Gravitational Search Algorithm for Optimization of Engineering Problem

PLATINUM JUBILEE
YEAR 2021-22

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PLATINUM JUBILEE
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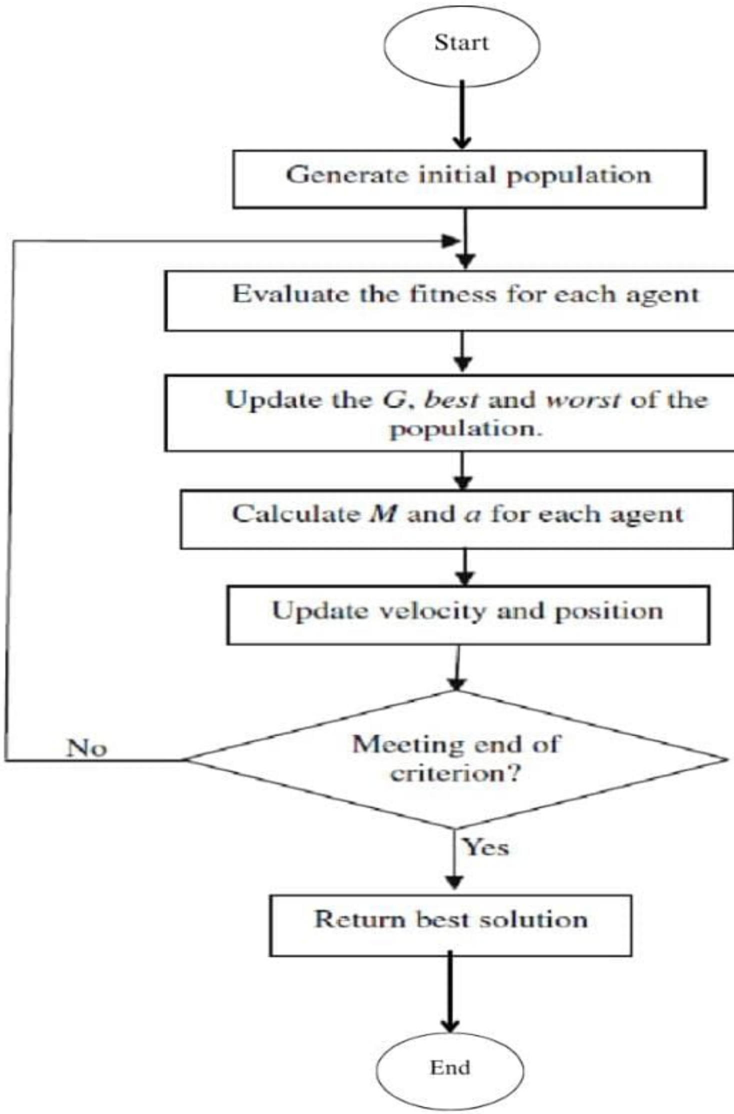
Introduction

The Gravitational Search Algorithm (GSA) is an optimization method inspired by gravity, where solutions are treated as masses that attract each other based on their weight and distance. Heavier solutions exert stronger forces, guiding the search towards optimal solutions. GSA balances exploration and exploitation, making it effective for solving complex global optimization problems.

Objectives

- To Study Gravitational Search Algorithms for optimization.
- To implement the algorithm for solving optimization problems.
- To compare the performance of the algorithms with other algorithms.

Flowchart:


Start
Generate initial population
Evaluate the fitness for each agent
Update the G, best and worst of the population.
Calculate M and a for each agent
Update velocity and position
Meeting end of criterion?
No
Yes
Return best solution
End

GSA Formulae:

1) Calculate the mass of each agent:

For each agent i, calculate its mass:

$$M_{ai} = M_{pi} = M_{ii} = M_i, \quad i = 1, 2, \ldots, N,$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)},$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)},$$

2) Update the gravitational constant G(t):

$$G(t) = G(t_0) \times \left(\frac{t_0}{t}\right)^{\beta}, \quad \beta < 1,$$

GSA Formulae:

3) Calculate total force acting on each agent:

For each agent i, calculate the force in each dimension d:

$$F_{ij}^{d}(t) = G(t)\frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon}\left(x_{j}^{d}(t) - x_{i}^{d}(t)\right),$$

4) Calculate acceleration and velocity of each agent:

For each agent i, calculate acceleration in each dimension d:

$$a_{i}^{d}(t) = \frac{F_{i}^{d}(t)}{M_{ii}(t)},$$

Pseudo code (GSA) :

1.Search Space Identification:
Define the bounds for the search space: lower bound (lb) and upper bound (ub).
MaxIter (maximum iterations), PopSize (population size), and problem dimensions.

2.Randomized Initialization:
Initialize a population of solutions Xi randomly within the search space [lb, ub].
Initialize the velocity of each agent as zero.

3.Fitness Evaluation of Agents:
For each solution Xi in the population:
Evaluate the fitness of the solution using the objective function.

