# Principles of Engineering Sciences

Dept. of Mechanical-Mechatronics Engineering

Type: UG core

Branch: All

Semester: $3^{rd}$

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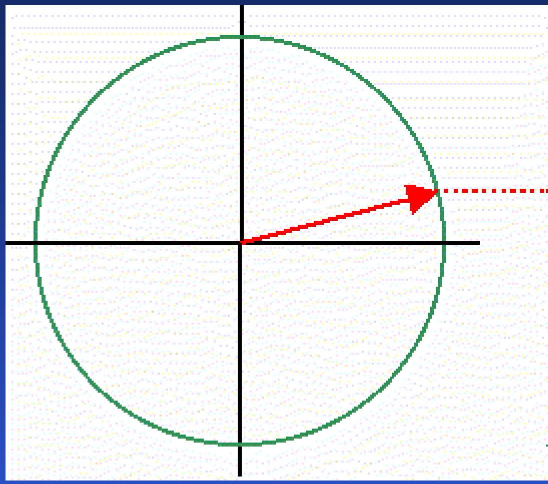
Syllabus:

- Three phase systems: Generation of three phase voltages- advantages of three phase systems, star and delta connection, 3 wire and four wire systems, relation between line and phase voltages & currents.
- Transformers: construction, types, emf equation, losses and efficiency
- DC Machines: DC generator- construction, types, emf equation, D.C motor- types, characteristics, applications.
- AC machines:  Three phase induction motor, single phase induction motor, three-phase alternator- construction, principle of operation, characteristics, methods of starting.

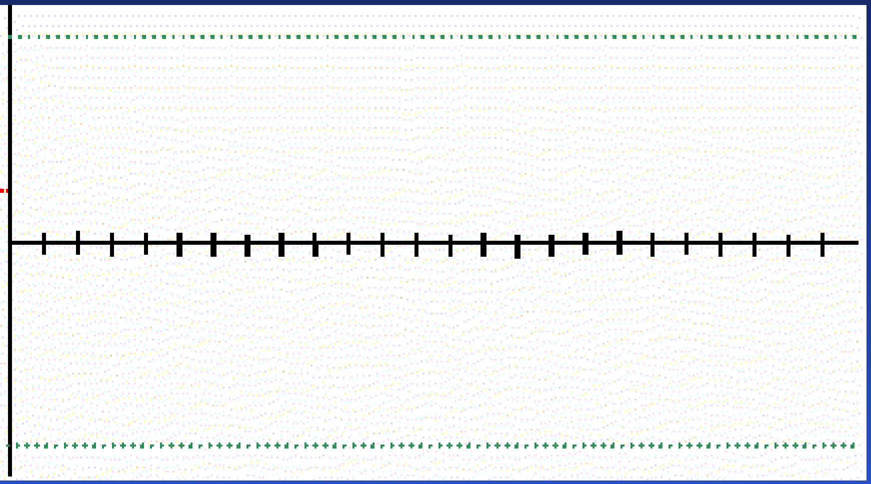
Text Books

1. Thereja B. L., *Fundamentals of Electrical Engineering and Electronics –I*, S. Chand & Co, New Delhi, 2012.
2. Chakrabarti A., et.al, *Basic Electrical Engineering,* TMH, 2012.
3. Hughes, *Electrical and Electronic Technology*, Pearson Education South Asia, 2011

