Z_Y = I_Y \times Z_Y$ = 16.4 156° × 20 45°

= 328 L210° V. (Ans.)

Voltage across $Z_B = I_B \times Z_B$ = 13.833 48° × 20 60°

= 276.67 108° V. (Ans.)

The phasor diagram is shown in Fig. 70.

Example 54. *The branch impedances of an unbalanced star-connected load are:* Z_R *= 20 30° Ω,* Z_Y *= 20 – 45° Ω,* Z_B *= 40 60° Ω. It is connected across a balanced 3-phase, 3-wire supply of 200 V. Find:*

(i) Line currents, and

(ii) Voltage across each impedance.

(iii) Use star-delta conversion method.

Solution. The unbalanced star-connected load and its equivalent delta-connected load are shown in Fig. 71.

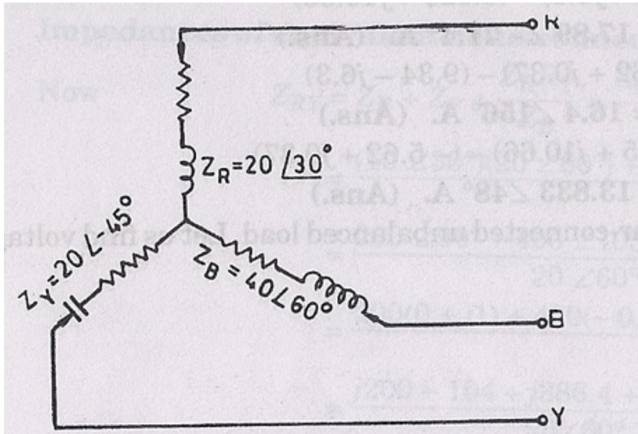


Fig. 71 (a)

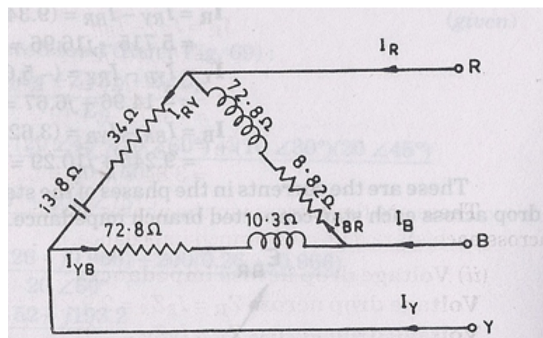


Fig. 71 (b)

Z_R = 20 30° Ω, Z_Y = 20 – 45° Ω, Z_B = 40 60° Ω

Now $Z_RZ_Y + Z_YZ_B + Z_BZ_R$

= (20 300)(20 – 45°) + (20 – 45°)(40 60°) + (40 60°)(20 30°)
= 400 – 15° + 800 15° + 800 90°

= 400(0.966 – j0.26) + 800(0.966 + j0.24) + j800

= 386.4 – j104 + 772.8 + j208 + j800 = 1159.2 + j904 = 1470 38°

Equivalent impedances of delta-connected load are:

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$= \frac{1470\angle 38^0}{40\angle 60^0} = 36.75\angle -22^0 = 36.75(0.927 - j0375) = 34 - j13.8$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$= \frac{1470\angle 38^0}{20\angle 30^0} = 73.5\angle 8^0 = 73.5(0.99 - j0.14) = 72.8 + j10.3$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

$$= \frac{1470\angle 38^0}{20\angle -45^0} = 73.5\angle 83^0 = 73.5(0.12 - j0.99) = 8.82 + j72.8$$

Taking E_{RY} as the difference vector and assuming sequence as $R \to Y \to B$,

$E_{RY} = 200\ \angle 0°$, $E_{YB} = 200\ \angle -120°$, $E_{BR} = 200\ \angle 120°$

$$E_{RY} = 200\angle 0^0,\ \ E_{YB} = 200\angle -120^0,\ \ E_{BR} = 200\angle 120^0$$

