PHYSICS OF THE UNIVERSE

END SEM EXAM April 2015

Duration : 03Hours **Total questions : 04** **Total Marks : 50**

NOTE: *PLEASE* $\boxed{\textit{BOX}}$ *ALL IMPORTANT STEPS &* $\boxed{\boxed{\textit{DOUBLE BOX}}}$ *ALL FINAL ANSWERS.*

Q 1. ***Relativity*** **[18 points]**

Consider the following expression for the metric $g_{\mu\nu}$ which describes the geometry outside a Black Hole of mass M :

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 Sin^2\theta \, d\phi^2$$

Note : M is the mass of the Black Hole and is a constant. (t, r, θ, ϕ) are the coordinates.

For this metric, determine the following connection coefficient $\Gamma^\gamma_{\beta\mu}$.

[You may use the expression : $\Gamma^\gamma_{\beta\mu} = \frac{1}{2} g^{\alpha\gamma} \left(\frac{\partial g_{\alpha\beta}}{\partial x^\mu} + \frac{\partial g_{\alpha\mu}}{\partial x^\beta} - \frac{\partial g_{\beta\mu}}{\partial x^\alpha}\right)$]

(i) Γ^t_{rt} (ii) Γ^r_{rr} (iii) Γ^r_{tt} (iv) $\Gamma^r_{\theta\theta}$

(v) $\Gamma^r_{\phi\phi}$ (vi) $\Gamma^\theta_{\theta r}$ (vii) $\Gamma^\phi_{\phi r}$ (viii) $\Gamma^\phi_{\theta\phi}$

(ix) $\Gamma^\theta_{\phi\phi}$

Q 2. ***Thermodynamics*** **[12 points]**

(a) Evaluate the energy density, ρ, of a gas of particles in the high temperature limit (T >> m, T >> μ).

You may use the expression for

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{[(E^2 - m^2)^{1/2}]E^2 \, dE}{\exp\left[\frac{(E-\mu)}{T}\right] \pm 1}$$

Express the result in the form :

$$\rho = AgT^n$$

What is the value of n ?

(b) Evaluate the number density, n, of a gas of particles in the high T limit (T >> m, T >> μ).

You may use the expression :

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{[(E^2 - m^2)^{1/2}]E\, dE}{\exp\left[\frac{(E-\mu)}{T}\right] \pm 1}$$

Express the result in the form : $n = BgT^m$

What is the value of m ?

(c) Evaluate the pressure P of a gas of particles in the high T limit (T >> m, T >> μ).

You may use the expression for :

$$P = \frac{g}{6\pi^2} \int_m^\infty \frac{[(E^2 - m^2)^{3/2}]\, dE}{\exp\left[\frac{(E-\mu)}{T}\right] \pm 1}$$

Express the result as

$$P = CgT^q$$

What is the value of q ?

(d) For the high temperature limit, if we express the relationship between the energy density ρ and pressure P as

$$P = \omega\rho$$

then what is the value of ω ?

(e) At the same values of T and g, the ratio of energy densities of Bosons to Fermions, $\frac{\rho(Boson)}{\rho(Fermion)} > 1$: is this true (T) or false (F) ? Why?

(f) At the same values of T and g, the ratio of the number density, n of Bosons & Fermions, obeys the relationship: $\frac{n(Boson)}{n(Fermion)} > 1$: is this true (T) or fales (F) ? Why?

Q 3. *History of the Universe*

[12 points]

After the Quark Hadron Phase Transition, the only particle species left in large numbers are the pions (π^+, π^-, π^0), muons (μ^+, μ^-), electrons (e^-, e^+), neutrinos & the photons.

(a) Determine the effective degrees of freedom, g_*, corresponding to photons (γ). (They are massless spin 1 particles.)

(b) Determine the g_* corresponding to e^+ and e^-. (They are spin $1/2$ particles.)

(c) Determine the g_* corresponding to μ^+ and μ^-. (They are spin $1/2$ particles.)

(d) Determine the g_* corresponding to neutrinos ($\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$). Take these to be massless spin $1/2$ particles as per the Standard Model of Particle Physics.

(e) Determine the g_* corresponding to the pions (π^+, π^- & π^0). These are spin 0 particles.

(f) The g_* before the Quark Hadron Transition was 61.75. How many effective degrees of freedom g_*, disappeared in the Quark Hadron Phase Transition.

Q4. ***Evolution of the Universe*** **[8 points]**

A solution of the Friedmann Equations determining the evolution of the Universe can be written down in the form :

$$R = \frac{(1 - cos\theta)\Omega_0}{2(\Omega_0 - 1)}$$

and

$$H_0 t = (\theta - sin\theta) \times \frac{\Omega_0}{2(\Omega_0 - 1)^{3/2}}$$

where, R is the scale factor of the Universe, H_0 is the Hubble Constant, $\Omega_0 = \frac{\rho_0}{\rho_{critical}}$ and θ is the parameter in the above parametric solution. (The time changes as θ changes.)

(a) Determine the maximum size of the Universe and express it in terms of Ω_0.

(b) What is the value of time or the age of the Universe when the Universe reaches its maximum size? Express the answer in terms of H_0 and Ω_0. Call this time t_{MAX}.

(c) What is the value of time or the age of the Universe at which the Universe re-collapses to zero size. [Note this time should not be 0.]
Express this in terms of H_0 and Ω_0. Call this time t_{CRUNCH}.

(d) What is the equation relating t_{CRUNCH} to t_{MAX}. Express this in the simplest relationship possible.

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