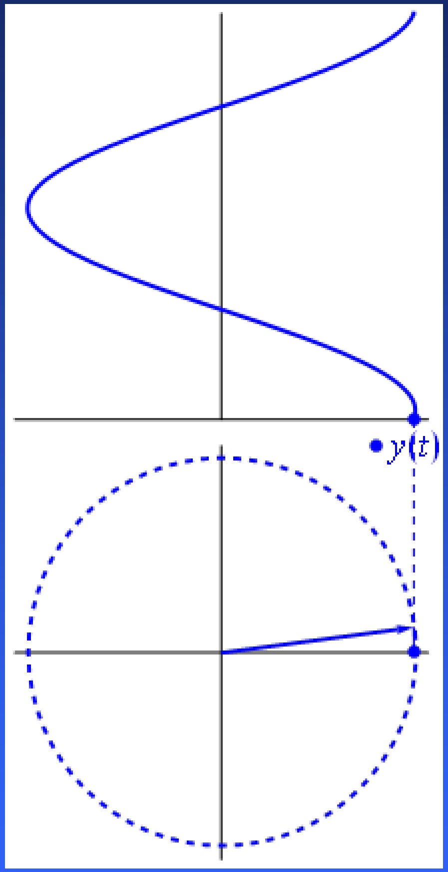
# Single-Phase signal

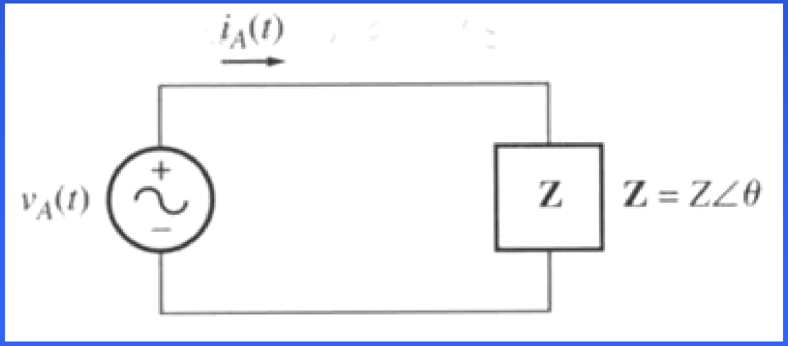


Rotating Phasor



Single phase signal

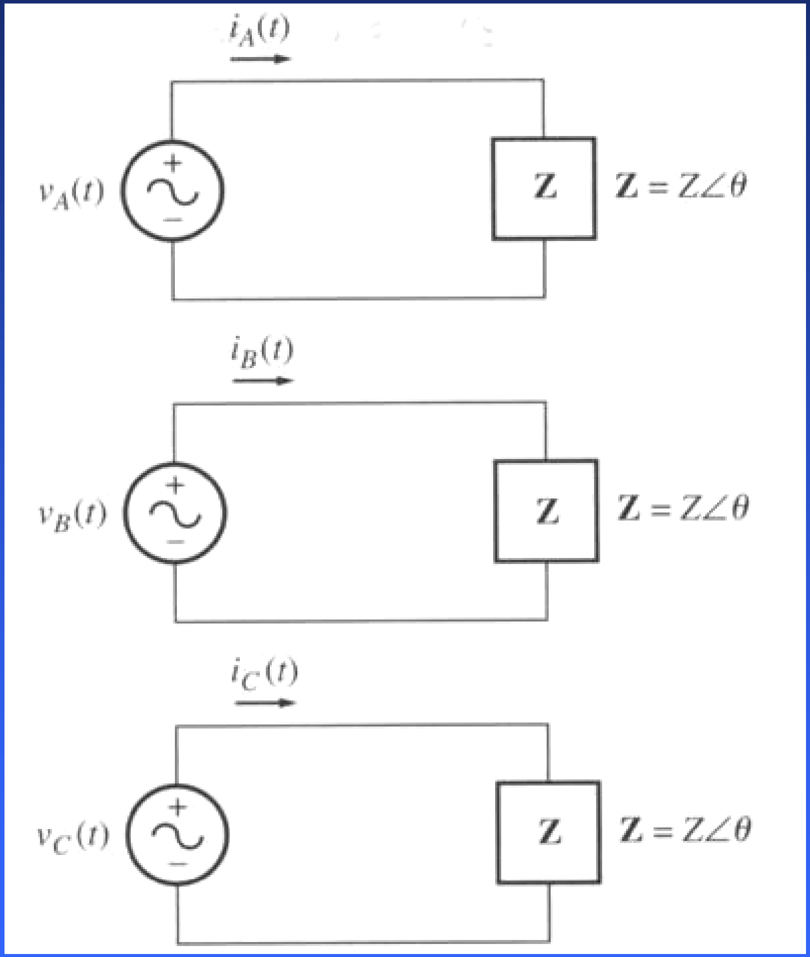


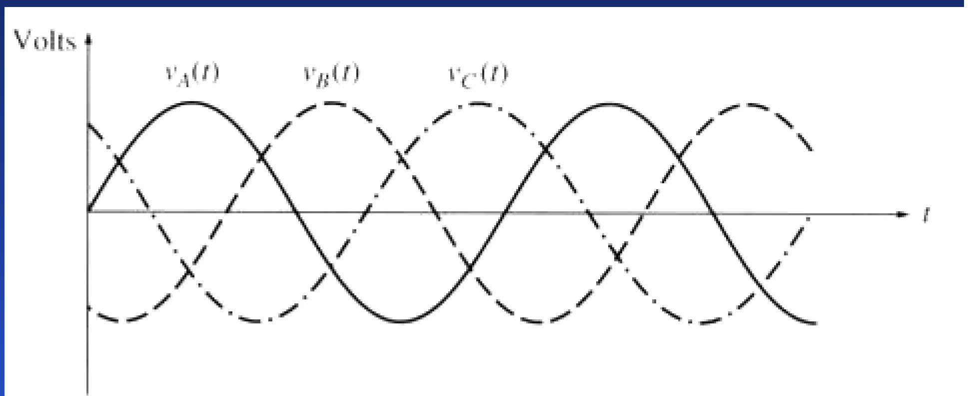



# Generating three-phase

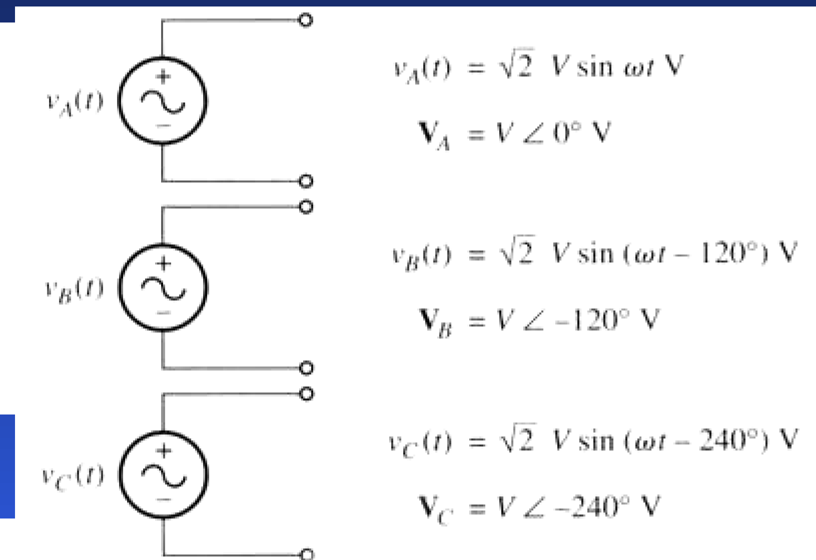
**Applying constraints**

1. Amplitude of $v_A$ $v_B$ $v_c$ are equal.
2. The voltages have a 120 ° phase difference between each other.
3. Connect identical loads to each supply.