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{200\angle 0^0}{36.75\angle -22^0} = 5.44\angle 22^0$$

$$= 5.44(0.927 + j0.375) = 5.04 + j2.04$$

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{200\angle -120^0}{73.5\angle 8^0} = 2.72\angle -128^0$$

$$= 2.72(-0.616 - j0.79) = -1.67 - j2.15$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{200\angle 120^0}{73.5\angle 83^0} = 2.72\angle 37^0 = 2.72(0.8 + j0.6) = 2.17 + j1.63$$

(i) **Line Currents:**

$I_R = I_{RY} - I_{BR}$

$= (5.04 + j2.04) - (2.17 + j1.63) = 2.87 + j0.41 = 2.9\ \angle 8.1°$ A. (Ans.)

$I_Y = I_{YB} - I_{RY} = (-1.67 - j2.15) - (5.04 + j2.04)$

$= -6.71 - j4.19 = 7.91\ \angle -148°$ A. (Ans.)

$I_B = I_{BR} - I_{YB} = (2.17 + j1.63) - (-1.67 - j2.15)$

(i) **Line Currents:**

$I_R = I_{RY} – I_{BR}$

$= (5.04 + j2.04) – (2.17 + j1.63) = 2.87 + j0.41 = 2.9\ 8.1°$ A. (Ans.)

$I_Y = I_{YB} – I_{RY} = (-1.67 – j2.15) – (5.04 + j2.04)$

$= – 6.71 – j4.19 = 7.91\ -148°$ A. (Ans.)

$I_B = I_{BR} - I_{YB} = (2.17 + j1.63) - (-1.67 - j2.15)$

$= 3.84 + j3.78 = 5.39\ 44.5°$ A. (Ans.)

These are the currents in the phases of star-connected unbalanced load. Let us find the drop across each star-connected branch impedance.

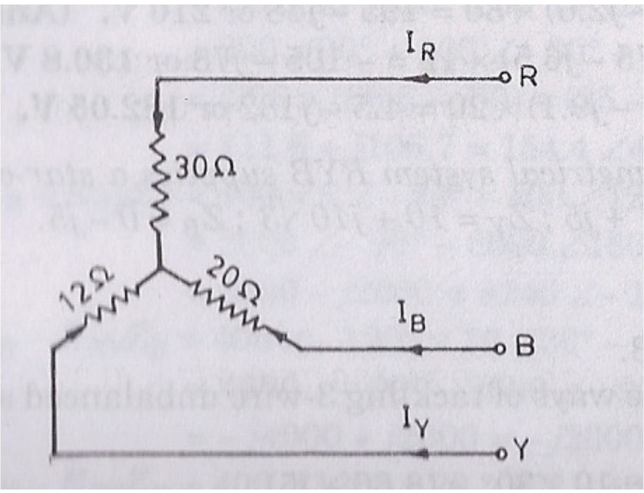
(ii) **Voltage drop across impedances:**

Voltage drop across $Z_R = I_R Z_R = 2.9\ 8.1° \times 20\ 30° = 58\ 38.1°$ V. (Ans.)

Voltage drop across $Z_Y = I_Y Z_Y = 7.91\ -148° \times 20 - 45° = 158.2\ -193°$ V. (Ans.)

Voltage across $Z_B = I_B Z_B = 5.39\ 44.5° \times 40\ 60° = 215.6\ 104.5°$ V. (Ans.)

Example 55. *A star-connected load consisting of non-inductive resistors of 30, 12 and 20 ohms connected to the R, Y and B lines respectively is fed by a 300 V (line), 3-phase supply. The phase sequence is RYB. Calculate the voltage across each resistor.*