4.Update Parameters (G(t), best(t), worst(t), and Mi(t)):
Update gravitational constant G(t) based on the current iteration t.
Identify the best solution best(t) and the worst solution worst(t) based on fitness values.
Calculate the mass Mi(t) for each agent using their fitness relative to the best and worst solutions.

Pseudo code (GSA) :

5.Calculation of Total Force:

For each solution Xi, calculate the total gravitational force acting on it due to all other solutions Xj in the population.

6.Calculation of Acceleration and Velocity:

Calculate the acceleration of each solution based on the total force acting on it.

Update the velocity of each solution based on the calculated acceleration.

7.Updating Agents' Position:

Update the position of each agent Xi using its velocity.

Ensure that the new position remains within the search space bounds [lb, ub].

8.Repeat Steps 3 to 7:

Continue updating fitness, force, acceleration, velocity, and position until the stop criteria (e.g., MaxIter iterations) is reached.

9.End:

Output the best solution found during the optimization process.

Problem Statements:

Sphere Function:

Used as a benchmark problem in optimization.

$$Min\ F(X) = \sum_{i=0}^{n} Xi^2$$

Gear Train Optimization:

Designing a gear train to achieve a specific gear ratio while minimizing the overall size, weight, or cost of the gears.

$$Minimize\ f(x) = [\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}]^2$$

Pressure Vessel Design:

Designing a cylindrical pressure vessel with hemispherical ends to minimize the cost. This includes material, welding and forming costs subject to various constraints.

$$Min\ f(x) = 0.6224\ x_1\ x_2\ x_3 + 1.7781 x_3\ {x_1}^2 + 3.1661 {x_4}^2 x_2 + 19.84 {x_1}^2 x_4$$

Problem 1:Sphere Function

Objective Function:

$$Min\ F(X) = \sum_{i=0}^{n} Xi^2$$

Range $-100 \leq Xi < 100$

Consider the population

X_1	X_2	**F(x)**
62.8907	25.6857	4614.9951
62.5171	-96.4914	131218.68
-89.1315	71.0286	12989.49
-29.2615	-89.49	8864.713
22.0788	0.4925	487.7159

Two step for GSA :Sphere function

Initial population :

x1	X2	F(X)
62.8907	25.6857	4614.9951
62.5171	-96.4914	131218.68
-89.1315	71.0286	12989.49
-29.2615	-89.49	8864.713
22.0788	0.4925	487.7159

Iteration I :

X1	X2	F(X)
61.789	24.859	4435.530
62.5171	-96.4914	131218.680
-88.221	70.311	12726.598
-28.499	-88.762	8690.7130
22.070	0.490	487.3122

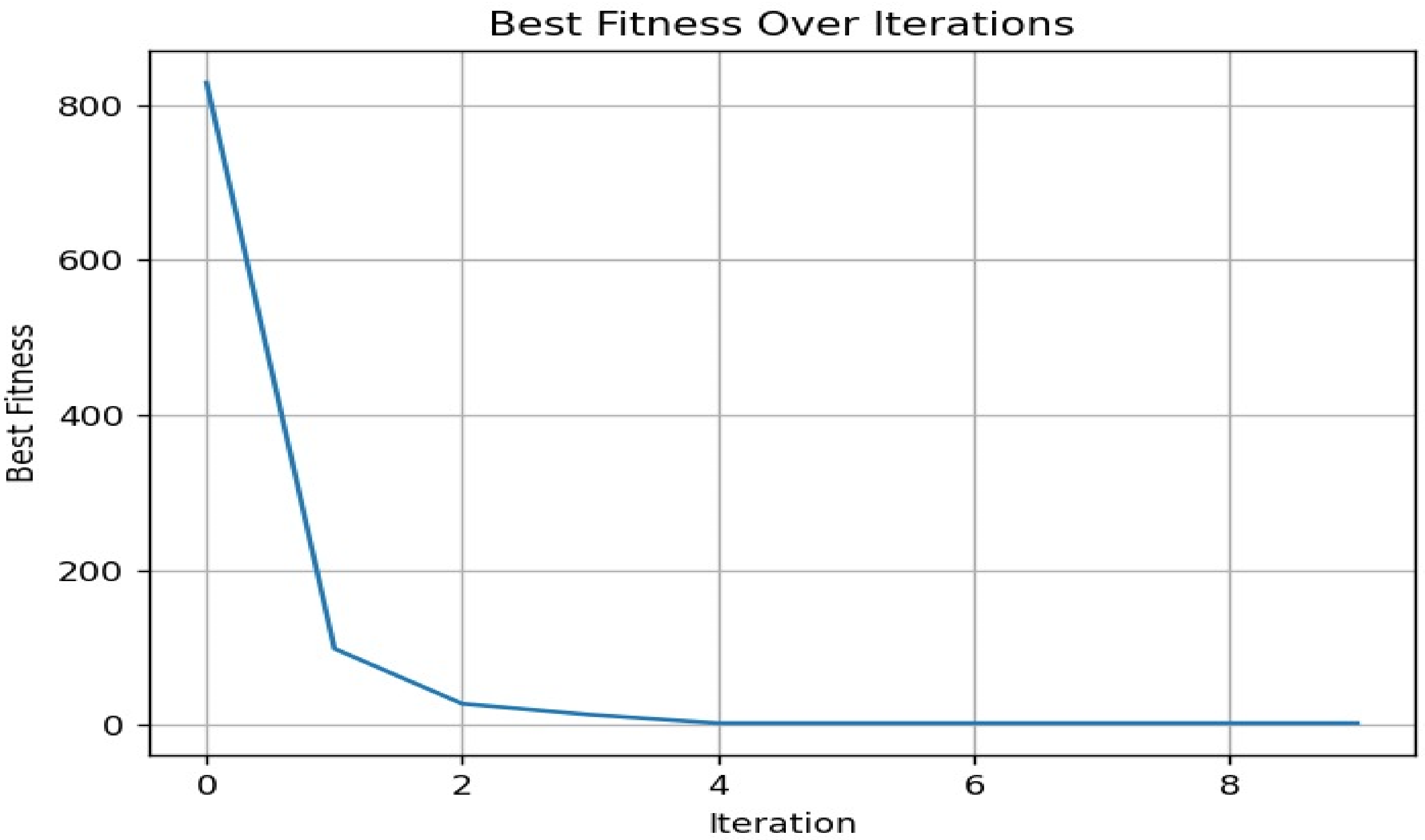
Iteration II :

