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FIR & IIR filter

Finite ~~i/p~~ impulse Response :-

$$y(n) = \sum_{k=0}^{q} b_q(k) z^{-k} + \sum_{l=1}^{p} a_p(l) z^{-l}$$

Take $q=0$, $p=1$.

$$y(n) = \sum_{k=0}^{q} b_q(k) x(n-k) + \sum_{l=1}^{p} a_p(l) y(n-l)$$

$$\boxed{y(n) = b_q(0) x(n) + a_p(1) y(n-1)}$$

$y(n-1)$ can have infinity so we cannot determine how ~~many impulses~~ are there in o/p response

$q = 1 \quad p = 0$

$$y(n) = b_q(0) x(n) + b_q(1) x(n-1)$$

Finite Impulse Response.

The response to any s/s.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$x(n)$ is causal

$$= \sum_{k=0}^{n} h(k) x(n-k)$$

$h(n)$ is causal.

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Requirement

(i) Linear phase

$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$ $\qquad y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

let's take $x(n) = A e^{j\omega n}$.

$$y(n) = A \sum_{k=-\infty}^{\infty} h(k)\, e^{j\omega(n-k)}$$

$$= A \left[\sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k} \right] e^{j\omega n}.$$

$$= H(\omega) \cdot x(n)$$

$$\angle H(\omega) = -\omega K.$$

$$\frac{\partial \angle H(\omega)}{\partial \omega} = -K \quad \rightarrow \text{constant}$$

Two freq ω_1 and ω_2 then o/p is scaled by $x(n)$ and phase is constant

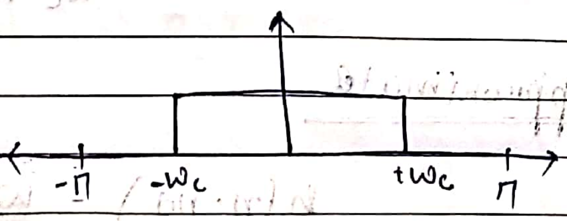
$H(\omega) = \omega^2 K.$

Differential

$2\omega K$ $\quad$ [different phase for diff i/p]

Filter Design :-

Ideal Filter :-



Discrete range. $0 - \pi$ max.

$x(t) = A \sin 2\pi f t$

$x(n) = A \sin 2\pi \left(\frac{f n}{f_s}\right)$

$f_s = 2f$ $\quad x(n) = A \sin \pi n$

$H(\omega) = 1$ for $\omega \le \omega_c$ cut of freq.

$\ 0$ otherwise

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$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)\, e^{j\omega n}\, d\omega$$

$\leftarrow \omega = 0$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\, 2\omega_c = \frac{\omega_c}{\pi}.$$

$\omega \neq 0$

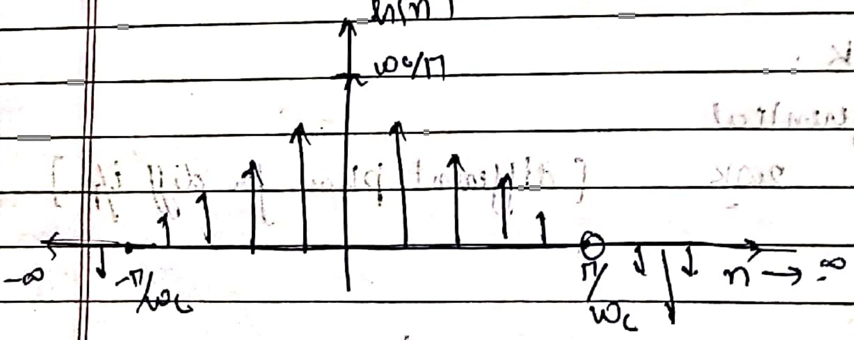
$$h(n) = \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi}\,\frac{e^{j\omega_c n} - e^{-j\omega_c n}}{jn}$$

$$= \frac{\omega_c \sin \omega_c n}{\pi n\, \omega_c}$$

$$\frac{\sin x}{x}$$

$$\frac{\omega_c}{\pi}\,\frac{\sin \omega_c n}{\omega_c n}.$$



Non causal + infinite impulse response.

Approximate

$$h(n-n_d) = \frac{\omega_c}{\pi}\,\frac{\sin \omega_c (n-n_d)}{\omega_c (n-n_d)}$$

shifted sinc we want sinc fn

approximating sinc fn.