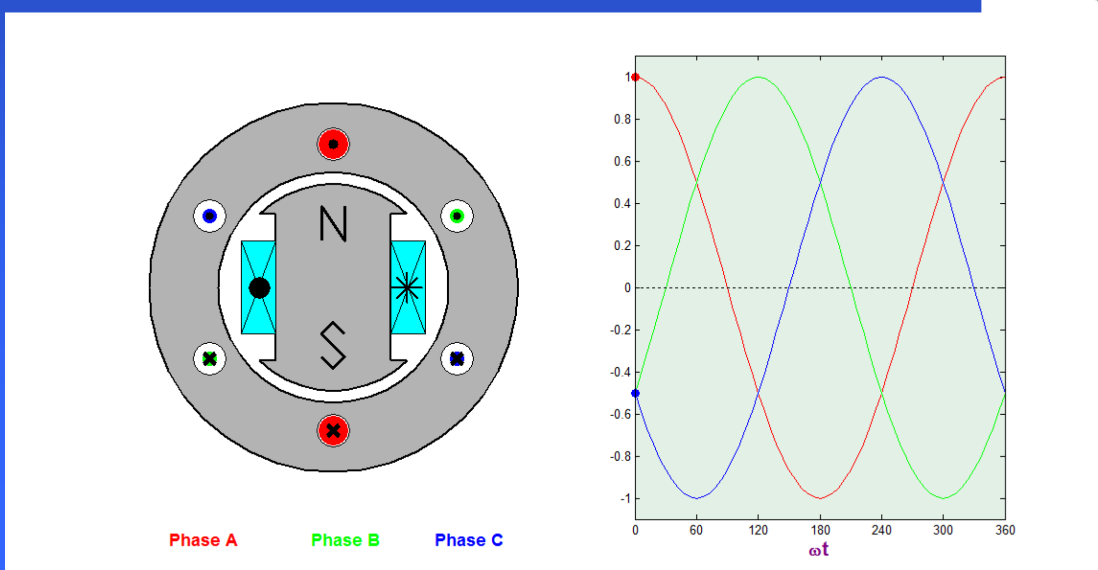




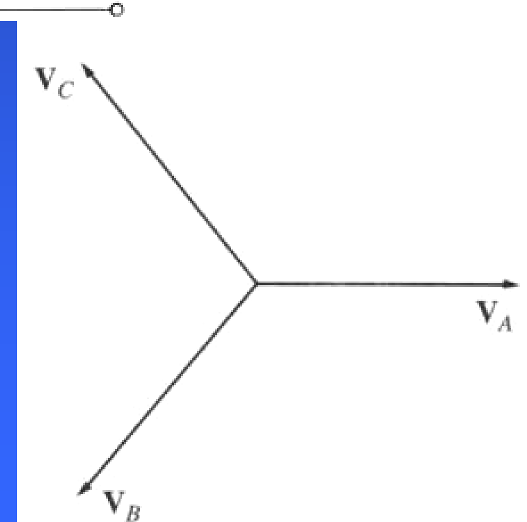
Volts
$v_A(t)$
$v_B(t)$
$v_C(t)$
$t$

$v_A(t)$
$v_A(t) = \sqrt{2}\ V \sin \omega t$ V
$\mathbf{V}_A = V \angle 0°$ V
$v_B(t)$
$v_B(t) = \sqrt{2}\ V \sin(\omega t - 120°)$ V
$\mathbf{V}_B = V \angle -120°$ V
$v_C(t)$
$v_C(t) = \sqrt{2}\ V \sin(\omega t - 240°)$ V
$\mathbf{V}_C = V \angle -240°$ V

N
S
Phase A
Phase B
Phase C
ωt

$\mathbf{V}_C$
$\mathbf{V}_A$
$\mathbf{V}_B$

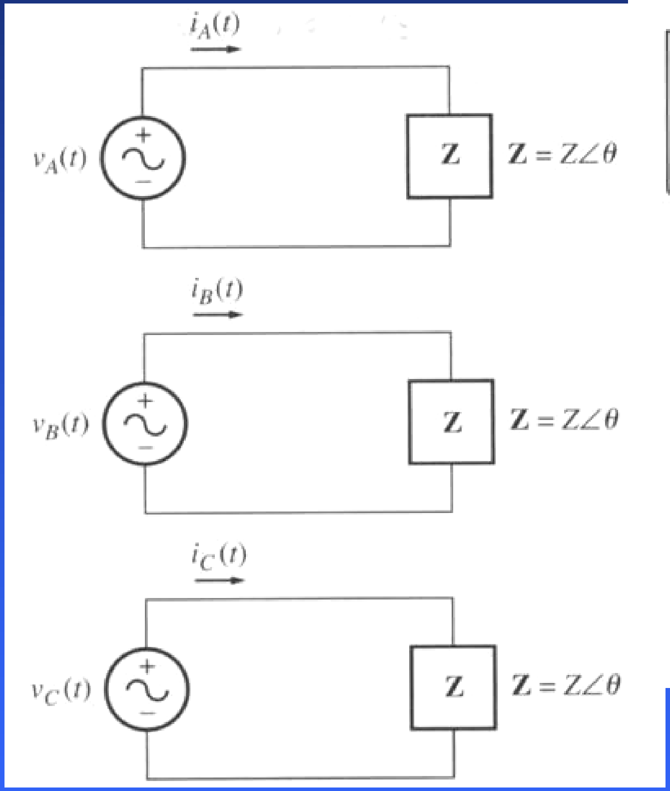
The current flowing to each load can be found as I = V/Z

