**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987

Ans. The data is normally distributed with *μ* = 45 and *σ* = 8

If X is the time taken to repair the customer’s car then to complete it in 1 hr we must have X ≤ 50.

The probability that the service manager cannot meet his commitment is P(X > 50)

= 1- P (X ≤ 50) = 1- P (Z ≤ (50-45)/8) where P (Z ≤ (50-45)/8) = 0.734

Hence, the probability that the service manager cannot meet his commitment will be

1-0.734 = 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans. The probability of no of employees greater than age of 44 is P(X>44) = 0.1586

& the probability of no of employees between age of 38 and 44 is P(38<X<44) = 0.3413.

Hence, the statement that more employees at the processing center is older than 44 than between 38 and 44 is false.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans. The probability of employees less than age of 30 is P(X<30) = 0.0912

So the no of employees with probability 0.912 of them being under age of 30 is

0.0912\*400= 36.48

Hence the statement is true.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discus s both their distributions and parameters.

Ans. If X~ *N*(μ1, σ12) and Y ~ *N*(μ2, σ22) are *iid* normal random variables then

X + Y ~ N(μ1+ μ2, σ12  + σ22) & X - Y ~ N(μ1- μ2, σ12  + σ22)

Also we know that Z=aX+bY where Z is linear combination of X &Y so,

Z~ N(aμ1+bμ2, a2 σ1 2+ b2 σ22) therefore, 2 *X*1 ~ N(2 μ, 4σ2)

& *X*1 + *X*2 ~ N(μ+ μ, σ2  + σ2) ~ N(2 μ, 2σ2)

2 *X*1 - (*X*1 + *X*2 ) = N(4μ, 6σ2)

4. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

Ans. Option D is correct. 48.5 & 151.5 are the two values of a & b such that the probability of

a random variable taking a value between them is 0.99

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
2. Specify the 5th percentile of profit (in Rupees) for the company
3. Which of the two divisions has a larger probability of making a loss in a given year?

Ans. The mean profit from two different divisions of a company is mean1+mean2 = 5+7 = 12

The mean profit in Rupees is 12\*45=Rs 540 Million

The std deviation in Rupees is √ (9+16) \* 45 = Rs 225 Million

A. The range that it contains 95% probability for the company’s annual profit is

Rs. (99.0081, 980.9918) in Millions.

B. To compute 5th percentile we use X= μ + Z σ

Hence, X = 540+ (-1.645)\*225 is approximately Rs 170 Million.

C. The probability of division 1 making a loss is 0.04779 whereas the

probability of division 2 making a loss is 0.04

Hence, division 1 has a larger probability of making loss in a given year.