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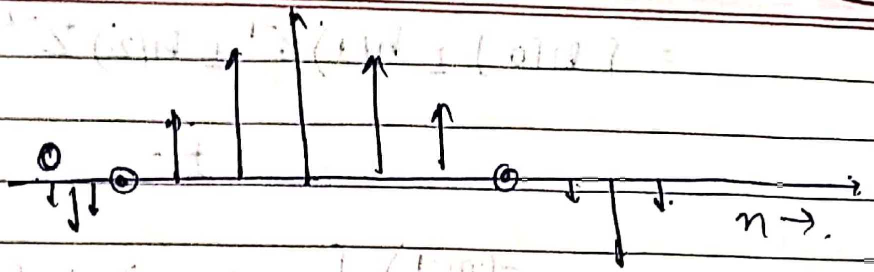
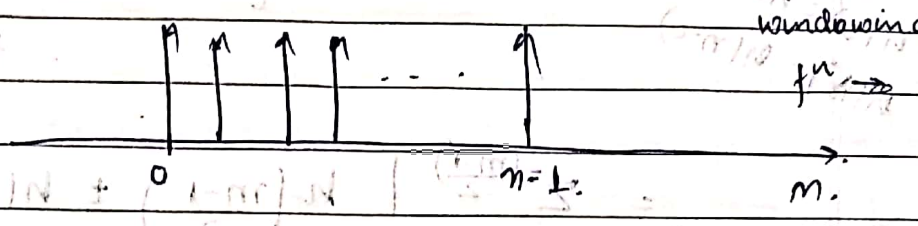


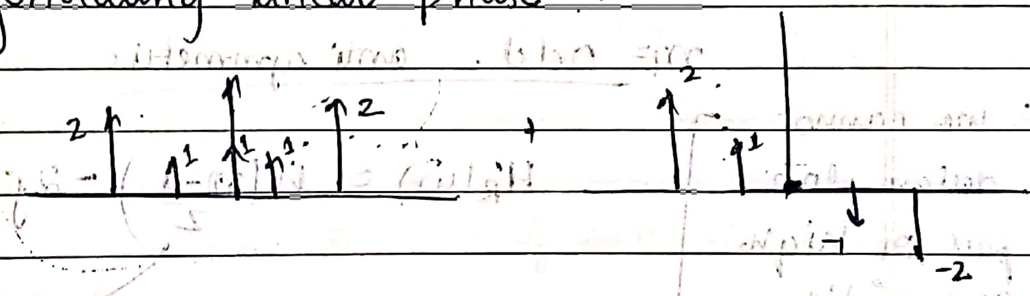
X



O/p will be Hurz and causal.

Linear phase cannot be generated using IIR :-
→ IIR → phase distortion.

Why FIR is generating linear phase :



$$h[n] = \frac{1}{2}[h_e[n] + h_o[n]]$$

Design a filter using symmetric sequences :

$$h[n] = \pm h[m-1-n] \quad ; \text{for } 0 \le n \le m-1$$

Z-transform

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

$$= \sum_{n=0}^{m-1} h[n] z^{-n}$$

[IIR filter : we cannot say it a made from symmetric sequence since we cannot say how many no of impulse are there in IIR]

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$$= [h(0) \pm h(1) z^{-1} \pm h(2) z^{-2} \pm \cdots \pm h\left(\frac{m-1}{2}\right) z^{-\left(\frac{m-1}{2}\right)}$$

$$\pm \cdots \pm h(n-1) z^{-(n-1)}]$$

Symmetric
$h(0) = h(n-1)$
$h(1) = h(n-2)$

$$= z^{-\left(\frac{m-1}{2}\right)} \left[h\left(\frac{m-1}{2}\right) \pm [h(0) z^{\frac{m-1}{2}} + h(n-1) z^{-\frac{m-1}{2}}] \right.$$

$$\left. \pm \cdots \pm [h(1) z^{\left(\frac{m-1}{2}-1\right)} \pm h(n-2) z^{-\left(\frac{n-1}{2}-1\right)}] \right]$$

$$= z^{-\left(\frac{m-1}{2}\right)} \left[h\left(\frac{m-1}{2}\right) \pm h(0) [z^{m-1/2} \pm z^{-\left(\frac{m-1}{2}\right)}] \right.$$

$$\left. \pm \cdots \pm h(1) [z^{\left(\frac{m-1}{2}-1\right)} \pm z^{-\left(\frac{m-1}{2}-1\right)}] \right]$$

$$= z^{-\left(\frac{m-1}{2}\right)} \left[h\left(\frac{m-1}{2}\right) \pm \sum_{k=0}^{\frac{m-1}{2}-1} h(k) [z^{\frac{m-1}{2}k} - z^{-\left(\frac{m-1}{2}\right)k}] \right]$$

m even $\rightarrow$ symmetric

m odd $\rightarrow$ antisymmetric.