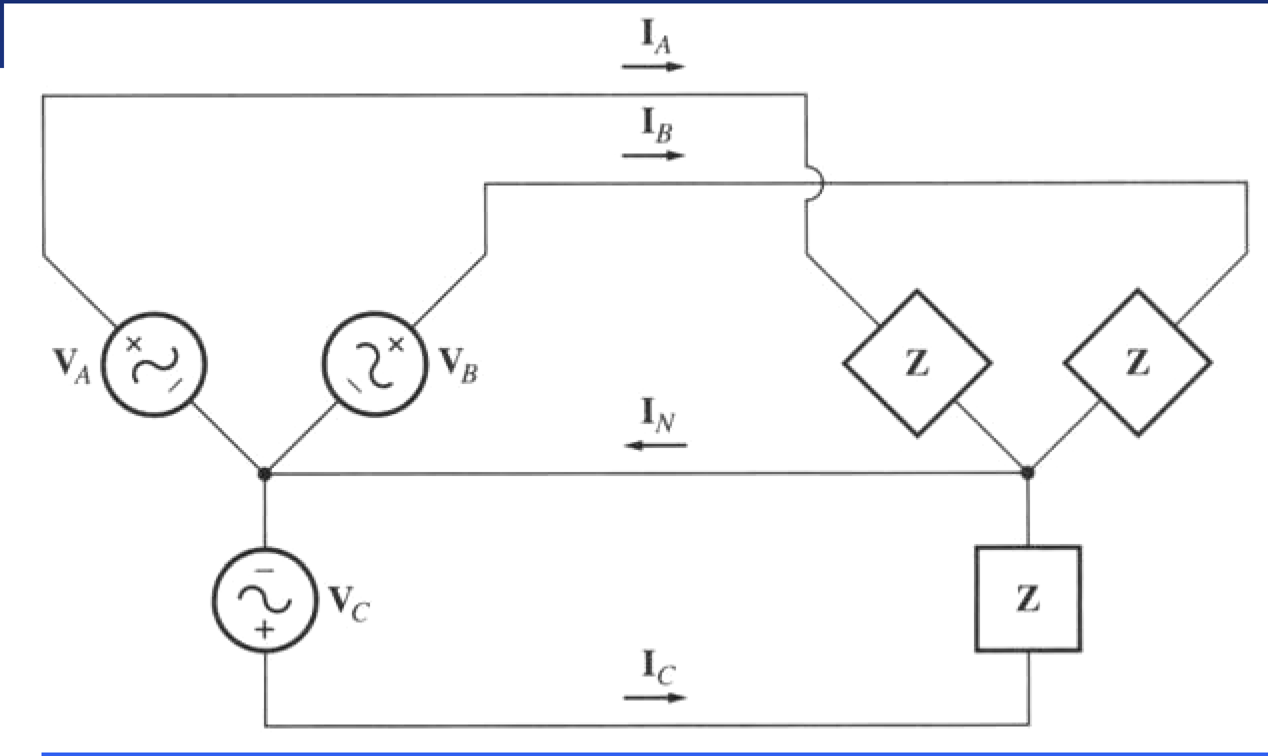
$$I_A = \frac{V\angle 0^0}{Z\angle \theta} = I\angle -\theta$$

$$I_B = \frac{V\angle -120^0}{Z\angle \theta} = I\angle -120-\theta$$

$$I_A = \frac{V\angle -240^0}{Z\angle \theta} = I\angle -240-\theta$$

Connect the grounds together using a single wire so the 3 circuits will have a single return path.




Therefore, $I_N = I_A + I_B + I_C$

$I_N = ?$

$$I_N = I_A + I_B + I_C = I\angle -\theta + I\angle -\theta - 120^0 + I\angle -\theta - 240^0$$
$$= I\cos(-\theta) + jI\sin(-\theta) + I\cos(-\theta - 120^0) + jI\sin(-\theta - 120^0) + I\cos(-\theta - 240^0) + jI\sin(-\theta - 240^0)$$
$$= I\left[\cos(-\theta) + \cos(-\theta - 120^0) + \cos(-\theta - 240^0)\right] + jI\left[\sin(-\theta) + \sin(-\theta - 120^0) + \sin(-\theta - 240^0)\right]$$
$$= I\left[\cos(-\theta) + \cos(-\theta)\cos(120^0) + \sin(-\theta)\sin(120^0) + \cos(-\theta)\cos(240^0) + \sin(-\theta)\sin(240^0)\right]$$
$$+ jI\left[\sin(-\theta) + \sin(-\theta)\cos(120^0) - \cos(-\theta)\sin(120^0) + \sin(-\theta)\cos(240^0) - \cos(-\theta)\sin(240^0)\right]$$

$$I_N = I\left[\cos(-\theta) - \frac{1}{2}\cos(-\theta) + \frac{\sqrt{3}}{2}\sin(-\theta) - \frac{1}{2}\cos(-\theta) - \frac{\sqrt{3}}{2}\sin(-\theta)\right]$$
$$+ jI\left[\sin(-\theta) - \frac{1}{2}\sin(-\theta) + \frac{\sqrt{3}}{2}\cos(-\theta) - \frac{1}{2}\sin(-\theta) - \frac{\sqrt{3}}{2}\cos(-\theta)\right]$$
$$= 0$$

The current through the neutral line can be found as zero as long as the load is balanced.