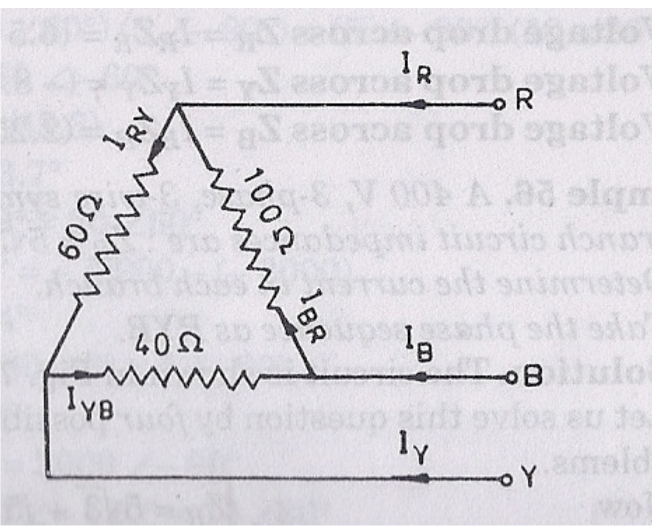


Fig.72

Using star-delta conversion method,

Impedances are :

$Z_R = 30\ \Omega$, $Z_Y = 12\ \Omega$, $Z_B = 20\ \Omega$

Now $Z_RZ_Y + Z_YZ_B + Z_{BR} = 30 \times 12 + 12 \times 20 + 20 \times 30 = 360 + 240 + 600 = 1200\ \Omega$

Equivalent impedances of delta-connected load are:

$$Z_{RY} = \frac{Z_RZ_Y + Z_YZ_B + Z_{BR}}{Z_R} = \frac{1200}{20} = 60\ \Omega$$

$$Z_{YB} = \frac{Z_RZ_Y + Z_YZ_B + Z_BZ_R}{Z_R} = \frac{1200}{30} = 40\ \Omega$$

$$Z_{BR} = \frac{Z_RZ_Y + Z_YZ_B + Z_BZ_R}{Z_Y} = \frac{1200}{12} = 100\ \Omega$$

Phase currents are:

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{300}{60} = 5\ A$$

$$I_{YB} = \frac{E_{YB}}{Z_{YR}} = \frac{300}{40} = 7.5\ A$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{300}{100} = 3\ A$$

Each current is in phase with its own voltage because the load is purely resistive.

To find line currents for delta-connection using vector algebra and choosing E_{RY} as the reference axis, we get

$I_{RY} = 5 + j0$

$I_{YB} = 7.5(- 0.5 – j0.866) = – 3.75 – j6.5$

$I_{BR} = 3(- 0.5 + j0.866) = – 1.5 + j2.6$

Line currents for delta-connection are:

$I_R = I_{RY} – I_{BR}$

$= (5 + j0) – (- 1.5 + j2.6) = (6.5 – j2.6)$ or 7 A in magnitude

$I_Y = I_{YB} - I_{RY}$

= (- 3.75 – j6.5) – (5 + j0) = – 8:75 – j6.5 or 10.9 A in magnitude
$I_B = I_{BR} = I_{YB}$ = (- 1.5 + j2.6) – (- 3.75 – j6.5)

= 2.25 – j9.1 or 9.37 A in magnitude

These line currents for delta-connection are the phase currents for star-connection.

Voltage drop across each limb of star-connected load is :

Voltage drop across Z_R = $I_R Z_R$ = (6.5 – j2.6) × 30 = 195 – j78 or **210 V. (Ans.)**
Voltage drop across Z_Y = $I_Y Z_Y$ = (- 8.75 -j6.5) × 12 = – 105 – j78 or **130.8 V. (Ans.)**
Voltage drop across Z_B = $I_B Z_B$ = (2.25 – j9.1) × 20 = 4.5 – j182 or **182.05 V. (Ans.)**

Example 56. *A 400 V, 3-phase, 3-wire symmetrical system R_{YB} supplies a star-connected load whose branch circuit impedances are:* $Z_R = 5\sqrt{3} + j5;\ Z_Y = 10 + j10\sqrt{3};\ Z_B = 0 - j5.$

Determine the current in each branch.
Take the phase sequence as RYB.

Solution. The circuit is shown in Fig. 73.