X1	X2	F(X)
60.978	23.902	4289.622
62.5171	-96.4914	131218.680
-87.450	69.599	12491.523
-27.789	-88.015	8519.369
21.961	0.483	482.518

Result : Sphere Function

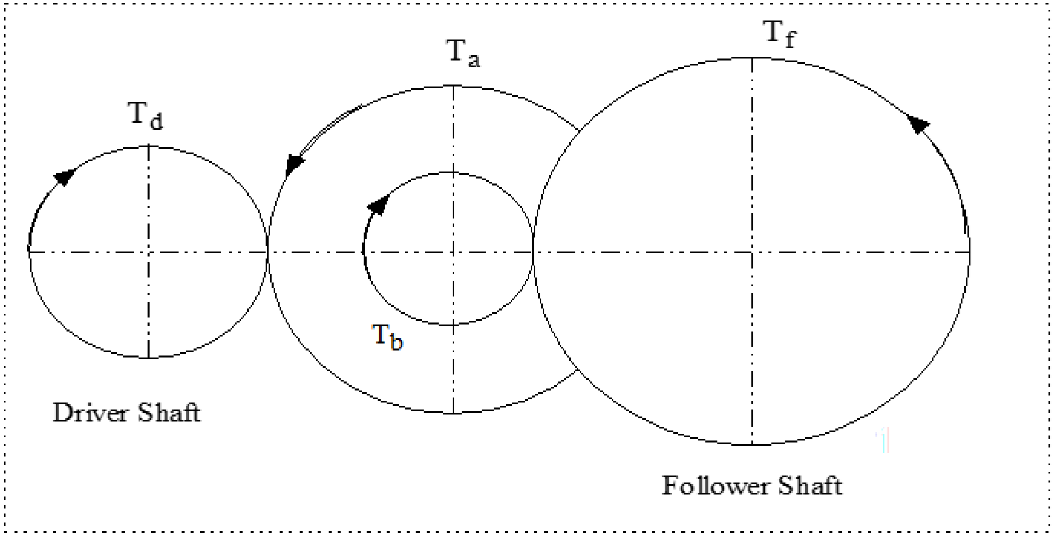
Iteration No.	X1	X2	Best fitness
1.	13.02207	25.6742	828.7427
2.	5.09018	8.53178	98.70134
3.	-3.09026	4.255626	27.660102
4.	0.19393	3.67610	13.55133
5.	-1.1724	-1.135607	2.664153
6.	-1.1724	-1.135607	2.664153
7.	-1.1724	-1.135607	2.664153
8.	-1.1724	-1.135607	2.664153
9.	-1.1724	-1.135607	2.664153
10.	-1.1724	-1.135607	2.664153

Graph :Sphere Function



Problem 2:Gear Train

Gear Train



$$Minimize\ \ f(x) = \left[\frac{1}{6.931} - \frac{x_1\, x_2}{x_3 x_4}\right]^2$$

Four dimensional problems and all four are strictly integers.