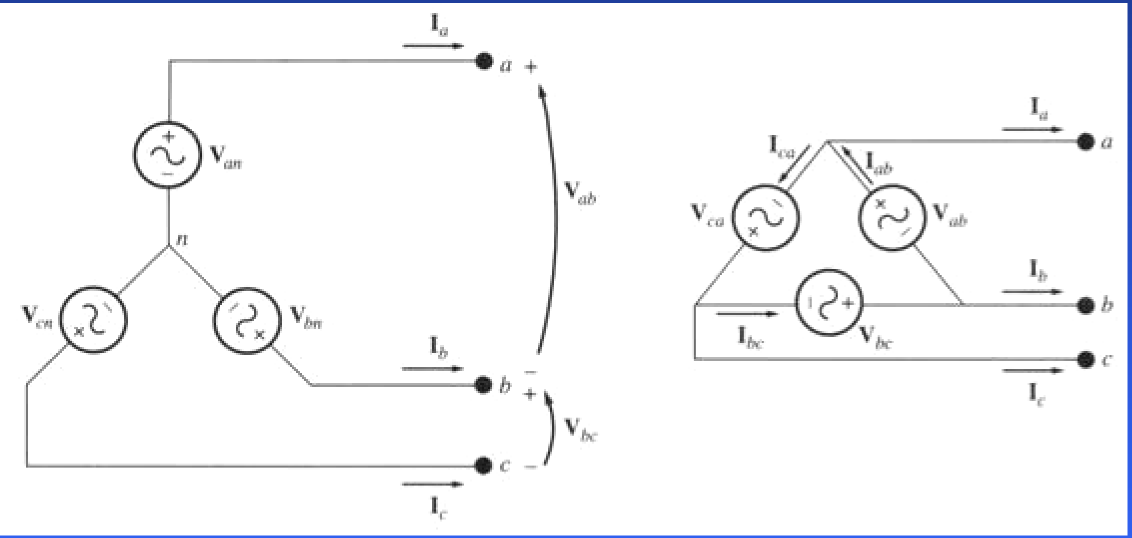
Therefore is the neutral line necessary?

In a balanced system, neutral is unnecessary!

# Definitions

- Represented as 3Φ
- 4 wires
  - 3 “active” phases, A(R), B(Y), C(B)
  - 1 “ground”, or “neutral”
- Color Code
  - Phase A Red
  - Phase B Yellow
  - Phase C Blue
  - Neutral Black
- Phase sequence(RYB) – order of phases in which individual voltages peak.

- There are two types of connections in three-phase circuits: Y (star) and Δ (delta)

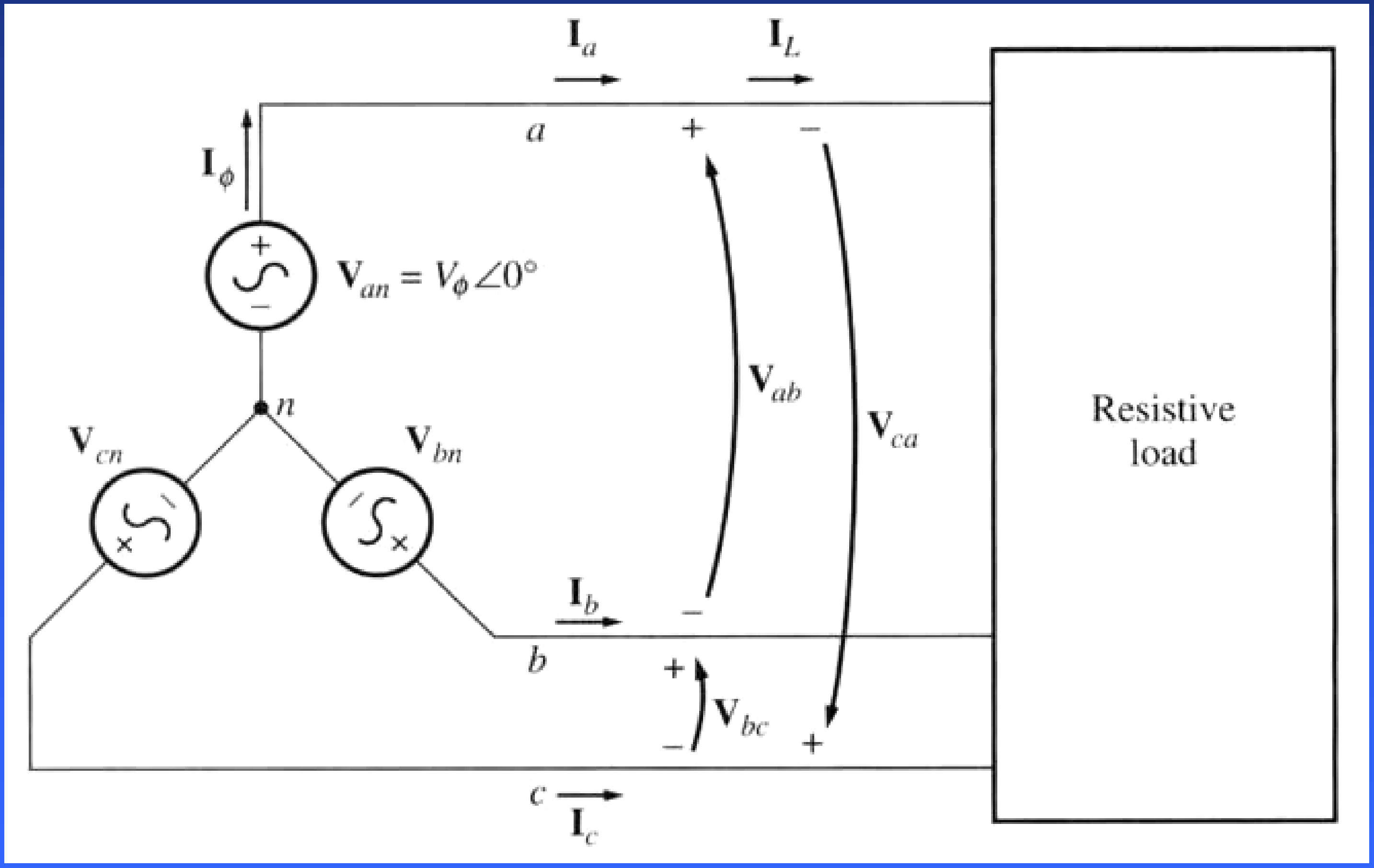



Phase quantities - voltages and currents in a given phase.
Line quantities – voltages between the lines and currents in the lines connected to the generators.