Let us solve this question by *four* possible ways of tackling 3-wire unbalanced star-connected load problems.

$$\text{Now} \qquad Z_R = 5\sqrt{3} + j5 = 10\angle 30^0 = (8.66 + j5)$$

$$Z_Y = 10 + j10\sqrt{3} = 20\angle 60^0 = (10 + j17.32)$$

$$Z_B = 0 - j5 = 5\angle -90^0 = -j5$$

$$\text{Also, let} \qquad E_{RY} = 400\angle 0^0 = (400 + j0)$$

$$E_{YB} = 400\angle -120^0 = (-200 - j346)$$

$$E_{BR} = 400\angle 120^0 = (-200 + j346)$$

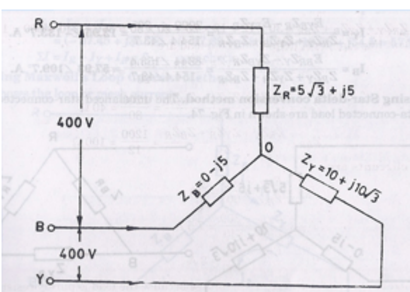


Fig.73

1. **By using Kirchhoff's law :**

We know that

We know that

$$I_R = \frac{E_{RY}}{Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}} - \frac{E_{BR}}{Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}}$$

$$= \frac{E_{RY} Z_B}{Z_R Z_B + Z_Y Z_B + Z_R Z_Y} - \frac{E_{BR} Z_Y}{Z_{BY} + Z_{RY} + Z_{BR}} \qquad \ldots (i)$$

$$I_Y = \frac{E_{YB} Z_Y}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R} - \frac{E_{RY} Z_B}{Z_{RY} + Z_{YB} + Z_{BR}} \qquad \ldots (ii)$$

and $$I_B = \frac{E_{BR} Z_Y}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R} - \frac{E_{RY} Z_B}{Z_{RY} + Z_{YB} + Z_{BR}} \qquad \ldots (iii)$$

Now $Z_R Z_Y + Z_Y Z_B + Z_B Z_R$

= (10 30°) (20° 60°) + (20 60°) (5 -90°) + (5 – 90°) (10 30°)

= 200 90° + 100 -30° + 50 -60°

= j200 + (86.6 – j50) + (25 – j43.3)

= 111.6 +j106.7 = 154.4 43.7°

$E_{RY} Z_B - E_{BR} Z_Y$ = 400 × 5 -90° – 400 120° × 20 60°

= 2000 -90° – 8000 180° = – j2000 – (- 8000)
= 8000 – j2000 = 8246 -14°

$E_{YB} Z_R - E_{RY} Z_B$ = 400 -120° × 10 30° – 400 0° × 5-90°
= 4000 -90° – 2000 -90°

=-j4000 + j2000 = – j2000 = 2000 -90°

$E_{BR} Z_Y - E_{YB} Z_R$ = 400 120° × 20 60° – 400 -120° × 10 30°

=8000180° – 4000 -90° = – 8000 + j4000 = 8944 153.4°
Current in each branch [from (i), (ii) and (iii)]

$$I_R = \frac{E_{RY} Z_B - E_{BR} Z_Y}{Z_R Z_Y + Z_Y Z_R + Z_R Z_R} = \frac{8246\angle -14^0}{154.4\angle 43.7^0} = \mathbf{53.4\angle -57.7^0\ A.}\ \ (\textit{Ans.})$$

$$I_Y = \frac{E_{YB} Z_R - E_{RY} Z_B}{Z_R Z_Y + Z_Y Z_R + Z_R Z_R} = \frac{2000\angle -90^0}{154.4\angle 43.7^0} = \mathbf{12.95\angle -133.7^0\ A.}\ \ (\textit{Ans.})$$

$$I_B = \frac{E_{BR} Z_Y - E_{YB} Z_R}{Z_R Z_Y + Z_Y Z_R + Z_R Z_R} = \frac{8944\angle 153.4^0}{154.4\angle 43.7^0} = \mathbf{57.93\angle -109.7^0\ A.}\ \ (\textit{Ans.})$$

2. By using Star-delta conversion method: The unbalanced star-connected load and its equivalent delta-connected load are shown in Fig 74.