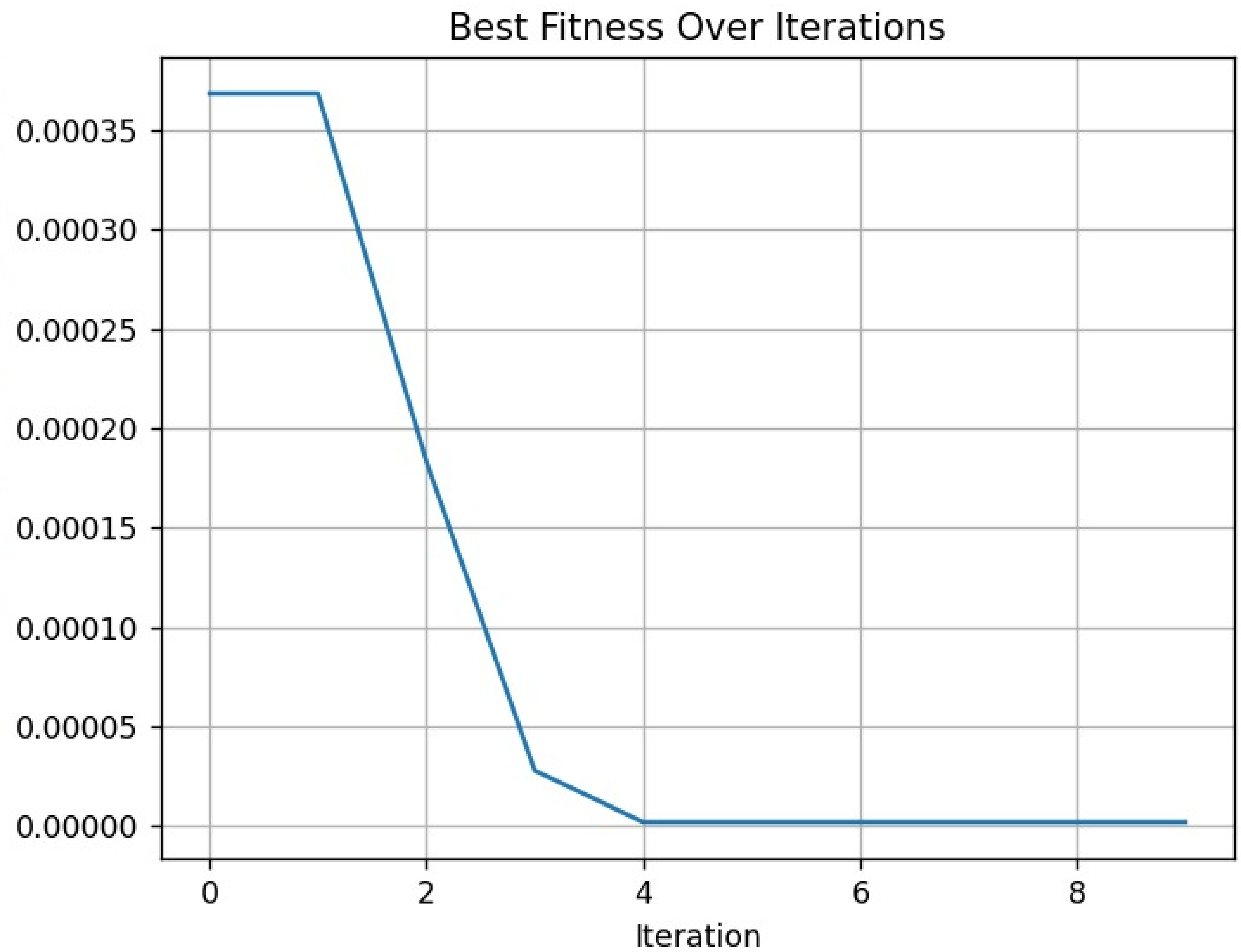
Subject to $12 \leq x_1, x_2, x_3, x_4 \leq 60$, all x_i's are integers

Where x1=Ta, x2=Tb, x3=Tc, x4=Td

Results: Gear Train

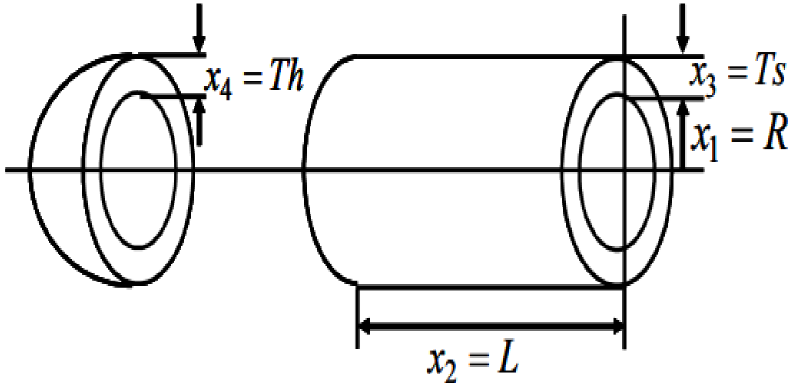
Iteration No.	X1	X2	X3	X4	Best Fitness
1.	56	31	14	50	3.681E-04
2.	56	31	14	50	3.681E-04
3.	51	14	32	56	1.822E-04
4.	40	19	12	40	2.767E-05
5.	60	12	27	38	1.946E-06
6.	60	12	27	38	1.946E-06
7.	60	12	27	38	1.946E-06
8.	60	12	27	38	1.946E-06
9.	60	12	27	38	1.946E-06
10.	60	12	27	38	1.946E-06