$\downarrow z = e^{j\omega}$

$m =$ odd. antisymmetric

$$H_R'(\omega) = h\left(\frac{m-1}{2}\right) - 2j \sum_{k=0}^{\frac{m-1}{2}} \sin\left(\omega\left(\frac{m}{2}\right)k\right)$$

$\omega = 0, \pi$ $m =$ odd.

$= 0$

$\therefore$ we cannot design low pass or high pass with $m =$ odd and antisymmetric.

$\omega = 0$ $= 0$

$\omega = \pi$ $= 0$.

center value and end value for anti symmetric is always zero

with $\pi/2$ we can design

Phase $\frac{\pi}{2} + \omega\left(\frac{m-1}{2}\right)$

constant

$m =$ even symmetric

Phase $\cancel{\frac{\pi}{2}} \cancel{+} \omega\left(\frac{m-1}{2}\right)$

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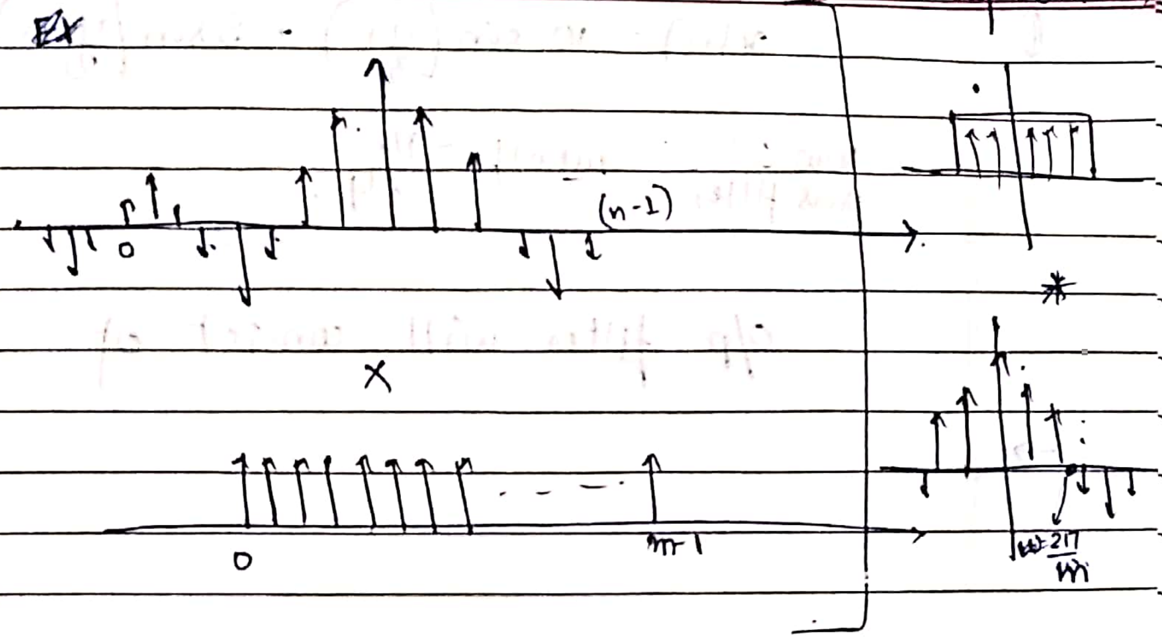
Time Domain — Freq Domain

Ex



$w(n) = 1 \quad 0 \le n \le M-1$

$\quad\;\; = 0 \quad$ otherwise.

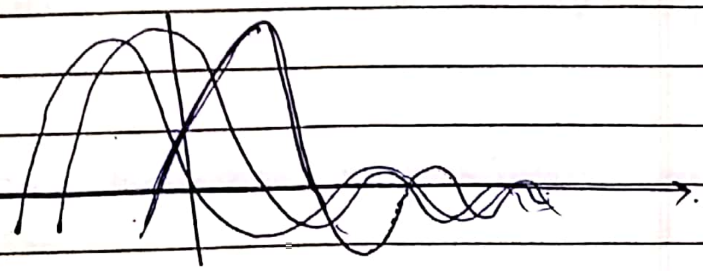
$$W(\omega) = \sum_{n=-\infty}^{\infty} w(n) e^{-j\omega n}$$

$$= e^{j\omega\left(\frac{M-1}{2}\right)} \frac{\sin \omega\left(\frac{M-1}{2}\right)}{\frac{\omega M}{2}}$$

$M \rightarrow$ no. of sample

↑ing $M \rightarrow$ More sharper

slope is control by M.



For Ideal Filter → IIR [Sharp response]

FIR → requiring More complex multiplication.

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$$x[n] = 10\sin\left(\frac{\pi}{3}n\right) + 5\sin\left(\frac{\pi}{2}n\right)$$

low pass filter - cutoff = $\pi/4$.

o/p filter will consist of.