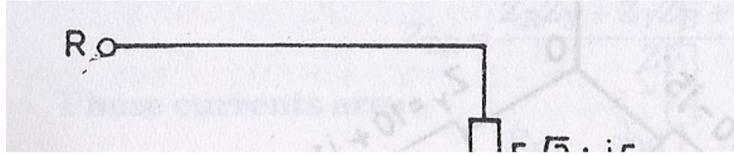


Fig.74 (a)

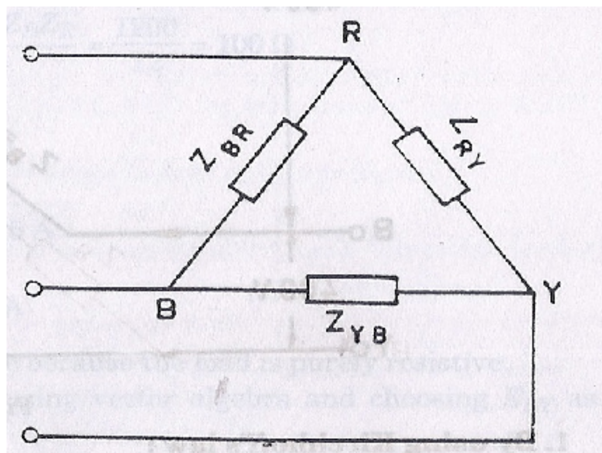


Fig.74 (b)

Now, $Z_R Z_Y + Z_Y Z_B + Z_B Z_R = 154.4\ 43.7°$ [Already calculated]

Equivalent delta-connected impedances are:

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} = \frac{154.4\angle 43.7^0}{5\angle -90^0} = 30.88\angle 133.7^0$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} = \frac{154.4\angle 43.7^0}{10\angle 30^0} = 15.44\angle 13.7^0$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} = \frac{154.4\angle 43.7^0}{20\angle 60^0} = 7.72\angle -16.3^0$$

Phase currents are:

$$I_{RY} = \frac{E_{RY}}{Z_{RY}} = \frac{400\angle 0^0}{30.88\angle 133.7} = 12.95\angle -133.7^0 = (-8.95 - j9.36)$$

$$I_{YB} = \frac{E_{YB}}{Z_{YB}} = \frac{400\angle -120^0}{15.44\angle 13.7} = 25.9\angle -133.7^0 = (-17.89 - j18.75)$$

$$I_{BR} = \frac{E_{BR}}{Z_{BR}} = \frac{400\angle 120^0}{7.75\angle -16.3^0} = 51.8\angle 136.3^0 = (-37.45 + j35.78)$$

Line currents for delta-connection are:

$I_R = I_{RY} - I_{BR}$

= (- 8.95 – j9.36) – (- 37.15 + j35.78)

= 28.5 – j15.11 = **53.38 57.7° A. (Ans.)**

$I_Y = I_{YB} - I_{RY}$

= (- 17.89 – j18.72) – (- 8.95 + j9.36)

= -8.94 – j9.36 = 12.94 133.7°

$I_B = I_{BR} - I_{YB}$

= (- 37.45 +j35.78) – (- 17.89 – j18.72) = – 19.56 + j54.5 = 57.9 109.7°

$\Sigma I = I_R + I_Y + I_B = 0$ ……… as a check.

3. By using Maxwell's Loop Current Method:

Fig. 75 shows the loop or mesh currents.