Graph :Gear Train



Problem 3 : Pressure Vessel

The problem is to minimize the cost of designing a pressure vessel by optimizing its dimensions



$$f(x)=0.6224\, x_1x_3x_4 + 1.7781\, x_3^2x_2 + 3.1661\, x_1^2x_4 + 19.84\, x_1^2x_3$$

$$0 \le x_1 \le 100$$

$$0 \le x_2 \le 100$$

$$10 \le x_3 \le 200$$

$$10 \le x_4 \le 200$$

Problem : Pressure Vessel

Pressure Vessel Design Constraints :

•**Radius-Shell Thickness** (g1):
$g1=-x1+0.0193\cdot x3\leq 0$
Ensures the radius is proportionate to the shell thickness.

•**Length-Shell Thickness** (g2):
$g2=-x2+0.00954\cdot x3\leq 0$
Controls the vessel's length relative to shell thickness.

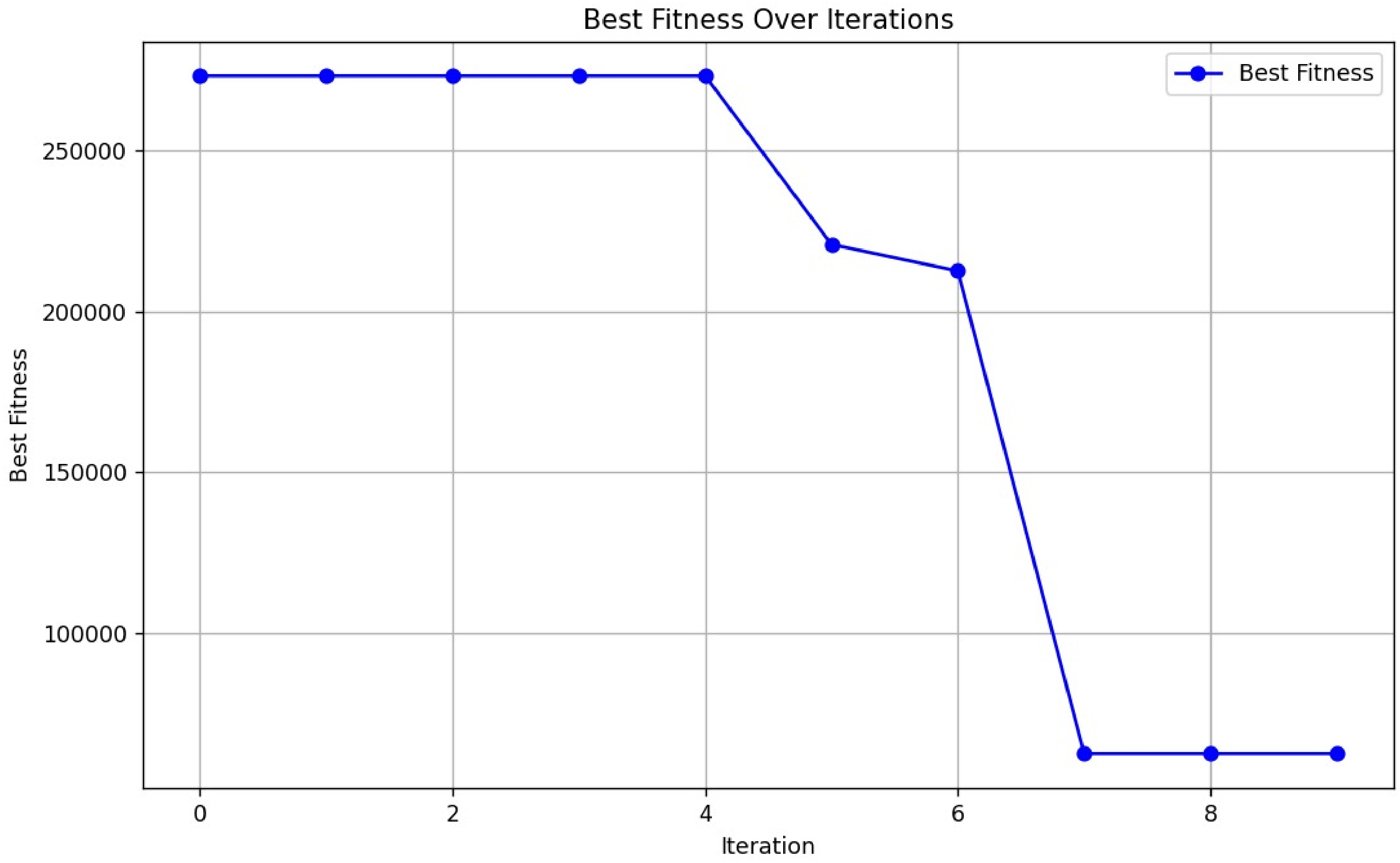
•**Material Volume** (g3):
$g3=1296000-(4/3)\cdot\pi\cdot x-\pi\cdot x3^2\cdot x4\leq 0$
Limits the total material volume for cost efficiency.