# Voltages and currents

1. Y-connection

Assuming a resistive load…



# Voltages and currents

1. Y-connection (cont)

$$V_{an} = V_{\phi} \angle 0^{0}$$

$$V_{bn} = V_{\phi} \angle -120^{0}$$

$$V_{cn} = V_{\phi} \angle -240^{0}$$

Since we assume a resistive load:

$$I_{a} = I_{\phi} \angle 0^{0}$$

$$I_{b} = I_{\phi} \angle -120^{0}$$

$$I_{c} = I_{\phi} \angle -240^{0}$$

# 1. Y-connection (cont 2)

The current in any line is the same as the current in the corresponding phase.

$$\boxed{I_L = I_\phi}$$

Voltages are:

$$V_{ab} = V_a - V_b = V_\phi \angle 0^0 - V_\phi \angle -120^0 = V_\phi - \left( -\frac{1}{2} V_\phi - j\frac{\sqrt{3}}{2} V_\phi \right) = \frac{3}{2} V_\phi + j\frac{\sqrt{3}}{2} V_\phi$$

$$= \sqrt{3} V_\phi \left( \frac{\sqrt{3}}{2} + j\frac{1}{2} \right) = \sqrt{3} V_\phi \angle 30^0$$

# Voltages and currents

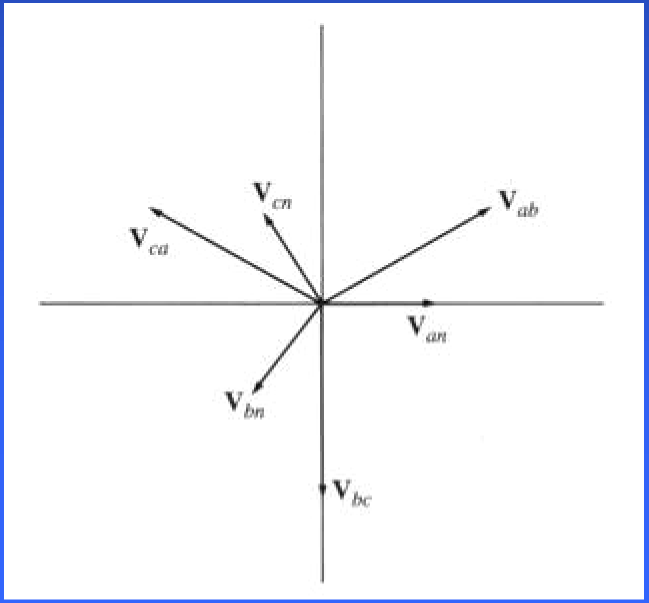
## 1. Y-connection (cont 3)

Magnitudes of the line-to-line voltages and the line-to-neutral voltages are related as:

$$V_{LL} = \sqrt{3} V_{\phi}$$

In addition, the line voltages are shifted by $30^0$ with respect to the phase voltages.

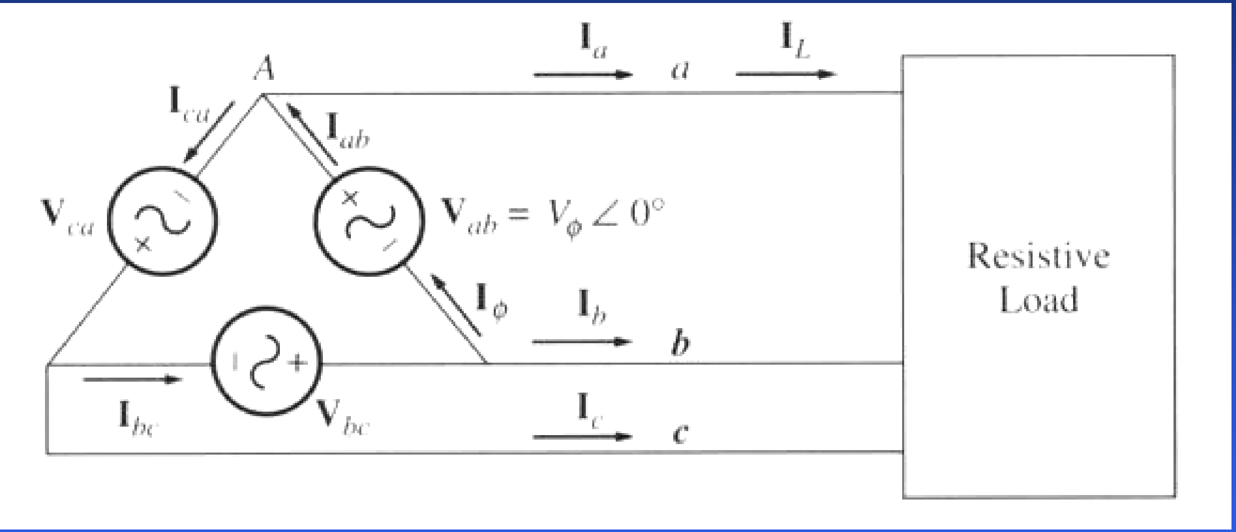
In a connection with *abc* sequence, the voltage of a line **leads** the phase voltage.