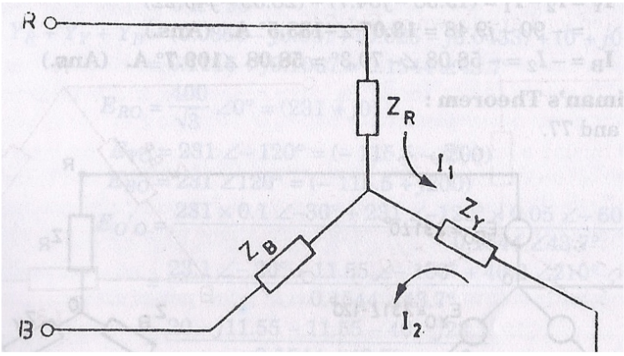


Fig. 75

It may be noted that $I_R = I_1$
$I_Y = I_2 - I_1$
and $I_B = -I_2$

Considering drops across R and Y arms, we get
$I_1Z_R + Z_Y(I_1 - I_2) = E_{RY}$

or $I_1(Z_R + Z_Y) - I_2Z_Y = E_{RY}$ …*(i)*

Similarly, considering the arms Y and B, we have
$(I_2 - I_1)\ Z_Y + I_2Z_B = E_{YB}$

$-I_1Z_Y + I_2(Z_Y + Z_B) = E_{YB}$

Solving for I_1 and I_2, we get ... *(ii)*

$$I_1 = \frac{E_{RY}(Z_Y + Z_B) + E_{YB}Z_Y}{(Z_R + Z_Y)(Z_Y + Z_B) - Z_Y^2}$$

$$I_2 = \frac{E_{YB}(Z_R + Z_Y) + E_{RY}Z_Y}{(Z_R + Z_Y)(Z_Y + Z_B) - Z_Y^2}$$

$$I_1 = \frac{400(10 + j12.32) + 400\angle -120 \times 20\angle 60^0}{(18.66 + j22.32)(10 + j12.32) - 400\angle 120^0}$$

$$= \frac{4000 + j4928 + 4000 - j6928}{(186.6 + j229.9 + j223.2 - 275) - (-200 + j346.4)}$$

$$= \frac{8000 - j2000}{111.6 + j106.7} = \frac{8246.2\angle -14^0}{154\angle 43.7^0}$$

$$= 53.5\angle 57.7^0 = (28.59 - j45.22)$$

$$I_2 = \frac{(-200 - j346)(18.66 + j22.32) + 400(10 + j17.32)}{111.6 + j106.7}$$

$$= \frac{-373.2 - j4464 - j6456.4 + 7723 + 4000 + j6928}{111.6 + j106.7}$$

$$= \frac{8000 - j4000}{111.6 + j106.7} = \frac{8944\angle -26.6^0}{154\angle 43.7^0} = 58.08\angle -70.3^0 = 19.58 - j54.7$$

$I_R = I_1 =$ **53.5 – 57.7° A. (Ans.)**

$I_Y = I_2 - I_1 = (19.58 - j54.7) - (28.59 - j45.22)$
= – 90 – j9.48 = **13.07 -133.5° A. (Ans.)**

$I_B = -I_2 =$ – 58.08 -70.3° = **58.08 109.7° A. (Ans.)**

4. Using Milliman's Theorem:

Refer Figs. 76 and 77.

According to Milliman's theorem, the voltage of the *load* star point 0′ with respect to the star point or neutral 0 of the generator or supply (normally zero potential) is given by

$$E_{o'o} = \frac{E_{RO} Y_R + E_{YO} Y_Y + E_{BO} Y_B}{Y_R + Y_Y + Y_B}$$

where E_{RO}, E_{YO} and E_{BO} are the phase voltages of the generator or 3-phase supply.

Voltage across each phase of the load is
$E_{RO'} = E_{RO} - E_{O'O}$

$E_{YO'} = E_{YO} - E_{O'O}$

$E_{BO'} = E_{BO} - E_{O'O}$

Now, $I_{RO'} = (E_{RO} - E_{O'O})$

$I_{YO'} = (E_{YO} - E_{O'O})\ Y_Y$

$I_{BO'} = (E_{BO} - E_{O'O})\ Y_B$

$$Here, \quad Y_R = \frac{1}{10\angle 30^0} = 0.1\angle -30^0 = (0.866 - j0.05)$$

$$Y = \frac{1}{\quad} = 0.05\angle -60^0 = (0.025 - j0.0433)$$

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