•**Head Thickness** (g4):
$g4=x4-240\leq 0g$
Caps the maximum head thickness.

Results : Pressure Vessel(GSA)

Iteration No.	X1	X2	X3	x4	Best Fitness
1.	5.9900	11.2705	88.2955	122.186	273194.59
2.	5.9900	11.2705	88.2955	122.186	273194.59
3.	5.9900	11.2705	88.2955	122.186	273194.59
4.	5.9900	11.2705	88.2955	122.186	273194.59
5.	5.9900	11.2705	88.2955	122.186	273194.59
6.	3.2720	14.6954	83.6849	98.7694	220744.02
7.	1.8207	15.6855	83.9451	98.7043	212485.58
8.	5.2059	0.6243	55.5541	109.3241	62357.25
9.	5.2059	0.6243	55.5541	109.3241	62357.25
10.	5.2059	0.6243	55.5541	109.3241	62357.25

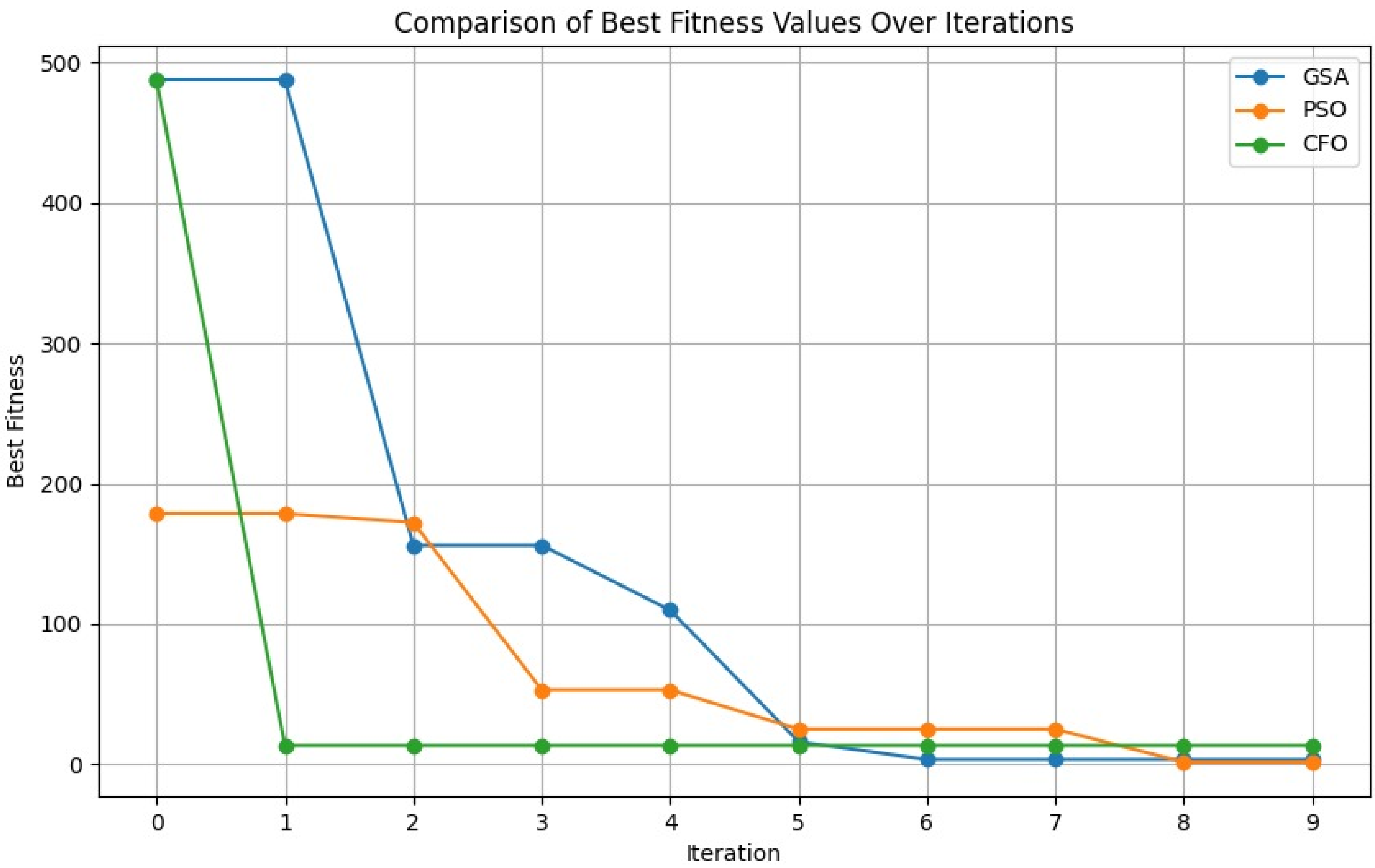
Graph:Pressure Vessel



Comparison Study: Sphere Function

Iteration No.	GSA	PSO	CFO
1.	487.7159	178.7396	487.7159
2.	487.7159	178.7396	13.60771
3.	156.087	172.2776	13.60771
4.	156.087	53.0352	13.60771
5.	109.8577	53.0352	13.60771
6.	16.1193	25.0171	13.60771
7.	3.4749	25.0171	13.60771
8.	3.4749	1.5974	13.60771
9.	3.4749	1.5974	13.60771
10.	3.4749	1.5974	13.60771

