



$$50 = I_1 + I_2$$

$$I_2 = 30 \cdot I_3$$

$$40 = I_3 + 50$$

$$60 = I_1 + 50$$

$$50 = I_1 + I_2$$

$$30 = I_2 + I_3$$

$$40 = I_3 + 50$$

$$60 = I_1 + 50$$

Solve :-

$$I_1 = 60 - 50$$

$$\boxed{I_1 = 10}$$

$$I_3 = 40 - 50$$

$$\boxed{I_3 = -10}$$

$$30 = I_2 + I_3$$

$$30 = I_2 + (-10)$$

$$I_2 = 30 + 10$$

$$\boxed{I_2 = 40}$$

$$\boxed{I_4 = 50}$$

Matrix $AX = B$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 40 \\ -10 \\ 50 \end{bmatrix} = \begin{bmatrix} 50 \\ 30 \\ 40 \\ 60 \end{bmatrix}$$

Norms of unknown variable i.e., $[10 \ 40 \ -10 \ 50]$

① Euclidean Norm (L^2 Norm)

$$\|X\|_2 = (10^2 + 40^2 + (-10)^2 + 50^2)^{1/2}$$

$$= (100 + 1600 + 100 + 2500)^{1/2}$$

$$= (4300)^{1/2}$$

$$\|X\|_2 \approx 65.57$$

② 1-Norm

$$\begin{aligned}\|X\|_1 &= |10| + |40| + |-10| + |50| \\ &= 10 + 40 + 10 + 50 \\ \|X\|_1 &= 110\end{aligned}$$

③ Infinity Norm

$$\begin{aligned}\|X\|_\infty &= \max(|x_1|, |x_2|, |x_3|, |x_4|) \\ &= \max(|10|, |40|, |-10|, |50|) \\ &= 50\end{aligned}$$

Rank of A & A/B

$$A/B = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 1 & 0 & 0 & 1 & 60 \end{array} \right] \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \\ \rightarrow R_4 \end{array}$$

$$R_4 - R_1$$

$$A/B = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & -1 & 0 & 1 & 10 \end{array} \right]$$

$$R_2 + R_4$$

$$A/B = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 0 & 1 & 1 & 40 \end{array} \right]$$

$$R_4 - R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\therefore \text{Rank}(A) = \text{Rank}(A|B) = 3$$

$$\text{as } \text{Rank}(A) = \text{Rank}(A|B) < n \quad \boxed{\because n=4}$$

\therefore Infinitely many solutions

$$\begin{aligned} \text{Rank Nullity} &= \text{No of columns} - \text{Rank}(A) \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 50 \\ 0 & 1 & 1 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 0 & 30 \\ 0 & 0 & 1 & 1 & 40 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$