# Voltages and currents

1. Δ-connection

assuming a resistive load:




$$V_{ab} = V_\phi \angle 0^0 \qquad I_{ab} = I_\phi \angle 0^0$$

$$V_{bc} = V_\phi \angle -120^0 \qquad I_{bc} = I_\phi \angle -120^0$$

$$V_{ca} = V_\phi \angle -240^0 \qquad I_{ca} = I_\phi \angle -240^0$$

# Voltages and currents

1. Δ-connection (cont)

$$\boxed{V_{LL} = V_\phi}$$

The currents are:

$$I_a = I_{ab} - I_{ca} = I_\phi \angle 0^0 - I_\phi \angle 240^0 = I_\phi - \left( -\frac{1}{2} I_\phi + j\frac{\sqrt{3}}{2} I_\phi \right)$$

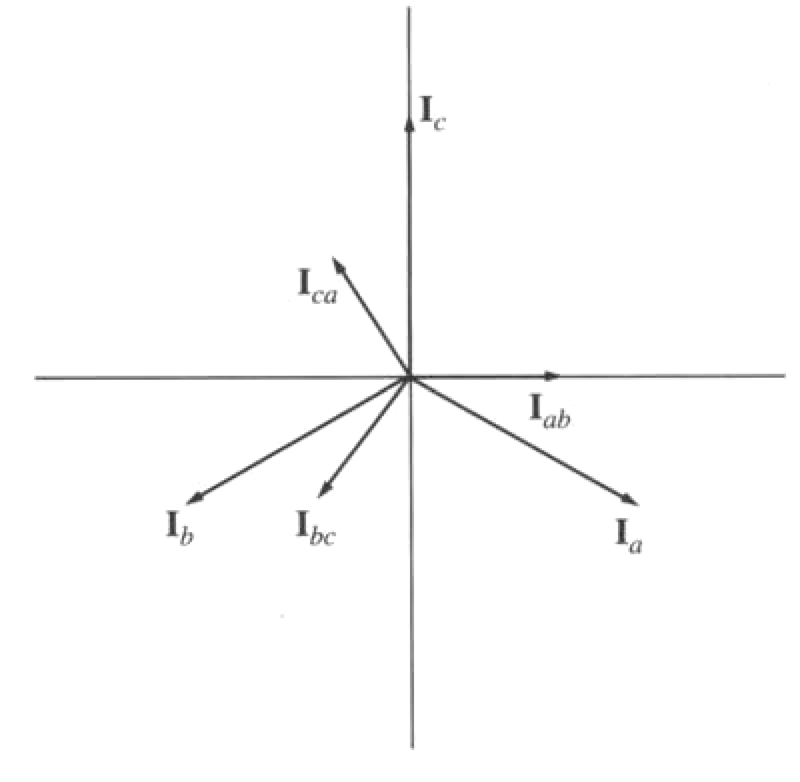
$$= \frac{3}{2} I_\phi - j\frac{\sqrt{3}}{2} I_\phi = \sqrt{3} I_\phi \left( \frac{\sqrt{3}}{2} - j\frac{1}{2} \right) = \sqrt{3} I_\phi \angle -30^0$$

The magnitudes:

$$\boxed{I_L = \sqrt{3} I_\phi}$$

# Voltages and currents

For the connections with the *abc* phase sequences, the current of a line **lags** the corresponding phase current by $30^0$ (see Figure below).



# Power relationships

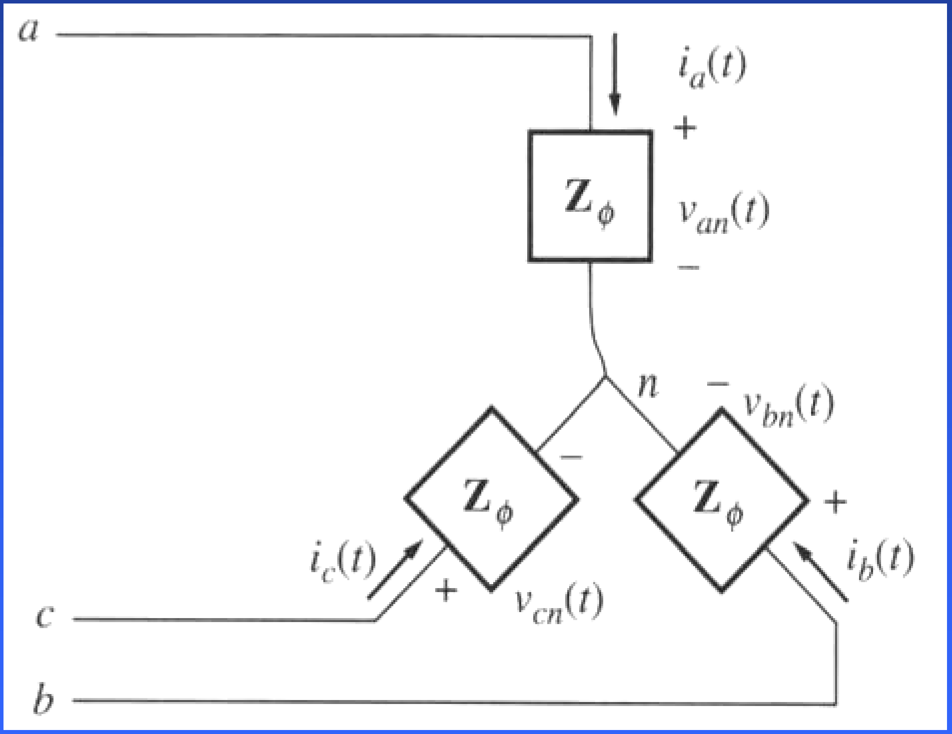
For a balanced Y-connected load with the impedance $Z_\phi = Z\angle\theta^0$:

and voltages:

$$v_{an}(t) = \sqrt{2}V\sin\omega t$$
$$v_{bn}(t) = \sqrt{2}V\sin(\omega t - 120^0)$$
$$v_{cn}(t) = \sqrt{2}V\sin(\omega t - 240^0)$$