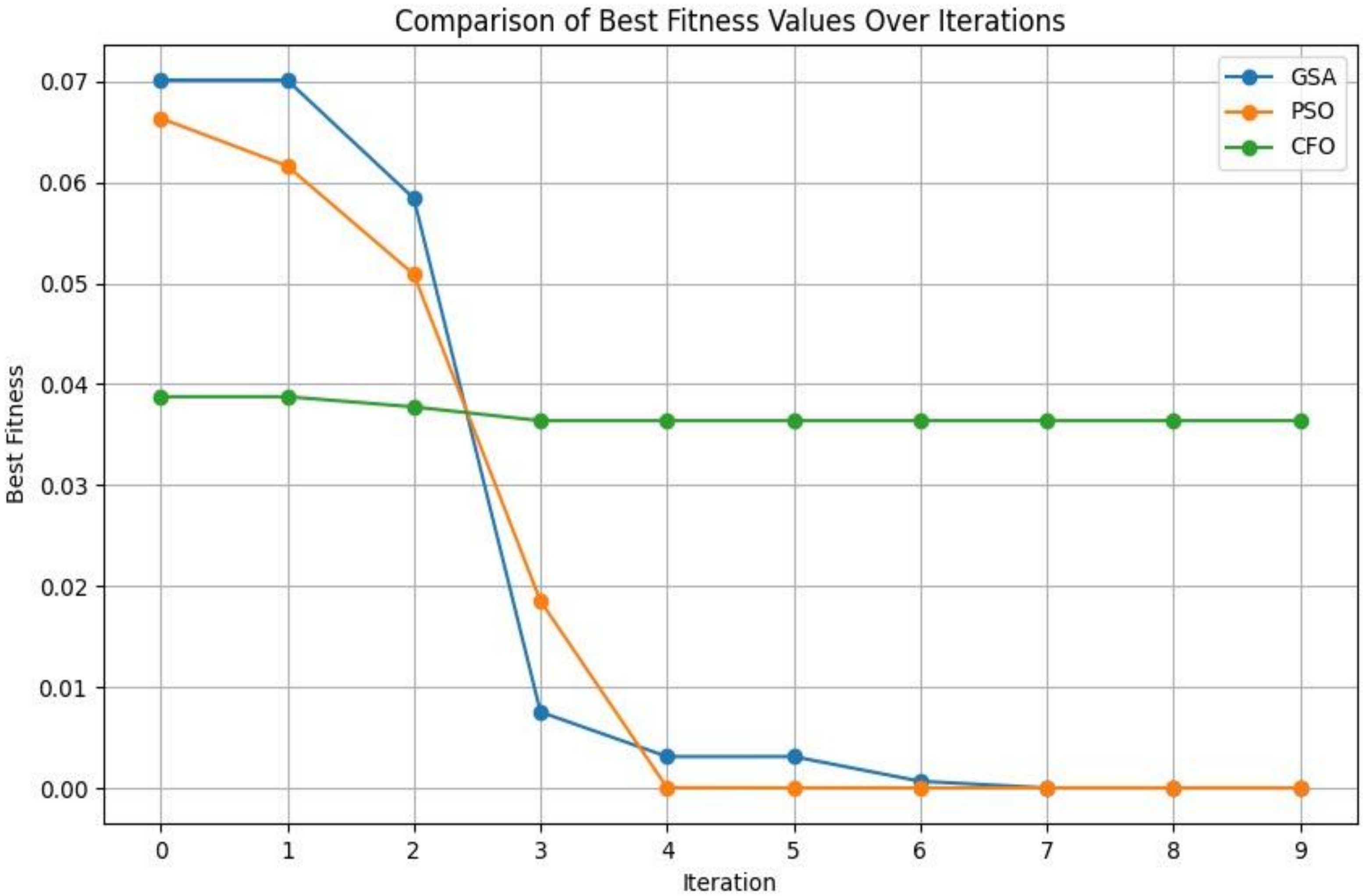
Comparison Graph of Sphere Function:



Comparison Study: Gear Train

Iteration No.	GSA	PSO	CFO
1.	0.07012	0.06634	0.03875
2.	0.07012	0.06161	0.03875
3.	0.05839	0.05087	0.03774
4.	0.00752	0.01851	0.03638
5.	0.00310	2.96244 e-05	0.03638
6.	0.00310	3.22279 e-06	0.03638
7.	0.00066	3.22279 e-06	0.03638
8.	1.38114	3.22279 e-06	0.03638
9.	1.38114	3.22279 e-06	0.03638
10.	1.38114	3.10691	0.03638

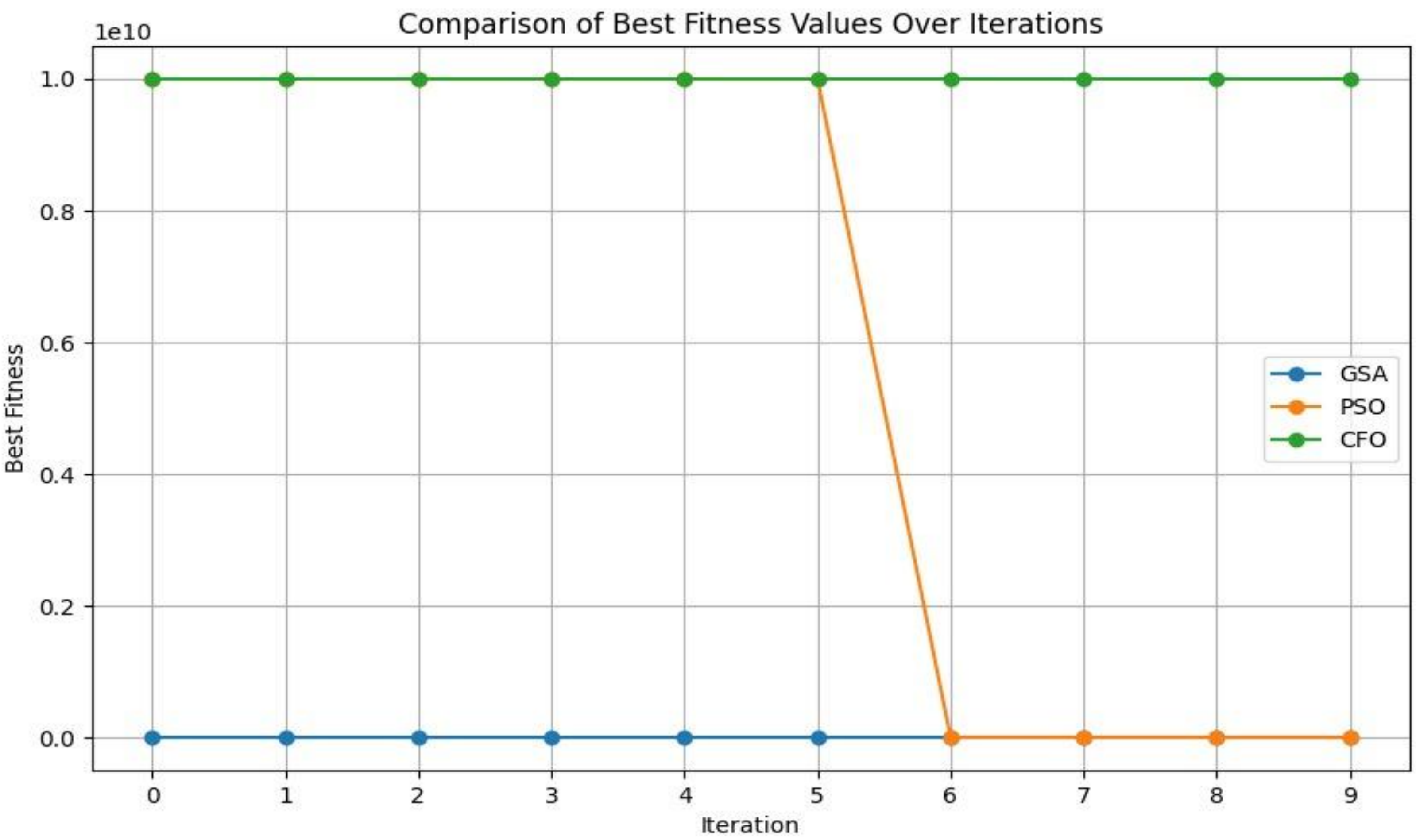
Comparison Graph of Gear Train :



Comparison Study: Pressure Vessel

Iteration No.	GSA	PSO	CFO
1.	302474.304	10000000000.0	10000000000.0
2.	302474.304	10000000000.0	10000000000.0
3.	302474.304	10000000000.0	10000000000.0
4.	302474.304	10000000000.0	10000000000.0
5.	302474.304	10000000000.0	10000000000.0
6.	302474.304	10000000000.0	10000000000.0
7.	302474.304	453646.94	10000000000.0
8.	302474.304	376187.11	10000000000.0
9.	302474.304	260031.15	10000000000.0
10.	302474.304	260031.15	10000000000.0

Comparison Graph of Pressure Vessel :



Conclusion of comparative study :

1. CFO converges the fastest, reaching optimal fitness by the first iteration and remaining stable throughout.

2. PSO converges more slowly, stabilizing around the fifth iteration.

3. GSA starts with the highest fitness value and converges at a similar rate to PSO, stabilizing around the fourth iteration.

4. All three algorithms eventually achieve similar optimal fitness values.

5. CFO is the most efficient in terms of speed, outperforming both PSO and GSA in convergence time.

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Thank You