The currents can be found:

$$i_a(t) = \sqrt{2}I\sin(\omega t - \theta)$$
$$i_b(t) = \sqrt{2}I\sin(\omega t - 120^0 - \theta)$$
$$i_c(t) = \sqrt{2}I\sin(\omega t - 240^0 - \theta)$$



# Power relationships

The instantaneous power is:

$$\boxed{p(t) = v(t)i(t)}$$

Therefore, the instantaneous power supplied to each phase is:

$$p_a(t) = v_{an}(t)i_a(t) = 2VI\sin(\omega t)\sin(\omega t - \theta)$$
$$p_b(t) = v_{bn}(t)i_b(t) = 2VI\sin(\omega t - 120^0)\sin(\omega t - 120^0 - \theta)$$
$$p_c(t) = v_{cn}(t)i_c(t) = 2VI\sin(\omega t - 240^0)\sin(\omega t - 240^0 - \theta)$$

Since

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

# Power relationships

Therefore

$$p_a(t) = VI\left[\cos\theta - \cos(2\omega t - \theta)\right]$$

$$p_b(t) = VI\left[\cos\theta - \cos(2\omega t - 240^0 - \theta)\right]$$

$$p_c(t) = VI\left[\cos\theta - \cos(2\omega t - 480^0 - \theta)\right]$$

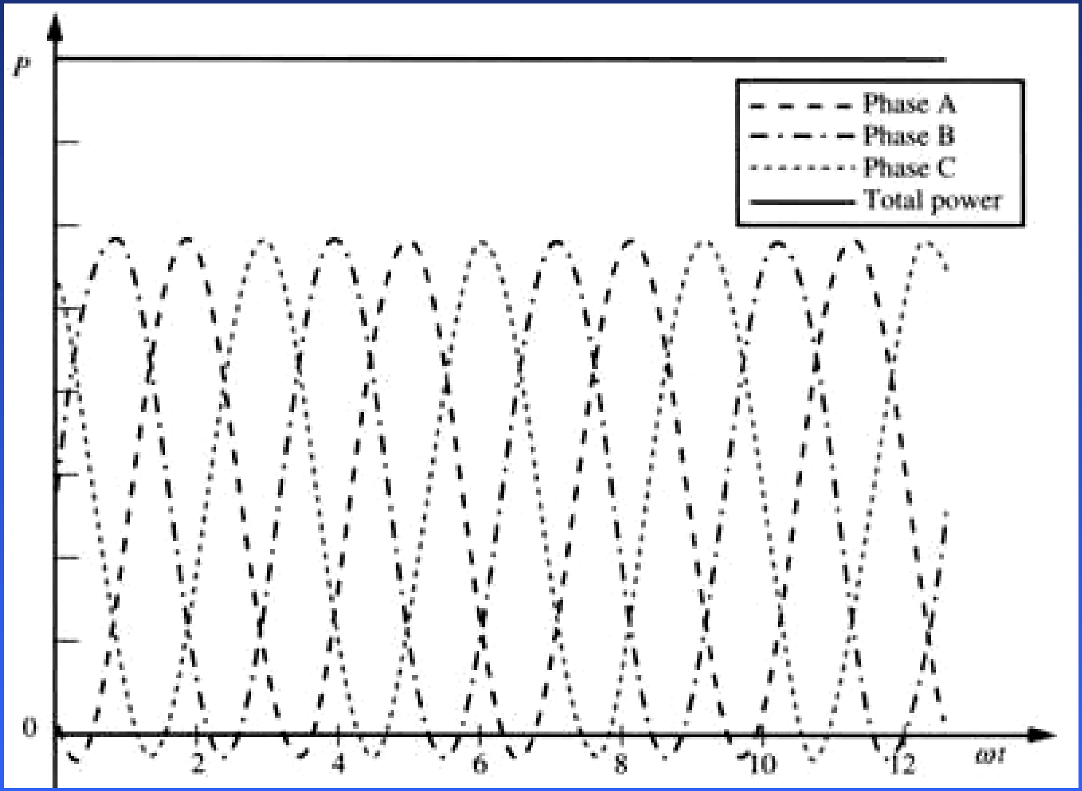
The total power on the load

$$p_{tot}(t) = p_a(t) + p_b(t) + p_c(t) = 3VI\cos\theta$$

# Power relationships

The instantaneous power in phases.

The total power supplied to the load is constant.



# Power relationships

Line quantities: Y-connection.

Power consumed by a load: $P = 3V_\phi I_\phi \cos\theta$

Since for this load $I_L = I_\phi \;\; and \;\; V_{LL} = \sqrt{3}V_\phi$

Therefore: $P = 3\dfrac{V_{LL}}{\sqrt{3}} I_L \cos\theta$

Finally: $\boxed{P = \sqrt{3}V_{LL} I_L \cos\theta}$

Note: these equations are valid for balanced loads only.

# Power relationships

Line quantities: $\Delta$-connection.

Power consumed by a load: $P = 3V_{\phi} I_{\phi} \cos\theta$

Since for this load $I_L = \sqrt{3} I_{\phi} \;\; and \;\; V_{LL} = V_{\phi}$

Therefore: $P = 3 \frac{I_L}{\sqrt{3}} V_{LL} \cos\theta$

Finally: $\boxed{P = \sqrt{3} V_{LL} I_L \cos\theta}$

Same as for a Y-connected load!

Note: these equations were derived for a balanced load.

# Problems